Refining Two-Loop Corrections to Trilinear Higgs Couplings

Johannes Braathen, Felix Egle and **Alain Verduras Schaeidt**

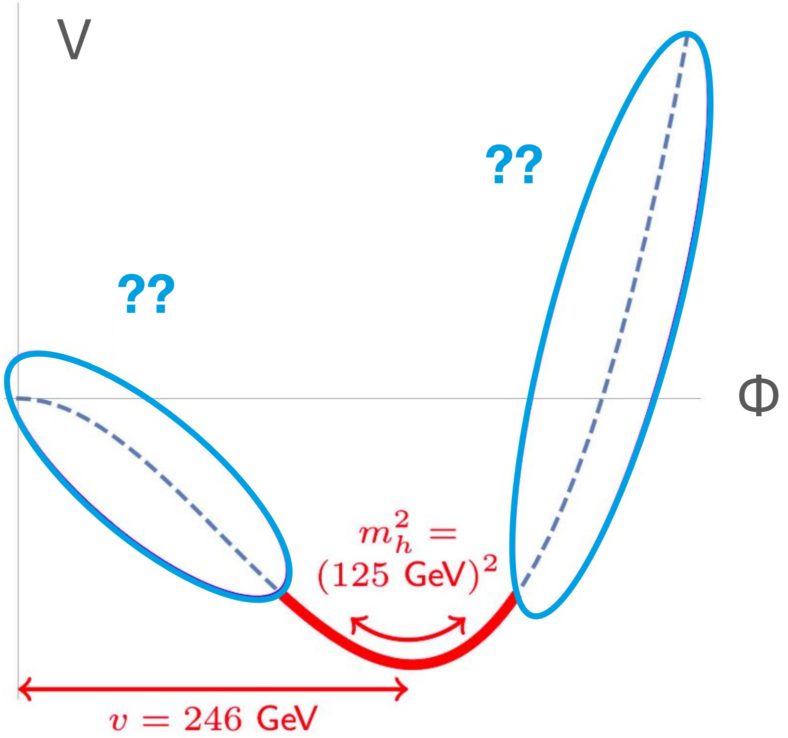
Planck 2025

Padova, 27-05-25

Motivation

- Why trilinear Higgs couplings?
  - Not measured: good portal for new physics
  - Large impact in hh production
  - It has an impact in the history of the universe: BAU and SFOEWPT
- Why loop corrections?
  - BSM effects enter at 1 loop level they can be larger than tree level
- Why alignment limit?
  - Fulfil experimental constraints
  - Maintain it at loop level

Following: [G. Degrassi, P. Slavich, '23]

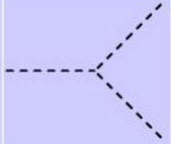
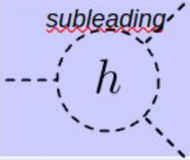
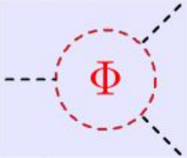
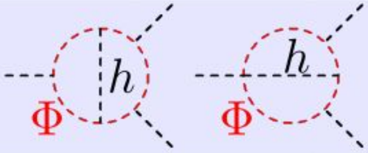
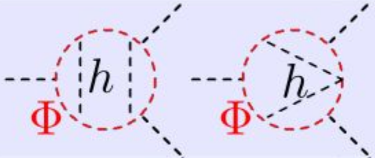
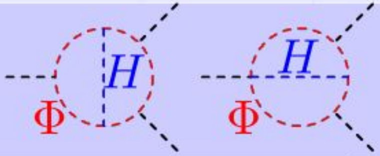
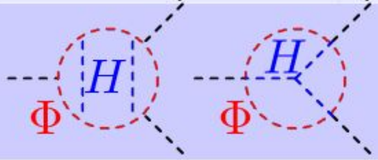
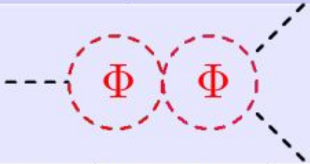
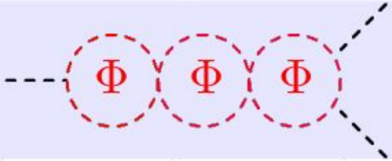


Taken from J. Braathen

Why two loops?

One loop corrections can be much larger than tree level due to BSM couplings

One loop corrections are LO in BSM and two loop are NLO

Coupling/Order	0L	1L	2L	3L
g_{hhhh}		subleading 	subleading	subleading
$g_{(h)h\Phi\Phi}$ $\left[g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}\right]$	-			
$g_{(h)H\Phi\Phi'}$ [$g_{(h)G\Phi\Phi'}$ case similar]	-	-		
$g_{\Phi\Phi\Phi'\Phi'}$ [2 BSM scalars of species Φ, 2 of species Φ']	-	-		

What means refining?

Previous set-up: [J. Braathen, S, Kanemura, '19] Loop-induced deviations from alignment were neglected (as subleading)

MODEL → Alignment limit $\Lambda_6=0$ → Renormalization → Loop corrections → **No Λ_6 counter term**

Our set-up:

MODEL → Renormalization → Loop corrections → Alignment limit $\Lambda_6=0$ → **Yes Λ_6 counter term**

The Λ_6 counter term ensures the alignment limit after loop corrections

THDM

General model

Doublets: $$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + \sigma_1 i) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + \sigma_2 i) \end{pmatrix}$$

$$\boxed{\begin{aligned} V(\Phi_1,\Phi_2) = {} & \tilde{M}_{11}^2|\Phi_1|^2 + \tilde{M}_{22}^2|\Phi_2|^2 - \tilde{M}_{12}^2(\Phi_2^\dagger\Phi_1 + \text{h.c.}) + \frac{\Lambda_1}{2}|\Phi_1|^4 + \frac{\Lambda_2}{2}|\Phi_2|^4 + \\ & + \tilde{\Lambda}_3|\Phi_1|^2|\Phi_2|^2 + \tilde{\Lambda}_4|\Phi_2^\dagger\Phi_1|^2 + \left[\frac{\tilde{\Lambda}_5}{2}(\Phi_2^\dagger\Phi_1)^2 + (\tilde{\Lambda}_6|\Phi_1|^2 + \tilde{\Lambda}_7|\Phi_2|^2)\Phi_1^\dagger\Phi_2 + \text{h.c}\right] \end{aligned}}$$

General basis: $$v_1, v_2, \tilde{M}_{11}^2, \tilde{M}_{22}^2, \tilde{M}_{12}^2, \tilde{\Lambda}_1, \tilde{\Lambda}_2, \tilde{\Lambda}_3, \tilde{\Lambda}_4, \tilde{\Lambda}_5, \tilde{\Lambda}_6, \tilde{\Lambda}_7$$

THDM

softly broken $\mathbb{Z}_2$ basis

Doublets: $\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + \sigma_1 i) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + \sigma_2 i) \end{pmatrix}$

$$\boxed{\begin{aligned} V(\Phi_1,\Phi_2) = {} & \tilde{M}_{11}^2|\Phi_1|^2 + \tilde{M}_{22}^2|\Phi_2|^2 - \tilde{M}_{12}^2(\Phi_2^\dagger\Phi_1 + \text{h.c.}) + \frac{\Lambda_1}{2}|\Phi_1|^4 + \frac{\Lambda_2}{2}|\Phi_2|^4 + \\ & + \tilde{\Lambda}_3|\Phi_1|^2|\Phi_2|^2 + \tilde{\Lambda}_4|\Phi_2^\dagger\Phi_1|^2 + \left[\frac{\tilde{\Lambda}_5}{2}(\Phi_2^\dagger\Phi_1)^2 + (\cancel{\tilde{\Lambda}_6|\Phi_1|^2} + \cancel{\tilde{\Lambda}_7|\Phi_2|^2})\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \end{aligned}}$$

General basis: $v_1, v_2, \tilde{M}_{11}^2, \tilde{M}_{22}^2, \tilde{M}_{12}^2, \tilde{\Lambda}_1, \tilde{\Lambda}_2, \tilde{\Lambda}_3, \tilde{\Lambda}_4, \tilde{\Lambda}_5, \cancel{\tilde{\Lambda}_6}, \cancel{\tilde{\Lambda}_7}$

$\mathbb{Z}_2$ mass basis: $v, \alpha, \beta, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, M^2$ Tad. eqs. solved for: $\tilde{M}_{11}^2, \tilde{M}_{22}^2$

THDM

Higgs basis

Doublets:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + \sigma_1 i) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(\cancel{v_2} + \phi_2 + \sigma_2 i) \end{pmatrix}$$

$$\boxed{\begin{aligned} V(\Phi_1,\Phi_2) = {} & \tilde{M}_{11}^2|\Phi_1|^2 + \tilde{M}_{22}^2|\Phi_2|^2 - \tilde{M}_{12}^2(\Phi_2^\dagger\Phi_1 + \text{h.c.}) + \frac{\Lambda_1}{2}|\Phi_1|^4 + \frac{\Lambda_2}{2}|\Phi_2|^4 + \\ & + \tilde{\Lambda}_3|\Phi_1|^2|\Phi_2|^2 + \tilde{\Lambda}_4|\Phi_2^\dagger\Phi_1|^2 + \left[\frac{\tilde{\Lambda}_5}{2}(\Phi_2^\dagger\Phi_1)^2 + (\tilde{\Lambda}_6|\Phi_1|^2 + \tilde{\Lambda}_7|\Phi_2|^2)\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \end{aligned}}$$

General basis: $v_1, \cancel{v_2}, \tilde{M}_{11}^2, \tilde{M}_{22}^2, \tilde{M}_{12}^2, \tilde{\Lambda}_1, \cancel{\tilde{\Lambda}_2}, \tilde{\Lambda}_3, \tilde{\Lambda}_4, \tilde{\Lambda}_5, \tilde{\Lambda}_6, \tilde{\Lambda}_7$

FC~~N~~C

$\tilde{\Lambda}_2 = \tilde{\Lambda}_2(\tilde{\Lambda}_7)$

Higgs basis: $v, \cancel{v_{BSM}}, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, M_{22}^2, \Lambda_6, \Lambda_7$

Tad. eq. for: $\tilde{M}_{11}^2, \tilde{M}_{12}^2$

Effective potential approach

$$\boxed{V_{eff} \equiv V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)}}$$

Loop factor:

$$\kappa = \frac{1}{16\pi^2}$$

Leading contributions from top quark and BSM scalars:

-One loop:

$$V^{(1)}(\phi) = -3m_t^2(\phi)\left(\overline{\log} m_t^2(\phi) - \frac{3}{2}\right) + \sum_{\phi_i} \frac{n_{\phi_i} m_{\phi_i}^4(\phi)}{4}\left(\overline{\log} m_{\phi_i}^2(\phi) - \frac{3}{2}\right)$$

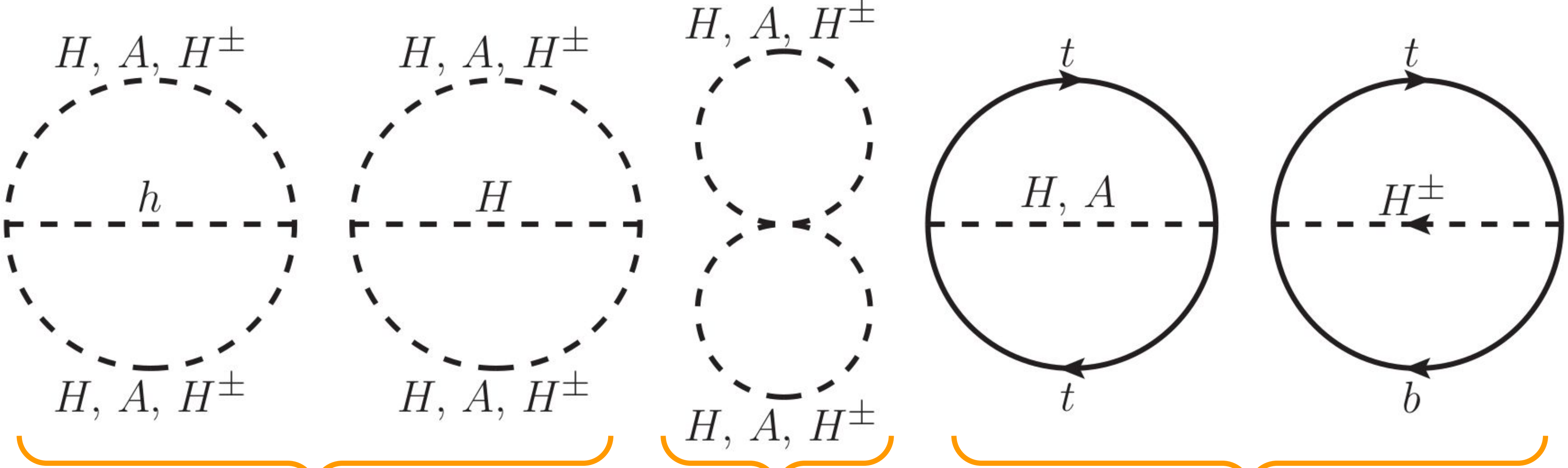
$$\phi = h, H$$

-Two loops:

$$V^{(2)}(\phi) = V_{SSS}^{(2)}(\phi) + V_{SS}^{(2)}(\phi) + V_{FFS}^{(2)}(\phi)$$

Effective potential approach

Figure from: [J. Braathen, S, Kanemura, '19]

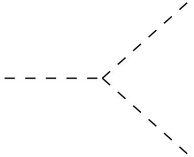


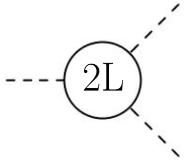
$$V^{(2)}_{SSS}(\phi) \qquad V^{(2)}_{SS}(\phi) \qquad V^{(2)}_{FFS}(\phi)$$

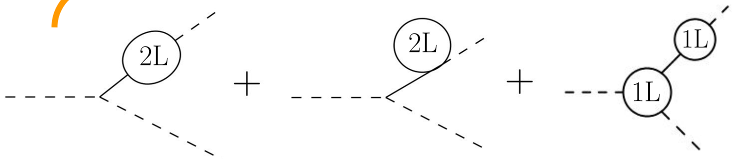
$$\phi = h, H$$

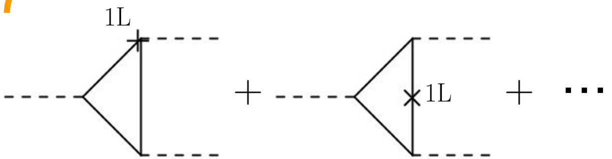
Diagrammatic approach

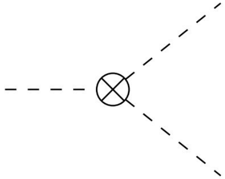
$$\hat{\lambda}_{ijk} = \lambda^{(0)}_{ijk} + \delta^{\text{gen}}\lambda_{ijk} + \delta^{\text{WFR}}\lambda_{ijk} + \delta^{\text{sub}}\lambda_{ijk} + \delta^{\text{CT}}\lambda_{ijk}$$











Definition of the countertems

Parameters for which we need one-loop counterterms:

$$v, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, m_t^2, M_{22}^2, \Lambda_6, t_h, t_H$$

Parameters for which we need two-loop counterterms:

$$m_h^2$$

Definition of the countertems

Parameters for which we need one-loop counterterms:

$$v, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, m_t^2, M_{22}^2, \Lambda_6, t_h, t_H$$

Parameters for which we need two-loop counterterms:

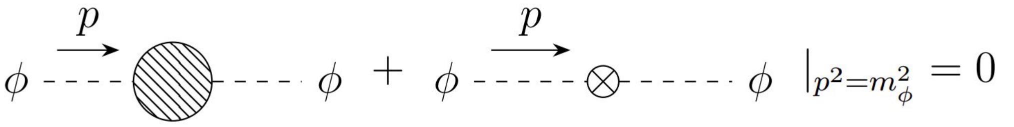
$$m_h^2$$

Field and mass renormalisation

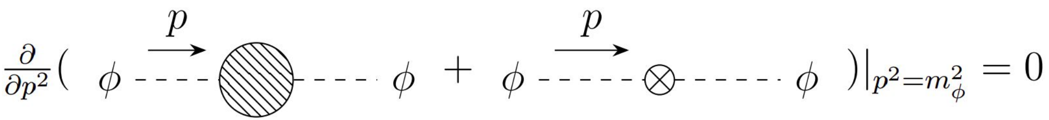
In the renormalisation procedure of the 2-point functions we fix two renormalization conditions following the OS scheme. To determine the Λ_6 counterterm we will use a modified version of the third OS condition.

OS conditions:

$$\mathrm{Re}[\hat{\Sigma}_{hh}(m_h^2)] = \mathrm{Re}[\hat{\Sigma}_{HH}(m_H^2)] = 0$$



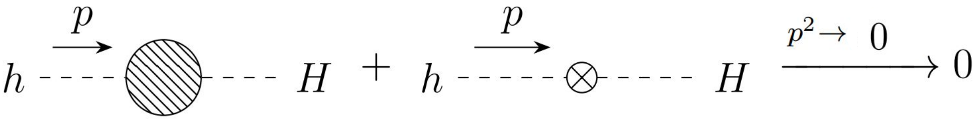
$$\mathrm{Re}\left[\frac{\partial \hat{\Sigma}_{hh}(p^2)}{\partial p^2}\bigg|_{p^2=m_h^2}\right] = \mathrm{Re}\left[\frac{\partial \hat{\Sigma}_{HH}(p^2)}{\partial p^2}\bigg|_{p^2=m_H^2}\right]$$



$\phi = h, H$

Modified OS condition:

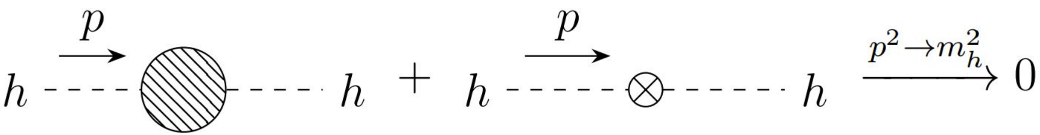
$$\mathrm{Re}[\hat{\Sigma}_{hH}(\ 0\ )] = \mathrm{Re}[\hat{\Sigma}_{Hh}(\ 0\ )] = 0$$



Mass counterterms

Taking the **first condition**, expanding it and imposing the condition at every loop order we can obtain the one and two loop **mass counterterms:**

Condition:



Counterterms:

$$\left\{ \begin{aligned} (\delta m_h^2)^{(1)} &= \Sigma_{hh}^{(1)}(m_h^2) - \delta T_{hh}^{(1)}\,, \\ (\delta m_h^2)^{(2)} &= \Sigma_{hh}^{(2)}(m_h^2) - \delta T_{hh}^{(2)} - (\delta D_{hh}^2)^{(1)} \delta Z_{hh}^{(1)} \end{aligned} \right\}$$

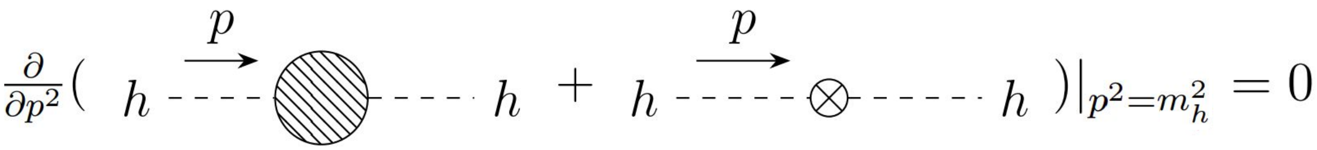
We can follow the same procedure for all the particles which masses need a counterterm:

$$h, H, A, H^{\pm}, t$$

External leg corrections

Taking the **second condition**, expanding it and imposing the condition at every loop order we can obtain the one and two loop **diagonal Z factor counterterms:**

$$\mathrm{Re}\left[\frac{\partial\hat{\Sigma}_{hh}(p^2)}{\partial p^2}\bigg|_{p^2=m_h^2}\right] = 0$$



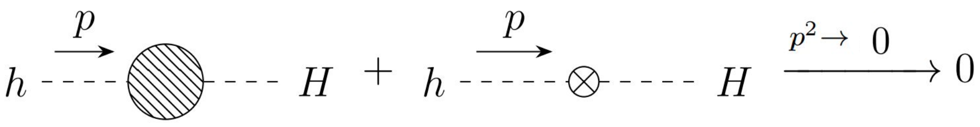
We can follow the same procedure for all the particles involved in the computation:

$$h, H, A, H^{\pm}$$

Λ_6 counterterm

Taking the **third condition**, expanding it and imposing the condition at every loop order we can obtain the one and two loop **Λ_6 counterterms**:

$$\mathrm{Re}[\hat{\Sigma}_{hH}(\ 0\ )] = \mathrm{Re}[\hat{\Sigma}_{Hh}(\ 0\ )] = 0$$



This condition is equivalent to impose that the mass matrix of the Higgs even sector is diagonal at tree level and to every loop order for 0 external momenta:

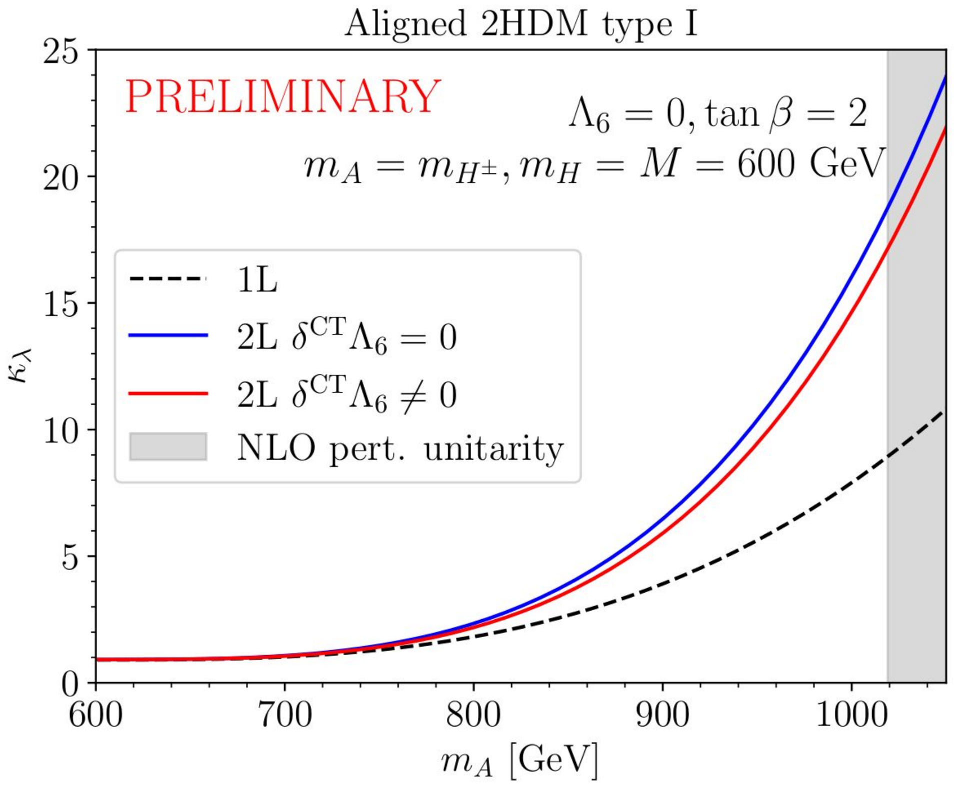
$$\hat{\mathcal{M}}^2 = \hat{M}^2 - \hat{\Sigma}_\phi(0) = \begin{pmatrix} M^2_{SM} & \Lambda_6 v^2 \\ \Lambda_6 v^2 & M^2_{BSM} \end{pmatrix} - \begin{pmatrix} \hat{\Sigma}_{hh}(0) & \hat{\Sigma}_{hH}(0) \\ \hat{\Sigma}_{Hh}(0) & \hat{\Sigma}_{HH}(0) \end{pmatrix}$$

Results for λ_{hhh}

$$\boxed{\kappa_\lambda = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM},(0)}}}$$

In the λ_{hhh} there is no effect from $\delta^{CT}\Lambda_6$ at the one loop level

At the two loop level we find a deviation between both approaches up to an 8%



Summary

We use the **Higgs Basis** of the THDM to have a better **control of the alignment** limit.

We compute the **two-loop corrections** following **two different approaches.**

We impose the **alignment limit** not only at tree level but also at **every loop level** and we take into account the renormalisation of Λ_6 at one- and two-loop orders.

For the $\boldsymbol{\lambda_{hhh}}$ **coupling** at the two loop level we find a **deviation** after the refining up to an **8%**

Thank you for your attention

Checks of the results

Compare **our counterterms** with the **RGEs of SARAH**

Check that the diagrammatic result is **UV finite**

Cross check the results between the **diagrammatic and the effective potential approach**

Cross check results with the **literature**

Vev counterterm

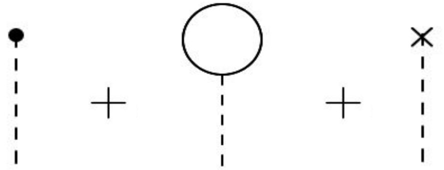
We express the EW vev in terms of the electroweak parameters and we follow the same procedure that for the SM:

Definition of the EW vev: $$v^2 = \frac{m_W^2}{\pi \alpha_{EM}} \left(1 - \frac{m_W^2}{m_Z^2}\right)$$

$$\boxed{\frac{\delta v}{v} = \frac{1}{2} \left(\frac{s_W^2 - c_W^2}{s_W^2} \frac{\mathrm{Re}[\Sigma_{WW}(m_W^2)]}{m_W^2} + \frac{c_W^2}{s_W^2} \frac{\mathrm{Re}[\Sigma_{ZZ}(m_Z^2)]}{m_Z^2} - \frac{d}{dp^2} \Sigma_{\gamma\gamma}(p^2) \Big|_{p^2=0} - \frac{2 s_W}{c_W} \frac{\Sigma_{\gamma Z}(0)}{m_Z^2} \right)}$$

Tadpole counterterms

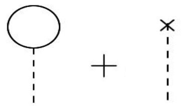
$$\boxed{\hat{t}_i = t_i^{(0)} + \delta t_i + \delta t_i^{\mathrm{CT}} = 0}$$



OS/standard scheme

$$t_i^{(0)} = 0$$

$$\left\{ \delta t_i + \delta t_i^{\mathrm{CT}} = 0 \right\}$$



For each loop order

Now we do not have to include tadpole diagrams in the calculation

Conditions to avoid FCNCs

The $\mathbb{Z}_2$ symmetry is imposed to avoid the flavor changing neutral currents (FCNC) but in the Higgs basis we do not impose this symmetry.

Comparing the Higgs basis with the $\mathbb{Z}_2$ symmetric case

$$\left\{ \begin{aligned} \Lambda_2 &= \Lambda_1 + 2(\Lambda_6 + \Lambda_7)\cot 2\beta, \\ \Lambda_3 + \Lambda_4 + \Lambda_5 &= \Lambda_1 + 2\Lambda_6 \cot 2\beta - (\Lambda_6 - \Lambda_7)\tan 2\beta \end{aligned} \right\}$$

Conditions from: [J. Bernon, J.F. Guinon, H.E. Haber, Y. Jiang, S. Kraml, '15]

$$\boxed{\Lambda_2 \overset{\text{alignment}}{=} \frac{m_h^2}{v^2} - \frac{2v^2\Lambda_7^2}{2M_{22}^2 + m_h^2 - 2m_H^2}}$$