Planck - May 2025

Randomness and Neutrino Flavour

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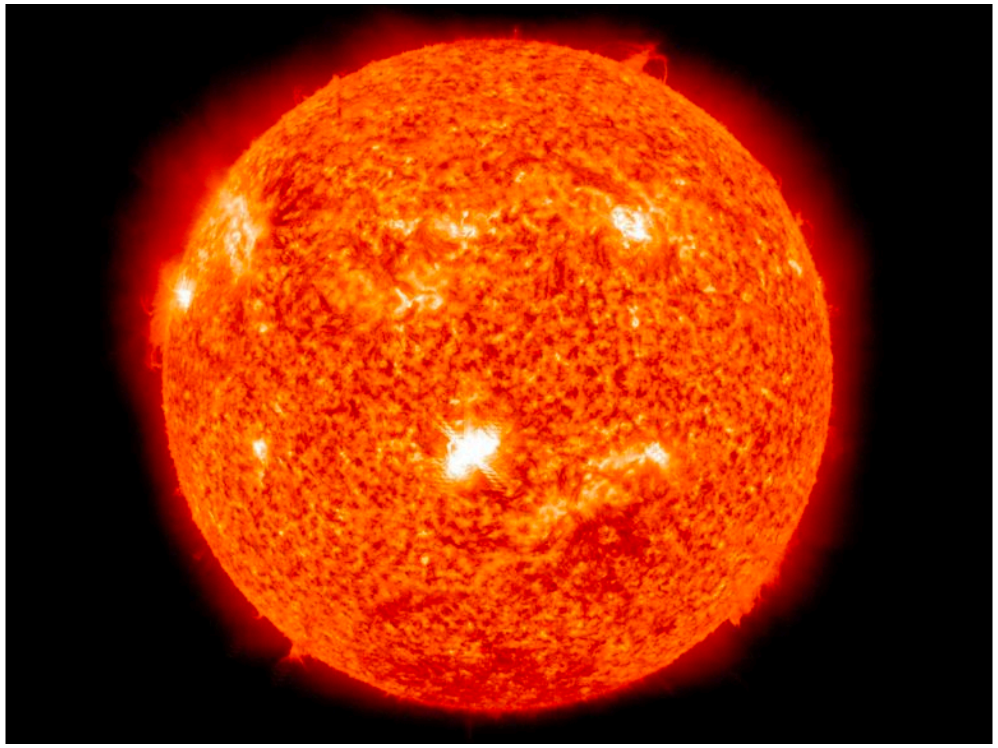
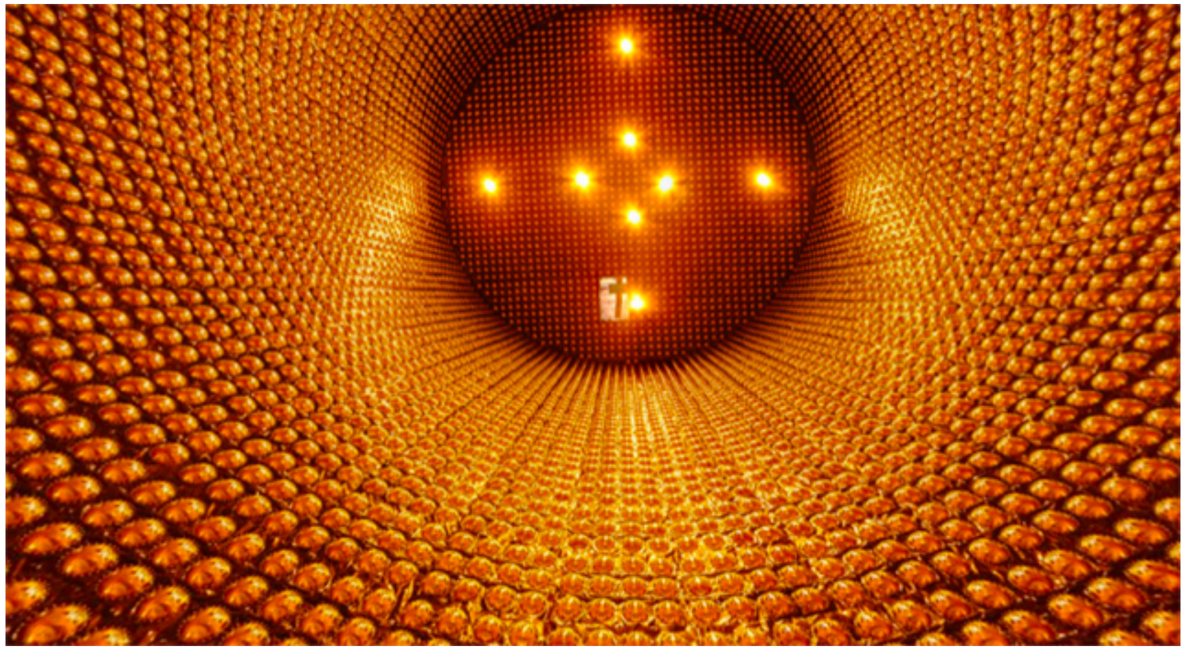
With Aadarsh Singh (in prepration)

Congratulations to My
Mentor, teacher, guide and
a great friend

Prof. Antonio Masiero

Thank you very much !





$$m_{\nu 2} \sim \sqrt{\Delta m^2_{\odot}} \sim 0.008 \text{ eV}$$

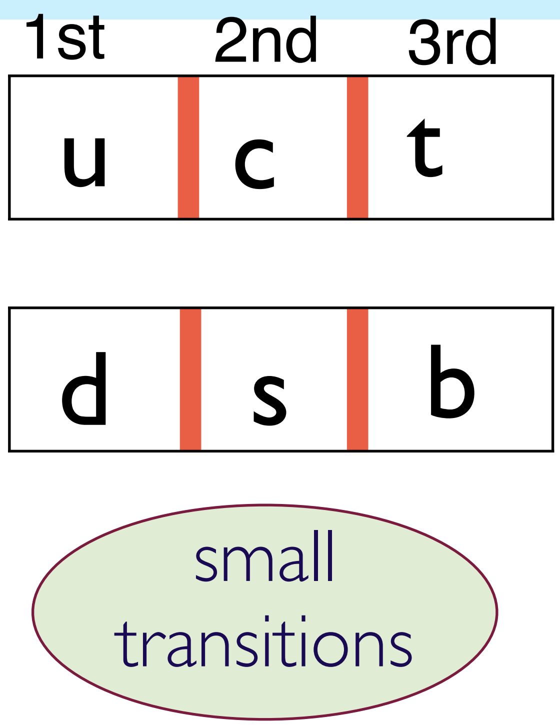
$$m_{\nu_3} \sim \sqrt{\Delta m^2_{\textbf{ATM}}} \sim 0.05 \textbf{ eV}$$

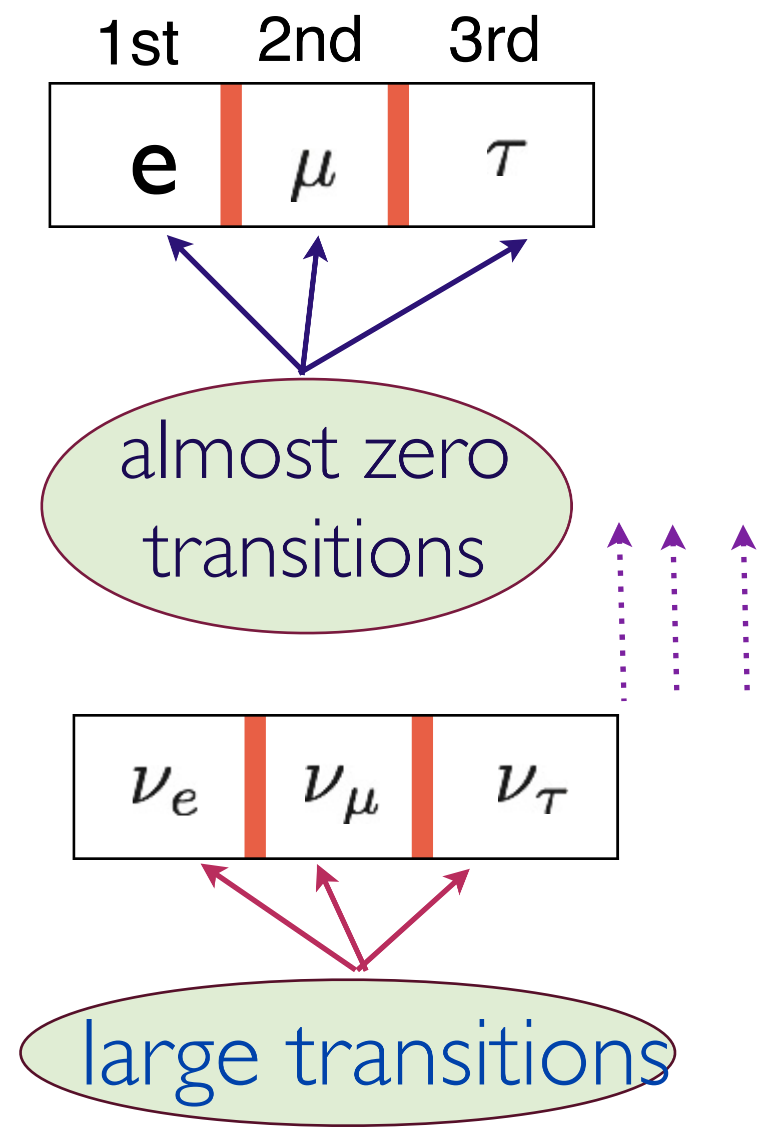
Neutrinos Oscillate and this can be explained by tiny Sub eV masses

(assuming normal hierarchy)

Neutrino Mixing angles

Inter-generational transitions are small for quarks and almost zero for charged leptons





Information of large mixing can pass from the neutrino sector to other sectors giving severe constraints on the model parameter space

Clockwork For Dirac Neutrinos

Choi and Im, JHEP 2016
Kaplan and Rattazzi, 2016

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm Clockwork} + \mathcal{L}_{\rm int} \ ,$$

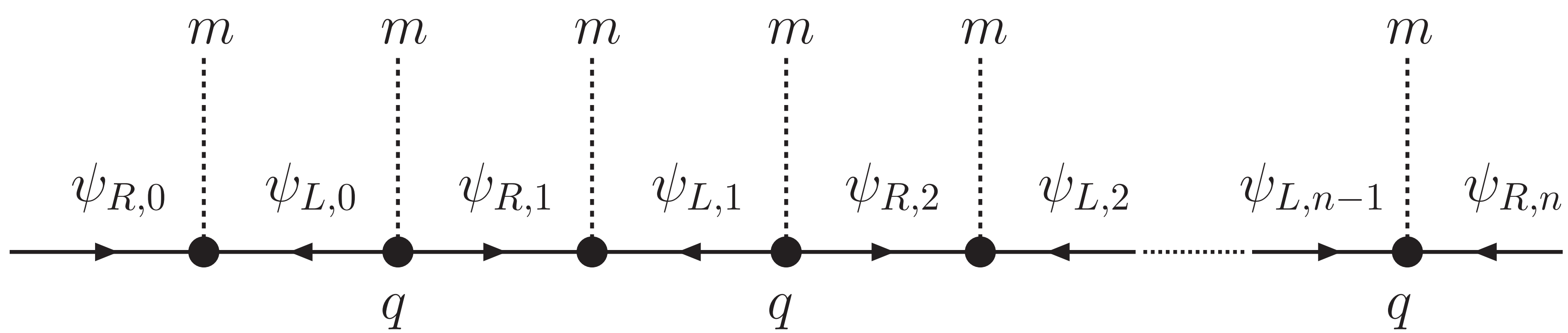
Giudice and McCoullough,2016

The clockwork sector contains (0,1,...n-1) left handed chiral fields and (0,1,....n) right handed chiral fields.

$$H_{ij}^{CW} = m\delta_{ij} + qm\ \delta_{i+1,j}$$

$$\mathcal{L}_{\rm int} = -Y\widetilde{H}\overline{L}_L\psi_{Rn} \ ,$$

We begin with one generation and the generalise to N generations.



$$\mathcal{L} = \mathcal{L}_{\text{Kin}} - m \sum_{j=0}^{n-1} \left(\overline{\psi}_{L,j} \psi_{R,j} - q\, \overline{\psi}_{L,j} \psi_{R,j+1} + H.c \right) \equiv \mathcal{L}_{\text{Kin}} - \left(\overline{\psi}_L M_\psi \psi_R + H.c \right)$$

$$M_\psi = m \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ 0 & 1 & -q & \cdots & & 0 \\ 0 & 0 & 1 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & -q & 0 \\ 0 & 0 & 0 & \cdots & 1 & -q \end{pmatrix}$$

one zero mode, and n Dirac fermions

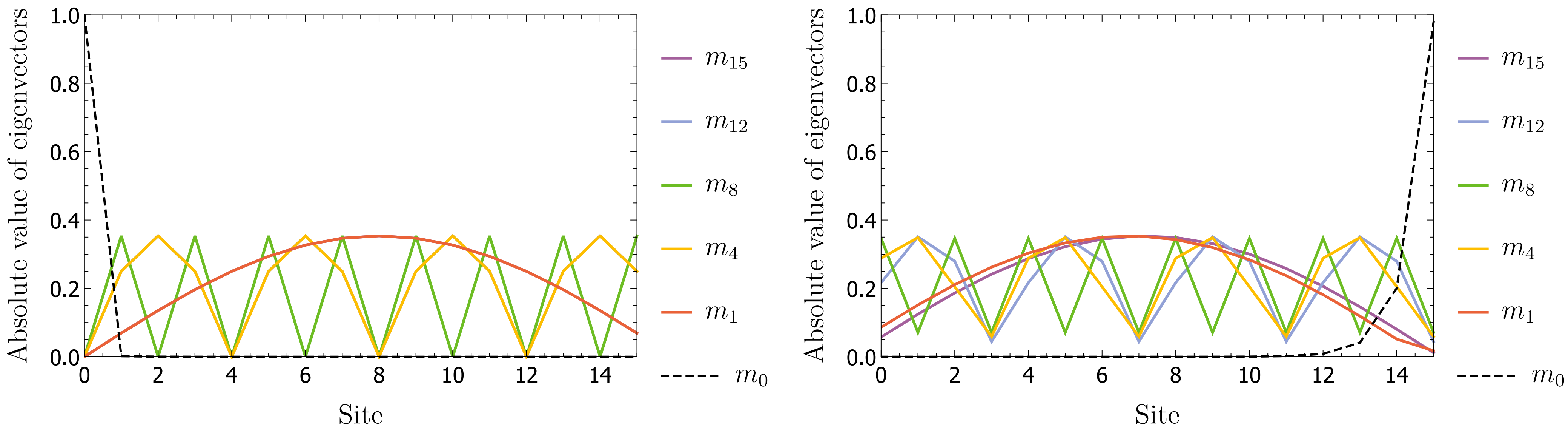
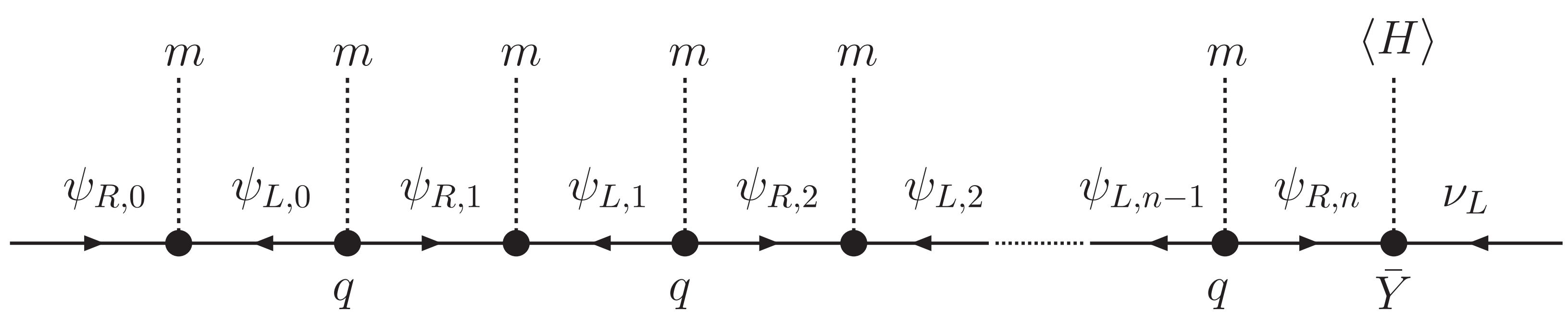


Figure 2. Composition of the left-handed (left panel) and right-handed (right panel) mass eigenmodes in terms of the original clockwork fields in the uniform clockwork model with $N = 15$, $m = v$, $q = 4.887$, and $y = 0.01$.

Hong, Kurup and Perelstein, JHEP010 (2019) 073

After EW symmetry breaking from the interaction term



$$\mathcal{L}_{\rm int} = -Y \widetilde{H} \overline{L}_L \psi_{Rn} \ ,$$

$$m_\nu \approx v Y_0$$

$$m_\nu^D = \begin{array}{c} \\ \nu_L \\ N_{L1} \\ N_{L2} \\ \vdots \\ N_{Ln} \end{array} \begin{array}{c} \begin{array}{ccccc} N_{R0} & N_{R1} & N_{R2} & \cdots & N_{Rn} \end{array} \\ \begin{pmatrix} vY_0 & vY_1 & vY_2 & \cdots & vY_n \\ 0 & M_1 & 0 & \cdots & 0 \\ 0 & 0 & M_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & M_n \end{pmatrix} \end{array} .$$

$$Y_0 \equiv Y(u_R)_n = \frac{Y}{q^n}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}} \ ,$$

$$Y_k \equiv Y(U_R)_{nk} = Y\sqrt{\frac{2}{(n+1)\lambda_k}}\left[q \sin\frac{nk\pi}{n+1}\right] \ , \qquad k = 1, \ldots, n \ .$$

Kushwaha, Ibarra and Vempati, 2017

a kind of multi-degenerate-seesaw mechanism for Dirac neutrinos, where large n reduces the neutrino mass

At least two clockworks for two mass scales.

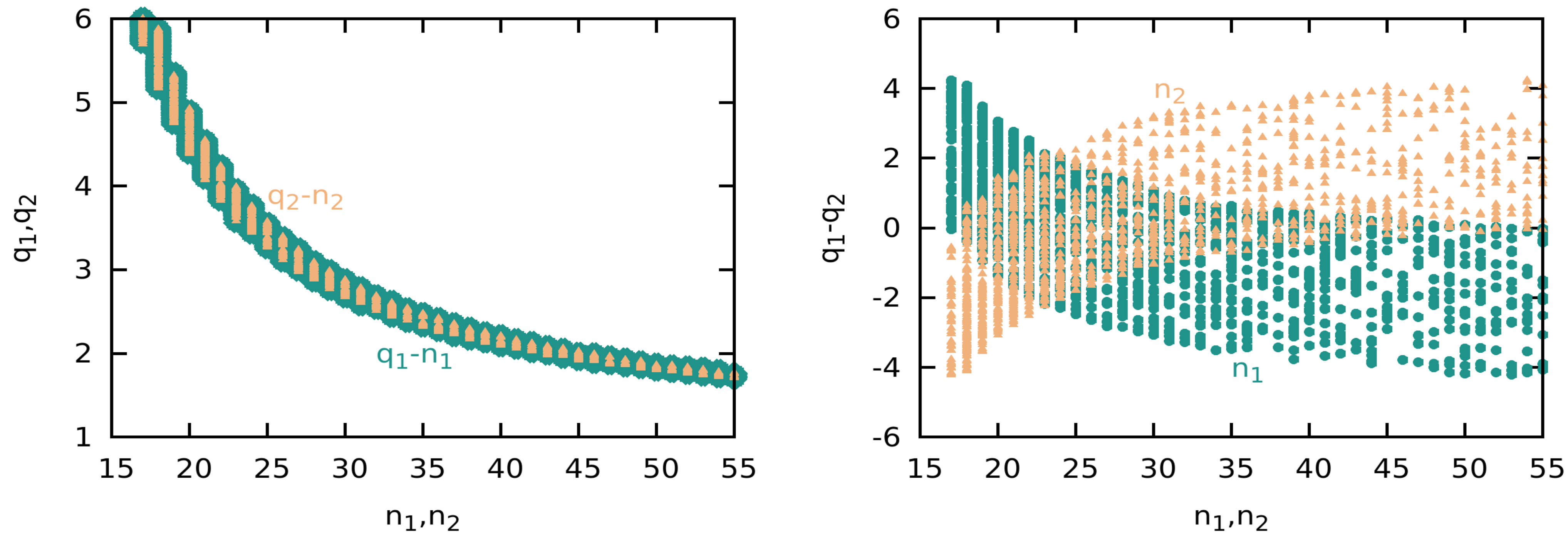


Figure 2: Values of q_1 and q_2 (left panel) and difference between them (right panel), as a function of n_1 and n_2, compatible with the measured values of the neutrino mass splittings and mixing angles within 1σ, for a scenario with two clockwork generations.

Results with three clockworks similar

Kushwaha, Ibarra and Vempati,2017

"Anderson localisation" in 4D

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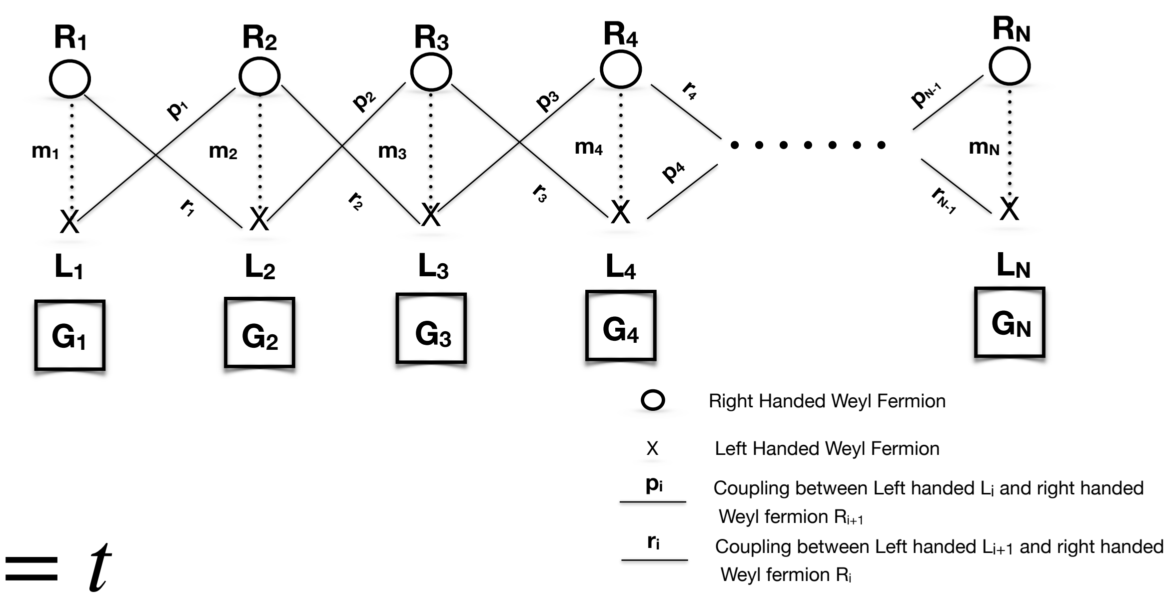
$$\mathcal{L}_{NP} = \mathcal{L}_{kin} - \sum_{i,j=1}^{n} \overline{L_i}\mathcal{H}_{i,j}R_j + h.c.$$

Deconstruction Model



$$\mathcal{H}_{i,j} = \epsilon_i \delta_{i,j} - t_i(\delta_{i+1,j} + K\delta_{i,j+1})$$

$$M_{mass} = \begin{bmatrix} 0 & M_A \\ M_A & 0 \end{bmatrix} \qquad K = 1, t_i = t$$

$$M_A = \begin{bmatrix} \epsilon_1 & -t & 0 & \dots & 0 \\ -t & \epsilon_2 & -t & \dots & 0 \\ 0 & -t & \epsilon_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & -t & \epsilon_N \end{bmatrix}$$

Sutherland and Craig, 2017

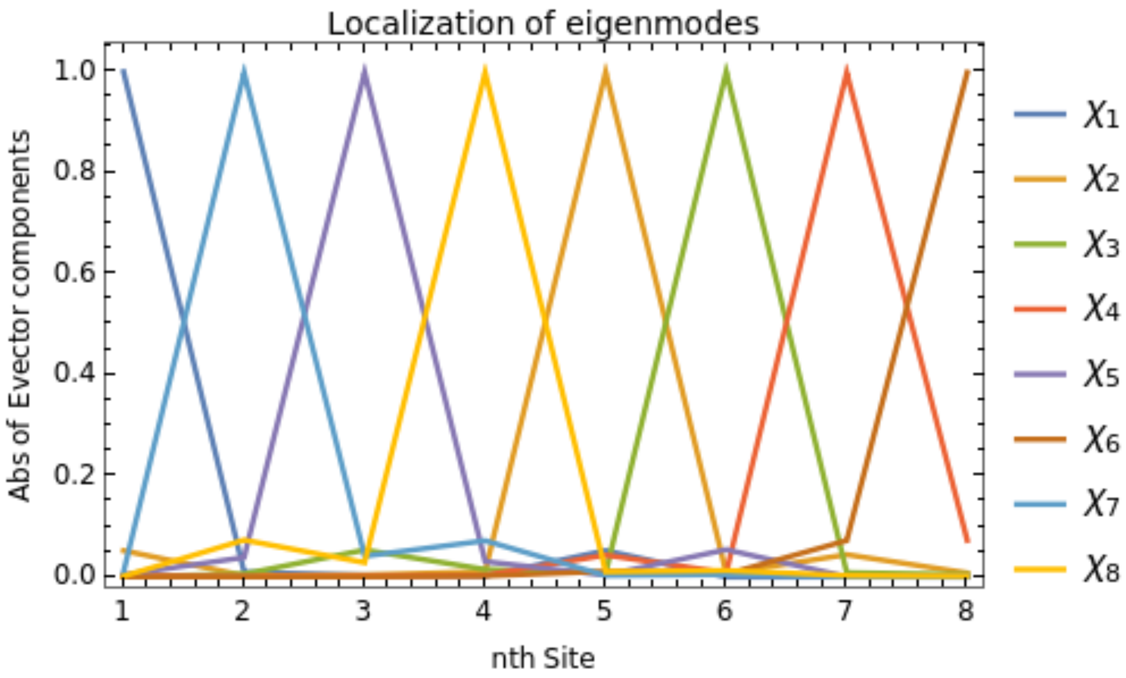
$$\epsilon_i \in [2t, W + 2t] \approx [0, W]$$
$$W \gg t$$

Strong localisation limit

$$L\left(m_i^2, t, W\right) \sim \left(\ln \frac{W}{2t} - 1\right)^{-1}$$

Exponential hierarchies can be generated

For t=1, W = 3, N = 8



1) All modes are localised
2) With every iteration the position of the localisation changes.

Neutrino Masses with Anderson Localisation

Craig and Sutherland 2017

$$\mathcal{L}_{int.} = Y_1 \bar{\nu}_L H R_1 + Y_2 \bar{\nu}_R H L_n + h.c.$$

$$ermion = \begin{bmatrix} 0 & v_1^1 & v_1^2 & v_1^3 & \ldots & v_1^n \\ v_n^1 & \lambda_1 & 0 & 0 & \ldots & 0 \\ v_n^2 & 0 & \lambda_2 & 0 & \ldots & 0 \\ v_n^3 & 0 & 0 & \lambda_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ v_n^n & 0 & 0 & 0 & \ldots & \lambda_n \end{bmatrix}$$

$$m_0 \approx \sum_{i=1}^{n} \frac{v_1^i v_n^i}{\lambda_i} \propto \sum_{i=1}^{n} v^2 \frac{e^{-\frac{n}{L_n}}}{\lambda_i}$$

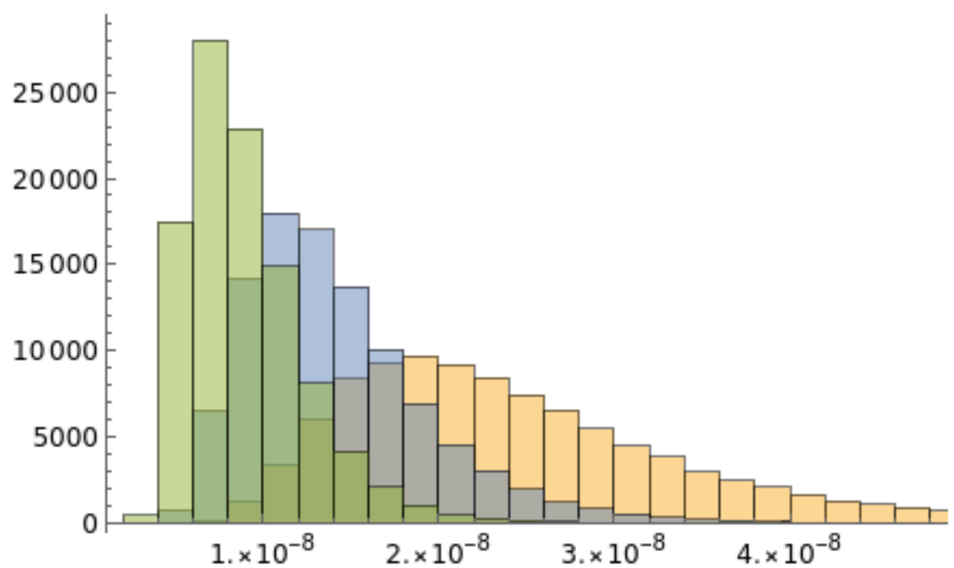
For one generation this looks good !! Tiny neutrino masses can be generated !

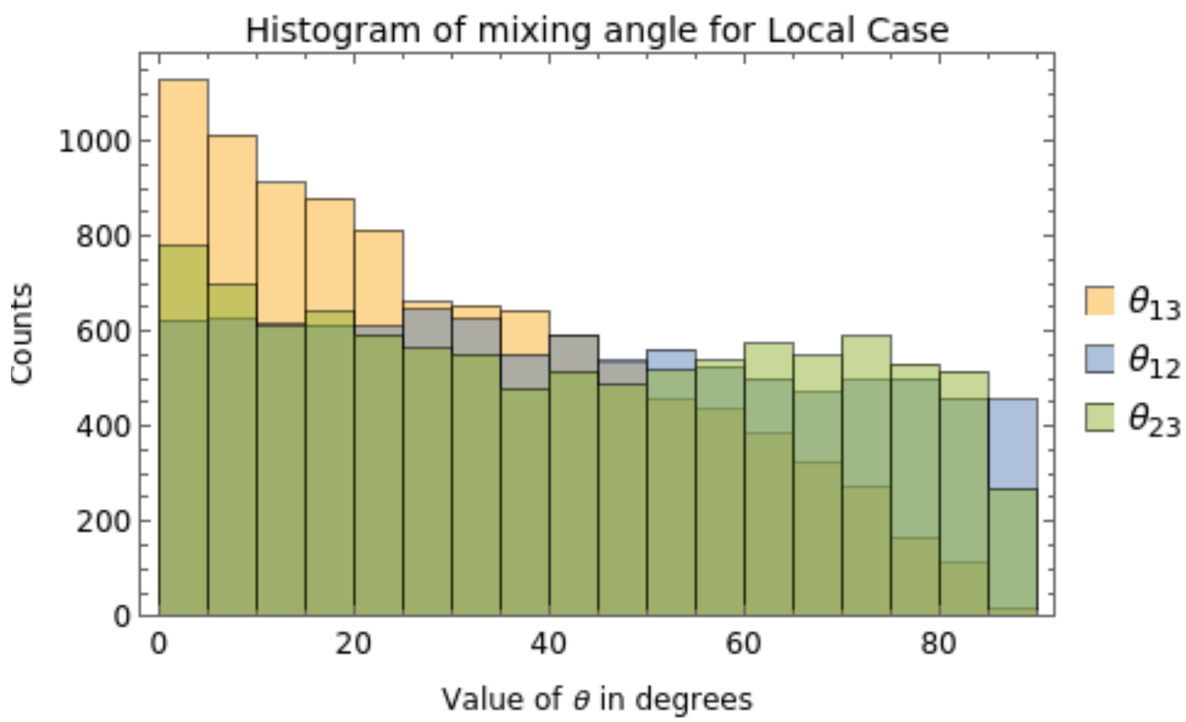
1) What about Flavour Mixing ?
2) What about generalisations in "geometry"

Dirac Case

Straight forward generalisation to three generations

$$\mathcal{L}_{int.} = Y_1 \bar{\nu}_L H R_1 + Y_2 \bar{\nu}_R H L_n + h.c.$$





O(1) eV neutrino masses

Mixing angles are anarchical.

Completely Non-local

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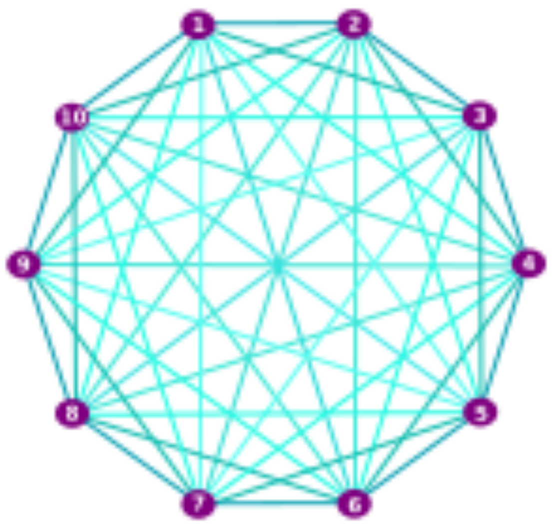
Ji Ji fan et.al

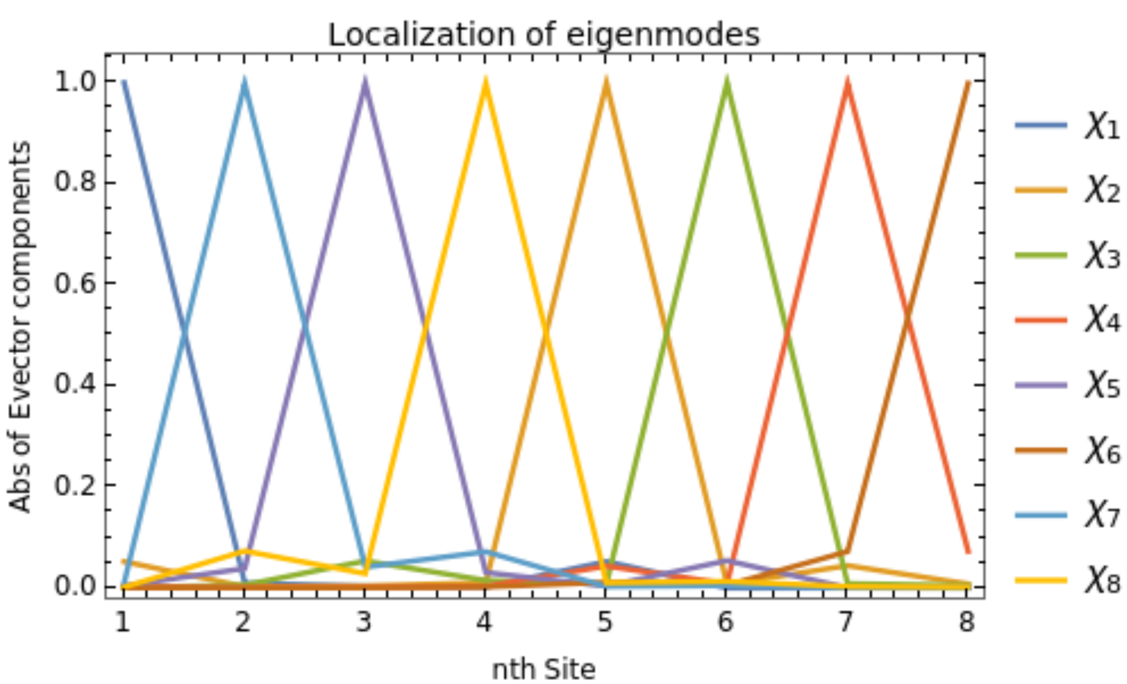


- Lattice Site

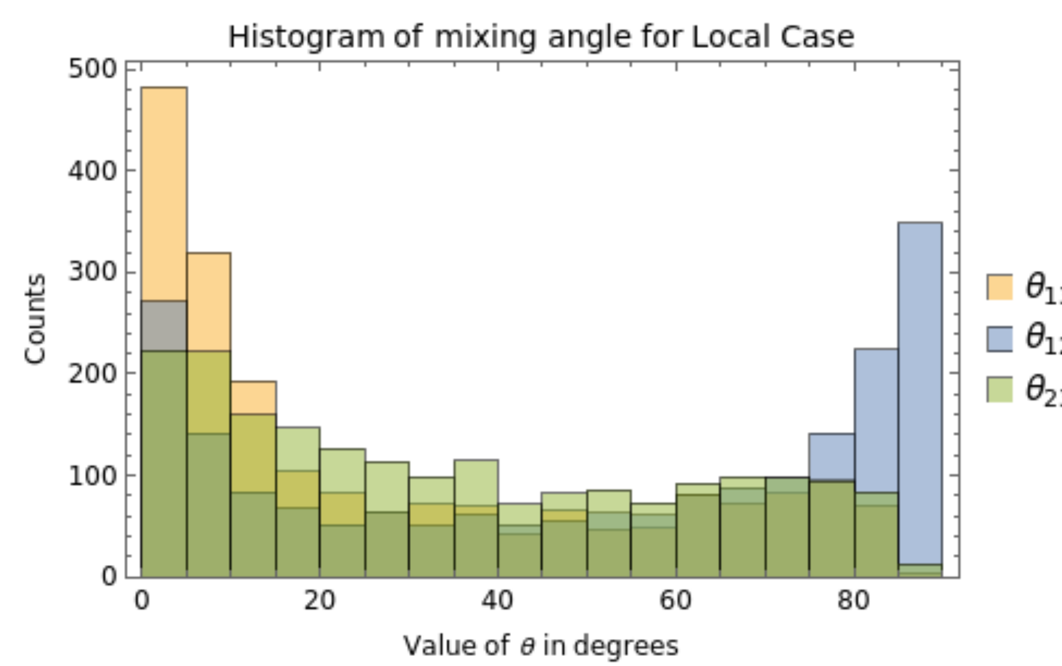
- Link Field

$$\mathcal{L}_{long-range} = L_{Kin} - \sum_{i,j=1}^{N} \overline{L_i}\epsilon_{i,j}R_j - \sum_{i,j=1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(1-\delta_{i,j}\right)R_j + h.c.$$

$$M_{long-range} = \begin{bmatrix} \epsilon_1 & \frac{g}{b} & \frac{g}{b^2} & \cdots & \frac{g}{b^{N-1}} \\ \frac{g}{b} & \epsilon_2 & \frac{g}{b} & \cdots & \frac{g}{b^{N-2}} \\ \frac{g}{b^2} & \frac{g}{b} & \epsilon_3 & \cdots & \frac{g}{b^{N-3}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{g}{b^{N-1}} & \cdots & \cdots & \frac{g}{b} & \epsilon_N \end{bmatrix}$$



$\epsilon_i = 2\mathrm{W},\ \mathrm{g} = 1,\ \mathrm{N} = 8$ b=0.7, W = 4



Singh and Vempati

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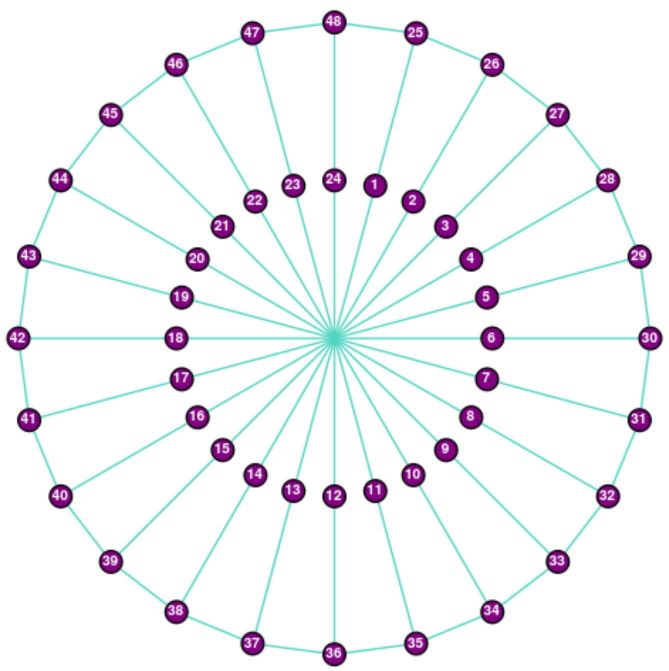
Pietersen Graph



$$L_{Petersen} = L_{Kin} - \sum_{i,j=1}^{N} \overline{L_i} \epsilon_{i,j} R_j - \sum_{i,j=1}^{N/4} \overline{L_i} \frac{g}{b^{|i-j|}} \left(\delta_{i,j+N/4} + \delta_{i+N/4,j}\right) R_j$$
$$- \sum_{i,j=1}^{N/2} \overline{L_i} \frac{g}{b^{|i-j|}} \left(\delta_{i,j+N/2} + \delta_{i+N/2,j}\right) R_j - \sum_{i,j=N/2+1}^{N} \overline{L_i} \frac{g}{b^{|i-j|}} \left(\delta_{i,j+1}\right) R_j$$
$$- \sum_{i,j=N/2+1}^{N} \overline{L_i} \frac{g}{b^{|i-j|}} \left(\delta_{i+1,j}\right) R_j + h.c.$$

$$M_{Petersen} = \begin{bmatrix} \epsilon_1 & 0 & \frac{g}{b^2} & 0 & \frac{g}{b^4} & 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 & \frac{g}{b^2} & 0 & \frac{g}{b^4} & 0 & 0 \\ \frac{g}{b^2} & 0 & \epsilon_3 & 0 & 0 & 0 & \frac{g}{b^4} & 0 \\ 0 & \frac{g}{b^2} & 0 & \epsilon_4 & 0 & 0 & 0 & \frac{g}{b^4} \\ \frac{g}{b^4} & 0 & 0 & 0 & \epsilon_5 & \frac{g}{b} & 0 & \frac{g}{b^3} \\ 0 & \frac{g}{b^4} & 0 & 0 & \frac{g}{b} & \epsilon_6 & \frac{g}{b} & 0 \\ 0 & 0 & \frac{g}{b^4} & 0 & 0 & \frac{g}{b} & \epsilon_7 & \frac{g}{b} \\ 0 & 0 & 0 & \frac{g}{b^4} & \frac{g}{b^3} & 0 & \frac{g}{b} & \epsilon_8 \end{bmatrix}$$

Partially non local

Anarchic mixing angles

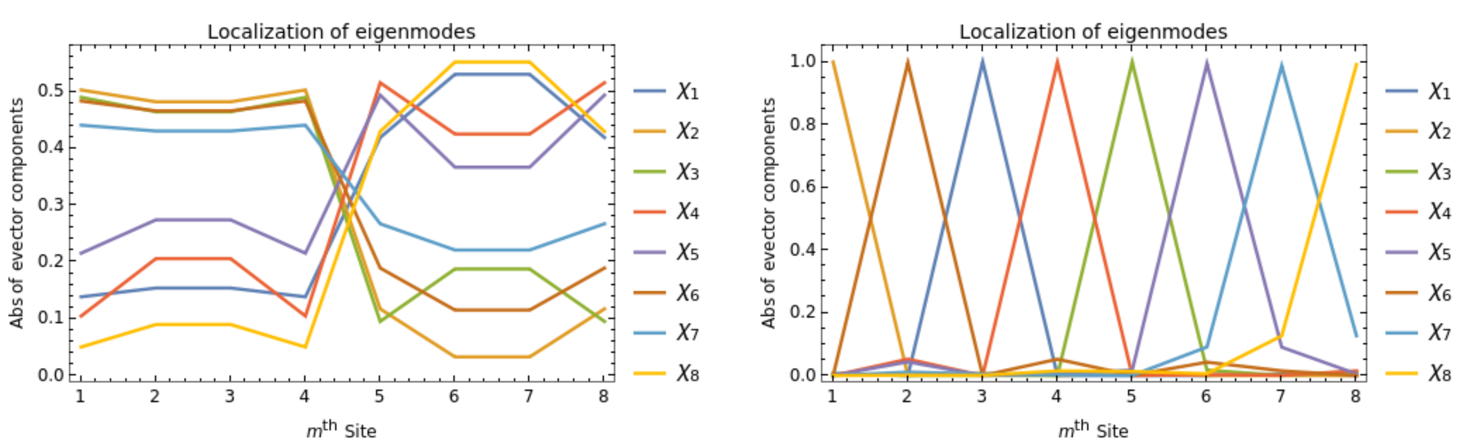
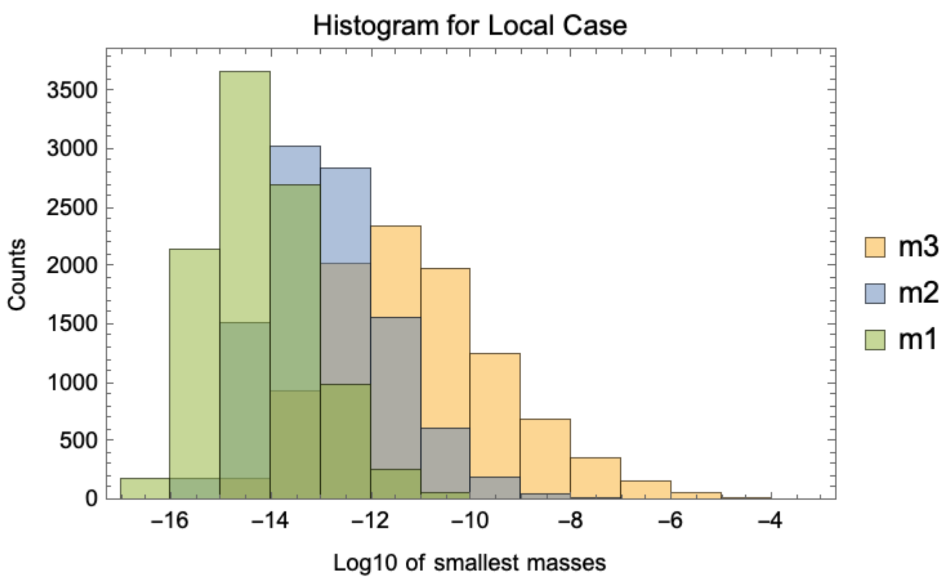


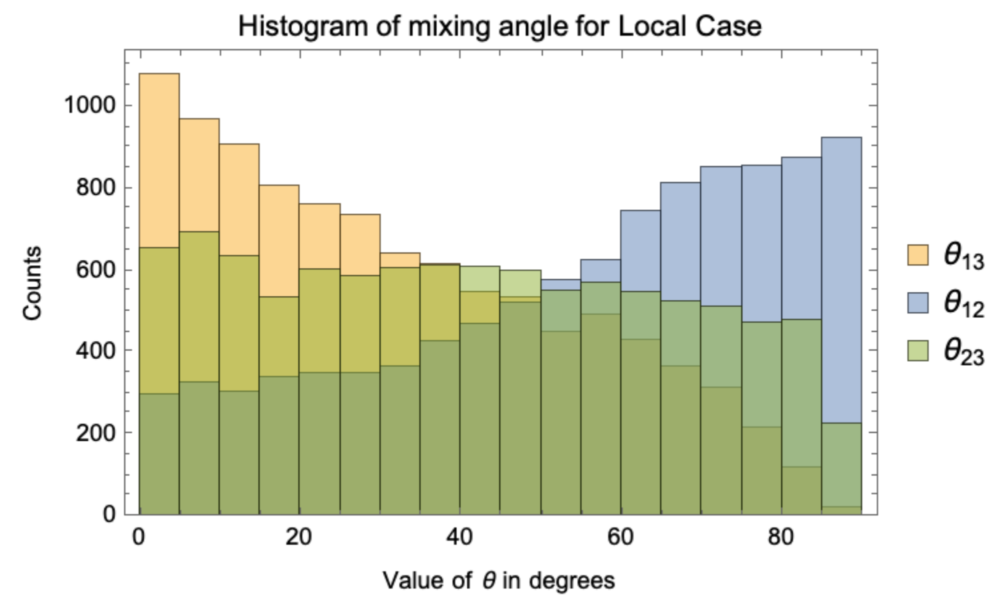
Fig.6 - Mass modes of Petersen graph with uniform sites (left) and random sites(right) for N = 8, W = 5, g = 1/4 and b = 1.4.

Singh and Vempati,

Majorana Case

$$\mathcal{L}_{NP} = L_{kin} - t\bar{L}_1\Psi - \sum_{i,j=1}^{n} \overline{L_i}\mathcal{H}_{i,j}R_j - W\Psi\Psi + h.c.$$





Hierachial neutrino masses with but anarchical mixing angles.

Strong Localisation Limit :

$$\mathcal{H}_{i,j} = \epsilon_i \delta_{i,j} - t_i(\delta_{i+1,j} + K\delta_{i,j+1})$$

-independent of geometry of the mass Chain

- Some universal features for neutrino masses and mixing.

Hierachial masses and anarchical mixing angles

Mixing Type	Local	Non-local	Petersen
Ψ Yukawa Mixing	Small mix	Small mix	Small mix
Site Flavour Mixing	Large mix, Anarchical	Large mix, Anarchical	Large mix, Anarchical
In this scenario, results are independent of underlying graph connectivity.			

But may be strong localisation is not needed at all !

Role of Geometry : Weak Disorder

Scenario	N	ϵ_i	t_i
Local	9	[W-t, W+t]	t
Non-local	14	[W-t, W+t]	t
Petersen	12	[W-t, W+t]	t

$$W \gg t$$

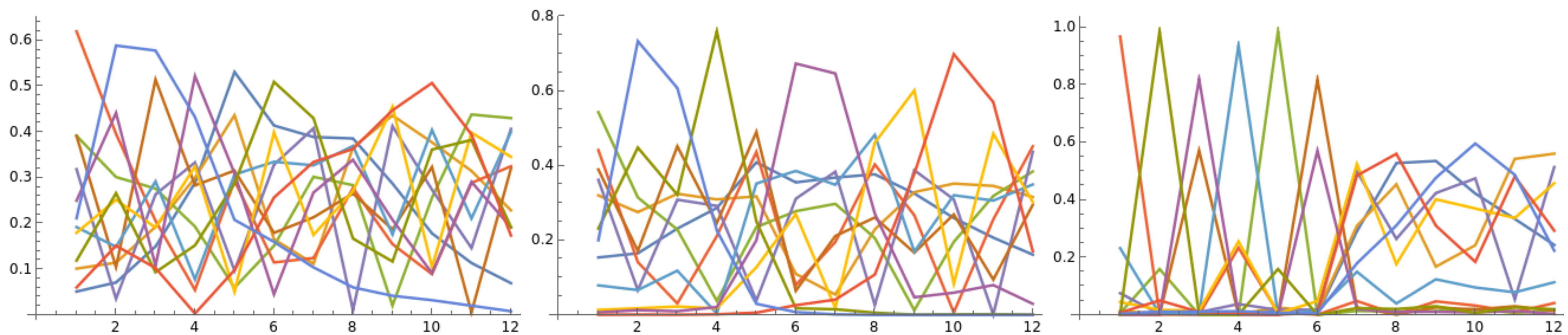


FIG. 15: Figure shows the wavefunctions for three geometries 1) local (left), 2) nonlocal (middle) and 3) Petersen (right) with $\epsilon_i \in [W-t, W+t]$, W = 6 TeV, b = 2 and t_i = t = 0.2 TeV, N = 12.

No strong localisation of the modes !! But still it works !

Neutrino masses in weak disorder :

$$\mathcal{L}_{int.} = Y_1 \bar{\nu}_L H R_1 + Y_2 \bar{\nu}_R H L_n + h.c.$$

Unitarity comes in to play !

$$\begin{aligned}
m_0 &\approx v^2 \sum_{i=1}^{n} \frac{v_1^i v_n^i}{\lambda_i} \\
&= v^2 \sum_{i=1}^{n} \frac{v_1^i v_n^i}{\epsilon} \frac{\epsilon}{\lambda_i} \\
&= v^2 \frac{1}{\epsilon} \sum_{i=1}^{n} v_1^i v_n^i (1 - x_i)^{-1} \\
&= \frac{v^2}{\epsilon} \sum_{i=1}^{n} v_1^i v_n^i + \frac{v^2}{\epsilon} \sum_{i=1}^{n} v_1^i v_n^i x_i + \ldots \\
m_0 &\approx \frac{v^2}{\epsilon} \sum_{i=1}^{n} v_1^i v_n^i x_i
\end{aligned}$$

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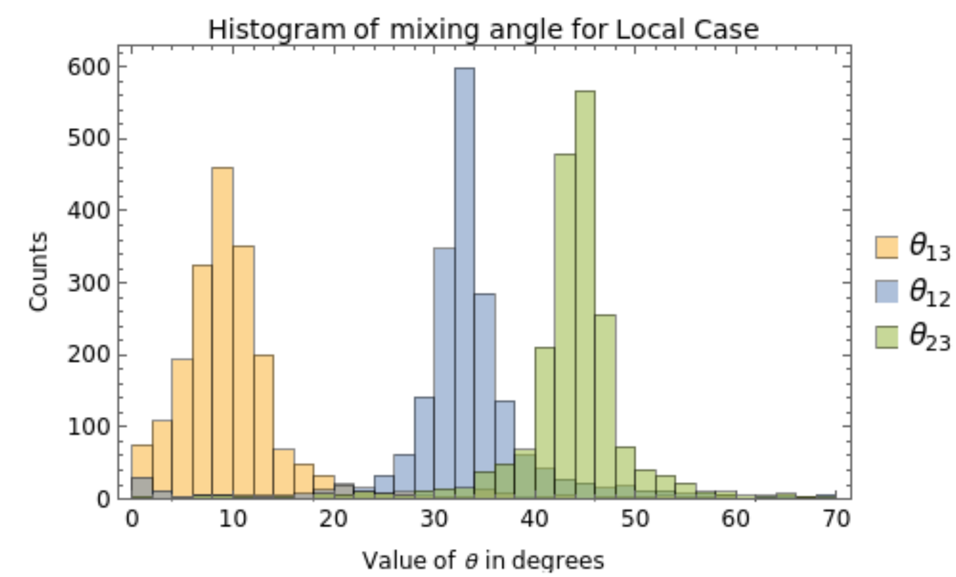
$$\mathcal{L}_{int.} = Y_1 \bar{\nu}_L H R_1 + Y_2 \bar{\nu}_R H L_n + h.c.$$

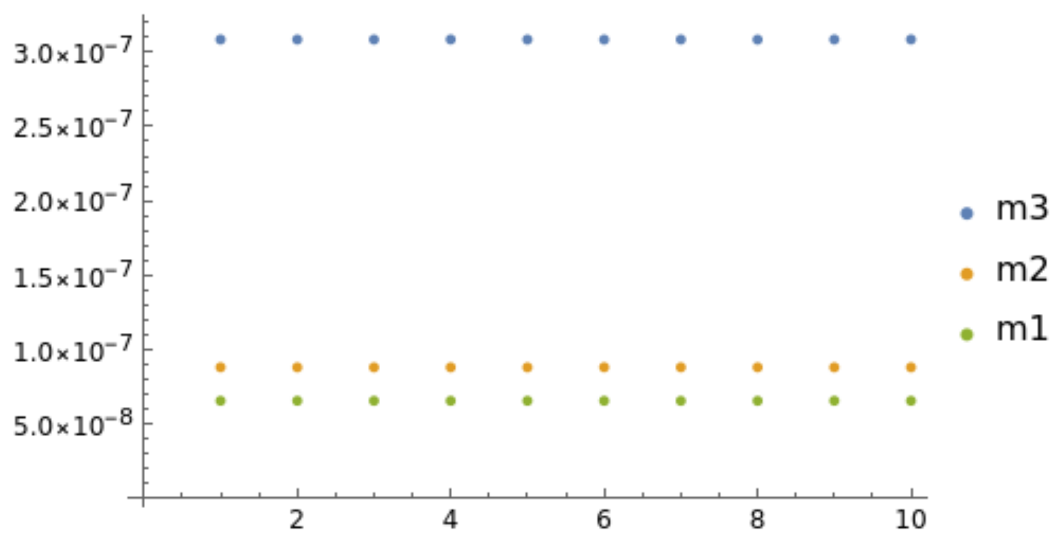
$$W \gg t$$

Scenario	N	ϵ_i	t_i
Local	9	[W-t, W+t]	t
Non-local	14	[W-t, W+t]	t
Petersen	12	[W-t, W+t]	t

Dirac Scenario : Local Lattice
(only nearest neighhour)

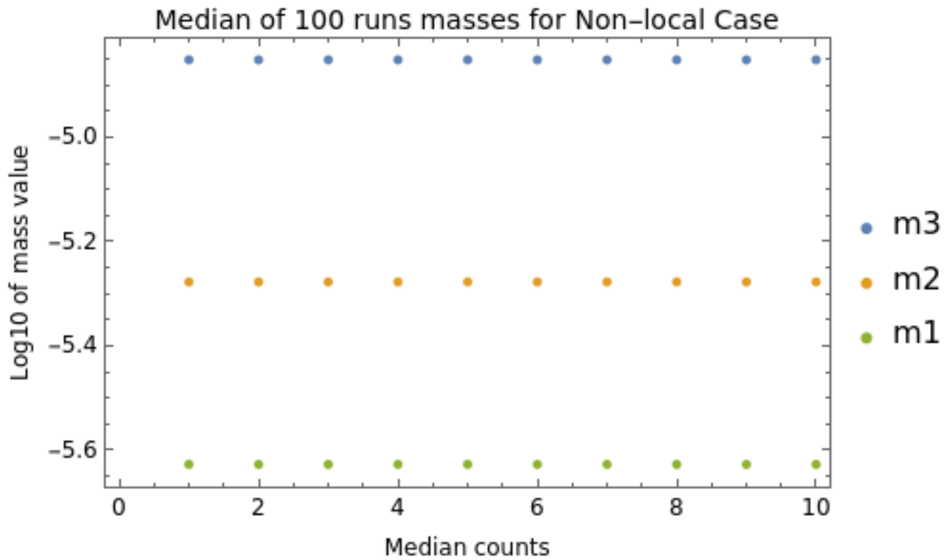


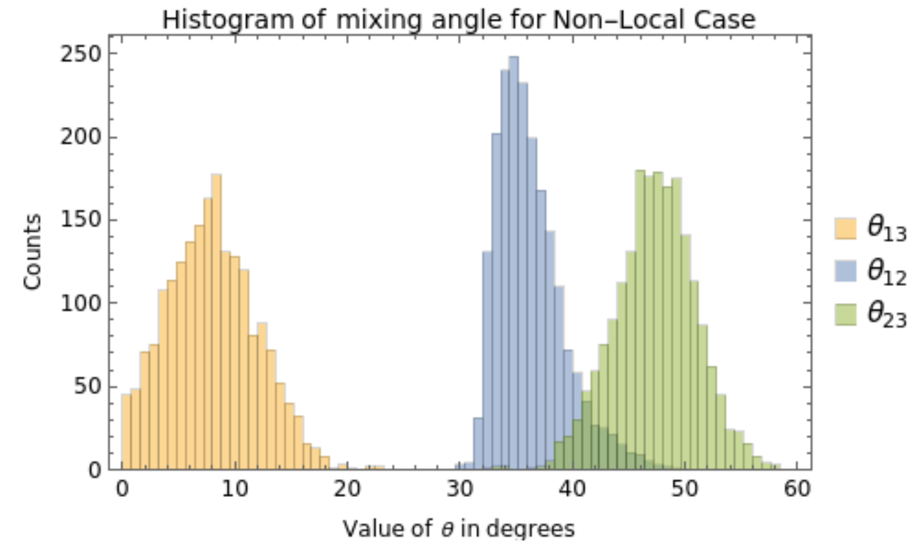
Assuming Y's are not anarchical



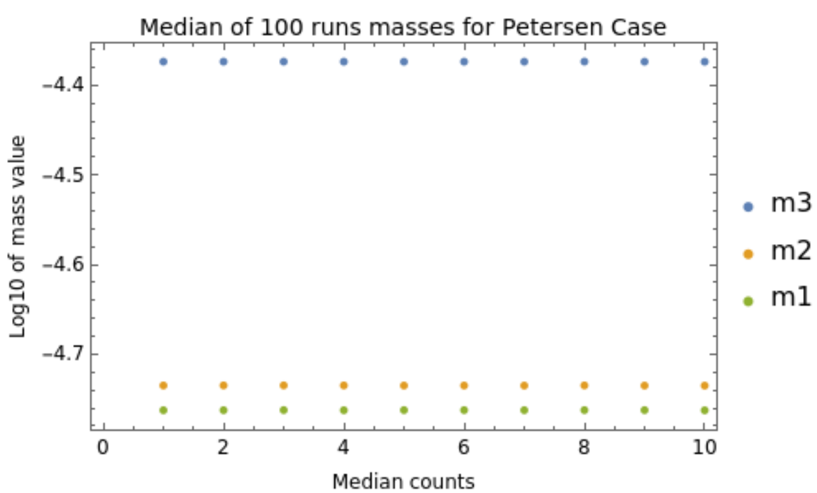
Mixing angles are "localised".

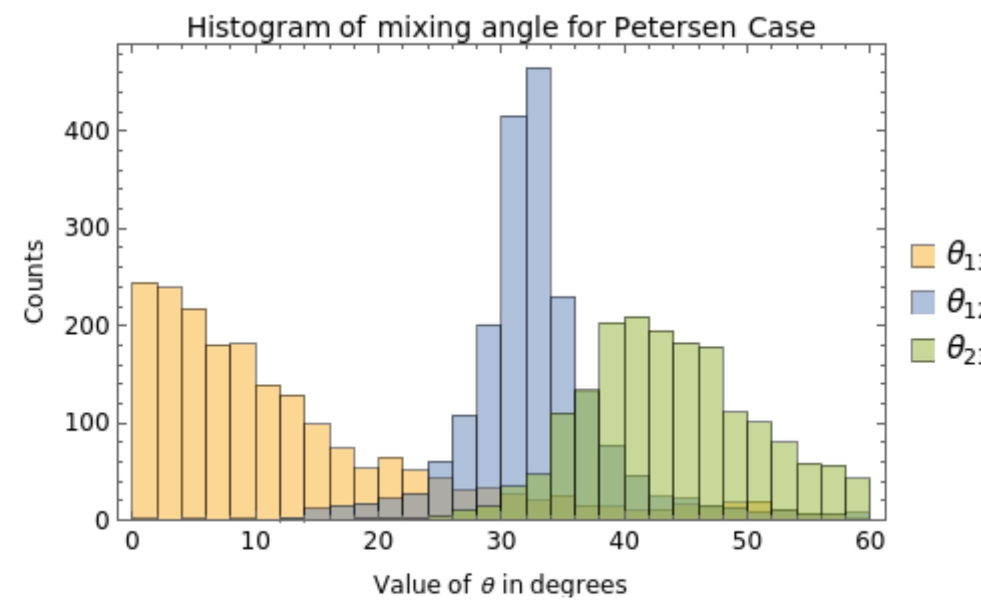
Fully non-local





Partially non-local





Hierarchial neutrino masses with anarchic mixing angles
is a feature of the strong localisation regime independent of the
type of geometry, couplings (non-local, partially local etc.)

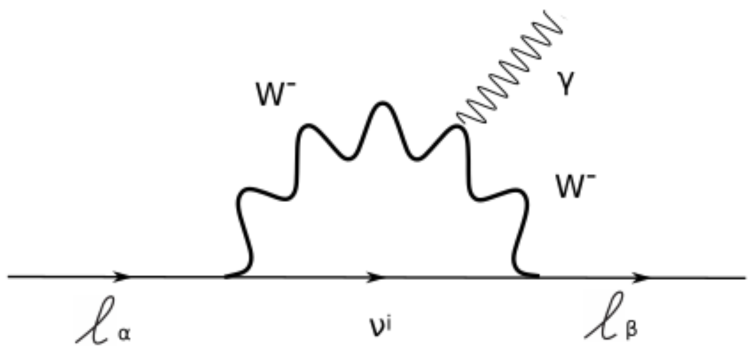
In the case of strong disorder in couplings (t) parameter,
geometry does play a mild role, but mixing angles are
mostly anarchic, except one !.

Strong disorder-> strong localisation while nice to explain large hierarchies
is not necessarily useful to explain flavour mixings as it leads to
anarchical mixing angles.

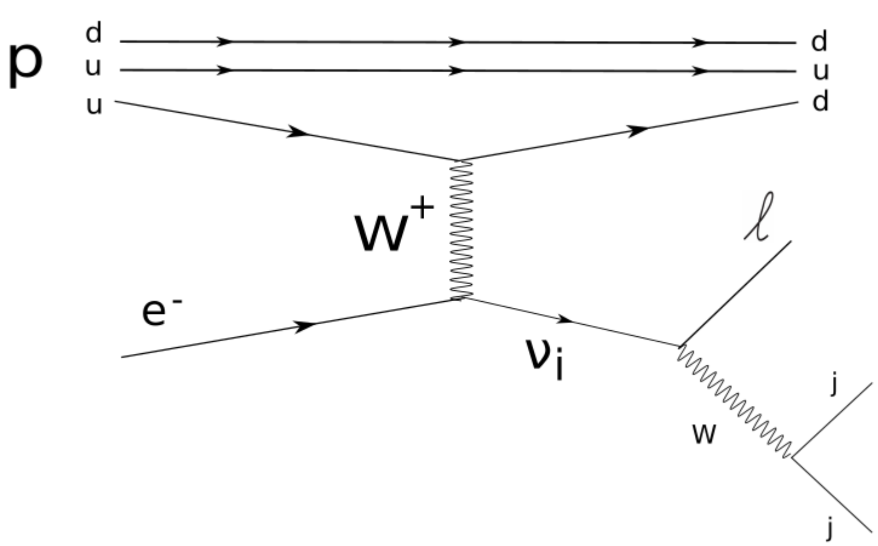
Weak disorder is sufficient to generate models which give
reasonable hierarchies in masses and more importantly flavour mixing !

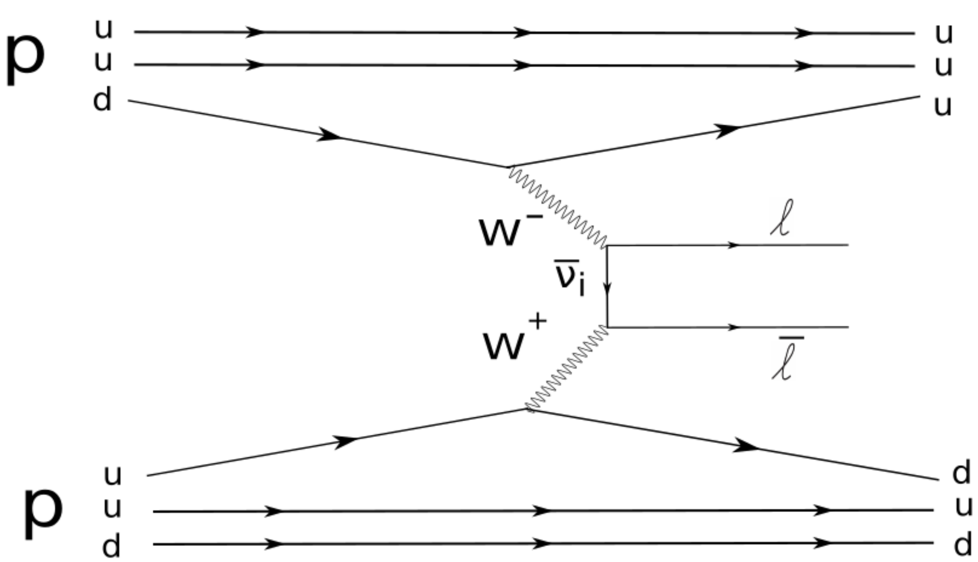
Phenomenology

Phenomenological Signatures depend on the geometry of the mass chain



Models	$pp \to \ell\bar{\ell}jj$ in pb		$pe^- \to \ell njj$ in pb		
	14(TeV)	100(TeV)	6(TeV)	9(TeV)	15(TeV)
Local	0.4508	2.413	4.786×10^{-5}	0.0096	0.08389
Petersen	0.4501	2.402	2.507×10^{-5}	0.01511	0.1597
Non-local	0.4422	2.261	12.38×10^{-5}	0.02083	0.1725





Back up slides

Anderson localisation in particle physics

Craig Sutherland
2017

Using randomness in couplings to generate exponential hierarchies.
Applications to neutrino masses

Sources of randomness :

(I) stringy landscapes — Balasubramaniam et.al

(II) dark sectors — Dienes, kumar et.al

(1) Generalised Clockwork

$$L_{CW} = L_{kin} - \sum_{i=1}^{n} \bar{\psi}_{L_i} H_{ij} \psi_{R_j} + H.C$$

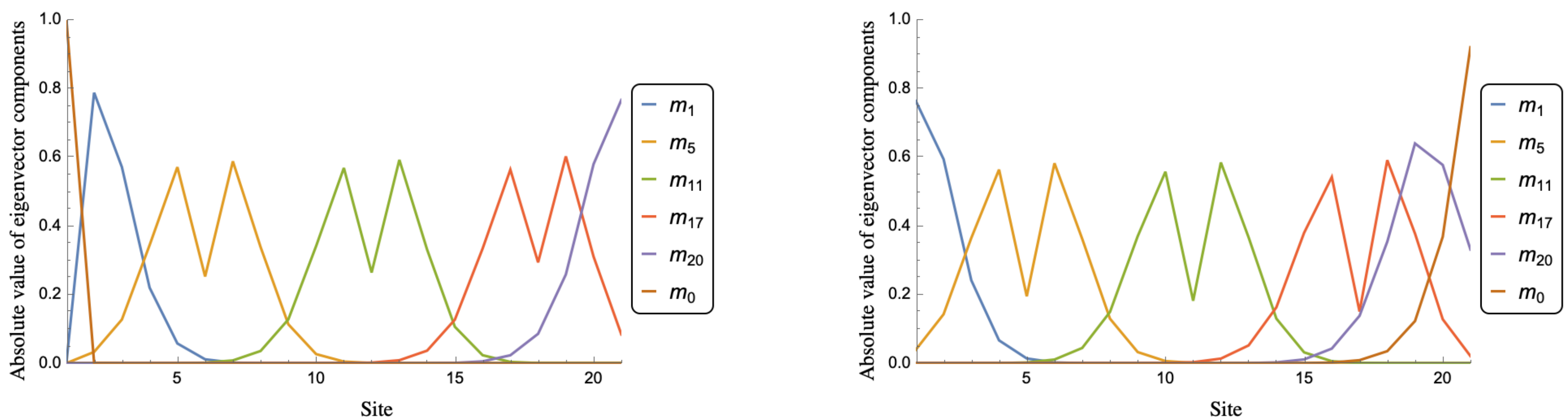
$$H_{ij} = m_i \, \delta_{ij} + q_i m_i \, \delta_{i+1,j}$$

Tiny Dirac neutrino masses !

Zero Mode !

Hong, Kurup, Perelstein

Localisation possible for regions of parameters (no large hierarchies)

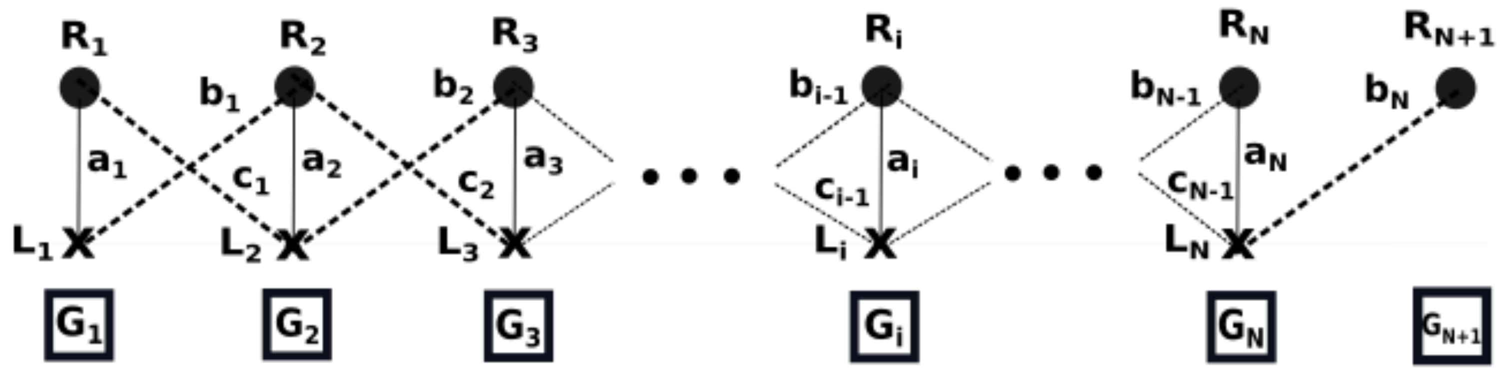


Plot 1(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

(2) Two Sided Clockwork

$$L_{CW} = L_{kin} - \sum_{i=1}^{n} \bar{\psi}_{L_i} H_{ij} \psi_{R_j} + H.C \qquad H_{ij} = m(\delta_{ij} + q_i \delta_{i+1,j} + q' \delta_{i,j+1})$$

Tiny Dirac neutrino masses !



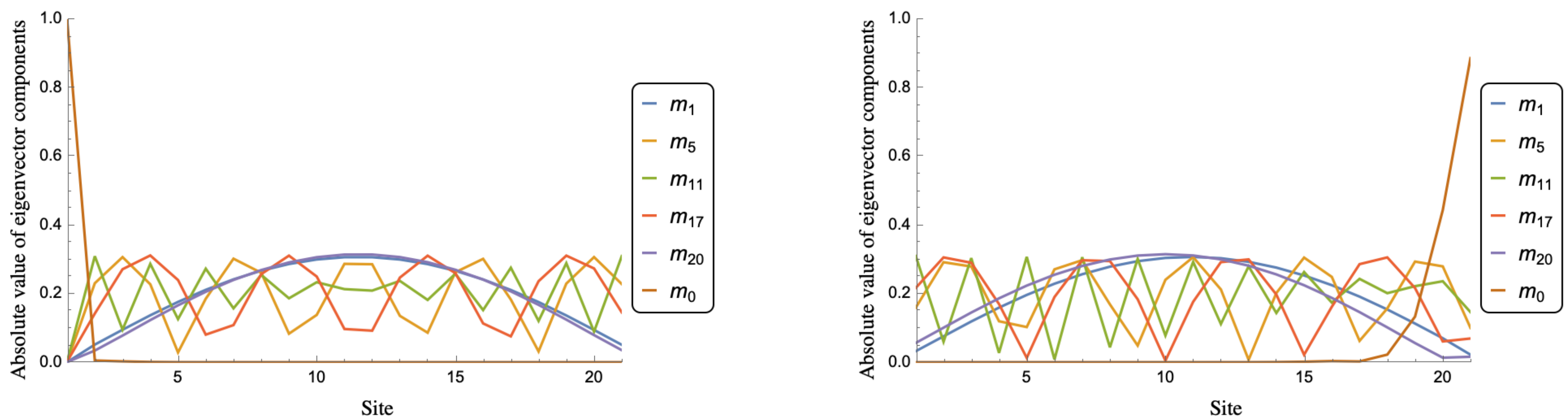
Deconstruction Model
Linear Moose

Zero Mode !

X - Left Handed CW Fermion

● - Right Handed CW Fermion

Localisation possible for regions of parameters (no large hierarchies)



Plot 2(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

Extremely efficient localisation with randomness/disorder

Gero Gresdroff

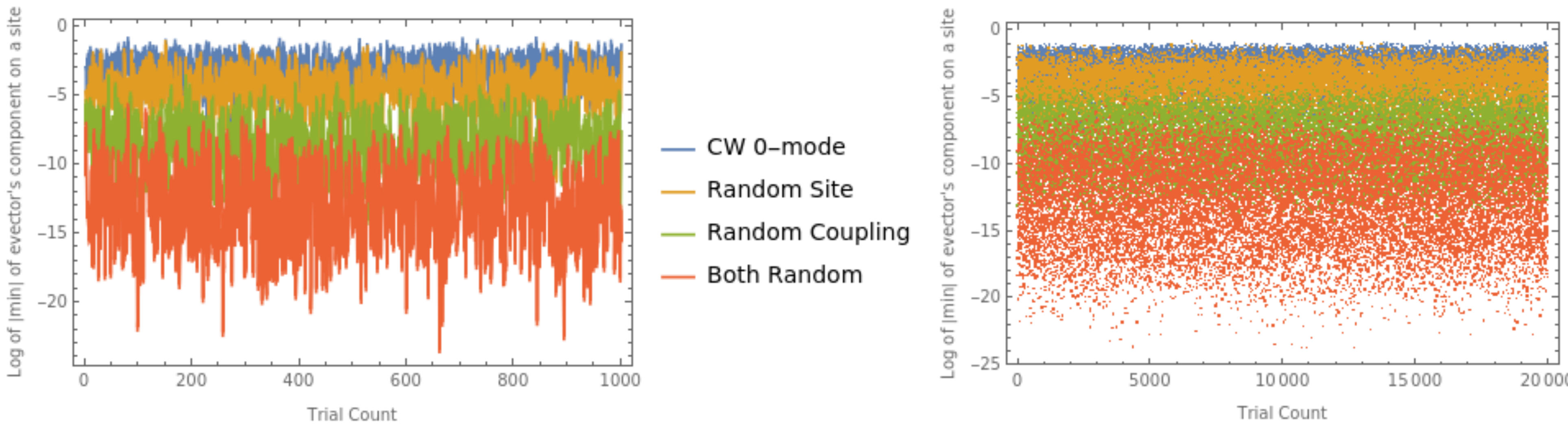
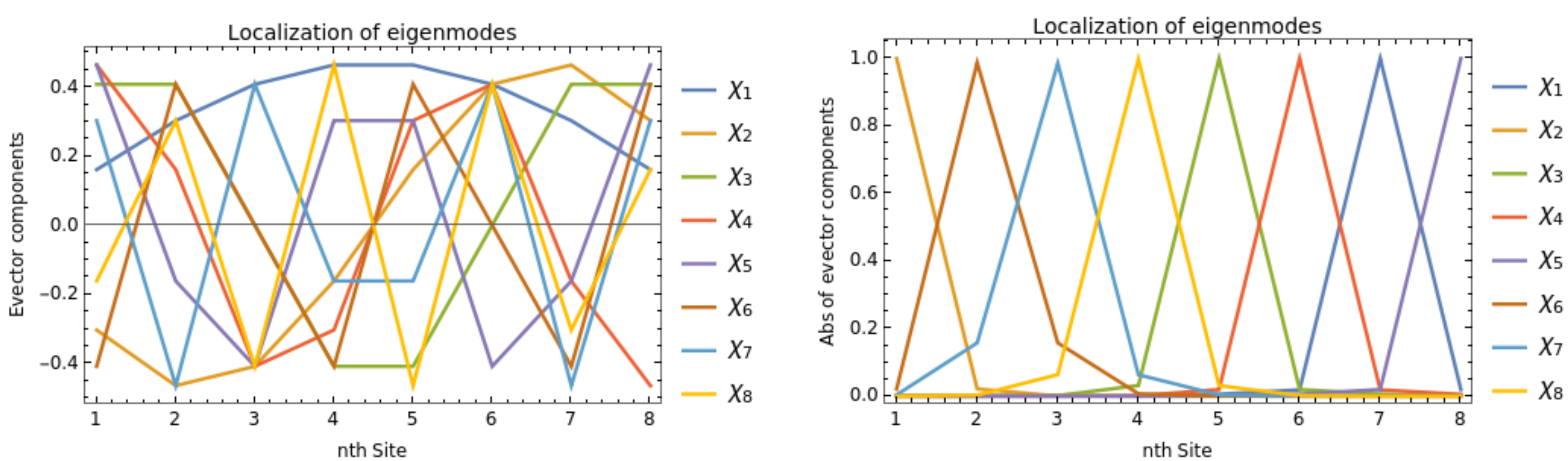


Fig.6 - Figure shows the Log of minimum component 0-mode of CW and lightest mode of disorder models achieved with n = 10 sites.



All modes localised

Fig.2 - Mass modes of Local lattice with uniform sites ϵ_i = W & $t_i = t$ (left) and random sites $t_i = t$ & $\epsilon_i \in$ [2W, -2W] (right) for W = 4 and t = 1/4 with N = 8 sites..

Singh and vempati to appear

(3) Non-local Interactions

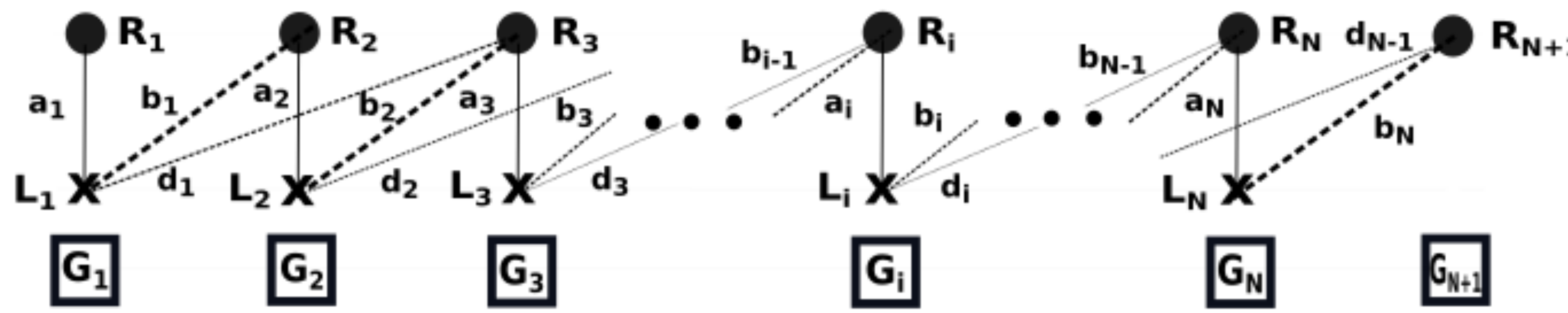
Planck - May 2025

$$\mathcal{H}_{i,j} = a_i \delta_{i,j} + b_i \delta_{i+1,j} + d_i \delta_{i+2,j}$$

Tiny Dirac neutrino masses !

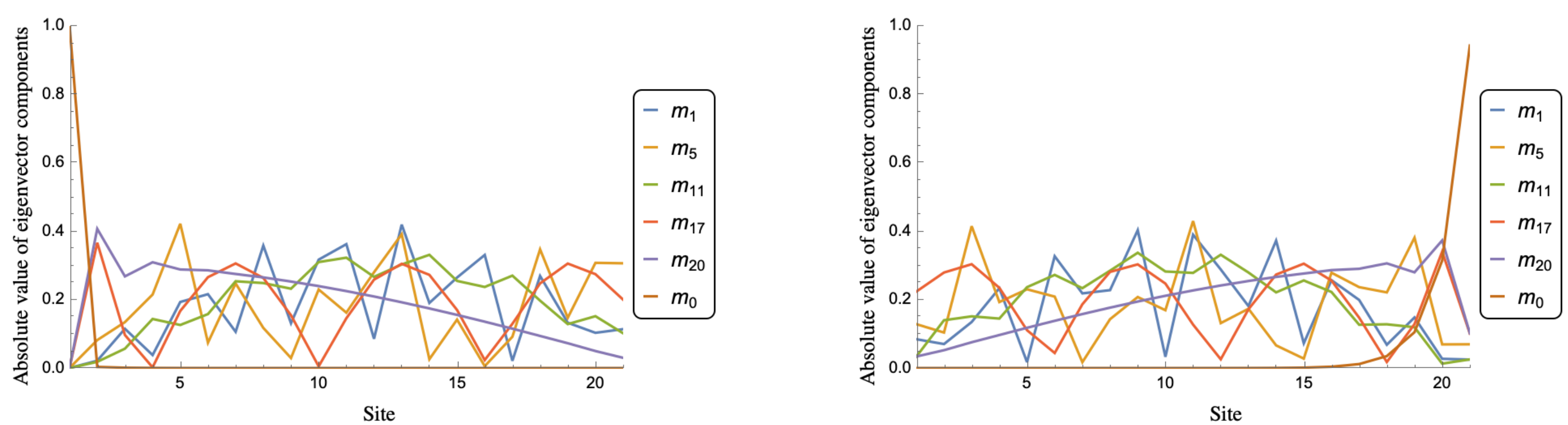
Zero Mode !

Localisation possible for regions of parameters (no large hierarchies)



X - Left Handed CW Fermion

● - Right Handed CW Fermion

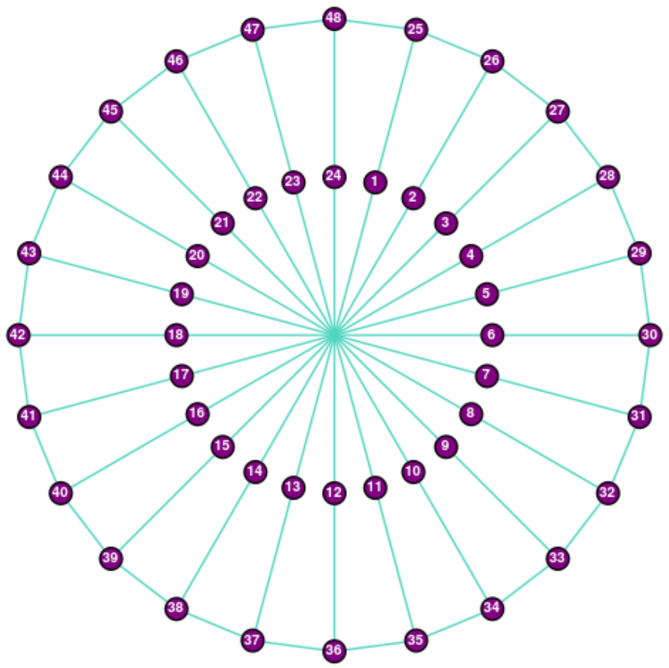


Plot 3(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

Singh and vempati, 2024

Planck - May 2025

Pietersen Graph



$$
\begin{aligned}
L_{Petersen} = L_{Kin} &- \sum_{i,j=1}^{N} \overline{L_i}\epsilon_{i,j}R_j - \sum_{i,j=1}^{N/4} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+N/4} + \delta_{i+N/4,j}\right)R_j \\
&- \sum_{i,j=1}^{N/2} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+N/2} + \delta_{i+N/2,j}\right)R_j - \sum_{i,j=N/2+1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+1}\right)R_j \\
&- \sum_{i,j=N/2+1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i+1,j}\right)R_j + h.c.
\end{aligned}
$$

Anarchical Mixing angles !

Partially non local

$$
M_{Petersen} = \begin{bmatrix}
\epsilon_1 & 0 & \frac{g}{b^2} & 0 & \frac{g}{b^4} & 0 & 0 & 0 \\
0 & \epsilon_2 & 0 & \frac{g}{b^2} & 0 & \frac{g}{b^4} & 0 & 0 \\
\frac{g}{b^2} & 0 & \epsilon_3 & 0 & 0 & 0 & \frac{g}{b^4} & 0 \\
0 & \frac{g}{b^2} & 0 & \epsilon_4 & 0 & 0 & 0 & \frac{g}{b^4} \\
\frac{g}{b^4} & 0 & 0 & 0 & \epsilon_5 & \frac{g}{b} & 0 & \frac{g}{b^3} \\
0 & \frac{g}{b^4} & 0 & 0 & \frac{g}{b} & \epsilon_6 & \frac{g}{b} & 0 \\
0 & 0 & \frac{g}{b^4} & 0 & 0 & \frac{g}{b} & \epsilon_7 & \frac{g}{b} \\
0 & 0 & 0 & \frac{g}{b^4} & \frac{g}{b^3} & 0 & \frac{g}{b} & \epsilon_8
\end{bmatrix}
$$

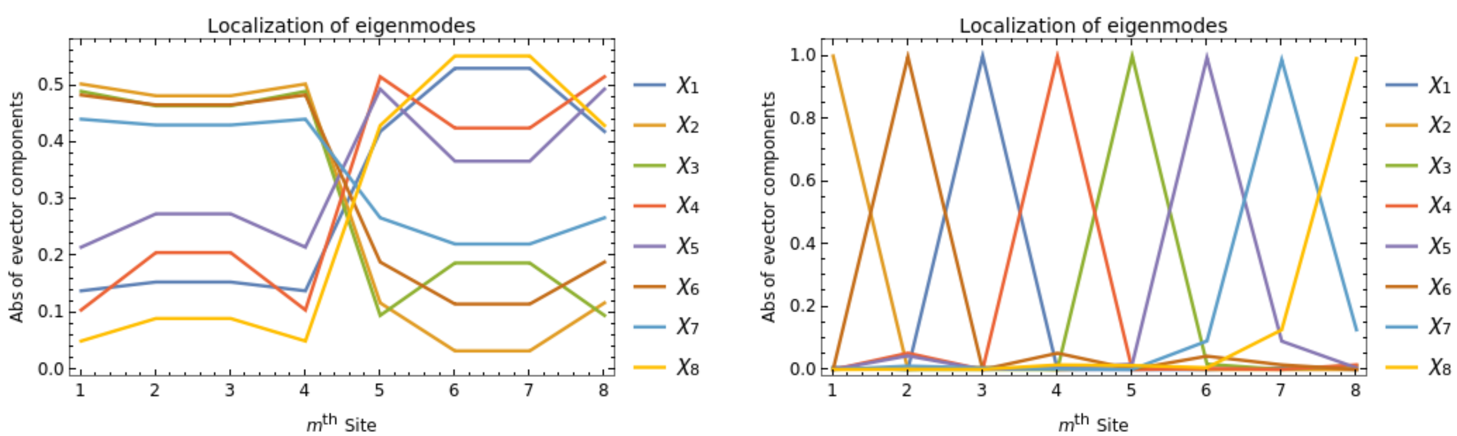
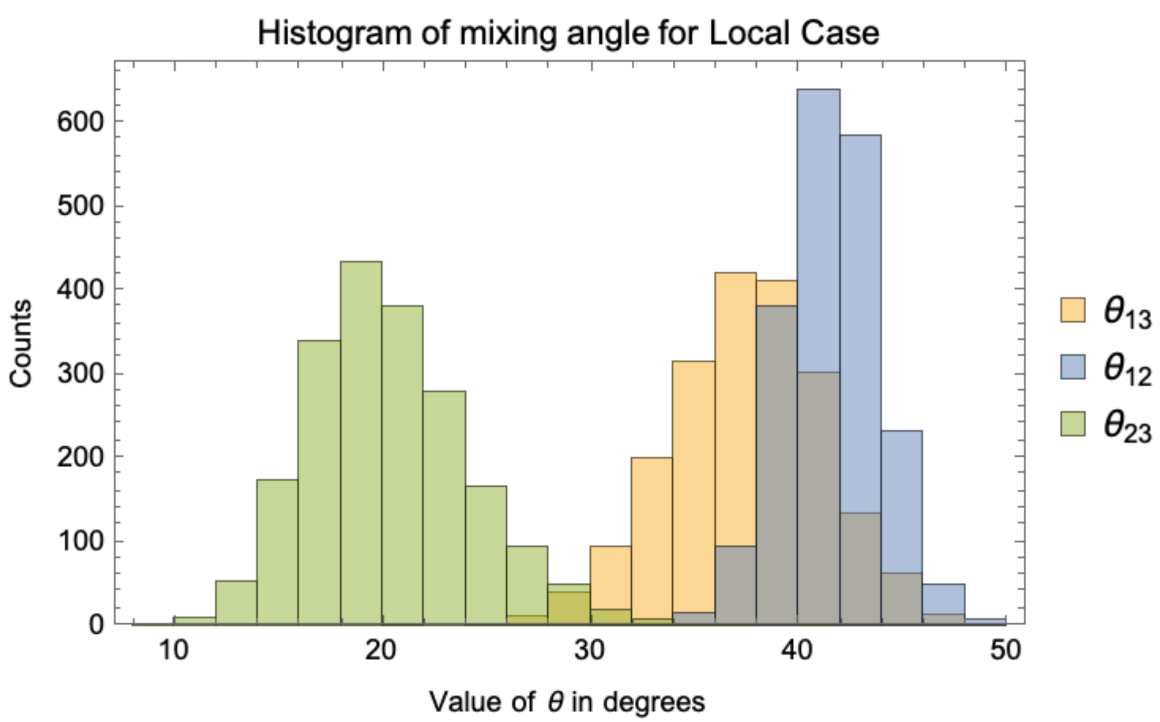
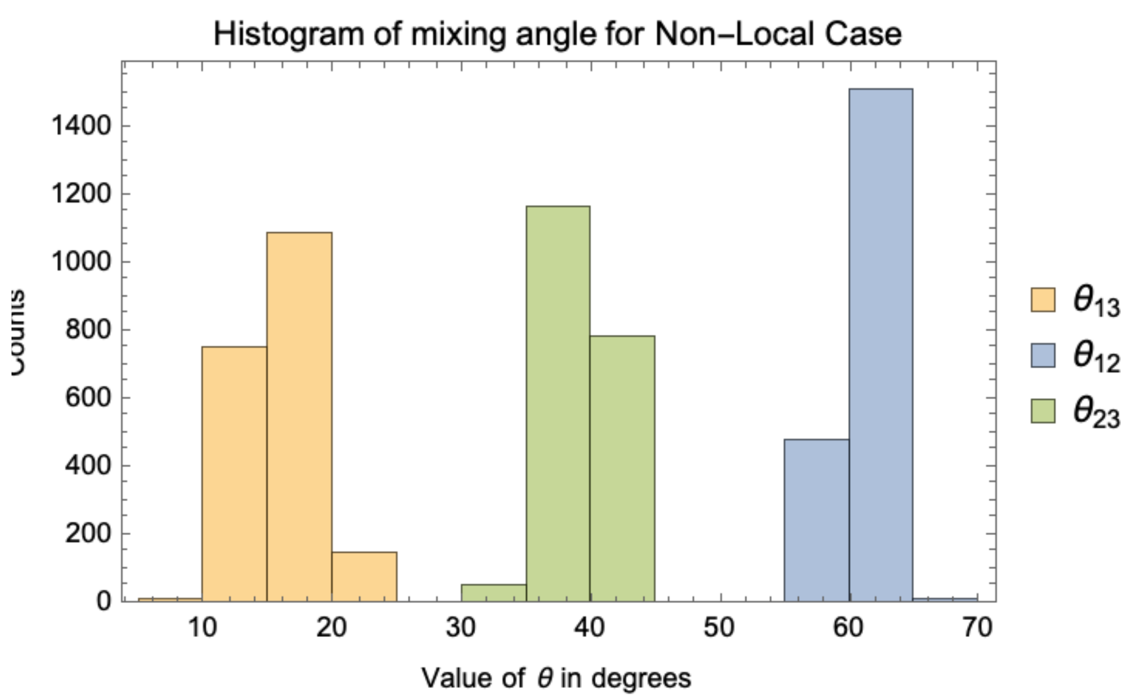


Fig.6 - Mass modes of Petersen graph with uniform sites (left) and random sites(right) for N = 8, W = 5, g = 1/4 and b = 1.4.

Singh and Vempati,

For the Majorana case, we get similar " localisation"





Outlook

Randomness in couplings can lead to exponentially hierarchal couplings.

In the regime of strong coupling, the geometry of the mass chains does not matter significantly. They predict hierarchal neutrino masses and anarchical mixing angles for both Dirac or Majorana scenarios.

In the weak coupling regime, geometry does play a role and can be chosen carefully to ``localise" the mixing angles.

Experimental signatures become weaker for non-local /partially non-local cases compared to local case.

Majorana Case

The gears have large couplings as before.

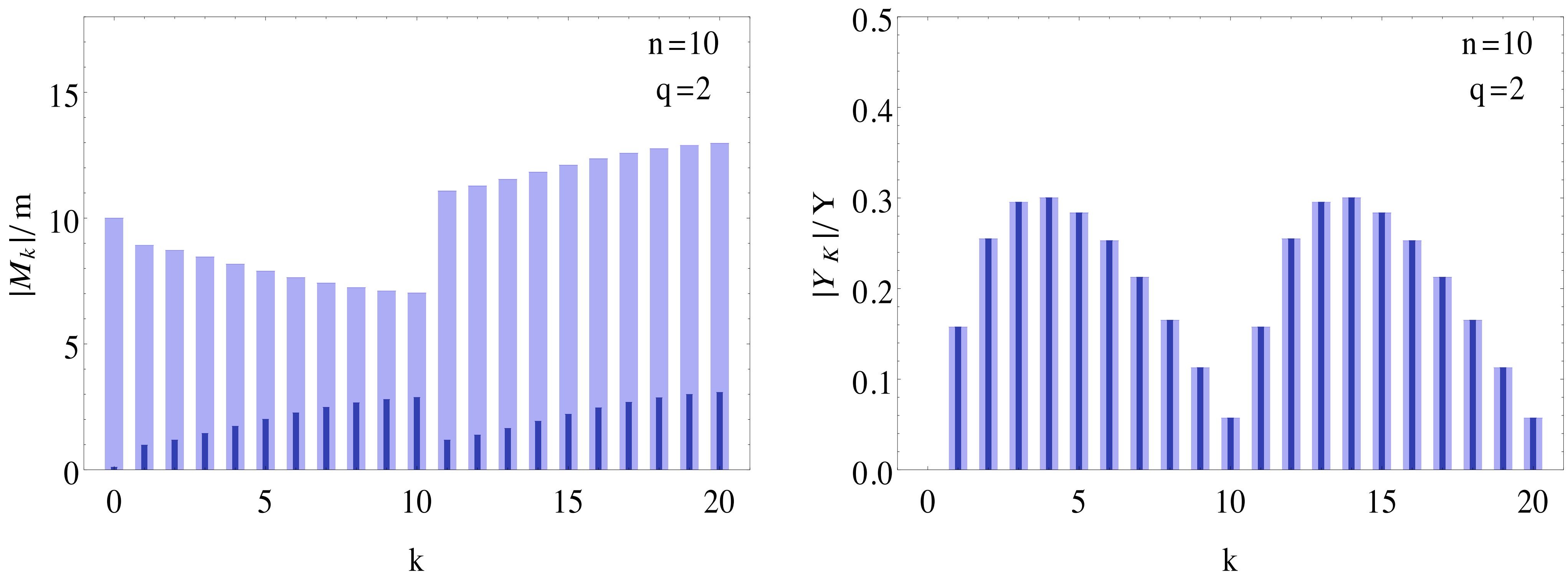


Figure 3: Majorana masses (left panel) and Yukawa couplings (right panel) of the singlet fermions of the clockwork sector, normalized respectively to m and Y, for the specific case $n = 10$, $q = 2$ and $\widetilde{q} = 0.1$ (dark blue) or $\widetilde{q} = 10$ (light blue).

Generalisation with Majorana Masses for the New Fermions

$$\mathcal{L}_{\rm Clockwork} = \mathcal{L}_{\rm kin} - \sum_{i=0}^{n-1}\left(m_i \overline{\psi}_{Li}\psi_{Ri} - m_i' \,\overline{\psi}_{Li}\psi_{Ri+1} + {\rm h.c.}\right) - \sum_{i=0}^{n-1}\frac{1}{2}M_{Li}\overline{\psi^c_{Li}}\psi_{Li} - \sum_{i=0}^{n}\frac{1}{2}M_{Ri}\overline{\psi^c_{Ri}}\psi_{Ri} \ ,$$

$m_i = m$, $m_i' = mq$ $M_{Ri} = M_{Li} = m\widetilde{q}$ for all i.

related works:
Hambye et. al
Park et. al

$$\mathcal{M} = m\begin{pmatrix} \widetilde{q} & 0 & \cdots & 0 & 1 & -q & \cdots & 0 \\ 0 & \widetilde{q} & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \widetilde{q} & 0 & 0 & 0 & -q \\ 1 & 0 & \cdots & 0 & \widetilde{q} & 0 & \cdots & 0 \\ -q & 1 & \cdots & 0 & 0 & \widetilde{q} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -q & 0 & 0 & 0 & \widetilde{q} \end{pmatrix} ,$$

$$\begin{aligned} M_0 &= m\widetilde{q} \ , \\ M_k &= m\widetilde{q} - m\sqrt{\lambda_k} \ , \qquad k = 1, \ldots, n \ , \\ M_{n+k} &= m\widetilde{q} + m\sqrt{\lambda_k} \ , \qquad k = 1, \ldots, n \ , \end{aligned}$$

No Zero mode !!

$$\lambda_k \equiv q^2 + 1 - 2q\cos\frac{k\pi}{n+1} \ .$$

can be diagonalised the matrix

$$\mathcal{U} = \begin{pmatrix} \vec{0} & \frac{1}{\sqrt{2}} U_L & -\frac{1}{\sqrt{2}} U_L \\ \vec{u}_R & \frac{1}{\sqrt{2}} U_R & \frac{1}{\sqrt{2}} U_R \end{pmatrix} .$$

$$\vec{0}_j = 0 \ , \qquad j = 1, ..., n \ ,$$

$$(u_R)_j = \frac{1}{q^j} \sqrt{\frac{q^2 - 1}{q^2 - q^{-2n}}} \ , \qquad j = 1, ..., n \ ,$$

$$(U_L)_{jk} = \sqrt{\frac{2}{n+1}} \sin \frac{jk\pi}{n+1} \ , \qquad j, k = 1, ..., n \ ,$$

$$(U_R)_{jk} = \sqrt{\frac{2}{(n+1)\lambda_k}} \left[q \sin \frac{jk\pi}{n+1} - \sin \frac{(j+1)k\pi}{n+1} \right] \ , \quad j = 0, .., n, \quad k = 1, ..., n \ ,$$

under the universality assumption, the presence of the Majorana masses does not change the mixing matrices !!.

The purely majorana mass mode has same features as the zero mode

Generalisation with Majorana Masses for the New Fermions

$\tilde{q} \ll q$

$q \ll \tilde{q}$

pseudo-Dirac Masses

Phenomenology unexplored

Perhaps ICECUBE

Normal Seesaw like scenario

$$m_\nu \approx \sum_k \frac{Y_k^2 v^2}{M_k} \, .$$

Neutrino mass limits push the gear masses to GUT scale.

Sterile neutrino phenomenology needs to be explored

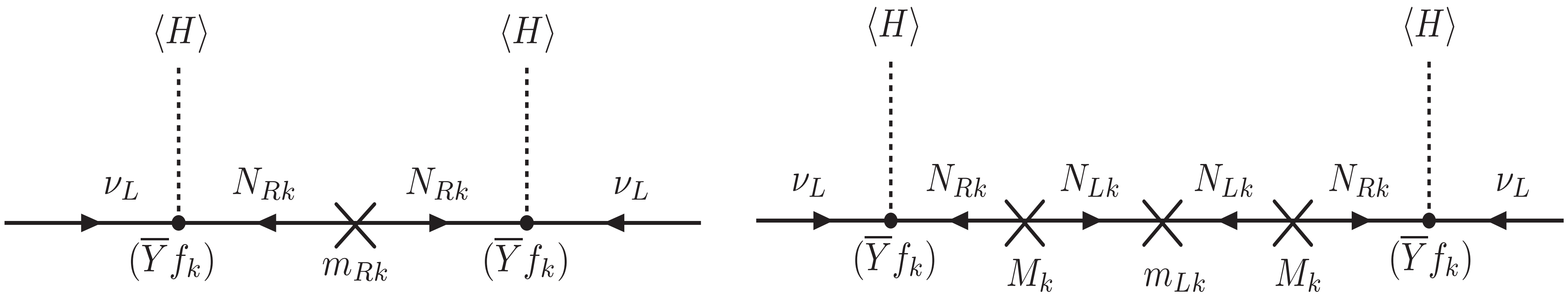


Figure 9: Neutrino Mass at tree level in Majorana Case.

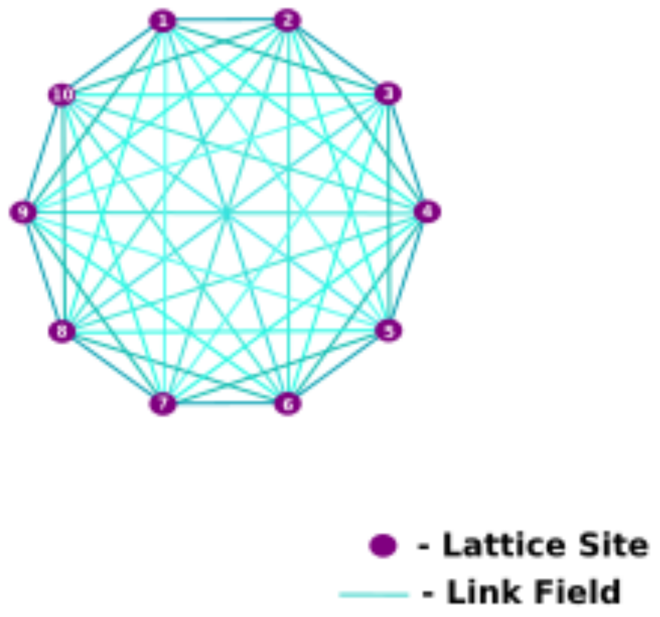
Gear masses are pushed to the GUT scale as they give large corrections to the neutrino masses.

In this case, no signals at the weak scale due to "gears", the new fermions.

Back Up

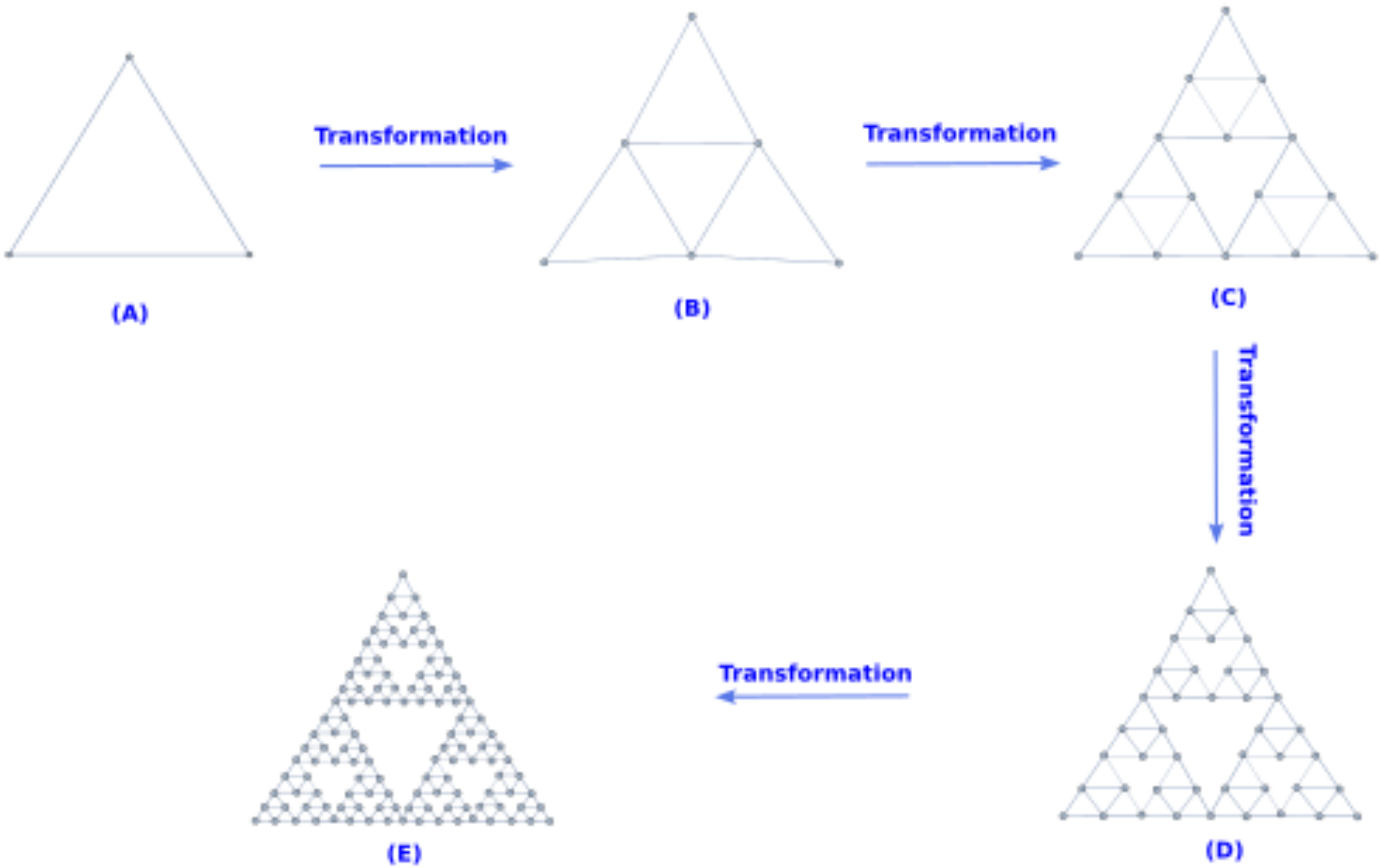
$$M_{fermion} = \begin{bmatrix} 0 & v_1^1 & v_1^2 & v_1^3 & \dots & v_1^n \\ v_n^1 & \lambda_1 & 0 & 0 & \dots & 0 \\ v_n^2 & 0 & \lambda_2 & 0 & \dots & 0 \\ v_n^3 & 0 & 0 & \lambda_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ v_n^n & 0 & 0 & 0 & \dots & \lambda_n \end{bmatrix}$$

$$m_0 \approx \sum_{i=1}^{n} \frac{v_1^i v_n^i}{\lambda_i} \propto \sum_{i=1}^{n} v^2 \frac{e^{-\frac{n}{L_n}}}{\lambda_i}$$



Non Local and Two Dimensional Graphs

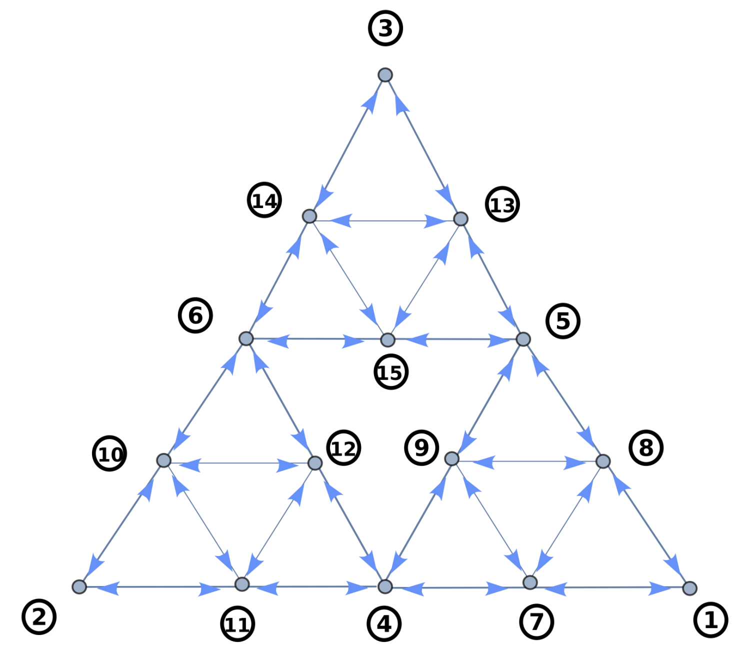
Sierpinski Fractal Graphs

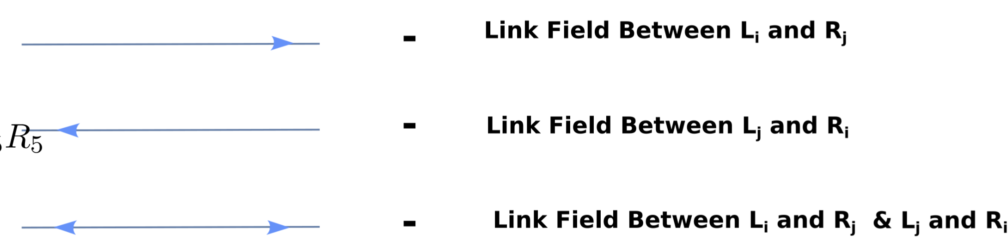


Example with 15 vertices :

- three zero modes !

-localisation of the zero modes

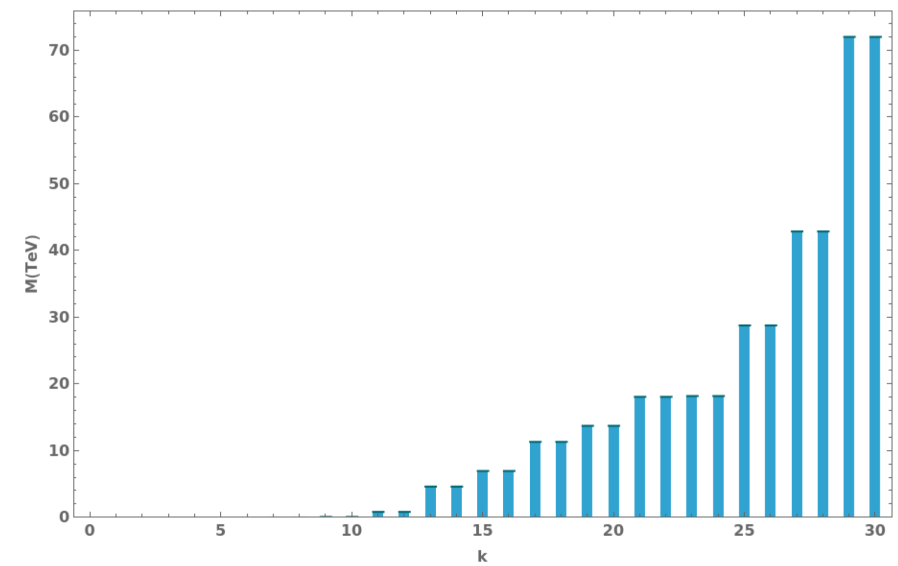


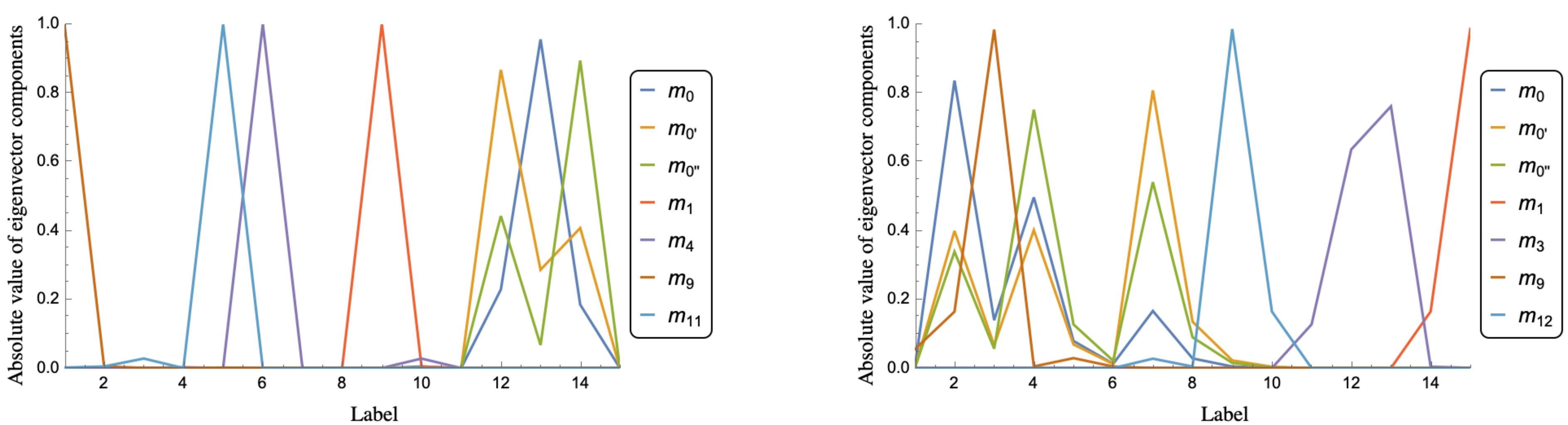


$$
\begin{aligned}
\mathcal{L}_{NP} =& \mathcal{L}_{kin} - \sum_{i,j=1}^{15} m_i \overline{L_i}\delta_{i,j}R_j + m\Big(\overline{L_1}q_{1,7}R_7 + \overline{L_1}q_{1,8}R_8 + \overline{L_7}q_{7,4}R_4 + \overline{L_7}q_{7,9}R_9 + \overline{L_7}q_{7,8}R_8 + \overline{L_8}q_{8,5}R_5 \\
& + \overline{L_8}q_{8,9}R_9 + \overline{L_4}q_{4,9}R_9 + \overline{L_4}q_{4,11}R_{11} + \overline{L_4}q_{4,12}R_{12} + \overline{L_9}q_{9,5}R_5 + \overline{L_5}q_{5,13}R_{13} + \overline{L_5}q_{5,15}R_{15} + \\
& \overline{L_2}q_{2,10}R_{10} + \overline{L_2}q_{2,11}R_{11} + \overline{L_{10}}q_{10,6}R_6 + \overline{L_{10}}q_{10,12}R_{12} + \overline{L_{10}}q_{10,11}R_{11} + \overline{L_{11}}q_{11,12}R_{12} + \overline{L_6}q_{6,12}R_{12} \\
& + \overline{L_6}q_{6,14}R_{14} + \overline{L_6}q_{6,15}R_{15} + \overline{L_3}q_{3,13}R_{13} + \overline{L_3}q_{3,14}R_{14} + \overline{L_3}q_{3,15}R_{15} + \overline{L_{13}}q_{13,14}R_{14} + \overline{L_{14}}q_{14,15}R_{15}\Big) \\
& + m\overline{L_i}q_{i\leftrightarrow j}R_j + h.c.
\end{aligned}
$$

One graph for all the three neutrinos !!

$$
M_{Fractal} = \begin{pmatrix}
2m & mf & mf^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{m}{f} & 2m & mf & mf^2 & mf^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{m}{f^2} & \frac{m}{f} & 2m & 0 & mf^2 & mf^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{m}{f^2} & 0 & 2m & mf & 0 & mf^3 & mf^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{m}{f^3} & \frac{m}{f^2} & \frac{m}{f} & 2m & mf & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{m}{f^3} & 0 & \frac{m}{f} & 2m & 0 & 0 & mf^3 & mf^4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{m}{f^3} & 0 & 0 & 2m & mf & 0 & 0 & mf^4 & mf^5 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & 0 & 0 & 0 & mf^4 & mf^5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{m}{f^3} & 0 & 0 & 2m & mf & 0 & 0 & 0 & mf^5 & mf^6 \\
0 & 0 & 0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & 0 & 0 & mf^3 & mf^4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & 0 & 2m & mf & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & mf & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & 0 & \frac{m}{f^3} & 0 & \frac{m}{f} & 2m & mf & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & mf \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^6} & 0 & 0 & 0 & 0 & \frac{m}{f} & 2m
\end{pmatrix}
$$





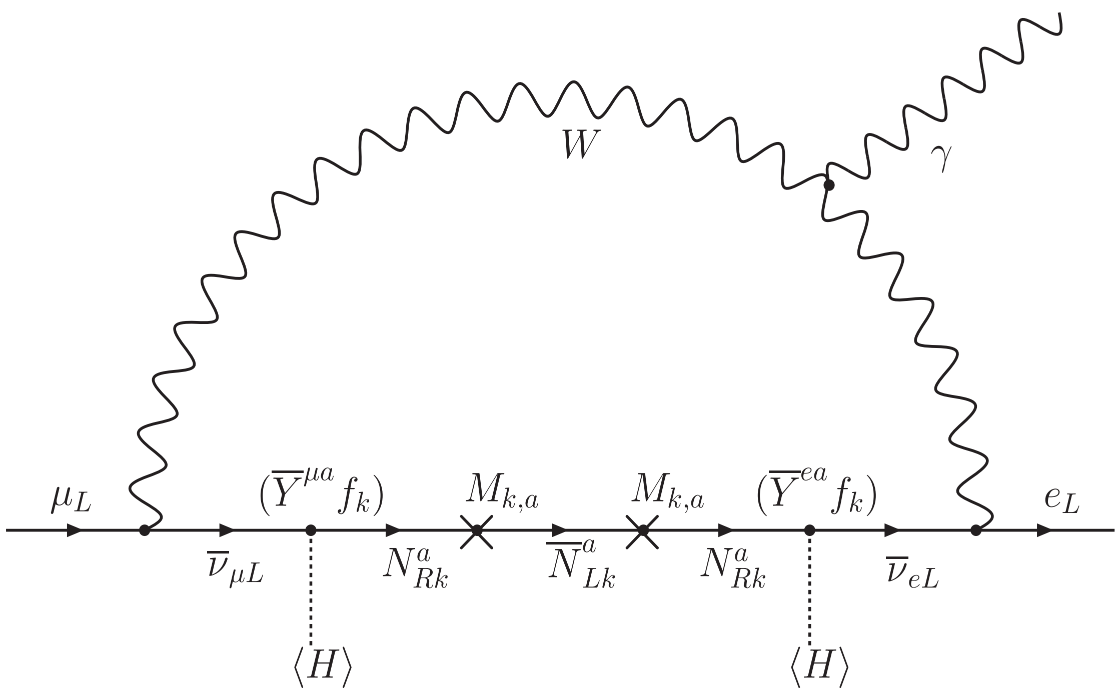
Plot 2(B) - Left plot shows the absolute value of components of left-handed mass eigenvectors and the right plot for the right-handed mass eigenvector.

Phenomenology

Putting in the full Standard Model (leptonic sector)

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{Clockwork}} + \mathcal{L}_{\text{int}} \ ,$$

Exchange of clockwork gears leads to lepton flavour violation.



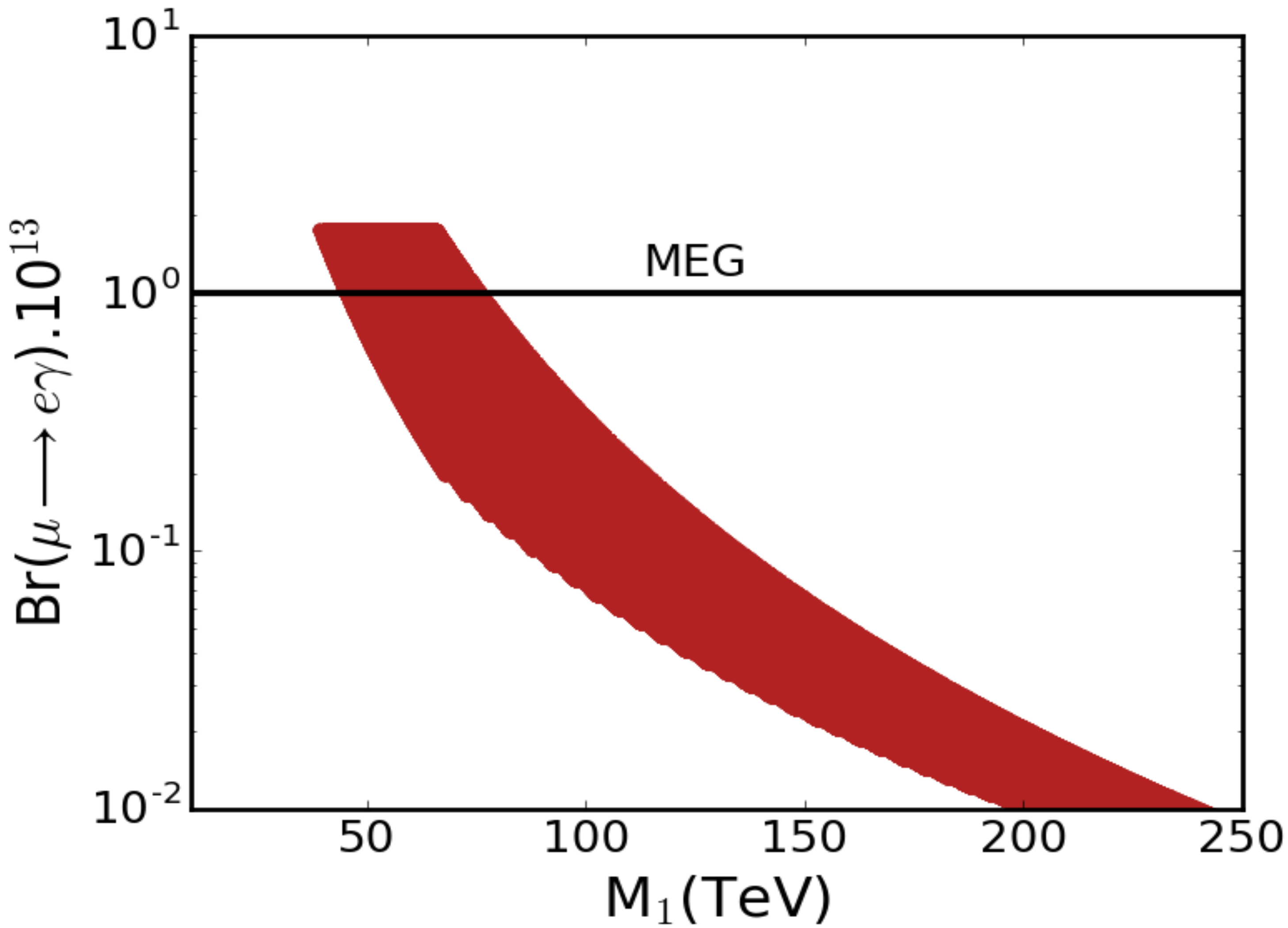
Consider for example a rare process, which has not yet Been discovered...similar to rare flavour violating processes in the Hadronic sector.

$$\mu \to e + \gamma$$

But there are strong limits on it

$$B\,(\mu \to e\gamma) \simeq \frac{3\alpha_{\text{em}} v^4}{8\pi} \left| \sum_{\alpha=1}^{N} \sum_{k=1}^{n_\alpha} \frac{Y_k^{e\alpha} Y_k^{\mu\alpha}}{M_k^{\alpha\,2}} F(x_k^\alpha) \right|^2 ,$$

$$F(x) \equiv \frac{1}{6(1-x)^4}(10 - 43x + 78x^2 - 49x^3 + 4x^4 - 18x^3 \log x) \ ,$$



Present limit of around 40 TeV !!

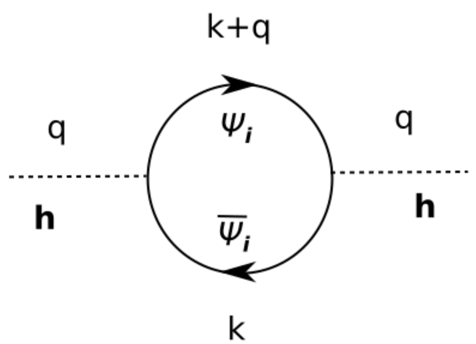
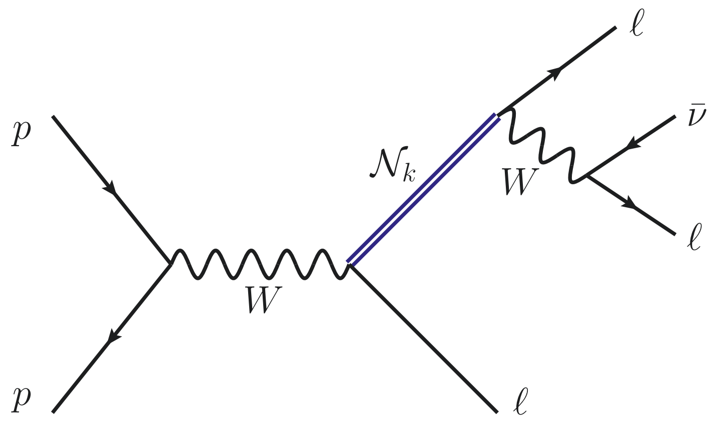
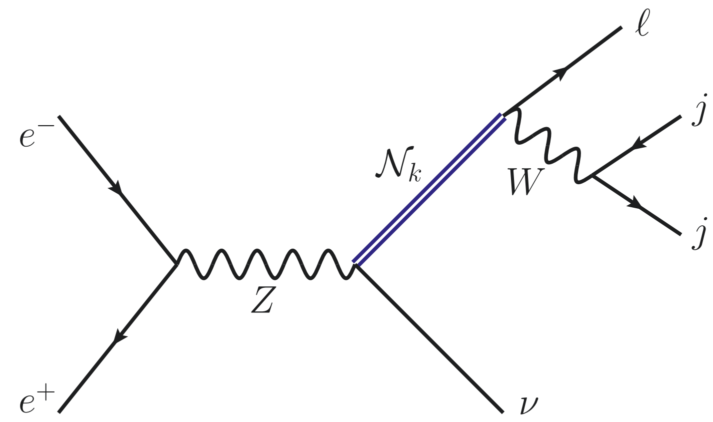


Fig. 6 - These Feynman diagrams show the 1-loop contribution of fermions in Higgs mass radiative corrections. The Left diagram shows it for the same fermions in the loop with y_{ii} coupling and the right diagram shows it for different fermions in the loop with y_{ij} coupling strength.

Higgs corrections !!

LFV at colliders





A lot of things still left to be explored.

Conclusions

We presented localisation in models which are “finite” not equavilent to extra dimensions and they provide interesting phenomena.

Theoretically it means that the Standard Model has to be extended

New Particles and/or Additional Symmetry
or both

Closely related to the question
whether neutrinos are Majorana or Dirac

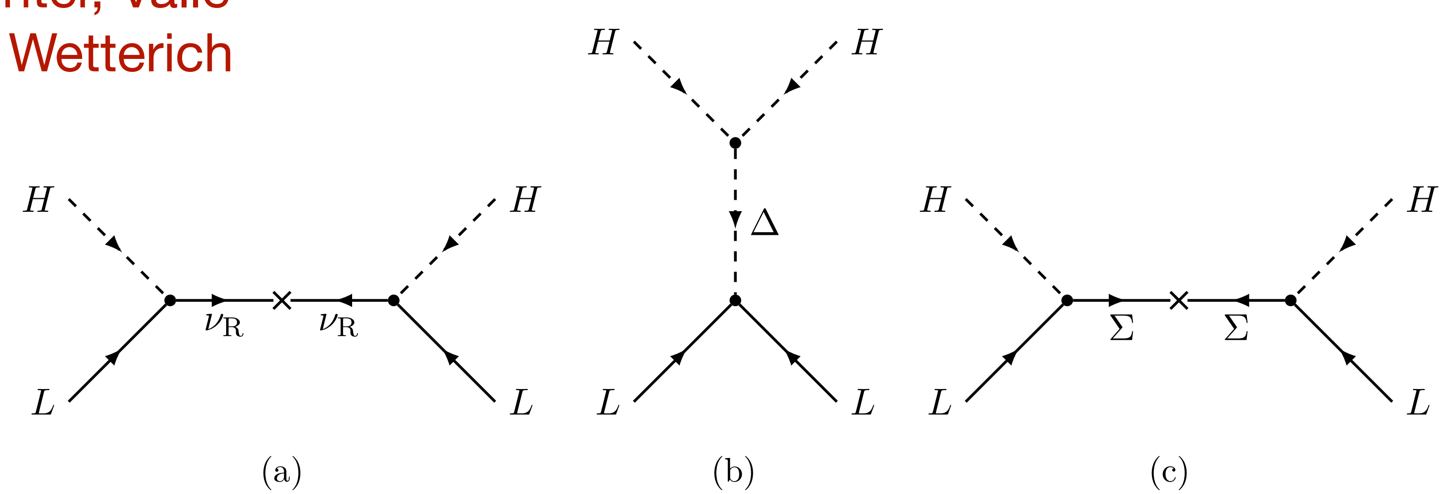
Effective Theory : $$\mathcal{L} = \mathcal{L}_{SM} + \frac{c}{\Lambda} L\tilde{H}L\tilde{H}$$

Weinberg, 1977

- Typically a large mass/small vev is required to generate the small masses
- in seesaw like mechanisms
- Can fit naturally in GUTs

Minkowski
Senjanovic, Mohapatra,
GellMann, Ramond, Slansky
Yanagida, Schechter, Valle
Lazarides, Shafi, Wetterich

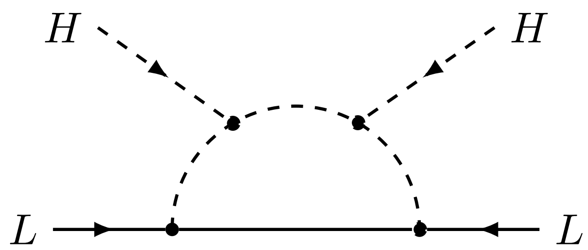
S. F. King, 2003
A. De Gouvea, 2016
Davidson et.al, 2008
Cai, et.al, 2018



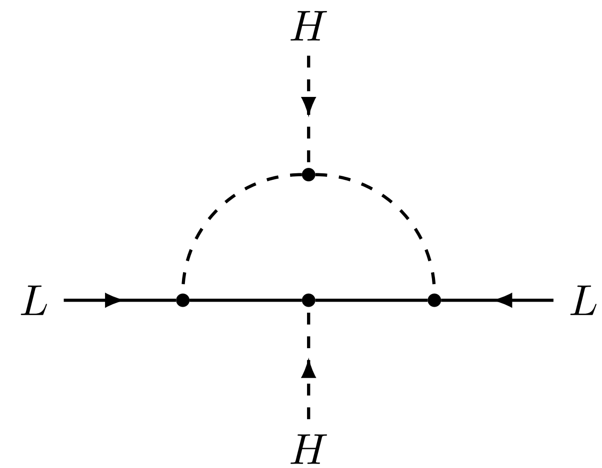
Type I **Type II** **Type III**

Radiative Mechanisms

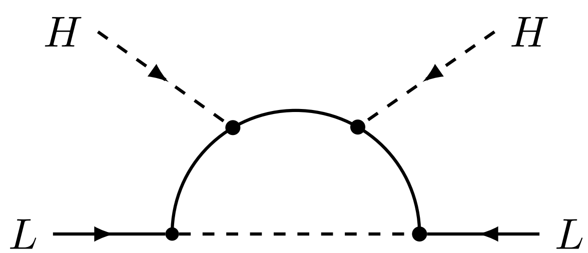
E. Ma,
Babu
Zee



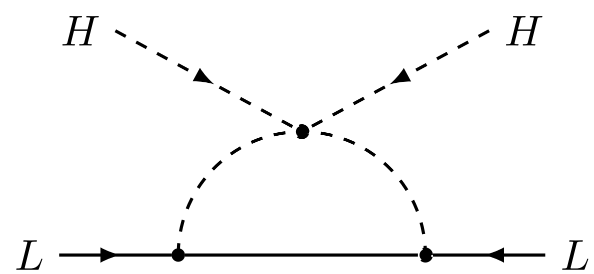
(a) T1-i



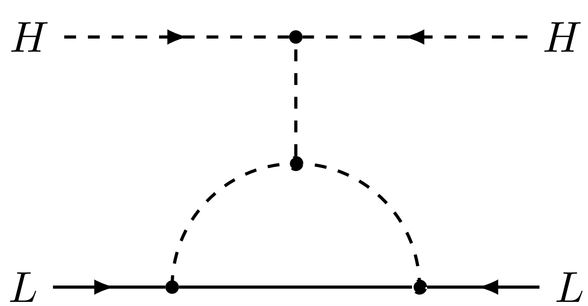
(b) T1-ii



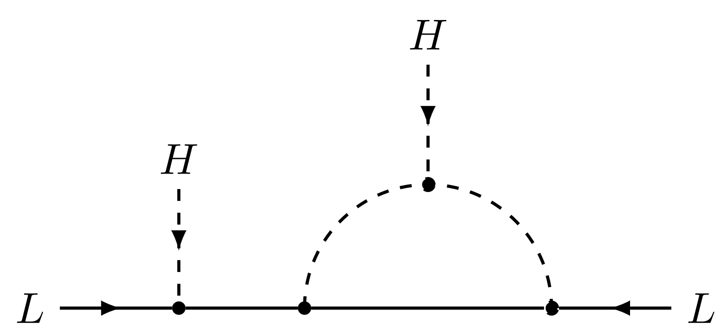
(c) T1-iii



(d) T3



(e) T4-2-i



(f) T4-3-i

Babu Leung, 2001
Cai et.al 2017

Hierarchies in the parameters of the theory ! !

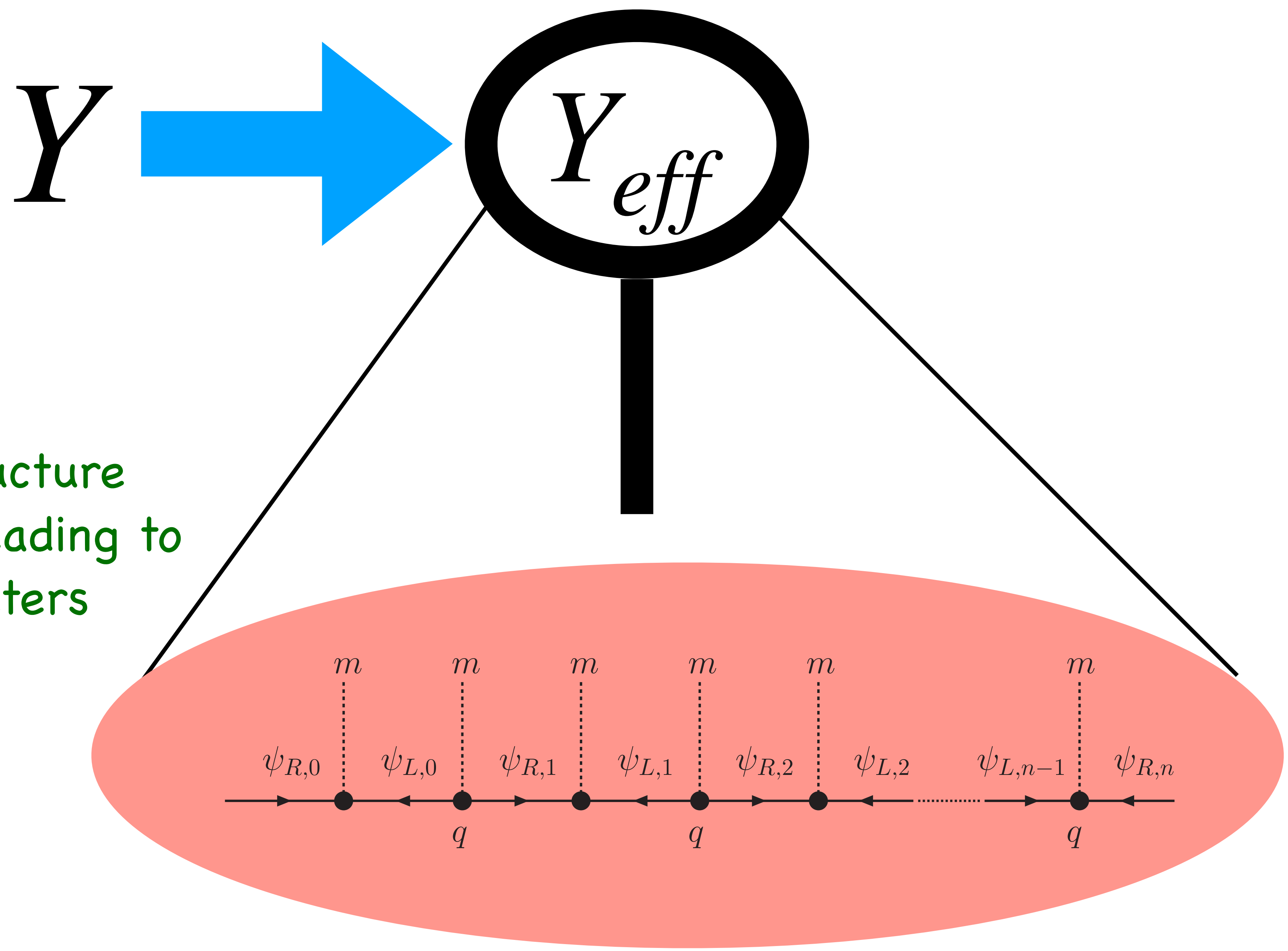
A (LOFTIER) GOAL

Generate Hierarchies of masses and other parameters starting with O(1) parameters of the theory

And testable !

Consider Dirac Masses

$$\mathcal{L}_{SM} + Y \bar{\nu}_L \nu_R \tilde{H}$$



A deeper heavier structure
With O(1) parameters, leading to
hierarchial parameters

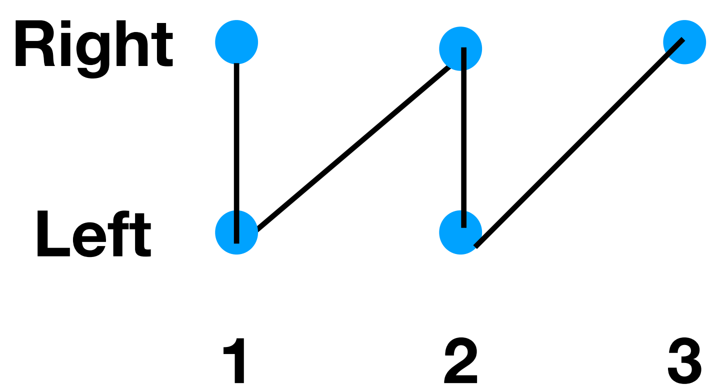
Generating small couplings: Toy Model

consider mixing within five new (chiral) fermions in 4D

$$\chi_{L1}, \chi_{L2}, \chi_{R1}, \chi_{R2}, \chi_{R3}$$

nearest neighbour masses

$$\mathcal{L}_{\rm CW} = \mathcal{L}_{kin} + m\ \bar{\chi}_{L1}\chi_1 + qm\ \bar{\chi}_{L1}\chi_{R2} + m\bar{\chi}_{L2}\chi_{R2} + qm\ \bar{\chi}_{L2}\chi_{R3}$$



$$M_\psi = m \begin{pmatrix} 1 & -q & 0 \\ 0 & 1 & -q \end{pmatrix}$$

Has an unbroken U(1) global symmetry (with all the phases equal)

$$U(2)_L \times U(3)_R \to \Pi_{i=1} U(1)_i \to U(1)_{\rm CW}$$

$$M_\psi^\dagger M_\psi = m^2 \begin{pmatrix} 1 & -q & 0 \\ -q & 1+q^2 & -q \\ 0 & -q & q^2 \end{pmatrix}$$

eigenvalues $0, m^2\lambda_1, m^2\lambda_2$

$$\lambda_1 = 1 - q + q^2$$

$$\lambda_2 = 1 + q + q^2$$

$$\psi_L = U_L N_L\,, \qquad \psi_R = U_R N_R$$

one massless chiral fermion
and two Dirac fermions

$$U_R = \begin{pmatrix} \lambda_0 & -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{\lambda_1}} & -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{\lambda_2}} \\ \frac{1}{q}\lambda_0 & \frac{1}{\sqrt{2}}\frac{q-1}{\sqrt{\lambda_1}} & \frac{1}{\sqrt{2}}\frac{q+1}{\sqrt{\lambda_2}} \\ \frac{1}{q^2}\lambda_0 & \frac{1}{\sqrt{2}}\frac{q}{\sqrt{\lambda_1}} & -\frac{1}{\sqrt{2}}\frac{q}{\sqrt{\lambda_2}} \end{pmatrix}$$

$$\lambda_0 = \frac{q^2}{\sqrt{q^4+q^2+1}}$$

$$q > 1$$

The zero eigenvalue field has a suppressed component of the third field, it is getting localised on the 1st site.

Clockwork mechanism : Toy model

$Y \sim \mathcal{O}(1)$

$$\mathcal{L}_{CW} + Y L \tilde{H} \chi_{R3}$$

Diagonalising $\downarrow$ identifying zero mode with ν_R

$$\mathcal{L}_{CW} + \frac{Y}{q^2} L \tilde{H} \nu_R + \ldots$$

The coupling of the zero mode is suppressed by a power factor

$q \sim 3,$ It is an order of magnitude

interactions
with new Dirac fermions
need not be suppressed.

They are in fact O(1)

Clockwork Mechanism

Choi and Im, JHEP 2016

Kaplan and Rattazzi, 2016

Introduced to solve transplanckian excitations required by the inflaton field in Relaxion models (axion fields)

Giudice and McCoullough,2016

Fermion fields and spin 1,2 fields in a clockwork

noted the application to neutrino masses

Starting from O(1) couplings, clockwork is a mechanism which generates exponentially small couplings naturally in the theory.
(This is for the symmetry protected zero mode)

It provides a natural framework to understand
Dirac Neutrinos which require
tiny couplings.