Superadditivity at Large Charge

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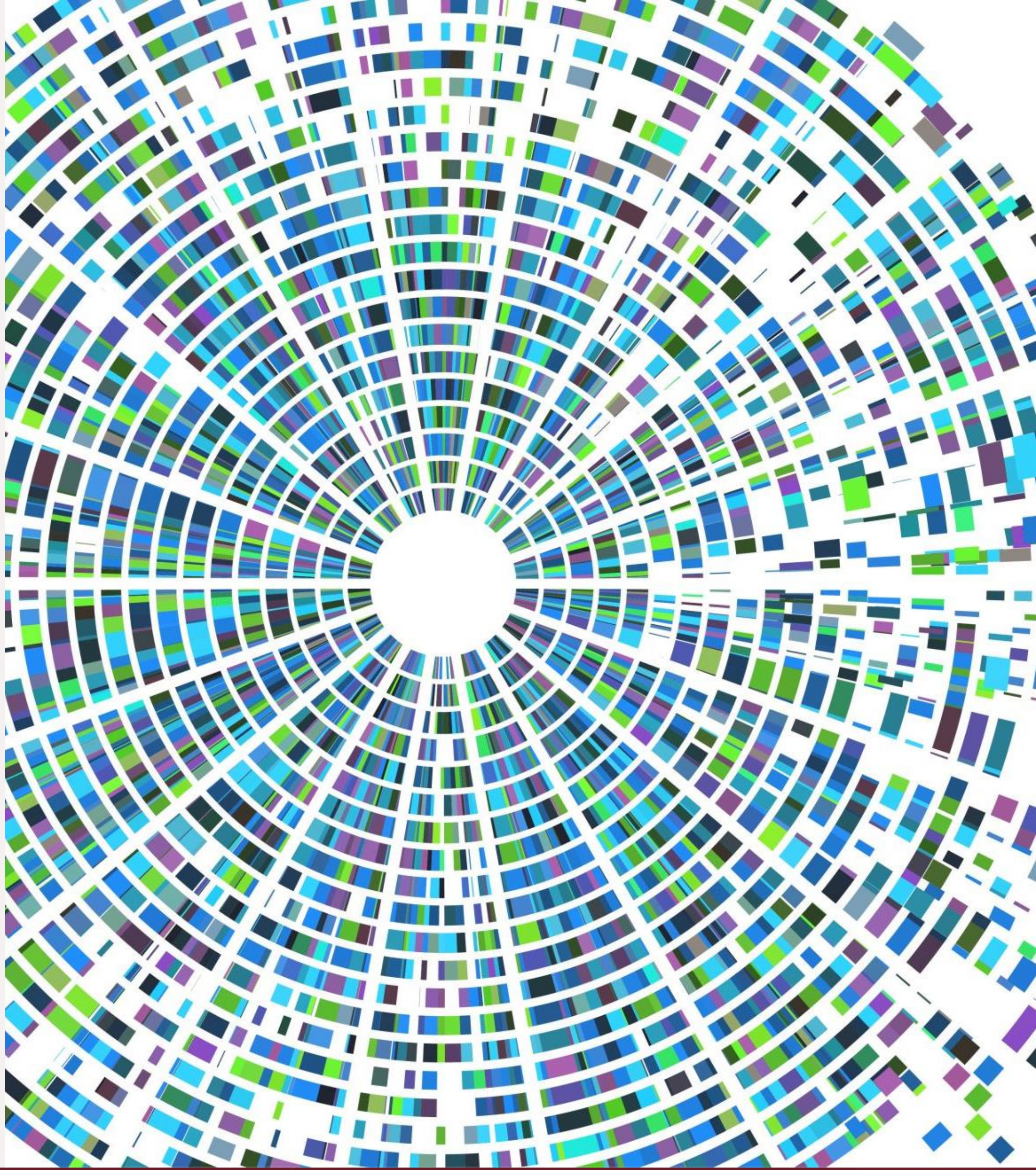
hep-th/2503.16603



Superadditivity from Weak Gravity

- Gravity is weakest force → natural generalization to Positive Binding Conjecture
- Exists a particle with positive self-binding energy from electric repulsion
- AdS/CFT dual: Dimension vs. Charge is Superadditive:
- Abelian Charge Convexity Conjecture for CFTs with U(1) global symmetry

$$\Delta(n_1 q_0 + n_2 q_0) \geq \Delta(n_1 q_0) + \Delta(n_2 q_0)$$

Superfluid ETF

- To calculate scaling dimensions of large charge operators, compute two-point function on Euclidean cylinder
- Path integral generically dominated by superfluid saddle
- Theory around saddle is an EFT for superfluid mode χ + other DOFs

$$\langle\psi_q|e^{-HT}|\psi_q\rangle = \mathcal{Z}^{-1}\int \mathcal{D}\phi_i\mathcal{D}\bar{\phi}_i\, e^{-\int_0^T \mathrm{d}\tau\,\mathrm{d}^{d-1}\theta\,\mathcal{L}_{\text{eff}}}$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + i\frac{q}{\Omega_{d-1}R^{d-1}}\dot{\chi}$$

Bottom-Up construction

- Diffeomorphism + Weyl → Conformal
- Then just need a spurion to get correct superfluid symmetry breaking pattern

$$SO(d+1,1) \times U(1)_Q \to SO(d) \times D'$$

$$\chi = \mu t + \pi$$

$$g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$$

$$\hat{g}_{\mu\nu} \equiv (g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi)g_{\mu\nu} = (\partial\chi)^2 g_{\mu\nu}$$

$$S = \int \mathrm{d}^d x \sqrt{\hat{g}} \Big(c_1 + c_2 \hat{\mathcal{R}} + c_3 \hat{\mathcal{R}}_{\mu\nu} \hat{\partial}^\mu \chi \hat{\partial}^\nu \chi + \cdots \Big)$$

Result with only Goldstone

- Legendre transform gives cylinder Hamiltonian, and thus scaling dimension
- Obtain universal power law up to higher curvature terms

$$H = \int \mathrm{d}^d x \left(\dot{\chi} \frac{\partial \mathcal{L}}{\partial \dot{\chi}} - \mathcal{L} \right) \implies \Delta = \mu q - L$$

$$\dot{\chi} = \mu \qquad \text{and} \qquad q = \frac{\partial L}{\partial \mu}$$

$$\Delta \sim q^{d/(d-1)} + \cdots$$

Adding a light mode

- Thus need another light mode to have a chance to violate superadditivity
- Couple it such that chemical potential fixed on saddle → linear leading behavior
- But checking theory cutoff, problem from curvature term… but seems essential

$$\mathcal{L} = \sqrt{\hat{g}}\left[b_1 + \hat{\phi}^2\left(1 + \frac{N^2}{(d-1)(d-2)}\hat{\mathcal{R}}\right) + a_1(\partial\hat{\phi})^2\right]$$

$$\Delta = Nq - b_1\Omega_{d-1}N^d$$

$$\mathcal{L} \supset N^2\hat{\phi}^2\left[5\dot{\pi}^2 + \ddot{\pi}^2\right]$$

R = 1, d = 4
from now on

Dilaton construction

- If we promote light field to a dilaton, can build more Weyl-invariant metrics
- Can quickly find strongly-superadditivity violating EFTs with high cutoffs

$$\hat{g}_\alpha = |\partial\chi|^\alpha \phi^{2-\alpha}\, g$$

$$\mathcal{L} = \frac{1}{12}\sqrt{\hat{g}_0}\,\hat{\mathcal{R}}_0 + \sqrt{\hat{g}_\alpha} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\phi^2 + \phi^{4-2\alpha}|\partial\chi|^{2\alpha}$$

$$\Delta \propto q^{\frac{2\alpha}{1+\alpha}}$$

$$\alpha > 1/2$$

Towards a proof and Conclusion

- At large charge, EFT is useful to understand scaling dimension behavior as a function of charge
- With only a Goldstone, displays universal superadditive behavior
- Possible to change with addition of a light mode, but generically breaks EFT
- Can get **sub**additive behavior with a dilaton, however must be exact (even with small breaking will revert to superadditivity)
- Outside SUSY, where all known examples are superadditive, perfectly flat dilaton potential seems unlikely
- Though not a proof, conjecture violation seems to require **infinite tuning**