Inverse bubbles from broken supersymmetry

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G. Barni, **SB**, M. Vanvlasselaer
[2406.01596] JCAP & [2503.01951]

Planck 2025 - 27.05.25

First order phase transitions

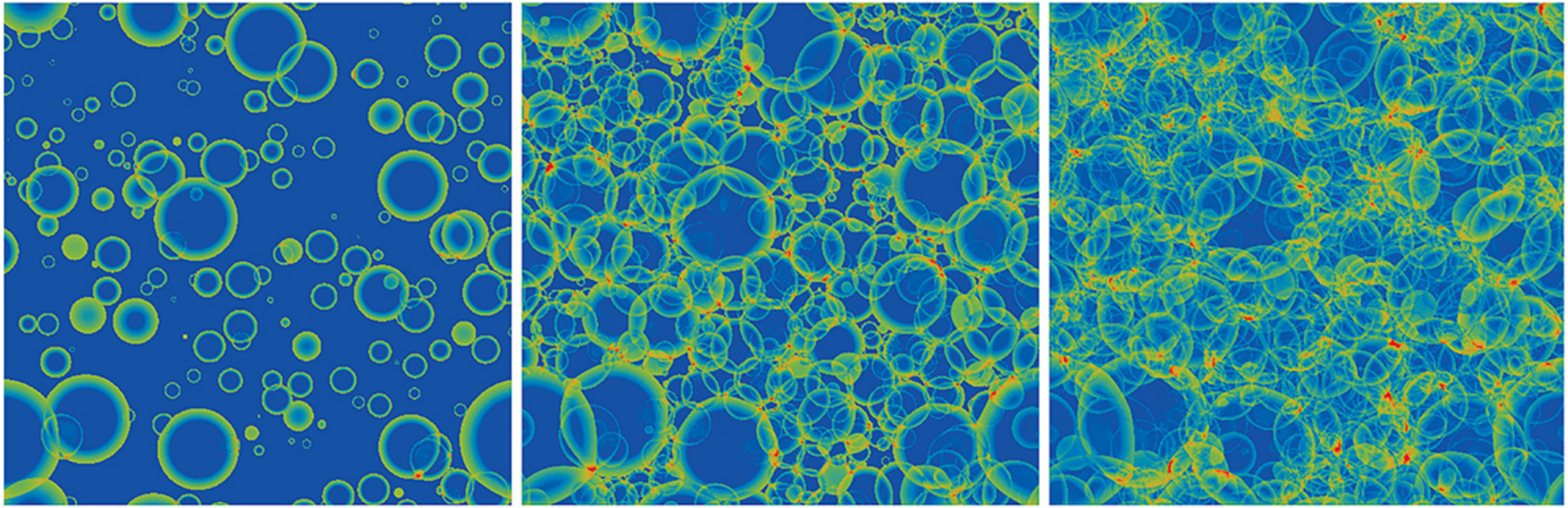
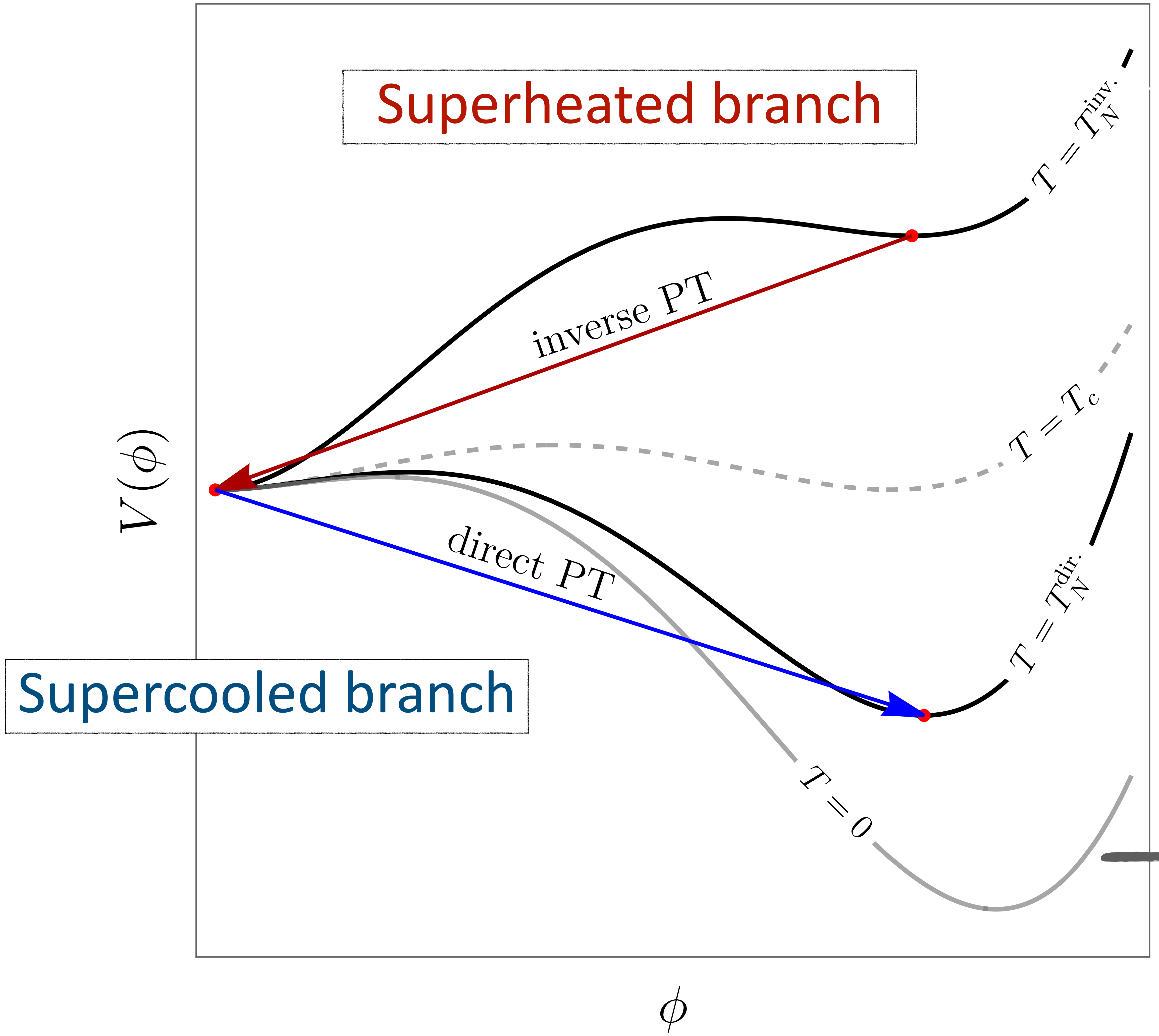


Fig. from Higgsless simulations [2010.00971]

First order phase transitions



Are these two phase transitions qualitatively the same?

The $T = 0$ potential matters!

Hydrodynamics

$$T^{\mu\nu}_{\phi} = (\partial^\mu \phi)\partial^\nu \phi - g^{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right], \qquad \text{(scalar field component)}$$

$$T^{\mu\nu}_{f} = (e_f + p_f)u^\mu u^\nu - g^{\mu\nu} p_f\,, \qquad \text{(plasma component)}$$

$$\nabla_\mu T^{\mu\nu} = \nabla_\mu \left(T^{\mu\nu}_{\phi} + T^{\mu\nu}_{f}\right) = 0$$

Hydrodynamics

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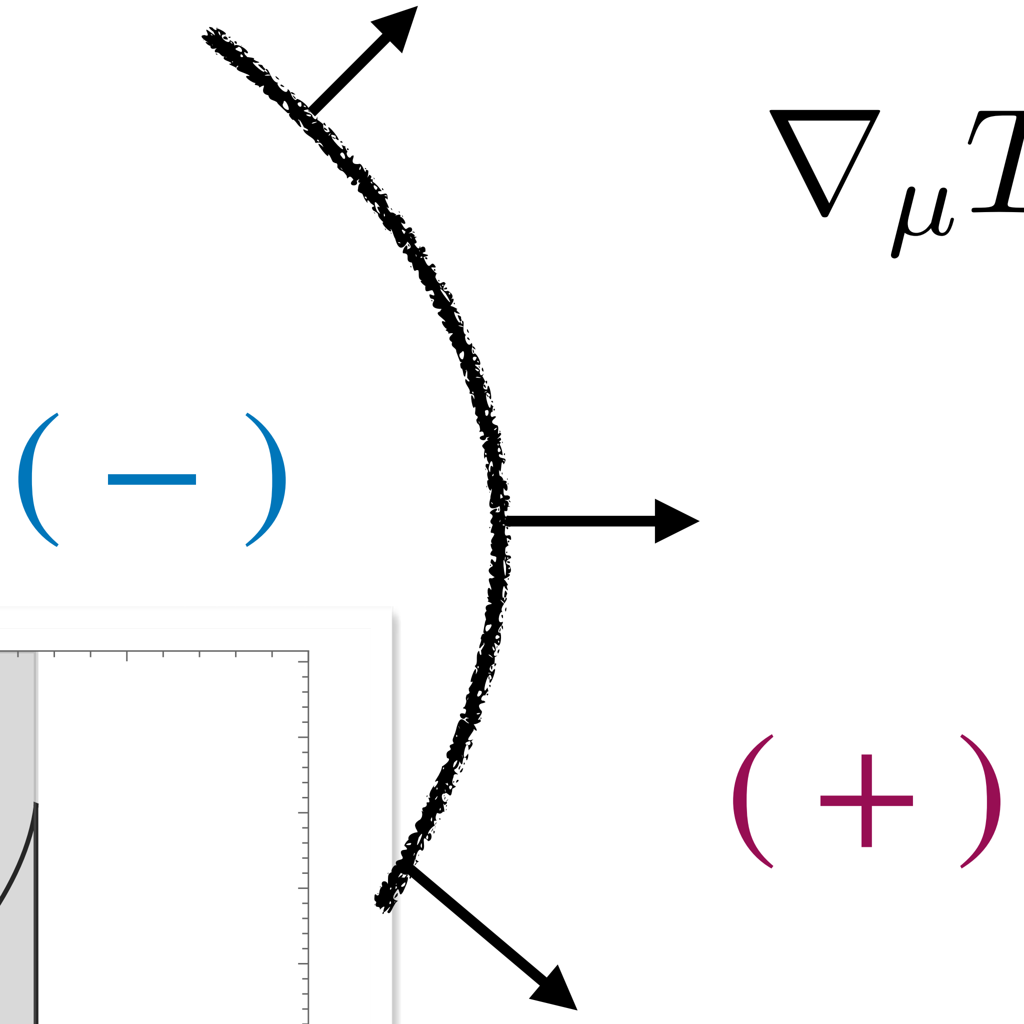
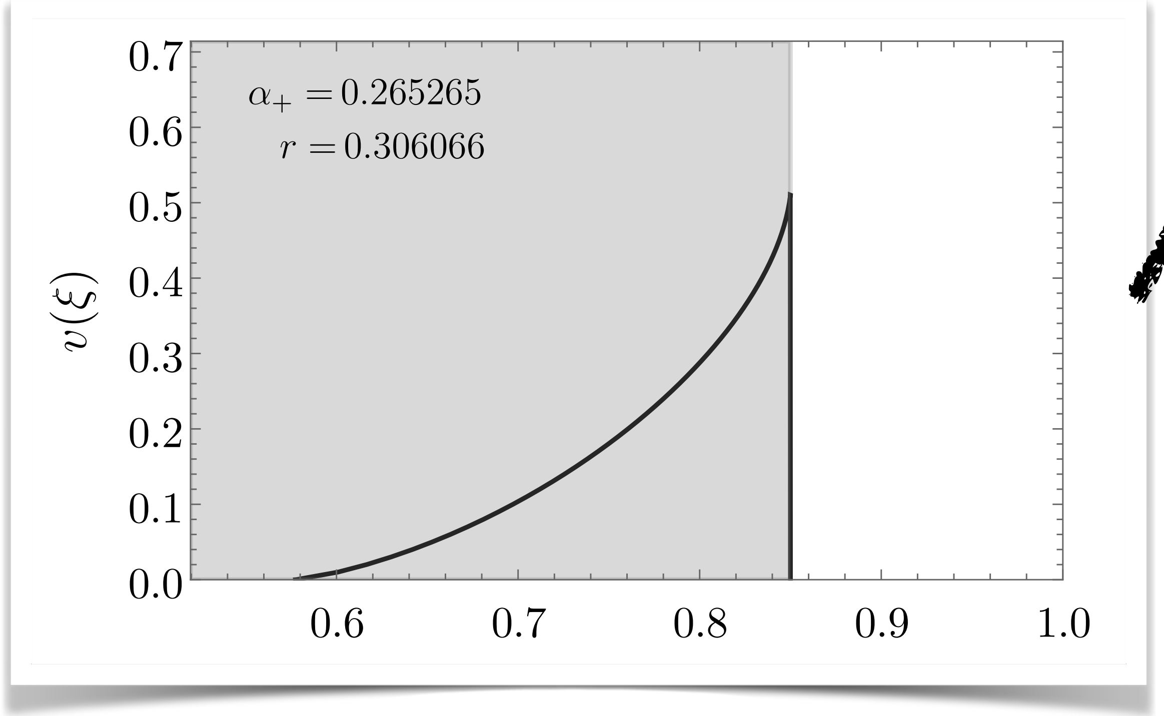


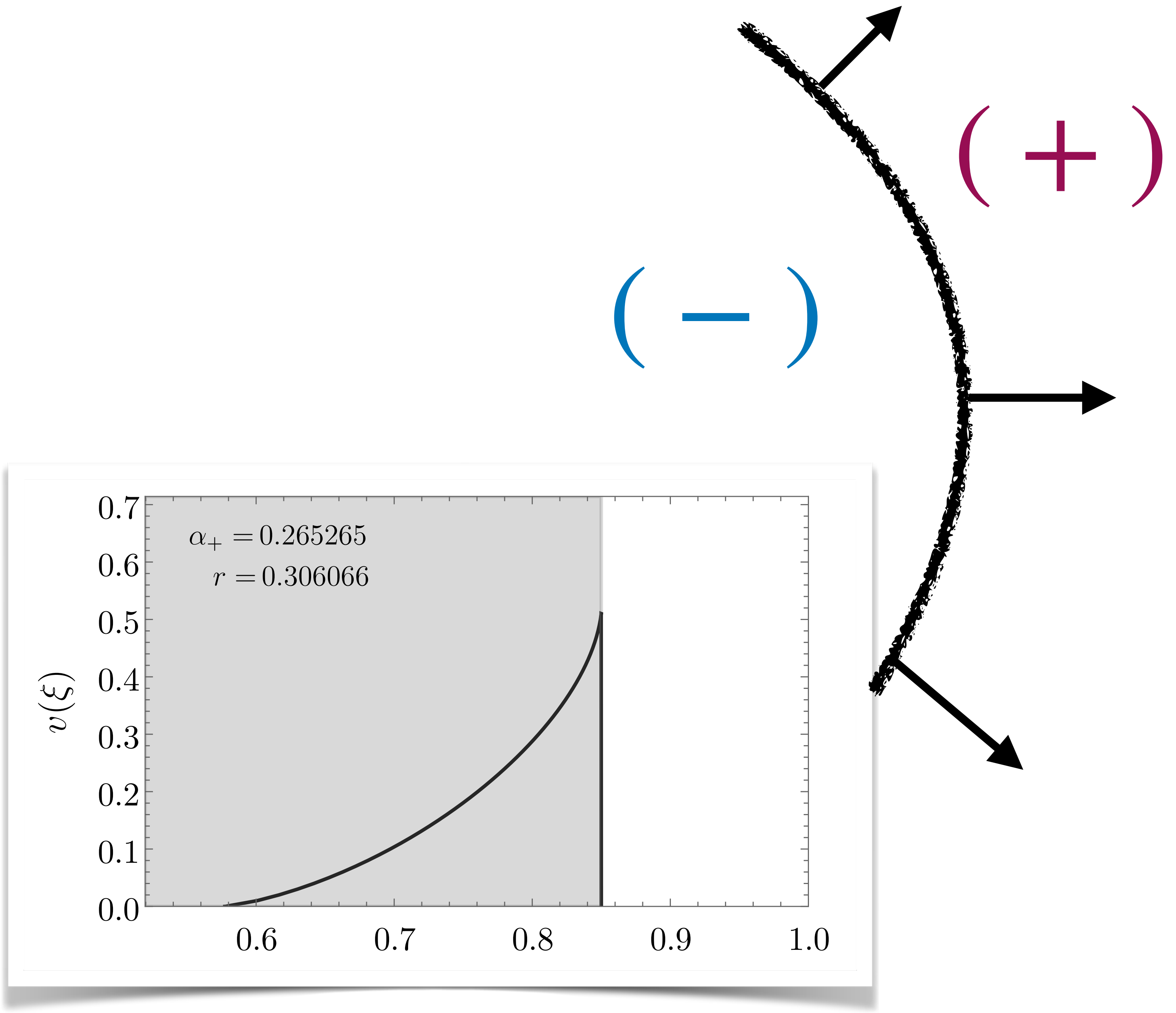
$$\nabla_{\mu}T^{\mu\nu} = \nabla_{\mu}\left(T^{\mu\nu}_{\phi} + T^{\mu\nu}_{f}\right) = 0$$



- This approach does not fully capture the physics at the interface where entropy production takes place

Matching conditions

- However, the wall interface fixes the thermodynamics on the two sides:



$$w_+\gamma_+^2 v_+ = w_-\gamma_-^2 v_- \,,$$
$$w_+\gamma_+^2 v_+^2 + p_+ = w_-\gamma_-^2 v_-^2 + p_-$$

$v_\pm$: fluid velocity ahead and behind the wall, in the wall frame

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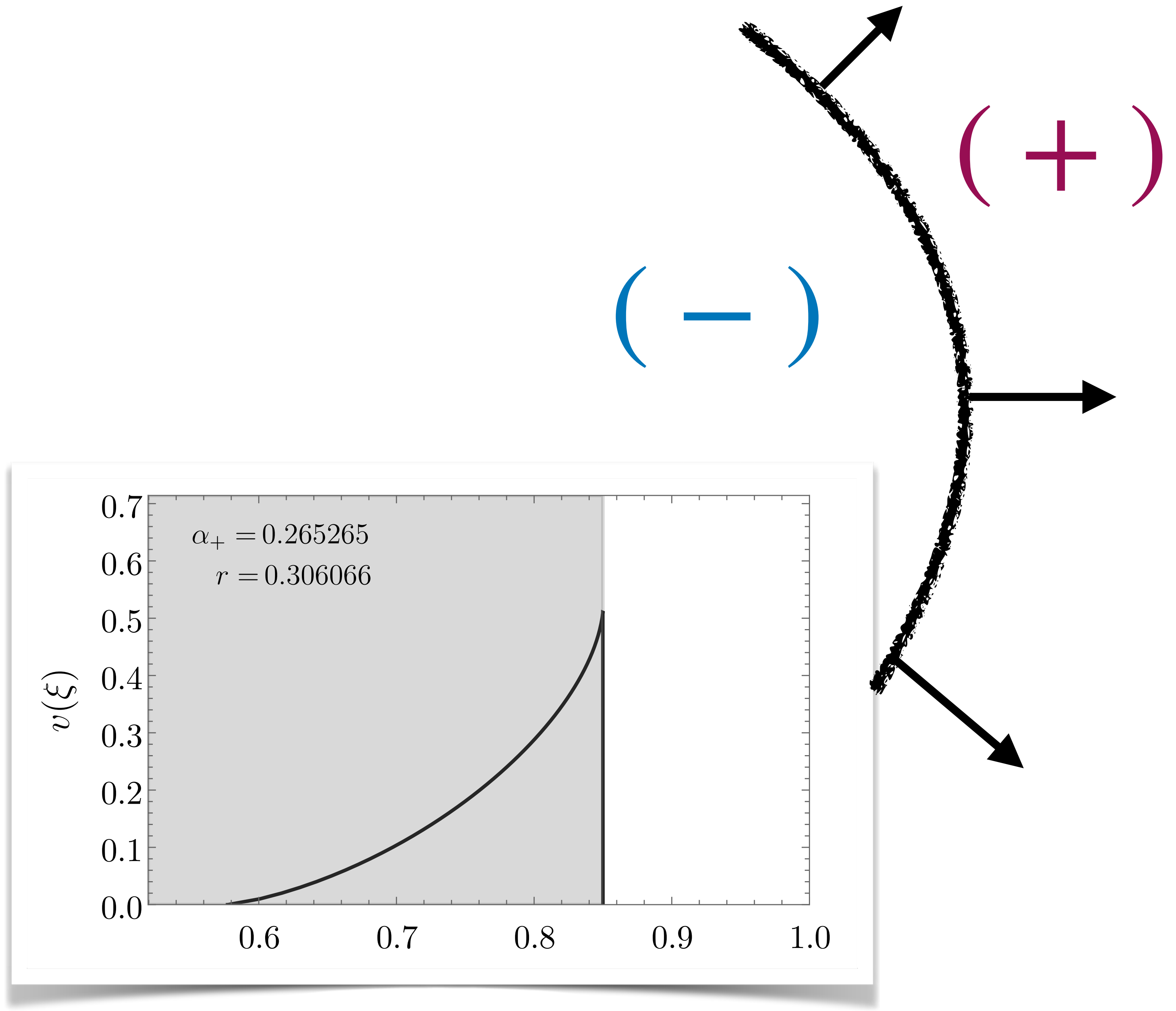

$$\frac{v_+}{v_-} \simeq \frac{(v_+ v_-/c_s^2 - 1) + 3\alpha_{\bar{\theta}}}{(v_+ v_-/c_s^2 - 1) + 3 v_+ v_- \alpha_{\bar{\theta}}}$$

$$\bar{\theta} \equiv e - p/c_s^2 \,, \qquad \alpha_{\bar{\theta}} \equiv \frac{D\bar{\theta}}{3 w_+}$$

Giese, Konstandin, van de Vis
[2004.06995] JCAP

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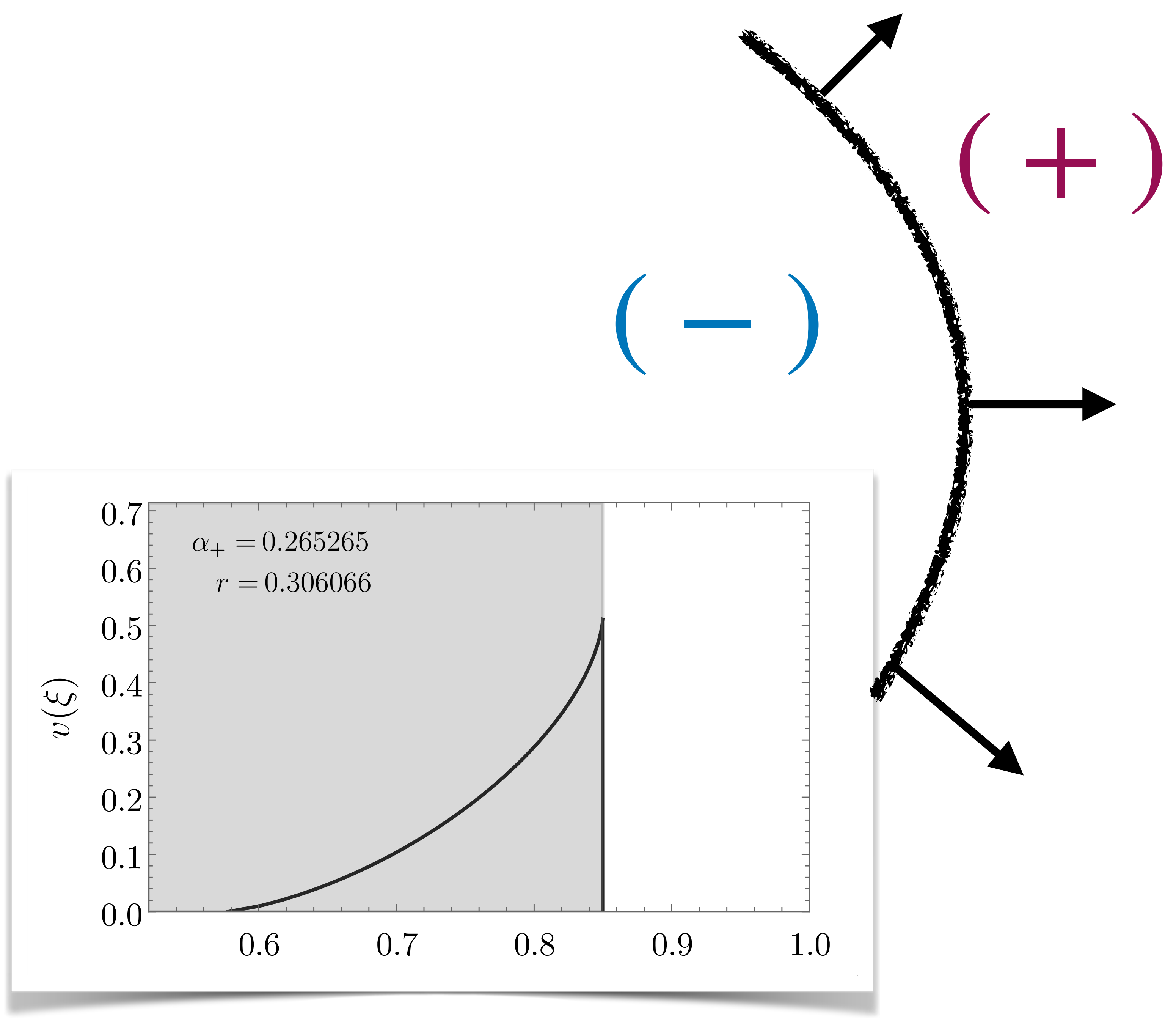

$$\frac{v_+}{v_-} \simeq \frac{(v_+v_-/c_s^2 - 1) + 3\alpha_{\bar\theta}}{(v_+v_-/c_s^2 - 1) + 3v_+v_-\alpha_{\bar\theta}}$$

- Simplified bag equation of state ($c_s^2 = 1/3$, $\alpha_{\bar\theta} = \alpha_+$):

$$p_+(T) = \frac{1}{3}a_+T^4 - \epsilon_+$$

$$p_-(T) = \frac{1}{3}a_-T^4 - \epsilon_-$$

$$\alpha_+ \equiv \frac{\epsilon_+ - \epsilon_-}{a_+T_+^4}$$

Matching conditions

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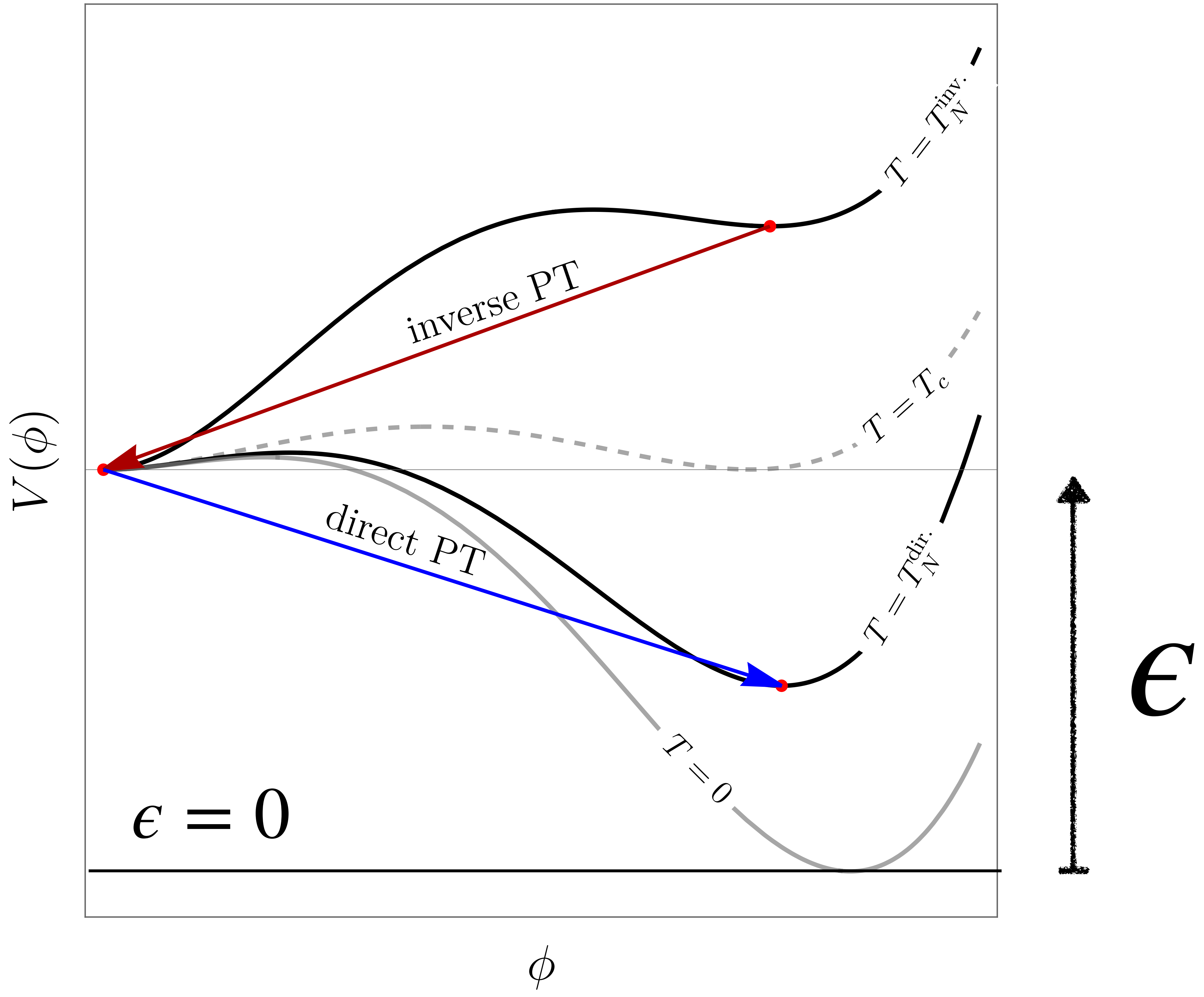

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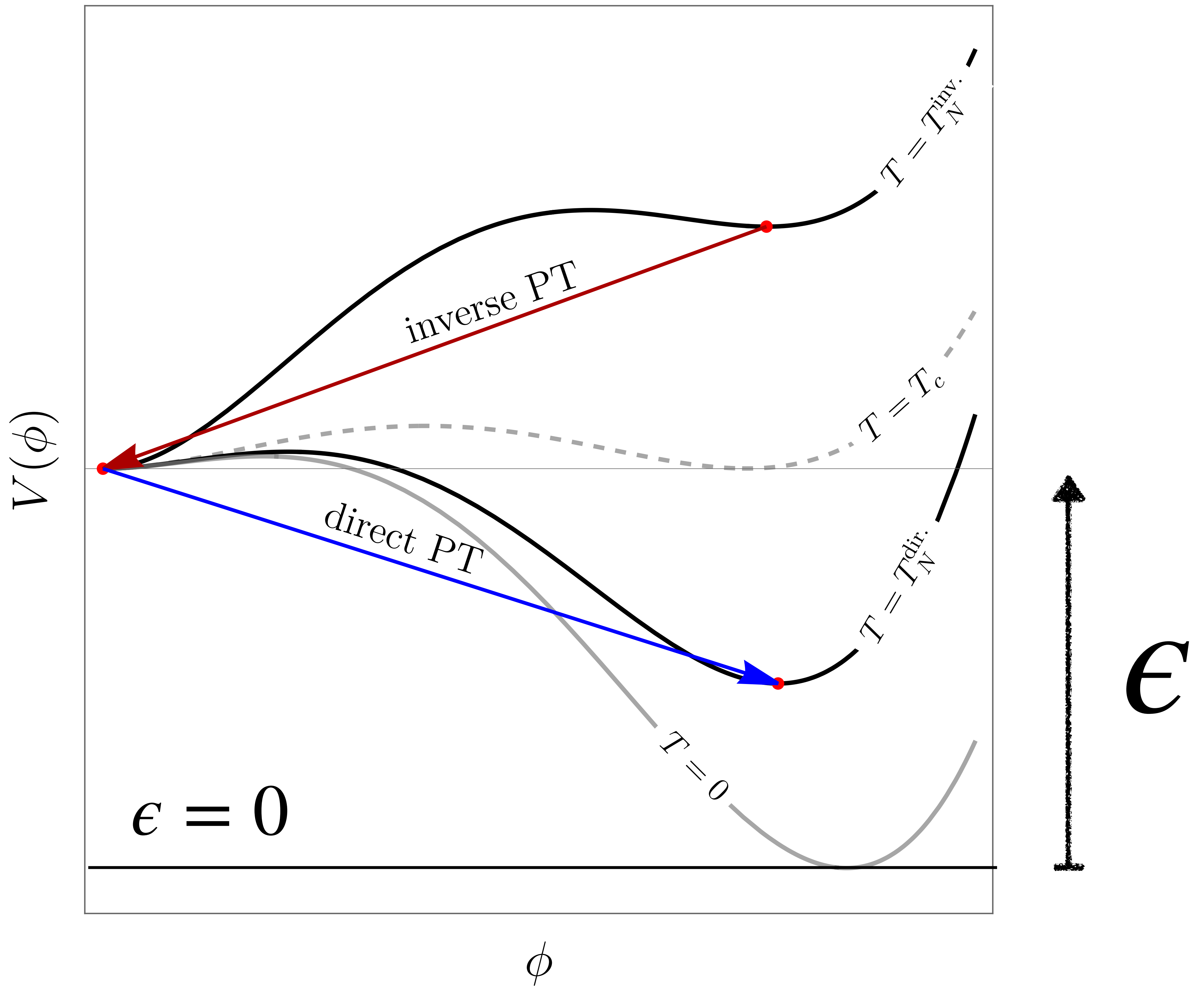

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Matching conditions

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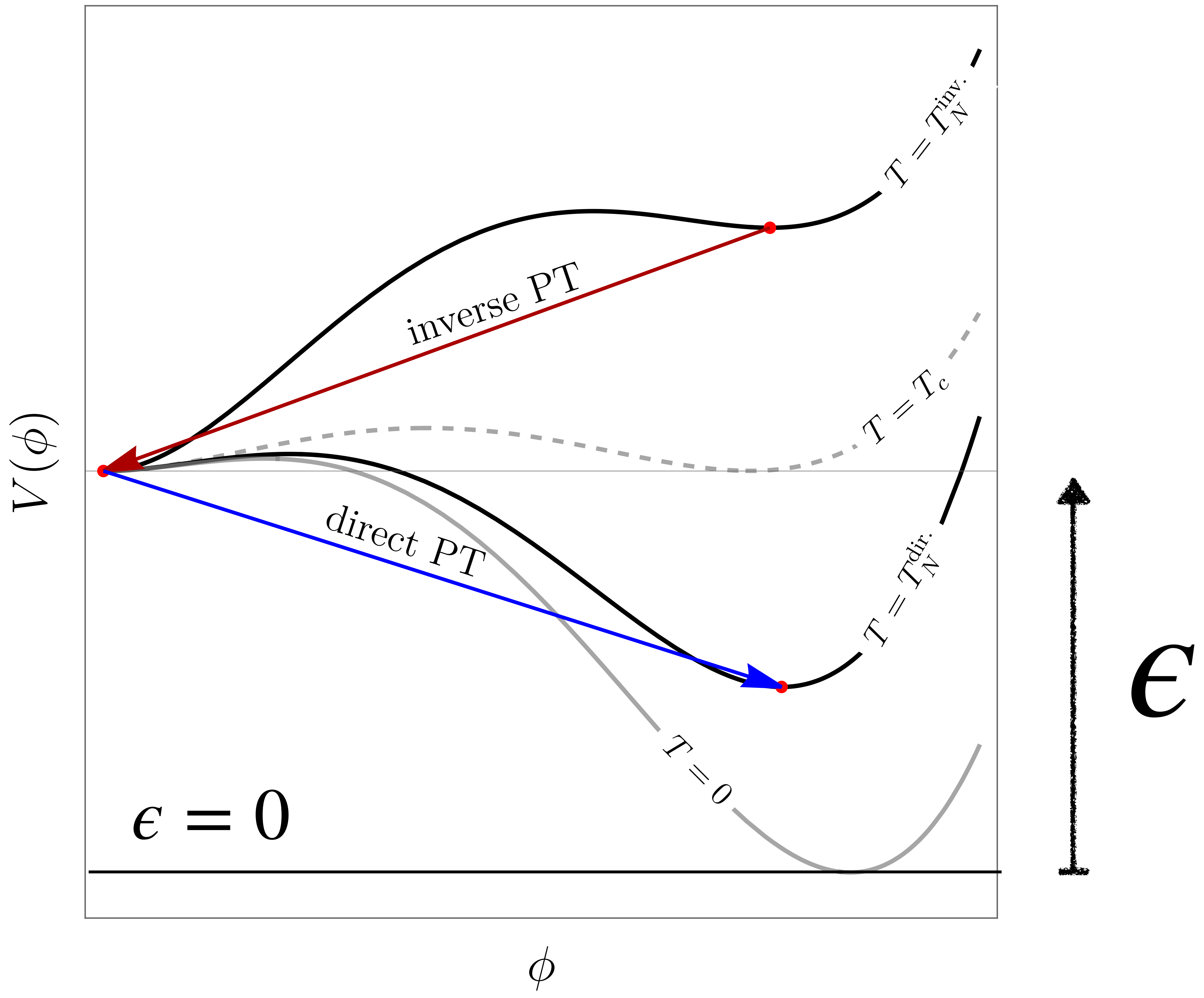

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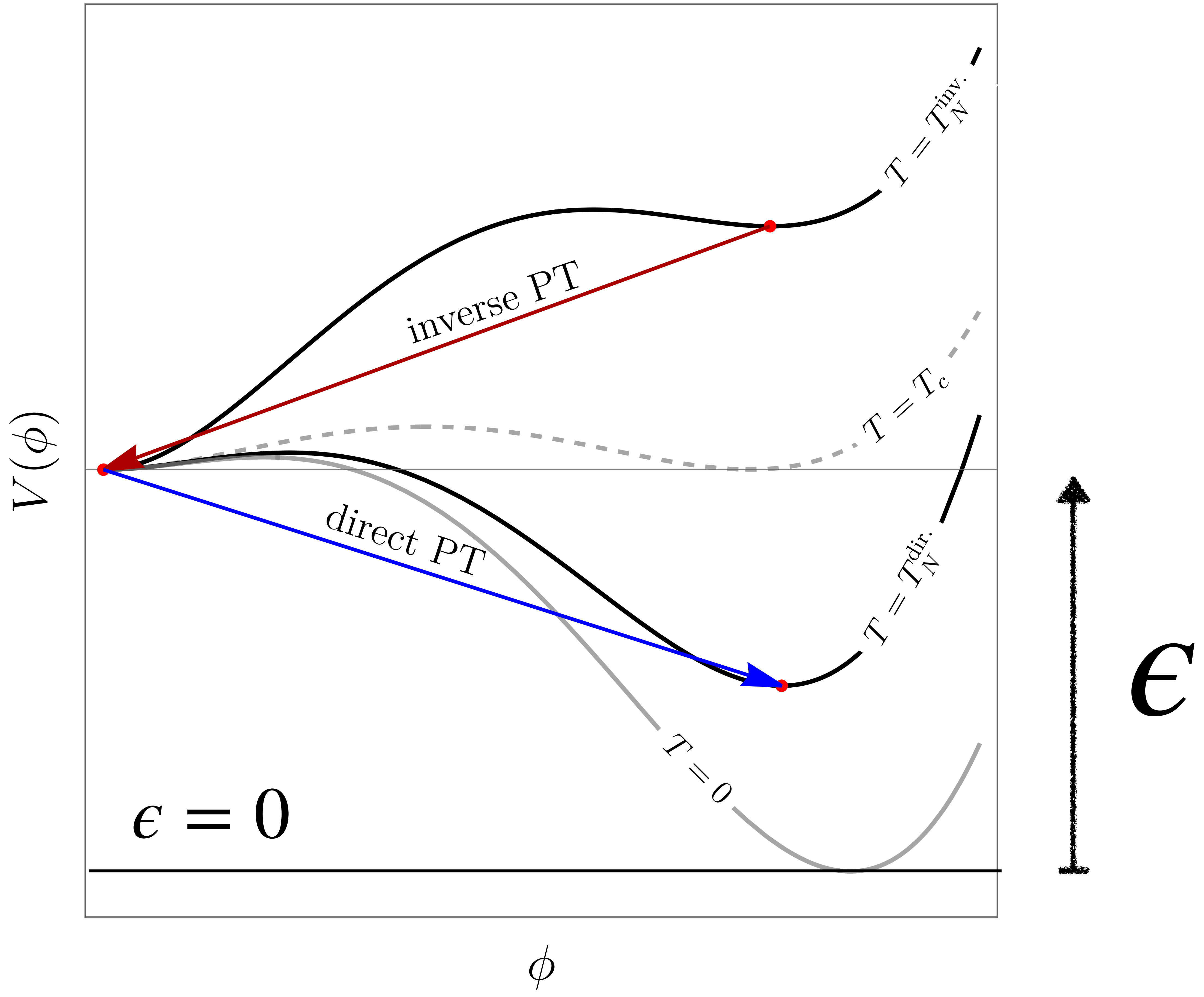

$$\alpha_+ \equiv \frac{\cancel{\epsilon_+} - \epsilon_-}{a_+ T_+^4} < 0$$

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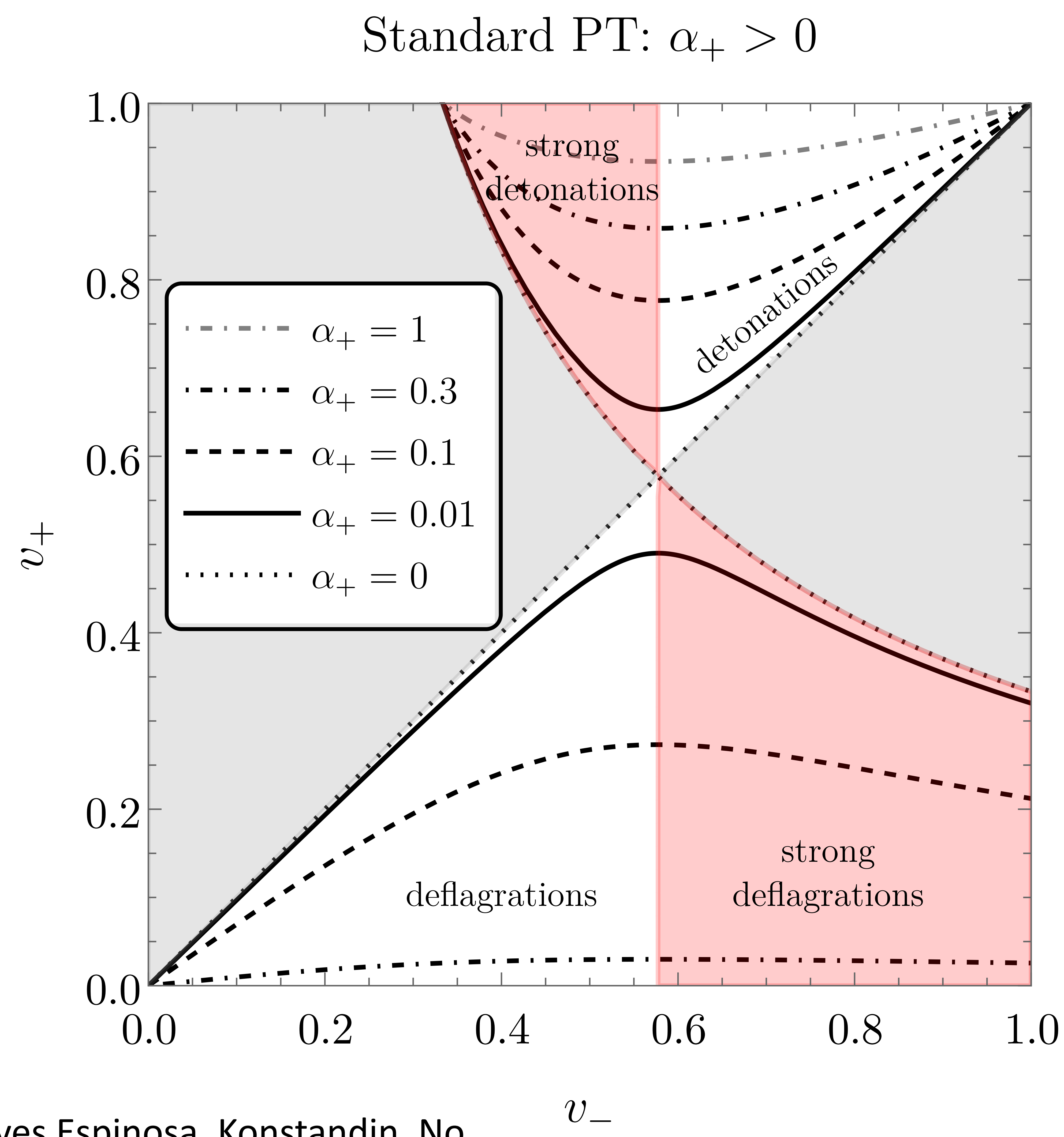
Matching conditions

Fig. from G. Barni, **SB**, M. Vanvlasselaer [2406.01596] JCAP



Left plot gives Espinosa, Konstandin, No, Servant [1004.4187] JCAP

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Fig. from G. Barni, **SB**, M. Vanvlasselaer
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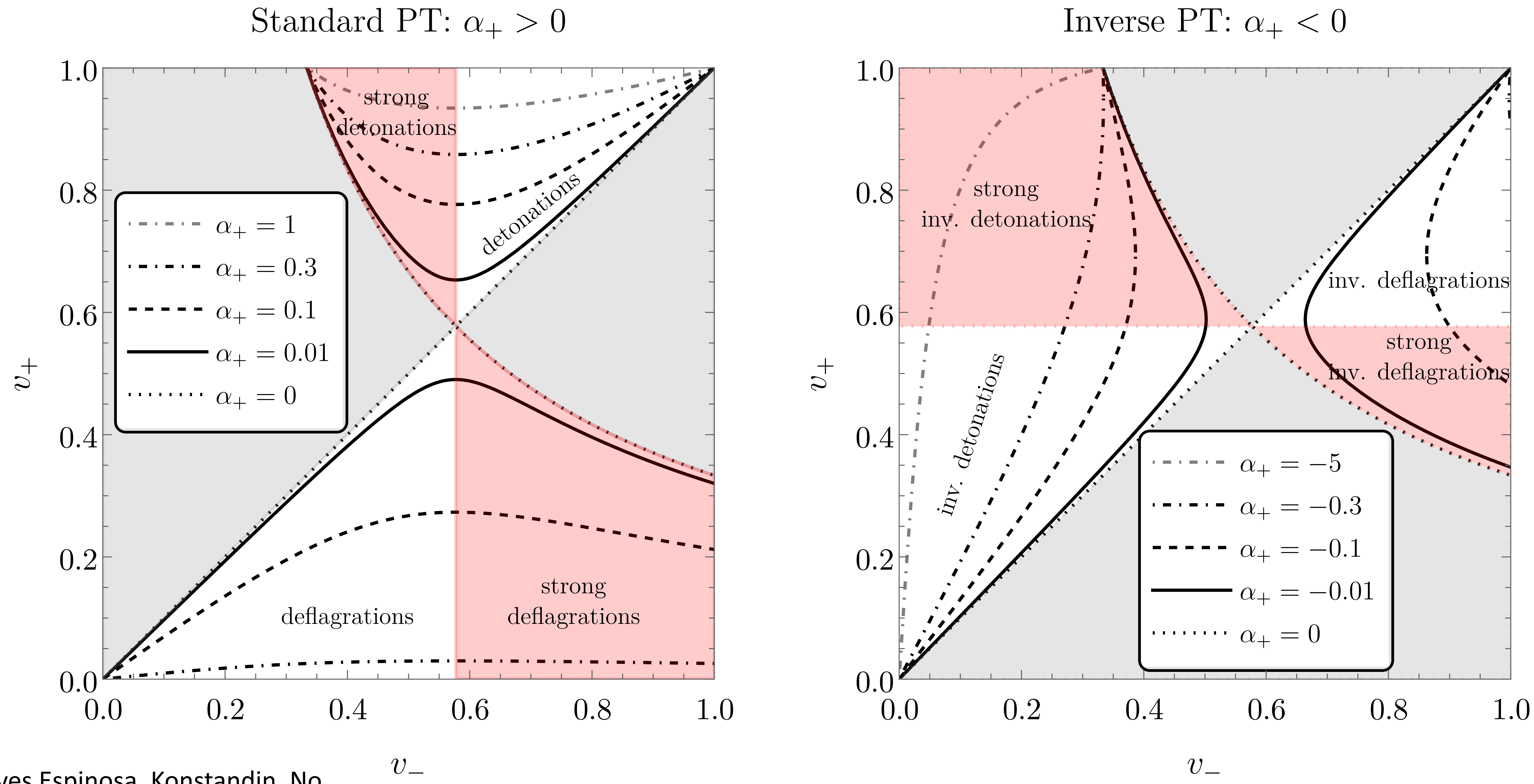


Left plot gives Espinosa, Konstandin, No, Servant [1004.4187] JCAP

Fluid equations

- Self similar profiles ($\xi = r/t$):

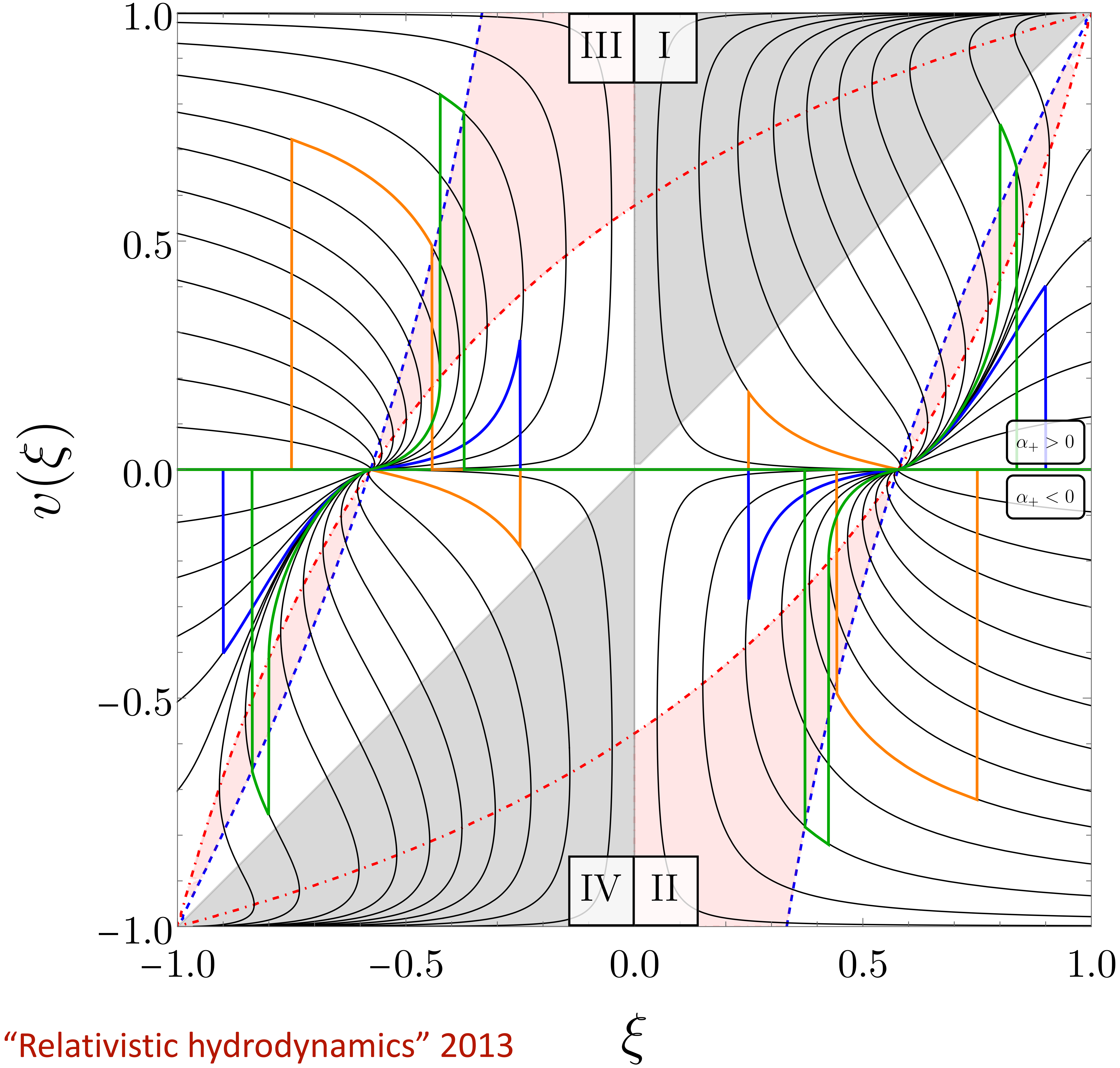
$$(\xi - v)\frac{\partial_\xi e}{w} = 2\frac{v}{\xi} + [1 - \gamma^2 v(\xi - v)]\partial_\xi v$$
$$(1 - v\xi)\frac{\partial_\xi p}{w} = \gamma^2(\xi - v)\partial_\xi v\,.$$

- Relate p, e to the constant sound speed:

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2_{p\to w}}{c_s^2} - 1\right]\partial_\xi v,$$

- Symmetric for:

$$\xi \to -\xi \qquad v \to -v$$



See also Rezzolla & Zanotti, "Relativistic hydrodynamics" 2013

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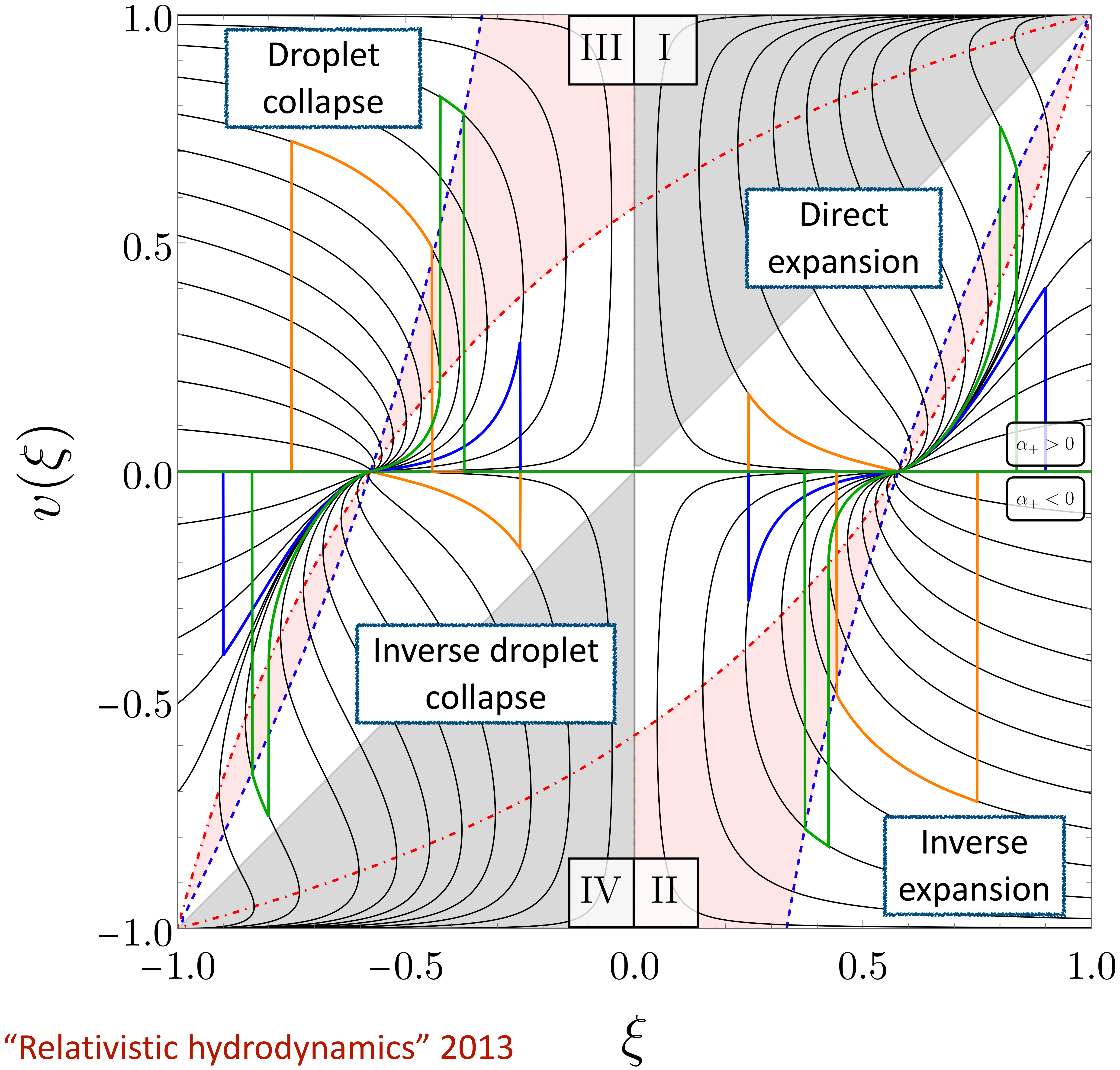

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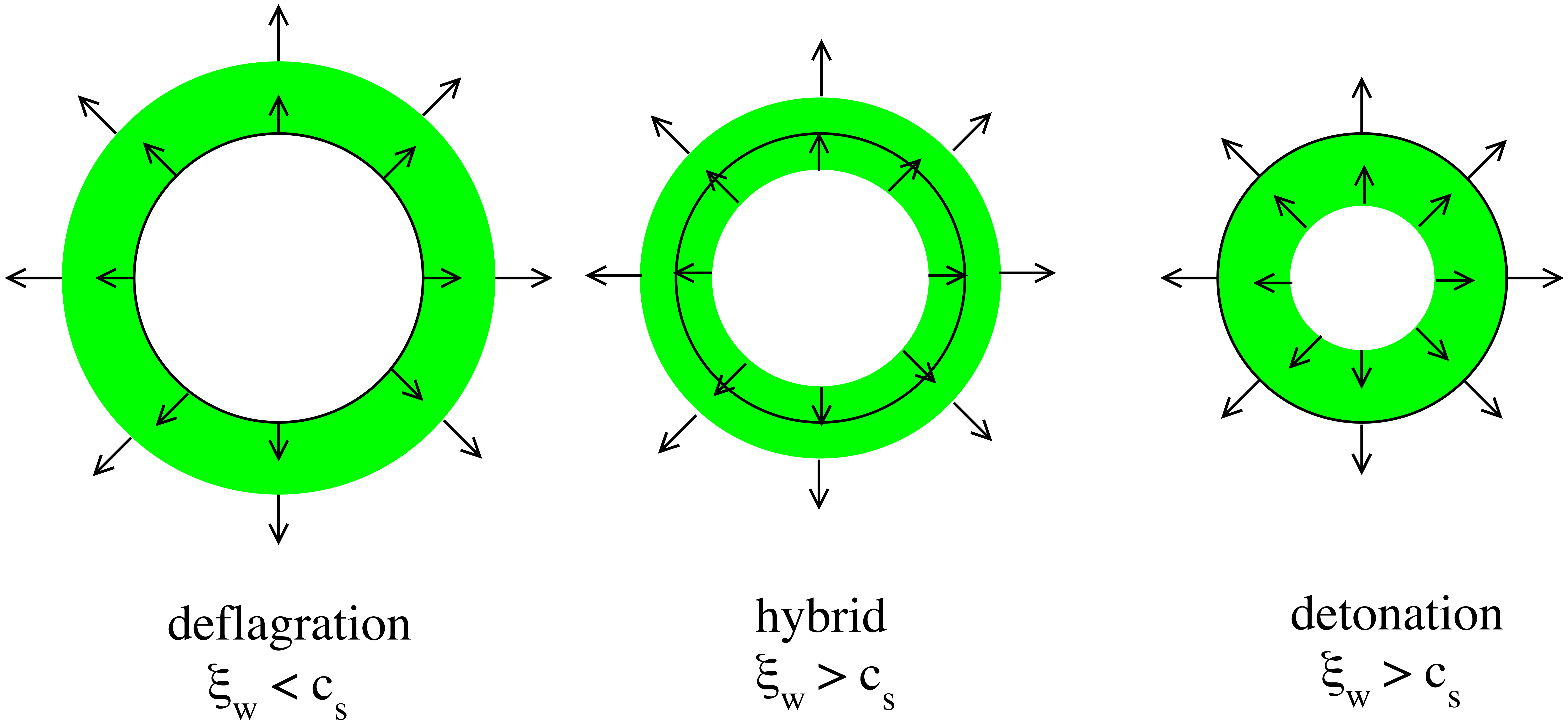
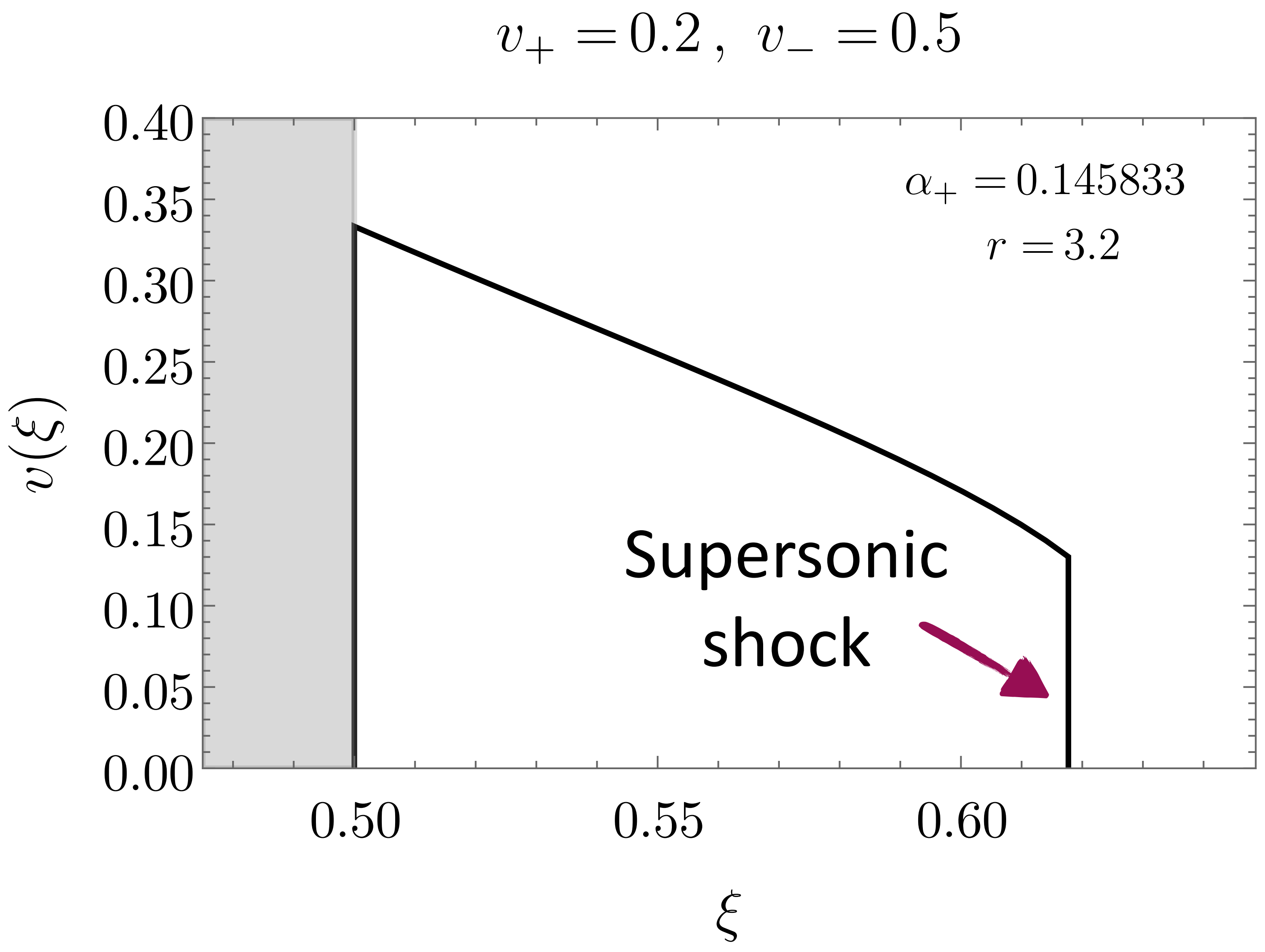
Direct bubble expansion



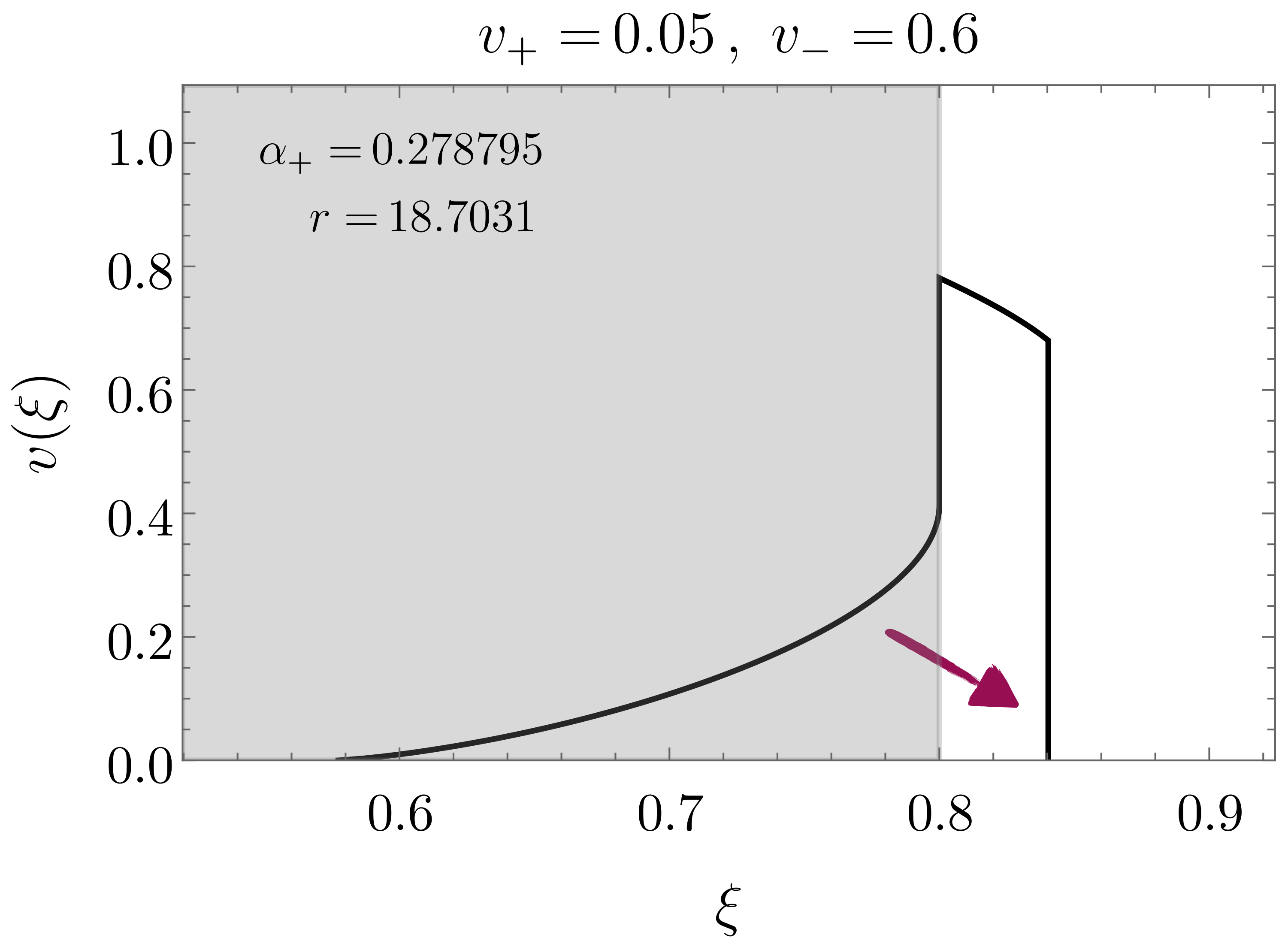
Espinosa, Konstandin, No, Servant
[1004.4187] JCAP

Direct bubble expansion

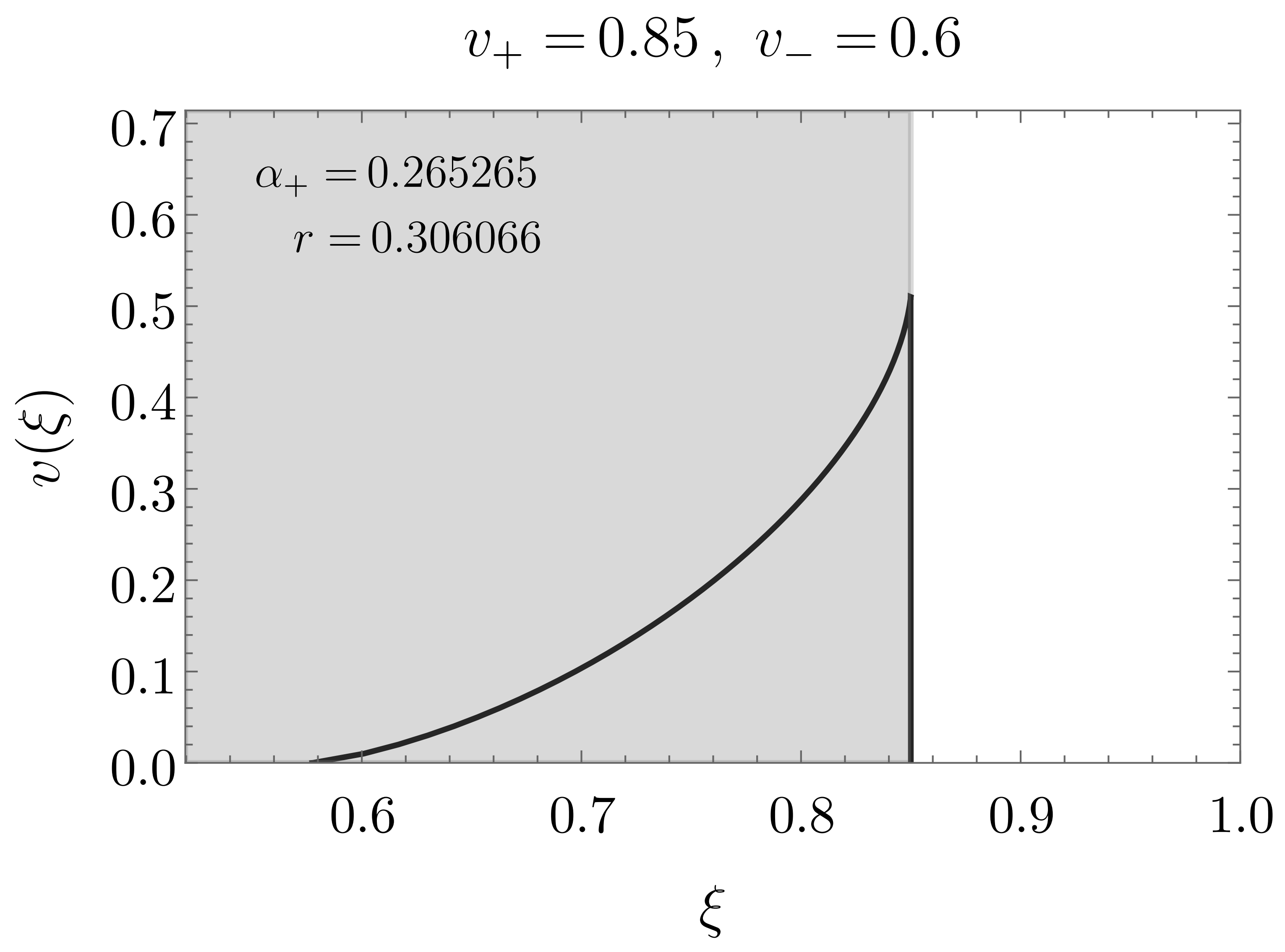
Deflagration



Hybrid

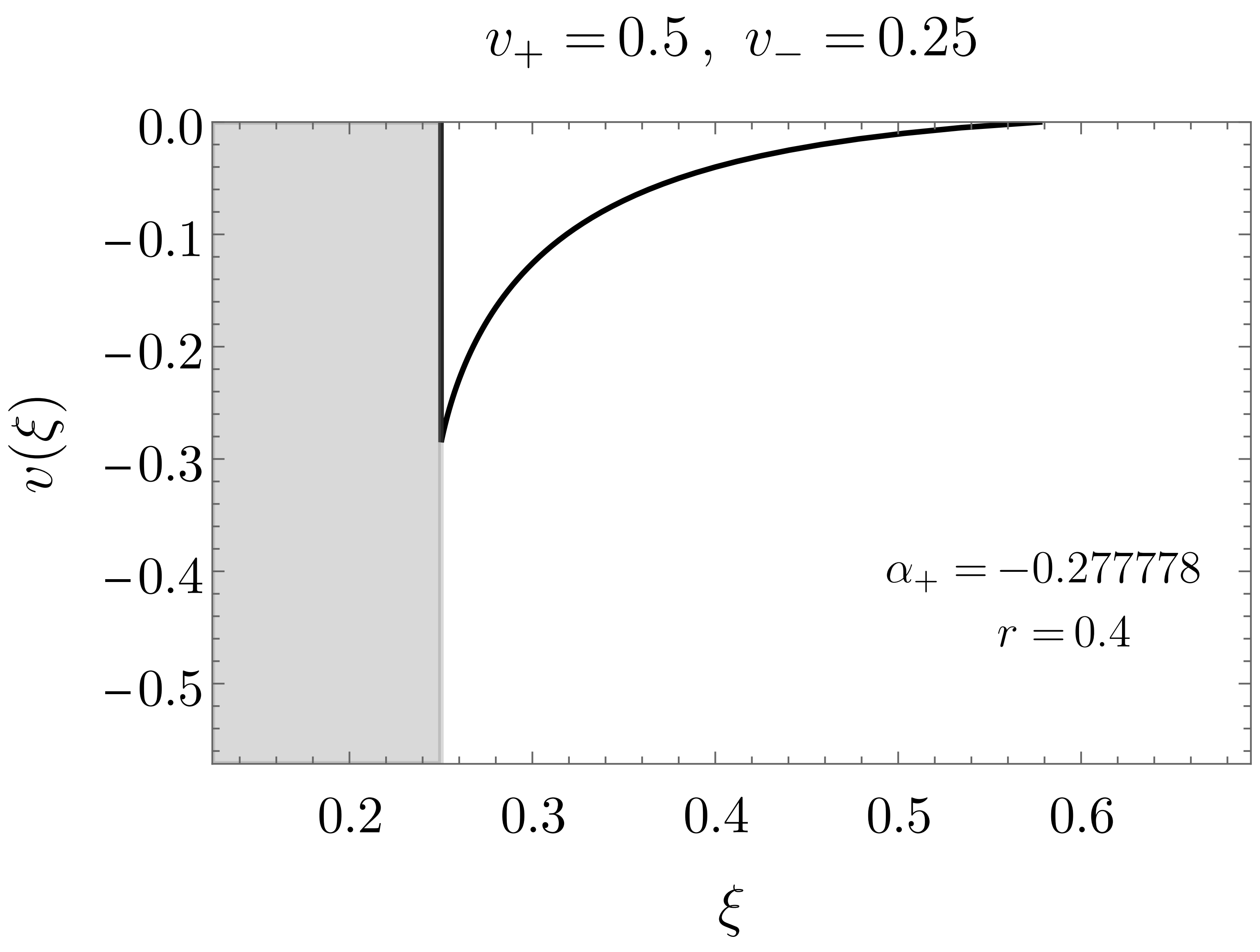


Detonation

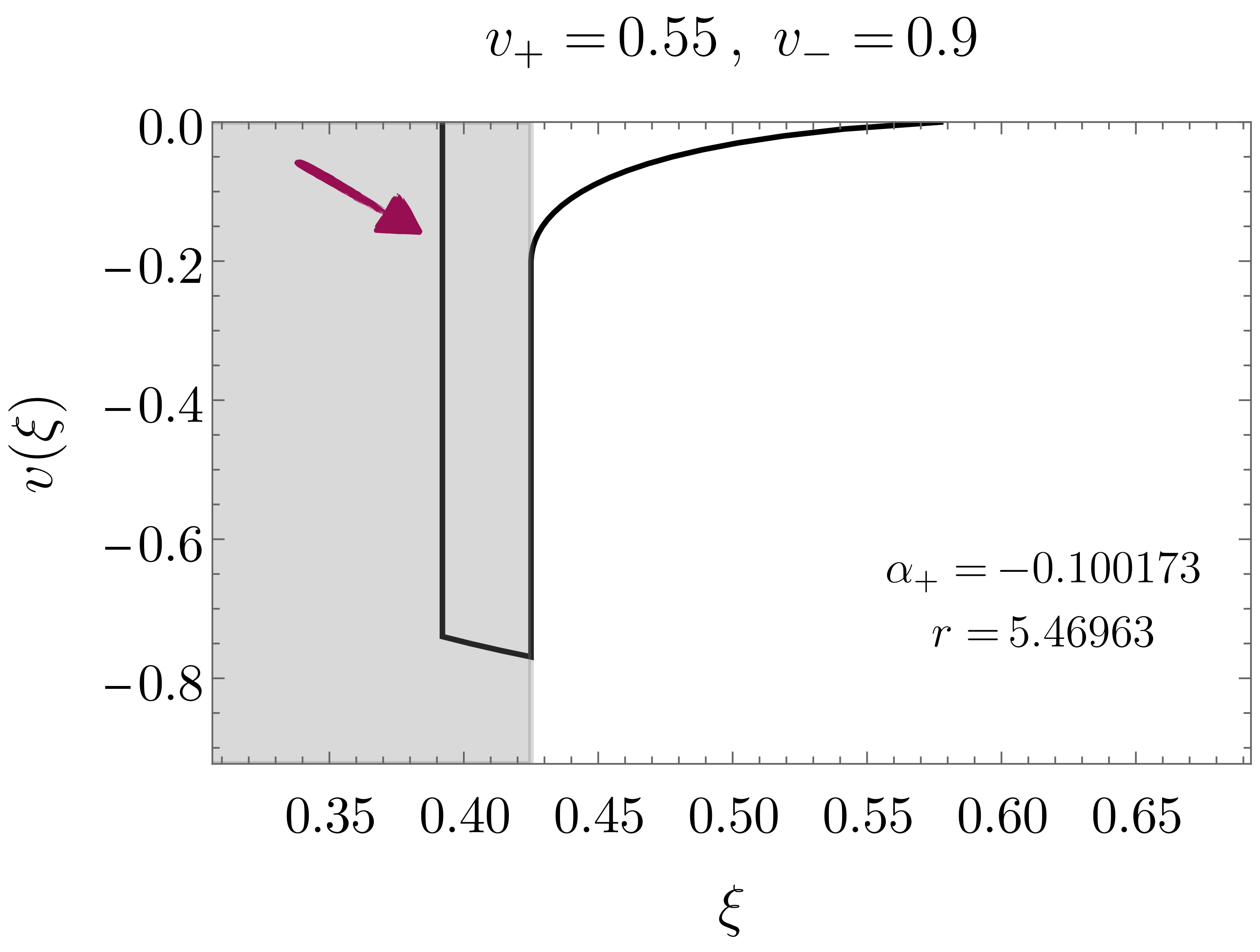


Inverse bubble expansion

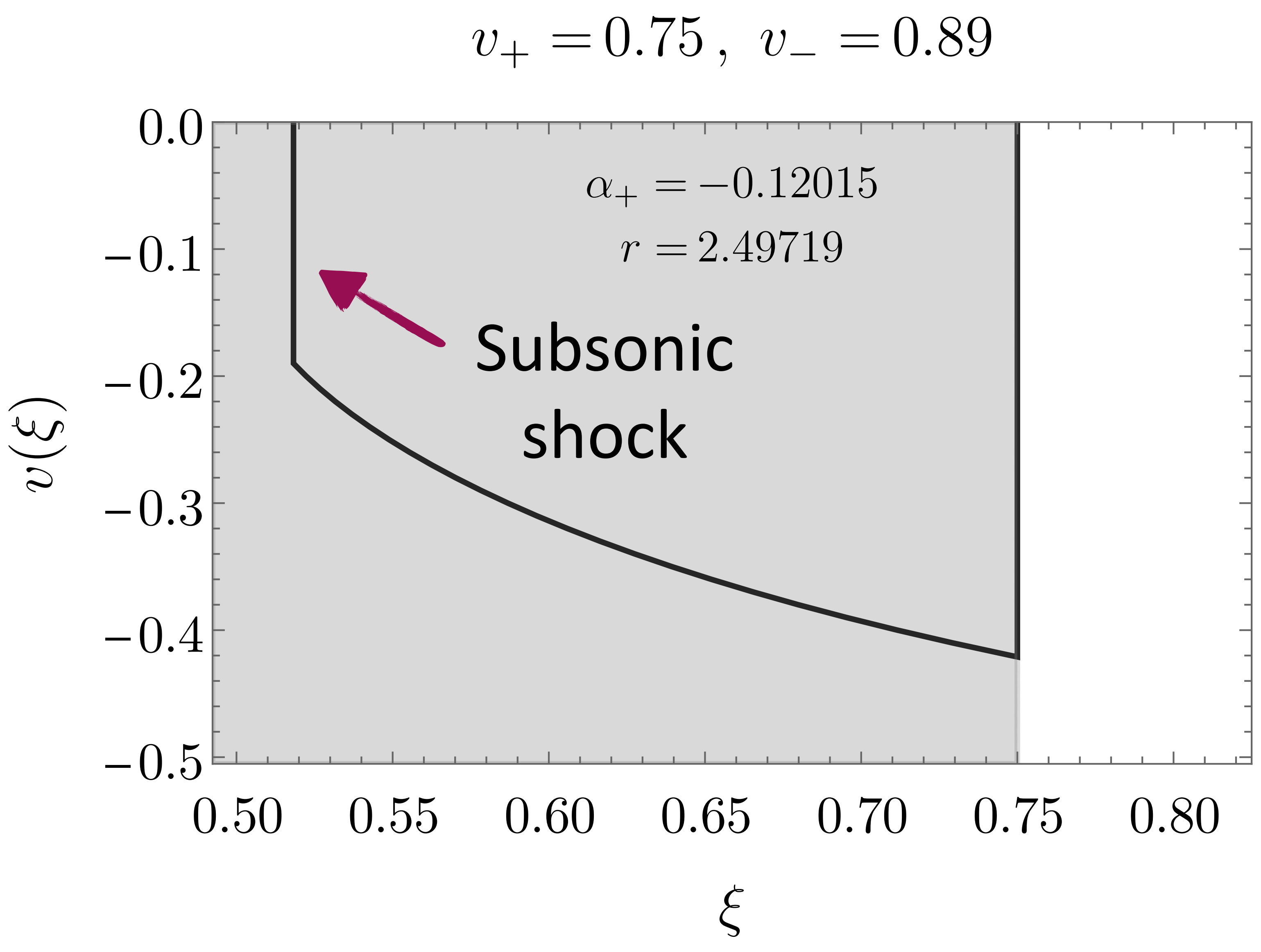
- Fluid is sucked in by the advancing bubble wall, or equiv. the bulk velocity is negative in the plasma frame



Anti-detonation



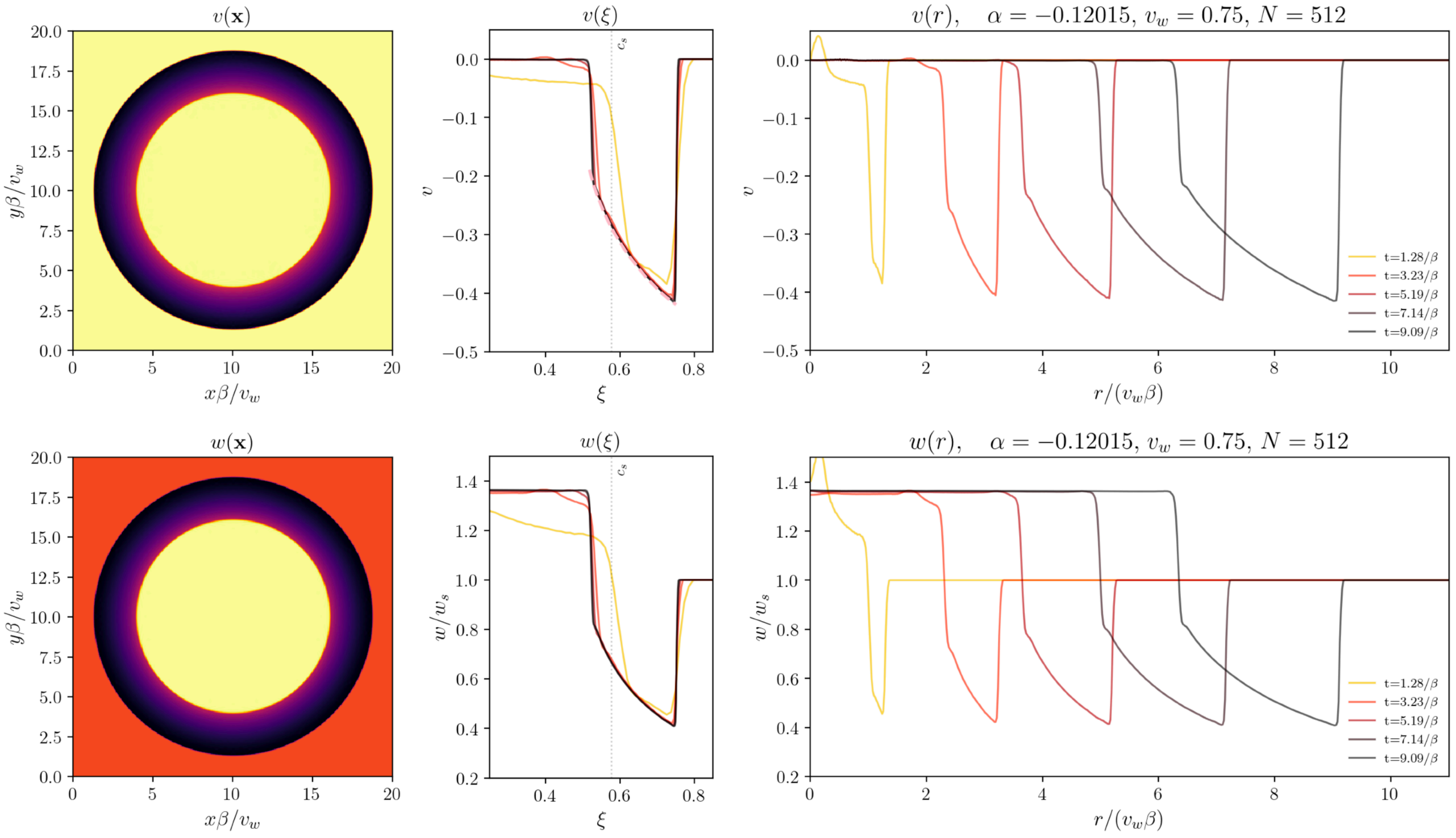
Anti-hybrid



Anti-deflagration

Higgsless simulation

Thanks to Isak Stomberg!



Energy budget (direct)

- From energy conservation it follows that

$$\underbrace{\frac{\xi_w^3}{3}\epsilon}_{\text{vacuum energy}} + \underbrace{\frac{3}{4}\int w_N \xi^2 d\xi}_{\text{initial thermal energy}} = \underbrace{\int \gamma^2 v^2 w \xi^2 d\xi}_{\text{fluid motion}} + \underbrace{\frac{3}{4}\int w \xi^2 d\xi}_{\text{final thermal energy}}$$

- Efficiency for converting vacuum energy into fluid motion:

$$\frac{\rho_{\text{kin}}}{\rho_{\text{tot}}} \equiv \kappa_{\text{direct}} \frac{\alpha_N}{1+\alpha_N} \qquad \kappa_{\text{direct}} = \frac{3}{\epsilon \xi_w^3}\int \gamma^2 v^2 w \xi^2 d\xi$$

Energy budget (inverse)

- From energy conservation it follows that

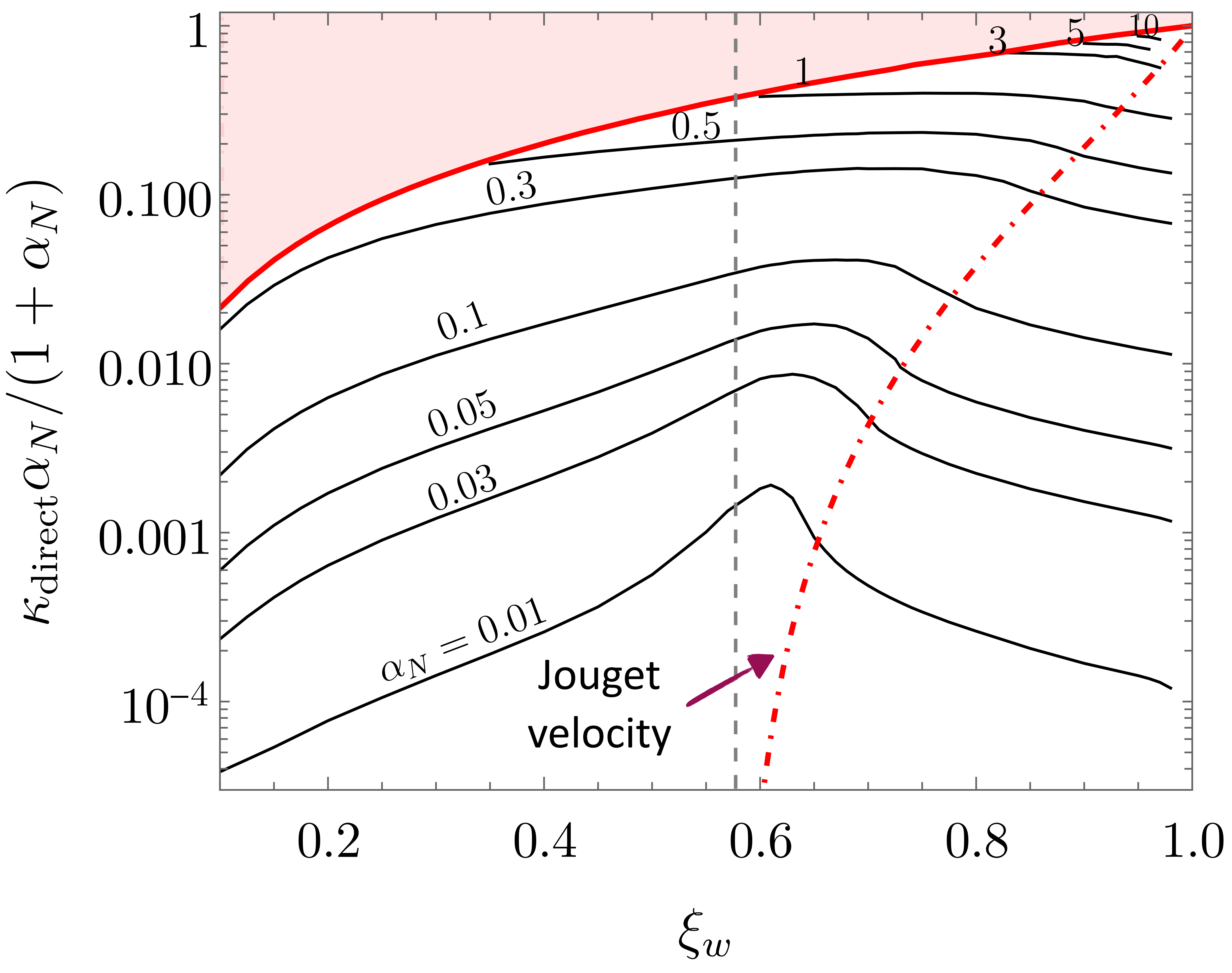
$$\underbrace{\frac{3}{4}\int w_N \xi^2 d\xi}_{\text{initial thermal energy}} = \underbrace{\frac{\xi_w^3}{3}\epsilon}_{\text{vacuum energy}} + \underbrace{\int \gamma^2 v^2 w \xi^2 d\xi}_{\text{fluid motion}} + \underbrace{\frac{3}{4}\int w \xi^2 d\xi}_{\text{final thermal energy}}$$

- Efficiency for converting enthalpy into fluid motion:

$$\kappa_{\text{inverse}} \equiv \frac{\rho_{\text{kin}}}{\rho_{\text{tot}}} = \frac{4}{\bar{v}^3}\int \xi^2 d\xi \, v^2 \gamma^2 \frac{w}{w_N} \qquad \bar{v} = \mathrm{Max}(\xi_w, c_s)$$

Comparison of efficiencies

- Fraction of the critical energy density that goes into bulk fluid motion:



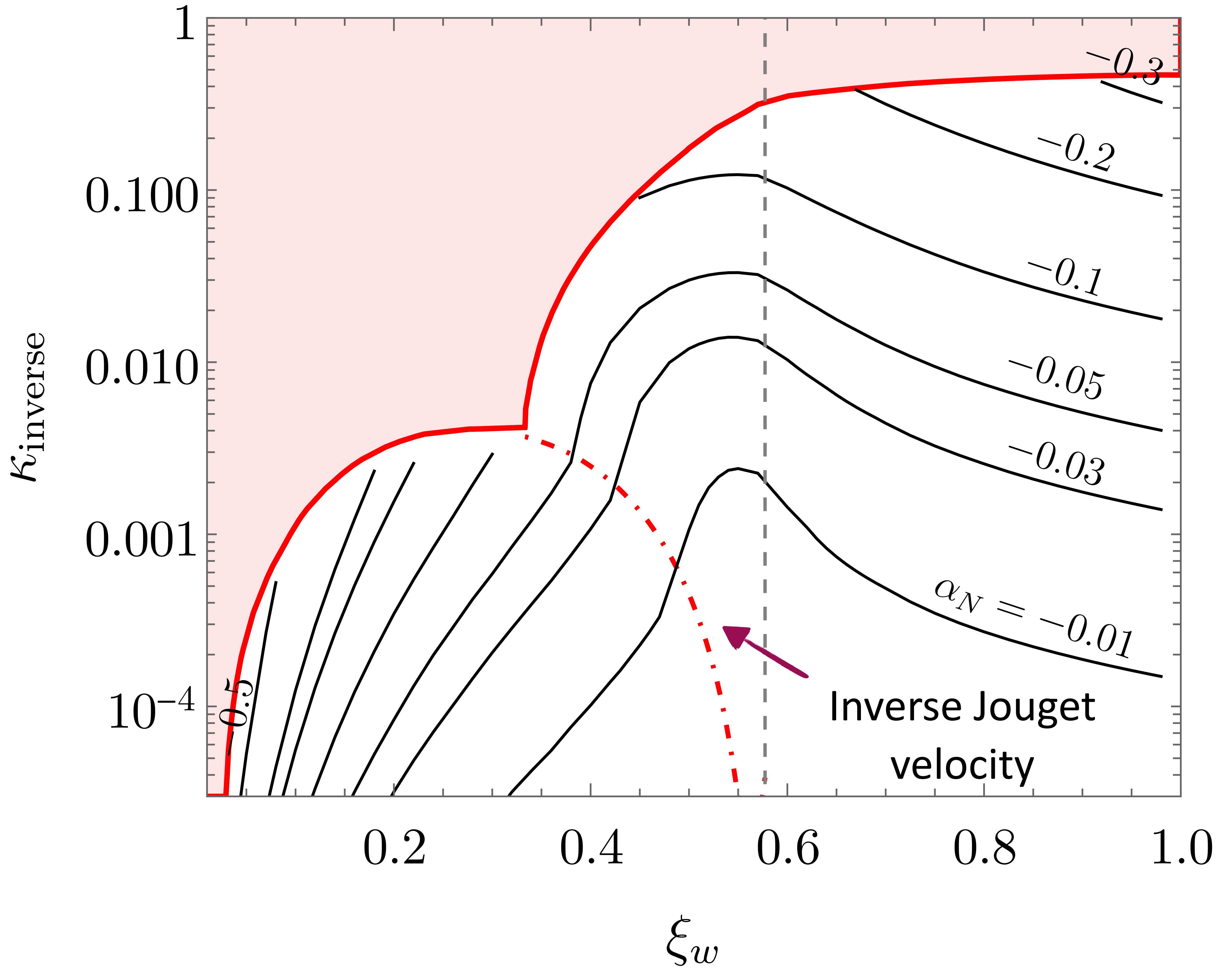


Fig. from G. Barni, **SB**, M. Vanvlasselaer [2406.01596] JCAP

Possible realizations

- Reheating (e.g. inflaton decay)
  Buen-Abad, Chang, Hook [2305.09712] PRD
- Heating by rarefaction waves
  Caprini, No [1111.1726] JCAP

  Hydrodynamics in G. Barni, **SB**, M. Vanvlasselaer [2406.01596] JCAP & Bea et al. [2406.14450]
- Heating after neutron star mergers
  J. Casalderrey-Solana, D. Mateos, M. Sanchez-Garitaonandia [2210.03171]
- **Supercooling** along flat directions (pseudomodulus)

  Barni, **SB**, Vanvlasselaer [2503.01951]

Inverse bubbles from SUSY breaking

- Chiral superfield X that controls SUSY and R-symmetry breaking:

$$X = \frac{x}{\sqrt{2}} e^{2ia/f_a} + \sqrt{2}\theta\tilde{G} + \theta^2 F$$

Pseudomodulus flat direction

- Superpotential of the vector-like O'Raifeartaigh model:

$$W = -FX + \lambda X \Phi_1 \tilde{\Phi}_2 + m(\Phi_1\tilde{\Phi}_1 + \Phi_2\tilde{\Phi}_2)$$

Craig, Levi, Mariotti, Redigolo [2011.13949] JHEP

Inverse bubbles from SUSY breaking

- Scalar potential:

$$V = |F - \lambda\phi_1\tilde{\phi}_2|^2 + |\lambda X\tilde{\phi}_2 + m\tilde{\phi}_1|^2 + |\lambda X\phi_1 + m\phi_2|^2 + |m\phi_1|^2 + |m\tilde{\phi}_2|^2$$

Scalar component

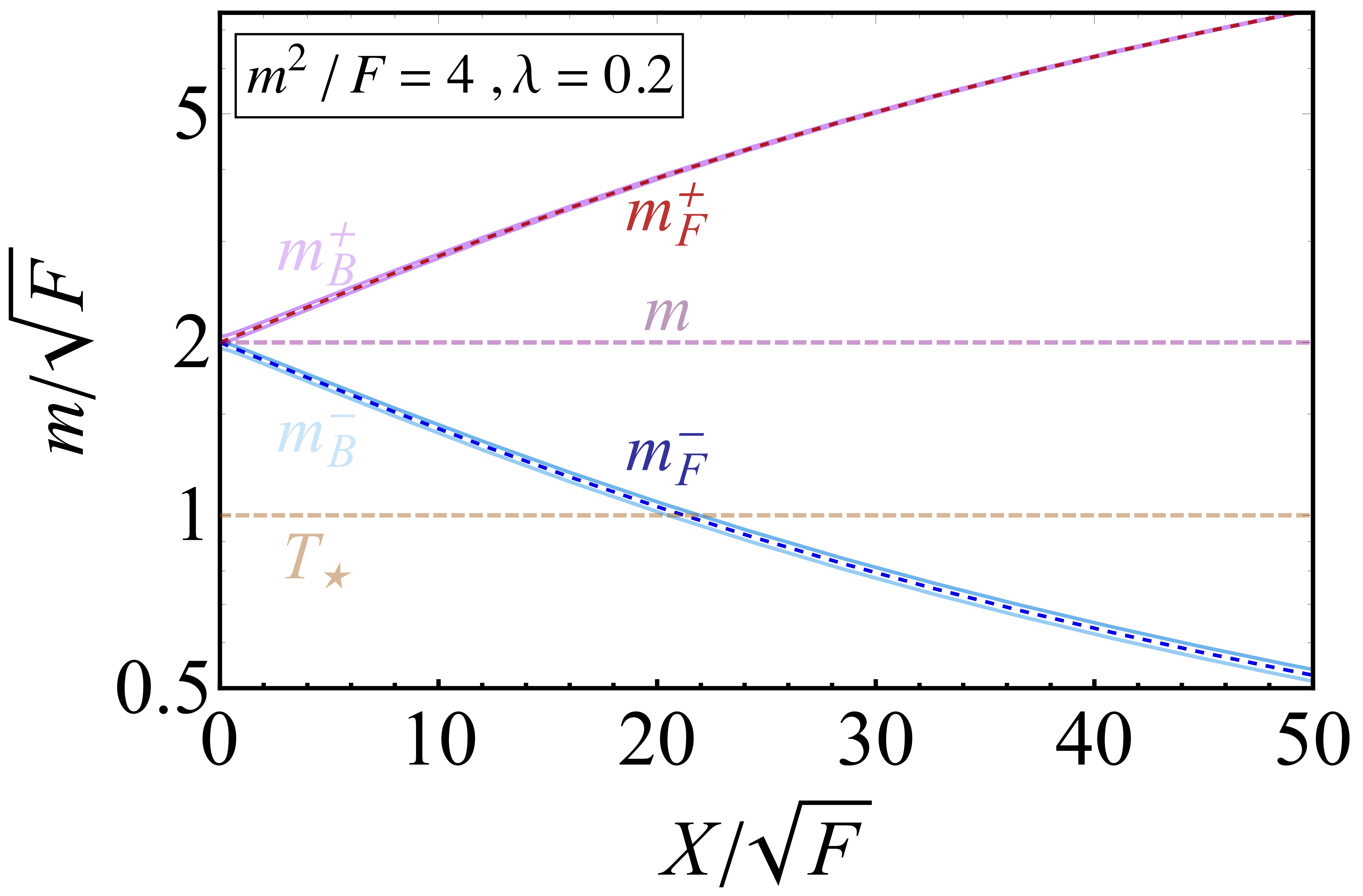
- In the vacuum $\phi_i = 0$ and $\tilde{\phi}_i = 0$ with X undetermined (tree-level flat direction)

- Quantum corrections lift the X direction, and stabilize it to $X = 0$ at zero temperature

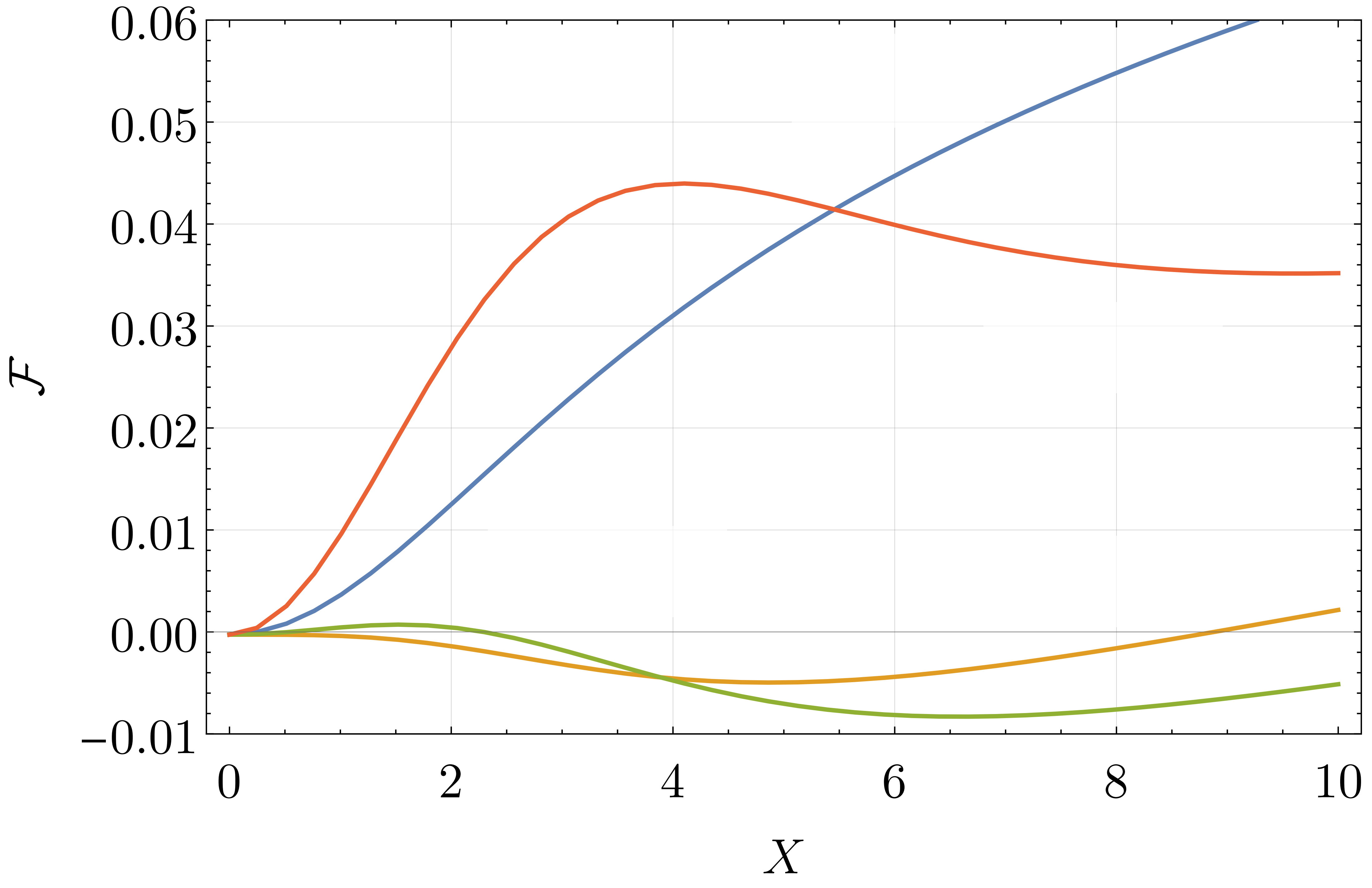
Craig, Levi, Mariotti, Redigolo [2011.13949] JHEP

Inverse bubbles from SUSY breaking

- Spectrum of the theory:



- Effective one-loop, finite-T potential for X:



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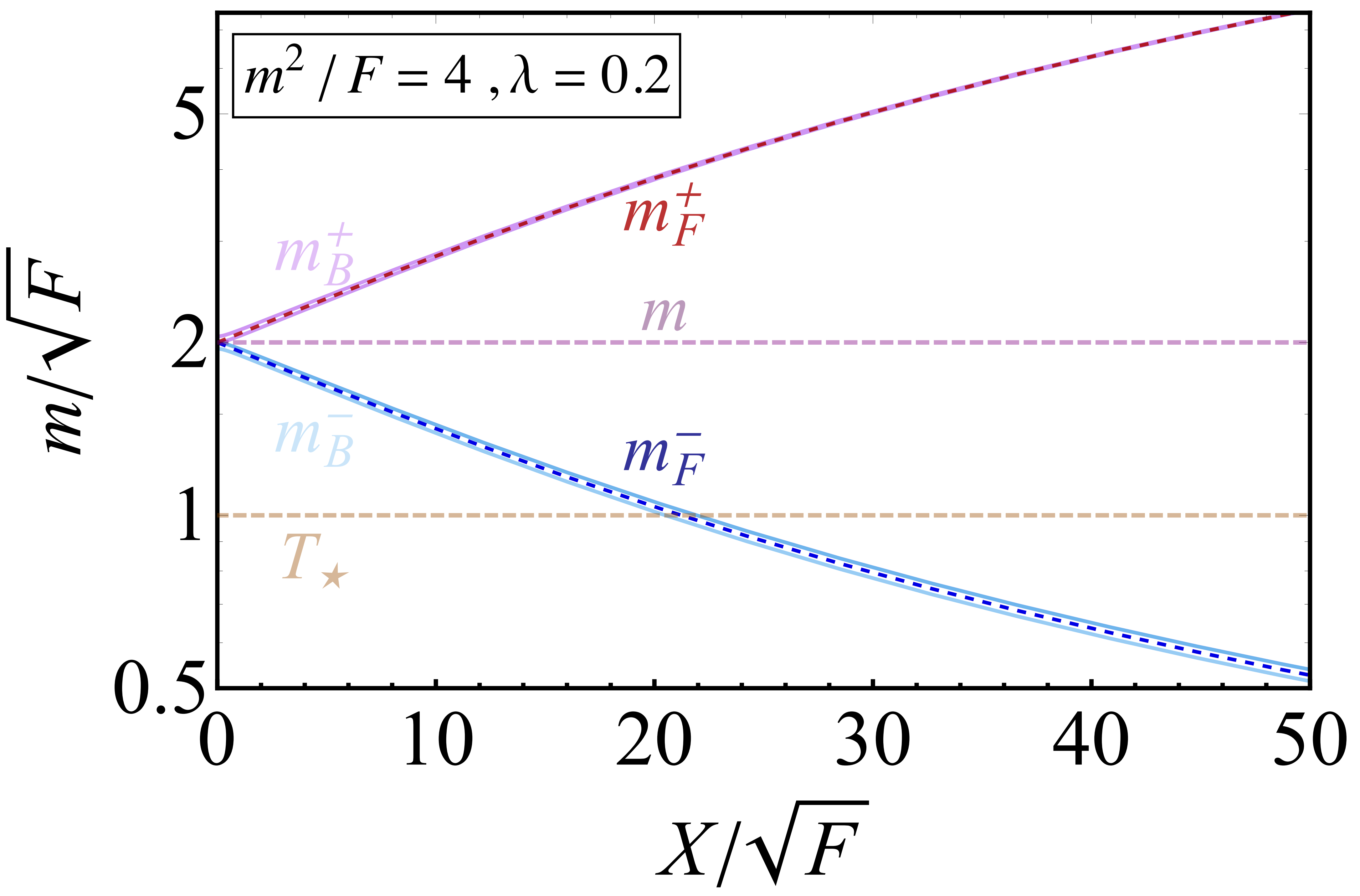

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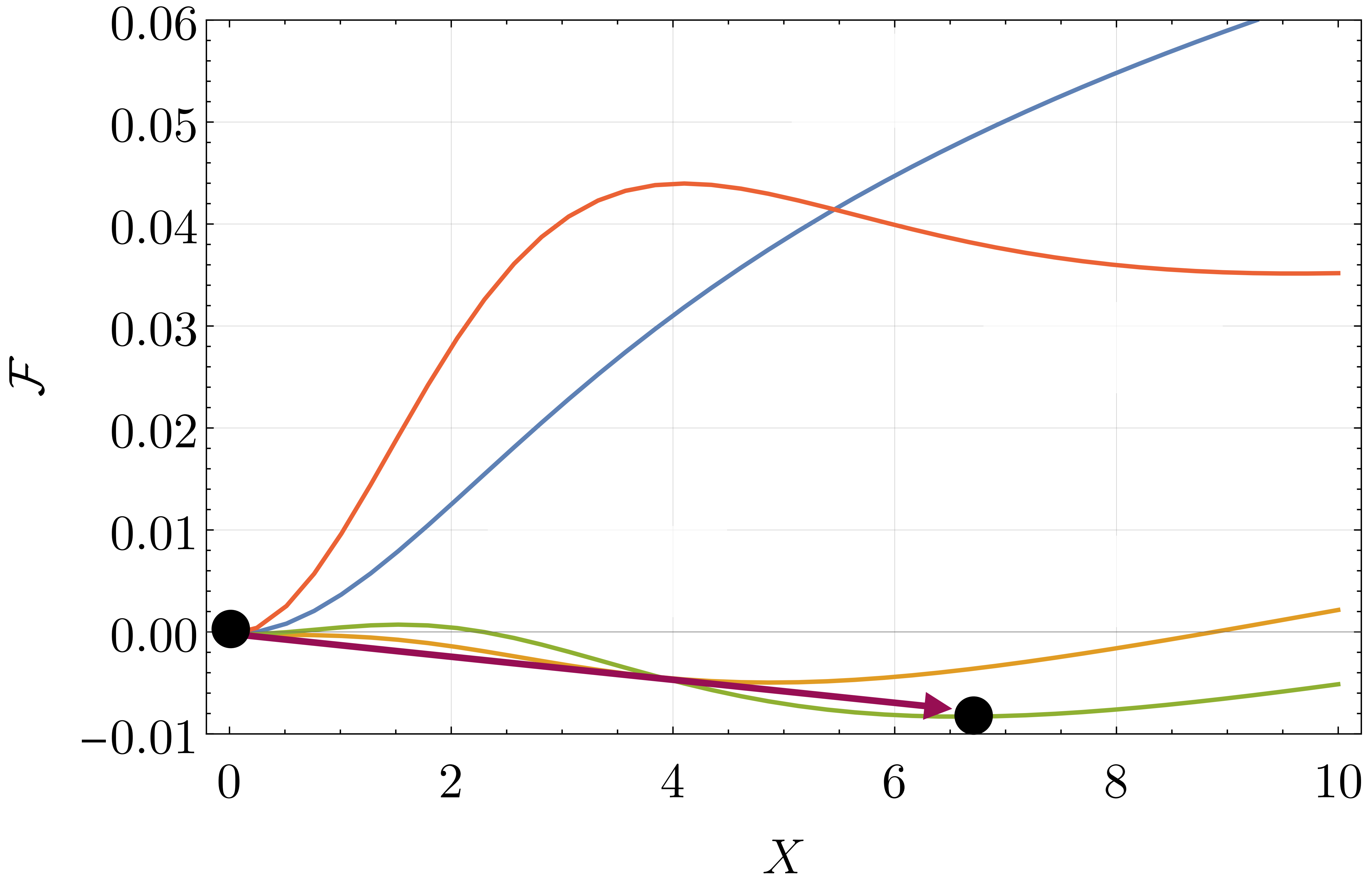


R-symmetry breaking first order PT

Craig, Levi, Mariotti, Redigolo [2011.13949] JHEP

Hydrodynamics of R-symmetry breaking

- System controlled by the free energy $\mathscr{F}(T) = -\,p(T)$ evaluated at 1-loop
- Solve the matching conditions without approximations in terms of $T_\pm$:

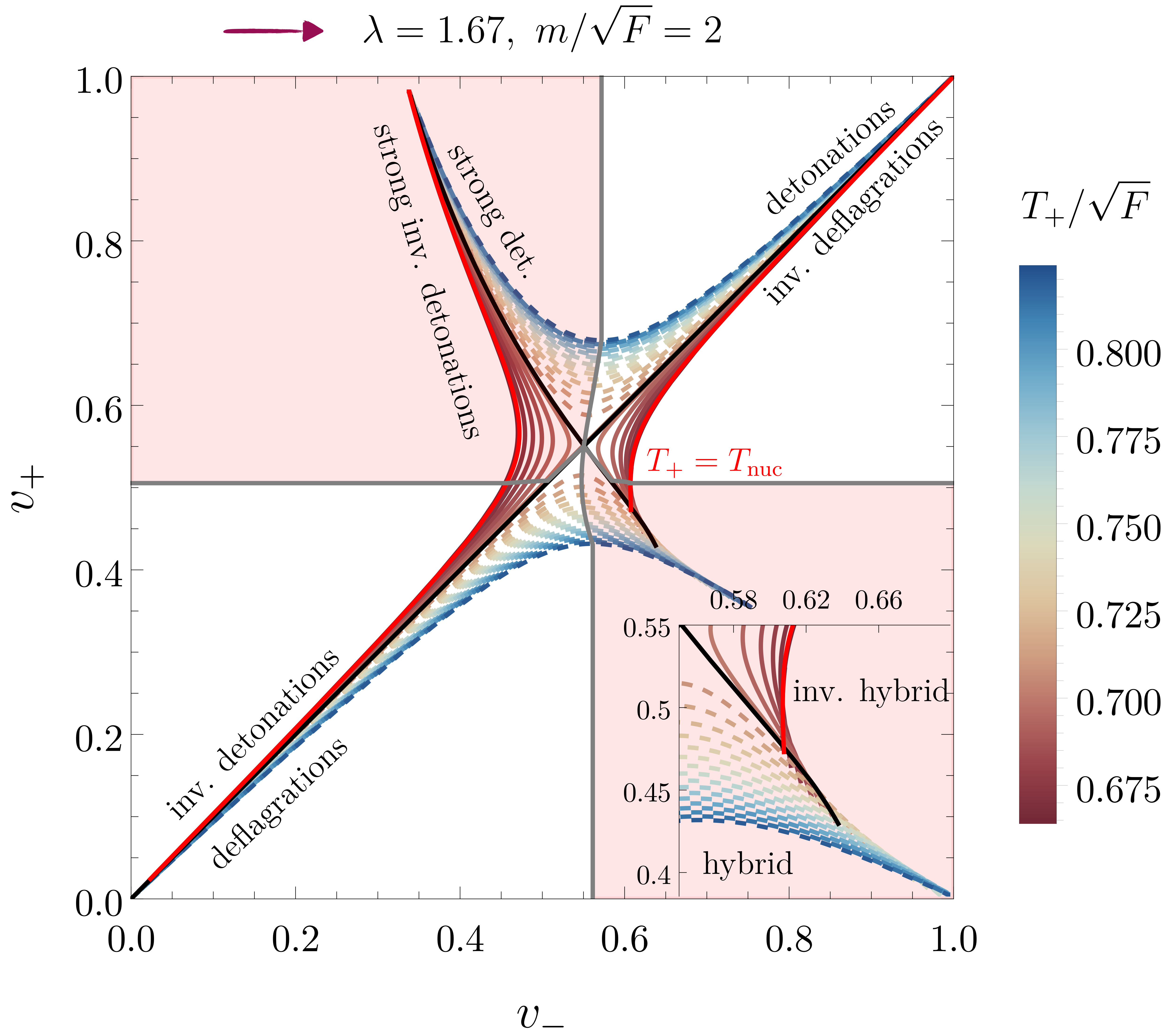
$$v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}\ , \qquad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}\ , \qquad e = T\,\frac{dp}{dT} - p$$

- Hydrodynamics controlled by the sign of a generalized pseudo-trace:

$$\alpha_\vartheta \equiv \frac{4D\vartheta}{3w_+(T_+)} \equiv \frac{4\left(De(T_+) - \frac{\delta e}{\delta p}(T_+, T_-)Dp(T_+)\right)}{3w_+(T_+)}$$

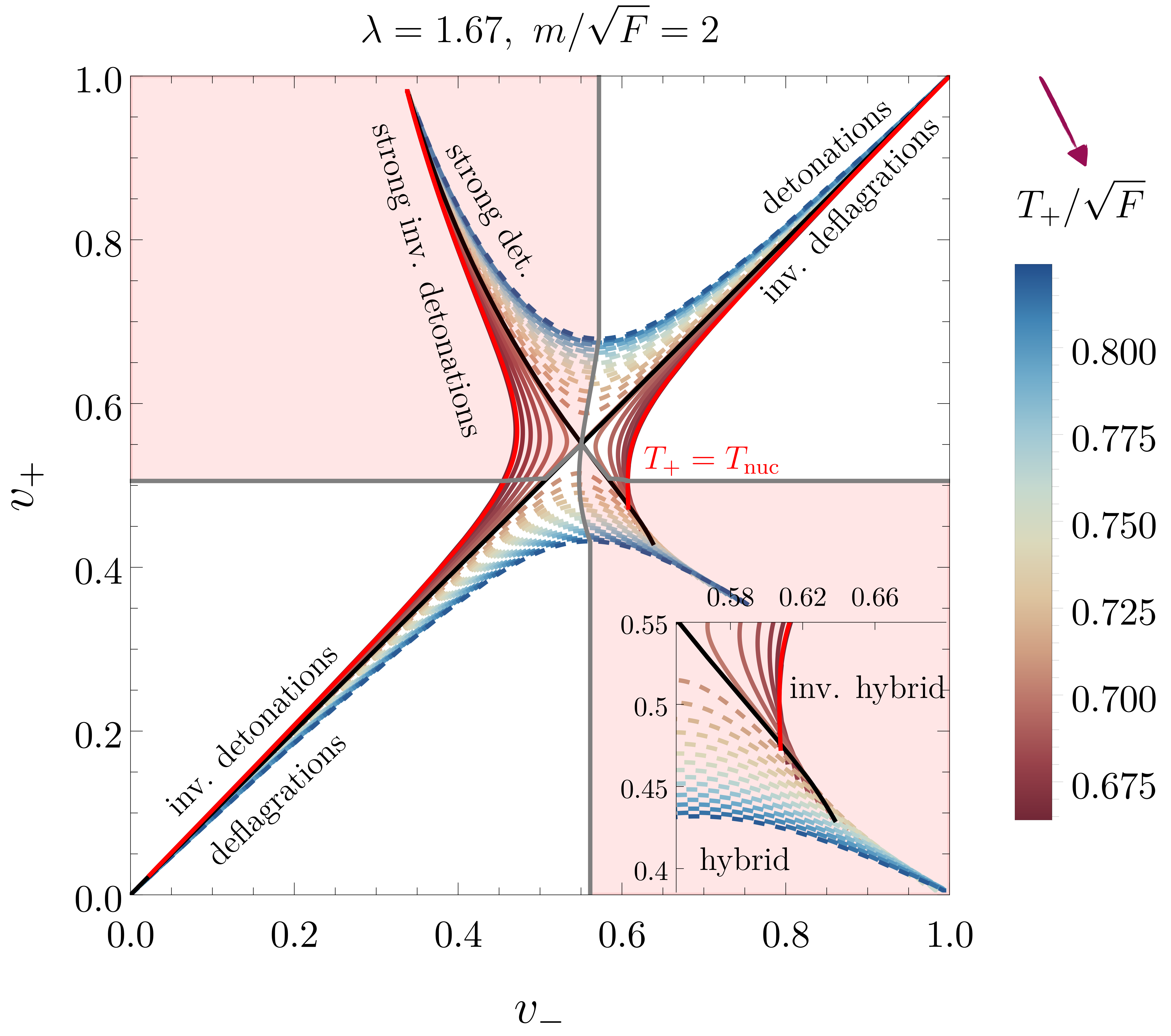
Hydrodynamics of R-symmetry breaking

- Fix particle physics parameters



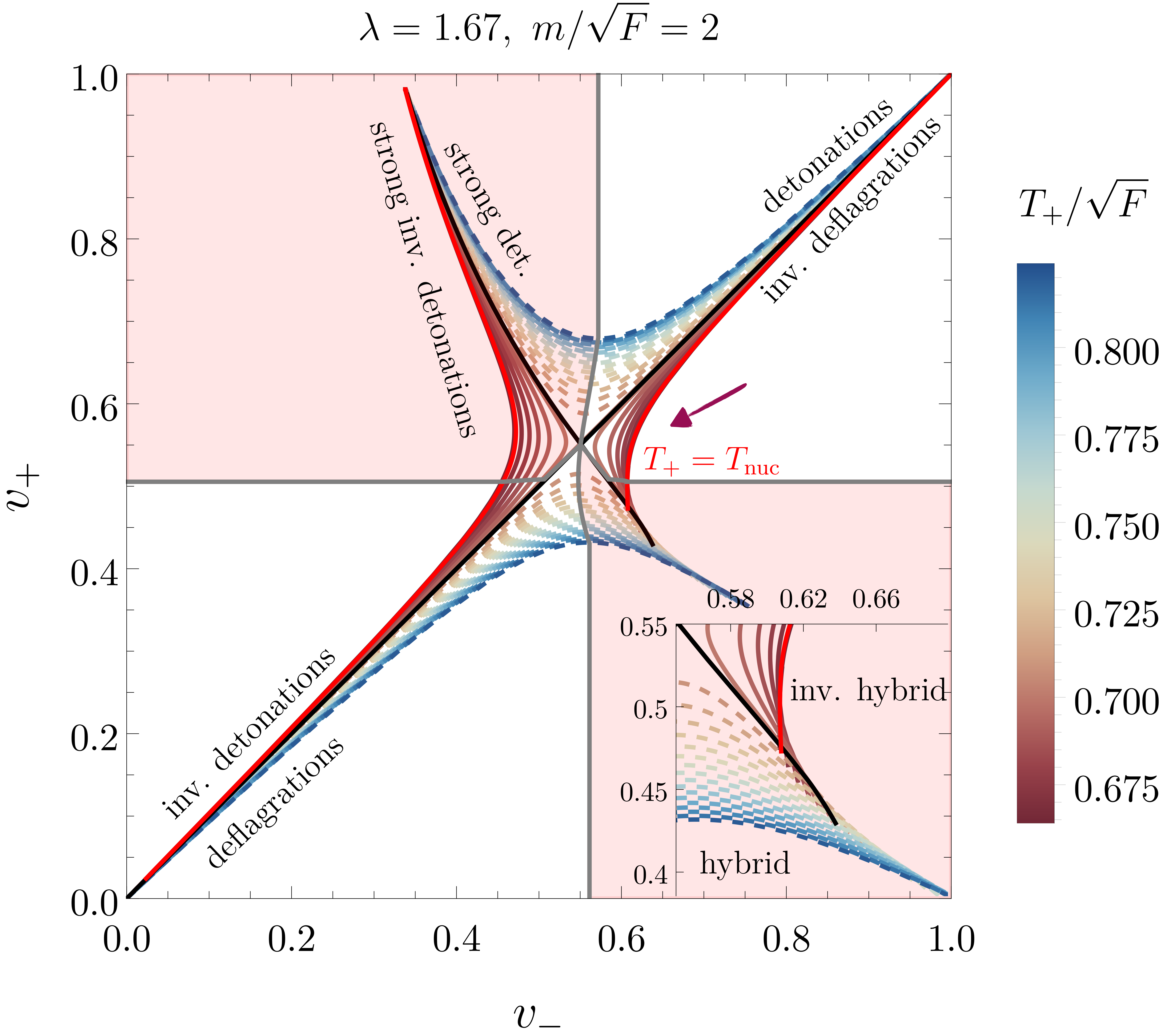
Hydrodynamics of R-symmetry breaking

- Fix particle physics parameters
- As a function of T_+ different (v_-, v_+) regions are populated



Hydrodynamics of R-symmetry breaking

- Fix particle physics parameters
- As a function of T_+ different (v_-, v_+) regions are populated
- Evaluation of the nucleation rate selects the right temperature and right branch



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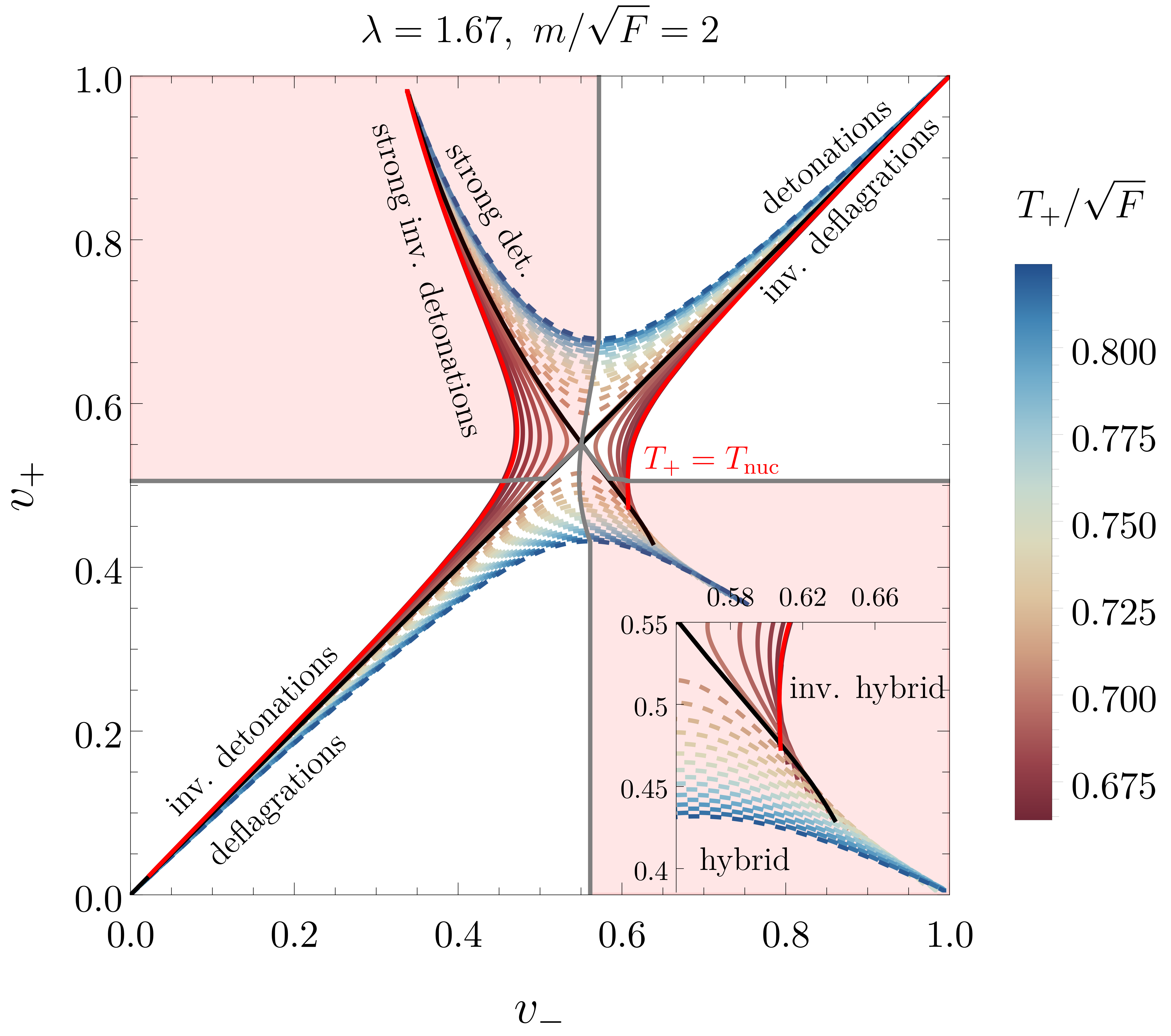
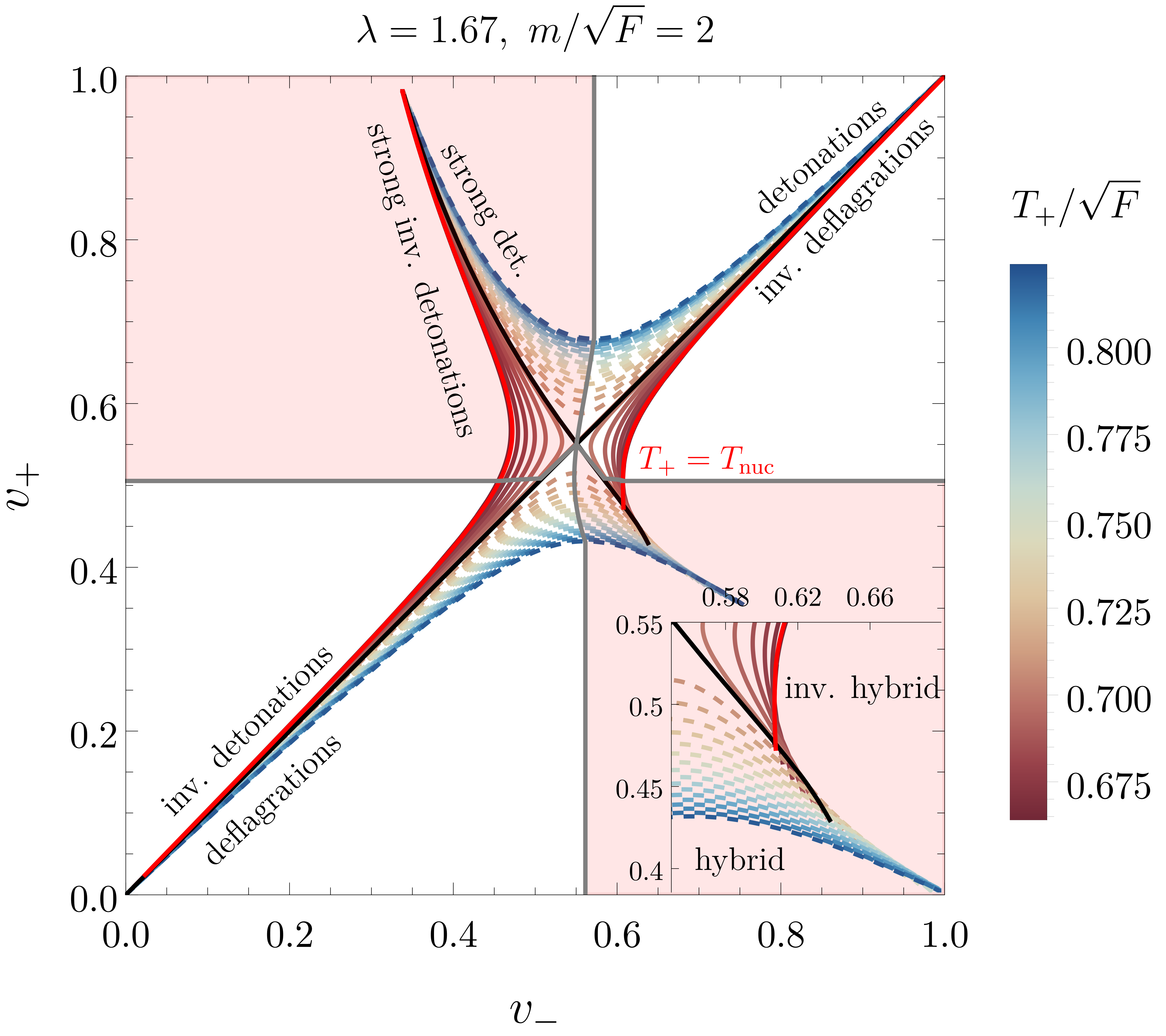


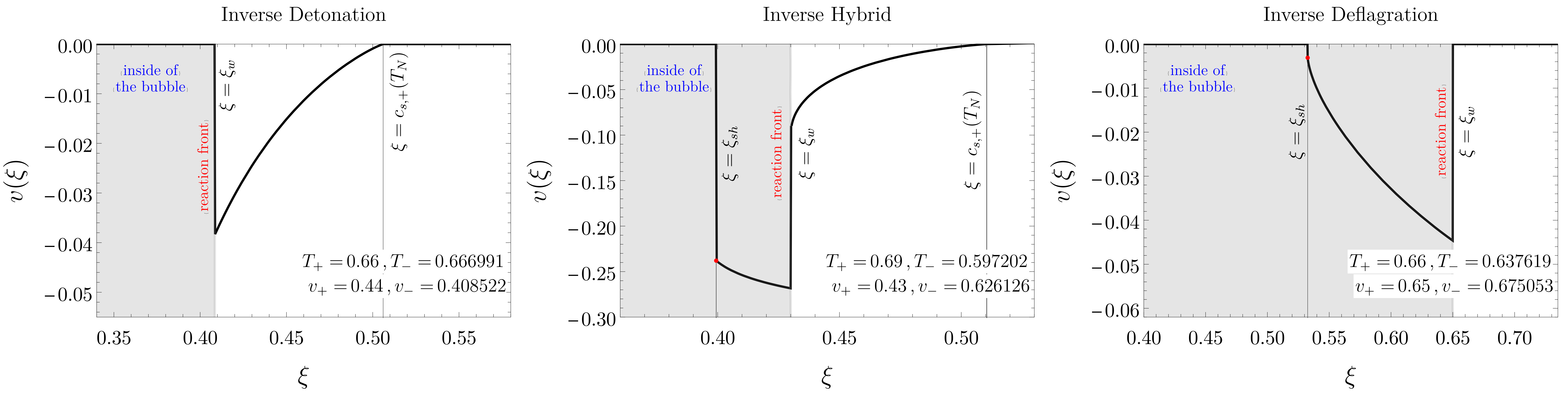
Hydrodynamics of R-symmetry breaking

- Fix particle physics parameters
- As a function of T_+ different (v_-, v_+) regions are populated
- Evaluation of the nucleation rate selects the right temperature and right branch
- Sign of α_ϑ controls the “inverseness”
- Given the wall velocity, hydro is fully specified



Hydrodynamics of R-symmetry breaking

- Full (1-loop) fluid profiles with temperature-dependent speed of sound:



Strength of inverse transitions

- Upper bound on the pseudotrace in terms of the change in the effective number of relativistic degrees of freedom in the two phases:

$$\alpha_\vartheta < 0 \;: \qquad \frac{3}{4}|\alpha_\vartheta| < \frac{\omega_-(T_+) - \omega_+(T_+)}{\omega_+(T_+)} = \frac{\Delta a_{\text{eff}}(T_+)}{a_{\text{eff},+}(T_+)}$$

vs

$$\alpha_\vartheta > 0 \;: \qquad \alpha_\vartheta \sim \frac{\Delta\epsilon}{a_{\text{eff},+}T_+^4}$$

(controlled by supercooling)

Summary & Outlook

- First hydrodynamical study of inverse phase transition (with negative latent heat) in the bag EoS: new fluid solutions!
- Fraction of initial energy converted into bulk fluid motion: key input for gravitational wave emission (GW)
- Explicit (SUSY) realization of this new hydrodynamics in standard cooling cosmology
- Non SUSY realizations?
- Evaluation of the bubble wall velocity?
- Characterization of the GW signal?

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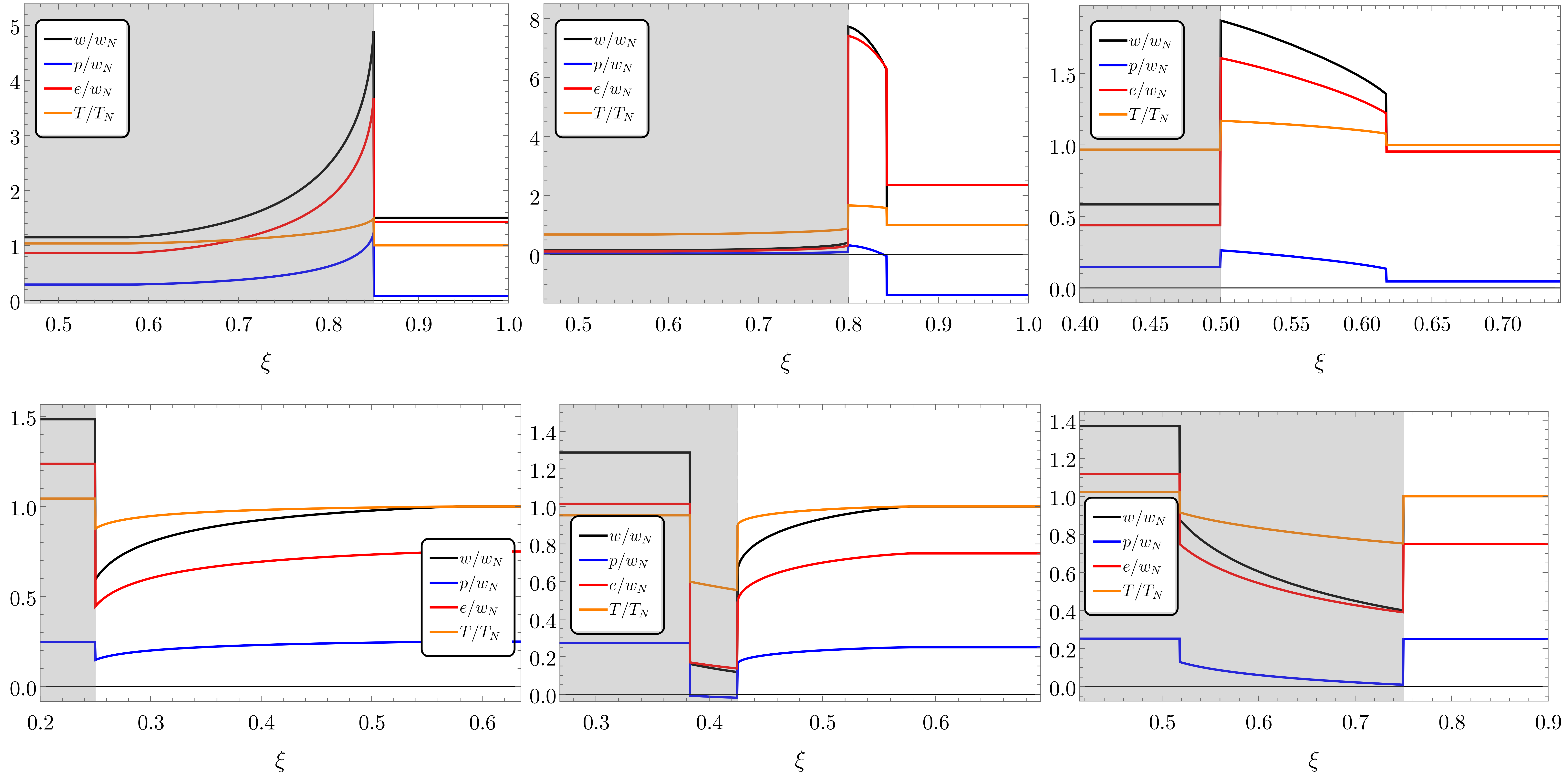
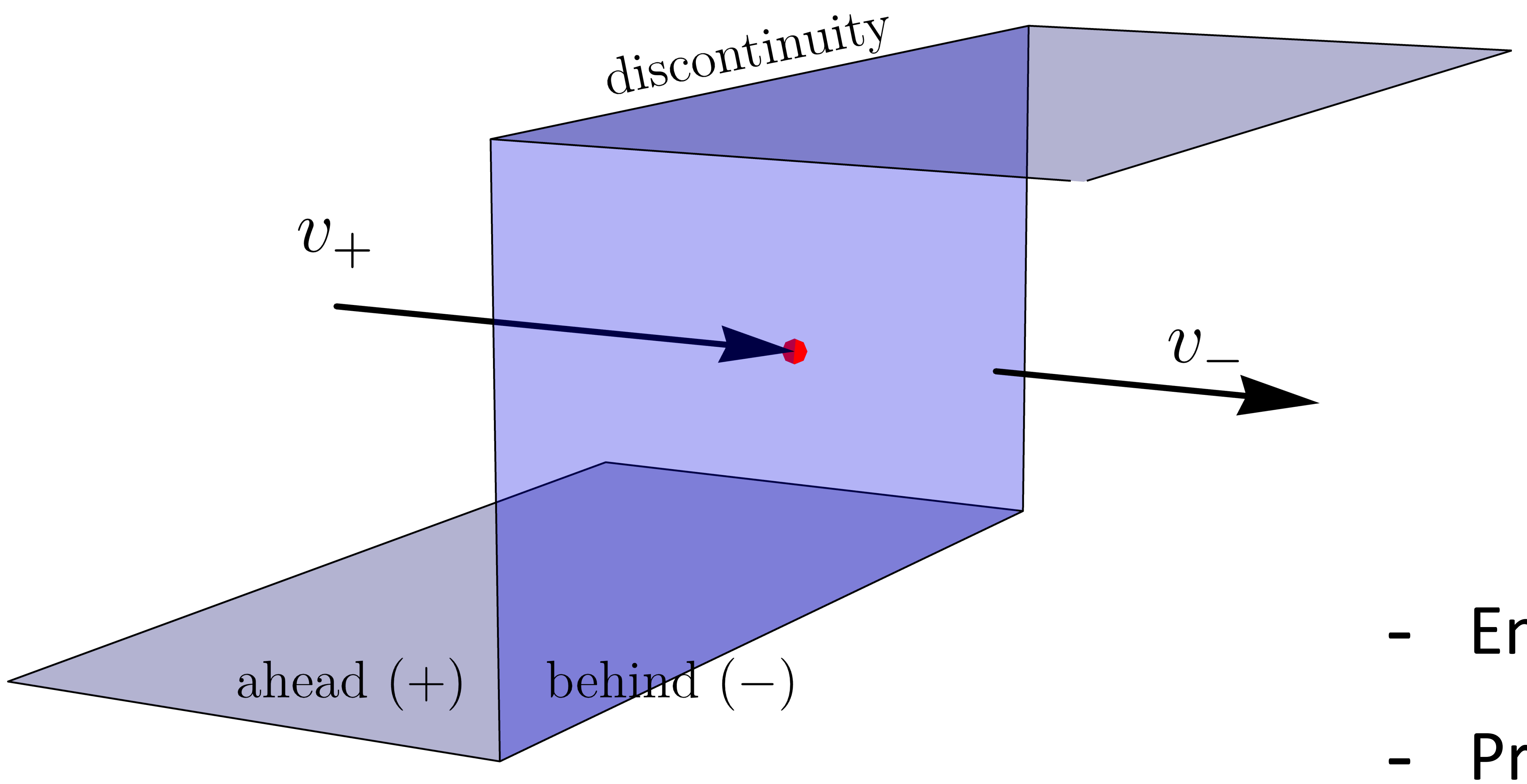
Thank you!

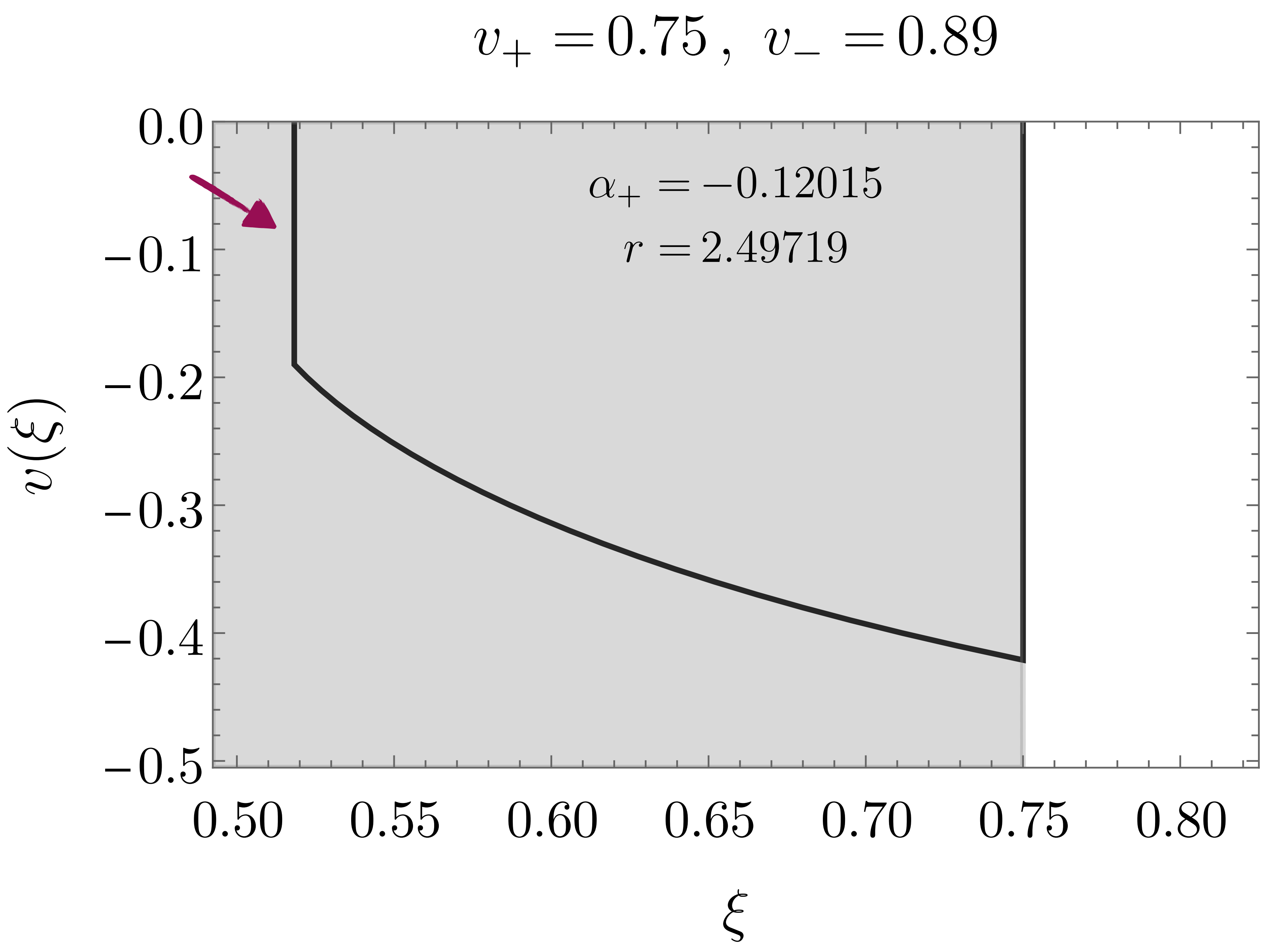
Backup



Subsonic shocks?

Condition for the shock: $v_+ v_- = \dfrac{1}{3}$

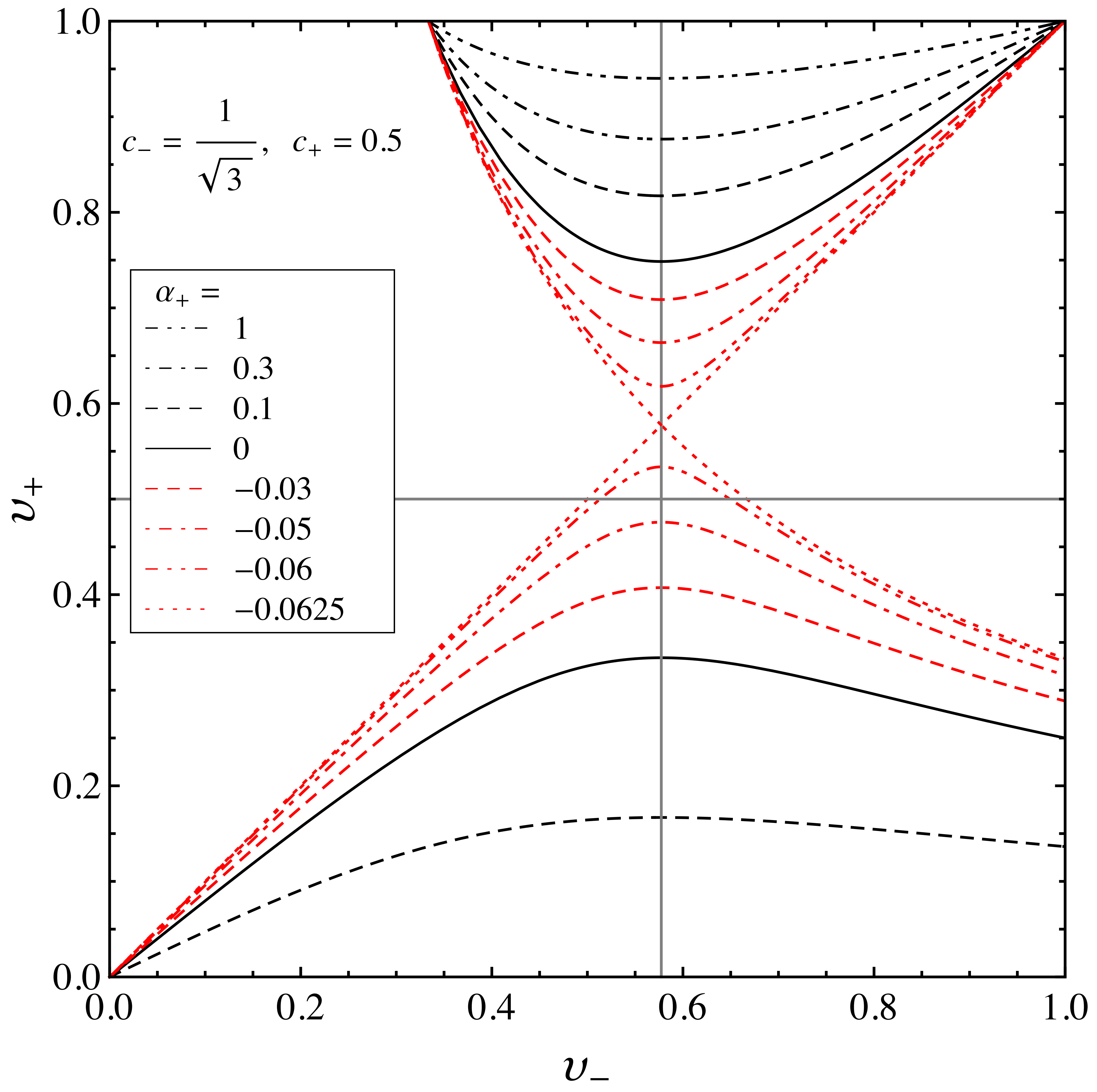




- Enthalpy must increase
- Pressure must increase
- Flow is supersonic ahead of the shock and subsonic behind

$\nu-$model?

Leitao, Mégevand [1410.3875] Nucl. Phys. B



- Modification of the bag EoS that allows for (constant) $c_s^2 \neq 1/3$
- $\alpha_+ < 0$ but the hydro looks like in the direct case…
- One should in fact look at the pseudo trace:

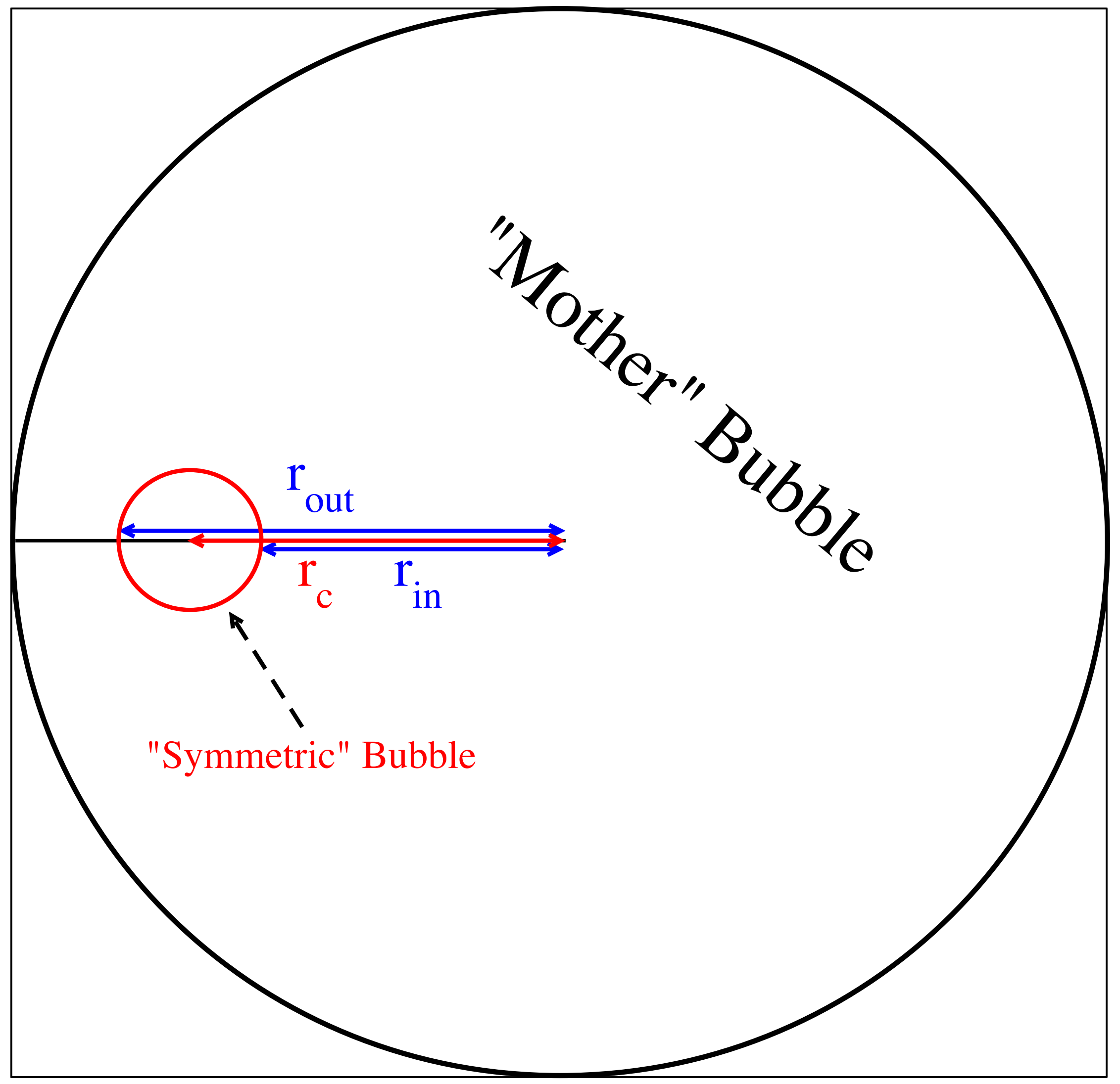
$$\alpha_{\bar{\theta}} = \frac{1}{12}\left(4 - \nu + \frac{3\epsilon\nu}{a_+ T_+^4}\right)$$

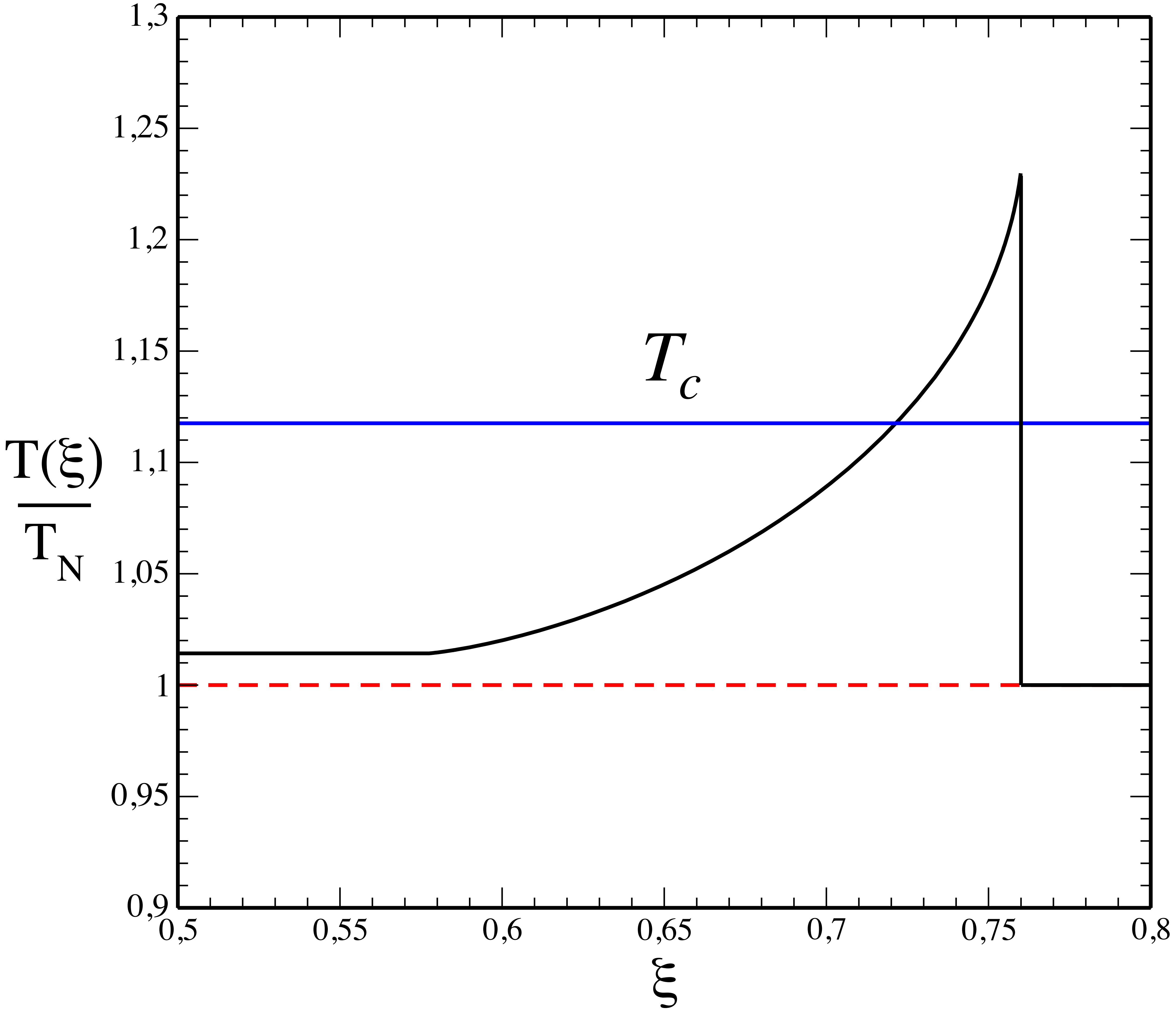
Giese, Konstandin, van de Vis [2004.06995] JCAP

Riding the rarefaction wave

Caprini, No [1111.1726] JCAP

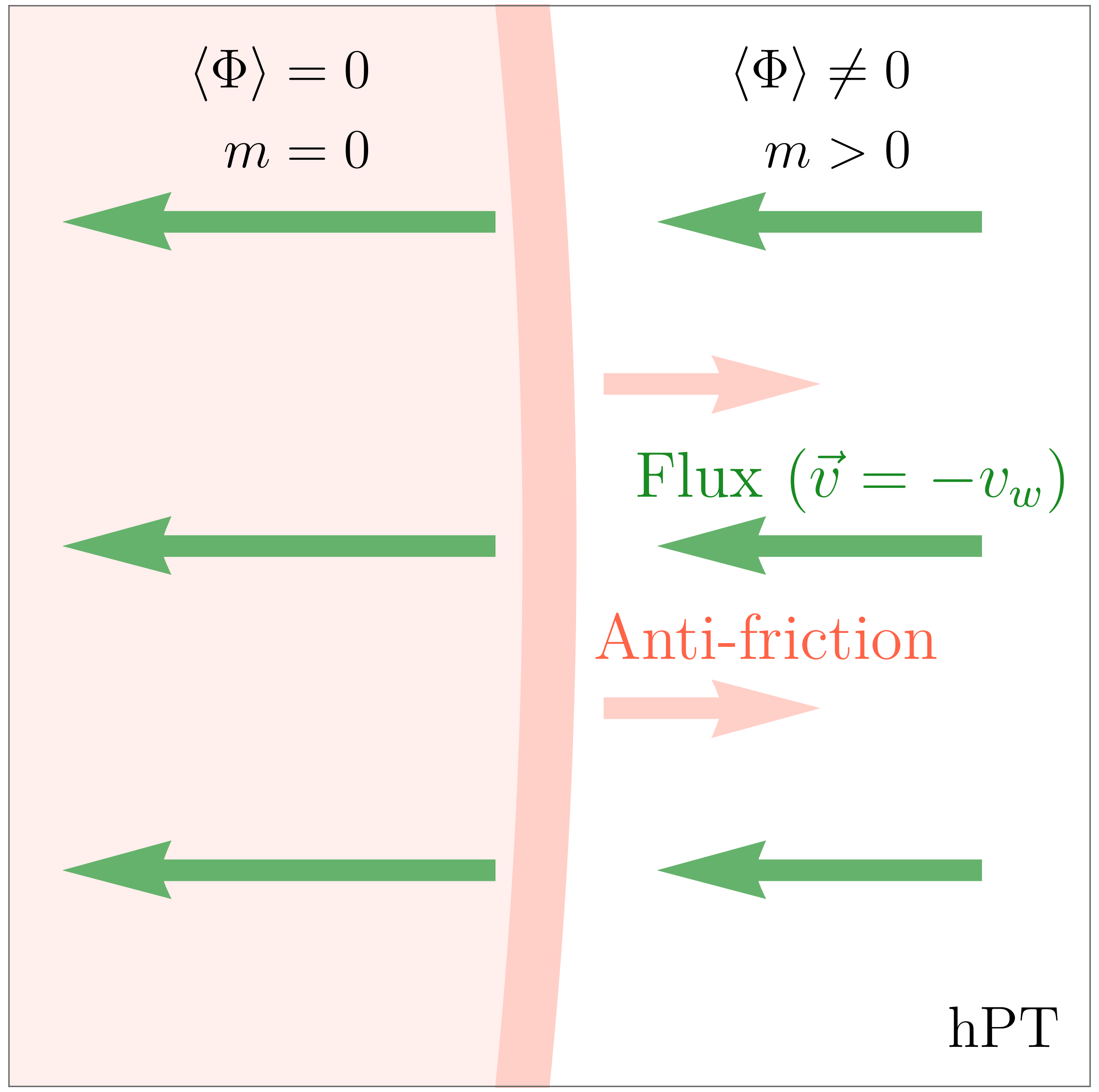
- Along the detonation front, T can increase compared to T_N
- If the temperature rises above T_c the old phase may become favorable



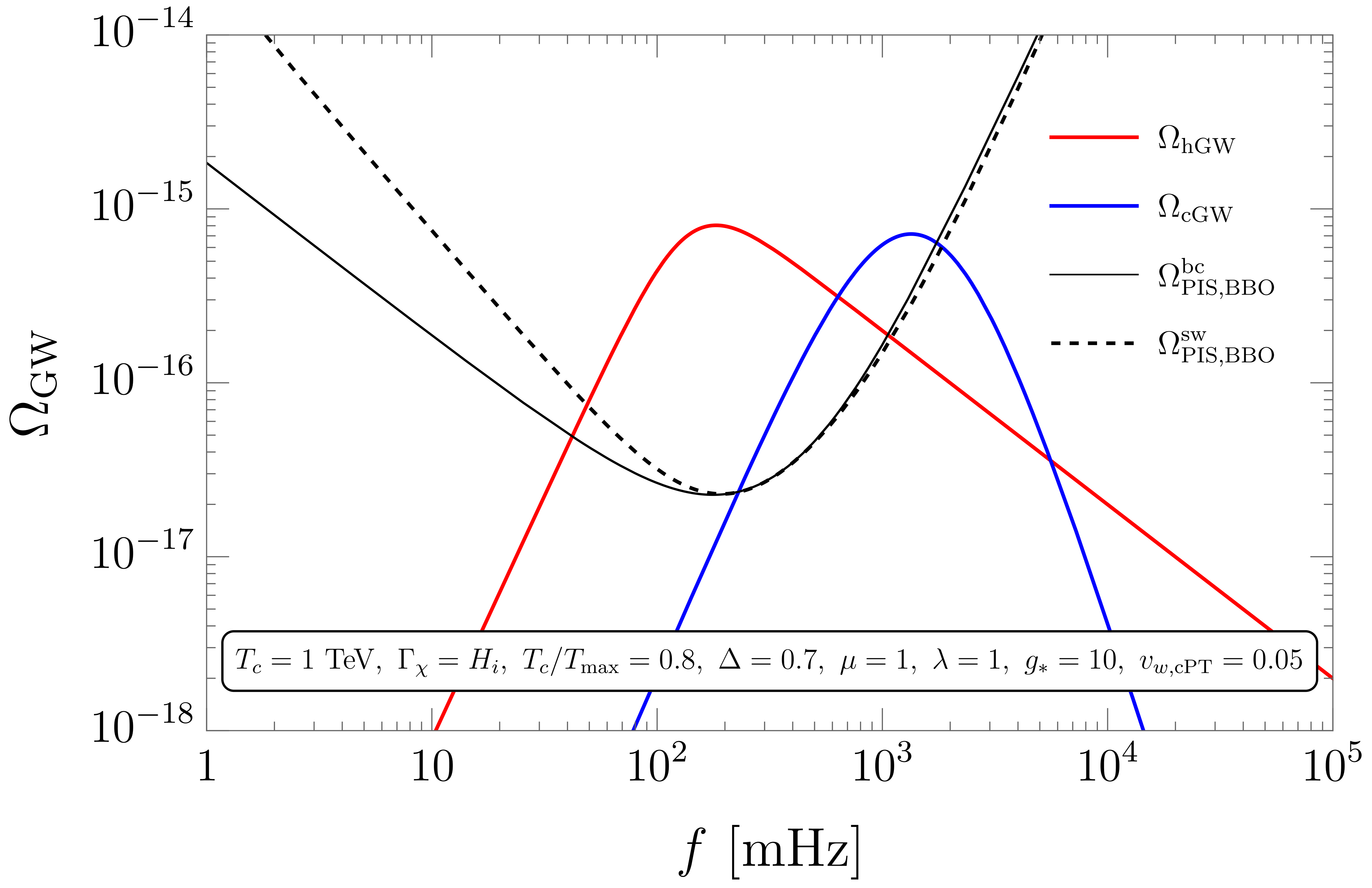


Reheating

- Particles loose mass when crossing the wall: “anti friction”



- **hPT** typically runaway (?) GWs from bubble collisions, while **cPT** terminal velocity



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