Gravitational waves production by inflationary transitions in U(1) aligned axion inflation

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Natural Inflation

In natural inflation, we have an axion with the following potential

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

K. Freese, J.A. Frieman and A.V. Olinto ‘90

Planck Results 2018

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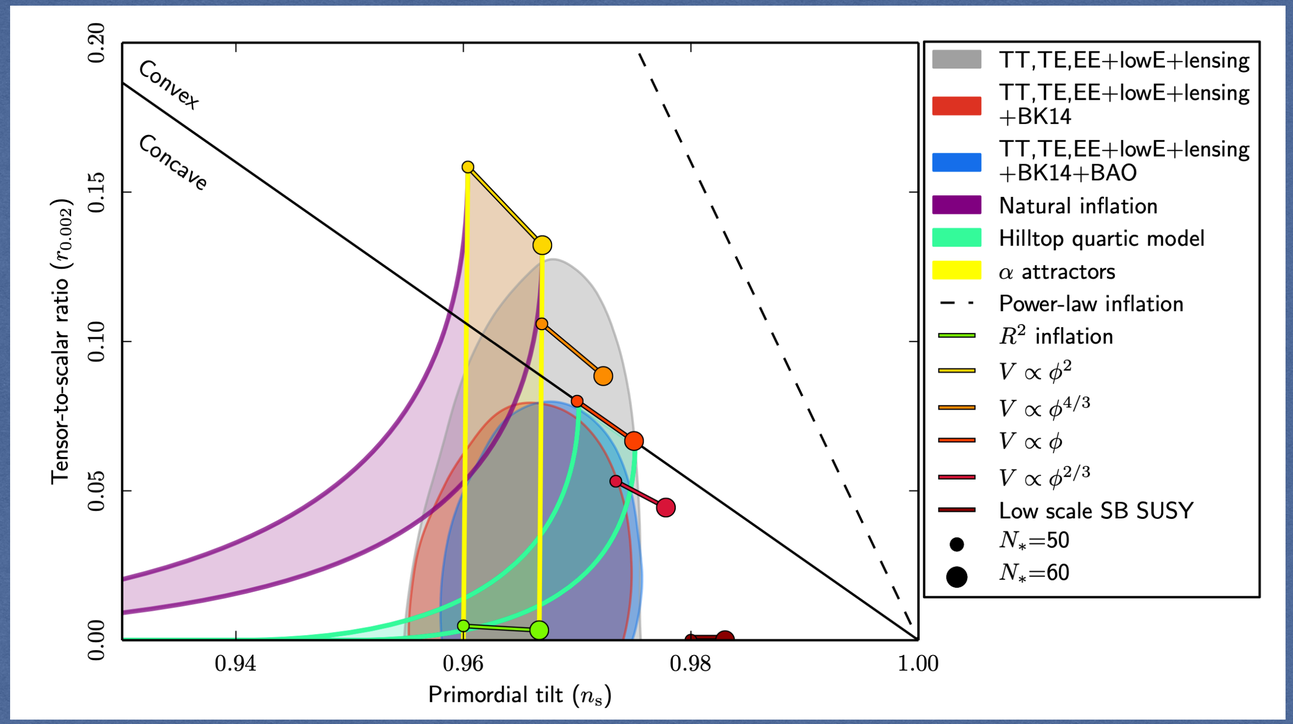
To lead to a viable phenomenology, we need(ed):

$f > M_{pl}$



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Aligned Natural Inflation

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$$\mathscr{L} = -\frac{1}{2}(\partial\theta)^2 - \frac{1}{2}(\partial\rho)^2 - \sum_{i=1,2} \Lambda_i^4 \left[1 - \cos\left(\frac{\theta}{f_i} + \frac{\rho}{g_i}\right)\right]$$

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When the alignment condition is satisfied: $\frac{f_1}{g_1} \simeq \frac{f_2}{g_2}$

$V(\theta, \phi) = V(\alpha\theta + \beta\rho) \Rightarrow$ The orthogonal direction becomes almost flat

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Starting from $f_i < M_{pl}$, we obtain an effective transplanckian direction!

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Symmetries of the model

The lagrangian possesses an internal symmetry given by

$$\begin{pmatrix}\theta\\ \rho\end{pmatrix} \to R \begin{pmatrix}\theta\\ \rho\end{pmatrix} \quad \begin{pmatrix}f_i\\ g_i\end{pmatrix} \to R \begin{pmatrix}f_i\\ g_i\end{pmatrix} \quad R \in SO(2)$$

Therefore, we introduce $SO(2)$ invariant parameters as

$$n_1 \equiv f_1^{-2} + g_1^{-2} \ , \ n_2 \equiv f_2^{-2} + g_2^{-2} \ , \ \mathscr{C} \equiv f_2^{-1} g_1^{-1} - f_1^{-1} g_2^{-1}$$

We also define

$$\Lambda^4 \equiv \Lambda_1^4 + \Lambda_2^4 \ , \ r_\Lambda \equiv \frac{\Lambda_2^4}{\Lambda_1^4}$$

The physical observables will depend only on the invariant quantities

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Trajectories in field space

There are two possible inflationary trajectories:

M. Peloso and C. Unal 2015

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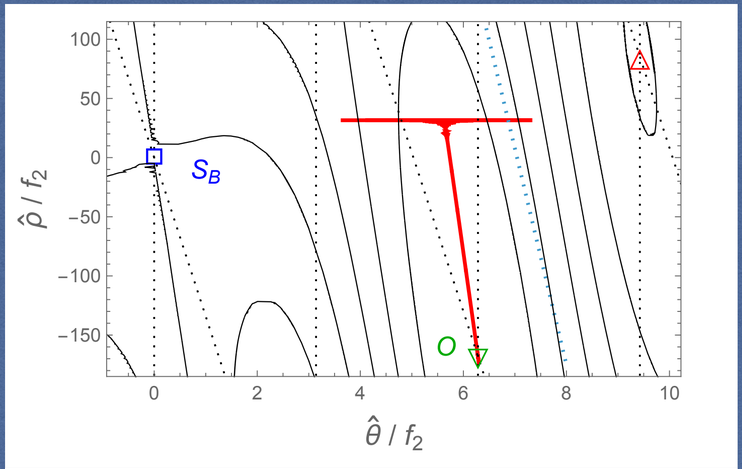

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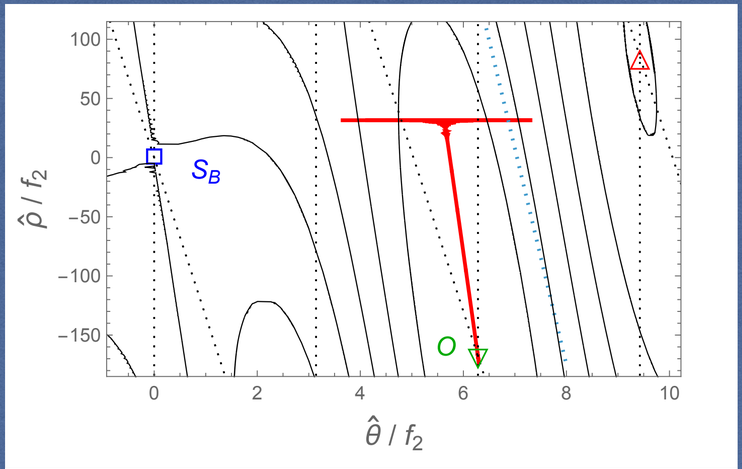

⇒

Phenomenology analogous to natural inflation, therefore ruled out!

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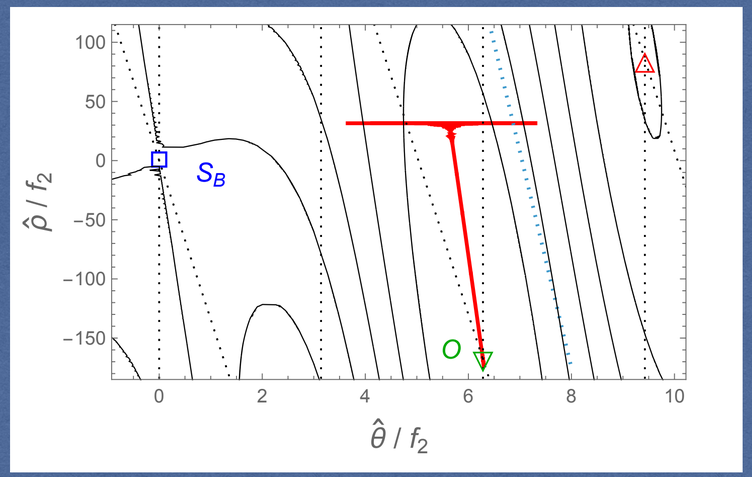

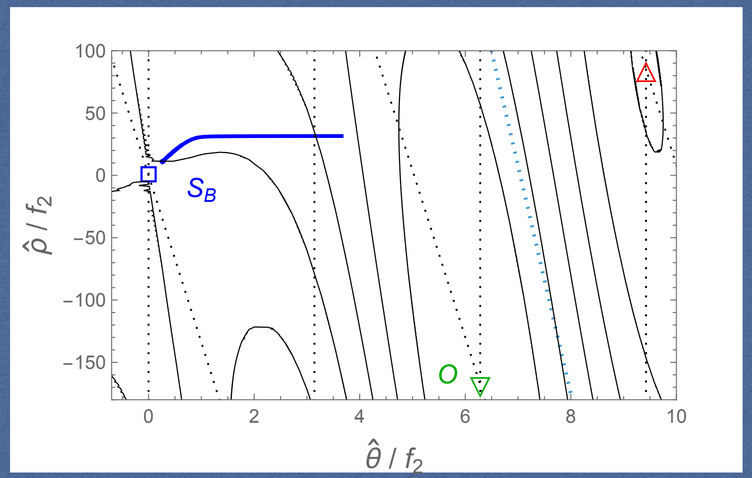

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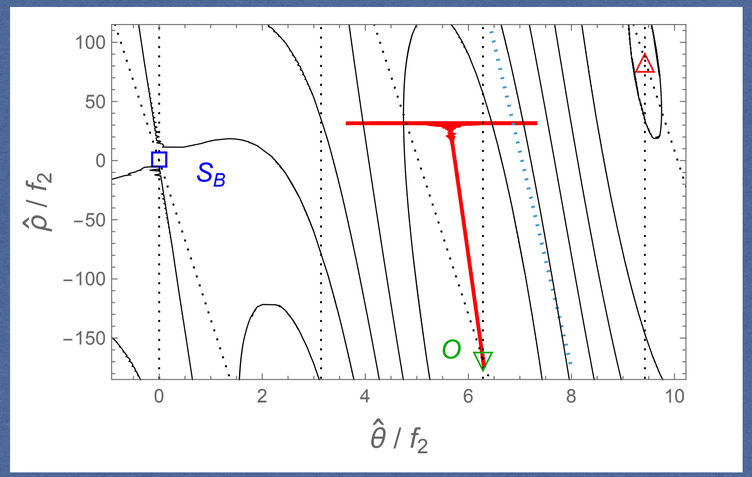

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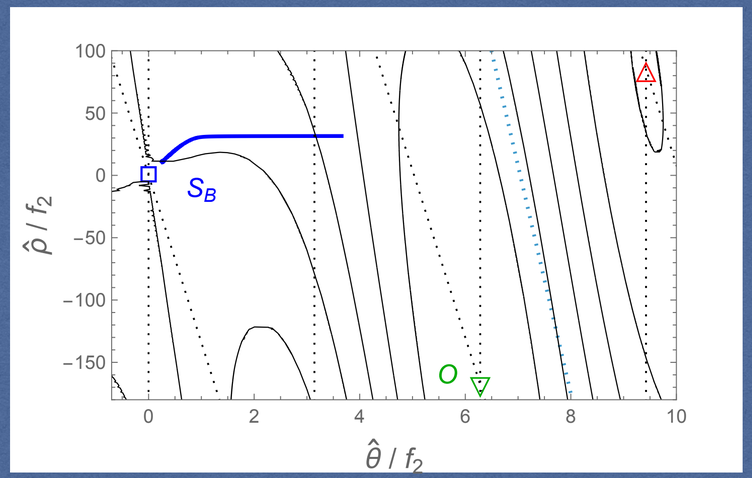

⇒ Suppressed tensor signal, therefore still viable!

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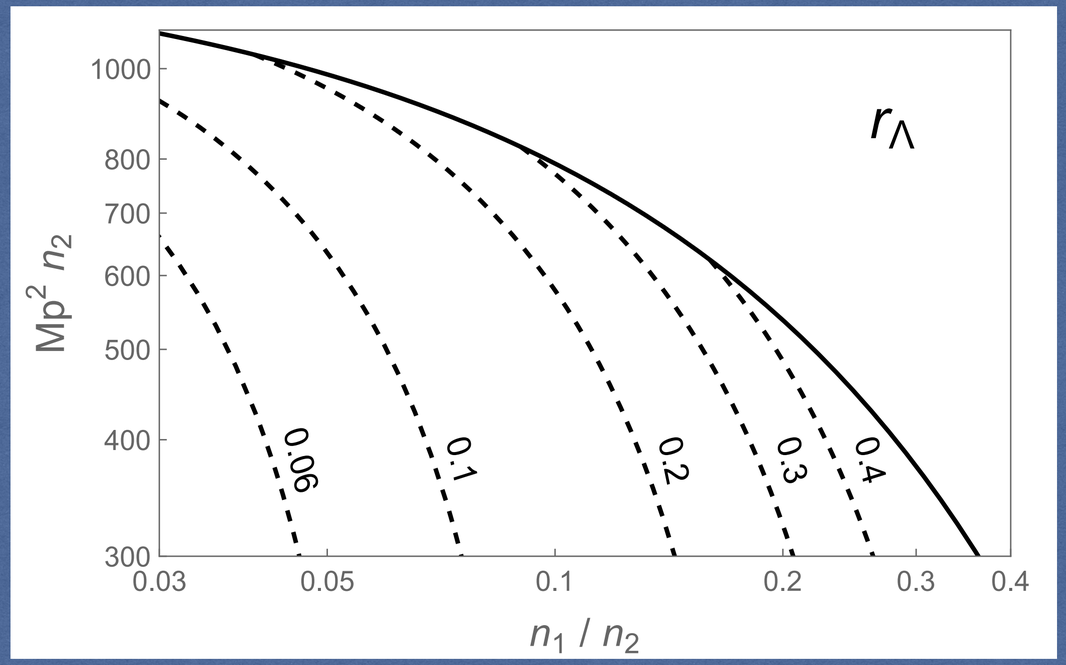
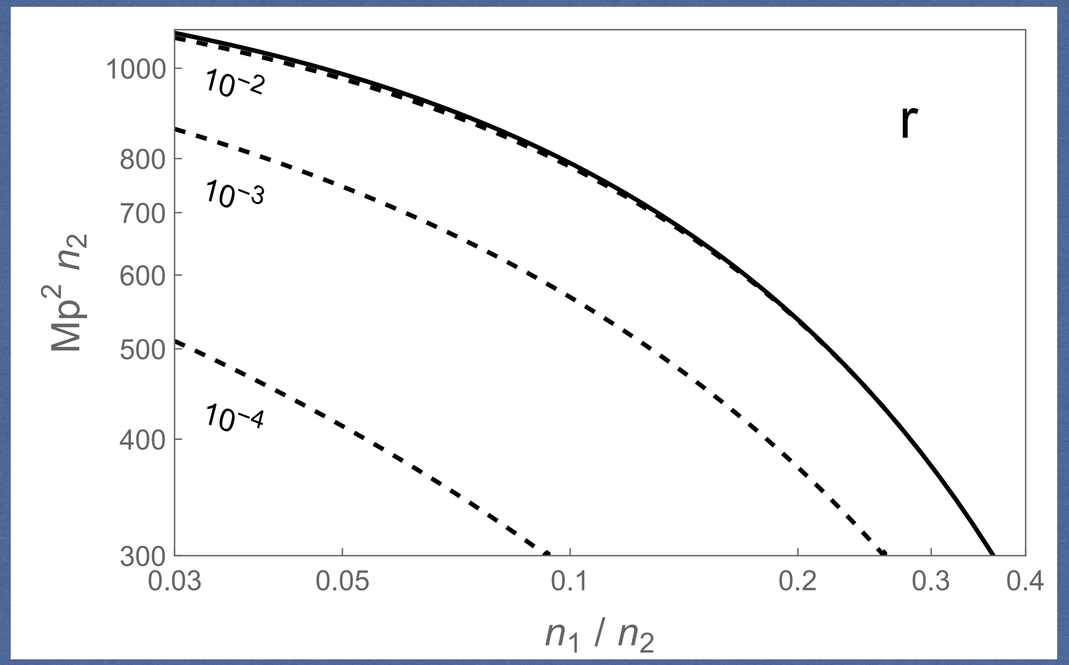
Metastable trajectories

Fixing the level of alignment and $n_s = 0.965$, we are left with 2 parameters that produce the following phenomenology





The maximum tensor-to-scalar ration is given by $r \simeq 10^{-2}$, which is below the current bound

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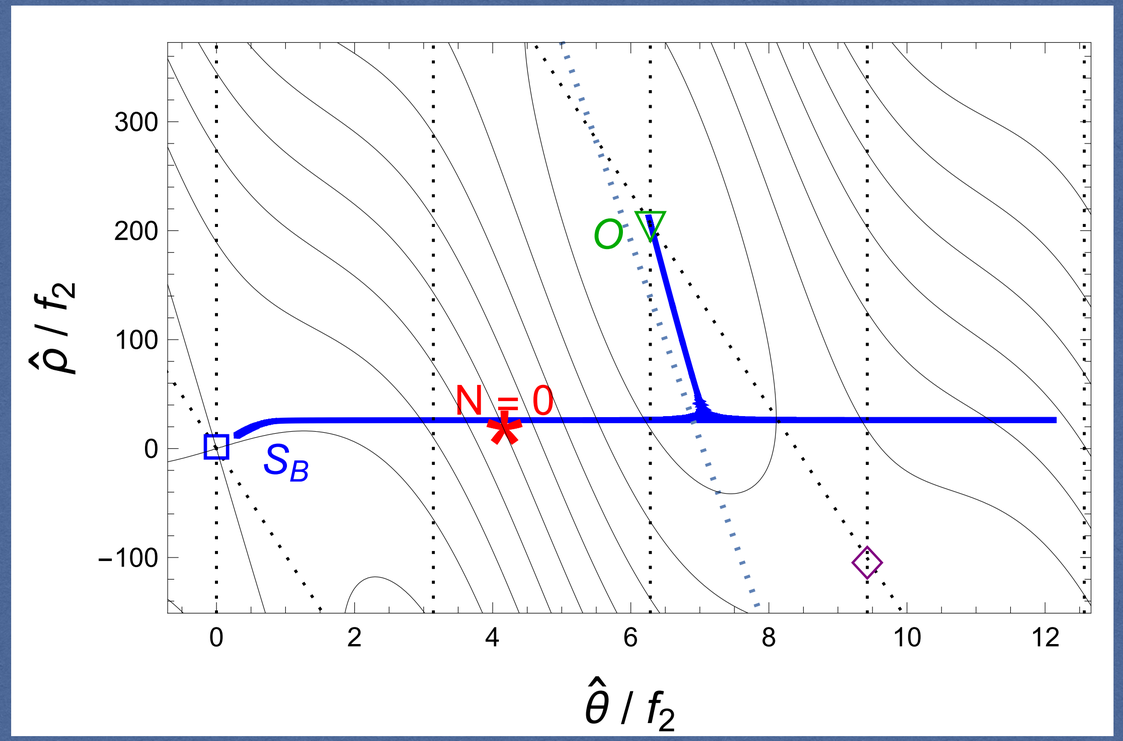
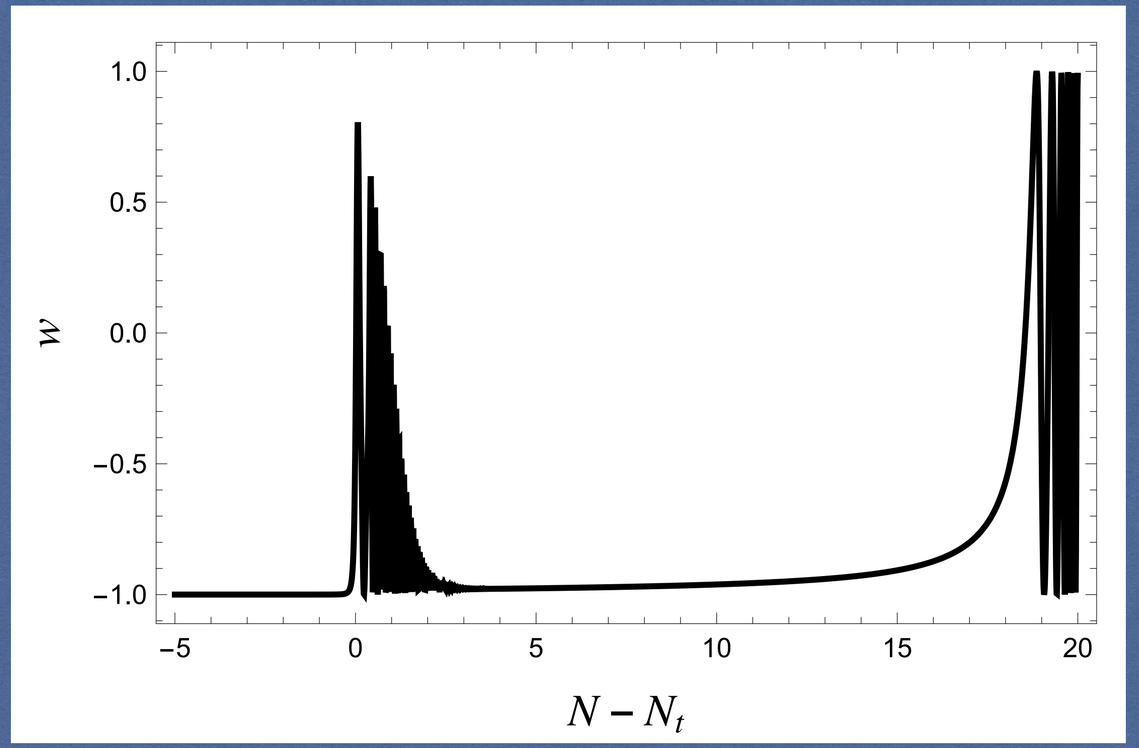
Inflationary transitions

There are trajectories which are composed by both inflationary paths





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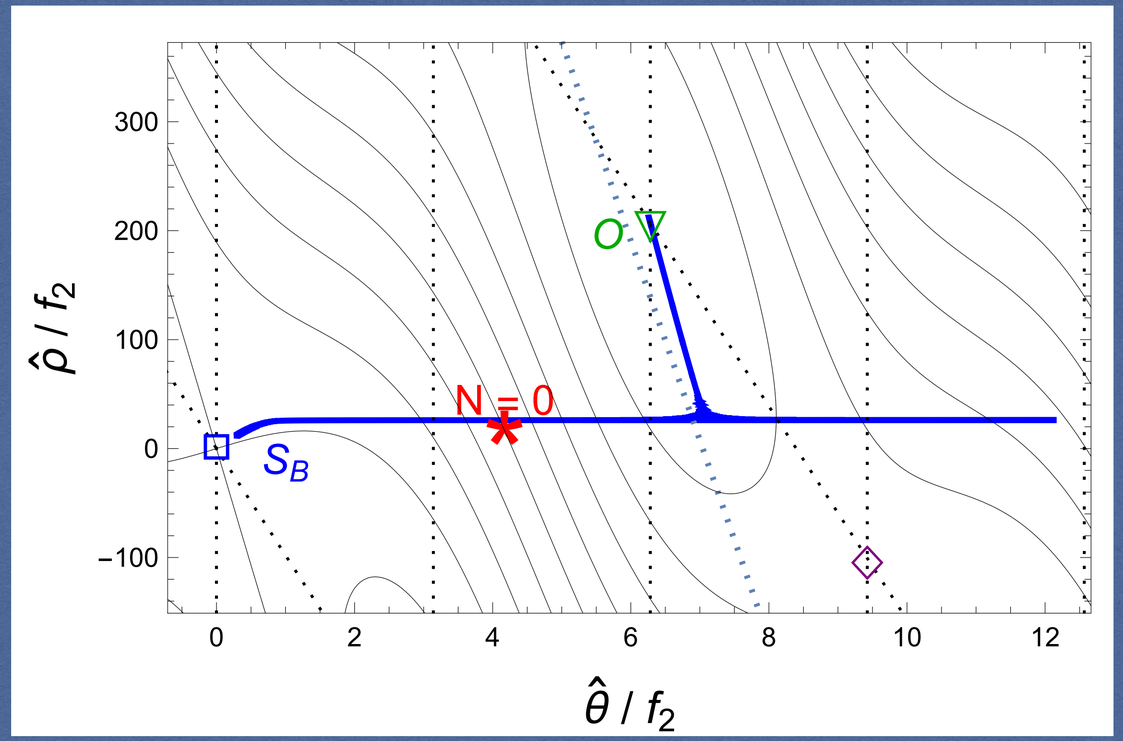

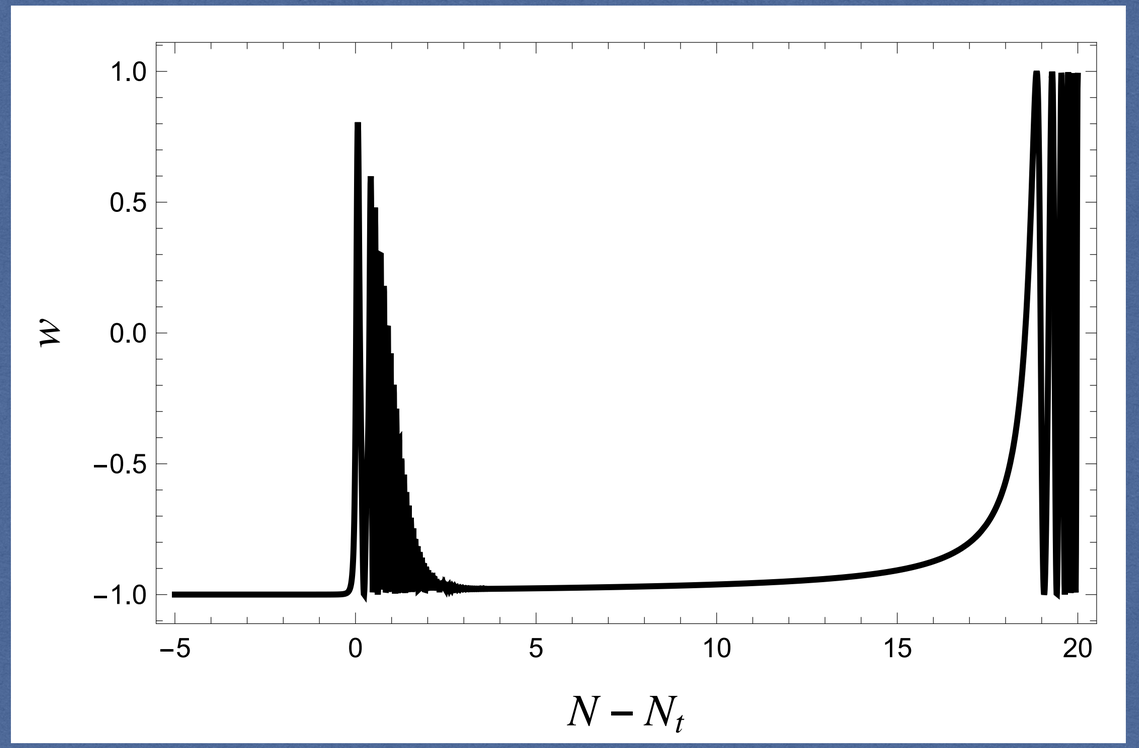

We have a transition between the two trajectories characterised by fast oscillations of the fields

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Duration of the second path

How general are these transitions between inflationary phases?

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We can estimate the number of e-folds in the second phase as:

$$\mathscr{L}_{\text{eff}}(\varphi,\dot{\varphi}) = \frac{1}{2}\dot{\varphi}^2 - V_{\text{eff}}(\varphi) \Rightarrow N_{\text{tot}} = \frac{\varphi_0(n_1,n_2,\gamma)^2}{4M_p^2} - \frac{1}{2}$$

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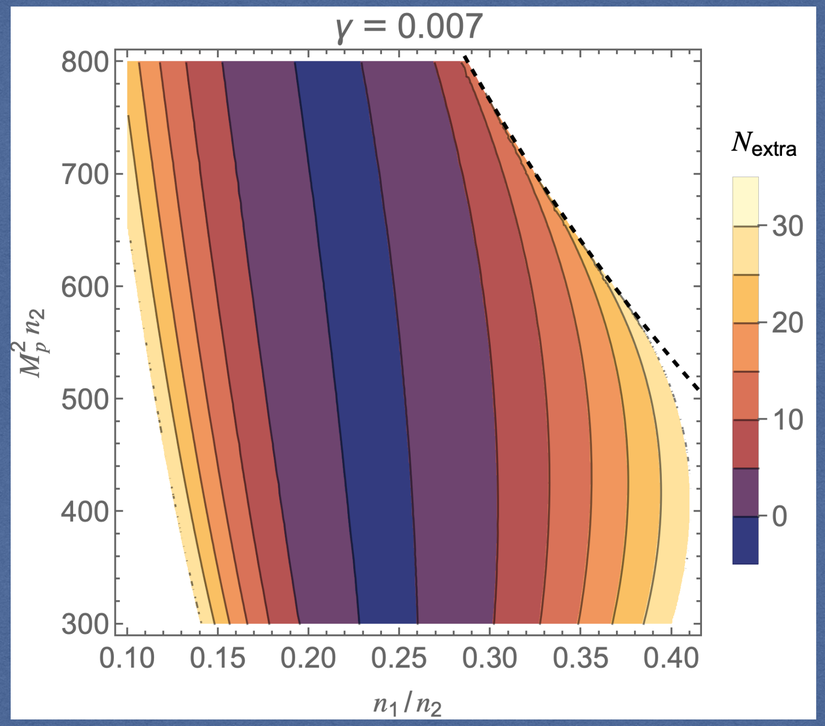

For general parameters ($\gamma \ll 1$), we have a non-negligible amount of e-folds in the second stage

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Phenomenology

Could the fast oscillations performed at the transition produce some observable phenomenology?

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Yes:

Fast oscillating axions $\Rightarrow$ Gauge fields production through $\mathscr{L} \supset g\frac{a}{f}F\tilde{F}$

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Yes:

Fast oscillating axions ⇒ Gauge fields production through $\mathscr{L} \supset g\frac{a}{f}F\tilde{F}$

In an inflating Universe, the gauge field production is controlled by the parameter:

$$\xi = \frac{\dot{a}}{2fH}$$

Phenomenology

We introduce a U(1) gauge field to study its production during the transition

$$\mathscr{L} = \mathscr{L}_{\text{al}} - \frac{1}{4}F^2 - \frac{1}{4}\left(\frac{\theta}{F} + \frac{\rho}{G}\right) F\tilde{F}$$

$$\tilde{F}^{\mu\nu} = \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} F_{\rho\sigma}$$

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The heavy eigenstate is

$$\psi = v_1\,\theta + v_2\,\rho, \quad v_1 \simeq 1 + \mathcal{O}(\gamma^2), \quad v_2 \simeq \mathcal{O}(\gamma^2)$$

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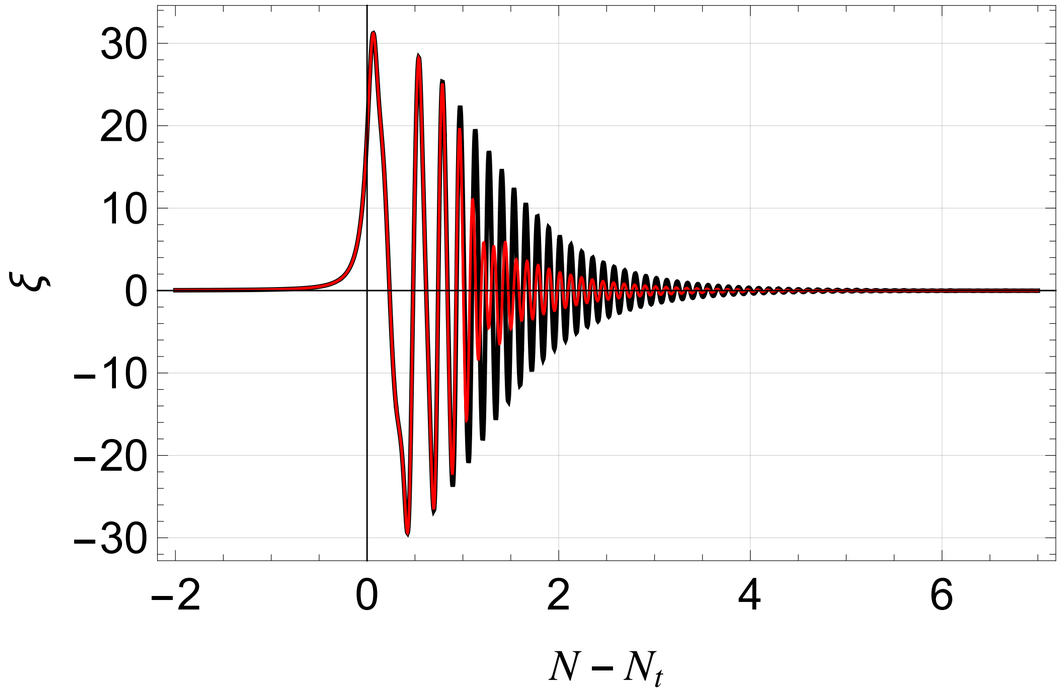
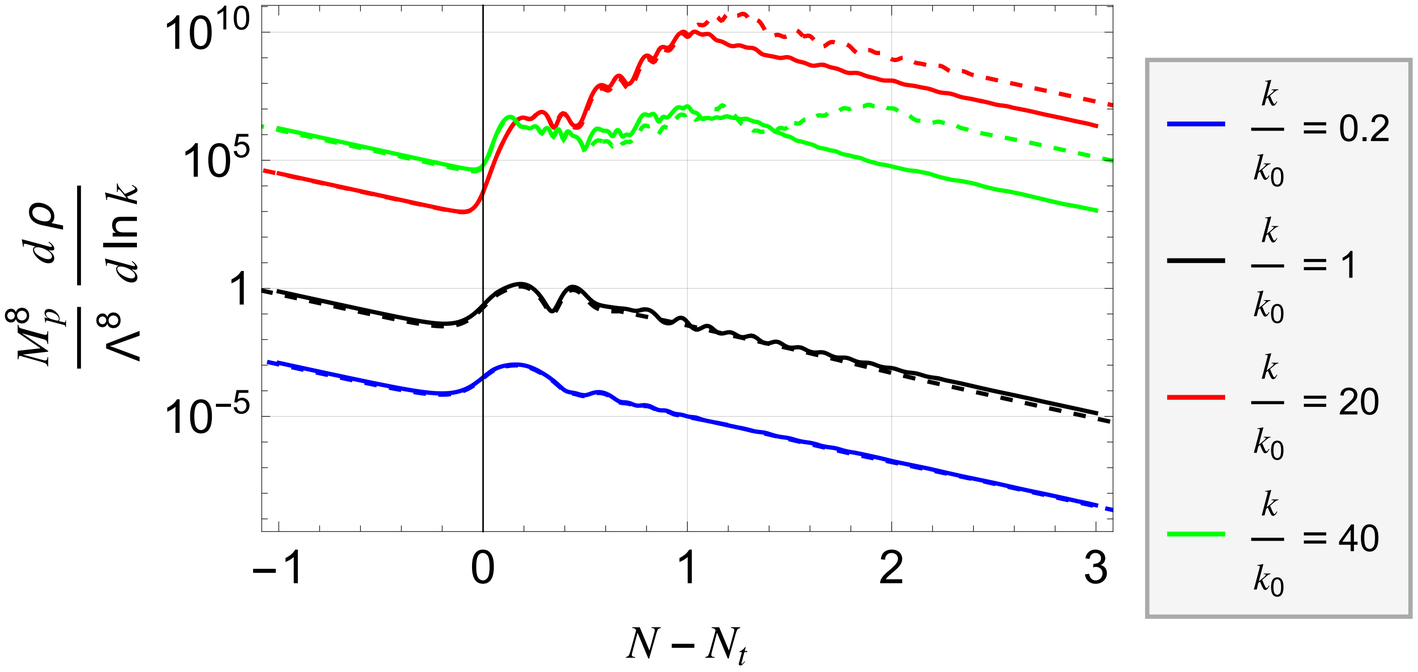
Therefore, we expect $G^{-1} \ll F^{-1}$ and we neglect its contribution in the lagrangian

$$\xi = \frac{1}{2H}\left(\frac{\dot{\theta}}{F} + \frac{\dot{\rho}}{G}\right) \simeq \frac{\dot{\theta}}{2HF}$$

Phenomenology

We computed numerically the gauge field production, with and without the inclusion of the back-reaction on the axions





As expected, there is a considerable production of gauge fields at the transition slightly reduced when taking into account the back-reaction

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GW production

The gauge field sources tensor perturbations through the 2nd order action

$$S_{\rm GW} = \int d^4x \left[\frac{M_p^2 a^2}{8} \left(\hat{h}_{ij}'^2 - \partial_k \hat{h}_{ij}^2 \right) + \frac{a^2}{2} \hat{h}_{ij} \hat{T}_{ij}^{\rm TT} \right]$$

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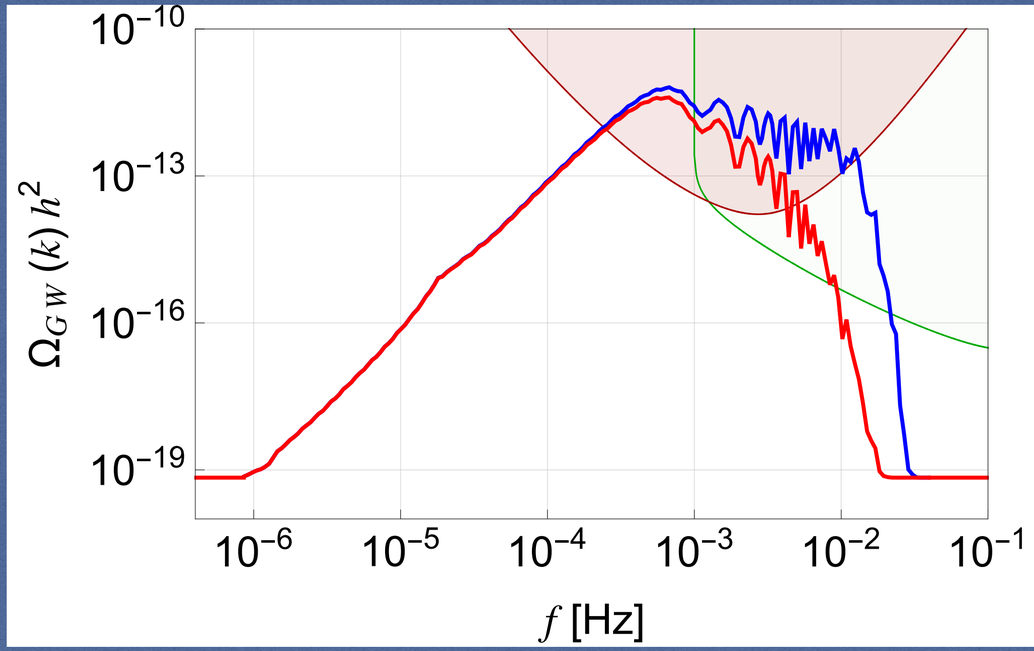
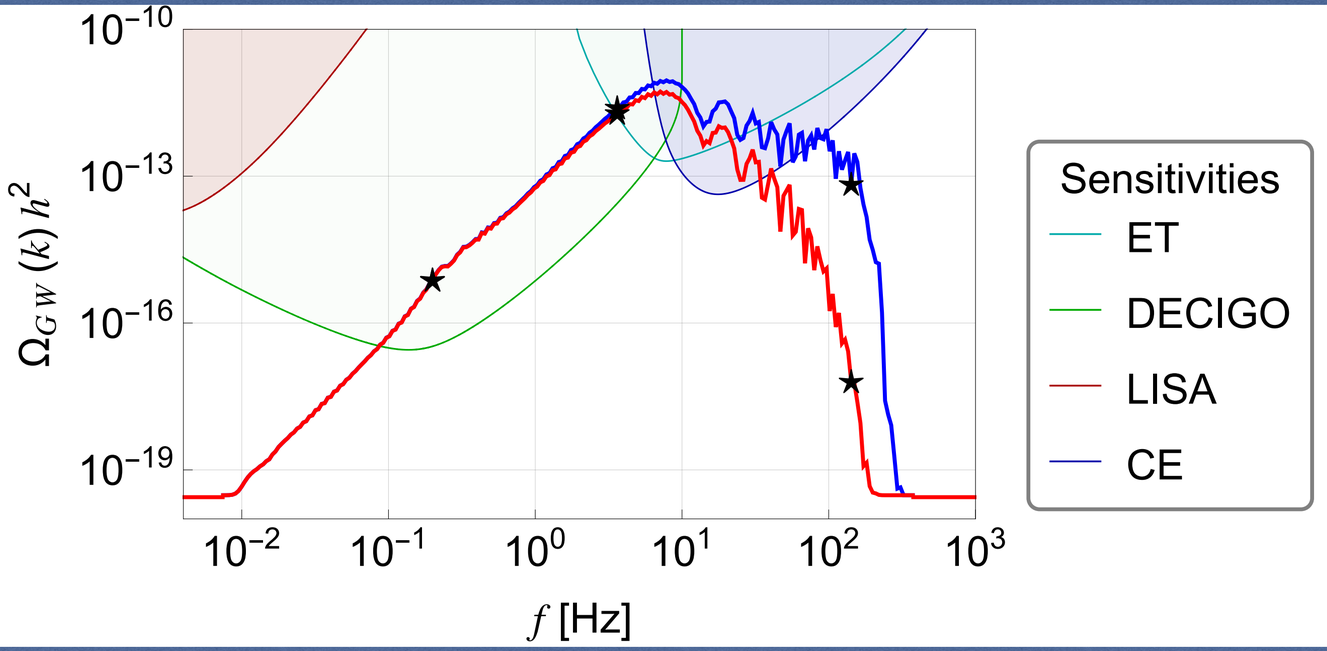
$$\Downarrow$$

$$\left\langle \hat{h}_\lambda(\tau, \vec{k}) \;\; \hat{h}_\lambda(\tau, \vec{k}') \right\rangle = \frac{2\pi^2}{k^3} P_\lambda(k) \; \delta^{(3)}(\vec{k} + \vec{k}') \rightarrow \text{Power spectrum}$$

GW production

We obtain a PS which is peaked at the transition scale, producing a characteristic signal





The possible signal is within the rich of the next GW experiments and would point directly at the transition scale

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Conclusions

We have seen that the aligned natural inflation model:

- Admists a richer class of inflationary paths than what was originally thought,
- These paths generally lead to a transition between two inflationary phases,
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Thank you for the attention!