Linear response theory for light dark matter-electron scattering in materials

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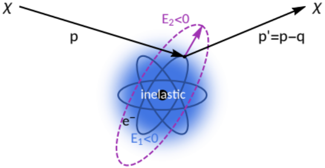
Overview

- We are interested in modelling DM-induced electronic transitions in materials,



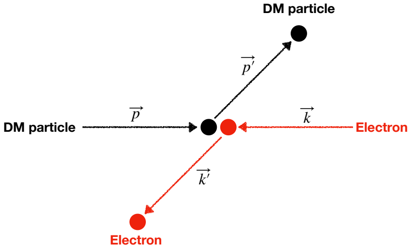
- To this end, we developed a framework that combines a non-relativistic **effective theory** for DM-electron interactions with **linear response theory**

  R. Catena and N. A. Spaldin, "Linear response theory for light dark matter-electron scattering in materials," Phys. Rev. Res. **6** (2024) no.3, 033230

- Our framework applies to general DM-material couplings, and can capture in-medium effects

Effective theory for dark matter-electron interactions

- Consider the scattering of a DM particle of mass $m_\chi \lesssim 1$ GeV by a free electron,



- In the non-relativistic limit, the process is characterised by a double separation of scales:

$$|\boldsymbol{q}|/m_e \ll 1\,, \qquad \boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p}'$$
$$|\boldsymbol{v}| \ll 1\,, \qquad \boldsymbol{v} = \boldsymbol{p}/m_\chi$$

- Its amplitude $\mathscr{M}_{\chi e}$ is invariant under Galilean boosts, translations and rotations

Effective theory for dark matter-electron interactions

- We construct an effective theory where DM and electrons are the relevant degrees of freedom
- The underlying symmetries are Galilean boosts, translations and rotations
- We then write $\mathscr{M}_{\chi e}$ as a power series in $|\boldsymbol{q}|/m_e \ll 1$ and $|\boldsymbol{v}| \ll 1$ where each term:

- only depends on the momenta and spins of the relevant degrees of freedom
- is invariant under Galilean boosts, translations and rotations

Effective theory for dark matter-electron interactions

- What is the predicted form for $\mathscr{M}_{\chi e}$ in our non-relativistic effective theory? We find:

$$\mathscr{M}_{\chi e}(\boldsymbol{q}, \boldsymbol{v}^{\perp}) = \sum_i c_i \langle \mathcal{O}_i \rangle$$

Matrix elements

Out of the four momenta $\vec{p}$, $\vec{p}\,'$, $\vec{k}$ and $\vec{k}\,'$ only two are independent: $\vec{q}$ and $\vec{v}^{\perp}$

Sum over operator type

Unknown coupling constants

Rotationally invariant operators in the DM-electron spin space

Examples of $\mathcal{O}_i$ operators:

$\mathcal{O}_1 = \mathbb{1}_\chi \mathbb{1}_e$, $\mathcal{O}_4 = \mathbf{S}_\chi \cdot \mathbf{S}_e$, $\mathcal{O}_7 = \mathbf{S}_\chi \cdot \boldsymbol{v}^{\perp}$, $\mathcal{O}_{11} = i\mathbf{S}_\chi \cdot \boldsymbol{q}/m_e$, ...

- $\mathscr{M}_{\chi e}$ can be written as a matrix element between free electron states of a potential $V_{\rm eff}^{ss'}$:

$$\langle \boldsymbol{k}', r' | V_{\rm eff}^{ss'} | \boldsymbol{k}, r\rangle = -\frac{\mathscr{M}_{\chi e}}{4 m_e m_\chi V^2} (2\pi)^3 \delta^{(3)}(\boldsymbol{k}' + \boldsymbol{p}' - \boldsymbol{k} - \boldsymbol{p})$$

- For example, for

$$\mathscr{M}_{\chi e} = c_1 \langle \mathbb{1}_\chi \mathbb{1}_e \rangle$$

we find

$$V_{\rm eff}^{ss'} = -\frac{1}{4 m_e m_\chi V} F_0^{ss'} e^{i \boldsymbol{q}\cdot \boldsymbol{r}_e}$$

where $\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p}'$, $F_0^{ss'} = c_1 \xi^{s'} \xi^{s}$, $\boldsymbol{r}_e$ is the electron position vector, and $e^{i\boldsymbol{q}\cdot\boldsymbol{r}_e} = n_0(-\boldsymbol{q})$

Dark matter as an external perturbation

- In general,

$$V_{\text{eff}}^{ss'} = -\frac{1}{4m_e m_\chi V} \sum_\alpha F_\alpha^{ss'}(\boldsymbol{q}) j_\alpha(-\boldsymbol{q}) \,,$$

  where α runs over the set of densities and currents DM can couple to in a material

- We find,

$$j_\alpha = (n_0, n_A, \boldsymbol{j}_5, \boldsymbol{j}_M, \boldsymbol{j}_E)$$

- where

  $n_0 \rightarrow$ electron density

  $n_A \rightarrow$ spin-momentum density $(\boldsymbol{\sigma}_e \cdot \nabla_{\boldsymbol{r}_e})$

  $\boldsymbol{j}_5 \rightarrow$ spin current $(\boldsymbol{\sigma}_e)$

  $\boldsymbol{j}_M \rightarrow$ paramagnetic current $(\nabla_{\boldsymbol{r}_e})$

  $\boldsymbol{j}_E \rightarrow$ **Rashba spin-orbit current** $(\boldsymbol{\sigma}_e \times \nabla_{\boldsymbol{r}_e})$

Dark matter as an external perturbation

- In the **interaction picture**,

$$V_{\text{eff}}^{ss'}(t) = -\frac{1}{4m_e m_\chi V} \sum_\alpha F_\alpha^{ss'}(\boldsymbol{q}) j_\alpha(-\boldsymbol{q})\, e^{i\Delta E_\chi t}\,,$$

where $\Delta E_\chi = q^2/(2m_\chi) - \boldsymbol{q}\cdot\boldsymbol{v}$.

- Equivalently,

$$V_{\text{eff}}^{ss'}(t) = -\sum_\alpha \int \mathrm{d}\boldsymbol{r}\, j_\alpha(\boldsymbol{r})\, S_\alpha^{ss'}(\boldsymbol{r}, t)\,,$$

where

$$S_\alpha^{ss'}(\boldsymbol{r}, t) = \frac{1}{4m_e m_\chi V}\, F_\beta^{ss'}(\boldsymbol{q})\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}}\, e^{i\Delta E_\chi t}\,.$$

Linear response to a dark matter perturbation

- DM-induced fluctuation in j_α

$$\langle \Delta j_\alpha(\boldsymbol{r}, t)\rangle = \sum_\beta \int_{-\infty}^{t} \mathrm{d}t' \int \mathrm{d}\boldsymbol{r}' \, \chi_{j_\alpha j_\beta}(\boldsymbol{r}-\boldsymbol{r}', t-t') \, S_\beta^{ss'}(\boldsymbol{r}', t') \, ,$$

- **Generalised susceptibility**:

$$\chi_{j_\alpha j_\beta}(\boldsymbol{r}-\boldsymbol{r}', t-t') = i\theta(t-t')\Big\langle \left[j_\alpha(\boldsymbol{r}, t), j_\beta(\boldsymbol{r}', t')\right] \Big\rangle \, .$$

- We evaluate $\chi_{j_\alpha j_\beta}(\boldsymbol{q}, \omega)$ by applying the equation of motion (EOM) method, namely:
- Acting with $\partial/\partial t$ on the above Eq.
- Using Heisenberg equations for j_α and j_β

Linear response to a dark matter perturbation

- An approximate solution to the equation of motion for $\chi_{j_\alpha j_\beta}(\boldsymbol{q},\omega)$ is given by,

$$\chi_{j_\alpha j_\beta}(\boldsymbol{q},\omega) = \Sigma_{j_\alpha j_\beta}(\boldsymbol{q},\omega) - \frac{\Sigma_{j_\alpha n_0}(\boldsymbol{q},\omega)\,U(\boldsymbol{q})\left[1-G(\boldsymbol{q})\right]\Sigma_{n_0 j_\beta}(\boldsymbol{q},\omega)}{1+U(\boldsymbol{q})\left[1-G(\boldsymbol{q})\right]\Sigma_{n_0 n_0}(\boldsymbol{q},\omega)},$$

- where

$\Sigma_{j_\alpha j_\beta}(\boldsymbol{q},\omega) \rightarrow$ Lindhard response function: $\Sigma_{j_\alpha j_\beta} \propto \sum_{ss'} \langle s'|j_\alpha|s\rangle\langle s|j_\beta|s'\rangle$

$U(q) \rightarrow$ Fourier transform of Coulomb potential

$G(q) \rightarrow$ Local-field factor

Linear response to a dark matter perturbation

- Geometric series expansion,

$$\chi_{j_\alpha j_\beta}(\boldsymbol{q},\omega) = \Sigma_{j_\alpha j_\beta}(\boldsymbol{q},\omega) + \sum_{\ell=0}^{\infty} A(\boldsymbol{q},\omega) R(\boldsymbol{q},\omega)^{\ell}$$

where

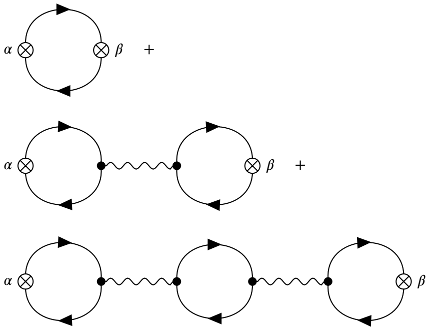
$$A(\boldsymbol{q},\omega) = \Sigma_{j_\alpha n_0}(\boldsymbol{q},\omega)\Sigma_{n_0 j_\beta}(\boldsymbol{q},\omega) \times U(\boldsymbol{q})\left[1 - G(\boldsymbol{q})\right]$$

and

$$R(\boldsymbol{q},\omega) = U(\boldsymbol{q})(1 - G(\boldsymbol{q}))\Sigma_{n_0 n_0}(\boldsymbol{q},\omega)$$

- Diagrammatic representation,

$\chi_{j_\alpha j_\beta}(\boldsymbol{q},\omega) \; =$



$+ \ldots$

Dark matter-induced electronic transition rate

- Rate formula

$$\Gamma = \frac{n_\chi V}{16 m_e^2 m_\chi^2} \sum_{\alpha\beta} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} \int \mathrm{d}\boldsymbol{v}\, f(\boldsymbol{v})\, \mathscr{F}_{\alpha\beta}(\boldsymbol{q}, \boldsymbol{v})\, \Delta\chi_{\alpha\beta}(\boldsymbol{q}, -\Delta E_\chi)\,,$$

- **DM form factor**:

$$\mathscr{F}_{\alpha\beta}(\boldsymbol{q}, \boldsymbol{v}) = \frac{1}{2} \sum_{ss'} F_\alpha^{ss'}(\boldsymbol{q}, \boldsymbol{v})\, F_\beta^{ss'*}(\boldsymbol{q}, \boldsymbol{v})$$

- **Material response function**:

$$\Delta\chi_{\alpha\beta}(\boldsymbol{q}, \omega) = -i[\chi_{j_\alpha^\dagger j_\beta} - \chi^*_{j_\beta^\dagger j_\alpha}](\boldsymbol{q}, \omega)$$

Dark matter-induced electronic transition rate

- $\chi_{j^\dagger_\alpha j_\beta}$ is causal and real in the time domain, and vanishes for $\omega \to \infty$ in the frequency domain

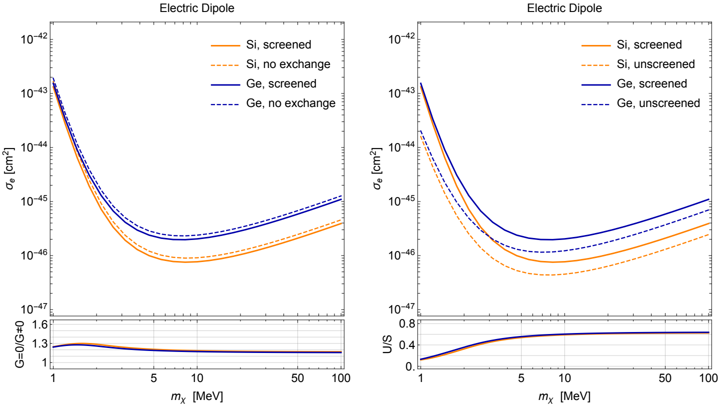
- Consequently, it obeys the **Kramers-Kronig relations**, which imply,

$$\int_0^{+\infty} \frac{\mathrm{d}\omega}{\omega} \, \mathrm{Im}\chi_{j^\dagger_\alpha j_\beta}(\boldsymbol{q}, \omega) = \frac{\pi}{2} \, \chi_{j^\dagger_\alpha j_\beta}(\boldsymbol{q}, 0) \, .$$

- We used the above to obtain a general **theoretical upper bound** on $\Gamma \leq \Gamma_{\mathrm{opt}}$

  R. Catena and M. Iglicki, "A general upper bound on the light dark matter scattering rate in materials," arXiv:2501.18261

First applications



$$\chi_{j_\alpha j_\beta}(\boldsymbol{q},\omega) = \Sigma_{j_\alpha j_\beta}(\boldsymbol{q},\omega) - \frac{\Sigma_{j_\alpha n_0}(\boldsymbol{q},\omega)\, U(\boldsymbol{q})\, [1 - G(\boldsymbol{q})]\, \Sigma_{n_0 j_\beta}(\boldsymbol{q},\omega)}{1 + U(\boldsymbol{q})\, [1 - G(\boldsymbol{q})]\, \Sigma_{n_0 n_0}(\boldsymbol{q},\omega)} \,,$$

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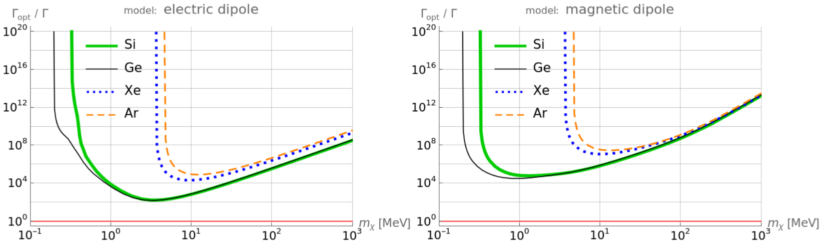
First applications



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Summary and Outlook

- We combined a non-relativistic **effective theory** for DM-electron interactions with **linear response theory** to describe DM-electron scattering in materials

- Our formalism:
  - applies to general DM - material couplings
  - fully accounts for in-medium effects
  - explicitly factorises particle from material physics inputs
  - implies an upper bound on the DM-induced electronic transition rate

- First results presented focusing on spin-unpolarised materials:
  - Screening important only in models where DM couples to the electron density

- Extension to spin-polarised materials is in progress