A general upper bound on the light DM scattering rate in materials

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talk based on arXiv:2501.18261
(with R. Catena; submitted to JCAP)

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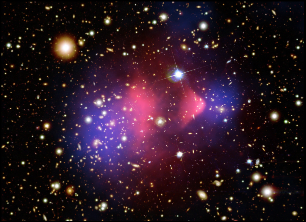
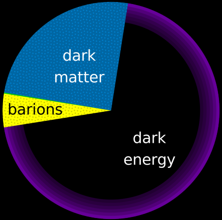
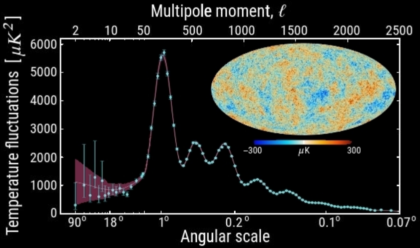
Dark matter

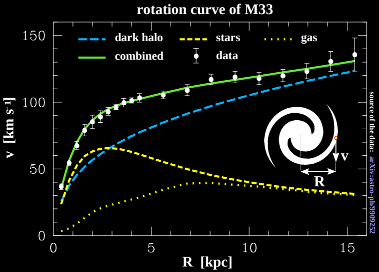


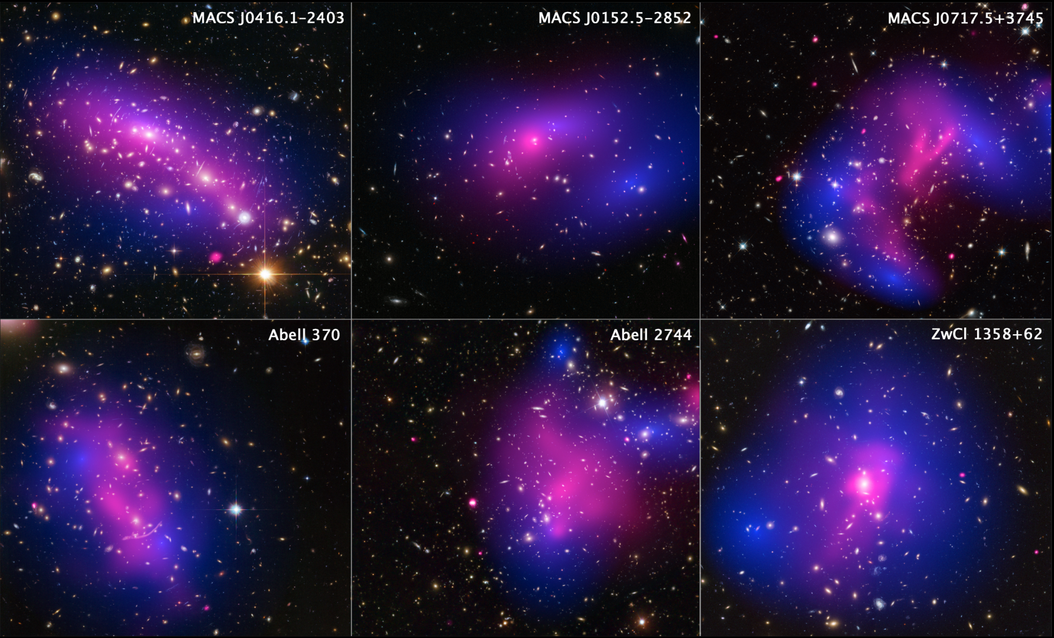
https://apod.nasa.gov/apod





http://sci.esa.int/planck
https://wiki.cosmos.esa.int/planck-legacy-archive





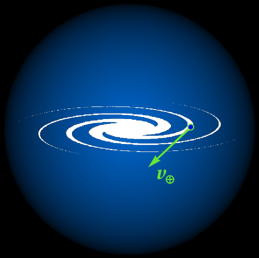
`https://hubblesite.org`

Local distribution of DM

Baxter et al., 2105.00599

$$n_{\mathsf{DM}} \simeq \frac{1\ \mathsf{GeV}}{m_{\mathsf{DM}}} \times 0.3\ \ \mathsf{cm}^{-3}$$

$$\rho(\boldsymbol{v}) = \mathcal{N}\ \exp\left[-\frac{(\boldsymbol{v}+\boldsymbol{v}_\oplus)^2}{v_0^2}\right]\theta(v_{\mathsf{esc}} - |\boldsymbol{v}+\boldsymbol{v}_\oplus|)$$

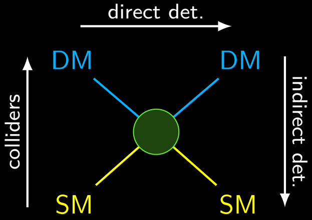


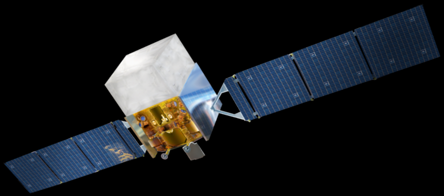
$$v_{\mathsf{esc}} = 544\,\mathsf{km/s}$$

$$v_\oplus = 250.5\,\mathsf{km/s}$$

$$v_0 = 238\,\mathsf{km/s}$$

How to detect particle dark matter?





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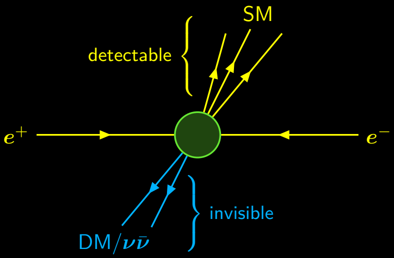
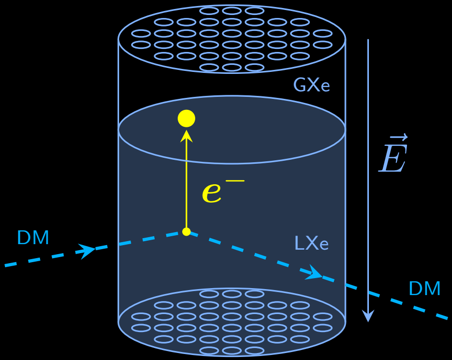
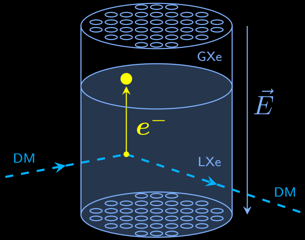




Direct detection experiments

- LUX-ZEPLIN, PandaX-4T, XENONnT, SuperCDMS, ...



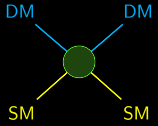
- WIMP paradigm: GeV+ range of masses
  - no success so far ⇒ **sub-GeV** DM?

- nuclear vs. electronic recoil of **non-relativistic** DM

$$\Delta E_{\mathrm{SM}} \leq \frac{4\,\mu}{(1+\mu)^2}\,E^{\mathrm{in}}_{\mathrm{DM}} \qquad \leftarrow \text{ maximized for } \mu \equiv m_{\mathrm{SM}}/m_{\mathrm{DM}} = 1$$

⇒ m_{SM} should be as close to m_{DM} as possible

⇒ **electrons** preferable for **light DM**



- what material to use?

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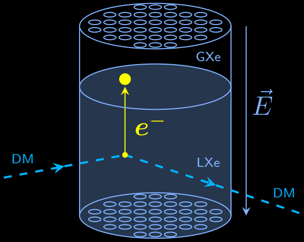

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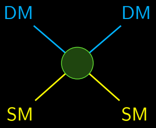


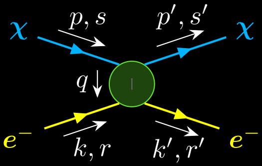
- what material to use?

Effective non-relativistic theory for spin-$^1/_2$ DM

Catena et al., 2105.02233

$$\boldsymbol{v}^\perp \equiv \frac{\boldsymbol{p}+\boldsymbol{p}'}{2\,m_\chi} - \frac{\boldsymbol{k}+\boldsymbol{k}'}{2\,m_e}$$

$$\boldsymbol{q}\cdot\boldsymbol{v}^\perp \xrightarrow{\text{en. cons.}} 0$$



$$\left.\begin{array}{l}\text{non-relativistic limit}\\ \text{Lorentz (Galilean) invariance}\end{array}\right\} \Rightarrow \quad \mathcal{M} = \textstyle\sum_i c_i\,\mathcal{O}_i$$

14 simple operators
in the leading order

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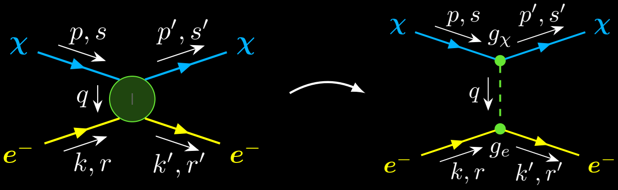

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14 simple operators
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- example: scalar coupling

$$\mathcal{M} \simeq \underbrace{-i\,\frac{g_\chi\,g_e}{q^2+M^2}\,4\,m_\chi m_e}_{c_1}\;\underbrace{\delta^{ss'}\delta^{rr'}}_{\mathcal{O}_1}$$

- other examples:

$$\mathcal{O}_4^{rr'ss'} = \frac{\boldsymbol{\sigma}^{rr'}}{2}\cdot\frac{\boldsymbol{\sigma}^{ss'}}{2}\,, \qquad \mathcal{O}_{15}^{rr'ss'} = \left[\left(\frac{\boldsymbol{\sigma}^{rr'}}{2}\times\frac{\boldsymbol{q}}{m_e}\right)\cdot\boldsymbol{v}^\perp\right]\left(\frac{\boldsymbol{\sigma}^{ss'}}{2}\cdot\frac{\boldsymbol{q}}{m_e}\right)$$

Linear response theory

Catena & Spaldin, 2402.06817

see Riccardo's talk

- another decomposition:

$$\mathcal{M} = \sum_i c_i\, \mathcal{O}_i$$

$$= \sum_a \underbrace{F_a^{ss'}(\boldsymbol{q},\boldsymbol{v}_\chi)}_{\text{DM model}} \times \underbrace{J_a^{rr'}(\boldsymbol{v}_e^\perp)}_{\text{electronic part}} \;, \qquad \boldsymbol{v}_\chi \equiv \frac{\boldsymbol{p}}{m_\chi} \;, \qquad \boldsymbol{v}_e^\perp \equiv \frac{\boldsymbol{k}+\boldsymbol{k}'}{2m_e}$$

- electronic operators:

$$J_{n_0}^{rr'} \equiv \delta^{rr'} \;, \quad J_{n_A}^{rr'} \equiv \boldsymbol{v}_e^\perp \cdot \sigma^{rr'} \;,$$

$$\boldsymbol{J}_{\boldsymbol{j}_5}^{rr'} \equiv \boldsymbol{\sigma}^{rr'} \;, \quad \boldsymbol{J}_{\boldsymbol{j}_M}^{rr'} \equiv \boldsymbol{v}_e^\perp\, \delta^{rr'} \;, \quad \boldsymbol{J}_{\boldsymbol{j}_E}^{rr'} \equiv -i\, \boldsymbol{v}_e^\perp \times \boldsymbol{\sigma}^{rr'}$$

- example: scalar coupling

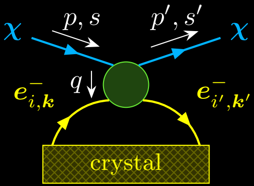
$$\mathcal{M} \simeq \overbrace{\underbrace{-i\,\frac{g_\chi\, g_e}{q^2+M^2}\,4\,m_\chi m_e}_{c_1}\,\phantom{\delta^{ss'}}}^{F_0}\!\!\!\!\!\!\underbrace{\delta^{ss'}\;\overbrace{\delta^{rr'}}^{J_0}}_{\mathcal{O}_1}$$

Int. rate for bounded electrons & generalized susceptibilities

Catena et al., 1912.08204, Catena & Spaldin, 2402.06817

see Riccardo's talk

- electronic states ≠ momentum eigenstates



$$\mathcal{M} = \sum_a \overbrace{F_a^{ss'}(\boldsymbol{q},\boldsymbol{v}_\chi)}^{\text{DM model}} \times \overbrace{J_a^{rr'}(\boldsymbol{v}_e^\perp)}^{\text{electronic part}}$$

$$\Big\downarrow \left|\begin{matrix}\text{integral over}\\ e^-\ \text{wave fun.}\end{matrix}\right|^2$$

$$|\widetilde{\mathcal{M}}_{\boldsymbol{ik}\to i'\boldsymbol{k}'}|^2 \simeq \sum_{ab} \underbrace{\mathcal{F}_{ab}(\boldsymbol{q},\boldsymbol{v}_\chi)}_{\text{DM model}} \times [\text{material response}]$$

- interaction rate per dark particle

$$\Gamma(\boldsymbol{v}_\chi) \sim \int \frac{d^3q}{(2\pi)^3} \underbrace{\overbrace{\sum_{ii'} \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}}^{\text{sum over electronic states}} |\widetilde{\mathcal{M}}_{\boldsymbol{ik}\to i'\boldsymbol{k}'}|^2\, \delta(\text{cons.})}$$

- total interaction rate

$$\Gamma = \frac{-i}{16 m_e^2 m_\chi^2} \int \frac{d^3q}{(2\pi)^3}\, d^3v\, \boldsymbol{\rho}(\boldsymbol{v}_\chi) \overbrace{\sum_{ab} \mathcal{F}_{ab}(\boldsymbol{q},\boldsymbol{v}_\chi)(\chi_{a^\dagger b} - \chi^*_{b^\dagger a})(\boldsymbol{q},\omega_{v,q})}^{\text{generalized susceptibilities}}$$

This work: theoretical upper bounds on the interaction rate

for the simplest case see: Lasenby & Prabhu, 2110.01587

- **Kramers-Kronig** relation: χ is analytic and causal, so

$$\int_0^\infty \frac{d\omega}{\omega} \Im \Delta\chi_i(\omega, q) = \frac{\pi}{2} \Delta\chi_i(0, q) \le \pi\, U(q)^{-1} \qquad \left(U(q) \equiv \frac{4\pi\alpha}{q^2} \right)$$

$$\varepsilon_i > 0$$

- conclusion: **upper bound**

material-dependent exact value →

material-independent upper bound →

$$\boxed{\begin{aligned} \Gamma &= \frac{1}{32\pi^2} \frac{1}{m_e^2 m_\chi^2} \sum_i \int dq\, q^2 \int d\omega\, f_i(\omega, q)\, \Im \Delta\chi_i(\omega, q) \\ \Gamma_{\text{opt}} &= \frac{1}{32\pi} \frac{1}{m_e^2 m_\chi^2} \sum_i \int dq\, q^2 \max_\omega \left[\omega\, f_i(\omega, q) \right] U(q)^{-1} \end{aligned}}$$

$$f_i(\omega, q) \equiv \boldsymbol{\rho}^{(0)}(\omega, q)\, \mathcal{F}_i^{(0)}(\omega, q) + \boldsymbol{\rho}^{(2)}(\omega, q)\, \mathcal{F}_i^{(2)}(\omega, q)$$

- truncated thermal local distribution of DM ⇒ $\boldsymbol{\rho}^{(0)}(\omega, q)$, $\boldsymbol{\rho}^{(2)}(\omega, q)$
- effective models of DM-e^- interactions ⇒ $\mathcal{F}_i^{(0)}(\omega, q)$, $\mathcal{F}_i^{(2)}(\omega, q)$
- material science ⇒ $\Im \Delta\chi_i(\omega, q)$

Results: methodology

material-**independent**

↙

$$f(m_\chi) \equiv \frac{\textbf{theor. upper bound}}{\textbf{true interaction rate}}$$

↖

material-**dependent**

- effective models of DM-e^- interactions
  - anapole
  - magnetic dipole
  - electric dipole

$$f(m_\chi) = 1$$

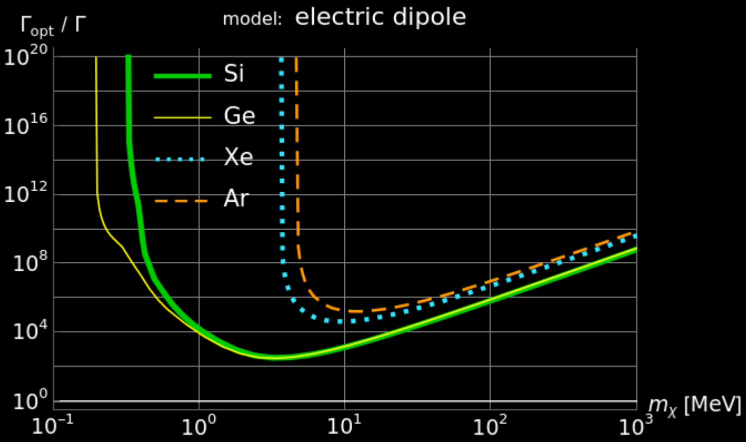
- which material is closest to **saturating the bound** for a given model?
  - Si, Ge, Xe, Ar
  - numerical data based on Catena et al., 2105.02233, 2210.07305

Results

electric dipole model

$$\mathcal{L}_{\text{int}} = \frac{g}{\Lambda}\, i\, \bar{\chi}\sigma^{\mu\nu}\gamma_5\chi\, F_{\mu\nu} \qquad \Rightarrow \qquad \mathcal{F}_{n_0 n_0} \propto \frac{m_\chi^2}{q^2}$$

(others vanish)



Results

magnetic dipole model

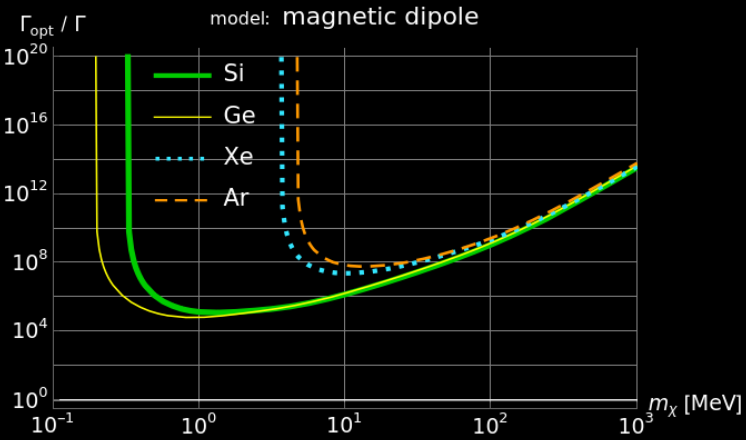
$$\mathcal{L}_{\text{int}} = \frac{g}{\Lambda}\,\bar{\chi}\sigma^{\mu\nu}\chi\,F_{\mu\nu} \qquad \Rightarrow$$

$$\mathcal{F}_{n_0 n_0} \propto 1 - 4\,v_q^2\,\frac{m_\chi^2}{q^2} + 4\,v^2\,\frac{m_\chi^2}{q^2}$$

$$\text{Tr}\mathcal{F}_{j_5 j_5} \propto 2\,\frac{m_\chi^2}{m_e^2}\,, \quad \text{Tr}\mathcal{F}_{j_M j_M} \propto 8\,\frac{m_\chi^2}{q^2}$$

$$\mathcal{F}_{j_M^k j_M^l}\,\frac{a_k a_l}{a^2} \propto 4\,\frac{m_\chi^2}{q^2}\,, \quad \boldsymbol{a}\cdot\boldsymbol{q} \equiv 0$$

(others vanish)



Results

anapole model

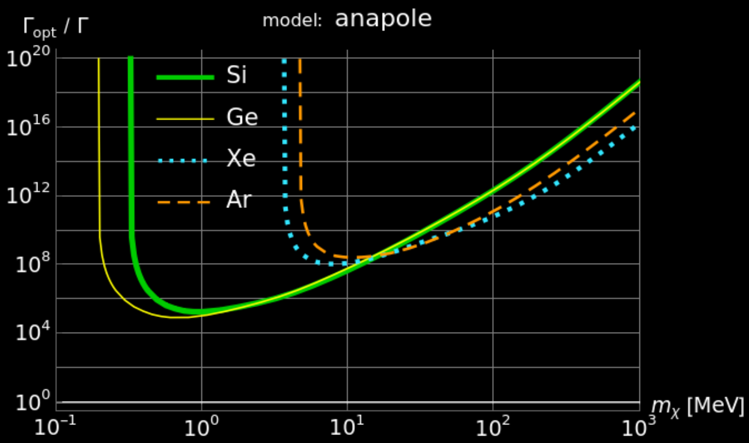
$$\mathcal{L}_{\text{int}} = \frac{g}{2\,\Lambda^2}\,\bar{\chi}\gamma^\mu\gamma_5\chi\,\partial^\nu F_{\mu\nu} \qquad \Rightarrow$$

$$\mathcal{F}_{n_0 n_0} \propto v^2 - \frac{q\,v_q}{2\,m_\chi}$$

$$\mathrm{Tr}\mathcal{F}_{j_5 j_5} \propto \frac{q^2}{2\,m_e^2}\,, \qquad \mathrm{Tr}\mathcal{F}_{j_M j_M} \propto 3$$

$$\mathcal{F}_{j_M^k n_0}\frac{q_k}{q} = \mathcal{F}_{n_0 j_M^k}\frac{q_k}{q} \propto \frac{q}{2m_\chi} - v_q$$

(others vanish)



Summary

- **effective approach** to non-relativistic DM-e^- interactions
    - small set of operators in the leading order
- **linear response theory**

$$[\text{interaction rate}] = \int [\text{DM model}] \times [\text{material response of the detector}]$$

- material response → generalized susceptibilities $\chi_{a^\dagger b}(\omega, q)$
- **Kramers-Kronig** relations

$$\chi: \text{ causal, analytic} \qquad \Rightarrow \qquad \int_0^\infty \frac{d\omega}{\omega}\, \mathfrak{I}\chi(\omega, q) = \frac{\pi}{2}\,\chi(0, q)$$

- material-independent **theoretical upper bound** on the interaction rate
- results:
    - solids typically better than nobles (excl. $m_\chi \gtrsim 20$ MeV in the anapole model)
    - all of them **far from the theoretical bound**
- outlook:
    - characterize an optimal response function? ← work in progress...
    - new materials?

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thank you!

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