Status of the $(g-2)_\mu$ puzzle

Gilberto Colangelo

UNIVERSITÄT BERN

AEC
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FOR FUNDAMENTAL PHYSICS

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Outline

Introduction: $(g-2)_\mu$ in the Standard Model

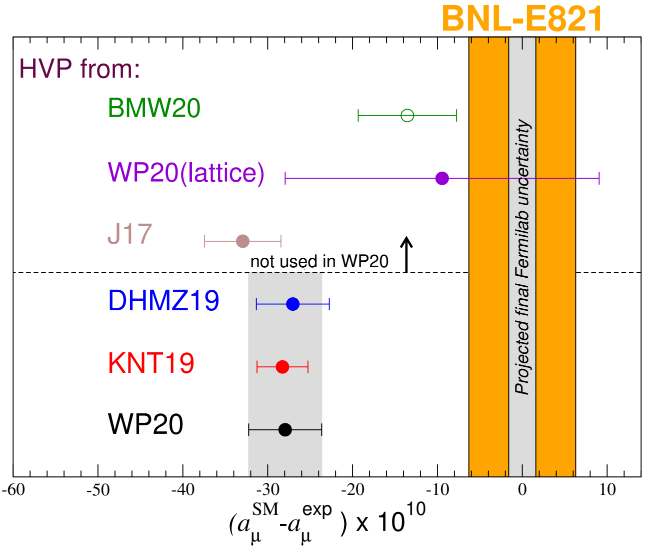
Hadronic light-by-light contribution
- Dispersive
- Lattice

Hadronic Vacuum Polarization contribution
- Dispersive

Conclusions and Outlook

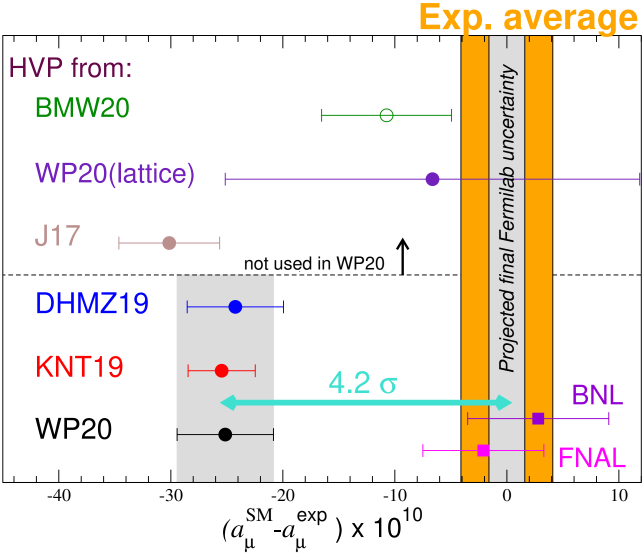
Present status of $(g-2)_\mu$: experiment vs SM

Before



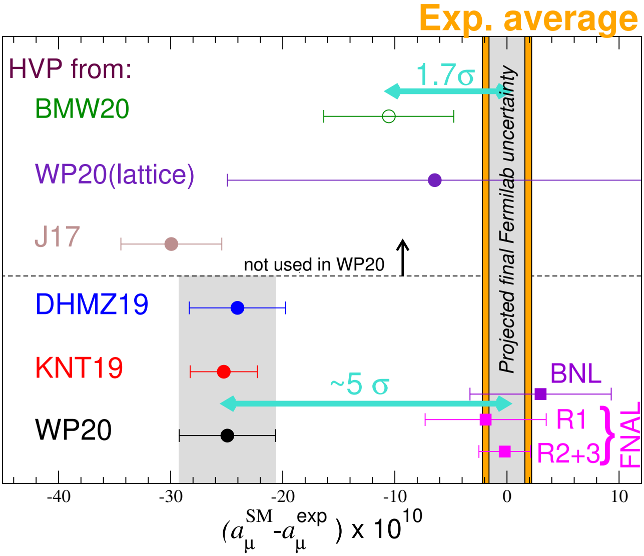
Present status of $(g-2)_\mu$: experiment vs SM

After the 2021 Fermilab result



Present status of $(g-2)_\mu$: experiment vs SM

After the 2023 Fermilab result



White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice, *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 059(22)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	249(48)

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HVP LO (lattice, *udsc*) → BMW(20)	7075(55)
HLbL (phenomenology)	92(19)
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White Paper (2020): $(g-2)_\mu$, experiment vs SM

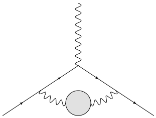
White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

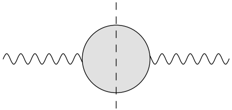
Muon $g-2$ Theory Initiative
Steering Committee:
GC
Michel Davier (vice-chair)
Aida El-Khadra (chair)
Martin Hoferichter
Laurent Lellouch
Christoph Lehner (vice-chair)
Tsutomu Mibe (J-PARC E34 experiment)
Lee Roberts (Fermilab E989 experiment)
Thomas Teubner
Hartmut Wittig

Rest of the talk: **White Paper 25**, `arXiv:2505.21476`

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$





- unitarity and analyticity $\Rightarrow$ dispersive approach
- $\Rightarrow$ direct relation to experiment: $\sigma_{\rm tot}(e^+e^- \to \text{hadrons})$
- e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
- alternative approach: lattice, now competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD, χQCD)

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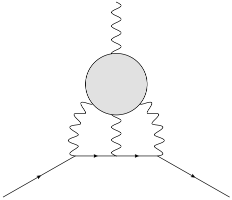
- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$
- Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)



- earlier: model-based—uncertainties difficult to quantify
- recently: dispersive approach $\Rightarrow$ data-driven, systematic treatment
- more recently: lattice QCD also competitive (Mainz, RBC/UKQCD, BMW)

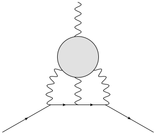
The 2×2 matrix of Hadronic Contributions

	dispersive	lattice
HLbL	??	??
HVP	??	??

The 2 $\times$ 2 matrix of Hadronic Contributions

	dispersive	lattice
HLbL	??	??
HVP	??	??

HLbL contribution: Master Formula



$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{48\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta: $Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$
The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$.

GC, Hoferichter, Procura, Stoffer (15)

- T_i: known kernel functions
- $\bar{\Pi}_i$ are amenable to a dispersive treatment:
  imaginary parts are related to measurable subprocesses

Improvements obtained with the dispersive approach

Contribution	PdRV(09) *Glasgow cons.*	N/JN(09)	J(17)	WP(20)	WP(25)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)	$91.2\ ^{+2.9}_{-2.4}$
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)	−9.1(1.0)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)	$65.7\ ^{+3.1}_{-2.6}$
scalars	–	–	–	} − 1(3) (scalars + tensors)	} 34.6(8.1) (scalars + tensors + axial vectors + u, d, s-loops / short-distance)
tensors	–	–	1.1(1)		
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)	
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)	
c-loop	2.3	–	2.3(2)	3(1)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)	103.3(8.8)

- significant reduction of uncertainties in the first three rows

  CHPS (17), Masjuan, Sánchez-Puertas (17) Hoferichter, Hoid, Holz, Kubis (18,24)

- more recent progress: resonances + short-distance constraints

  Lüdtke, Procura, Stoffer (23), Bijnens et al. (23,24), Hoferichter, Stoffer, Zillinger (25), Mager, (Cappiello), Leutgeb, Rebhan (23-25)

Improvements in the last rows

- large-Q_i (short-distance) behavior fixed by QCD

  @LO Melnikov, Vainshtein (04)

  @NLO Bijnens, Hermansson-Truedsson, (Laub), Rodríguez-Sánchez (20-24)

- smooth transition from hadronic to QCD OPE description is nontrivial Melnikov, Vainshtein (04), GC, Hagelstein, Hoferichter, Laub, Stoffer (20-21)

- axial-vectors (as well as tensors) play an important role and their dispersive treatment is subtle Hoferichter, Stoffer, Zillinger (24)

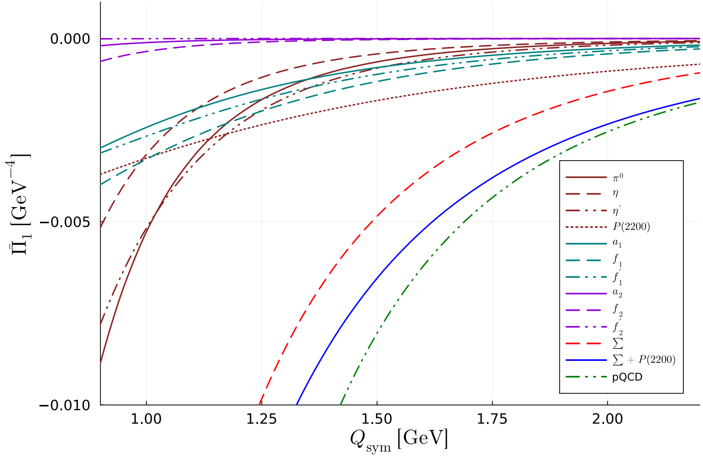
- hQCD has provided a useful guidance

  Mager, (Cappiello), Leutgeb, Rebhan (23-25)

  Regge models and DSE/BSE useful check too

  Masjuan, Roig, Sanchez-Puertas (22), Eichmann, Fischer, Weil, Williams (19-20)

Improvements in the last rows



Hoferichter, Stoffer, Zillinger (24)

Improvements in the last rows

$Q_0 = 1.5$ GeV

Region		Dispersive	hQCD	Regge	DSE/BSE
$Q_i > Q_0$		$6.2^{+0.2}_{-0.3}$	6.3(7)	4.8(1)	2.3(1.5)
Mixed	A, S, T	3.8(1.5)			
	OPE	10.9(0.8)			
	Effective pole	1.2			
	Sum	15.9(1.7)	13.5(2.4)	12.8(5)	10.1(3.0)
$Q_i < Q_0$	$A = f_1, f_1', a_1$	12.2(4.3)	13.1(1.5)	10.9(1.0)	8.6(2.6)
	$S = f_0(1370), a_0(1450)$	$-0.7(4)$			$-0.8(3)$
	$T = f_2, a_2$	$-2.5(8)$	2.9(4)		
	Other	2.0	8.0(9)	3.2(6)	2.8(6)
	Sum	11.0(4.4)	24.0(2.8)	14.1(1.2)	10.6(2.7)
Sum		33.2(4.7)	43.8(5.9)	31.7(1.6)	23.0(7.4)

Dispersive: Hoferichter, Stoffer, Zillinger (25), hQCD: Mager, (Cappiello), Leutgeb, Rebhan (25),

Regge: Masjuan, Roig, Sanchez-Puertas (22), DSE/BSE: Eichmann, Fischer, Weil, Williams (19,20)

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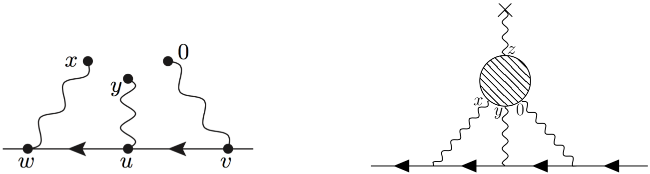
The 2 × 2 matrix of Hadronic Contributions

	dispersive	lattice
HLbL	✓	??
HVP	??	??

Master formula for HLbL lattice calculations



$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int d^4x\, d^4y\, \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(p,x,y)\; i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y),$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z\, z_\rho \left\langle j_\mu(x)\, j_\nu(y)\, j_\sigma(z)\, j_\lambda(0)\right\rangle_{\mathrm{QCD}} .$$

with $\mathcal{L}_{\ldots}(p,x,y)$ the analytically calculable QED kernel:

$$\mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(p,x,y) = \frac{1}{16m^2} \int d^4u d^4v d^4w G(w-x)\, G(u-y)\, G(v)\, e^{-ip\cdot(w-v)}$$
$$\times\, \mathrm{Tr}\left\{[\gamma_\rho,\gamma_\sigma]\,(-i\not{p}+m)\gamma_\mu S(w-u)\gamma_\nu S(u-v)\gamma_\lambda(-i\not{p}+m)\right\}$$

HLbL Lattice results

Collab.	$10^{11}\,a_\mu^{\mathrm{HLbL},\ell}$	$10^{11}\,a_\mu^{\mathrm{HLbL},s}$	$10^{11}\,a_\mu^{\mathrm{HLbL},c}$
Mainz/CLS	107.4(11.3)(9.2)(6.0)	−0.6(2.0)	2.8(5)
RBC/UKQCD	122.0(10.1)(9.5)	−0.0(2.2)(0.3)	–
BMW	122.6(11.6)	−1.7(8)(3)	2.73(27)

HLbL Lattice results

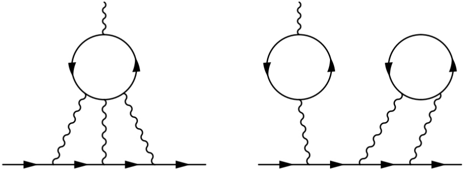


Figure 63: Illustration of the connected and (2+2) disconnected Wick-contraction diagrams for the quarks. The latter are not $SU(3)_F$ suppressed and turn out to be the dominant class of disconnected diagrams. Figure from Ref. [61].

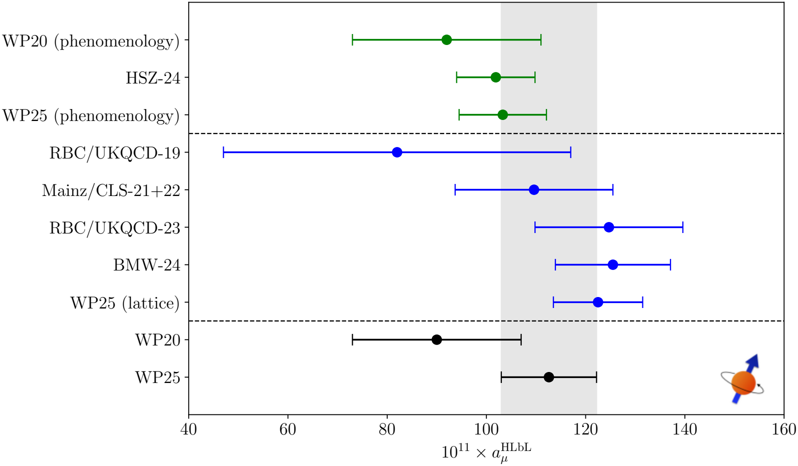
HLbL Lattice results

$[10^{-11}]$	RBC/UKQCD QED$_L$ [58]
$a_\mu^{\mathrm{HLbL},(4\ell)}$	$241.6(23.0)_{\mathrm{stat}}(51.1)_{\mathrm{syst}}$ [56.0]
$a_\mu^{\mathrm{HLbL},(2\ell+2\ell)}$	$-160.9(21.4)_{\mathrm{stat}}(39.9)_{\mathrm{syst}}$ [45.3]
$a_\mu^{\mathrm{HLbL},\ell}$	$82.3(30.7)_{\mathrm{stat}}(17.7)_{\mathrm{syst}}$ [35.4]

$[10^{-11}]$	Mainz/CLS [59]	RBC/UKQCD [61]	BMW [62]
$a_\mu^{\mathrm{HLbL},(4\ell)}$	see text	$257.0(13.3)_{\mathrm{stat}}(19.9)_{\mathrm{syst}}$ [23.9]	$220.1(13.0)_{\mathrm{stat}}(3.8)_{\mathrm{syst}}$
$a_\mu^{\mathrm{HLbL},(2\ell+2\ell)}$	see text	$-135.0(13.6)_{\mathrm{stat}}(12.1)_{\mathrm{syst}}$ [18.2]	$-101.1(12.4)_{\mathrm{stat}}(3.2)_{\mathrm{syst}}$
$a_\mu^{\text{no pion}}$	–	$54.0(9.4)_{\mathrm{stat}}(5.3)_{\mathrm{syst}}$ [10.8]	see text
$a_\mu^{\mathrm{HLbL},\ell}$	107.4(11.3)(9.2)(6.0)[15.8]	$122.0(10.1)_{\mathrm{stat}}(9.5)_{\mathrm{syst}}$ [13.8]	$122.6(11.5)_{\mathrm{stat}}(1.8)_{\mathrm{syst}}$

Table 31: Results for the light-quark contributions in various linear combinations, as well as the total $a_\mu^{\mathrm{HLbL},\ell}$.

HLbL: comparison dispersive/lattice

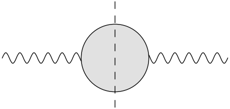


The 2 × 2 matrix of Hadronic Contributions

	dispersive	lattice
HLbL	✓	✓
HVP	??	??

HVP contribution: Master Formula

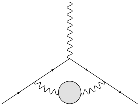
Unitarity relation: simple, same for all intermediate states



$$\mathrm{Im}\bar{\Pi}(q^2) \propto \sigma(e^+e^- \to \mathrm{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

Analyticity $\left[\bar{\Pi}(q^2) = \frac{q^2}{\pi}\int ds \frac{\mathrm{Im}\bar{\Pi}(s)}{s(s-q^2)}\right]$ $\Rightarrow$ Master formula for HVP

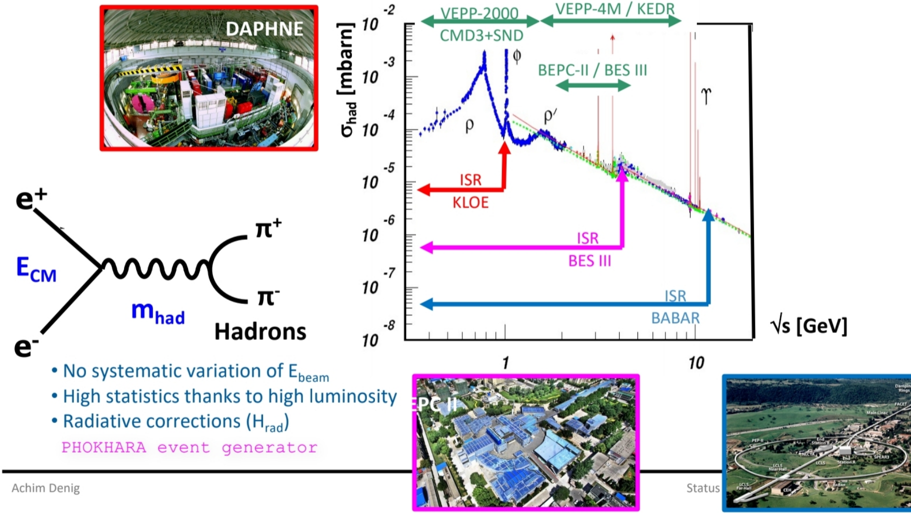
Bouchiat, Michel (61)



$$\Leftrightarrow \qquad a_\mu^{\mathrm{hvp}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}K(s)R(s)$$

$K(s)$ known, depends on m_μ and $K(s) \sim \frac{1}{s}$ for large s

HVP contribution: Master Formula



Slide by Achim Denig, MUonE Workshop, June 2024

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(3.38)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(1.45)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.30)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.55)	18.15(74)	−0.12
K^+K^-	23.08(0.44)	23.00(22)	0.08
K_SK_L	12.82(0.24)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.10)	4.58(10)	−0.17
Sum of the above	626.08(3.90)	623.62(2.27)	2.46
$[1.8, 3.7]\,\mathrm{GeV}$ (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)\,\mathrm{GeV}$	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(4.0)	692.8(2.4)	1.2

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For the dominant $\pi\pi$ channel more theory input can be used

Comparison between DHMZ19 and KNT19

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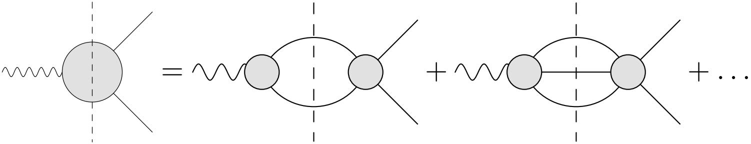
For the 3π and KK channels also Hoferichter, Hoid, Kubis, Stamen, Hariharan, Stoffer

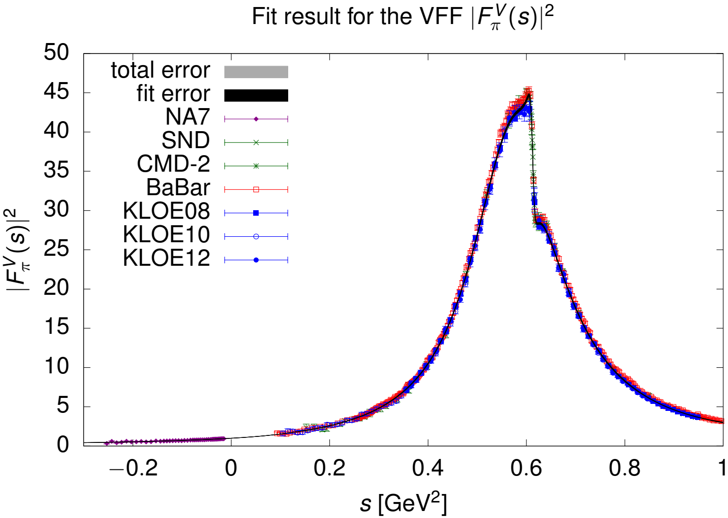
Omnès representation including isospin breaking



$$F_V(s) = \Omega_{\pi\pi}(s) \cdot G_\omega(s) \cdot \Omega_{\text{in}}(s)$$

Fit results

GC, Hoferichter, Stoffer (18)



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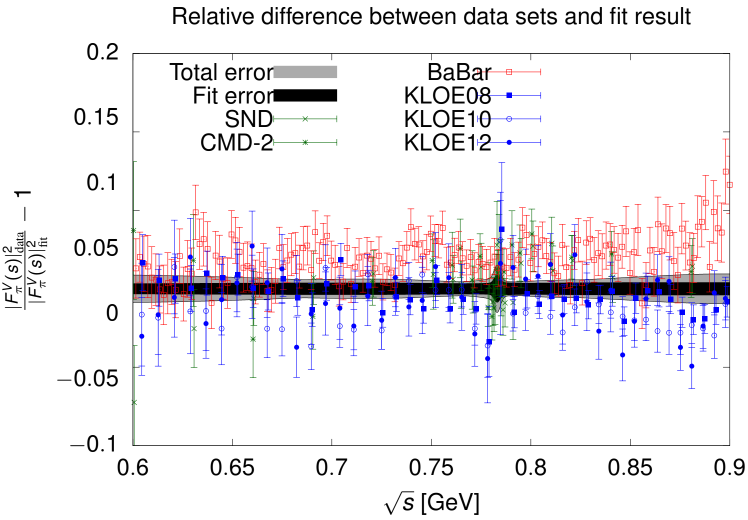

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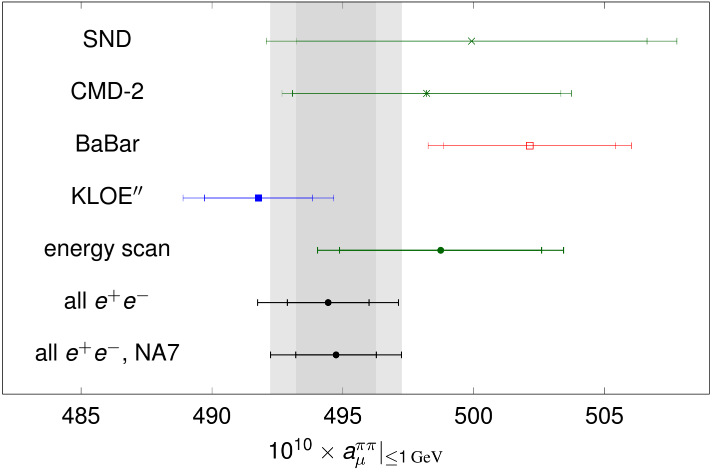
Result for $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$ from the VFF fits to single experiments and combinations



Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been so combined:

HHK=Hoferichter, Hoid, Kubis

- central values are obtained by simple averages (for each channel and mass range)
- the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- $1/2$ difference DHMZ$-$KNT (or BABAR$-$KLOE in the 2π channel, if larger) is added to the uncertainty

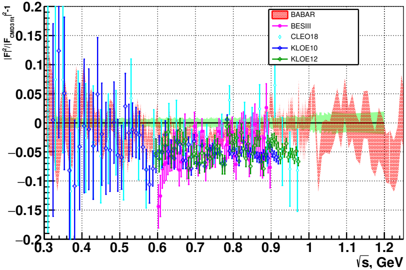
Final result:

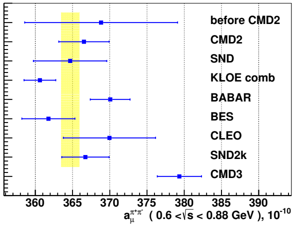
$$a_\mu^{\text{HVP, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10}$$
$$= 693.1(4.0) \times 10^{-10}$$

WP(20)

CMD-3 measurement of $e^+e^- \to \pi^+\pi^-$

F. Ignatov et al., CMD-3, arXiv:2302.08834

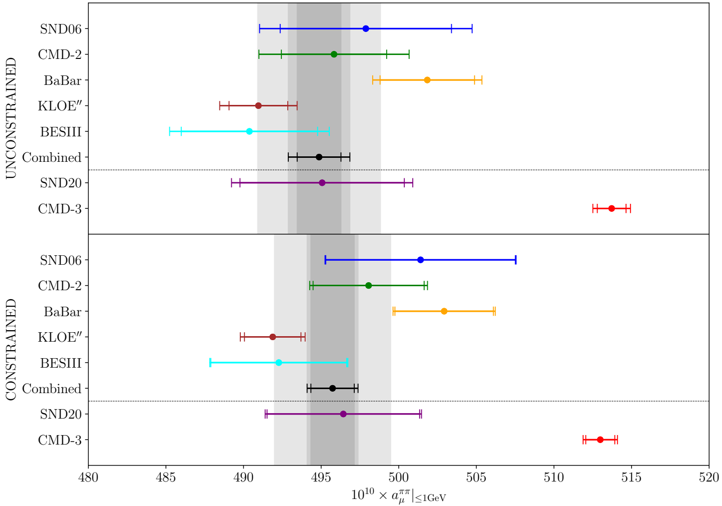




The comparison of pion form factor measured in this work with the most recent ISR experiments (BABAR [21], KLOE [18, 19], BES [22]) is shown in Fig. 34. The comparison with the most precise previous energy scan experiments (CMD-2 [12, 13, 14, 15], SND [16] at the VEPP-2M and SND [23] at the VEPP-2000) is shown in Fig. 35. The new result generally shows larger pion form factor in the whole energy range under discussion. The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of ρ-meson ($\sqrt{s} = 0.6 - 0.75$ GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. The source of this difference is unknown at the moment.

Comparison between CMD-3 and other experiments

Leplumey and Stoffer, arXiv:2501.09643



Comparison between CMD-3 and other experiments

Leplumey and Stoffer, arXiv:2501.09643

Discrepancy w/ CMD-3	$a_\mu^{\pi\pi}\big\|_{\leq 1\,\text{GeV}}$ unconstrained	constrained
SND06	2.0σ	1.8σ
CMD-2	3.3σ	3.7σ
BaBar	2.9σ	2.8σ
KLOE″	7.4σ	8.9σ
BESIII	4.2σ	4.5σ
SND20	3.0σ	3.2σ
Combination	4.4σ [7.3σ]	4.4σ [8.1σ]

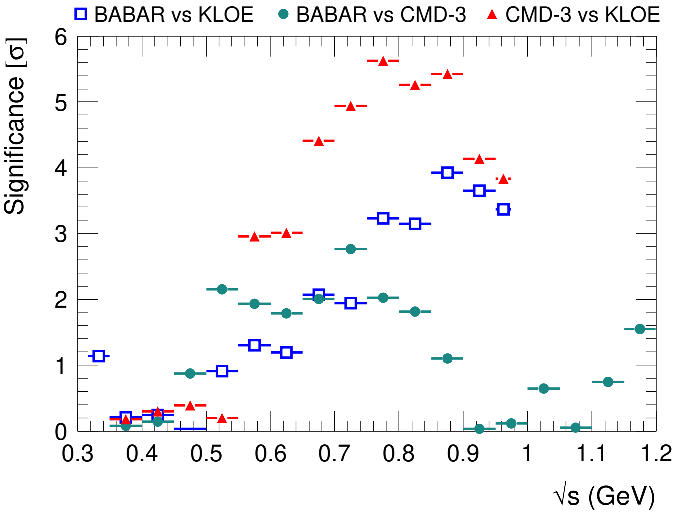
Uncertainties in brackets exclude KLOE-BaBar systematic eff.

Combination: NA7 + all data sets other than SND20 and CMD-3

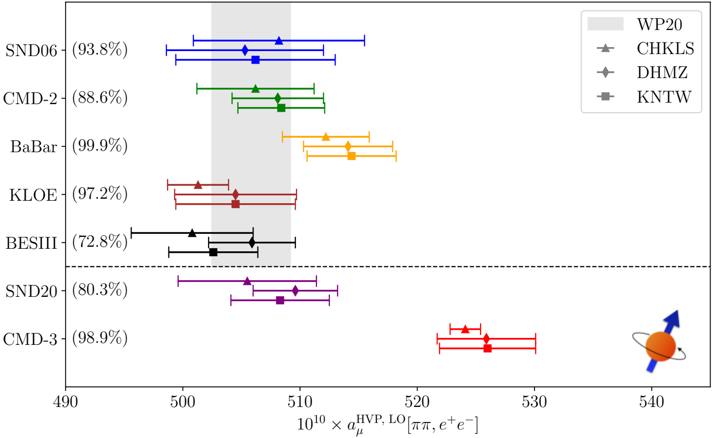
Comparison between CMD-3 and other experiments

Comparison according to DHLMZ

Davier, Hoecker, Lutz, Malaescu, Zhang (23)



Comparison between different e^+e^- experiments



Puzzle not just data-driven vs lattice

Di Luzio, Masiero, Paradisi, Passera (22)

Updates on IB corrections from $(g-2)_7$ @KEK 2024

- KLOE and BESIII have rebutted claims that higher-order radiative corrections might have solved the puzzle

  talks by A. Denig and G. Venanzoni @KEK24

- claim that initial/final radiation interference on the box diagram might impact significantly radiative-return experiments is under scrutiny F. Ignatov @STRONG2020 Zürich (23)

- reconsideration of τ decays as input for HVP in WP25

  TI Virtual workshops on Nov. 8 and Dec. 9

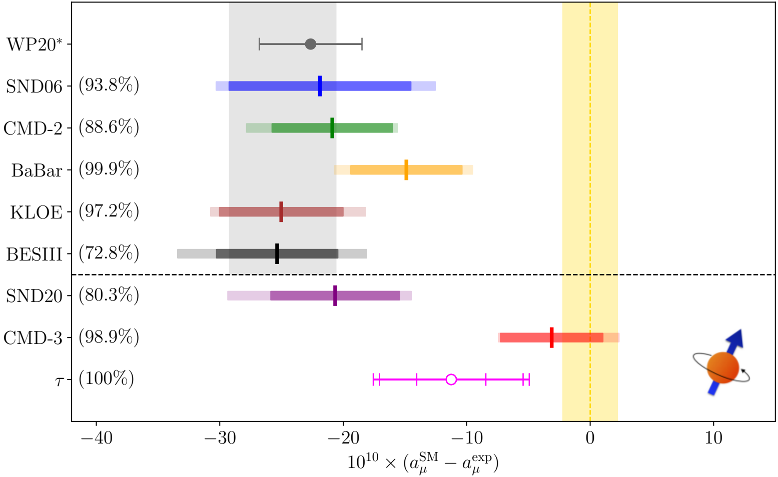
- analysis of IB for τ decays on the lattice is ongoing

  talk by M. Bruno @KEK24

- dispersive analysis of IB for τ decays is ongoing

  talk by M. Cottini @KEK24

Comparison between e^+e^-- and τ-based HVP



The 2 $\times$ 2 matrix of Hadronic Contributions

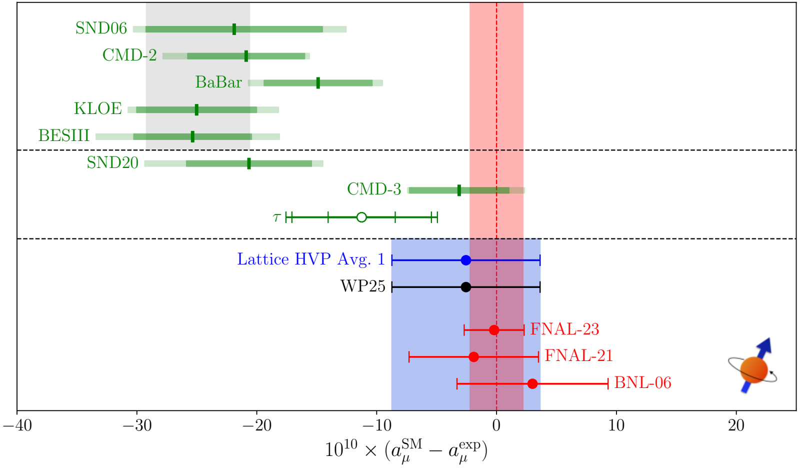
	dispersive	lattice
HLbL	✓	✓
HVP	?!?!?!	$\rightarrow$ talk by L. Lellouch

The 2 × 2 matrix of Hadronic Contributions

	dispersive	lattice
HLbL	✓	✓
HVP	?!?!?!	✓

More details in WP25 : `arXiv:2505.21476`

Summary plot WP25



Summary table WP25

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-, τ)	no estimates provided
HVP NLO (e^+e^-)	$-99.6(1.3)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice)	7132(61)
HLbL (phenomenology)	103.3(8.8)
HLbL NLO (phenomenology)	2.6(6)
HLbL (lattice)	122.5(9.0)
HLbL (phenomenology + lattice)	112.6(9.6)
QED	116 584 718.8(2)
EW	154.4(4)
HVP LO (lattice) + HVP N(N)LO (e^+e^-)	7045(61)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)
Total SM Value	116 592 033(62)
Experiment	116 592 059(22)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	26(66)

Summary Table WP20

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
EW	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 059(22)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	249(48)

Conclusions

- Dispersive evaluation of HLbL contribution:
  WP20 20% $\rightarrow$ WP25 $\sim$ 10% accuracy.
  Lattice calculations [Mainz/CLS(21), RBC/UKQCD(23), BMW24] agree with it

- WP20: 0.6% error of data-driven HVP contribution dominated the theory uncertainty

  Main contribution: $\pi\pi$ (<1 GeV) based on [CMD-2, SND, BaBar, KLOE, BES-III]
  Puzzle: results by CMD-3 (23) significantly higher!

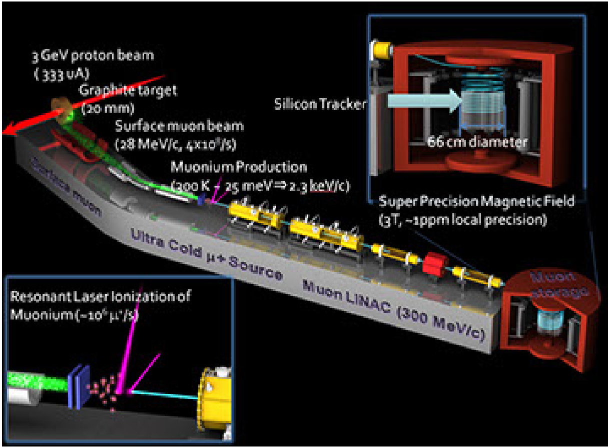
- Lattice calculations of HVP provide a consistent picture: agree with each other and with CMD-3 for all possible sub-quantities and the total: $a_\mu^{\rm SM}$(WP25) relies on Lattice HVP

$$a_\mu^{\rm SM} - a_\mu^{\rm exp} = 26(66) \cdot 10^{-11}$$

Outlook

- The Fermilab experiment aims to reduce the BNL uncertainty by a factor four $\Rightarrow$ final result will be announced next week
- Improvements on the SM theory/data side:
  - Situation for HVP data-driven urgently needs to be clarified:
    - New CMD-3 result—after thorough scrutiny—is a puzzle
    - Forthcoming measur./analyses: BaBar, Belle II, BESIII, KLOE, SND
    - Model-independent evaluation of RadCorr underway
    - MuonE will provide an alternative way to measure HVP
  - HVP lattice: very good agreement at present; more complete calculations are coming [Fermilab-MILC-HPQCD, ETMC]; some contributions (LD, IB, disc) need to be improved
  - HLbL: goal of $\sim$ 10% uncertainty (data-driven and lattice) has been achieved. Further improvements underway

Future: Muon $g-2$/EDM experiment @ J-PARC



Credit: J-PARC