The Schwinger effect during axion inflation

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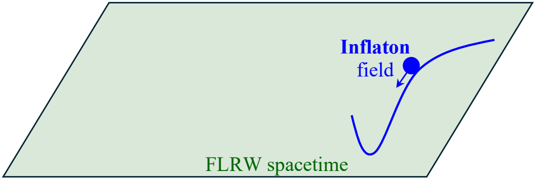
The 27th International Conference from the Planck Scale to the Electroweak Scale

PLANCK 2025

Overview

1. Introduction
2. Axion inflation
3. Schwinger effect during inflation
4. Gradient-expansion formalism
5. Numerical results
6. Conclusions

Motivation



[Starobinsky'80]

[Guth'81]

[Linde'82]

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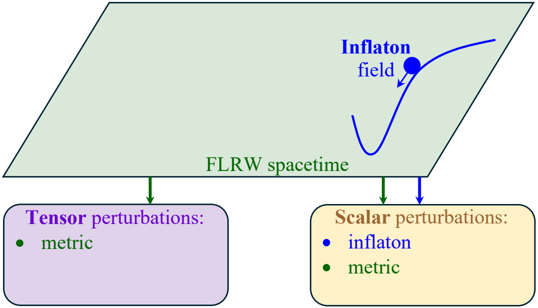
[Albrecht'82]

[Linde'83]

Motivation



[Mukhanov'81]
[Mukhanov'82]
[Guth'82]
[Hawking'82]
[Bardeen'82]

[Grishchuk'74]
[Starobinsky'79]
[Rubakov'82]
[Fabbri'83]
[Abbott'84]

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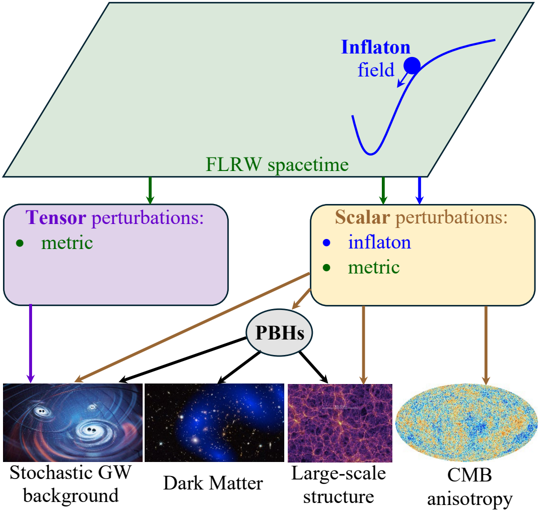


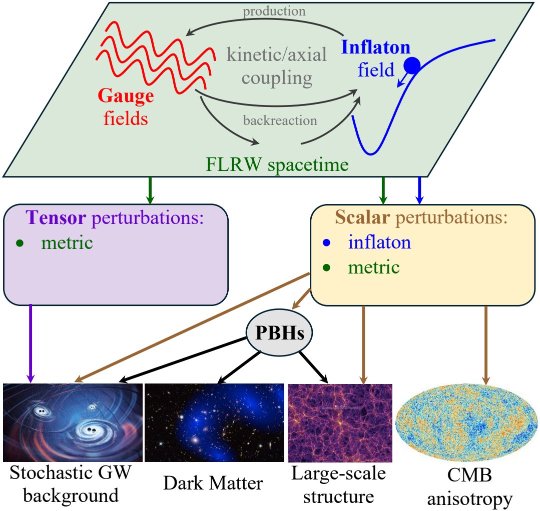
Credit: O. Shmahalo, D. Harvey, R. Massey, H. Ebeling,J.-P. Kneib, Millenium Simulation Project, NASA, ESA, Planck Collaboration

[Planck'18]
[BICEP/Keck'21]
[Reid'10]
[Cabass'22]

[NANOGrav'23]
[EPTA/InPTA'23]
[Reardon'23]
[Xu'23]

[Hawking'71]
[Carr'74]
[Carr'16]
[Carr'22]

Motivation



Credit: O. Shmahalo, D. Harvey, R. Massey, H. Ebeling,J.-P. Kneib, Millenium Simulation Project, NASA, ESA, Planck Collaboration

[Turner'88]

[Ratra'91]

[Garretson'92]

[Anber'06]

[Martin'08]

[Durrer'13]

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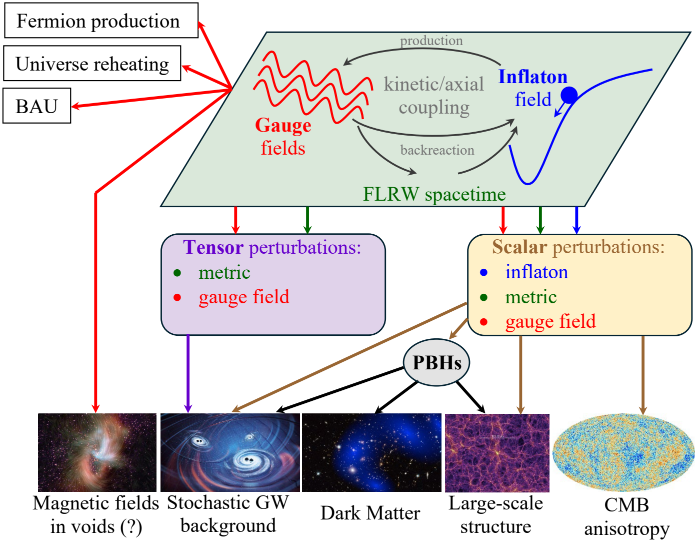

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[Neronov'10]
[Taylor'11]

[Anber'15]
[Adshead'16]
[Domcke'19]
[Domcke'23]

[Cuissa'19]
[Adshead'20]
[Figueroa'23]

[Kobayashi'14]
[Hayashinaka'16]
[Domcke'18]
[Sobol'20]

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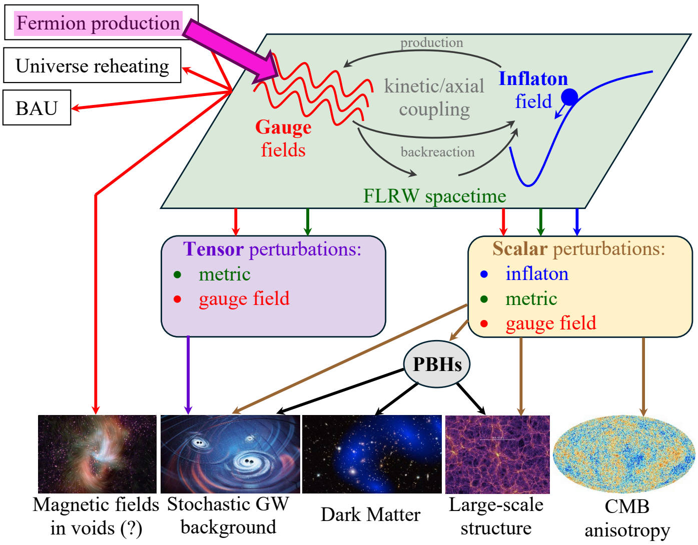

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Our aim:
To consistently describe the Schwinger pair production and its impact on the evolution of the gauge field itself.

Axion inflation

- For successful slow-roll inflation we need the inflaton potential to be **sufficiently flat**. However, the **radiative corrections** may **break** this **flatness** and spoil inflation.
- This usually happens unless the flatness of the potential is **protected by a shift symmetry** $\phi \to \phi + \text{const}$.
  E.g., natural inflation model [Freese et al., PRL **65** (1990)]
- Interaction terms with matter fields should also be shift-symmetric. The simplest choice for the gauge field is [Garretson et al., PRD **46** (1992)]

$$S_{GF} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\beta}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\right].$$

- Such scalar field ϕ is often called **axion** (or axion-like field).

Gauge-field generation during axion inflation

$$S=\int d^4x\sqrt{-g}\Big[\underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-V(\phi)}_{\text{pseudoscalar inflaton}}-\underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free gauge field}}-\underbrace{\frac{\beta}{4M_{\rm P}}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{axion coupling of GF to inflaton}}+\underbrace{\mathcal{L}_{ch}(A_\nu,\chi)}_{\text{charged field (Schwinger eff.)}}\Big]$$

Equations of motion:

- Friedmann eq.: $H^2=\frac{1}{3M_{\rm P}^2}\Big[\frac{1}{2}\dot{\phi}^2+V(\phi)+\frac{1}{2}\langle \boldsymbol{E}^2+\boldsymbol{B}^2\rangle+\rho_\chi\Big]$
- Klein-Gordon eq.: $\ddot{\phi}+3H\dot{\phi}+V'(\phi)=\frac{\beta}{M_{\rm P}}\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle$
- Maxwell equations:

$$\dot{\boldsymbol{E}}+2H\boldsymbol{E}-\frac{1}{a}\mathrm{rot}\,\boldsymbol{B}+\frac{\beta}{M_{\rm P}}\dot{\phi}\,\boldsymbol{B}+\boldsymbol{j}=0,$$

$$\dot{\boldsymbol{B}}+2H\boldsymbol{B}+\frac{1}{a}\mathrm{rot}\,\boldsymbol{E}=0,\quad \mathrm{div}\,\boldsymbol{E}=0,\quad \mathrm{div}\,\boldsymbol{B}=0.$$

- Eq. for charged particles: $\dot{\rho}_\chi+4H\rho_\chi=\boldsymbol{j}\cdot\boldsymbol{E}$.

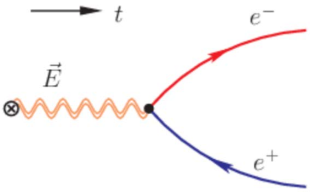
Schwinger effect during inflation

Strong electric component may lead to the Schwinger pair production [Schwinger, PR **82** (1951)].

Studied analytically [Domcke, JHEP **02** (2020)]

- De Sitter space
- Constant and **collinear** electric and magnetic fields
- Strong-field regime

$$E_{\rm th} \triangleq \frac{2m_e c^3}{e\hbar} \simeq 10^{18}\ \mathrm{V/m}.$$



$$|e\boldsymbol{E}| \gg H^2$$

$$t_E = \frac{1}{\sqrt{|eE|}} \ll H^{-1} = t_H$$

$$j = \frac{(e|Q|)^3}{6\pi^2}\frac{|B|E}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eQE|}\right).$$

Schwinger induced current: different representations

1) **Electric picture:**

$$j = \sigma_E E, \qquad \sigma_E = \frac{(e|Q|)^3}{6\pi^2}\frac{|B|}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eQE|}\right);$$

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$$j = \sigma_E E + \sigma_B B = \frac{(e|Q|)^3}{6\pi^2}\frac{|B|E}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eQE|}\right).$$

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No difference at the classical level. **But how to incorporate this current into the Maxwell equation for the quantum gauge field?**

Schwinger induced current: different representations

Typical approach: current is **linear in gauge-field operators** with **conductivities being classical functions** (depends on mean fields). Then, three pictures appear to be **inequivalent**!

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1) **Electric picture** [Kobayashi, JHEP **10** (2014); Gorbar, PRD **104** (2021)]:

$$\hat{\boldsymbol{j}} = \sigma_E \hat{\boldsymbol{E}}, \qquad \sigma_E = \frac{(e|Q|)^3}{6\pi^2}\frac{|B|}{H}\coth\Big(\frac{\pi|B|}{|E|}\Big)\exp\Big(-\frac{\pi m^2}{|eQE|}\Big);$$

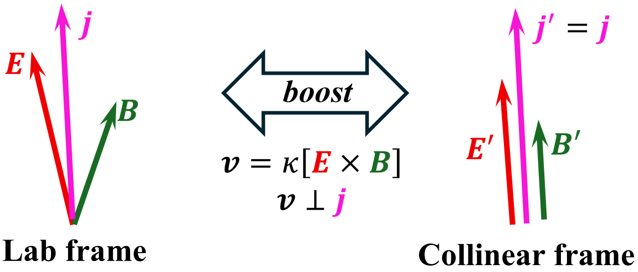
2) **Magnetic picture** [Domcke, JHEP **11** (2018); JHEP **02** (2020)]:

$$\hat{\boldsymbol{j}} = \sigma_B \hat{\boldsymbol{B}}, \qquad \sigma_B = \frac{(e|Q|)^3}{6\pi^2}\frac{|E|}{H}\coth\Big(\frac{\pi|B|}{|E|}\Big)\exp\Big(-\frac{\pi m^2}{|eQE|}\Big)\mathrm{sign}\,(EB);$$

3) **Mixed picture:**

$$\hat{\boldsymbol{j}} = \sigma_E \hat{\boldsymbol{E}} + \sigma_B \hat{\boldsymbol{B}} \qquad \sigma_E,\ \sigma_B - ?$$

Schwinger induced current: vector decomposition



$$\hat{\boldsymbol{j}} = \sigma_E \hat{\boldsymbol{E}} + \sigma_B \hat{\boldsymbol{B}}$$

$$\sigma_E = \left[\frac{(e|Q|)^3}{6\pi^2} \frac{|B'|}{H} \coth\left(\frac{\pi |B'|}{|E'|} \right) \exp\left(-\frac{\pi m^2}{|eQE'|} \right) \right] \gamma (1 - \kappa \boldsymbol{B}^2) \,;$$

$$\sigma_B = \left[\frac{(e|Q|)^3}{6\pi^2} \frac{|B'|}{H} \coth\left(\frac{\pi |B'|}{|E'|} \right) \exp\left(-\frac{\pi m^2}{|eQE'|} \right) \right] \gamma \kappa \, \boldsymbol{E} \cdot \boldsymbol{B} \,.$$

Relevant scales

1) **Tachyonic instability scale**:

$$k_h(t) = \max_{t' \leq t} \left(\max_k k : \Omega^2(k, t') < 0 \right).$$

Maximal momentum of the Fourier mode which undergoes (or underwent in the past) **tachyonic instability**: $\mathcal{A}_k'' + \Omega^2(k,t)\mathcal{A}_k = 0$, $\Omega^2 < 0$.

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2) **Pair-creation scale**:

$$k_{\rm S}(t) = a(t)\sqrt{|eQE'(t)|}$$

Modes with wavelengths much shorter than $\lambda_{\rm S} \sim 1/k_{\rm S}$ cannot feel the presence of a conducting medium.

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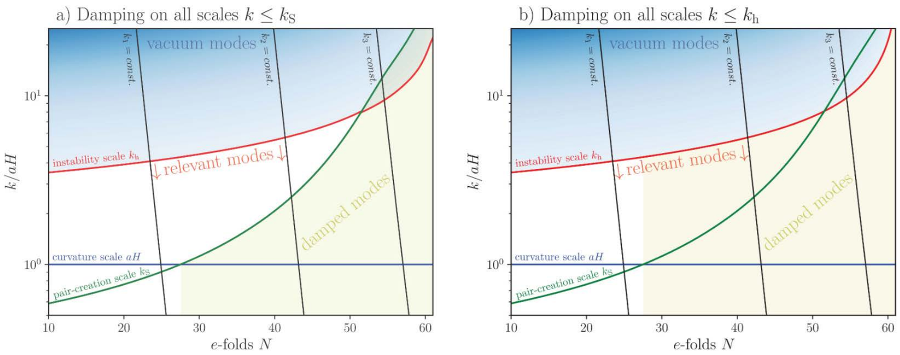
3) **Curvature (Hubble) scale**:

$$k_H(t) = a(t)H(t)$$

If $k_{\rm S} \ll k_H$, i.e., $|eE'| \ll H^2$, the Schwinger pair production is not effective, but also irrelevant for the gauge-field evolution.

Scale-dependent damping



One must track the evolution of all relevant scales and “turn on” the Schwinger conductivities in the right moments of time (depending on the momentum).

Gradient-expansion formalism

[Gorbar, Schmitz, OS, Vilchinskii, PRD **104** (2021); PRD **105** (2022)]

We introduce an infinite set of quantities:

$$\mathcal{E}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n \boldsymbol{E}\rangle\,, \quad \mathcal{G}^{(n)} = -\frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n \boldsymbol{B}\rangle\,, \quad \mathcal{B}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{B}\cdot \mathrm{rot}^n \boldsymbol{B}\rangle\,.$$

They satisfy the following chain of equations ($\xi \equiv \beta\dot{\phi}/(2HM_p)$):

$$\dot{\mathcal{E}}^{(n)} + (n+4)H\,\mathcal{E}^{(n)} - 4H\xi\,\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{E}}^{(n)}]_b,$$

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$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{B}}^{(n)}]_b.$$

Thus, we trade an **infinite number of Fourier-modes** for an **infinite set** of scalar **functions** in the coordinate space – **what's the gain?**

The chain can be truncated!

Boundary terms

Any function $X^{(n)}$ has the following spectral decomposition:

$$X = \int_0^{k_\mathrm{h}(t)} \frac{dk}{k} \frac{dX}{d\ln k}.$$

There are two sources of time dependence:

- The spectral density depends of $\mathcal{A}_\lambda(k, t)$ and its derivatives.
- The upper integration limit $k_\mathrm{h}(t)$ is time dependent!
  E.g., w/o Schwinger effect, $k_\mathrm{h}(t) = 2a(t)H(t)|\xi(t)|$.

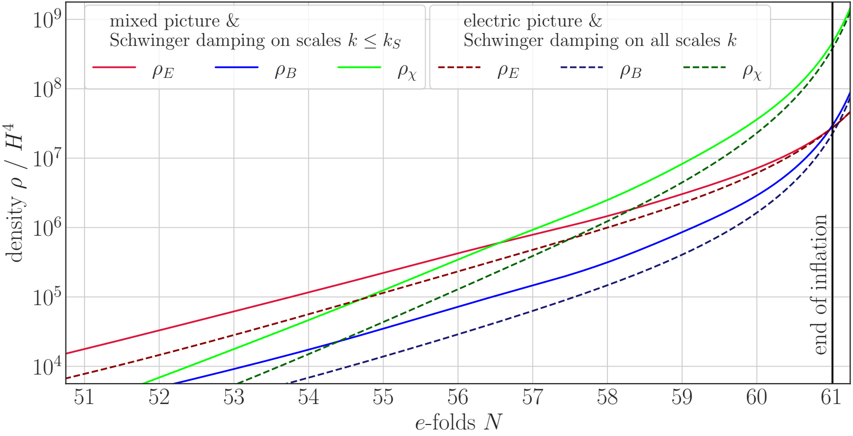
Boundary terms describe the latter time dependence, i.e., they take into account the fact that the **number of physically relevant modes grows in time** during inflation.

$$(\dot{X})_b = \left.\frac{dX}{d\ln\ k}\right|_{k=k_\mathrm{h}} \cdot \frac{d\ln k_\mathrm{h}}{dt}.$$

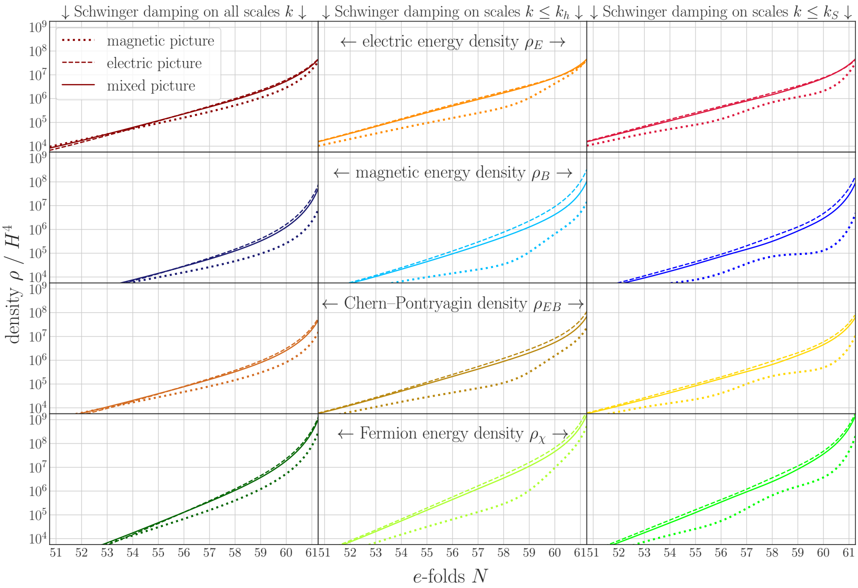
They are expressed in terms of Whittaker functions.

Comparison of different approaches: old vs new

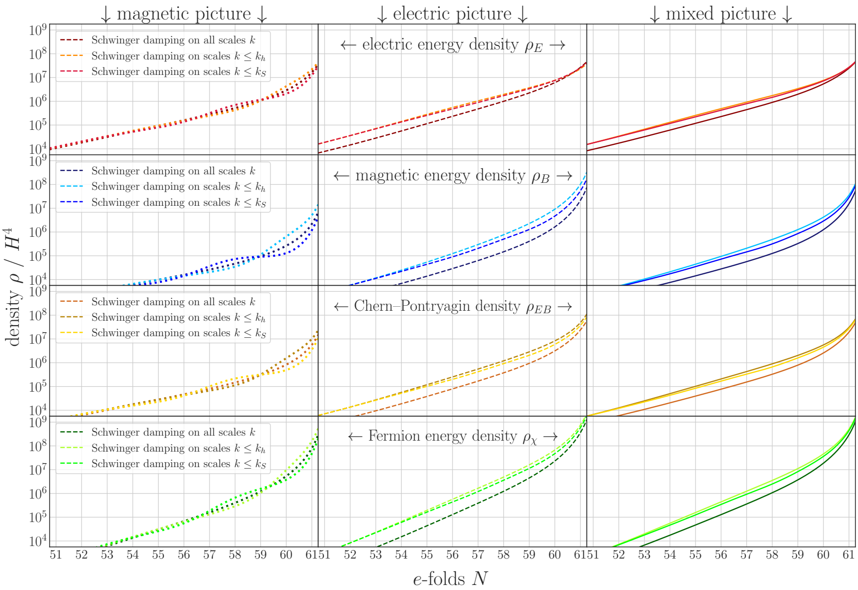
"Old" approach: $\boldsymbol{j} = \sigma_E \boldsymbol{E}$ (electric), damping of **all** gauge-field modes.
"New" approach: $\boldsymbol{j} = \sigma_E \boldsymbol{E} + \sigma_B \boldsymbol{B}$ (mixed), damping only for $k \le k_{\rm S}(t)$.



Comparison of different approaches: three pictures



Comparison of different types of scale-dependence



Conclusions

1. Schwinger production of charged particles **strongly alters the gauge-field dynamics** in axion inflation.
2. Taking into account that the electric and magnetic fields are in general not collinear, we derived the vector decomposition of the Schwinger-induced current in terms of these fields and determined the corresponding effective **electric and magnetic conductivities**.
3. We incorporated Schwinger damping of the gauge field **in a scale-dependent fashion** in the equations of motion.
4. In some cases, our new results differ from the old ones by **more than one order of magnitude**.

arXiv:2408.16538

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Thank you very much for your attention!

Peace to all of us!