Quantisation across the Bubble wall and Friction

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based on JHEP 05 (2024) 294 and JHEP 12 (2024) 056
with *Aleksandr Azatov, Rudin Petrossian-Byrne and Miguel Vanvlasselaer*

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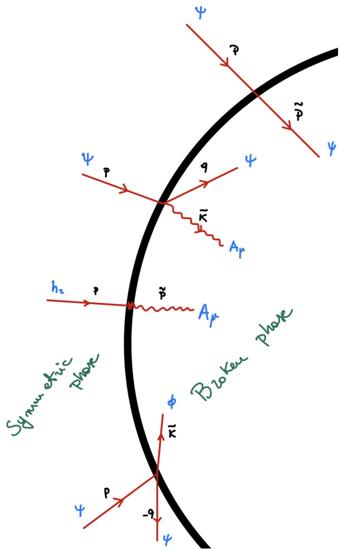
Outline

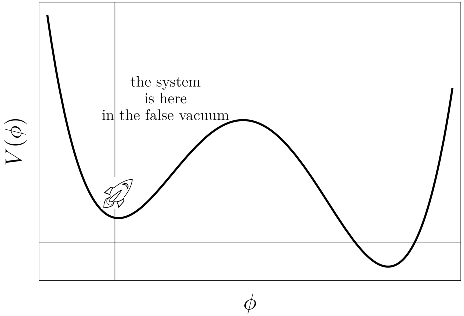
1. FOPT: Why, What & bubbles dynamic
2. Part I: Toolkit for quantisation across the wall
   - I. Complete basis of solutions to the spatially dependent EOM
   - II. Quantisation & construction of ‘In‘ and ‘Out‘ asymptotic states
   - III. Amplitudes
   - IV. Approximations (Step wall and WKB)
3. Part II: **Gauge-fixing** and spin-interpolation
   - I. EOM + gauge fixing
   - II. Interpolation between Higgs and longitudinal polarisation
4. Results & Conclusions

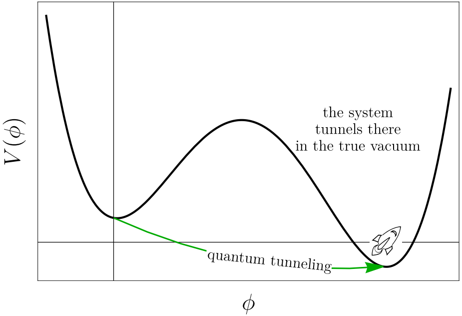


What?

FOPT: What? (from quantum field theory)

Let us consider a system described by the scalar potential $V(\phi)$

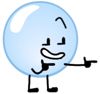




Tunneling decay rate of the false vacuum

$$\Gamma \sim A e^{-S_E},$$ Euclidean action

S_E computed on the sol. of the EOMs



The solutions with the least action are **spherically symmetric**

Coleman, Callan ('77)

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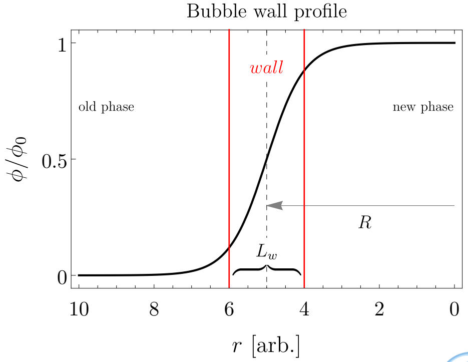

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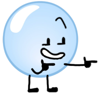

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Why?

FOPT: Why?

Today, there is no known FOPT of the fundamental interactions in 4d at $\mu = 0$ for any T!

apart from Higgs instability, but not conclusive...

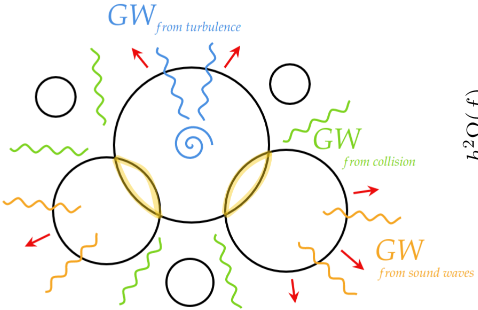
So why?

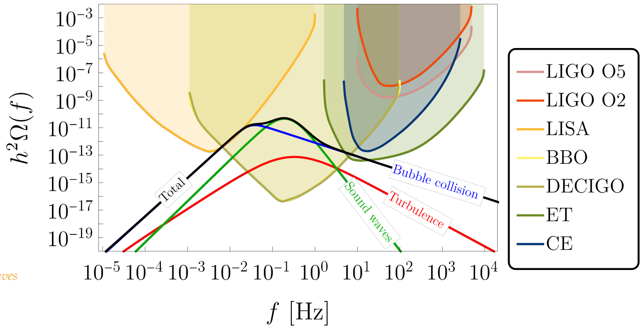
They frequently appear in **BSM** theories!

- **Many phenomenological consequences**: (EW) baryogenesis, dark matter, PBH, collider signatures, topological defects, axions, ... and **Gravitational Waves!**

FOPT: Why?

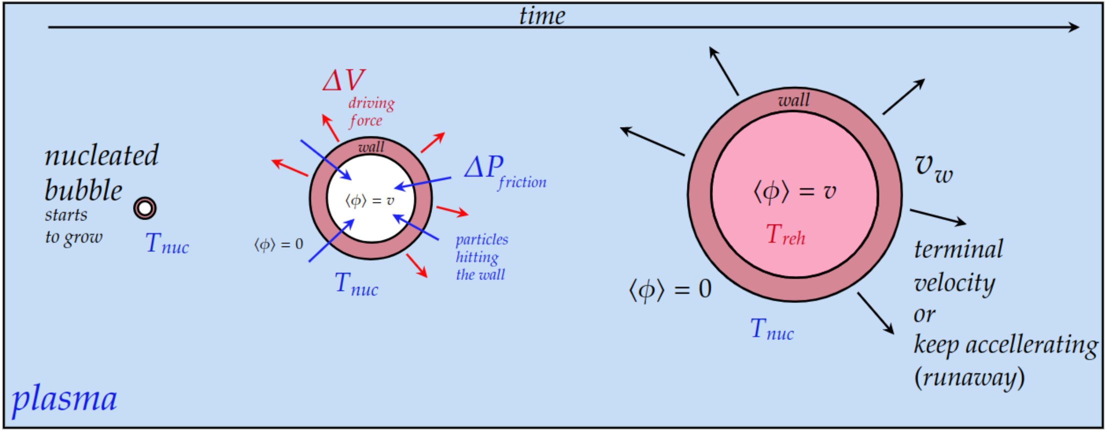
A GW stochastic background potentially observable at upcoming detectors
(even completely decoupled sectors become interesting)





How?

FOPT: How?



time
nucleated bubble starts to grow
T_{nuc}
ΔV
driving force
wall
$\Delta P_{friction}$
$\langle\phi\rangle = v$
$\langle\phi\rangle = 0$
particles hitting the wall
T_{nuc}
wall
$\langle\phi\rangle = v$
T_{reh}
v_w
terminal velocity or keep accellerating (runaway)
$\langle\phi\rangle = 0$
T_{nuc}
plasma

Interaction with the plasma

FOPT: Calculating pressure

- Work in the wall frame (ignore curvature). $\gamma_w(v_w)$ is the boost factor (velocity) of the wall.
- In the limit $\gamma_w \gg 1$ the plasma can be thought of **individual particles hitting the wall**.
- Translational symmetry broken $\longrightarrow$ $z-$momentum not conserved!

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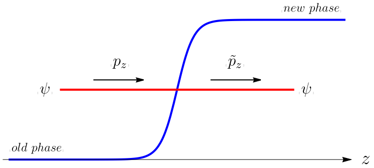
Incoming flux
$\gamma_w n v_w$



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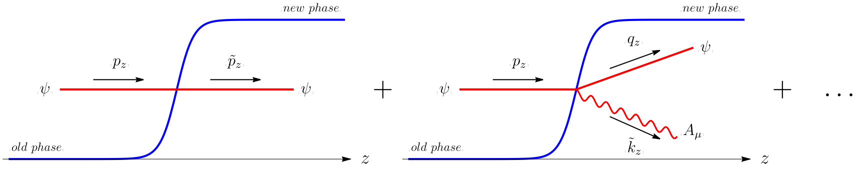

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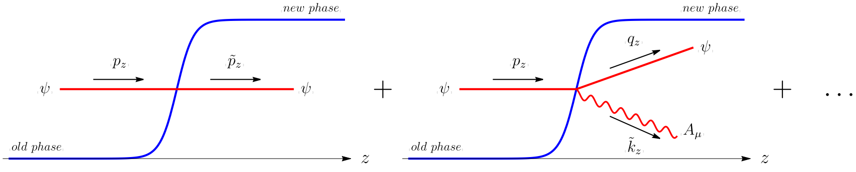

$$\mathcal{P} = \underbrace{\int \frac{d^3p}{(2\pi)^3}\frac{p^z}{p_0} f_i^{\rm eq}}_{\text{incoming flux}} \times \underbrace{\sum_f \int d\mathbb{P}_{i\to f}|\mathcal{M}|^{\in}\Delta p^z}_{\langle \Delta p^z\rangle} \sim (\gamma_w n v_w)\langle \Delta p\rangle$$

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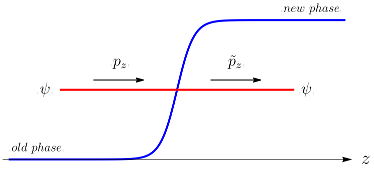
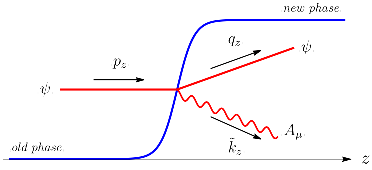


$+$



$+ \quad \dots$

$$\mathcal{P} \sim (\gamma_w n v_w)\langle \Delta p \rangle$$

$$\sim \Delta m^2 T_{\rm nuc}^2$$

Bödeker, Moore ('09)

$$\sim \frac{1}{16\pi^2} g^2 m_V \gamma_w T_{\rm nuc}^3$$

Bödeker, Moore ('17)
Azatov, Vanvlasselaer ('20)
Gouttenoire, Jinno, Sala ('21)

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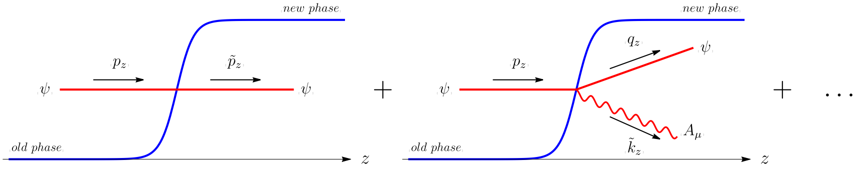

Weaknesses: $\begin{cases}\text{WKB approx. despite IR dominated emission} \\ \text{Longitudinal pol. not properly considered}\end{cases}$

Toolkit for Quantisation across the bubble wall

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1. Specify a model   example: $-\mathcal{L} \supset \frac{1}{2} m_\phi^2(z)\phi^2 + \frac{1}{2} m_\psi^2 \psi^2 + \frac{y}{2}\psi^2\phi$

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2. Solve the EOM — $(\Box + m_\psi^2)\psi = 0\ , \quad (\Box + m_\phi^2(z))\phi = 0$

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4. Compute the amplitude: $\langle f | \mathcal{S} | i \rangle \equiv (2\pi)^3 \delta^{(3)}(p^n - k^n - q^n) i\mathcal{M}_I$ $E,\ P_\perp^{\text{tot}}$ conserved

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6. Computation of the average exchanged

$$\langle \Delta p \rangle \sim \int d\mathbb{P}\, \Delta p\, |\mathcal{M}|^2$$

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5. Phase space integration: $\begin{cases} \text{IR} & \longrightarrow \quad \text{step wall approx } (k^z \lesssim L_w^{-1}) \\ \text{UV} & \longrightarrow \quad \text{WKB approx } (k^z \gtrsim L_w^{-1}) \end{cases}$
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$$\langle \Delta p \rangle \sim \sum_{L,R} \int^{k^z \lesssim L_w^{-1}} d^3k \; \Delta p \, |\mathcal{M}^{\text{step}}|^2 \; + \int_{k^z \gtrsim L_w^{-1}} d^3k \; \Delta p \, |\mathcal{M}^{\text{wkb}}|^2$$

Gauge-fixing
and spin-interpolation

Vector boson emission: Abelian Higgs model

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu H|^2 - V(\sqrt{2}|H|) + |D_\mu \psi|^2 - \frac{1}{2}m_\psi^2 \psi^2 + \text{gauge fixing}, \qquad D_\mu = \partial_\mu + igA_\mu$$

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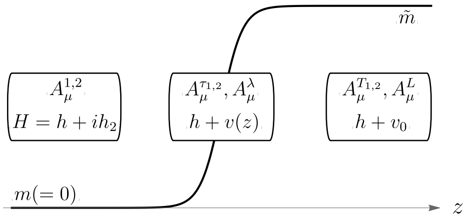
EOM: R_ξ gauge

$$\partial_z^2 v(z) = V'(v)$$

$$\Box h = -V''(v)h$$

$$\Box h_2 = -\xi g^2 v^2 h_2 - V'(v)\frac{h_2}{v} - 2g\partial_\mu v A^\mu$$

$$\partial_\nu F^{\mu\nu} = \frac{1}{\xi}\partial^\mu(\partial_\nu A^\nu) + g^2 v^2 A^\mu - 2gh_2\partial^\mu v$$



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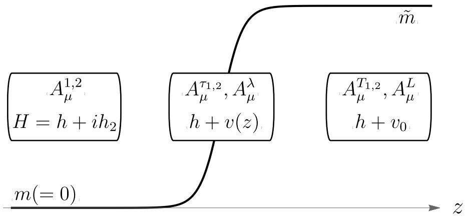
EOM: Unitary gauge $\xi \to \infty$

$$\partial_z^2 v(z) = V'(v)$$
$$\Box h = -V''(v)h$$
$$\partial_\nu F^{\mu\nu} = g^2 v^2(z) A^\mu \equiv m^2(z) A^\mu$$

(new!) Transversality condition $\begin{cases} \partial_\mu (m^2(z) A^\mu) = 0 \\ 3 \text{ propagating dofs} \end{cases}$



Vector boson emission: 'wall' polarisations

Generalized Lorenz condition: $\boxed{\partial_\mu(m^2(z)A^\mu) = 0}$

$k^\mu = (k_0, k_\perp, 0, k^z)$

- $A_z = 0 \to \boxed{\partial_\mu A^\mu = 0} \Rightarrow \tau-$**polarisations**:

$$\epsilon_\mu^{\tau_1} = (0,0,1,0), \qquad \epsilon_\mu^{\tau_2} = (k_\perp, k_0, 0, 0)/\sqrt{k_0^2 - k_\perp^2}$$

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- $A_z \neq 0 \Rightarrow \lambda-$**polarisation**: $A_\mu^\lambda = \partial_i a + A_z$ with $i = 0, 1, 2$ $E = \sqrt{k_0^2 - k_\perp^2}$

$$\epsilon_\mu^\lambda = \frac{k_\mu}{m(z)} \times \frac{k^z}{E} + \left(0, 0, 0, \frac{m(z)}{E}\right) \quad \to \quad A_\mu^\lambda = \partial_\mu a + \frac{m(z)^2}{E^2}(0, 0, 0, A_z)$$

(τ, λ) best suited for the problem, they differ from conventional (T, L) (agree only for $k_\perp = 0$).

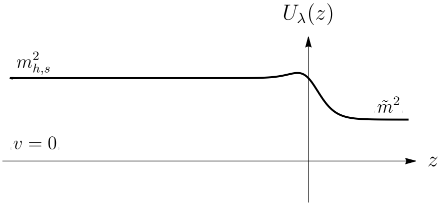
Vector boson emission: λ interpolates between h_2 and $A^{(\lambda)}$

EOM for A_z: $$\left[-E^2 - \partial_z^2 + m(z)^2 - 2\left(\frac{m'}{m}\right)\partial_z + 2\left(\frac{m'}{m}\right)^2 - 2\frac{m''}{m}\right] A_z = 0$$

Now defining $A_z = \dfrac{E}{m(z)}\lambda$ we get

$$\left[-E^2 - \partial_z^2 + U_\lambda(z)\right]\lambda = 0$$

Symmetric $\rightarrow$ Broken



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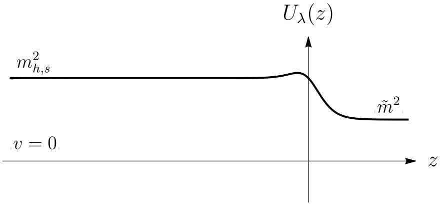

$$U_\lambda(-\infty) = m_{h,s}^2 \quad \xleftrightarrow[-\infty \leftarrow z]{z \rightarrow +\infty} \quad U_\lambda(+\infty) = \tilde{m}^2$$

Can be proven $\forall\ V(v)$

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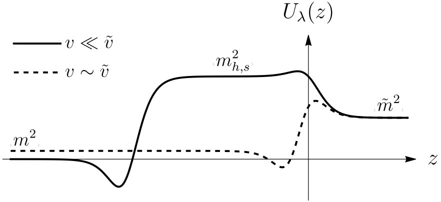

$$U_\lambda(-\infty) = m^2 \quad \overset{z\to+\infty}{\underset{-\infty\leftarrow z}{\longleftrightarrow}} \quad U_\lambda(+\infty) = \tilde{m}^2$$

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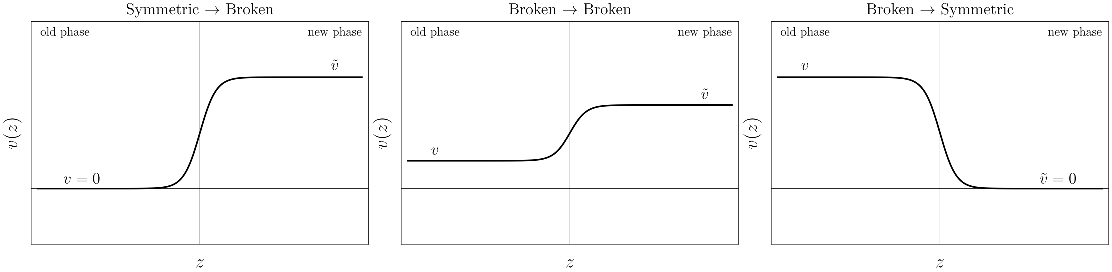
Types of PT

This formalism allows us to take care of all the possible types of PT



"*Quantisation Across Bubble Walls and Friction*"
JHEP 05 (2024) 294
A. Azatov, **GB**, R. Petrossian-Byrne, M. Vanvlasselaer

"*NLO friction in symmetry restoring phase transitions*"
2405.19447
A. Azatov, **GB**, R. Petrossian-Byrne

Results: Symmetric $\rightarrow$ Broken

$$\frac{\text{Symm.} \rightarrow \text{Broken}}{(v = 0, \quad \tilde{v} \neq 0)}$$

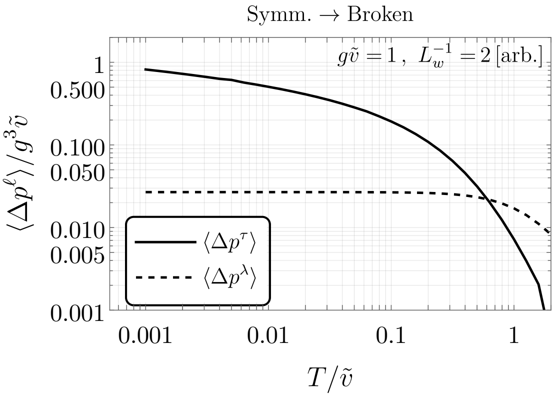
thin wall: $\tilde{m} L_w \ll 1$

$$\langle \Delta p^\tau \rangle \simeq g^3 \tilde{v} \log \frac{\tilde{v}}{T}$$

$$\langle \Delta p^\lambda \rangle \simeq g^3 \tilde{v} c_\lambda$$

No dependence on L_w

Relative importance of τ and λ contributions



Results: Broken → Broken

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$$\langle \Delta p^\tau \rangle \simeq g^3 \tilde{v} F_\tau \left(\frac{v}{\tilde{v}} \right)$$

$$\langle \Delta p^\lambda \rangle \simeq g^2 \left(\frac{v^2 - \tilde{v}^2}{v^2 + \tilde{v}^2} \right)^2 L_w^{-1}$$

λ enhanced by L_w^{-1} and can be dominant

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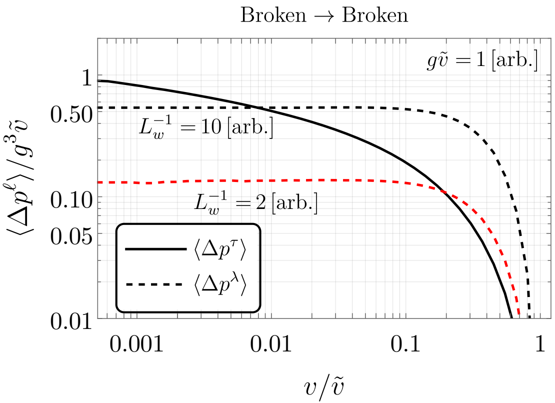

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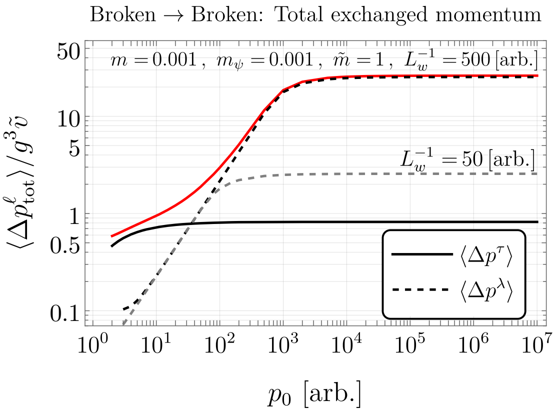


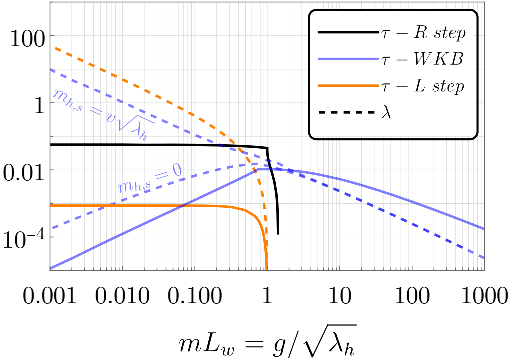
Results: Broken → Symmetric

Broken → Symmetric
$(v \neq 0, \quad \tilde{v} = 0)$

$$\langle \Delta p^\tau \rangle \simeq g^3 v \begin{cases} 4.4 & mL_w \ll 1 \\ (mL_w)^{-1} \log mL_w & mL_w \gg 1 \end{cases}$$

$$\langle \Delta p^\lambda \rangle \simeq g^3 v \begin{cases} 4(mL_w)^{-1} & mL_w \ll 1 \\ (mL_w)^{-1} & mL_w \gg 1 \end{cases}$$

λ enhanced by L_w^{-1} in thin wall limit



Summary

In all the three cases the asymptotic $\langle \Delta p_z \rangle$ is constant and the friction is $\mathcal{P} \sim \gamma_w \times \langle \Delta p_z \rangle$

Momentum transfer in the asymptotic limit $(p_0 \to \infty)$

	Symm. $\to$ Broken $(\tilde{m}L_w \ll 1)$	Broken $\to$ Broken $(\tilde{m}L_w \ll 1)$	Broken $\to$ Symm.
$\langle \Delta p^\tau \rangle \simeq$	$g^3 \tilde{v} \log \frac{\tilde{v}}{T}$	$g^3 \tilde{v} F_\tau \left(\frac{v}{\tilde{v}}\right)$	$g^3 v \begin{cases} 4.4 & mL_w \ll 1 \\ (mL_w)^{-1} \log mL_w & mL_w \gg 1 \end{cases}$
$\langle \Delta p^\lambda \rangle \simeq$	$g^3 \tilde{v} c_\lambda$	$g^2 \left(\frac{v^2 - \tilde{v}^2}{v^2 + \tilde{v}^2}\right)^2 L_w^{-1}$	$g^3 v \begin{cases} 4(mL_w)^{-1} & mL_w \ll 1 \\ (mL_w)^{-1} & mL_w \gg 1 \end{cases}$

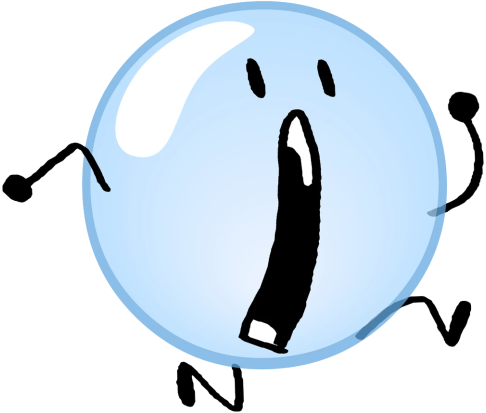
- λ always comparable wrt τ and can be the dominant contribution
- S$\to$B not sensitive to the wall width, L_w, while B$\to$B and B$\to$S, for λ, they are

Conclusions

1. We computed the **transition radiation emission on much more solid basis**, account for **longitudinal emission**, and analysed
   - **symmetric to broken**, aka symmetry–breaking PTs
   - **broken to broken** PTs
   - **broken to symmetric** aka symmetry–restoring PTs

2. We computed the **friction from transition radiation** in the most **minimal theory** (scalars) and in a **spontaneously broken Abelian gauge theory**.

3. We developed the tools for computing any particle process in such a background.
   (A step towards systematically studying QFT (EFT?) for broken translations)



Thanks for your attention!

Backup slides

Vertex interactions

[JCAP05(2017)025]: Bodeker, Moore

In the relativistic regime the friction can be computed as

$$\Delta\mathcal{P} = \int \frac{d^3p}{(2\pi)^3}\frac{p^z}{p_0} f_A^{\rm eq} \times \sum_{b,c} \int d\mathbb{P}_{a\to b,c}\Delta p^z$$

where b is soft and

$$\int d\mathbb{P}_{a\to b,c} \sim \int d^2k_\perp \int dx \, |\mathcal{M}(a\to b,c)|^2, \qquad x \equiv \frac{E_c}{E_a}$$

The matrix element is related to the interaction via

$$\mathcal{M}(a\to b,c) = \int dz \; \chi_a(z)\chi_b^*(z)\chi_c^*(z)V(z), \qquad V(z) : \text{vertex}$$

Soft singularity in x matters! $\rightarrow$ **emission of vector bosons!**

$a(p)\to b(k)c(p-k)$	$\lvert V^2 \rvert$
$S\to V_T S$ $F\to V_T F$ $V\to V_T V$	$4g^2C_2[R]\frac{1}{x^2}k_\perp^2$
$S\to V_L S$ $F\to V_L F$ $V\to V_L V$	$4g^2C_2[R]\frac{1}{x^2}m^2$
$F\to FV_T$	$2g^2C_2[R]\frac{1}{x}(k_\perp^2+m_b^2)$
$V\to FF$	$2g^2T[R]\frac{1}{x}(k_\perp^2+m_b^2)$
$S\to SV_T$	$4g^2C_2[R]k_\perp^2$
$F\to SF$	$y^2(k_\perp^2+4m_a^2)$
$S\to SS$	$\lambda^2\varphi^2$

(Non trivial) eingenmodes for step wall approximation

ϕ, ψ scalars: $-\mathcal{L} \supset \frac{1}{2} m_\phi^2(z)\phi^2 + \frac{1}{2} m_\psi^2 \psi^2 + \frac{y}{2}\psi^2\phi$

$\rightarrow$ $m_\psi = const$ does not feel the wall

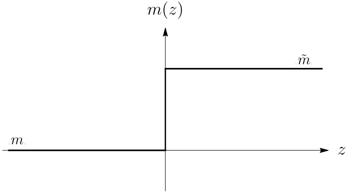
$\rightarrow$ while $m_\phi \equiv m_\phi(z)$ does

$\rightarrow$ EOM: $(\Box + m_\phi^2(z))\phi = 0$ and $\phi(z) = e^{-ik_0 t + ik_\perp x_\perp}\chi(z)$

$$\chi_R(z) = \begin{cases} e^{ik^z z} + r_k e^{-ik^z z} & z < 0 \\ t_k e^{i\tilde{k}^z z} & z > 0 \end{cases}$$

$$\chi_L(z) = \sqrt{\frac{k^z}{\tilde{k}^z}} \begin{cases} \frac{\tilde{k}^z}{k^z} t_k e^{ik^z z} & z < 0 \\ -r_k e^{i\tilde{k}^z z} + e^{-i\tilde{k}^z z} & z > 0 \end{cases}$$

where $r_k = \frac{\tilde{k}^z - k^z}{\tilde{k}^z + k^z}$ and $t_k = \frac{2k^z}{\tilde{k}^z + k^z}$



STEP WALL APPROX.
(valid in the IR)

Amplitudes & Phase Space (for step wall)

We are ready to compute the **amplitudes**

$$\mathcal{S} = \mathrm{T}\,\exp\left(-i\int d^4x \mathcal{H}_{\mathrm{Int}}\right) \qquad \mathcal{H}_{\mathrm{Int}} = -iy\psi^2(x)\phi(x)$$

$$\langle k_I^{\mathrm{out}}\, q|\, \mathcal{S}\, |p\rangle \equiv (2\pi)^3\delta^{(3)}(p^n - k^n - q^n)i\mathcal{M}_I \overset{\mathrm{tree}}{=} -i\int d^4x \langle k_I^{\mathrm{out}}\, q|\, \mathcal{H}_{\mathrm{Int}}\, |p\rangle$$

$$\boxed{\mathcal{M}_I \equiv \mathcal{M}(\psi \to \psi\phi_I) = y\int_{-\infty}^{\infty} dz\ \chi(p^z)\chi^*(q^z)\zeta_I^*(k^z)}$$

Then the **averaged exchanged momentum**

$$\begin{aligned}\langle \Delta p\rangle &= \langle \Delta p_R\rangle + \langle \Delta p_L\rangle \\ &= \int d\mathbb{P}_{\psi\to\psi\phi_{\zeta_R}}(p^z - q^z - \tilde{k}^z) + \int d\mathbb{P}_{\psi\to\psi\phi_{\zeta_L}}(p^z - q^z + k^z)\end{aligned}$$

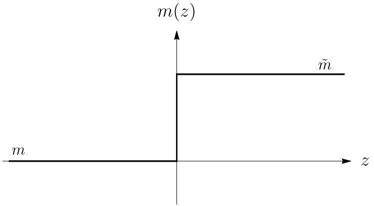
$$\int d\mathbb{P}_{\psi\to\psi\phi_I}\Delta p_I^z = \int_{k_{\mathrm{min}}^{z,I}}^{k_{\mathrm{max}}^z} \frac{dk_z}{2\pi}\frac{1}{2k_0}\int_0^{k_{\perp,\mathrm{max}}^2} \frac{dk_\perp^2}{4\pi}\cdot\frac{1}{2p^z}\left[\frac{1}{2|q^z|}|\mathcal{M}_I|^2\Delta p_I^z\right]_{q^z=\pm q_k^z}$$

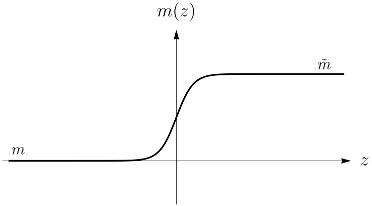
Beyond step wall → WKB

When does the step wall approximation break?

- If the z **momentum is large enough** $(k^z L_w \gtrsim 1)$ there will be **mostly transmission**! → WKB

$k^z \lesssim L_w^{-1}$ $\qquad$ $L_w^{-1} \lesssim k^z \leq k^z_{\max}$





Step wall: $\zeta_{R,L}$

WKB: $\chi_R(z) = \sqrt{\dfrac{k^z}{k^z(z)}}\, e^{-i\int_0^z dz' k^z(z') z'}$

- When $\Delta p L_w \gg 1$ then $\mathcal{M} \to 0$ $\qquad$ ($z-$momentum conservation is restored!)

Scalar emission: Quantisation

Having a complete set of states $\{\phi_{R,k^z}, \phi_{L,k^z}\}$ we can expand the field

$$\phi = \sum_{I=R,L} \int \frac{dk^3}{(2\pi)^3\sqrt{2k_0}} \left(a_{I,k^z}\phi_{I,k^z} + h.c.\right) , \qquad \begin{cases} [a_{I,k^z}, a^\dagger_{J,q^z}] = (2\pi)^3 \delta_{IJ}\delta^{(3)}(k-q) \\ [a_{I,k^z}, a_{J,q^z}] = [a^\dagger_{I,k^z}, a^\dagger_{J,q^z}] = 0 \end{cases}$$

We can define two types of states

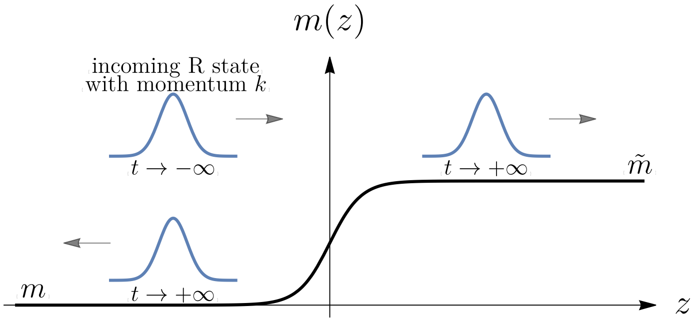
$$|k^z_R\rangle = \sqrt{2k_0}a^\dagger_{R,k^z}|0\rangle,$$
$$|k^z_L\rangle = \sqrt{2k_0}a^\dagger_{L,k^z}|0\rangle,$$

which should be thought as independent states in any process.

Scalar emission: complete basis for outgoing states

To compute $\langle \Delta p \rangle$ we need states with **definite final momentum**!

How we interpret the emission of a R movers?



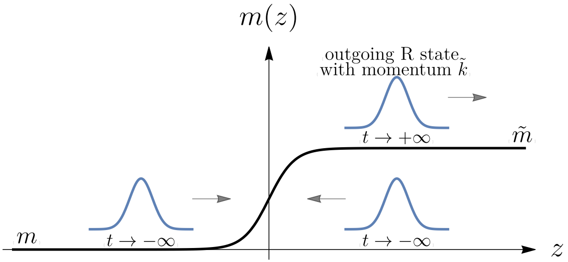
Definite initial momentum, but not $\hat{P}$ eigenstate for $t \to +\infty$

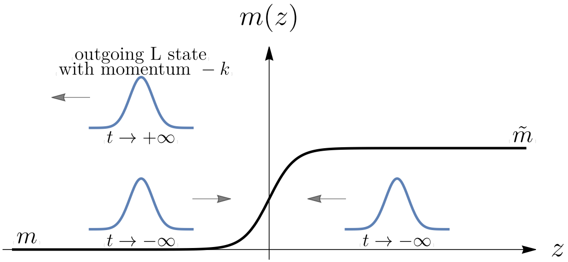
Scalar emission: complete basis for outgoing states

To compute $\langle \Delta p \rangle$ we need states with **definite final momentum**!
Then we define basis for outgoing states

$$|k_R^{\text{out}}\rangle = t_k^* \sqrt{\frac{\tilde{k}^z}{k^z}}\ |k_R^{\text{in}}\rangle - r_k^*\ |k_L^{\text{in}}\rangle,$$

$$|k_L^{\text{out}}\rangle = r_k^*\ |k_R^{\text{in}}\rangle + t_k^* \sqrt{\frac{\tilde{k}^z}{k^z}}\ |k_L^{\text{in}}\rangle\ \theta(\tilde{k})$$



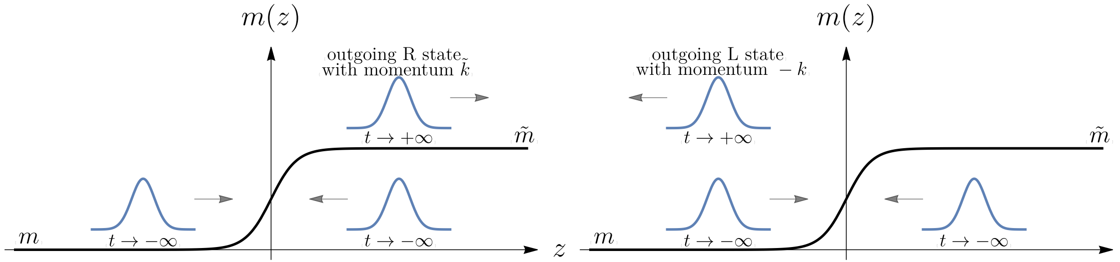


Scalar emission: complete basis for outgoing states

To compute $\langle \Delta p \rangle$ we need states with **definite final momentum**!
Then we define basis for outgoing states $\rightarrow \quad \{\zeta_{R,k^z}, \zeta_{L,k^z}\}$

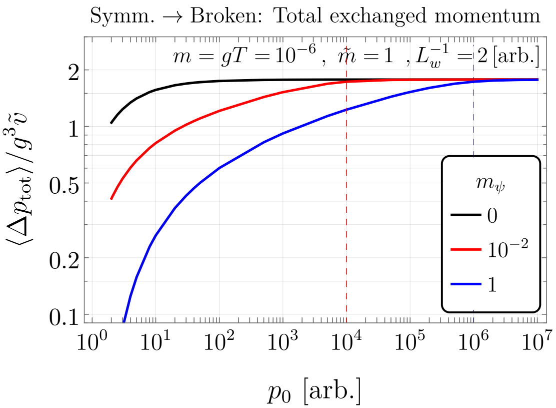
$$\zeta_R = t_k^* \sqrt{\frac{\tilde{k}^z}{k^z}}\, \chi_R - r_k^*\, \chi_L \equiv \chi_L^*,$$

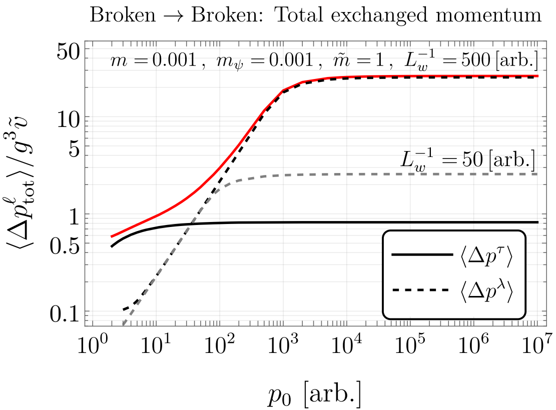
$$\zeta_L = r_k^*\, \chi_R + t_k^* \sqrt{\frac{\tilde{k}^z}{k^z}}\, \chi_L\, \theta(\tilde{k}) \equiv \chi_R^*$$



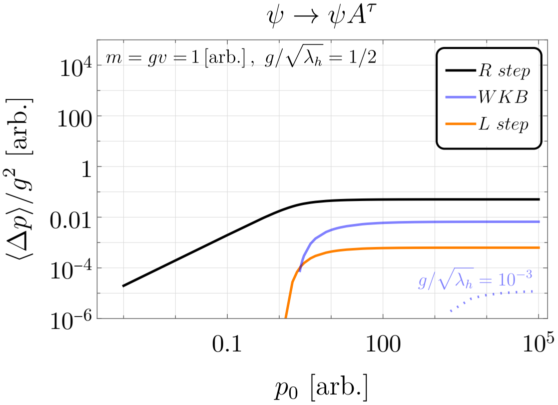
Results: Broken → Broken with transient regimes

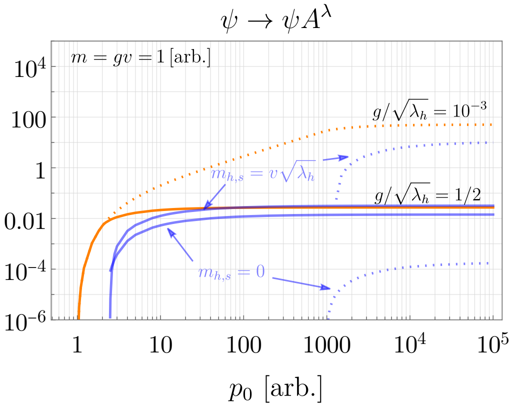
We are also able to capture transient regimes → ultimately matter to determine equilibrium velocity





Results: Broken → Symmetric with transient regimes





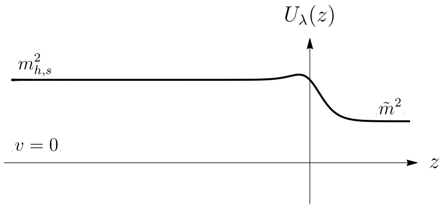
Vector boson emission: λ interpolates between h_2 and $A^{(\lambda)}$

EOM for A_z: $$\left[-E^2 - \partial_z^2 + m(z)^2 - 2\left(\frac{m'}{m}\right)\partial_z + 2\left(\frac{m'}{m}\right)^2 - 2\frac{m''}{m}\right] A_z = 0$$

Symmetric $\rightarrow$ Broken

Now defining $A_z = \dfrac{E}{m(z)}\lambda$ we get

$$\left[-E^2 - \partial_z^2 + U_\lambda(z)\right]\lambda = 0$$



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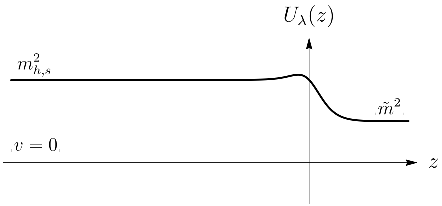
Symmetric $\to$ Broken



Now defining $A_z = \dfrac{E}{m(z)}\lambda$ we get

$$\left[-E^2 - \partial_z^2 + U_\lambda(z)\right]\lambda = 0$$

$$U_\lambda(-\infty) = m^2_{h,s} \quad \underset{-\infty\leftarrow z}{\overset{z\rightarrow+\infty}{\longleftrightarrow}} \quad U_\lambda(+\infty) = \tilde{m}^2$$

Can be proven $\forall\ V(v)$

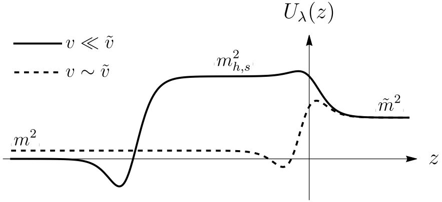
Vector boson emission: λ interpolates between h_2 and $A^{(\lambda)}$

EOM for A_z: $$\left[-E^2 - \partial_z^2 + m(z)^2 - 2\left(\frac{m'}{m}\right)\partial_z + 2\left(\frac{m'}{m}\right)^2 - 2\frac{m''}{m}\right] A_z = 0$$

Broken $\rightarrow$ Broken

Now defining $A_z = \dfrac{E}{m(z)}\lambda$ we get

$$\left[-E^2 - \partial_z^2 + U_\lambda(z)\right]\lambda = 0$$



$$U_\lambda(-\infty) = m^2 \quad \underset{-\infty\leftarrow z}{\overset{z\rightarrow+\infty}{\longleftrightarrow}} \quad U_\lambda(+\infty) = \tilde{m}^2$$

Can be proven $\forall\ V(v)$

Relations between (τ, λ) and (T, L)

Conventional pol. vectors:

$$\epsilon_{T_1} = (0,0,1,0), \quad \epsilon_{T_2} = \frac{1}{\sqrt{k_\perp^2 + k_z^2}}(0, k^z, 0, -k_\perp), \quad \epsilon_L = \frac{k_0}{m\sqrt{k_0^2 - m^2}}\left(\frac{k_0^2 - m^2}{k_0}, k_\perp, 0, k^z)\right)$$

$$\begin{pmatrix} \epsilon_{T_1} \\ \epsilon_{T_2} \\ \epsilon_L \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{k_0 k^z}{E\sqrt{k_z^2 + k_\perp^2}} & -\frac{k_0 m}{E\sqrt{k_z^2 + k_\perp^2}} \\ 0 & \frac{k_0 m}{E\sqrt{k_z^2 + k_\perp^2}} & \frac{k_0 k^z}{E\sqrt{k_z^2 + k_\perp^2}} \end{pmatrix} \begin{pmatrix} \epsilon_{\tau_1} \\ \epsilon_{\tau_2} \\ \epsilon_\lambda \end{pmatrix}$$

- For $k^z, E \gg k_\perp, m$ mixing between τ, λ scales as m/E
- We are interested in the sum of all contributions $\rightarrow$ all computations in (τ, λ) basis

Vector boson emission: step wall solution

$\tau-$**polarisations**: $A^\tau_\mu = \sum a^\tau_{1,2}\epsilon^{\tau_1,\tau_2}_\mu$, only τ_2 gives a contribution, then

$$a^{\tau_2}_{R,k^z} = e^{-ik_0t+ik_\perp x_\perp}\begin{cases} e^{ik^z z} + r_k e^{-ik^z z} & z<0 \\ t_k e^{i\tilde{k}^z z} & z>0 \end{cases}$$

$$a^{\tau_2}_{L,k^z} = e^{-ik_0t+ik_\perp x_\perp}\sqrt{\frac{k^z}{\tilde{k}^z}}\begin{cases} \frac{\tilde{k}^z}{k^z} t_k e^{ik^z z} & z<0 \\ -r_k e^{i\tilde{k}^z z} + e^{-i\tilde{k}^z z} & z>0 \end{cases}$$

where $k_0 = \sqrt{k_z^2 - m^2 - k_\perp^2}$, $\tilde{k}^z = \sqrt{k_z^2 + m^2 - \tilde{m}^2}$ and $E = \sqrt{k_0^2 - k_\perp^2}$.

Matching conditions: $\begin{cases} a^{\tau_2}|_{<0} = a^{\tau_2}|_{>0} \\ \partial_z a^{\tau_2}|_{<0} = \partial_z a^{\tau_2}|_{>0} \end{cases} \Rightarrow \begin{cases} r_k = \frac{\tilde{k}^z - k^z}{\tilde{k}^z + k^z} \\ t_k = \frac{2k^z}{\tilde{k}^z + k^z} \end{cases}$

Vector boson emission: step wall solution

$\lambda-$**polarisations**: $A_\mu^\lambda = \partial_\mu a + \frac{m(z)^2}{E^2}(0,0,0,A_z)$ where $A^z = \frac{E}{m(z)}\lambda$.

$$\lambda_{R,k^z} = e^{-ik_0 t + ik_\perp x_\perp} \begin{cases} e^{ik^z z} + r_k e^{-ik^z z} & z<0 \\ t_k e^{i\tilde{k}^z z} & z>0 \end{cases}$$

$$\lambda_{L,k^z} = e^{-ik_0 t + ik_\perp x_\perp} \sqrt{\frac{k^z}{\tilde{k}^z}} \begin{cases} \frac{\tilde{k}^z}{k^z} t_k e^{ik^z z} & z<0 \\ -r_k e^{i\tilde{k}^z z} + e^{-i\tilde{k}^z z} & z>0 \end{cases}$$

where $k_0 = \sqrt{k_z^2 - m^2 - k_\perp^2}$, $\tilde{k}^z = \sqrt{k_z^2 + m^2 - \tilde{m}^2}$ and $E = \sqrt{k_0^2 - k_\perp^2}$.

Matching conditions: $\begin{cases} \lambda v(z)|_{<0} = \lambda v(z)|_{>0} \\ \left.\frac{\partial_z \lambda}{v(z)}\right|_{<0} = \left.\frac{\partial_z \lambda}{v(z)}\right|_{>0} \end{cases} \Rightarrow \begin{cases} r_k = \frac{\tilde{v}^2 k^z - v^2 \tilde{k}^z}{\tilde{v}^2 k^z + v^2 \tilde{k}^z} \\ t_k = \frac{2k^z v \tilde{v}}{\tilde{v}^2 k^z + v^2 \tilde{k}^z} \end{cases}$

Vector boson emission: Quantisation

$$A^\mu = \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} \left(a^{\rm out}_{\ell,I,k} \; e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \; \zeta^\mu_{\ell,I,k}(z) + h.c. \right) \; ,$$

where $I = R, L$ and $\ell = \tau_1, \tau_2, \lambda$. The wave modes are constructed as follow

Outgoing states:

$$\zeta^\mu_{\tau_i,I,k} = \epsilon^\mu_{\tau_i} \; \chi^*_{\tau_i,I,k}(z) \; ,$$

$$\zeta^\mu_{\lambda,I,k} = \left(\frac{-ik^n \partial_z(v\lambda^*_{I,k})}{gEv^2}, \frac{E}{gv}\lambda^*_{I,k} \right) \overset{\text{on shell}}{=} \bar{\partial}^\mu \left(\frac{\partial_z(v\lambda^*_{I,k})}{Eg\; v^2} \right) + \frac{gv(z)}{E}\lambda^*_{I,k} \; \delta^\mu_z \; .$$

Anti–friction?

Anti–friction?

Is it possible to have negative friction?

Naive expectation: $\mathcal{P}_{\rm LO} \sim -m_i T_{\rm nuc}^2$ for particles loosing mass in symmetry restoring PT

Q: What can be negative in $\langle \Delta p \rangle = \int d\Pi_{\rm BTPH}\, |\mathcal{M}|^2\, \Delta p$?

BTPH: broken translation phase space

$$\Delta p \geq \Delta p_{\rm Min} = p^z - p^0 = -\frac{m_a^2}{2p^z} + \mathcal{O}\left(\frac{1}{p_z^2}\right) \quad \Longrightarrow \quad \langle \Delta p \rangle \geq \Delta p_{\rm Min} \int d\Pi_{\rm BTPH}\, |\mathcal{M}|^2 = \Delta p_{\rm Min}\, \mathbb{P}$$

where $\mathcal{M}$ and $\mathbb{P}$ are the matrix element and *total integrated probability* for the corresponding process.

Asymptotic friction will never dominate the LO contribution!

but nothing said about transient regimes ...

Phase space for vector emission: thermal masses

Phase space for vector emission: thermal masses (vectors)

When thermal corrections become important? We cut the phase space at momentum $|\vec{k}|^2 \sim g^2 T^2$, which is equivalent to use the following thermal masses for the vectors (symmetric $\rightarrow$ broken)

$$(\tau) : \begin{cases} \tilde{m} = m_\tau(z = +\infty) \approx g\sqrt{\tilde{v}^2 + T^2} \ , \\ m = m_\tau(z = -\infty) \approx gT \ , \end{cases} \quad (\lambda) : \begin{cases} \tilde{m} = m_\lambda(z = +\infty) \approx g\sqrt{\tilde{v}^2 + T^2} \ , \\ m = m_\lambda(z = -\infty) = m_{h,s}(T) \ . \end{cases}$$

gT is not the only scale possible. The self energy for transverse vectors receives ('magnetic mass') thermal corrections only at two loops of parametric order $\sim g^2 T$ from charged matter.
For broken to broken transitions the vector masses are, for both λ and τ fields

$$m \approx g\sqrt{v^2 + T^2} \ , \quad \tilde{m} \approx g\sqrt{\tilde{v}^2 + T^2} \ , \qquad \text{(broken to broken)} \ .$$

WKB approximation

KG eom in Fourier space: $$\chi''(z) + \frac{p_z^2}{\hbar^2}\chi(z) = 0$$

WKB ansatz: $\chi(z) = \exp\left[\frac{i}{\hbar}\left(S_0 + S_1\hbar + S_2\hbar^2 \dots\right)\right]$, put in the differential equation and match terms of the same order in $\hbar$.

$\implies$ Validity: $\hbar|S_0''(z)| \ll |S_0'(z)|$ & $2\hbar|S_0'S_1'| \ll |(\partial_z p^z(z))^2|$

$$\boxed{\chi(z) = \sqrt{\frac{p_s^z}{p^z(z)}}\exp\left[i\int_0^z d\hat{z}\, p^z(\hat{z}) + i\int_0^z d\hat{z}\left(\frac{1}{2}\left(\frac{\partial_z p^z}{p^z}\right)^2 - \frac{\partial_z^2 p^z}{4p_z}\right)\dots\right]}$$