Schwinger current in de Sitter Space

PLANCK25 - Padova

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Ongoing work with

M. Bastero Gil, P. B. Ferraz, L. Ubaldi, R. Vega Morales

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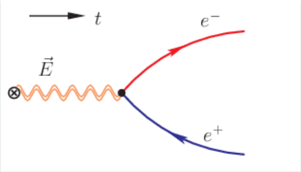
Spontaneous **particle creation**

- The Schwinger effect has been known for a long time

  Fritz, Sauter (1931); W. Heisenberg, H. Euler (1936); J. Schwinger (1951)

  - Pairs of charged particles and anti-particle created by background $\vec{E}$
  - Strong Electric fields are required
  - Effects are exponentially suppressed by the mass



$$\log \Gamma \propto \frac{m^2 c^3}{ehE} \quad \rightarrow \quad E_{CR} \simeq 1.32 \times 10^{18}\ \mathrm{Vm}^{-1}$$

- It can happen for constant $\vec{E}$, but requires time dependent vector potential $\vec{A}$

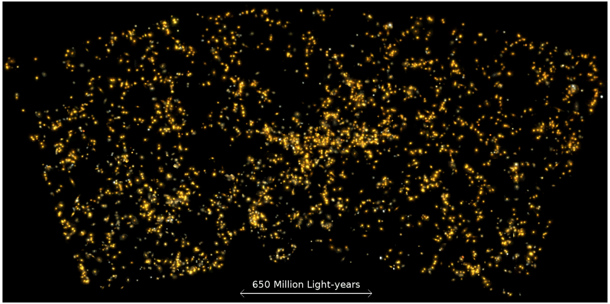
Spontaneous **particle creation**

- A similar effect can happen in Curved backgrounds

L. Parker (1966); S. W. Hawking (1975)

  - Particle production from vacuum under **time dependent gravitational field**
  - Effects are already important in cosmology
    - LSS might be seeded by accelerated expansion during inflation

Cosmological Schwinger effect, J. Martin 0704.3540



Sloan Digital Sky Survey, in Saraswati supercluster. Credit: IUCAA

Spontaneous **particle creation** by **time-varying backgrounds**

Combining the two examples: **Schwinger effect in de Sitter**

Inflationary Magnetogenesis

- Generate the **observed magnetic fields** present in voids our universe

T. Kobayashi, N. Afshordi 2014

Generation of Dark Sectors

- Candidates for non-thermal **dark matter** *M. Bastero-Gil, P. Ferraz, L. Ubaldi, R. Vega-Morales 2023*

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During inflation (Φ), in practice this could be realized with

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + V(\Phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{4f}\Phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_{ch}(\phi, A_\nu)\right],$$

- But no analytical solutions, difficult to test if (renormalization) results make sense

Scalar QED in de-Sitter

Forget about inflation

- Fix a de-Sitter background
- Set a constant electric field $\vec{E}$ (along z direction)

$$A_\mu = \frac{E}{H^2\tau}\delta^z_\mu, \qquad F_{\mu\nu}F^{\mu\nu} = -2E^2$$

$$S = \int d^4x\sqrt{-g}\left\{-g^{\mu\nu}\left(\partial_\mu - ieA_\mu\right)\phi^*\left(\partial_\nu + ieA_\nu\right)\phi - (m_\phi^2 + \xi R)\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\}$$

- Equations of motion

$$\phi_k'' + 2aH\phi_k + \omega_{\boldsymbol{k}}^2\phi_{\boldsymbol{k}} = 0 \quad \longrightarrow \quad \phi_k = \frac{\mathrm{e}^{-\pi\lambda r/2}}{a\sqrt{2k}} W_{i\lambda r,\mu}(2ik\tau)$$

$$\nabla^\nu F_{\mu\nu} = J_\mu^\phi \quad \text{with} \quad J_\mu^\phi = \frac{ie}{2}\left[\phi^\dagger\left(\partial_\mu + ieA_\mu\right)\phi - \phi\left(\partial_\mu - ieA_\mu\right)\phi^\dagger\right]$$

Renormalization (Scalar) QED in de-Sitter

- **Divergent expectation value**.

$$\left\langle 0 \left| J_z^{\phi} \right| 0 \right\rangle = \frac{2e}{a^4} \int \frac{d^3k}{(2\pi)^3} \left(k_z + eA_z\right) \left|\phi_k\right|^2 .$$

- **Renormalized** current

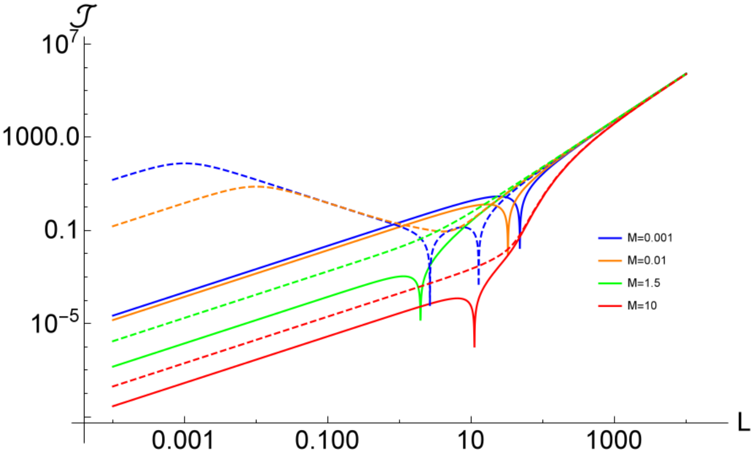
$$\langle J_z \rangle_{\rm ren} = aH \frac{e^2 E}{4\pi^2} \left[\frac{1}{6} \ln \frac{m_\xi^2}{H^2} - \frac{2\lambda^2}{15} + F_\phi(\lambda, \mu) \right] .$$

$$\lambda = \frac{eE}{H^2}, \qquad \mu^2 = \frac{9}{4} - \frac{m_\xi^2}{H^2} - \lambda^2 \quad \text{and} \quad m_\xi^2 = m_\phi^2 + 12\,\xi H^2 .$$

- Adiabatic Subtraction — *T. Kobayashi, N. Afshordi 2014*
- Point Splitting — *T. Hayashinaka, J. Yokoyama 2016*
- Pauli Villars — *M. Banyeres, G. Domenèch, J. Garriga 2018*

- Similar for **fermions** w. Adiabatic Subtraction — *T. Hayashinaka, T. Fujita, J. Yokoyama 2016*

Renormalization QED in de-Sitter



IR divergence $\sim \ln \frac{m}{H}$

Dashed: Scalars $\xi = 0$

Solid: Fermions

$\Downarrow$

Negative J when $\lambda < 1$

$= \lambda = \frac{eE}{H^2}$

T. Hayashinaka, T. Fujita, J. Yokoyama 2016

Revising the Renormalization

- Schwinger effect with **classical** $\vec{E}$
  - A_μ not quantized (charged particles do not accelerate in dS)
  - Only charged particles (ϕ/ψ) are quantized
    - No photon loops $\rightarrow$ ϕ/ψ propagator not corrected at loop level
    - Running of charge e $\Longleftrightarrow$ A_μ (from Ward Identity)

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    - Running of charge e $\Longleftrightarrow$ A_μ (from Ward Identity)
  - **Only one counter-term** in Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}\delta_3(F_{\mu\nu})^2 - eA_\mu J^\mu + \dots \ ,$$

And the corrected equations of motion will be

$$(\delta_3 + 1)\,\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle \,.$$

Revising the Renormalization

- For a constant electric field in de-Sitter ,

$$(\delta_3 + 1)\,\nabla^\nu F_{\mu\nu} = (\delta_3 + 1)\,(-2aHE\delta_\nu^z)\,.$$

- Define the **renormalized current**

$$\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle_{\mathrm{ren}}$$
$$\langle J_\mu \rangle_{\mathrm{ren}} = \langle J_\mu \rangle_{\mathrm{reg}} - (-2aHE\delta_\nu^z)\delta_{3\ \mathrm{reg}}\,.$$

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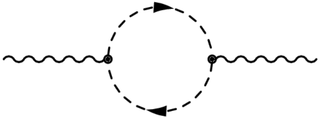
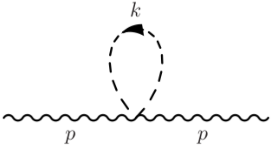
- To get ***physical*** **renormalized current**, **on-shell counter-term**!

$$\Pi(p^2 = m_A^2) = 0 \ \rightarrow\ \delta_3 = -e^2\Pi_2(m_A^2)$$

- With classical A_μ, Π_2 is fully defined (at one loop) by





Revising the Renormalization: Constant $\vec{E}$ in dS?

- Standard literature result is obtained with by treating Π_2 as in Minkowski

$$\delta_3 = -e^2\Pi_2(p^2 = 0) \;\rightarrow\; \delta_3^{PV} = -\frac{e^2}{48\pi^2}\ln\frac{\Lambda^2}{m^2}$$

- This results in $\ln m/H$ term that creates **negative conductivies** when $m \ll H$

- But does this condition actually hold for our setting?

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$$S = -\int d^4x\sqrt{-g}\,\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \;\rightarrow\; g^{\alpha\nu}g^{\beta\sigma}\nabla_\alpha F_{\nu\sigma} = 0\,.$$

Taking $A_\mu = \frac{E}{H^2\tau}\delta_\mu^z$, in e.o.m. we find

$$g^{\alpha\nu}g^{\beta\sigma}\nabla_\alpha F_{\nu\sigma} = -2a^{-4}\frac{E}{\tau^3H^2}\delta_i^z \neq 0\,.$$

- Just a kinetic term is not compatible with constant $\vec{E}$ in de-Sitter

Revising the Renormalization: Constant $\vec{E}$ in dS?

- Introduce an effective mass in Lagrangian

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2 A_\mu A^\mu\right).$$

e.o.m. gives

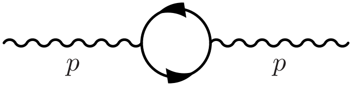
$$-a^{-4}2\frac{E}{\tau^3 H^2}\delta_i^z - m_A^2 a^{-2}\frac{E}{\tau H^2}\delta_i^z = 0 \;\rightarrow\; m_A^2 = -2H^2.$$

- Get effective tachyonic mass
- Interpreted as effective source that ensures that $\vec{E}$ is not diluted with expansion
- **Consistency with constant electric field** background implies

$$\Pi(p^2 = m_A^2) = 0 \;\rightarrow\; \delta_3 = -e^2\Pi_2(p^2 = -2H^2)$$

Revising the Renormalization: Constant $\vec{E}$ in dS?

Computing δ_3 as in Minkowski with the external momentum fixed by $p^2 = m_A^2 = -2H^2$



- Corrected $\ln m/H$ factor $\rightarrow$ Currents in the massless limit become **finite**
- But for fermions and conformal scalars ($\xi = 1/6$), **when** $eE \ll H^2$ **they are negative**

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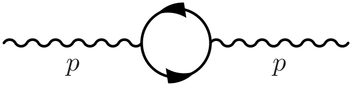

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- Minkowski propagators in the loop are not accurate
- Do not capture correctly **IR effects**

- We try a correction as exact de-Sitter does not seem doable (to us)

Scalars $m^2 \to m^2 + \xi R$ **Fermions** $m^2 \to m^2 + \frac{1}{4}R$

Finally δ_3

- Pauli-Villars to regularize both δ_3 and $\langle J_\mu \rangle$

$$\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle_{ren} = \langle J_\mu \rangle^{PV}_{reg} - (-2aHE\delta_\nu{}^z)\delta_3^{PV}$$

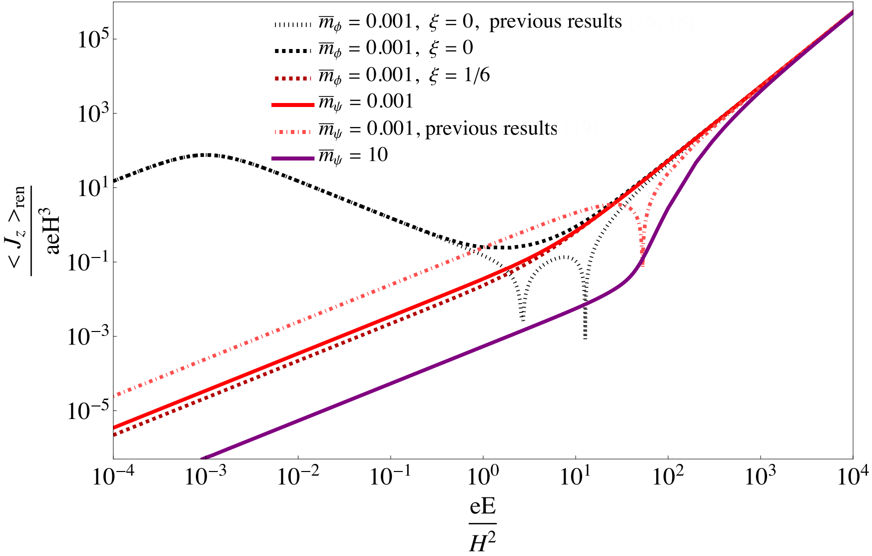
With

$$\delta_3 = \left(\frac{e}{12\pi}\right)^2 \left(3\ln\left(\frac{m^2}{\Lambda^2}\right) - 12\left(\frac{m}{H}\right)^2 + 6\left(2\left(\frac{m}{H}\right)^2 + 1\right)^{3/2} \coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2 + 1}\right) - 8\right)$$

- We find the **renormalized** current to be

$$\left\langle J_z^\phi \right\rangle^{PV}_{\mathrm{ren}} = aH\frac{e^2 E}{4\pi^2}\left[\frac{1}{3}\ln\frac{m}{H} - \frac{4}{9} - \frac{2}{3}\left(\frac{m}{H}\right)^2 - \frac{2\lambda^2}{15} + F_\phi \right.$$
$$\left. + \frac{\left(1 + 2\left(\frac{m}{H}\right)^2\right)^{3/2}}{3} \coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2 + 1}\right)\right]$$

Results


$\overline{m}_\phi = 0.001,\ \xi = 0,$ previous results
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$\overline{m}_\psi = 10$
$\frac{< J_z >_{\rm ren}}{\rm aeH^3}$
$\frac{\rm eE}{H^2}$
10^{5} 10^{3} 10^{1} 10^{-1} 10^{-3} 10^{-5}
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4}

Conclusions

- We have revised Schwinger pair production for constant **E** in de-Sitter
- Obtain the renormalised Lagrangian and correspondent parameters
- Imposed physical renormalisation conditions
- We were able to address and clarify literature's negative conductivities in $H > m$ case
  - **Unphysical** result comes from **wrong physical** conditions
  - Minkowski propagators are inadequate for IR behavior
- Obtained an UV and IR divergence free Schwinger current.

Backup

Revising PV

M. Banyeres, G. Domenèch, J. Garriga 2018

- An arbitrary number of additional **auxiliary fields are introduced to cancel divergences**
- The mass of these extra fields will then be sent to infinity, making them **non-dynamical**

Introduce **3** fields $$\sum_{i=0}^{3}(-1)^i = 0 \quad \text{and} \quad \sum_{i=0}^{3}(-1)^i m_i^2 = 0,$$

$$m_0 = m\,, \quad m_2^2 = 4\Lambda^2 - m^2 \quad \text{and} \quad m_1^2 = m_3^2 = 2\Lambda^2, \quad \Lambda \to \infty$$

The **regularized** current $$\langle J_z \rangle_{\rm reg} = \lim_{\Lambda\to\infty} \sum_{i=0}^{3} (-1)^i \langle J_z \rangle_i \,.$$

$$\left\langle J_z^\phi \right\rangle_{\rm reg} = aH\frac{e^2 E}{4\pi^2} \lim_{\Lambda\to\infty}\left[\frac{1}{6}\ln\frac{\Lambda^2}{H^2} - \frac{2\lambda^2}{15} + F_\phi(\lambda,\mu,r)\right]$$

- $\ln \Lambda/H$ **divergence** to be reabsorbed with **renormalization** of the charge

$$\langle J_\mu \rangle_{reg} = (\delta_3 + 1)\,\nabla^\nu F_{\mu\nu}$$
$$\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle_{ren} = \langle J_\mu \rangle_{reg}^{PV} - (-2aHE\delta_\nu^{\ z})\delta_3^{PV}$$