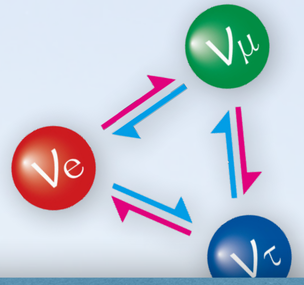
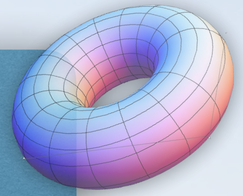
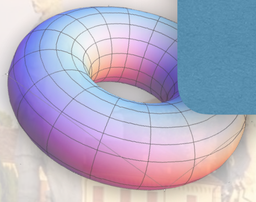
(2) INFN Istituto Nazionale di Fisica Nucleare



(1) ROMA TRE UNIVERSITÀ DEGLI STUDI



A new modular mechanism for neutrino masses from low-scale seesaw



Matteo Parriciatu (1)(2)

from a work in preparation with A. Granelli, D. Meloni, J.T. Penedo, S.T. Petcov

May 27th, 2025 Padova

$3\,\nu$ Paradigm

$$\underbrace{|\nu_\alpha\rangle}_{\text{Interaction basis}} = \sum_{i=1}^{3} U^*_{\alpha i} \underbrace{|\nu_i\rangle}_{\text{Mass basis}}$$

$$\theta_{12} \quad \theta_{13} \quad \theta_{23}$$

$$\delta_{CP} \quad \alpha_{21}\ , \alpha_{31}$$

Majorana

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \times \mathrm{diag}(1, e^{i\alpha_{21}}, e^{i\alpha_{31}})$$

$c_{ij} \equiv \cos\theta_{ij}\ ,\ s_{ij} \equiv \sin\theta_{ij}$ Pontecorvo - Maki - Nakagawa - Sakata

$$P(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3}\sum_{j=1}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \exp\left(-i\frac{\Delta m^2_{ij} L}{2E}\right)$$

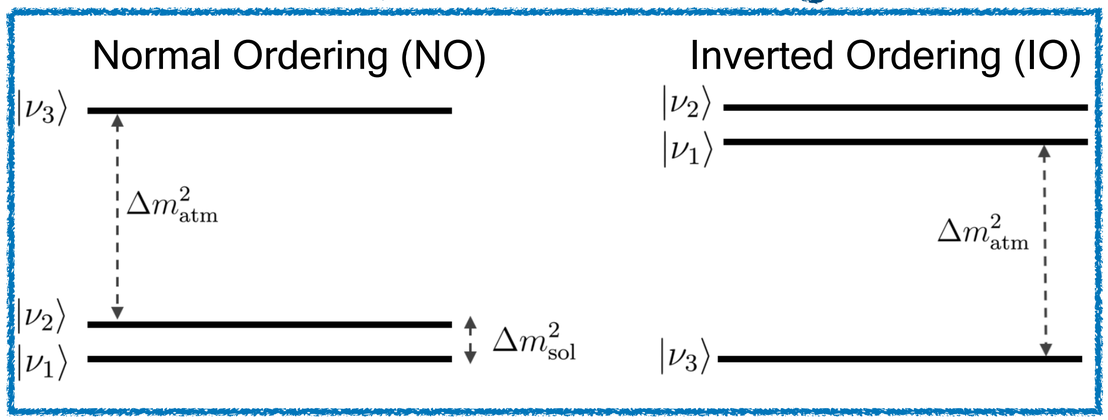
Sign determined from MSW

$$|\Delta m^2_{\text{sol}}| \equiv |m_2^2 - m_1^2| \qquad |\Delta m^2_{\text{atm}}| \equiv |m_3^2 - m_{1,2}^2|$$

$$r \equiv \frac{\Delta m^2_{\text{sol}}}{|\Delta m^2_{\text{atm}}|} \approx 1/30$$

Undetermined sign

?

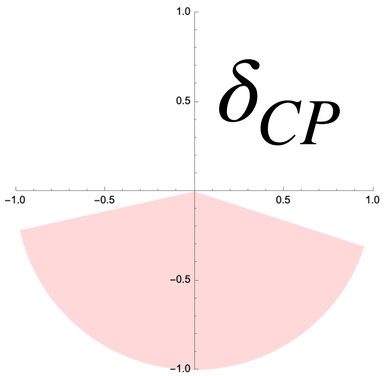


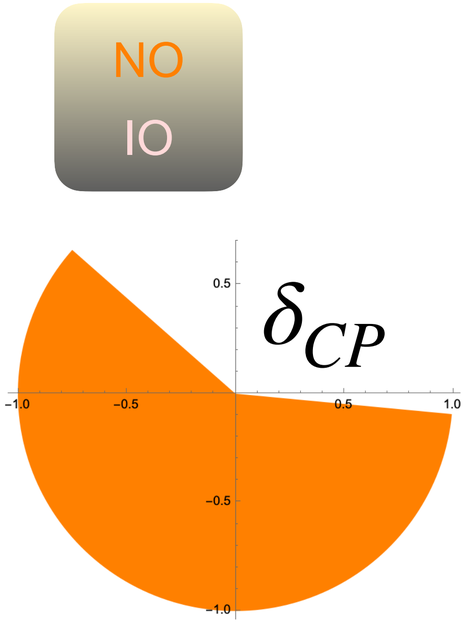
Best fit values

F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

$$\theta_{12} \approx 33.4^\circ \quad , \quad \theta_{13} \approx 8.5^\circ \quad , \quad \theta_{23} \approx 42.4^\circ \qquad (\mathbf{NO})$$
$$\theta_{12} \approx 33.4^\circ \quad , \quad \theta_{13} \approx 8.5^\circ \quad , \quad \theta_{23} \approx 48.9^\circ \qquad (\mathbf{IO})$$

Two big and one small angles





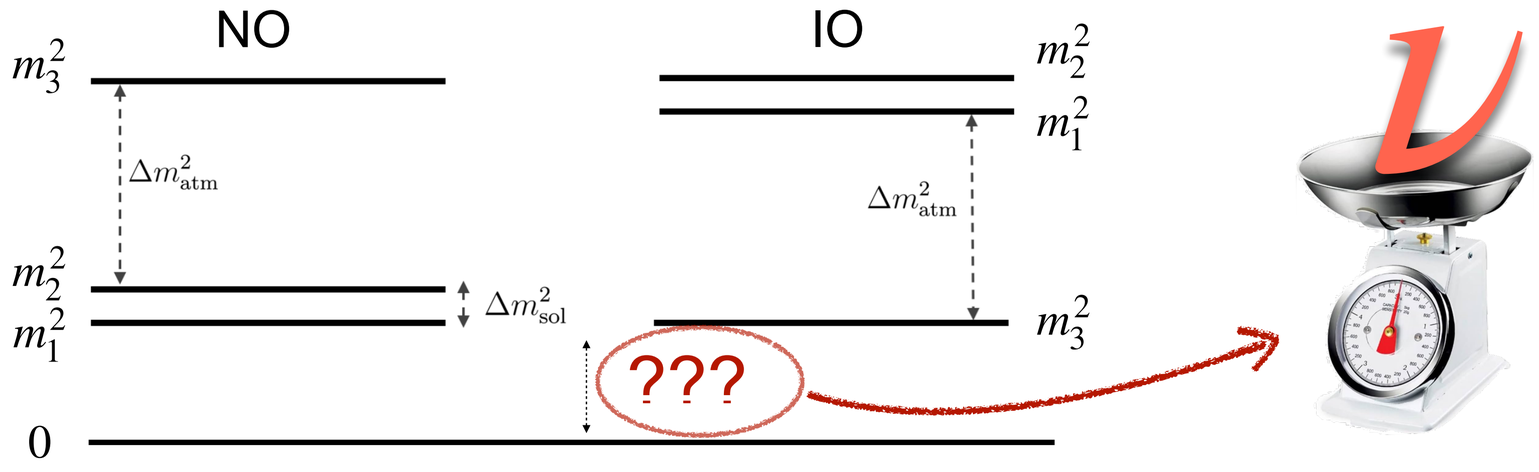
3σ ranges from
F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

$$J_{\rm CP} = c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13} \sin\delta_{\rm CP}$$

Violation if

$$\frac{\delta_{CP}}{\pi} \neq \{0,1\}$$

CP-conserving still allowed at 2σ (NO)



$$\sum_i^3 m_i < 0.115 \text{ eV} \qquad (95\% \text{ C.L.})$$

A. Shadab et al.

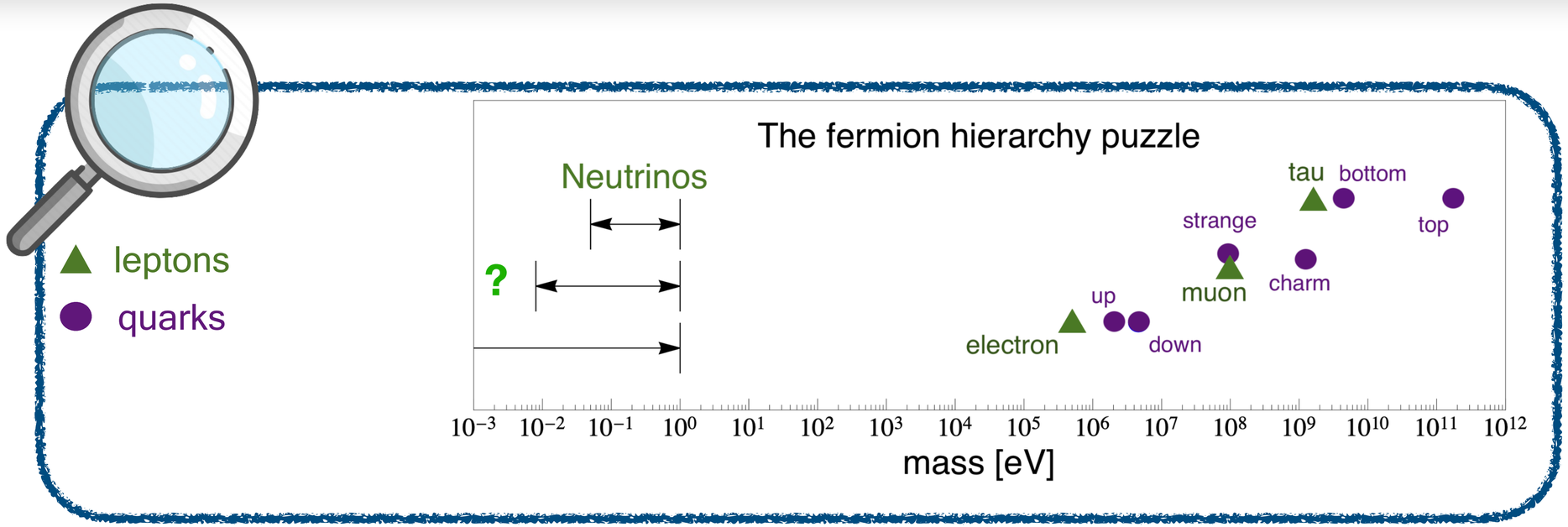
Phys. Rev. D **103** (Apr, 2021)

Cosmology

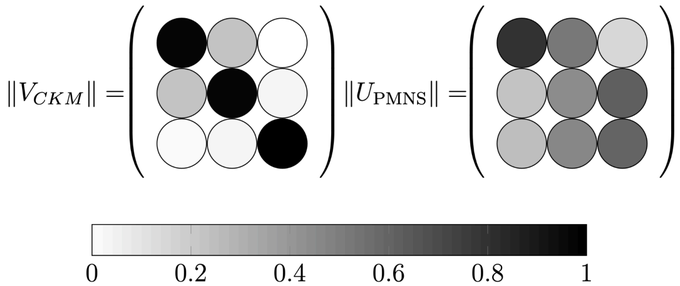
$$\sqrt{\sum_i m_i^2 |U_{ei}|^2} < 0.45 \text{ eV}$$

Science 388 (2025) 6743

KATRIN

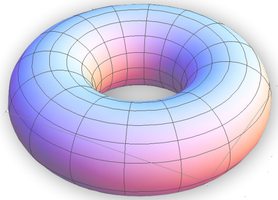


$$m_e/m_\mu \simeq \frac{1}{200} \qquad m_\mu/m_\tau \simeq \frac{1}{17}$$



D. Meloni [1709.02662v2]

F. Feruglio
[1706.08749]



$\tau \equiv$ modulus

$$(y_{jk}\bar{L}_j H \ell_{Rk} + \text{h.c.})$$

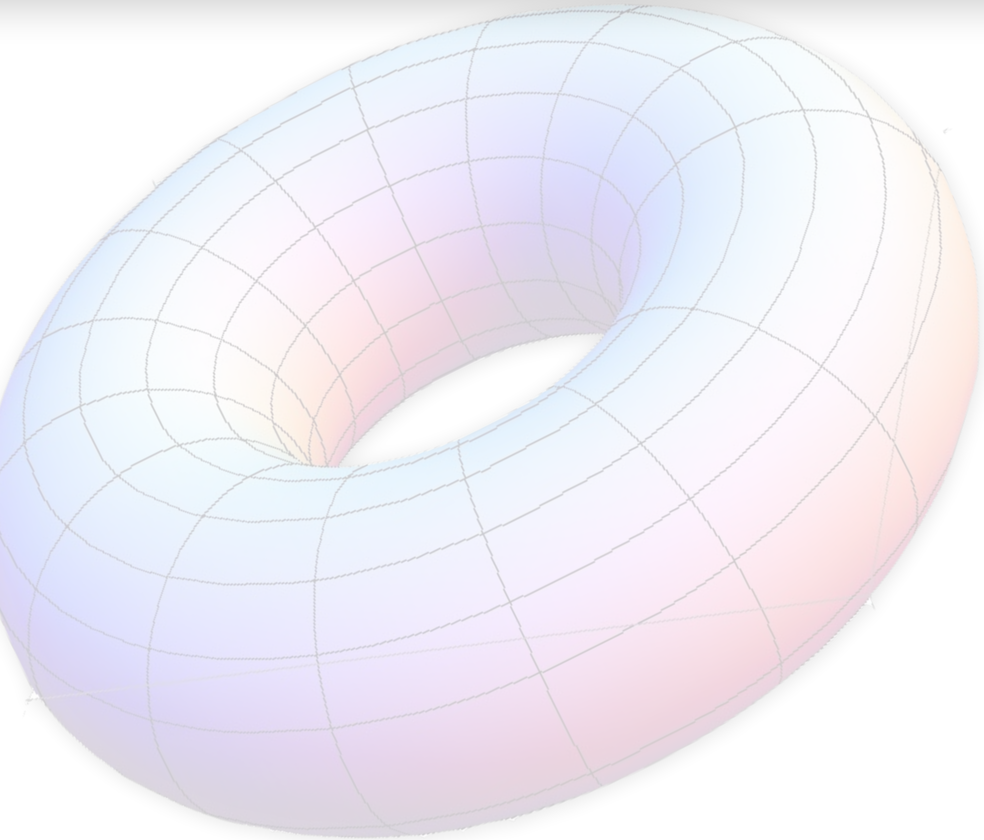
Free structureless parameters in the SM → **Modular Invariance**

Predictivity ✓

User-friendly ✗

$Y(\tau)$

Modular forms:
less free parameters



A substantial amount of successful modular models

For reviews, see

T. Kobayashi, M. Tanimoto (2307.03384)

G.-J. Ding, S.F. King (2311.09282)

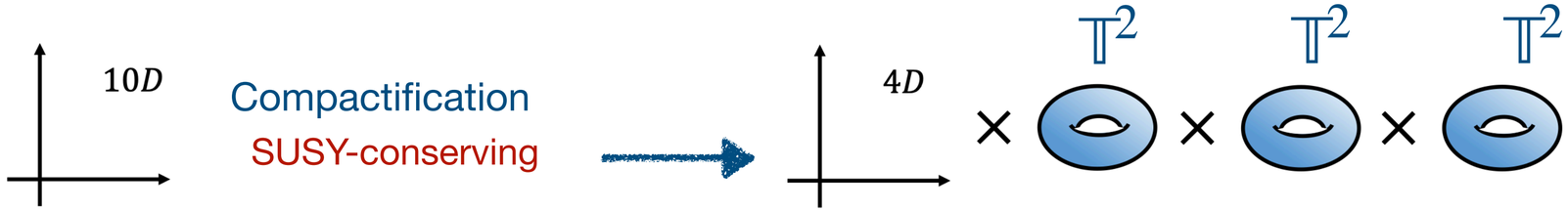
Attempts include:

Both lepton and quark sectors ✓

Strong CP problem ✓ (2305.08908)

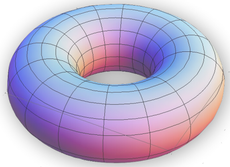
Inflation ✓ (2405.06497)

Baryogenesis ✓ (2504.03506)



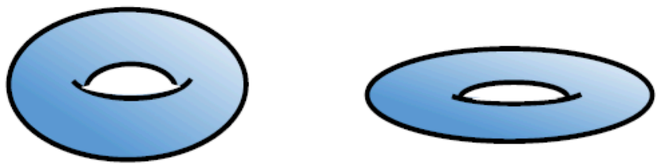
Discrete lattice Λ

$\omega_1, \omega_2 \in \mathbb{C}$

$\mathbb{C}/\Lambda \equiv$ 

$$\tau \equiv \frac{\omega_2}{\omega_1}$$

Modulus



$\tau = \tau_1$ $\tau = \tau_2$

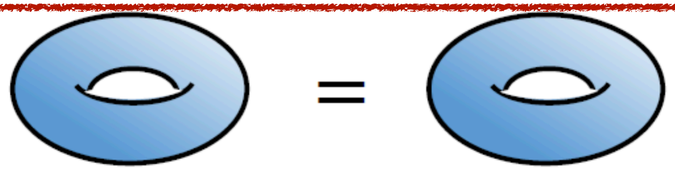
Change basis?

$$\begin{pmatrix}\omega_2' \\ \omega_1'\end{pmatrix} = \gamma \begin{pmatrix}\omega_2 \\ \omega_1\end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d\end{pmatrix}\begin{pmatrix}\omega_2 \\ \omega_1\end{pmatrix} \qquad a, b, c, d \in \mathbb{Z}$$

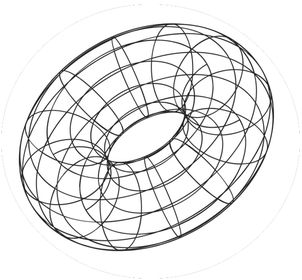
$\gamma \in$ SL$(2,\mathbb{Z}) \equiv$ Modular group $\equiv \Gamma$

$$\tau \equiv \frac{\omega_2}{\omega_1} \quad \xrightarrow{\text{SL}(2,\mathbb{Z})} \quad \tau' = \frac{a\,\omega_2 + b\,\omega_1}{c\,\omega_2 + d\,\omega_1} = \boxed{\frac{a\tau + b}{c\tau + d}}$$

$\tau \to \tau'$



$\tau \qquad \tau'$



$$\int d^4x\, d^6y\, \mathscr{L}_{10D} \implies \int d^4x\, \mathscr{L}_{EFT}$$

What is a modular form? $Y(\tau)$

- $Y(\gamma(\tau)) = (c\tau + d)^k\, Y(\tau)$

$$c, d \in \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \gamma$$

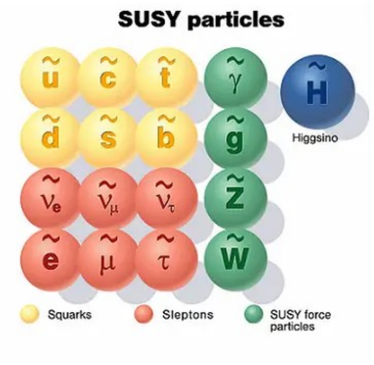
$$a, b, c, d \in \mathbb{Z} \quad , \quad ad - bc = 1$$

Holomorphic in: $\{\tau \in \mathbb{C} \,|\, \mathrm{Im}(\tau) > 0\}$

- "Weight" k>0

Very constraining!

Superfields transformations



$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} \qquad \gamma \in \Gamma \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}$$

Usual matter fields

Unitary irrep. of $\Gamma_N \subset \Gamma$

$N = 1, 2, 3, \ldots$ “Level"

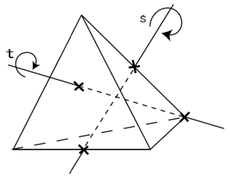
Finite modular group

$$\Gamma_N$$

for $N \leq 5$ isomorphic to

$N = 2$	$N = 3$	$N = 4$	$N = 5$
S_3	A_4	S_4	A_5

non-abelian discrete groups



- $\mathcal{W}(\Phi) = \sum (Y_{I_1 \ldots I_n}(\tau)\, \varphi^{(I_1)} \ldots \varphi^{(I_n)})_{\mathbf{1}}$ $\quad$ $\mathscr{W}(\Phi)$ modular invariant if:

$$\begin{cases} \rho \otimes \rho_{I_1} \otimes \rho_{I_2} \ldots \otimes \rho_{I_n} \supset \mathbf{1} & \longrightarrow \text{Usual} \\ k_Y = k_{I_1} + k_{I_2} + \ldots + k_{I_n} & \longrightarrow \text{Novelty} \end{cases}$$

$$Y_{I_1 \ldots I_n}(\tau) \to (c\tau + d)^{k_Y} \rho(\gamma) Y_{I_1 \ldots I_n}(\tau) \qquad \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)}$$

Yukawa: modular forms of weight k_Y $\qquad$ Superfields with modular charges $-k_I$

$$(c\tau + d)^{k_Y} (c\tau + d)^{-\sum k_{I_n}} = 1$$

Symmetry breaking

Γ

$< \tau > \equiv$ VEV

Γ

$Y(\tau)$

uniquely determined by τ

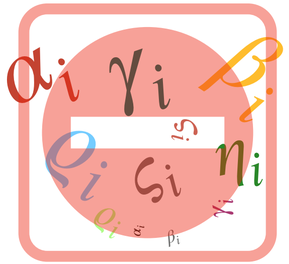
Fourier $\sim \sum_{n}^{\infty} a_n q^n$

$$q \equiv e^{2\pi i \tau}$$

Lepton mass matrices

$$\mathrm{M}_e \sim \sum_i \alpha_i \begin{pmatrix} f_{11}(\tau) & f_{12}(\tau) & \dots \\ \dots & \dots & \dots \\ \dots & \dots & f_{33}(\tau) \end{pmatrix}$$

$f_{ij} \equiv$ pre-determined functions of τ



$\alpha_i \equiv$ limited number of free parameters ✓

Impose CP symmetry on the model

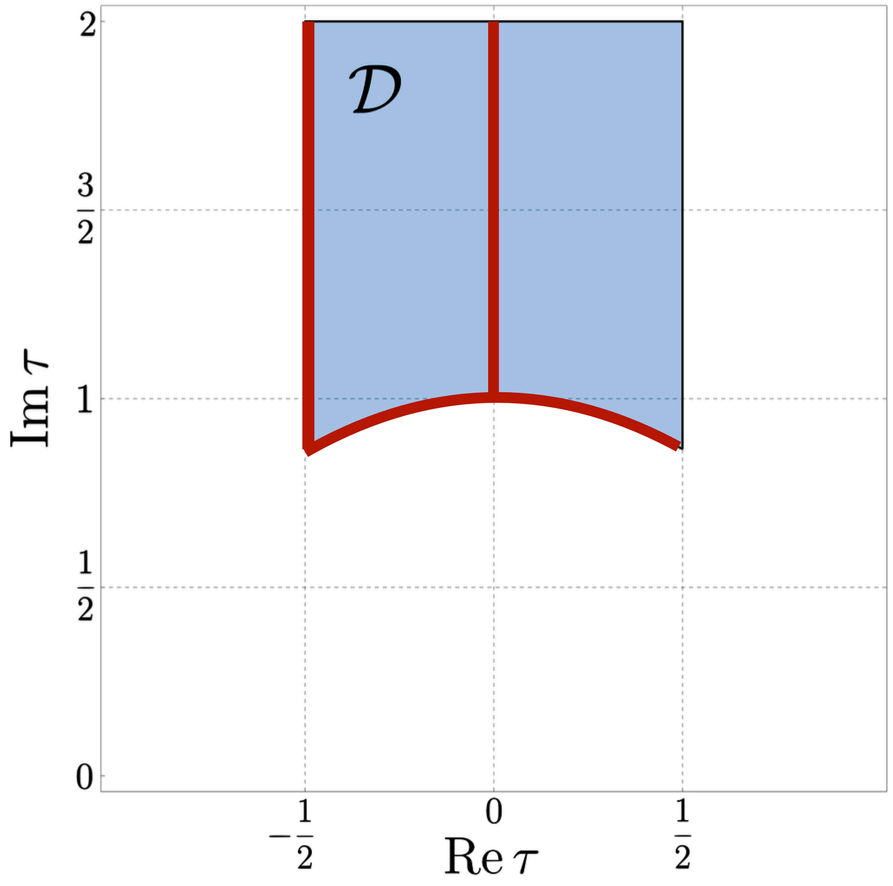
- gCP $\Rightarrow$ $\alpha_i \in \mathbb{R}$

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

Only source of CPV in the model is the VEV of τ

$$< \tau > = \mathrm{Re}\,\tau + i\,\mathrm{Im}\,\tau$$



- Every $\tau \notin \mathscr{D}$ can be mapped in $\tau' \in \mathscr{D}$ through Γ transformation

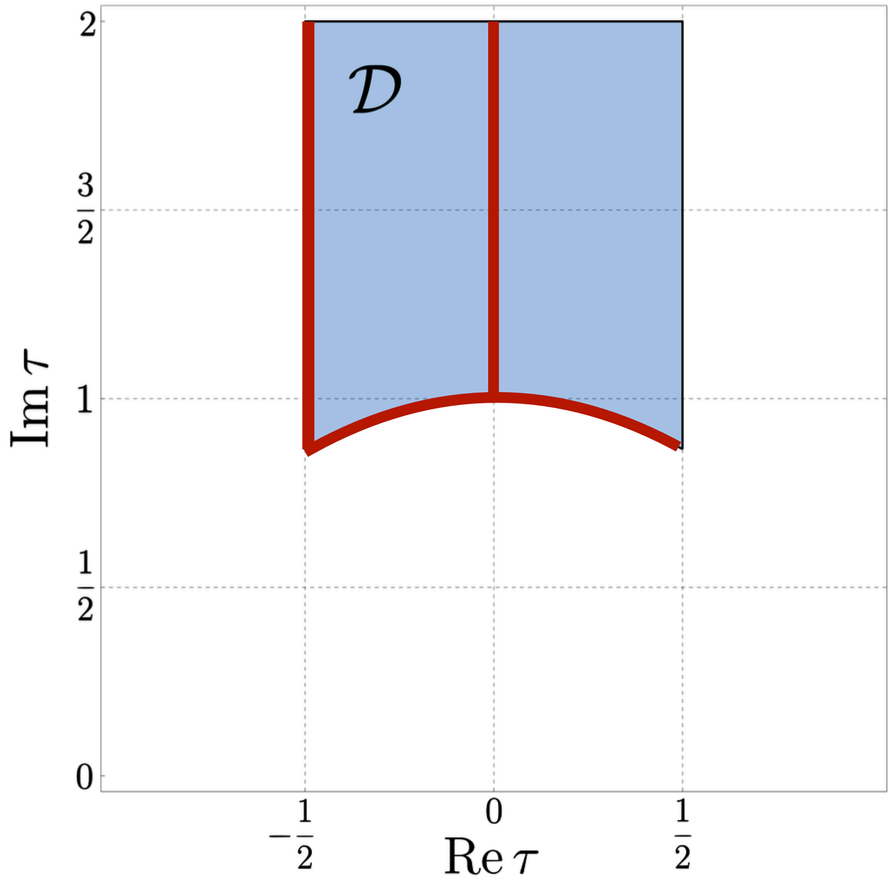
- CP conserving values

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

Only source of CPV in the model is the VEV of τ

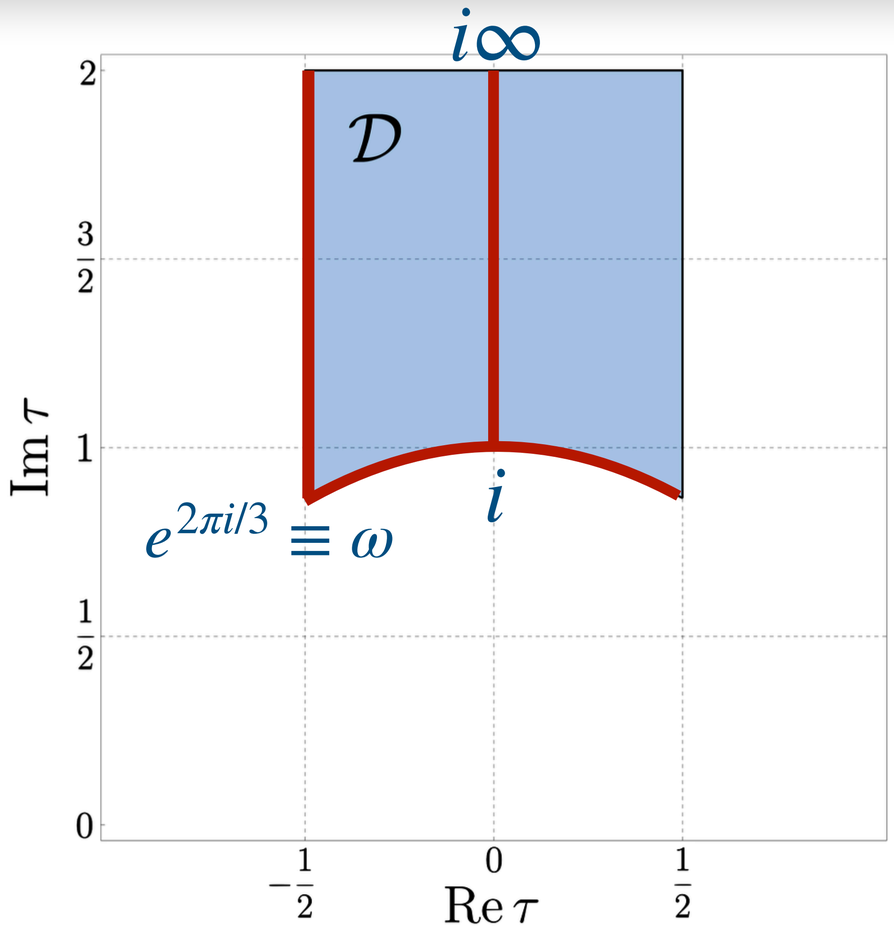
$$\mathcal{D} = \left\{ \tau \in \mathbb{C} \,:\, \mathrm{Im}\,\tau > 0\,,\ \ |\mathrm{Re}\,\tau| \leq \frac{1}{2}\,,\ \ |\tau| \geq 1 \right\}$$



$$\mathcal{D} = \left\{ \tau \in \mathbb{C} \,:\, \mathrm{Im}\,\tau > 0\,,\ \ |\mathrm{Re}\,\tau| \leq \frac{1}{2}\,,\ \ |\tau| \geq 1 \right\}$$

- Usually the hierarchies put by hand (e.g. $y_e \ll y_\mu \ll y_\tau$) in the free parameters
- Froggatt-Nielsen approaches need additional scalars
- The modular way: no new scalars, only τ needed!

2102.07488 Novichkov, Penedo, Petcov



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2102.07488 Novichkov, Penedo, Petcov

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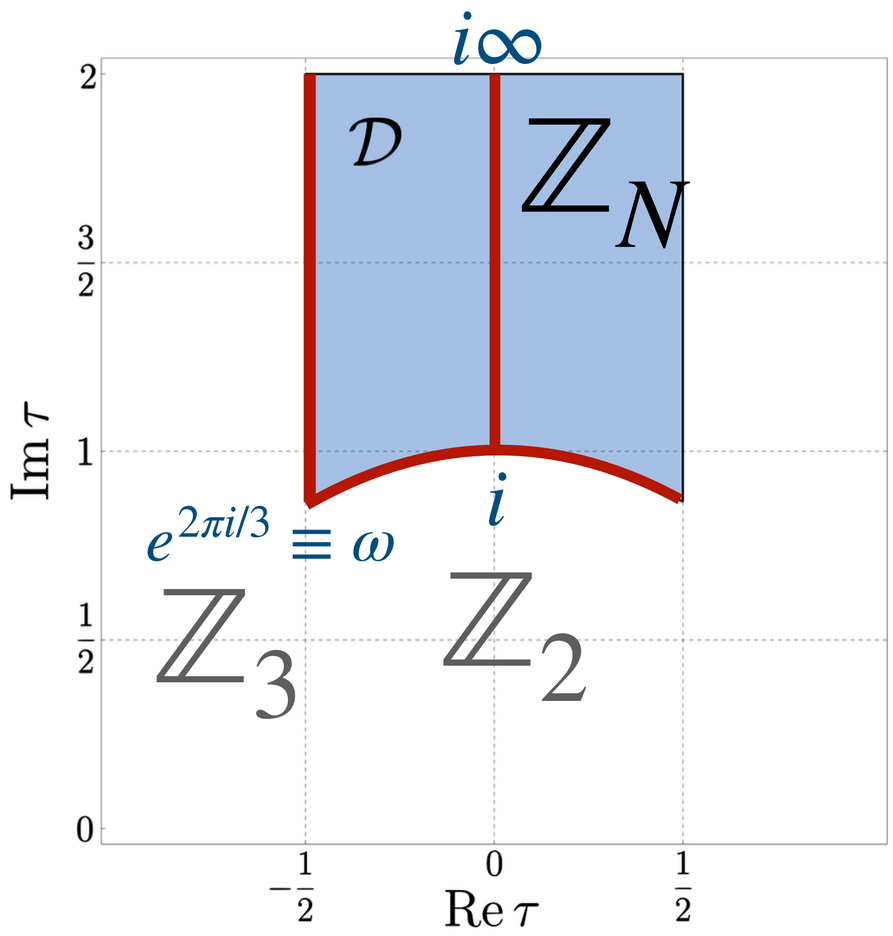


- The modular way: no new scalars, only τ needed!

  2102.07488 Novichkov, Penedo, Petcov

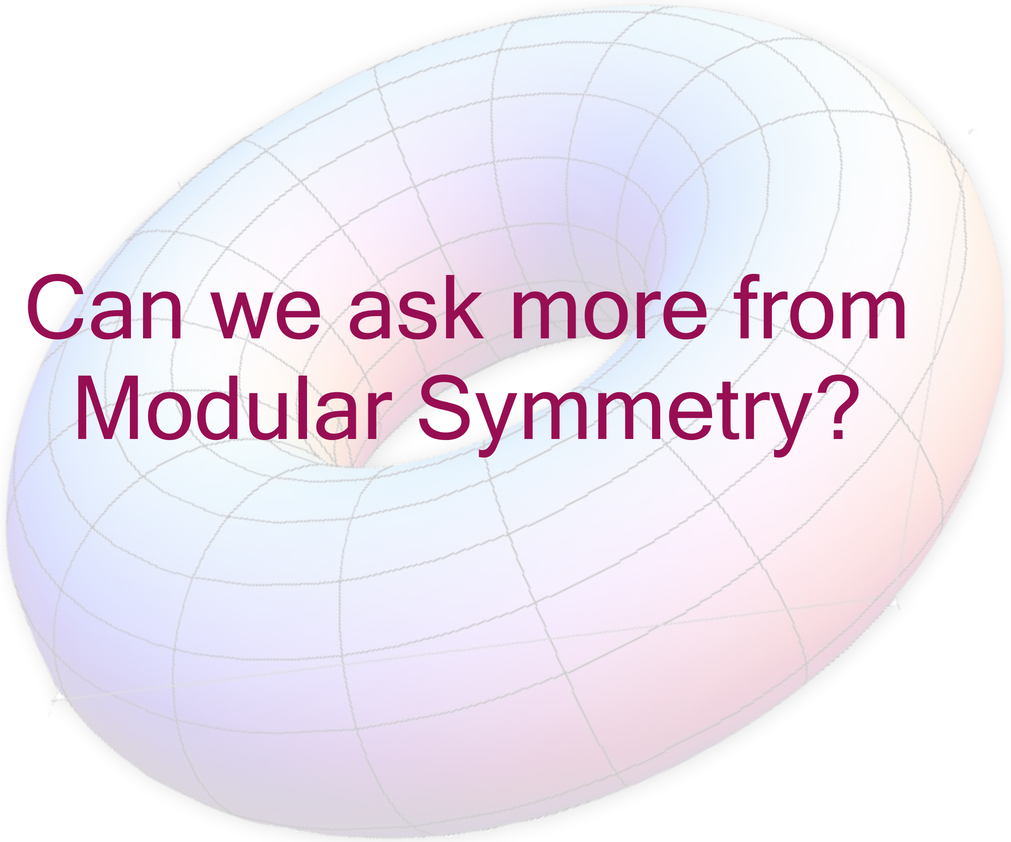
- $\tau = \tau_{\rm sym}$ $\qquad \epsilon = |\tau - \tau_{\rm sym}| > 0$

$$M \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \rightarrow \quad M \sim \begin{pmatrix} 1 & \epsilon^{\cdots} & \epsilon^{\cdots} \\ \epsilon^{\cdots} & \epsilon^{\cdots} & \epsilon^{\cdots} \\ \epsilon^{\cdots} & \epsilon^{\cdots} & \epsilon^{\cdots} \end{pmatrix}$$

Adapted from J.T.Penedo's slides at DISCRETE 2021

- Mass hierarchies: e.g. $(1,\ \epsilon\ ,\ \epsilon^2)$

- The pattern depends on both Γ'_N and the weights



Can we ask more from Modular Symmetry?

- Can "ϵ" also participate in new physics phenomenology?

Introducing
Modular-Symmetry-Protected seesaw

- Can "ϵ" also participate in new physics phenomenology?

Ingredients:

- The familiar symmetry-protected low-scale type-I seesaw

  see e.g. S. Antusch, O. Fischer 1502.05915

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 0 & y_{11}\,v_u & 0 & 0 \\ 0 & 0 & 0 & y_{21}\,v_u & 0 & 0 \\ 0 & 0 & 0 & y_{31}\,v_u & 0 & 0 \\ \hline y_{11}\,v_u & y_{21}\,v_u & y_{31}\,v_u & 0 & M_N & 0 \\ 0 & 0 & 0 & M_N & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_3 \end{array}\right)$$

L-conserving full 6x6 neutrino mass matrix

- $m_\nu = 0$ in this L conserving matrix

Ingredients:

- The familiar symmetry-protected low-scale type-I seesaw

  see e.g. S. Antusch, O. Fischer 1502.05915

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 0 & y_{11}\,v_u & 0 & 0 \\ 0 & 0 & 0 & y_{21}\,v_u & 0 & 0 \\ 0 & 0 & 0 & y_{31}\,v_u & 0 & 0 \\ \hline y_{11}\,v_u & y_{21}\,v_u & y_{31}\,v_u & 0 & M_N & 0 \\ 0 & 0 & 0 & M_N & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_3 \end{array}\right)$$

$$m_\nu \;=\; -m_D\, M_R^{-1}\, m_D^T$$

L-conserving full 6x6 neutrino mass matrix

- Modular Symmetry mimics L-symmetry in the symmetric points $\tau = \{\omega, i\infty\}$

- L broken by $\epsilon \equiv |\tau - \tau_{sym}|$

Ingredients:

- Modular Symmetry mimics L-symmetry in the symmetric points $\tau = \{\omega, i\infty\}$

- Massless neutrinos in the limit of L-symmetry

$$
\begin{array}{c}
\qquad\qquad\qquad\qquad\qquad\qquad m_0 \\
\left(\begin{array}{ccc|ccc}
0 & 0 & 0 & y_{11}\, v_u & 0 & 0 \\
0 & 0 & 0 & y_{21}\, v_u & 0 & 0 \\
0 & 0 & 0 & y_{31}\, v_u & 0 & 0 \\
\hline
y_{11}\, v_u & y_{21}\, v_u & y_{31}\, v_u & 0 & M_N & 0 \\
0 & 0 & 0 & M_N & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & M_3
\end{array}\right) M_0
\end{array}
$$

- pseudo-Dirac pair of mass M_N

Ingredients:

$$m_\nu = -m_D M_R^{-1} m_D^T$$

- Small neutrino masses controlled also by $\epsilon \equiv |\tau - \tau_{sym}|$

$$m_D = m_0 + \epsilon^d m_1 \qquad M_R = M_0 + \epsilon^r M_1$$

Controlled by modular forms

Ingredients:

- Small neutrino masses controlled by $\epsilon \equiv |\tau - \tau_{sym}|$

$$m_i^\nu \sim \max(|\epsilon|^r, |\epsilon|^d) \times \frac{y^2 v_u^2}{M}$$

$y \sim \mathcal{O}(10^{-6})$ and $M \sim$ GeV

- Mass splitting of the heavy pseudo-Dirac pair also controlled by $\epsilon \equiv |\tau - \tau_{sym}|$

$$\Delta M \sim |\epsilon|^r \times M\,. \qquad \text{If } M_1 \neq 0$$

$$\Delta M \sim |\epsilon|^d \times \frac{y^2 v_u^2}{M} \qquad \text{If } M_1 = 0$$

The pay-off

- Charged-leptons mass hierarchies from the same ϵ that breaks L-symmetry, gives $m_\nu \neq 0$, $\Delta M \neq 0$!
- Limited number of free parameters
- HNL phenomenology at colliders (low-scale seesaw) and at intensity frontier

An example

- Charged-leptons mass hierarchies controlled by the same ϵ which breaks L-symmetry, and gives $m_\nu \neq 0$, $\Delta M \neq 0$!

Model	Group	$\tau_{\rm sym}$	L	E^c	N^c	ΔM
A	A_4	ω	$(\mathbf{1},+2)\oplus(\mathbf{1}',+4)\oplus(\mathbf{1}'',+6)$	$(\mathbf{3},0)$	$(\mathbf{3},0)$	$\sim m_\nu$
B			$(\mathbf{1}',+1)\oplus(\mathbf{1}'',+3)\oplus(\mathbf{1},+5)$	$(\mathbf{3},+1)$	$(\mathbf{3},+1)$	$\sim \epsilon M$
C	S_4'	$i\infty$	$(\hat{\mathbf{1}},+2)\oplus(\mathbf{1},+3)\oplus(\hat{\mathbf{1}},+4)$	$(\mathbf{3},+1)$	$(\mathbf{3},+1)$	$\sim \epsilon^2 M$
D			$(\hat{\mathbf{1}}',+2)\oplus(\mathbf{1},+3)\oplus(\hat{\mathbf{1}}',+4)$			

Let's focus on this

Notation: $\psi \sim (\rho, k_\psi)$

An example: model C

Group
$\Gamma'_4 \simeq S'_4$

L	E^c	N^c	ΔM
$(\hat{\mathbf{1}},+2)\oplus(\mathbf{1},+3)\oplus(\hat{\mathbf{1}},+4)$	$(\mathbf{3},+1)$	$(\mathbf{3},+1)$	$\sim\epsilon^2 M$

$\tau_{sym} \simeq i\infty$

$$W = (Y_e)_{ij} L_i E^c_j H_d + (Y_D)_{i\kappa} L_i N^c_\kappa H_u + \frac{1}{2}(M_R)_{\kappa\rho} N^c_\kappa N^c_\rho$$

An example: model C

Group
$\Gamma'_4 \simeq S'_4$

L	E^c	N^c	ΔM
$(\hat{\mathbf{1}},+2)\oplus(\mathbf{1},+3)\oplus(\hat{\mathbf{1}},+4)$	$(\mathbf{3},+1)$	$(\mathbf{3},+1)$	$\sim \epsilon^2 M$

$\tau_{sym} \simeq i\infty$

$$M_R = \Lambda \begin{pmatrix} \frac{2}{\sqrt{3}}\left(Y^{(2)}_{\mathbf{2}}\right)_1 & 0 & 0 \\ 0 & \left(Y^{(2)}_{\mathbf{2}}\right)_2 & -\frac{1}{\sqrt{3}}\left(Y^{(2)}_{\mathbf{2}}\right)_1 \\ 0 & -\frac{1}{\sqrt{3}}\left(Y^{(2)}_{\mathbf{2}}\right)_1 & \left(Y^{(2)}_{\mathbf{2}}\right)_2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & \epsilon^2 & 1 \\ 0 & 1 & \epsilon^2 \end{pmatrix}$$

$$\epsilon \sim q^{1/4} = e^{i\pi\tau/2}$$

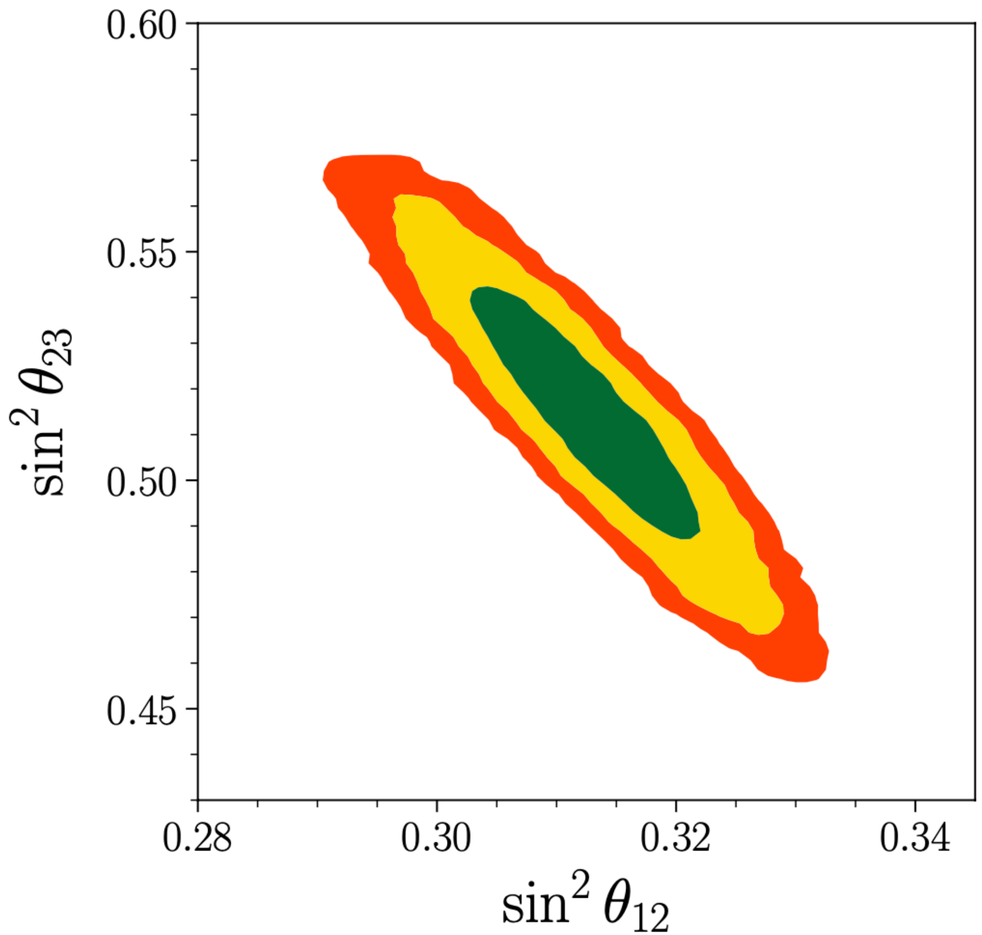
L-conserving for $\epsilon \to 0$

$$Y_D = \begin{pmatrix} g_1\left(Y^{(3)}_{\hat{\mathbf{3}}'}\right)_1 & g_1\left(Y^{(3)}_{\hat{\mathbf{3}}'}\right)_3 & g_1\left(Y^{(3)}_{\hat{\mathbf{3}}'}\right)_2 \\ g_2\left(Y^{(4)}_{\mathbf{3}}\right)_1 & g_2\left(Y^{(4)}_{\mathbf{3}}\right)_3 & g_2\left(Y^{(4)}_{\mathbf{3}}\right)_2 \\ g_3\left(Y^{(5)}_{\hat{\mathbf{3}}'}\right)_1 & g_3\left(Y^{(5)}_{\hat{\mathbf{3}}'}\right)_3 & g_3\left(Y^{(5)}_{\hat{\mathbf{3}}'}\right)_2 \end{pmatrix}$$

$$Y_e = \begin{pmatrix} \alpha_1\left(Y^{(3)}_{\hat{\mathbf{3}}'}\right)_1 & \alpha_1\left(Y^{(3)}_{\hat{\mathbf{3}}'}\right)_3 & \alpha_1\left(Y^{(3)}_{\hat{\mathbf{3}}'}\right)_2 \\ \alpha_2\left(Y^{(4)}_{\mathbf{3}}\right)_1 & \alpha_2\left(Y^{(4)}_{\mathbf{3}}\right)_3 & \alpha_2\left(Y^{(4)}_{\mathbf{3}}\right)_2 \\ \alpha_3\left(Y^{(5)}_{\hat{\mathbf{3}}'}\right)_1 & \alpha_3\left(Y^{(5)}_{\hat{\mathbf{3}}'}\right)_3 & \alpha_3\left(Y^{(5)}_{\hat{\mathbf{3}}'}\right)_2 \end{pmatrix}$$

$$\sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

Pheno: model C (Inverted Ordering)



1σ

2σ

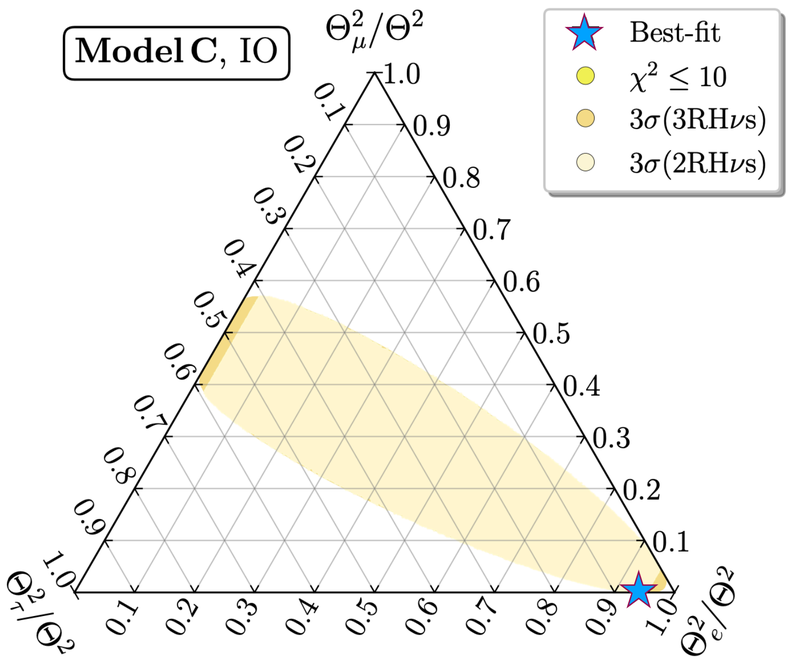
3σ

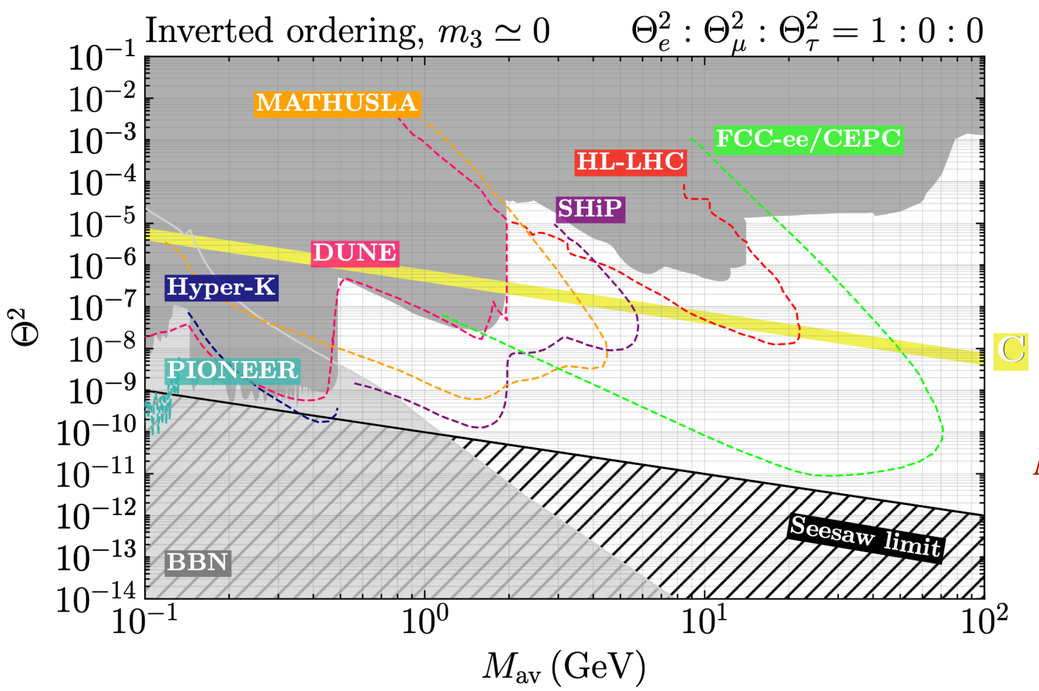
6 dimensionless parameters

9 in total (includes Λ and τ)

Pheno: model C (Inverted Ordering)

$$\Theta_{\alpha j} \equiv v_u \hat{Y}_{\alpha j}/M_j. \qquad \Theta_\alpha^2 \equiv \sum_{j=1}^{3} |\Theta_{\alpha j}|^2 \quad \text{and} \quad \Theta^2 \equiv \sum_{\alpha=e,\mu,\tau} \Theta_\alpha^2.$$





$$M_{av} = \frac{\sum_i M_i}{3}$$

Where to go from here?

Smallness of light neutrino masses

$m_\nu \to 0$ as $\epsilon \to 0$

$m_e/m_\mu \simeq \frac{1}{200}$

$m_\mu/m_\tau \simeq \frac{1}{17}$

$$\epsilon \equiv |\tau - \tau_{sym}|$$

$\Delta M \sim |\epsilon|^r \times M\,.$

L-conserving for $\epsilon \to 0$

CP conserving (in gCP mode)

$\delta_{CP}, \alpha_{21}, \alpha_{31} \to 0$ as $\epsilon \to 0$

Where to go from here?

$$\epsilon \equiv |\tau - \tau_{sym}|$$

Is spontaneous CP violation too much to ask?

$$\delta_{CP}, \alpha_{21}, \alpha_{31} \to 0 \quad \text{as} \quad \epsilon \to 0$$

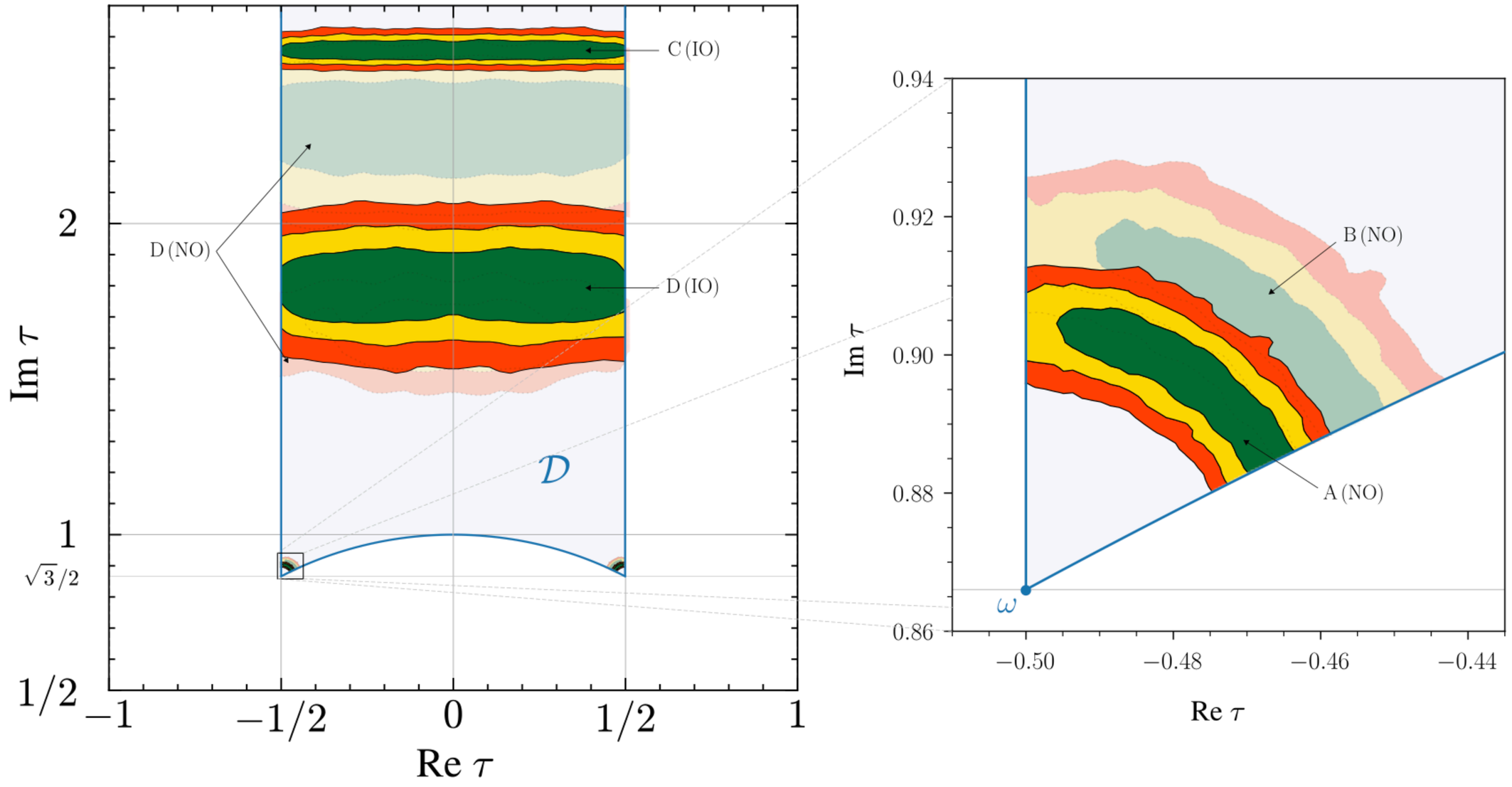
Thank you for the attention

Roma Tre Neutrino ν Group

SCAN ME

BACKUP SLIDES

Fundamental domain of the benchmarks



The numerics of the benchmarks

Model (ordering)	A (NO)	B (NO)	C (IO)	D (NO)	D (IO)
$\mathrm{Re}\,\tau$	$-0.472^{+0.017}_{-0.028}$	$-0.475^{+0.036}_{-0.024}$	all	all	all
$\mathrm{Im}\,\tau$	$0.892^{+0.019}_{-0.042}$	$0.912^{+0.015}_{-0.062}$	$2.55^{+0.07}_{-0.04}$	$2.31^{+0.25}_{-0.82}$	$1.68^{+0.37}_{-0.24}$
$\hat{\alpha}_2/\hat{\alpha}_1$	$-0.286^{+0.674}_{-0.106}$	$0.590^{+0.125}_{-0.083}$	$-0.593^{+0.102}_{-0.078}$	$0.31^{+1.06}_{-1.05}$	$-0.058^{+0.348}_{-0.084}$
$\hat{\alpha}_{3(,1)}/\hat{\alpha}_1$	$-0.160^{+0.145}_{-0.481}$	$-0.0502^{+0.0501}_{-0.1540}$	$0.0339^{+0.0026}_{-0.0024}$	$0.33^{+2.08}_{-0.12}$	$-1.38^{+0.70}_{-0.85}$
$\hat{\alpha}_{3,2}/\hat{\alpha}_1$	$0.0202^{+0.0094}_{-0.0514}$	$-0.0185^{+0.0037}_{-0.0062}$	—	$0.20^{+4.25}_{-0.79}$	$-0.71^{+2.30}_{-2.39}$
$\hat{g}_2/\hat{g}_1$	$0.712^{+0.052}_{-0.076}$	$0.219^{+0.011}_{-0.008}$	$0.0799^{+0.0137}_{-0.0125}$	$1.56^{+1.30}_{-1.19}$	$2.55^{+1.29}_{-0.61}$
$\hat{g}_{3(,1)}/\hat{g}_1$	$0.491^{+0.196}_{-0.461}$	$0.336^{+0.062}_{-0.161}$	$13.2^{+1.1}_{-1.2}$	$0.129^{+0.011}_{-0.073}$	$0.0651^{+0.0172}_{-0.0163}$
$\hat{g}_{3,2}/\hat{g}_1$	$0.214^{+0.119}_{-0.551}$	$0.289^{+0.018}_{-0.020}$	—	$0.853^{+0.077}_{-0.734}$	$0.798^{+0.224}_{-0.231}$
$v_d\,\hat{\alpha}_1$, GeV	$0.594^{+0.070}_{-0.121}$	$0.363^{+0.047}_{-0.055}$	$0.176^{+0.005}_{-0.007}$	$0.0689^{+0.0132}_{-0.0427}$	$0.0384^{+0.0126}_{-0.0061}$
$v_u^2\,\hat{g}_1^2/\Lambda$, eV	$0.0328^{+0.0145}_{-0.0070}$	$0.0568^{+0.0192}_{-0.0120}$	$0.00032^{+0.00005}_{-0.00003}$	$0.0184^{+0.0069}_{-0.0066}$	$0.0120^{+0.0108}_{-0.0035}$
m_e/m_μ	$0.0048^{+0.0006}_{-0.0005}$	$0.0047^{+0.0006}_{-0.0005}$	$0.0048^{+0.0005}_{-0.0006}$	$0.0047^{+0.0005}_{-0.0006}$	$0.0048^{+0.0005}_{-0.0006}$
m_μ/m_τ	$0.0560^{+0.0115}_{-0.0114}$	$0.0571^{+0.0114}_{-0.0121}$	$0.0577^{+0.0109}_{-0.0134}$	$0.0550^{+0.0129}_{-0.0105}$	$0.0572^{+0.0099}_{-0.0117}$
r	$0.0297^{+0.0022}_{-0.0021}$	$0.0297^{+0.0023}_{-0.0021}$	$0.0300^{+0.0023}_{-0.0021}$	$0.0299^{+0.0021}_{-0.0023}$	$0.0302^{+0.0020}_{-0.0023}$
$\sin^2\theta_{12}$	$0.307^{+0.034}_{-0.030}$	$0.308^{+0.032}_{-0.030}$	$0.312^{+0.019}_{-0.018}$	$0.310^{+0.030}_{-0.034}$	$0.308^{+0.032}_{-0.034}$
$\sin^2\theta_{13}$	$0.0220^{+0.0017}_{-0.0014}$	$0.0221^{+0.0014}_{-0.0014}$	$0.0222^{+0.0016}_{-0.0014}$	$0.0222^{+0.0016}_{-0.0016}$	$0.0224^{+0.0013}_{-0.0015}$
$\sin^2\theta_{23}$	$0.506^{+0.067}_{-0.065}$	$0.507^{+0.065}_{-0.062}$	$0.519^{+0.049}_{-0.058}$	$0.512^{+0.070}_{-0.065}$	$0.507^{+0.070}_{-0.030}$
m_1, eV	$< 10^{-4}$	$< 10^{-4}$	$0.0491^{+0.0002}_{-0.0002}$	$0.00267^{+0.00048}_{-0.00132}$	$0.0620^{+0.0025}_{-0.0032}$
m_2, eV	$0.00864^{+0.00028}_{-0.00029}$	$0.00865^{+0.00029}_{-0.00028}$	$0.0499^{+0.0001}_{-0.0002}$	$0.00907^{+0.00028}_{-0.00044}$	$0.0626^{+0.0024}_{-0.0032}$
m_3, eV	$0.0501^{+0.0001}_{-0.0002}$	$0.0501^{+0.0002}_{-0.0002}$	$< 10^{-4}$	$0.0502^{+0.0002}_{-0.0002}$	$0.0379^{+0.0038}_{-0.0054}$
$\Sigma_i m_i$, eV	$0.0588^{+0.0001}_{-0.0001}$	$0.0588^{+0.0001}_{-0.0001}$	$0.0990^{+0.0003}_{-0.0004}$	$0.0619^{+0.0006}_{-0.0017}$	$0.163^{+0.009}_{-0.012}$
$m_{\beta\beta}$, meV	$1.49^{+0.31}_{-0.28}$	$1.49^{+0.27}_{-0.28}$	$17.8^{+1.7}_{-1.8}$	$2.06^{+0.98}_{-0.22}$	$60.0^{+2.4}_{-3.1}$
δ	$\simeq 0, \pi$	$\simeq 0$	$\simeq 0$	$\simeq \pi$	$\simeq 0, \pi$
$\alpha_{21(23)}$	$\simeq \pi$	$\simeq \pi$	$\simeq 0$	$\simeq \pi$	$\simeq 0$
α_{31}	—	—	—	$\simeq \pi$	$\simeq \pi$
$\lvert\epsilon(\tau)\rvert$	$0.0218^{+0.0047}_{-0.0046}$	$0.0292^{+0.0076}_{-0.0067}$	$0.0182^{+0.0013}_{-0.0018}$	$0.0267^{+0.0700}_{-0.0087}$	$0.0713^{+0.0320}_{-0.0312}$
$(m_e/m_\tau)/\lvert\epsilon\rvert^{2,3}$	$0.563^{+0.167}_{-0.111}$	$0.317^{+0.123}_{-0.088}$	$45.8^{+3.1}_{-3.1}$	$14.0^{+26.0}_{-13.7}$	$0.76^{+2.85}_{-0.50}$
$(m_\mu/m_\tau)/\lvert\epsilon\rvert$	$2.57^{+0.05}_{-0.08}$	$1.96^{+0.20}_{-0.24}$	$3.16^{+0.39}_{-0.52}$	$2.06^{+0.96}_{-1.53}$	$0.801^{+0.517}_{-0.200}$
$\min N\sigma$	0.412	0.411	0.548	0.488	0.362
max par. ratio	49.6	54.1	165	12.2	39.1

Potential shortcomings

Bottom-up problems

Kähler potential

Normalisation of the modular forms

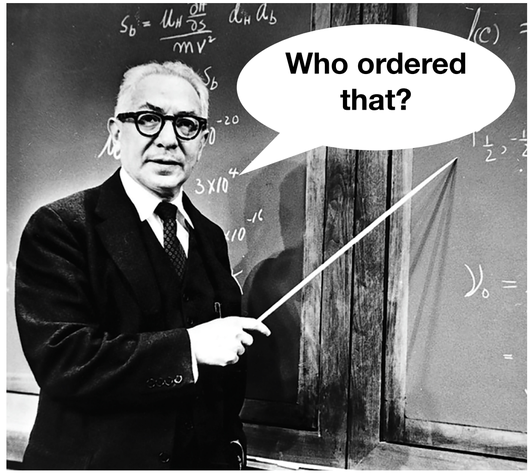
Modulus stabilization

SUSY-dependent theory

Three “copies”, different masses

ν_e ν_μ ν_τ

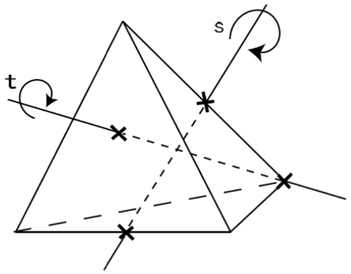
e μ τ



Isidor I. Rabi

non-abelian discrete symmetries

$S_3 \quad A_4 \quad S_4 \quad A_5 \ldots\ldots$



$$\mathscr{W}_{Yukawa} \supset \frac{\alpha}{\Lambda} E^c (L\phi_i)_{\mathbf{1}} H_d$$

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimaximal compatible with data until 2012

$\theta_{13} = 0 \quad \theta_{12} \simeq 35^\circ \quad \theta_{23} = 45^\circ$

Not zero

F. P. An *et al.*, “Observation of electron-antineutrino disappearance at daya bay,” *Phys. Rev. Lett.* **108** (Apr, 2012)

Shortcomings of the traditional approach

EFT with scalar "flavons" ϕ_i

✗ $U_{\text{PMNS}} = U^0_{\text{TBM}} + \ldots \text{corrections}$

$\theta_{13} = 0$ — Automatic

$\theta_{13} \approx 8.5^\circ\,?$ → $\delta\theta_{23}\,,\ \delta\theta_{12} \approx 8.5^\circ$

✗ $\alpha_i\ \gamma_i\ \beta_i\ \varrho_i\ \varsigma_i\ \eta_i\ \ldots$

✗ $V(\phi_i) \to$ Mess!

The Modular symmetry approach

Modular-invariant SUSY action

$$\mathcal{S} = \int d^4x \int d^2\theta d^2\bar{\theta}\, K(\Phi, \bar{\Phi}) + \left[\int d^4x \int d^2\theta\, \mathcal{W}(\Phi) + \text{h.c.}\right]$$

Kähler potential

- Gives the kinetic terms after the modulus acquires a VEV
- A minimalistic form is chosen

Superpotential

- Holomorphic function of superfields
- Encodes the Higgs Yukawa interactions

$$\sigma \equiv \Lambda_\tau \tau$$

The superfields transform as:

$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases} , \quad \text{with} \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_N$$

Action is invariant if, under Γ_N:

$$\begin{cases} \mathcal{W}(\Phi) \to \mathcal{W}(\Phi) \\ K(\Phi, \bar{\Phi}) \to \underbrace{K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi})}_{\text{Kähler transformation}} \end{cases}$$

$\theta, \bar{\theta}$ Grassmann spinor coordinates

$\Phi = (\tau, \varphi)$ Chiral superfields

φ Usual matter supermultiplets

$\rho(\gamma)$ Unitary representation of Γ_N

The Kähler potential...

- Minimalist choice for the Kähler

$$K(\Phi,\bar{\Phi}) = -h\Lambda_\tau^2 \log(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\varphi^{(I)}|^2$$

$\Lambda_\tau \equiv$ dimensions of mass

Satisfies $K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + f(\Phi) + f(\bar{\Phi})$ under Γ_N

$h \equiv$ positive constant

- In a bottom-up approach, this is just a choice

- Corrections of the Kähler potential can spoil the predictivity of the model

M.-C. Chen, S. Ramos-Sánchez, and M. Ratz, "A note on the predictions of models with modular flavor symmetries," *Physics Letters B* **801** (Feb, 2020) 135153.

- This question is an open one

The Modular symmetry approach

The group generators

- Finite modular group can be defined: $\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}) \middle| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\mathrm{mod}\ N) \right\}$$

subgroups of Γ N=1,2,3…called “level”

$$\overline{\Gamma} \equiv \Gamma/\{\pm\mathbb{I}\}$$
$$\overline{\Gamma}(N) \equiv \Gamma(N)/\{\pm\mathbb{I}\}$$

- Generators S and T of the modular group Γ_N

$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau + 1 \qquad S^2 = T^N = (ST)^3 = \mathbb{I}$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

- S_3 Generators S and T satisfy:

$$S^2 = T^2 = (ST)^3 = \mathbb{I}$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I},$$

Backup slides

Numerical procedure

- Define a "figure of merit", i.e. chi-square for every set of parameters $l(p_i) \equiv \sqrt{\chi^2(p_i)}$

$$\chi^2(p_i) = \sum_{j=1}^{6} \left(\frac{q_j(p_i) - q_j^{\text{b-f}}}{\sigma_j} \right)^2$$

$$p_i = \{\tau, \beta/\alpha, \gamma/\alpha, ..., g'/g, g_p/g, ...\}$$

$$q_j = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, m_e/m_\mu, m_\mu/m_\tau, r\}$$

- Define a "potential" with a given temperature T and a threshold

$$V(p_i) = \begin{cases} l(p_i) & , \quad l(p_i) \leq l_{\max} \\ +\infty & , \quad \text{otherwise} \end{cases}$$

P. P. Novichkov, J. T. Penedo, S. T. Petcov, and A. V. Titov, "Modular S_4 models of lepton masses and mixing," (2019)

- At iteration "t", generate a new point from a Gaussian centred on the previous one

→

- Accept the new point with a probability given by:

$$P_\alpha = \min[1, \exp(V(p_i^{(t)}) - V(p_i'))/T]$$

Non-standard interactions

tests of modulus couplings

G-J. Ding, FF, 2003.13448

non standard neutrino interactions

$$\mathcal{L} = i \sum_{f=e,e^c,\nu} \bar{f}\, \bar{\sigma}^\mu \partial_\mu f + \frac{1}{2}\partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha - \frac{1}{2} M_\alpha^2 \varphi_\alpha^2 - (m_e + \mathcal{Z}_\alpha^e \varphi_\alpha) e^c e - \frac{1}{2}\nu (m_\nu + \mathcal{Z}_\alpha^\nu \varphi_\alpha)\nu + h.c. + \dots .$$

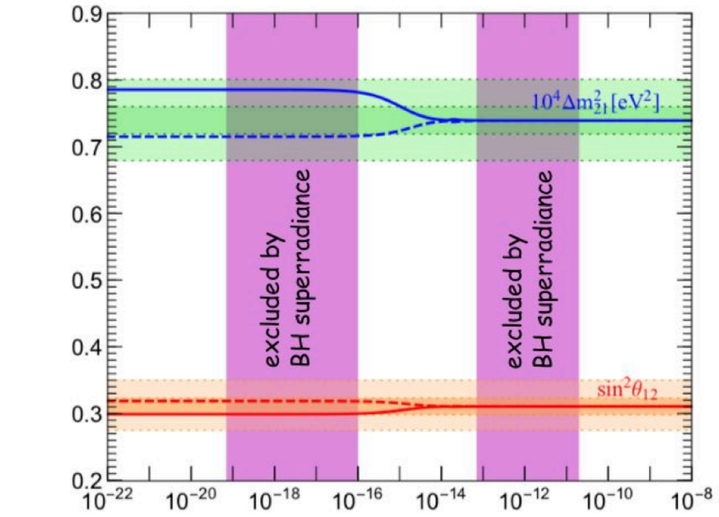
$$\tau = \langle \tau \rangle + \frac{\varphi_u + i\,\varphi_v}{\sqrt{2}}$$

in medium with non-zero electron number density

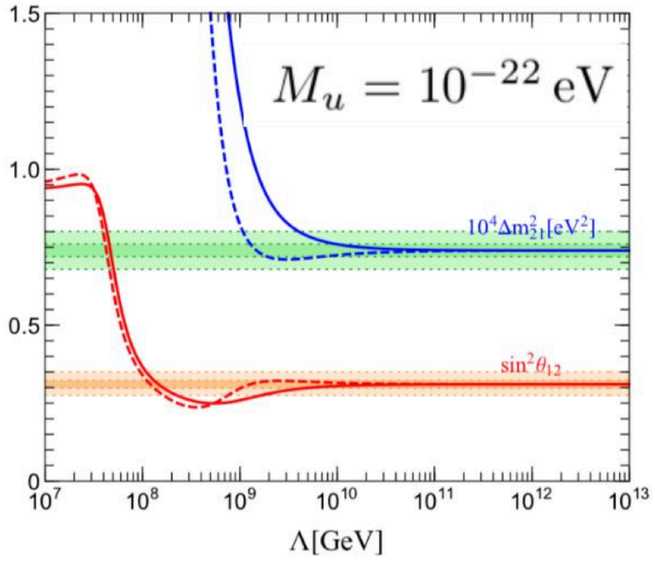
small, unless the modulus is very light

$$\delta m_\nu(0) = -n_e^0 \frac{\mathrm{Re}(\mathcal{Z}^e)\mathcal{Z}^\nu}{M^2(R)} ,$$

in the sun:



$\Lambda = 5 \times 10^9$ GeV



from Feruglio's slides at Mod. Symmetry Bethe Workshop