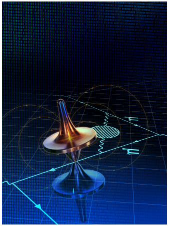
Probing the Standard Model to ~~0.37~~ 0.35 ppm with the muon anomalous magnetic moment

Laurent Lellouch

CNRS & Aix-Marseille U.

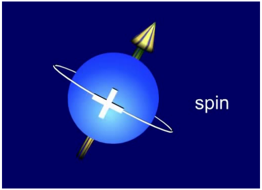


Budapest-Marseille-Wuppertal collaboration [BMW] & DMZ

2407.10913 ⟶ BMW-DMZ '24 (or this work)
Nature 593 (2021) ⟶ BMW '20
PRL 121 (2018) 022002 (Editors' Selection) ⟶ BMW '17
Aoyama et al., Phys. Rep. 887 (2020) 1-166 ⟶ WP '20
Aliberti et al., 2505.21476 ⟶ WP '25

Muons are tiny magnets

A massive elementary particle w/ electric charge and spin behaves like a tiny magnet



(← Silver Swan)

Magnetic moment of the muon

$$\vec{\mu}_\mu = \pm g_\mu \frac{e}{2m_\mu} \vec{S}$$

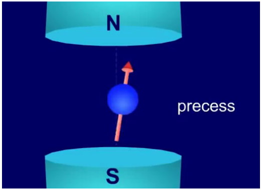
g_μ = Landé factor

In uniform magnetic field $\vec{B}$, $\vec{S}$ precesses w/ angular frequency

$$\omega_S = g_\mu \frac{e}{2m_\mu} |\vec{B}|$$

7×10^6 rotations per second for $|\vec{B}| = 1.45$ T

(Silver Swan →)



- $a_\mu \equiv (g_\mu - 2)/2$ can be **measured** & **calculated** very, very ... precisely

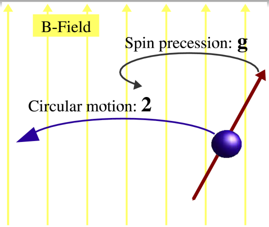
Key point:

- measurement = SM prediction ?

  → Yes: another victory for the SM

  → No: we have uncovered new fundamental physics

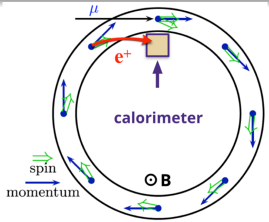
Measurement principle for a_μ



Precession determined by

$$\vec{\mu}_\mu = 2(1+a_\mu)\frac{Qe}{2m_\mu}\vec{S}$$

$$\vec{d}_\mu = \eta_\mu \frac{Qe}{2m_\mu}\vec{S}$$



$$\vec{\omega}_{a\eta} = \vec{\omega}_a + \vec{\omega}_\eta \simeq -\frac{Qe}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right] - \eta_\mu \frac{Qe}{2m_\mu}\left[\vec{E} + \vec{\beta}\times\vec{B}\right]$$

- Experiment measures very precisely $\vec{B}$ with $|\vec{B}| \gg |\vec{E}|$ &

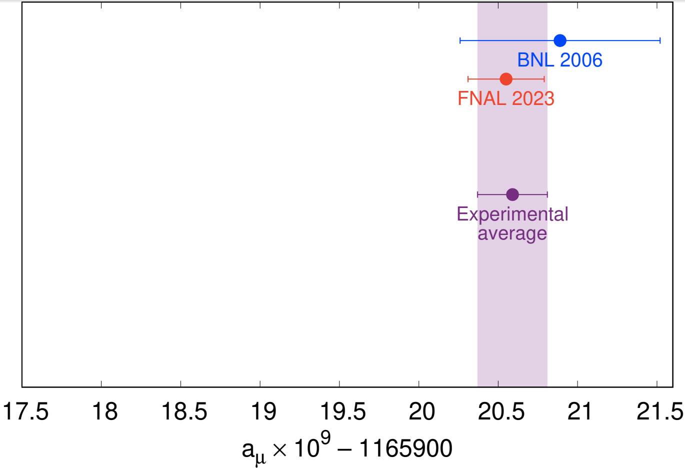
$$\Delta\omega \equiv \omega_S - \omega_C \simeq \sqrt{\omega_a^2 + \omega_\eta^2} \simeq \omega_a$$

since $d_\mu = 0.1(9)\times 10^{-19} e\cdot\mathrm{cm}$ (Benett et al '09)

- Consider either magic $\gamma = 29.3$ (CERN/BNL/Fermilab) or $\vec{E} = 0$ (J-PARC)

$$\to \Delta\omega \simeq a_\mu B \frac{e}{m_\mu}$$

a_μ: present experimental status



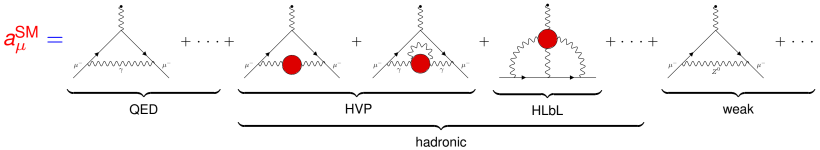
$$a_\mu^{\text{expt}} = 11\,659\,205.9\,(2.2) \times 10^{-10} \qquad [0.19\,\text{ppm}]$$

Based only on $\sim 25\%$ of Fermilab data!!

Expect $\sigma_{a_\mu^{\text{expt}}}$ reduced by ~ 2 (next week!!!)

a_μ^{SM}: standard model prediction of WP '20

For comparable precision $\rightarrow$ all three interactions and all SM particles



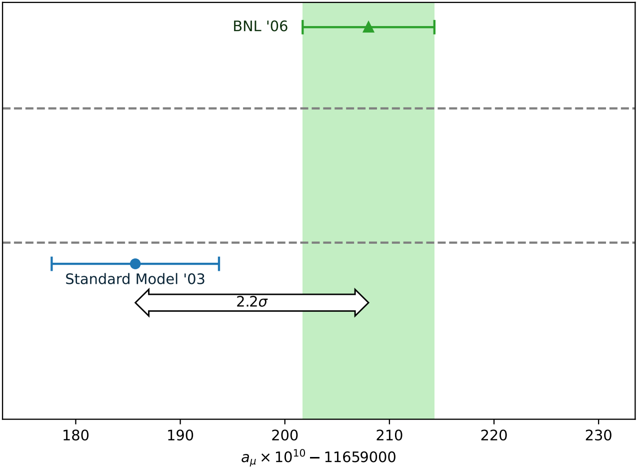
Hadronic contributions involve low-energy, nonperturbative QCD:

- Data-driven: unitarity, analyticity, short-distance QCD and data $\rightarrow$ dominated WP '20 averages
- Lattice: massively-parallel numerical simulations in QCD $\rightarrow$ play increasingly important role

SM contribution	$a_\mu^{\mathrm{contrib.}} \times 10^{10}$	Ref.
QED [5 loops]	11658471.8931 ± 0.0104	[Aoyama '19, WP '20]
EW [2 loops]	15.36 ± 0.10	[Gnendiger '15, WP '20]
HVP LO (Lattice)	711.6 ± 18.4	[WP '20]
HVP Tot. (e^+e^-, pheno)	684.5 ± 4.0	[WP '20]
HLbL. (lattice)	$7, .9 \pm 3.5$	[WP '20]
HLbL Tot. (pheno + lattice)	9.2 ± 1.8	[WP '20]
SM [0.37 ppm]	11659181.0 ± 4.3	[WP '20]

- HVP $\sim 75 \times$ HLbL
- $\sigma^2_{\mathrm{HVP}}/\sigma^2_{a_\mu} \sim 87\%$ & $\sigma^2_{\mathrm{HLbL}}/\sigma^2_{a_\mu} \sim 18\%$
- Focus mostly on HVP

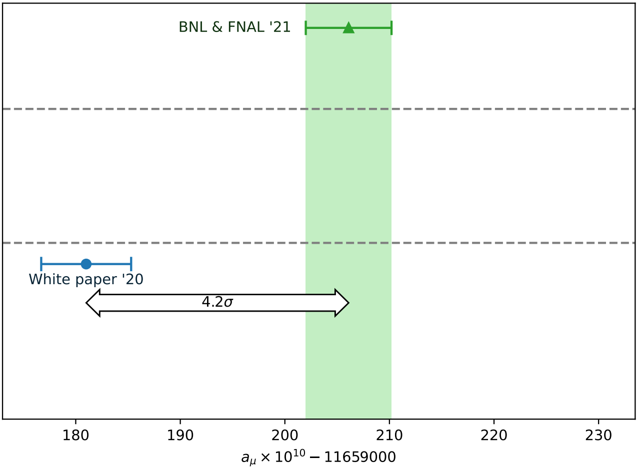
Experiment vs SM: 2006



Tension too small to claim new physics, too large to ignore:

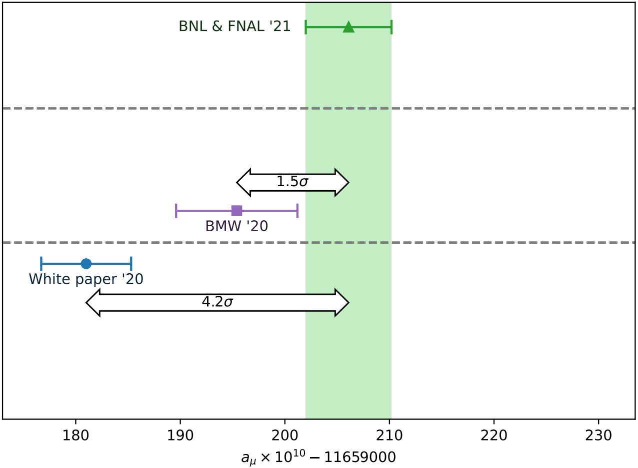
- → New Fermilab Muon $g-2$ experiment
- → Flurry of lattice and data-driven work on HVP and HLbL
- → *Muon $g-2$ Theory Initiative* $\geq$ 2017

Experiment vs SM: April 2021



Strong evidence for new fundamental physics

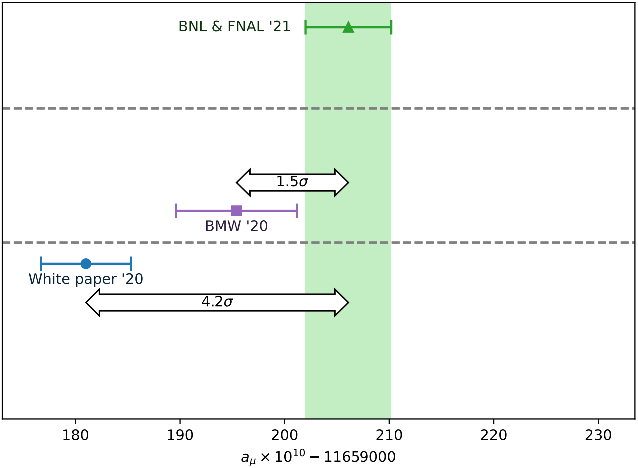
Experiment vs SM: April 2021



Overall picture changes:

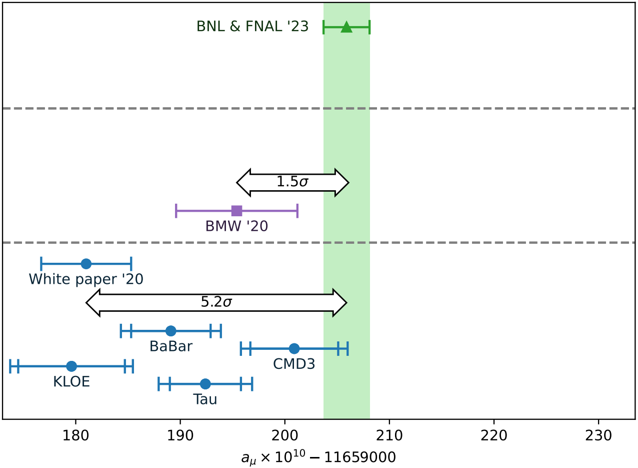
- Complete lattice calculation of HVP contribution in BMW '20 suggests SM still in the race
- Computation of sub-contribution in BMW '20 shows 3.7σ tension between lattice and data-driven: both cannot be correct

Experiment vs SM: April 2021



"if the result of the present [work] is right, when taken at face value, the consequences for particle physics would be quite dramatic. [. . .] several different and independent experiments [. . .] would have to be all wrong in a fundamental way." [an eminent colleague]

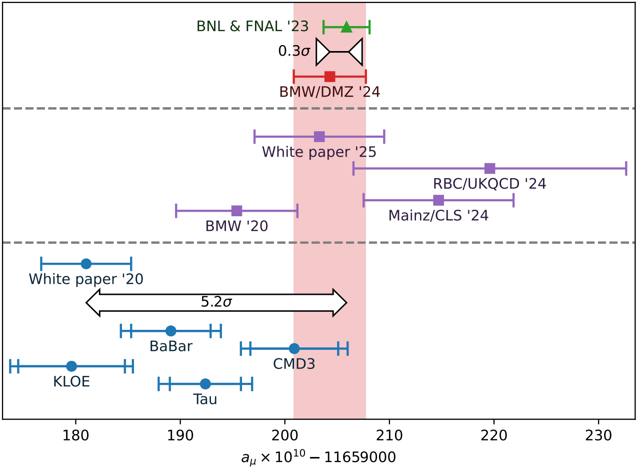
Experiment vs SM: 2023



New measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ in CMD-3 '23 further challenges WP '20 data-driven consensus

- $\to$ $a_\mu^{\text{LO-HVP}}$ prediction agrees w/ BMW '20 one to w/in 1.5σ
- $\to$ re-assessment of data-driven results further challenges consensus [DHLMZ '23]

Experiment vs SM: 27 May 2025



New calculation of $a_\mu^{\text{LO-HVP}}$ + WP '25 indicates SM confirmed to 0.35 ppm !

→ confirmed by two lattice calculations [RBC '24, Mainz '24] to w/in 1.5σ

→ WP '25 is consolidated combination of subcontributions from many lattice calculations

[RBC/UKQCD' 18, 23, 24; ETM '19, 22, 24; BMW '20, 24; LM '20; ABGP '22; Mainz '22, 24, 24; SL '24; FHM '24, 24]

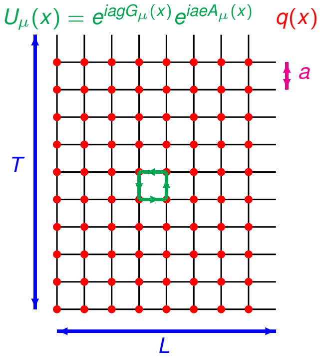
What is lattice QCD (LQCD) + QED?

To describe low-energy, strong (& electromagnetic) interaction phenomena w/ sub-% precision
$\rightarrow$ QCD + QED requires $\geq$ 132 numbers at every spacetime point
$\Rightarrow$ infinitely dense number of numbers in our continuous spacetime
$\Rightarrow$ must temporarily "simplify" the theory to calculate (regularization)
$\Rightarrow$ Lattice gauge theory $\longrightarrow$ mathematically sound definition of QCD (beyond PT) & QED:

- UV (& IR) cutoff $\rightarrow$ well defined functional integral in Euclidean spacetime:

$$\begin{aligned}\langle O\rangle &= \int \mathcal{D}U\mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\, e^{-S_G-\int \bar{q}D[M]q}\, O[U,A,q,\bar{q}] \\ &= \int \mathcal{D}U\mathcal{D}A\, e^{-S_G}\, \det(D[M])\, O[U,A]_{\mathrm{Wick}}\end{aligned}$$

- $\mathcal{D}U\mathcal{D}A\, e^{-S_G}\det(D[M]) \geq 0$ & finite # of dofs
  $\rightarrow$ evaluate numerically using stochastic methods



L(QCD+QED) is really QCD+QED: must tune $m_q \rightarrow m_q^{\mathrm{ph}}$ & $\Lambda_{\mathrm{QCD}} \rightarrow \Lambda_{\mathrm{QCD}}^{\mathrm{ph}}$, $e \rightarrow e^{\mathrm{ph}}$, $a \rightarrow 0$ (after renormalization), $L, T \rightarrow \infty$ (and stats $\rightarrow \infty$)

HUGE conceptual and numerical ($10^{10} \rightarrow 10^{11}$ dofs) challenge

Our particle "accelerators"

Such computations require some of the world's most powerful supercomputers

- Today: up to $\sim 4 \times 10^{17}$ flop/s on LUMI
- Soon in Europe: exaflop supercomputers ($> 10^{18}$ flop/s)
- Many many thanks to GENCI, GCS, EuroHPC and the European taxpayer

Method and challenges

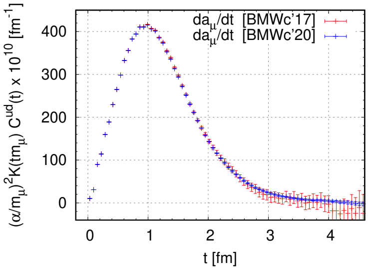
Compute on $T \times L^3$ Euclidean-time lattice w/ spacing a [Bernecker et al '11]

$$C_L(t) = \frac{a^3}{3} \sum_{i=1}^{3} \sum_{\vec{x}} \langle J_i(x) J_i(0) \rangle$$

w/ $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$

Then [$K(tm_\mu)$ known kinematical function]

$$a_\mu^{\text{LO-HVP}} = \lim_{\substack{a \to 0 \\ L, T \to \infty}} \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{a}{m_\mu^2}\right) \sum_{t=0}^{T/2} K(tm_\mu)\, \mathrm{Re}\, C_L(t)$$



(a) Statistical uncertainties of light and disconnected contributions at large t

(b) Finite V (and T) corrections on $I = 1$ contribution

(c) Continuum limits ($a \to 0$)

(e) Tuning of physical point $\leftrightarrow$ very precise determination QCD parameters: scale and m_u, m_d, m_s, m_c masses

(f) Must include small effects from QED and $m_u \neq m_d$ (SIB)

Strategy for improvement

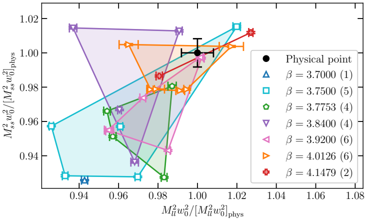
- 28 large-scale simulations including new finer ("Monster") lattices: $a = 0.064$ fm $[96^3 \times 144]$ $\longrightarrow a = 0.048$ fm $[128^3 \times 192]$
  - $\rightarrow$ 80% nearer continuum limit (in a^2)
  - $\rightarrow$ reduces $a \rightarrow 0$ uncertainty
  - $\rightarrow$ improves tuning to physical point

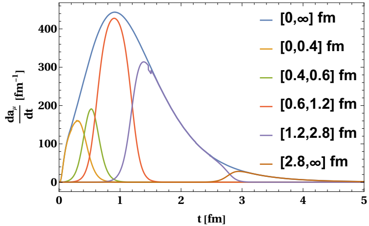
- Break up lattice computation into optimized set of windows: $0-0.4$, $0.4-0.6$, $0.6-1.2$, $1.2-2.8$ fm
  [window idea in RBC/UKQCD '18]

  Continuum extrapolate $I = 0$ instead of disconnected
  - $\rightarrow$ better control over $a \rightarrow 0$ limit
  - $\rightarrow$ overall reduction of uncertainties

- Data-driven evaluation of tail: $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ [proposed and used w/ 1 fm $\rightarrow \infty$ [RBC/UKQCD '18])
  - $\rightarrow$ reduces FV correction $[L: 6.3 \rightarrow \infty]$ $\div 3$ $18.5(2.5) \rightarrow 9.3(9)$, i.e. cv $\div 2$ & err
  - $\rightarrow$ reduces long-distance (LD) noise

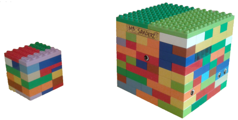
- Calculation fully blinded





[plot made w/ KNT '18 data set]

"ref" $\rightarrow$



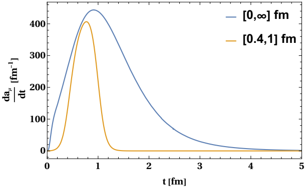
$\leftarrow$ "big"

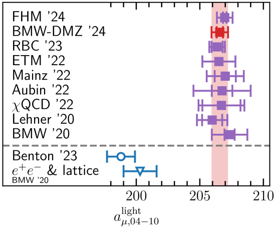
July 12, 2024: unblinding

Preprint uploaded to arXiv on July 15, 2024

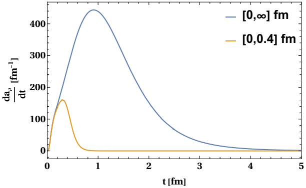
Benchmarking of lattice calculation: windows

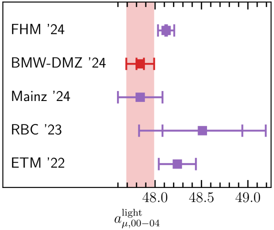
$0.4 \to 1\,\mathrm{fm}$



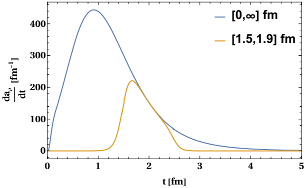


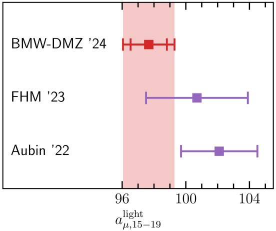
$0 \to 0.4\,\mathrm{fm}$





$1.5 \to 1.9\,\mathrm{fm}$



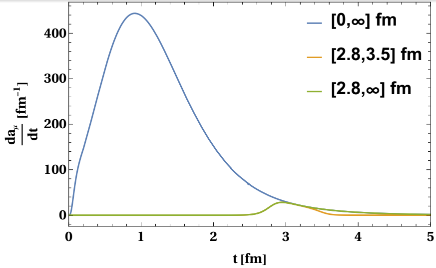


Connected light $= ud$ contribution to long-distance window ($1 \to \infty\,\mathrm{fm}$):

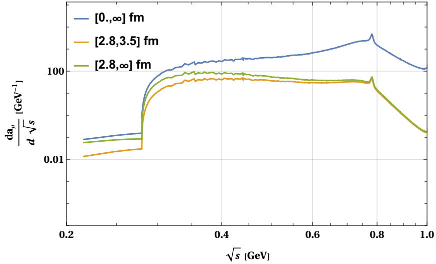
411.4[4.9] [RBC/UKQCD '24] ; 410.7[5.9] [Mainz '24, BMW world]

401.2[4.3] [FHM '24]

Tail contribution from $\sigma(e^+e^- \to \text{hadrons})$



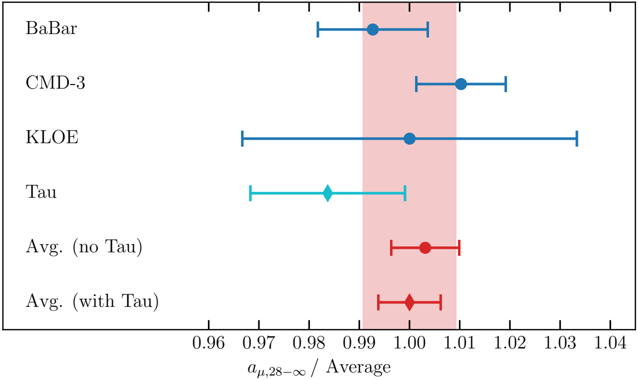
$$C(t) = \frac{1}{24\pi^2}\int_0^\infty ds\sqrt{s}\,R(s)\,e^{-|t|\sqrt{s}}$$



- Lattice computation up to $t = 2.8\,\mathrm{fm}$: $> 95\%$ of final result for $a_\mu^{\text{LO-HVP}}$
- Tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ computed using $e^+e^- \to$ hadrons for $t > 2.8\,\mathrm{fm}$: $< 5\%$ to final result for $a_\mu^{\text{LO-HVP}}$
- Tail dominated by cross section below ρ peak: $\sim 65\%$ for $\sqrt{s} \leq 0.55\,\mathrm{GeV}$
- All measurements of $\sigma(e^+e^- \to \text{hadrons})$ agree well in that region
- Partial tail $a_{\mu,28\text{-}35}^{\text{LO-HVP}}$ ($2.8\,\mathrm{fm} < t \leq 3.5\,\mathrm{fm}$) for comparison with lattice dominated by cross section below ρ peak: $\sim 55\%$ for $\sqrt{s} \leq 0.55\,\mathrm{GeV}$

[illustrative plots made w/ KNT '18 data set]

Data-driven tail



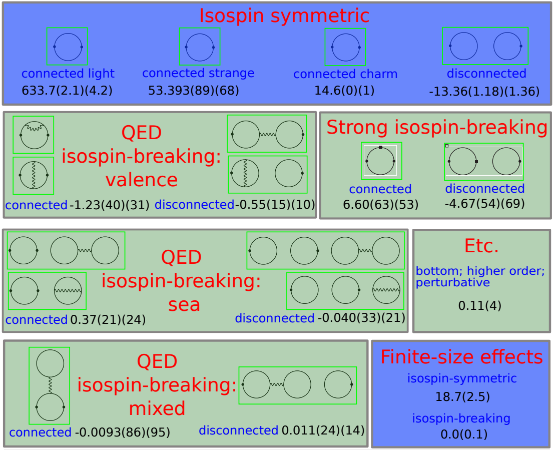
- Window from $2.8 \to \infty$ fm
- Only $\lesssim 5\%$ of final result for a_μ
- Data-driven results agree well
- Correlated average taken w/ and w/out τ: $\chi^2/\text{dof} = 1.0$ and 0.8
- Final number: average w/ τ, and systematic = full difference τ/no-τ added linearly

$$a_{\mu,28\text{-}\infty}^{\text{LO-HVP}} = 27.59(17)(9)[26]$$

- Davies et al '24 & Stoffer et al '25 find less agreement possibly leading to larger uncertainty ($0.26 \to\sim 0.8$) by imposing constraints
- Currently investigating
- Negligible impact on final uncertainty for $a_\mu^{\text{LO-HVP}}$

Improved in new work

Some checks in new work



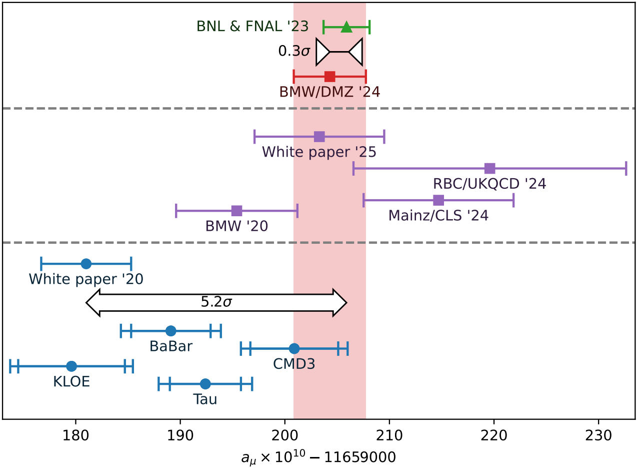
$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

2024 $\downarrow$ Lattice + tail

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 714.1(2.2)_{\text{stat}}(2.5)_{\text{sys}}[3.3]_{\text{tot}}$$

Corresponds to a 4.6 per mil total uncertainty

Experiment vs SM: 27 May 2025



New calculation of $a_\mu^{\text{LO-HVP}}$ + WP '25 indicates SM confirmed to 0.35 ppm !

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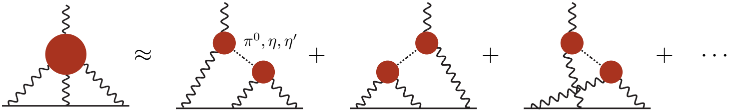
[Special thank to Antoine Gérardin]

Pseudoscalar-pole contributions



[Jegerlehner & Nyffeler '09]

$$a_\mu^{\mathrm{HLbL};P} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \left\{ w_1(Q_1, Q_2, \tau) \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2)\, \mathcal{F}_{P\gamma^*\gamma^*}(-Q_2^2, 0) \right.$$
$$\left. + \; w_2(Q_1, Q_2, \tau) \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\, \mathcal{F}_{P\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0) \right\}$$

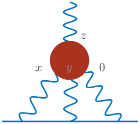
with $P = \pi^0,\ \eta,\ \eta'$

Direct calculation: based on coordinate-space approach developed by Mainz group

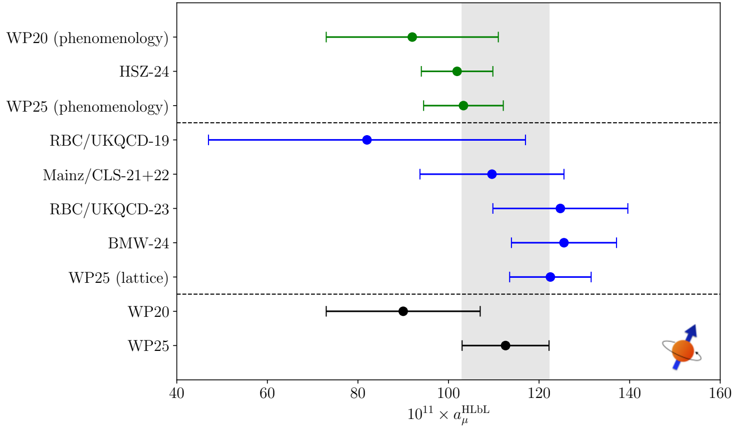
$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int \mathrm{d}^4 y \int \mathrm{d}^4 x\, \mathcal{L}_{[\rho,\sigma],\mu\nu\lambda}(x,y)\, i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)\,,$$

with

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int \mathrm{d}^4 z\, z_\rho \langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0) \rangle$$



HLbL summary



$$a_\mu^{\text{HLbL}, \pi^0, \eta, \eta'}|_{\text{BMW '23}} = 8.51(0.47)(2.3)[2.3] \times 10^{-10}$$

$$a_\mu^{\text{HLbL}}|_{\text{BMW '24}} = 12.55(1.15)(0.19)[1.17] \times 10^{-10}$$

$$a_\mu^{\text{HLbL}}|_{\text{WP '20}} = 9.0[1.7] \times 10^{-10} \quad \to \quad a_\mu^{\text{HLbL}}|_{\text{WP '25}} = 11.26[0.96] \times 10^{-10}$$

Conclusions and outlook

- HVP & HLbL:
  - WP '20: SM prediction dominated by data-driven methods
  - WP '25: SM prediction dominated by lattice calculations, w/ consolidated averages from many independent calculations
- New calculation of $a_\mu^{\text{LO-HVP}}$ to 0.46% ...
- ... + WP '25 indicates that SM confirmed to 0.35 ppm
- Decades of perturbative QED & EW calculations combined w/ fully nonperturbative QCD (+QED) ones predict g_μ to 0.35 ppb !
- Stringent, single test of the complete SM (all particles & interactions)

Conclusions and outlook

- $\tau^\pm \to \pi^\pm\pi^0\nu_\ell$ decays are making a comeback

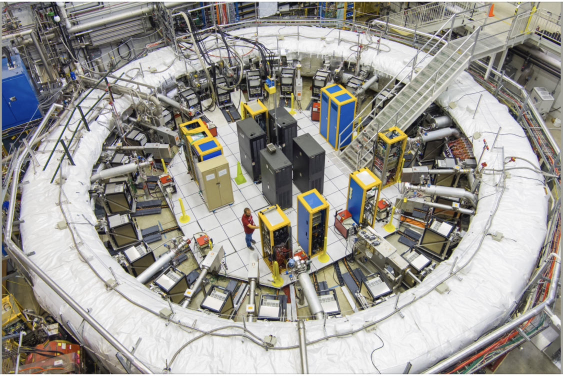
- Eagerly await

  - Fermilab $\sim$ 0.1? ppm measurement of a_μ next week!!
  - J-PARC entirely new method for a_μ measurement
  - More lattice results for complete $a_\mu^{\text{LO-HVP}}$ expected soon
  - New BABAR, KLOE, BES III, BELLE-II, SND-2 $e^+e^- \to$ hadrons analyses (and data) for $e^+e^- \to$ hadrons & $\tau^\pm \to \pi^\pm\pi^0\nu_\ell$
  - MUonE @ CERN for spacelike HVP

Tuesday, June 3 2025 @ 17h CET

Fermi National Accelerator Laboratory
6d ·

The muon g-2 experiment at Fermilab will unveil their final result on the muon's magnetic moment in a scientific seminar on June 3, 2025.
Learn more: https://muon-g-2.fnal.gov #gminus2



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a_μ^{expt} to 0.1? ppm