# BUBBLE NUCLEATION IN STRONGLY COUPLED QUANTUM FIELD THEORIES

## WITH HOLOGRAPHY

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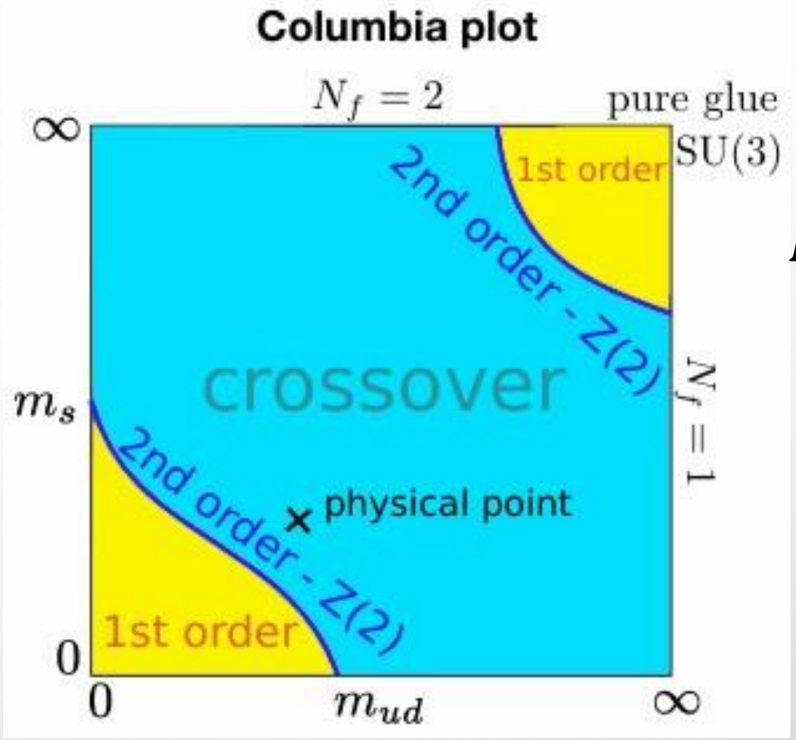
# DARK SU(N) YANG-MILLS THEORY


$$n_f = 0$$

$$M_G \sim (6-7)\Lambda_d$$

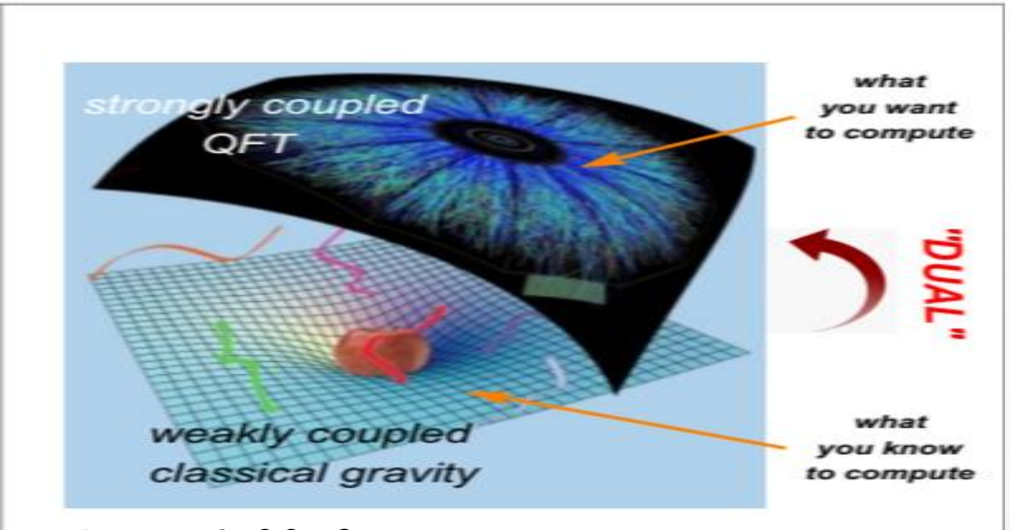
$$n_f > 0$$

Jana Gunther 2020

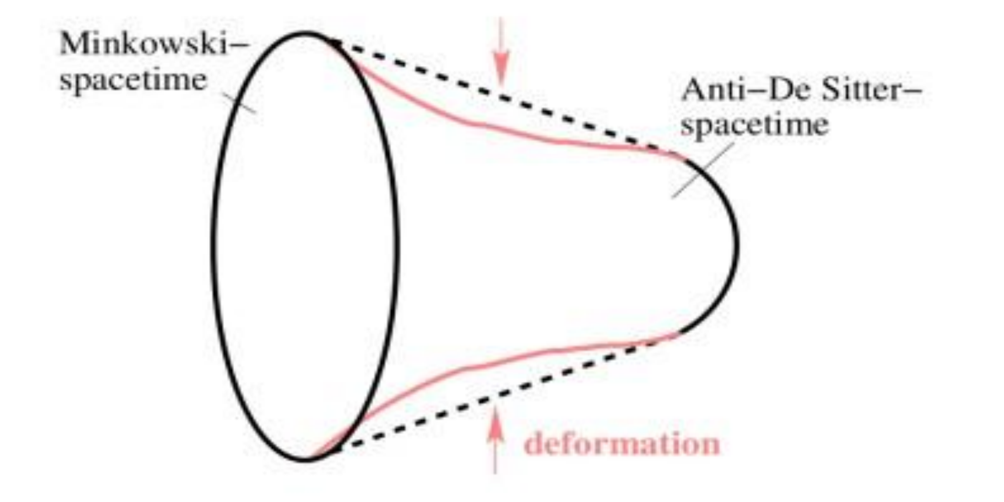
$$M_\pi \sim \Lambda_d * m_q$$

- Non-Abelian SU(N) dark sector, confinement scale $\Lambda_d$, with $n_f$ light/massless quarks.
- However, the confinement phase transition takes place when the theory is strongly coupled

# How to proceed with FOPTs at strong coupling ?




Baggioli 2019




Deformation of $AdS_5, \mathcal{S}_5$
Breaking of Conformal/SUSY.

We will make use of Improved Holographic QCD

$$\mathcal{S}_5 = -M_p^3 N_c^2 \int d^5 x \sqrt{g} \left( R - \frac{4}{3} (\partial \Phi)^2 + V(\Phi) \right) + 2 M_p^3 \int_{\partial \mathcal{M}} d^4 x \sqrt{h} \mathcal{K}$$

| | |
|---|---|
| • AdS5 Einstein Dilaton Gravity | • 4D "QFT"/CFT |
| • Radial 5-D coordinate r | • RG Scale |
| • Scalar field $\lambda = e^{\Phi}$ | • T'hooft coupling $\lambda_t = N_c g_{YM}^2$ |
| • Different Geometries | • Phases of YM theory |

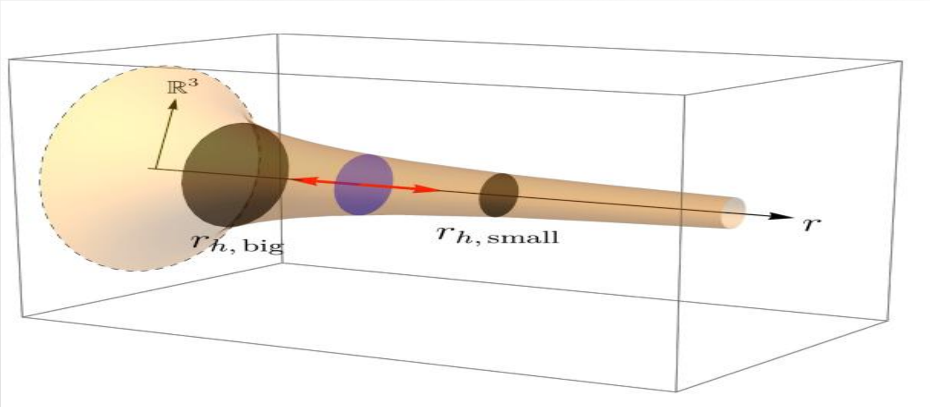
Main Idea: Use the Gauge/Gravity Dualtiy to construct a 4D QFT which resembles SU(3) Yang Mills Theory out of a D+1 Gravitational Theory in asymptotically AdS5

# Background Solutions, Thermodynamics

## Asymptotically AdS Vacuum Solutions T=0.

$$\boldsymbol{b}(\boldsymbol{r}) \approx \frac{\mathbf{l}}{\boldsymbol{r}}\left(1+\frac{4}{9}\frac{1}{\boldsymbol{\log \Lambda r}}+\cdots\right)$$

$$\boldsymbol{\lambda}(\boldsymbol{r}) \sim -\frac{1}{\mathbf{log}\,\boldsymbol{\Lambda r}}$$




## Thermal Graviton Gas Solution Confined Phase

$$ds^2 = b_0^2(r)(dr^2 - dt^2 + dx^m dx_m)$$

$$\Phi = \Phi_0(r), \qquad r \in (0, \infty)$$

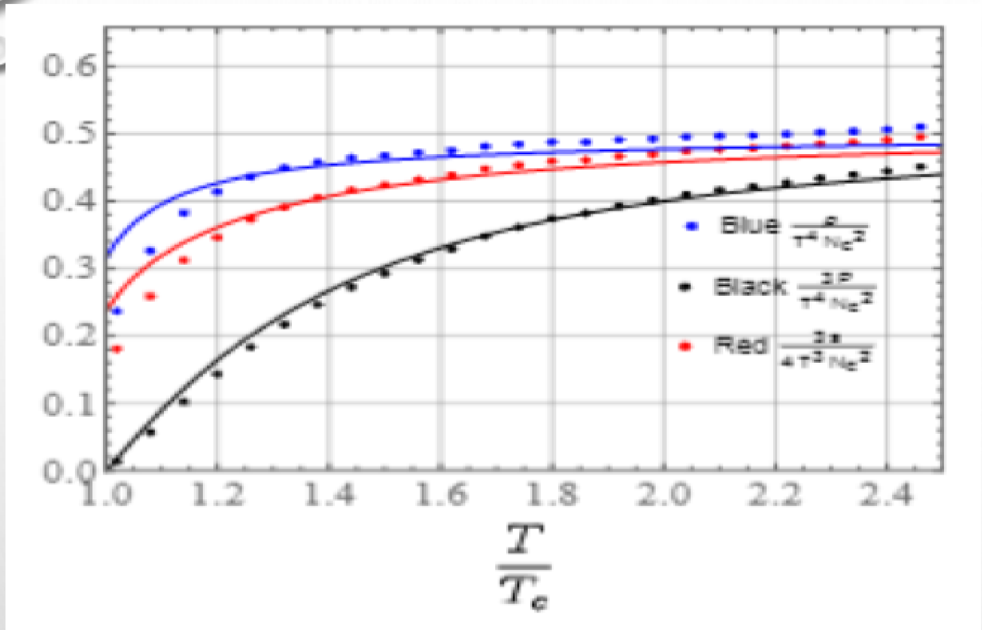
## AdS Schwarzchild BH solution Deconfined Phase

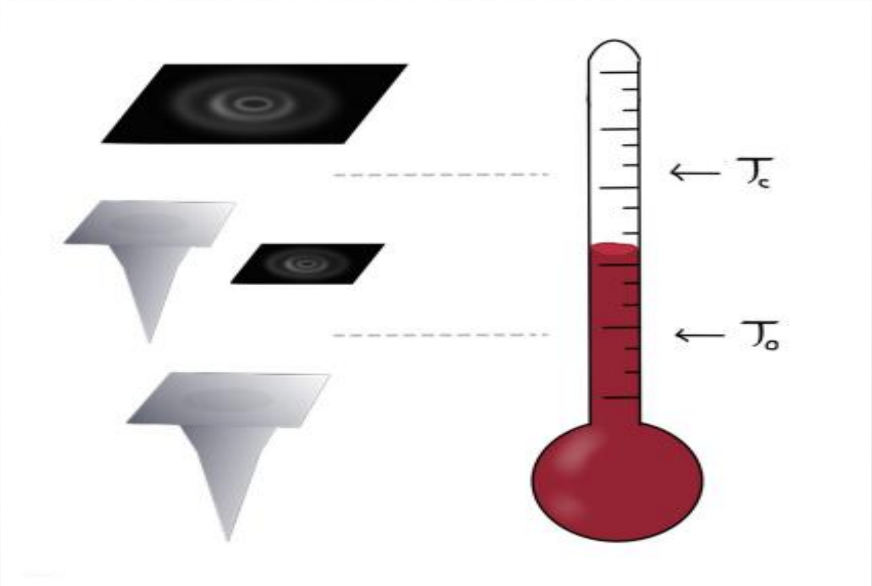
$$ds^2 = b^2(r)\left(\frac{dr^2}{f(r)} - f(r)dt^2 + dx^m dx_m\right)$$

$$\Phi = \Phi(r), \qquad r \in (0, \mathrm{r_h}), \qquad f(r_h) = 0$$

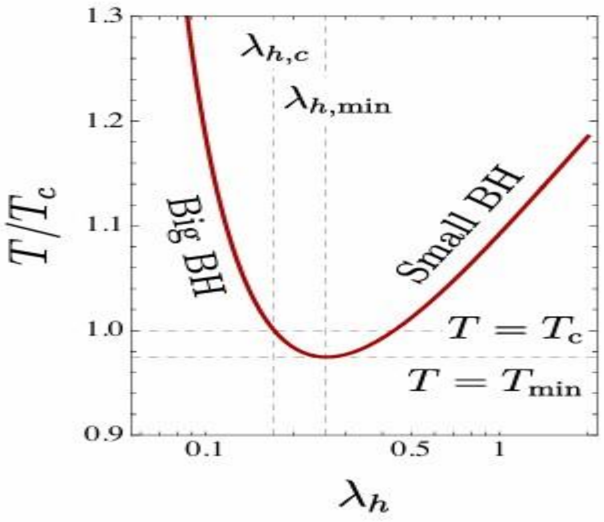
- Temperature, Time periodicity $\tau \to \tau + \frac{1}{T}$.
- $\mathrm{S} = \frac{\text{Area}}{4\mathrm{G}_5} = 4\pi \mathrm{M_p^3 N_c^2 V_3 b(r_h)^3}$
- $\mathcal{F} = \frac{\beta}{V_3}(\mathcal{S}_{dc} - \mathcal{S}_{cn.})$
- $T_h \equiv \frac{|\dot{f}(r_h)|}{4\pi} = T$
- TD of 4D Theory <-> Geometry of 5D.
- TD Relations holds!

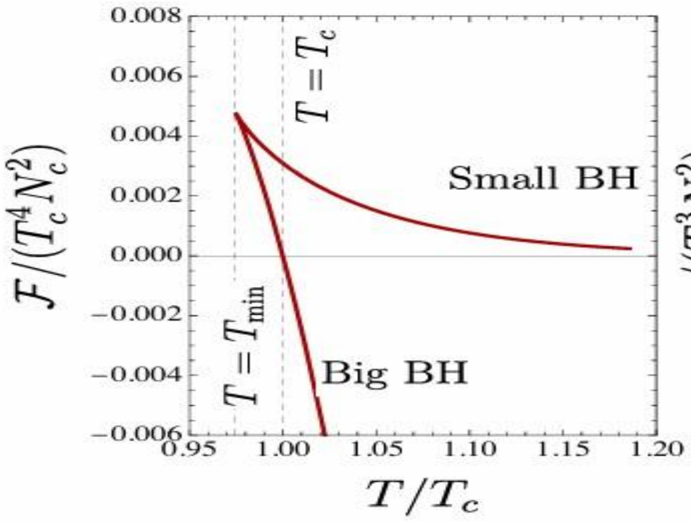
# Equilibrium Thermodynamics

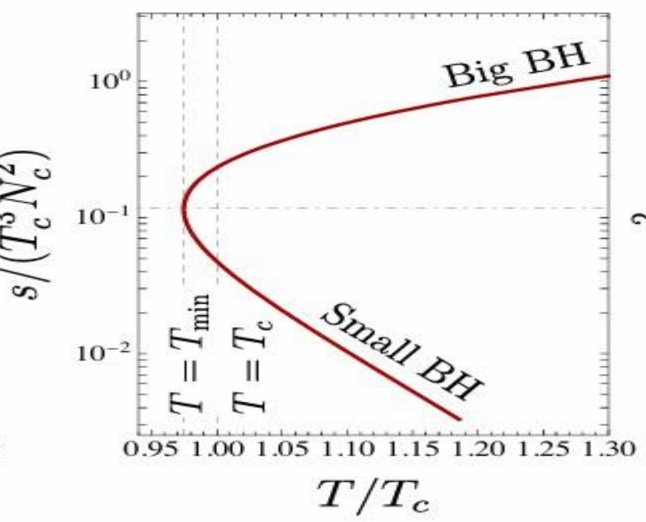


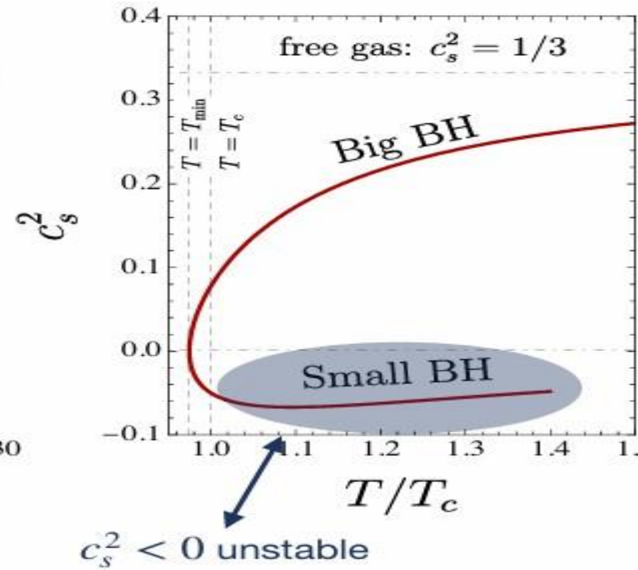




Oliver Dewolfe 2013

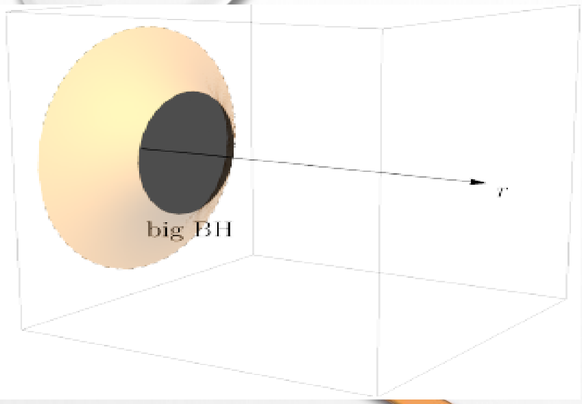


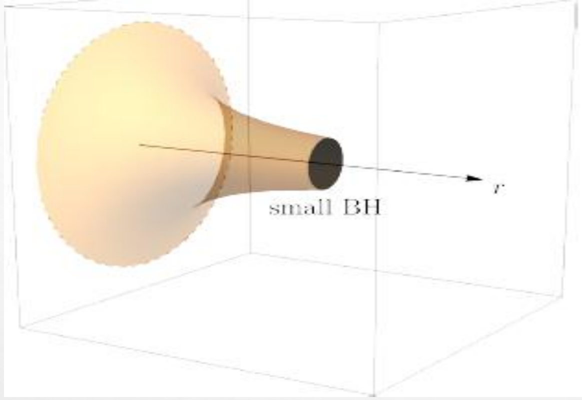



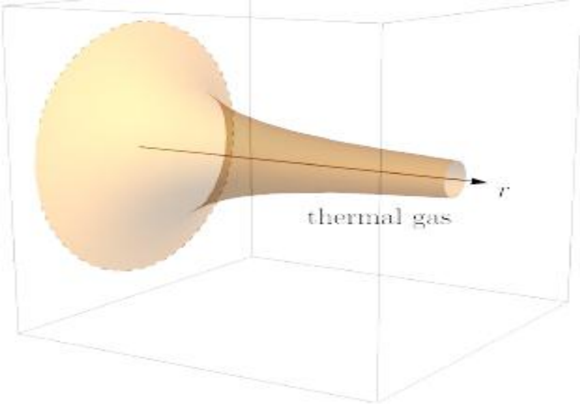



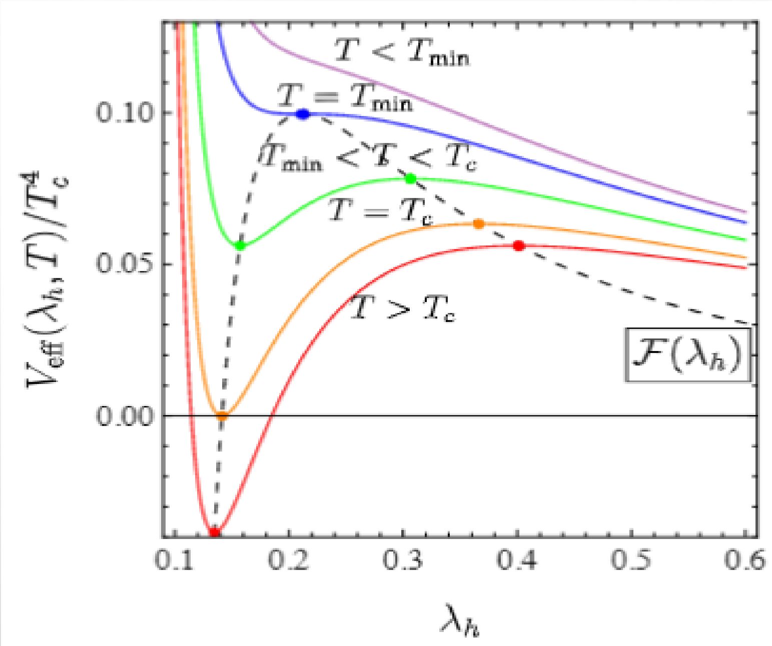
# Confinement PT (Hawking Page PT) & Effective Potential




- Interpolate between BBH and TG
- Choose an order parameter $\boldsymbol{r_h, \lambda_h}$




Calculation of the evaporation of a BBH/ Nucleation of Thermal Gas

$$\Gamma = Ae^{-(\mathcal{S}_{\mathcal{BH}} - \mathcal{S}_{\mathcal{TG}})}$$

Membrane Paradigm

Violate the condition

$T_h \neq T$

1. BH not in global thermal eq.
2. Conical singularity

Confined Phase at zero

$V_{\text{eff}}(\lambda_h, T) \rightarrow 0, \lambda_h$-> $\infty$

$$V_{\text{eff}}(\lambda_h, T) = \mathcal{F}(\lambda_h) - 4\pi M_p^3 N_c^2 b(\lambda_h)^3 \left(1 - \frac{T_h}{T}\right)$$

# PT Parameters & GW's in SU(3) YM

Kinetic term normalization: $c\,\frac{N_c^2}{16\pi^2}\left(\vec{\nabla}\lambda_h\right)^2$

Effective action for $\mathcal{O}(3)$ tunneling configurations

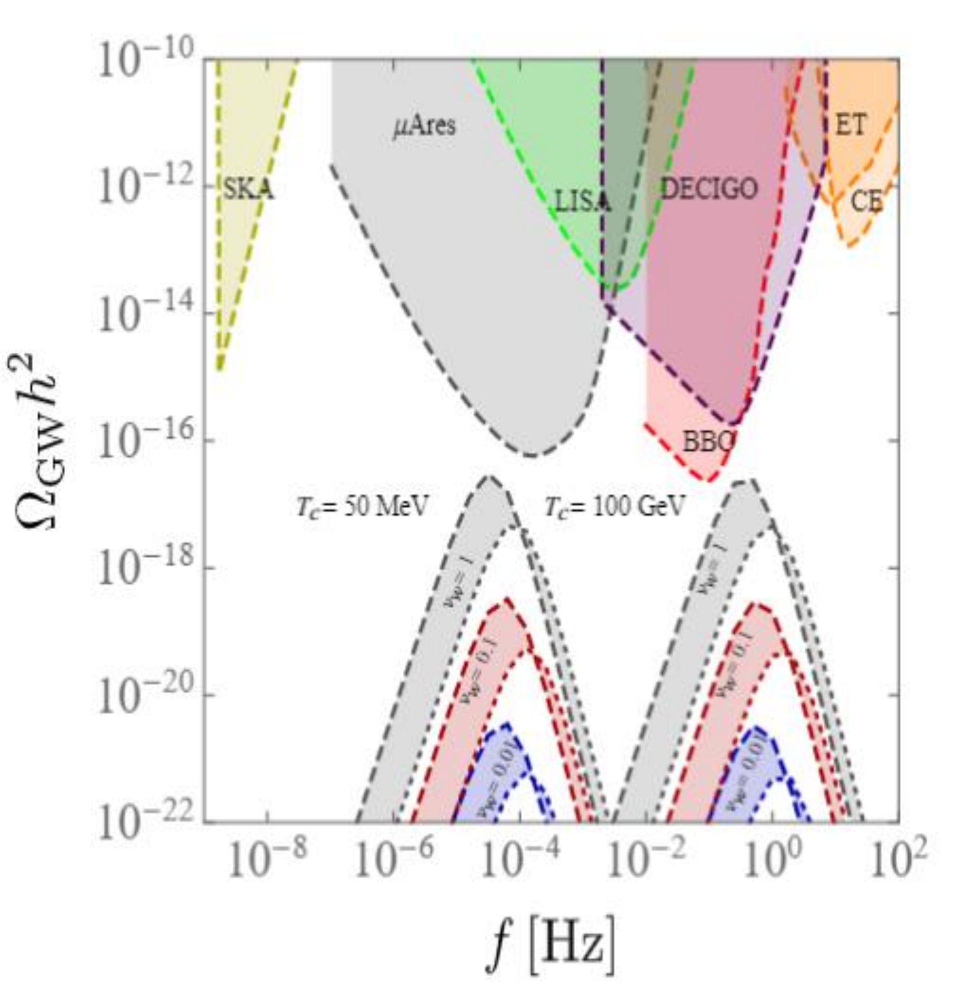
$$\mathcal{S}_\mathcal{B} = \frac{4\pi}{T}\int dr\ r^2\left[c\,\frac{N_c^2}{16\pi^2}\left(\partial_r\lambda_h(r)\right)^2 + V_{\text{eff}}(\lambda_h(r),T)\right]$$

Bubble Nucleation Rate

$\Gamma = T^4\left(\frac{\mathcal{S}_\mathcal{B}}{2\pi}\right)^{3/2} e^{-\mathcal{S}_\mathcal{B}}$, Nucleation $\Gamma \approx H^4$.

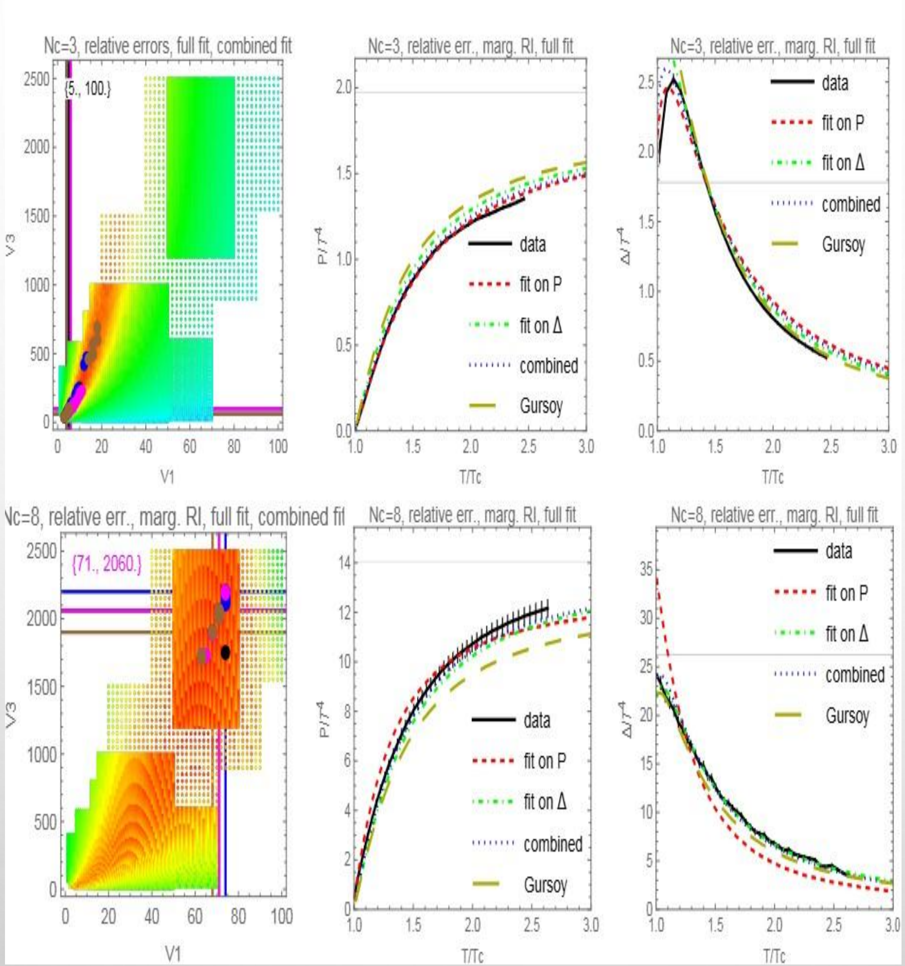
$$T_n \approx T_p \approx 0.99T_c$$

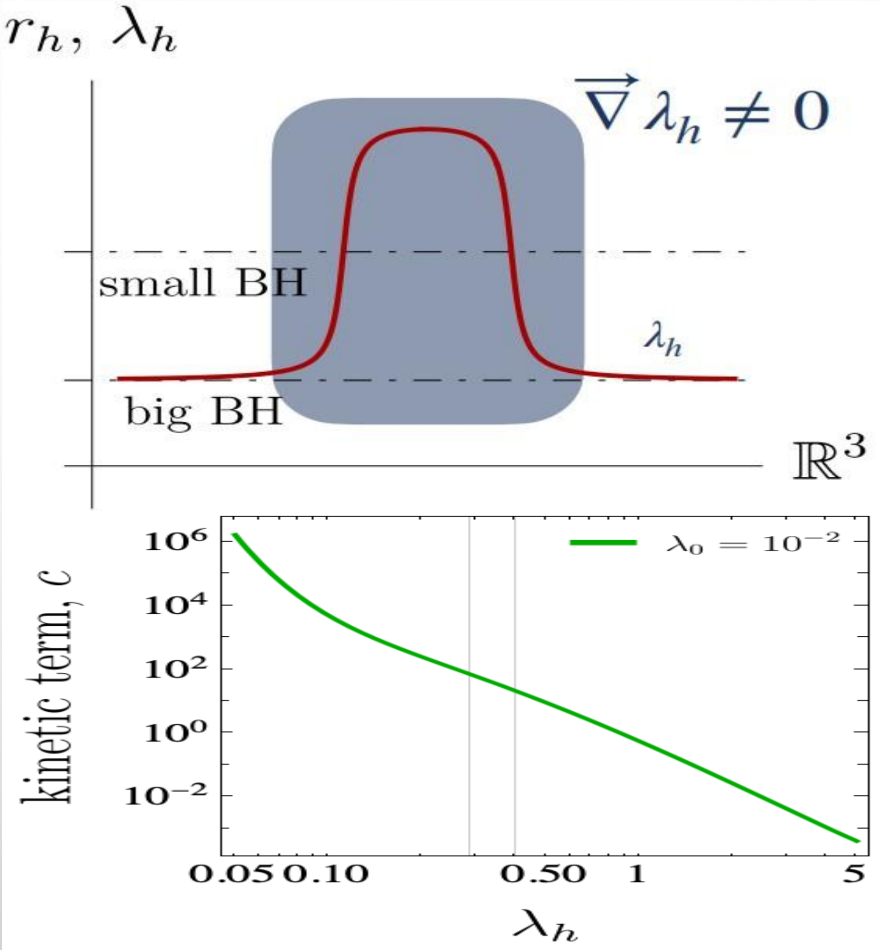
- PT stength (energy released)
  - $\alpha = \frac{4}{3}\frac{\Delta\theta}{\Delta w} = \frac{1}{3}\frac{\Delta\rho - 3\Delta p}{\Delta w} \sim 0.34$
- Inverse PT rate
  - $\frac{\beta}{H} = T\left(\frac{d\mathcal{S}_\mathcal{B}}{dT}\right) \sim 10^5$



# What we are doing now?

- Doing our own fits to lattice data.
- Studying the impact of this onto the PT parameters
- Estimating the kinetic term in this theory
- Studying Large N behaviour on PT parameters
- Employing more realistic estimates for the wall velocity




$$S_{eff} = \frac{4\pi}{T} N_c^2 \int d^3x\, \frac{T^2}{16\pi^2}\left(\frac{11}{32}\frac{s}{T^3} + \frac{3}{8}\frac{\mathcal{F}}{T^4}\right)\left(1 - r_h\frac{Q'(r_h)}{Q(r_h)}\right)^2\left(\frac{d\log r(\lambda_h)}{d\lambda_h}\right)^2\left(\vec{\nabla}\lambda_h(\vec{x})\right)^2 + V_{eff}(\lambda_h(\vec{x}), T)$$

# Verdict on GWs from SU(N) YM (Preliminary Upper bound!)

Limited amount of Supercooling
$T_{min} \sim 0.975 T_c$

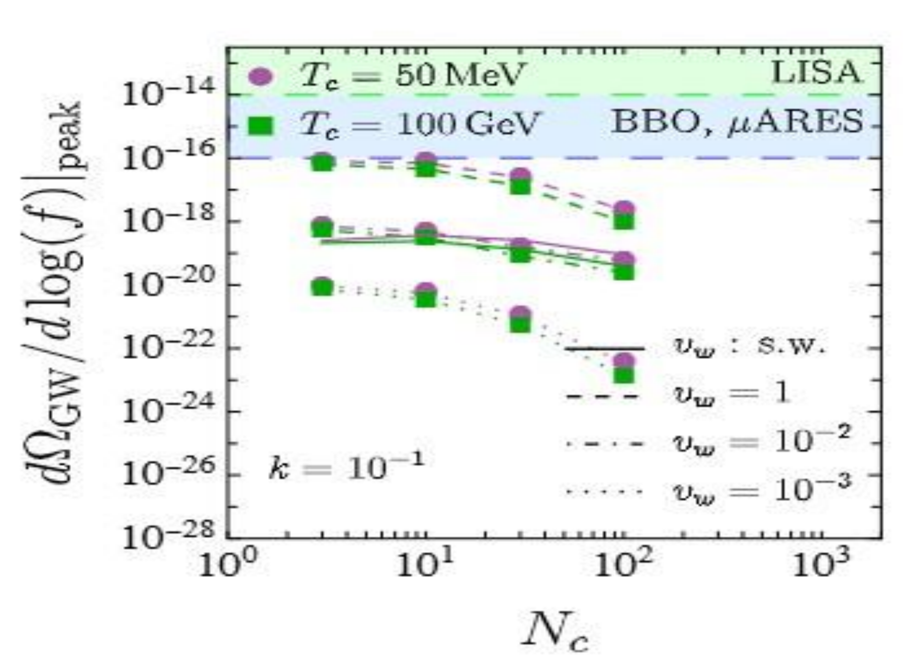
Competing effects of large Kinetic Term and Number of Colors

$\Omega_{GW}(f_p)h^2 \sim 10^{-18}$

Wall Velocity Estimates indicate $v_w \sim 0.1$

Small amplification in α by 15%.

Bounce Calculation breakdown at $T \sim 0.977 T_c$ , $\frac{\beta}{H} \rightarrow 0$, Need corrections as barrier vanishes




Study of GW from Spinodal Instability an Interesting Possibility here! $N_c \sim 100$

# Conclusions & Outlook

## Conclusions

- Duration of supercooling is minor in strongly coupled QFTs we are aware of
- Holography potentiates quantitative predictions for PTs at strong coupling.
- GWs may be our only chance to ever discover Dark Sectors (CDM).

## Outlook

- Bubble wall Velocities
- Incorporating Quarks
- Incorporating an axion/QCD Theta Angle
- Impact of out of equilibrium Hydrodynamics on PT
- Spinodal Instability