Naturalness and Generalized Symmetries

Seth Koren

they/them

University of Notre Dame

PLANCK
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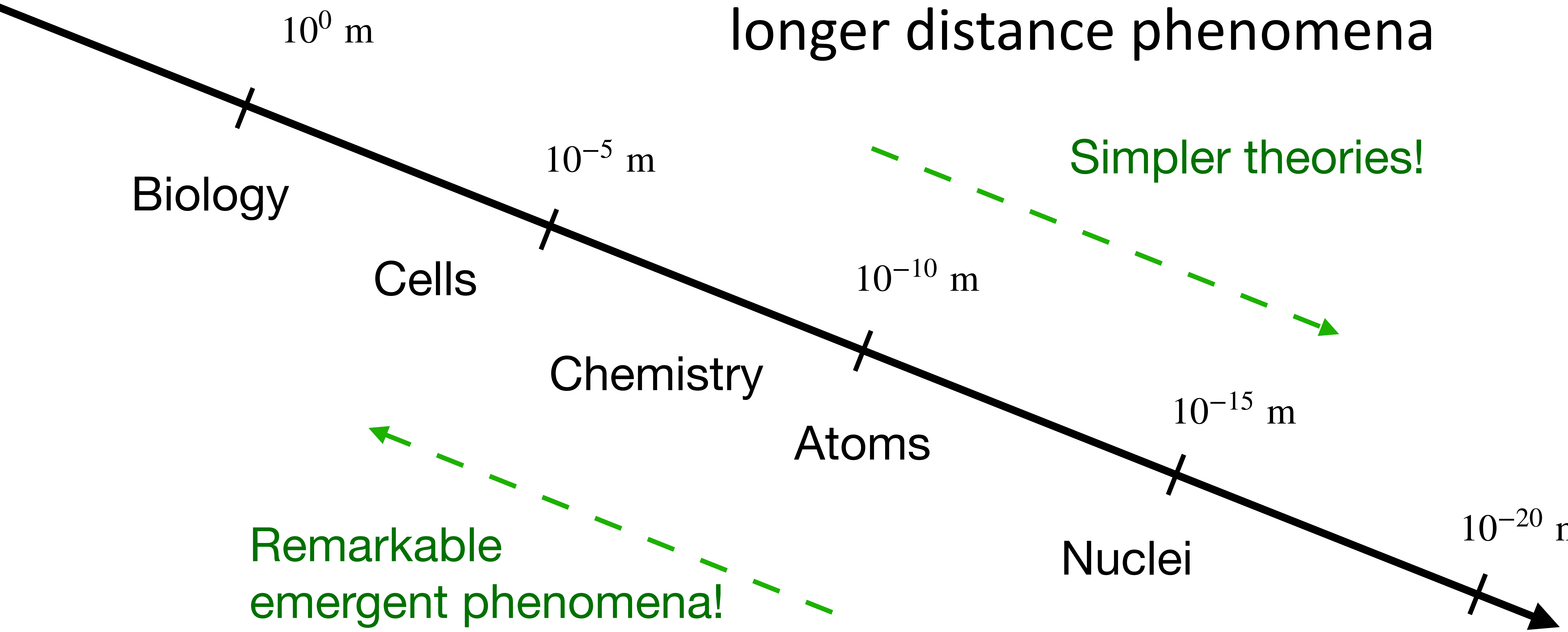
Padova, Italy

The strategy of reductionism: At smaller distances there is a simpler, more predictive theory of longer distance phenomena



The Standard Model of Particle Physics

At each stage we need both theory and experiment to determine the shorter distance theory!

Important bottom-up guidance comes from symmetries!

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	L	$\bar{e}$
$U(3)_L$	$\overline{\square}$	$-$
$U(3)_e$	$-$	$\square$

Nonzero SM yukawas explicitly break $U(3)^2$ flavor symmetry

$$\mathscr{L}_{\rm SM} \supset (y_e)^i{}_j H L_i \bar{e}^j$$

$(y_e)^i{}_j$ nongeneric; e.g. eigenvalues $y_e \ll y_\mu \ll y_\tau \ll 1$

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$U(3)_e$	$-$	$\square$	$\overline{\square}$

’t Hooft: Assign a ‘spurious’ charge. This formally restores invariance, and QFT doesn’t ‘know’ this charge assignment is fake

Simply by covariance under this spurious symmetry!

$$\delta(y_e)^i{}_j \propto (y_e)^i{}_j$$

Technical Naturalness

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But no hints as to how you ended up with small parameters!

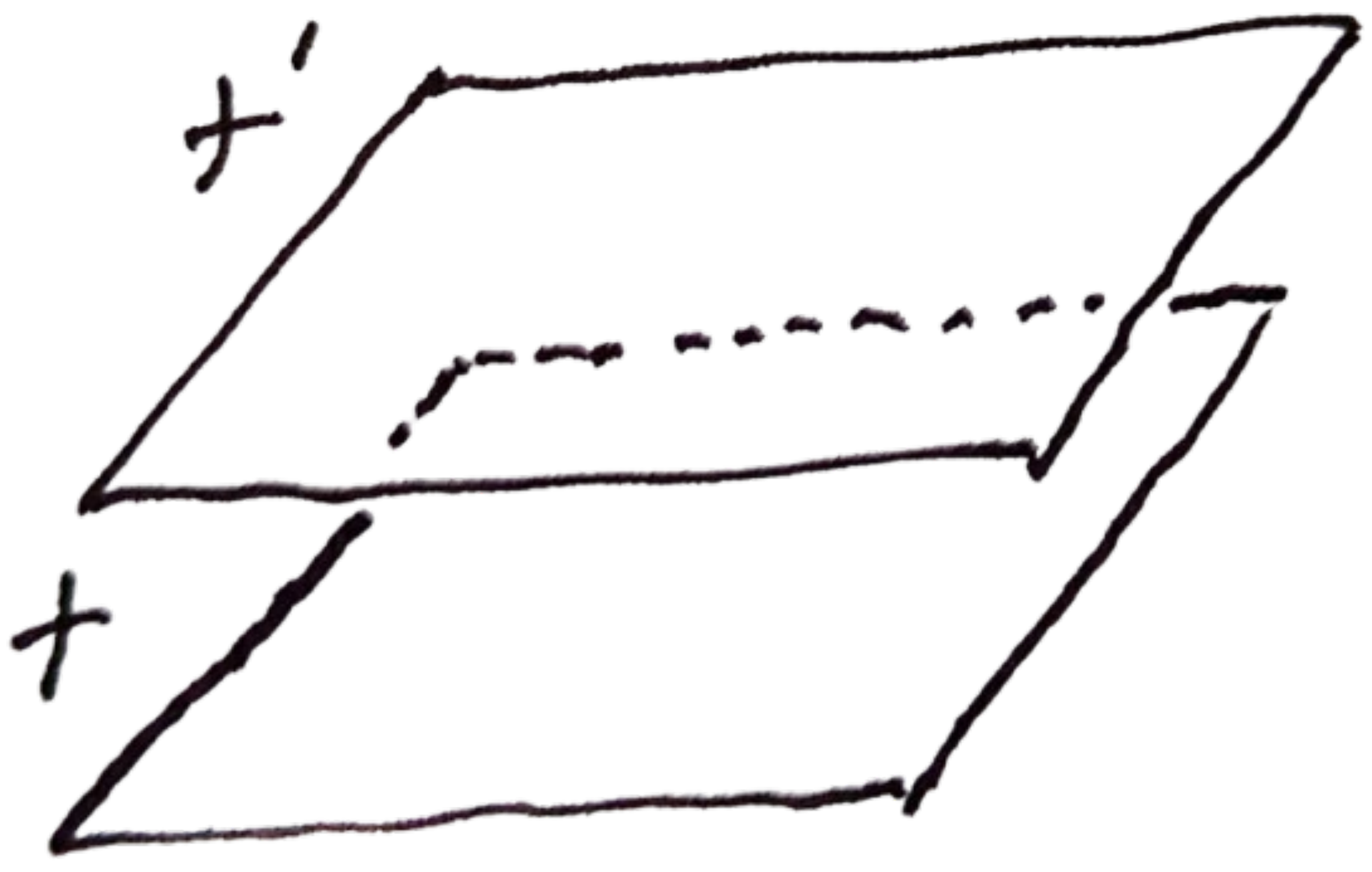
Could we get any further guidance from symmetries?

Global Symmetry of Point Operators

Noether 1918: A continuous global symmetry gives a current J_μ with $\partial_\mu J^\mu = \overrightarrow{\nabla} \cdot \vec{J} - \dot{J}_0 = 0$, and

$$Q(\mathscr{M}_{\text{space}}, t) = \int_{\mathscr{M}_{\text{space}}} J^0 dx^1 dx^2 dx^3$$

is conserved under time translations



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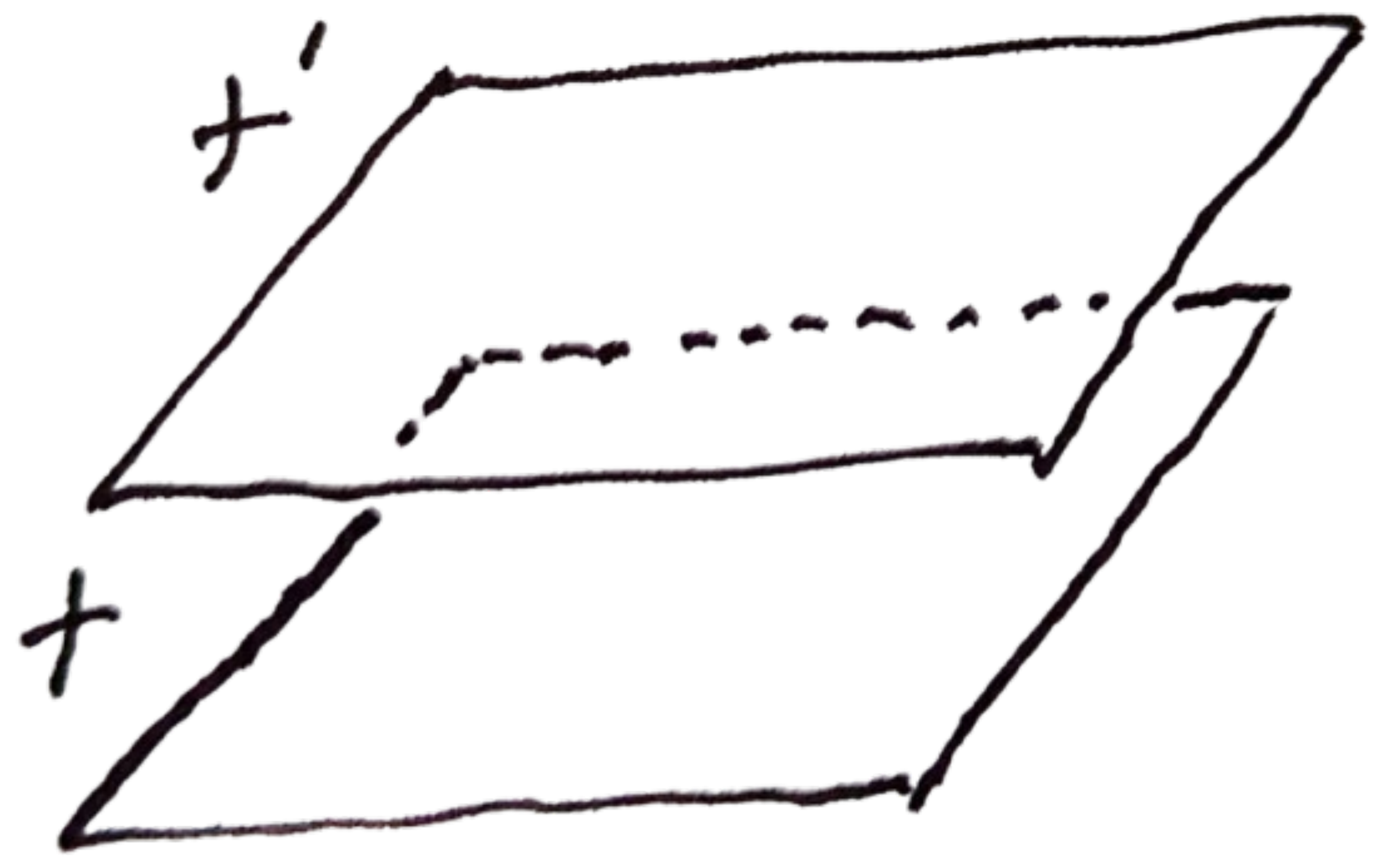

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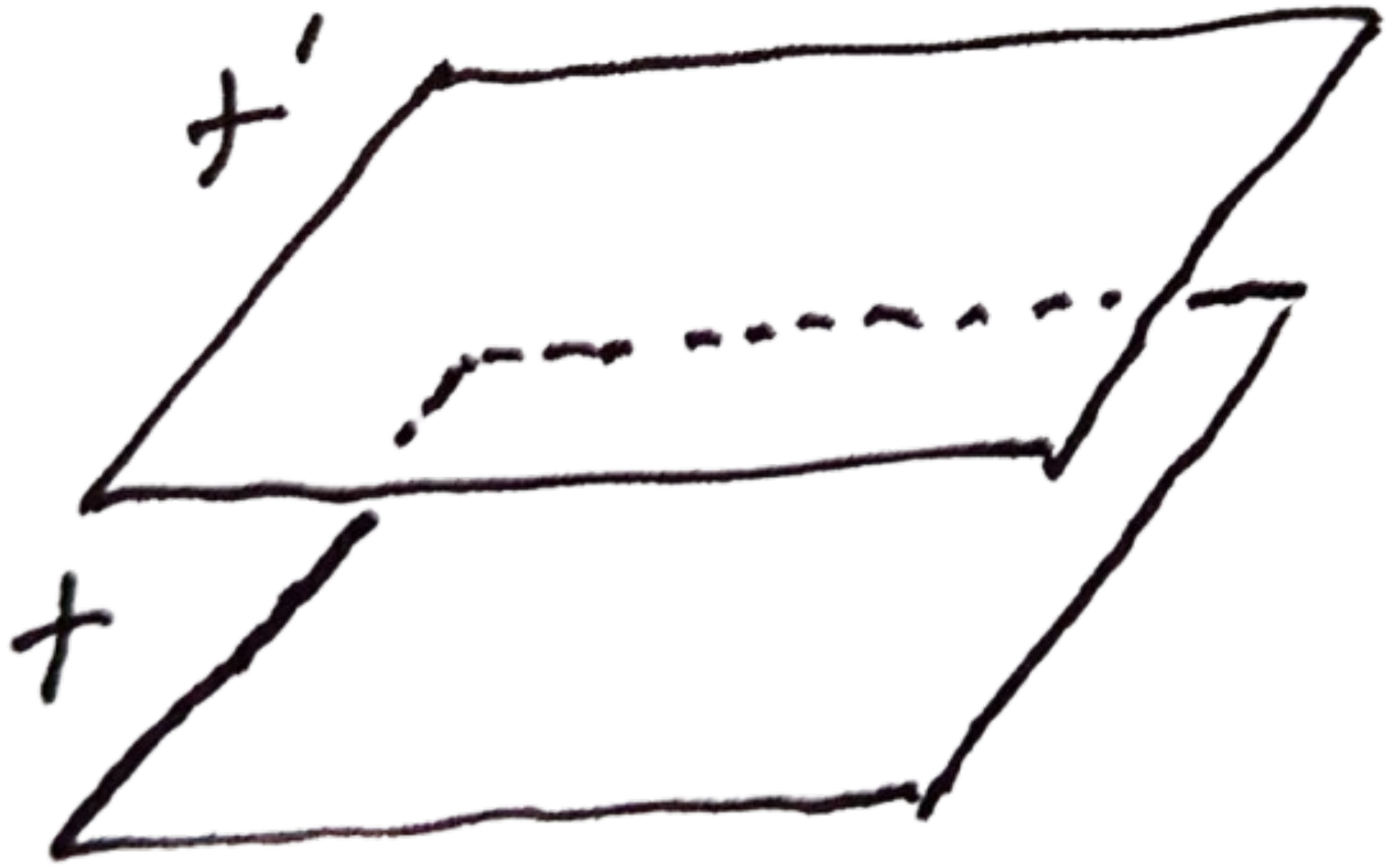


But why is this so non-covariant?

We can do better with some basic ideas from *topology*.

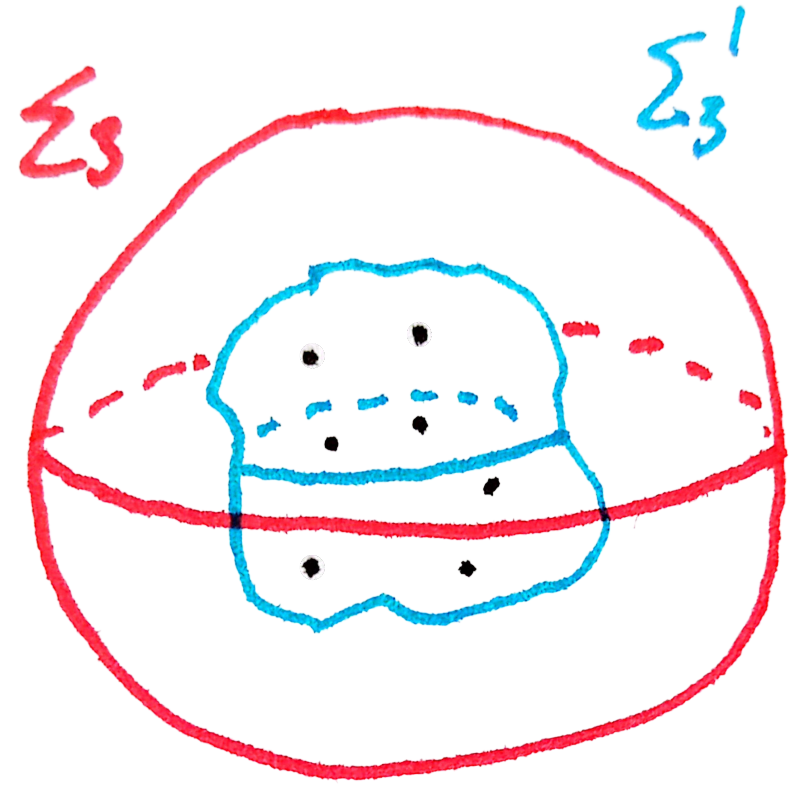
Recall topology deals with properties that do not change under smooth deformations: 'global' rather than 'local'

Henry Segerman *Topology Joke*

Symmetry detecting operators

Generalized Noether surfaces detect symmetry

charges of point operators $Q(\Sigma_3) = \int_{\Sigma_3} J^\mu \hat{n}_\mu d^3x$



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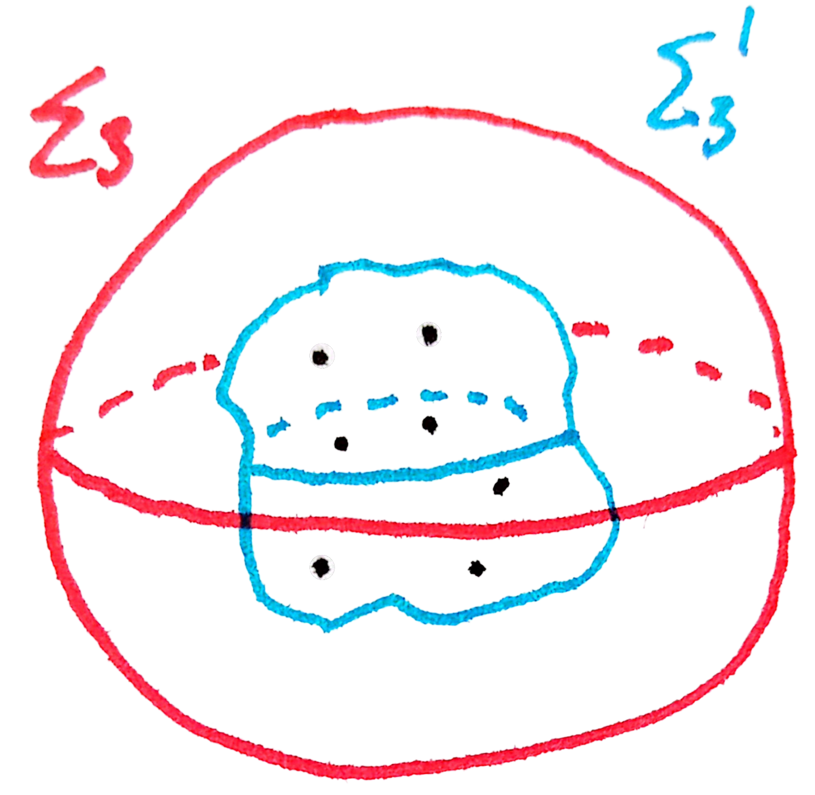
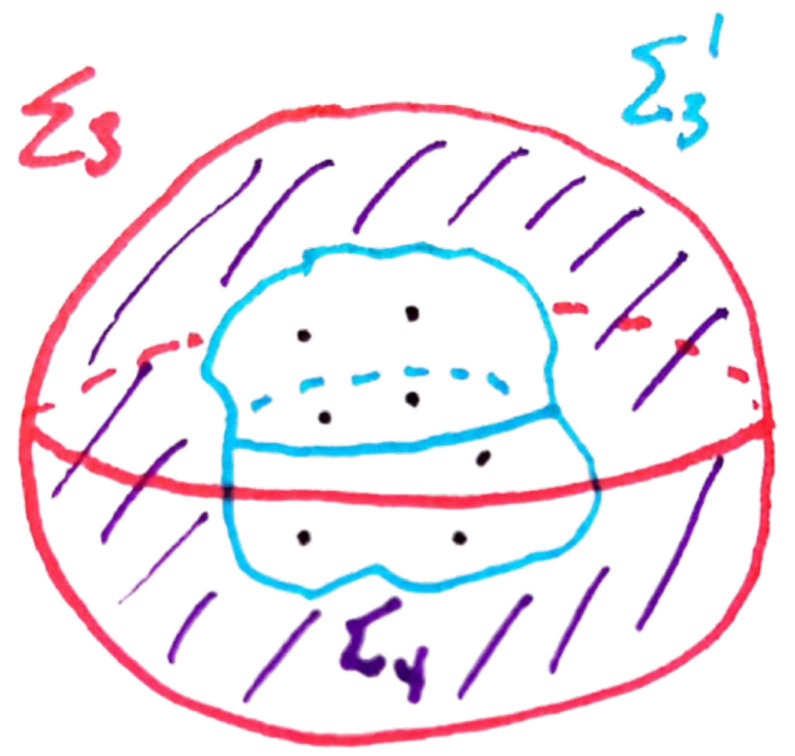


Time translation upgraded to topological invariance

$$Q(\Sigma_3) - Q(\Sigma_3') = \int_{\Sigma_3} J^\mu \hat{n}_\mu d^3x - \int_{\Sigma_3'} J^\mu \hat{n}_\mu d^3x = \int_{\Sigma_4} \partial_\mu J^\mu d^4x = 0$$



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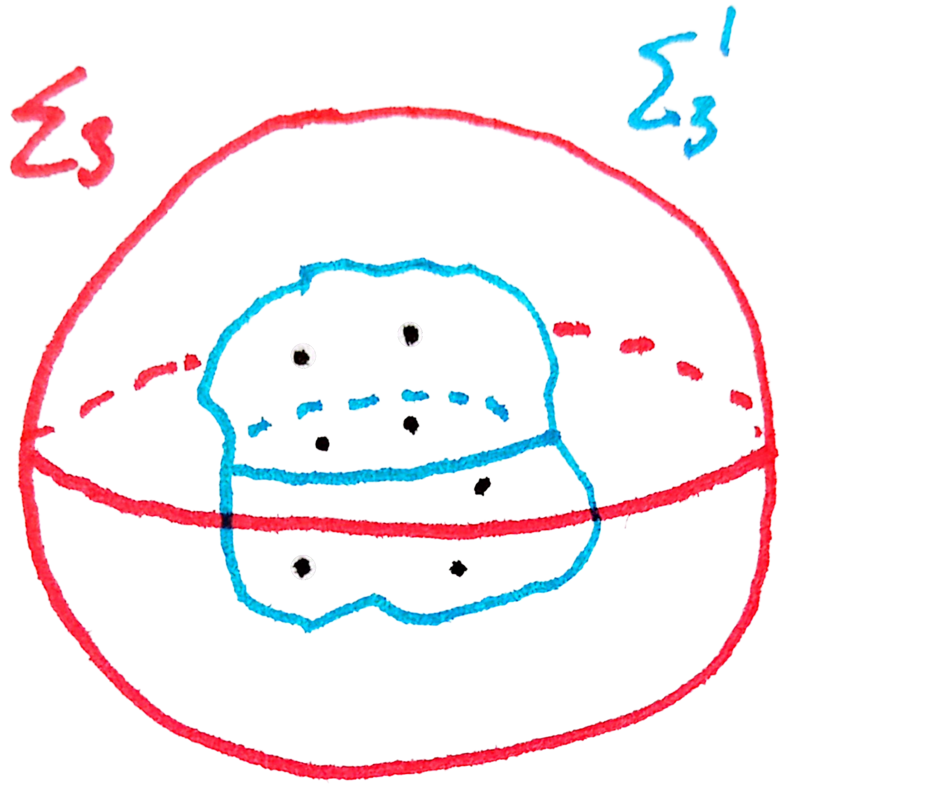

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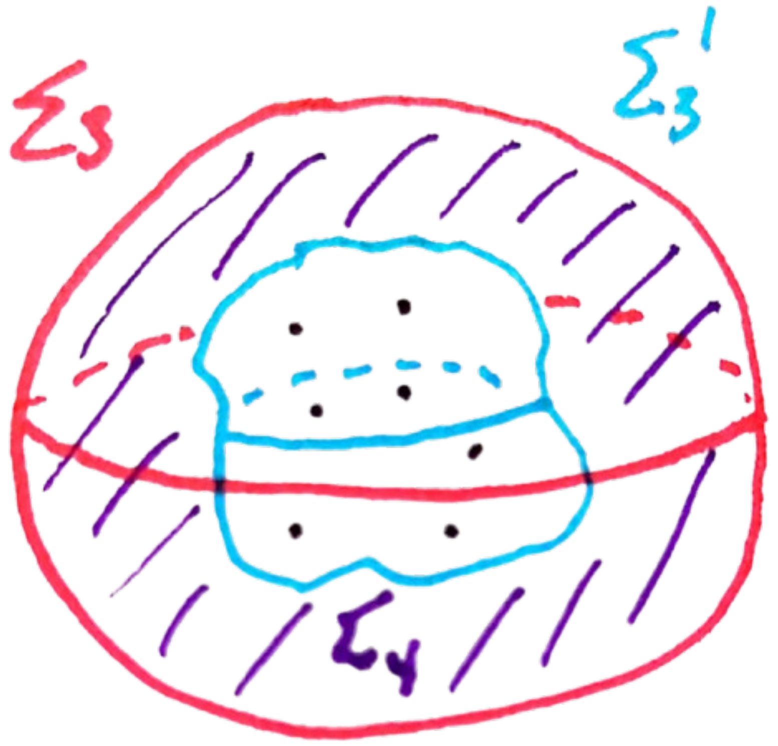

Symmetry ~ Topological surface operator!

Gaiotto, Kapustin, Seiberg, Willett '14

Generalized Noether Charges

What about topological operators on $d < 3$-dimensional surfaces?

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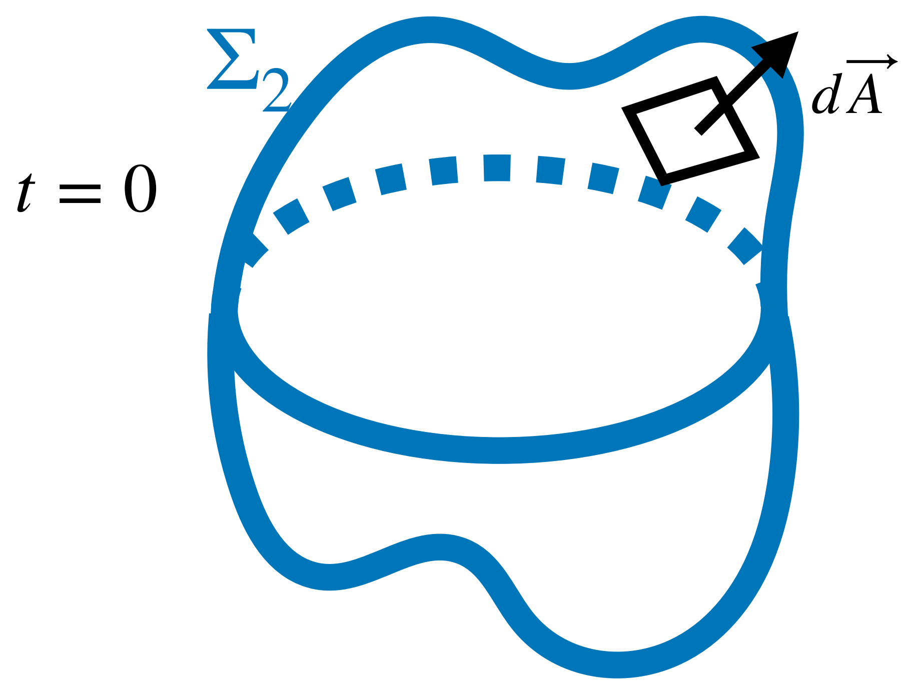
You're familiar with a topological 2-surface operator in Maxwell theory!



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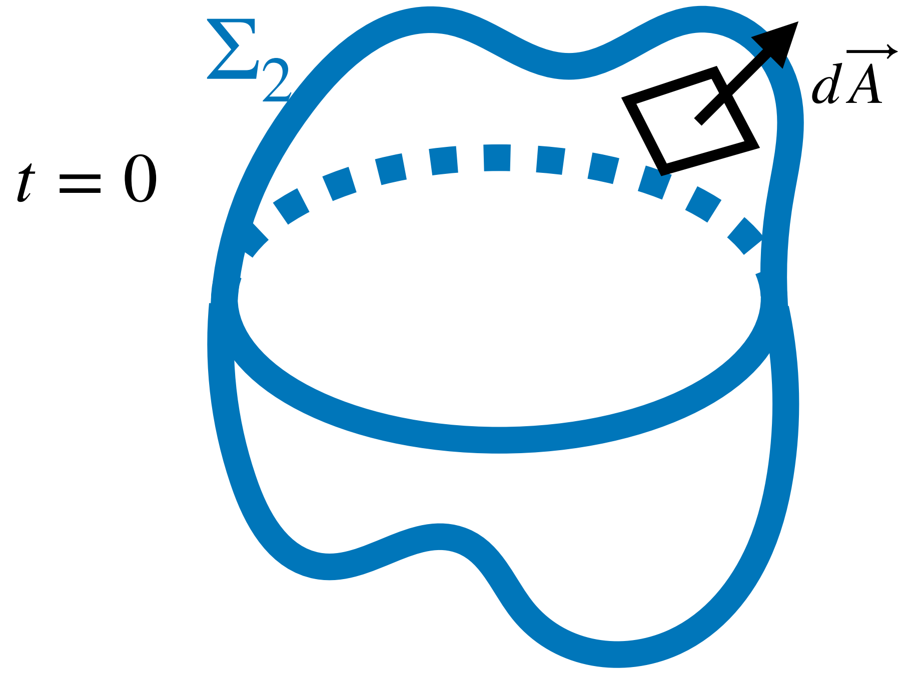

$$Q(\Sigma_2) = \int_{\Sigma_2} \vec{E} \cdot d\vec{A} = \int_{\Sigma_2} F_{\mu\nu}\hat{n}^\mu \hat{n}^\nu d^2x$$

$$Q(\Sigma_2) - Q(\Sigma_2') = \int_{\Sigma_3} \partial_\mu F^{\mu\nu}\hat{n}_\nu d^3x = 0$$

Gauss' law is the existence of a topological 2-surface ~ global symmetry!

What should be the charged object be now?

What dimensional operators could ‘link with’ this 2-dim surface?

$$\text{Link}\left(\Sigma_p, \Sigma_{d-p-1}\right) \in \mathbb{Z}$$

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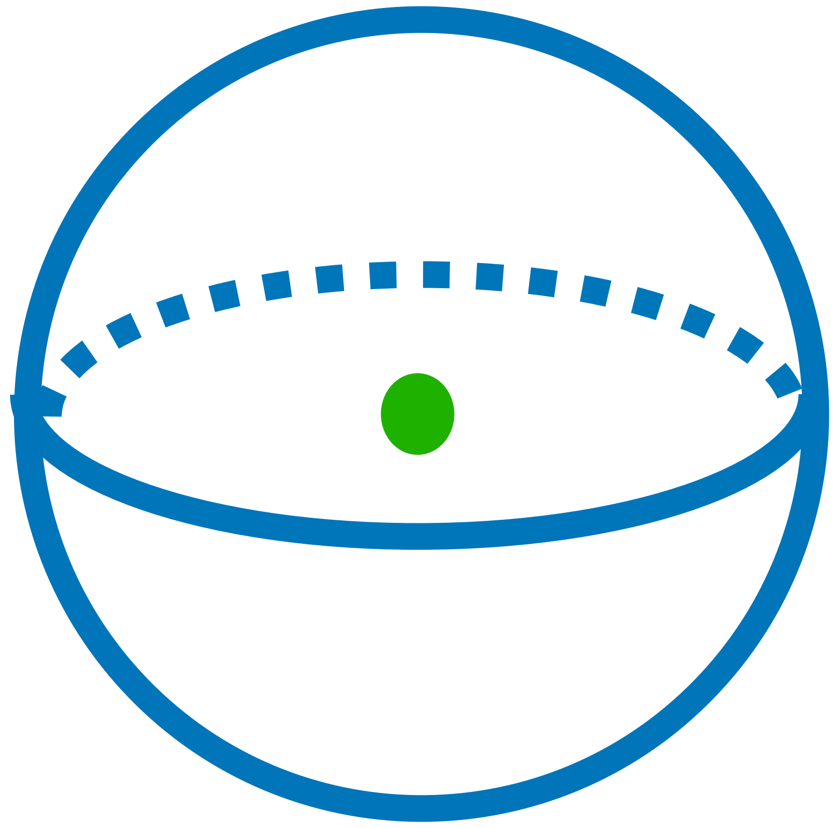
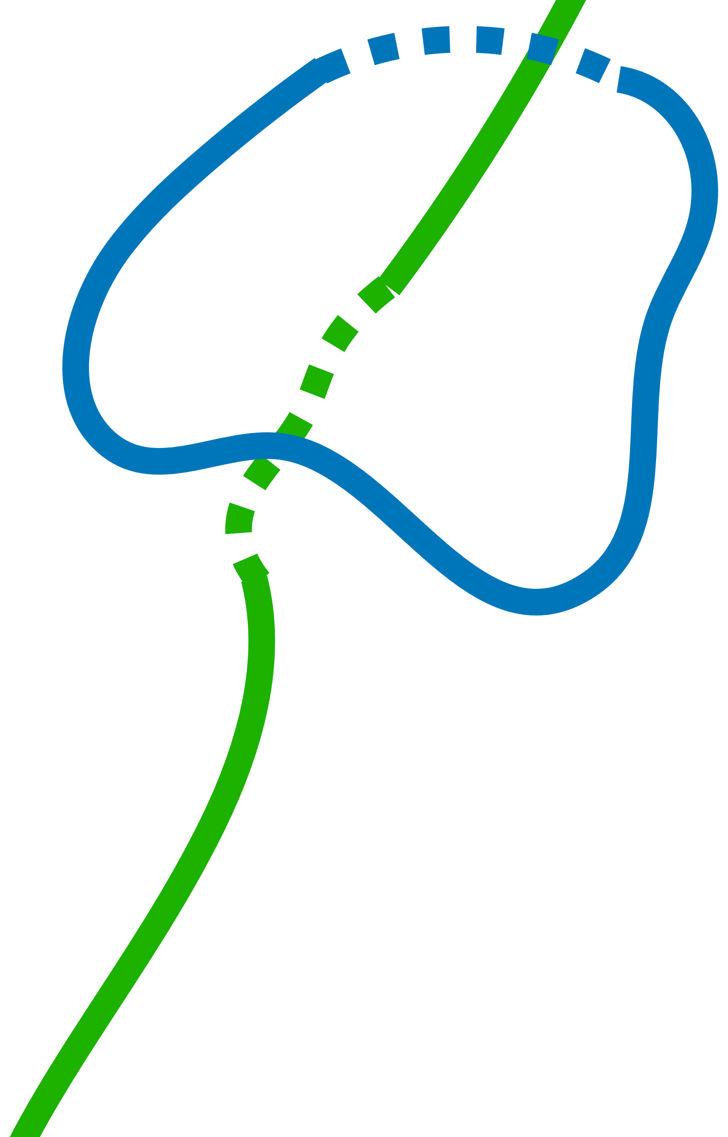
Digression because d=3 space is easier for us primates

$p = 0, d - p - 1 = 2$



$p = 1, d - p - 1 = 1$



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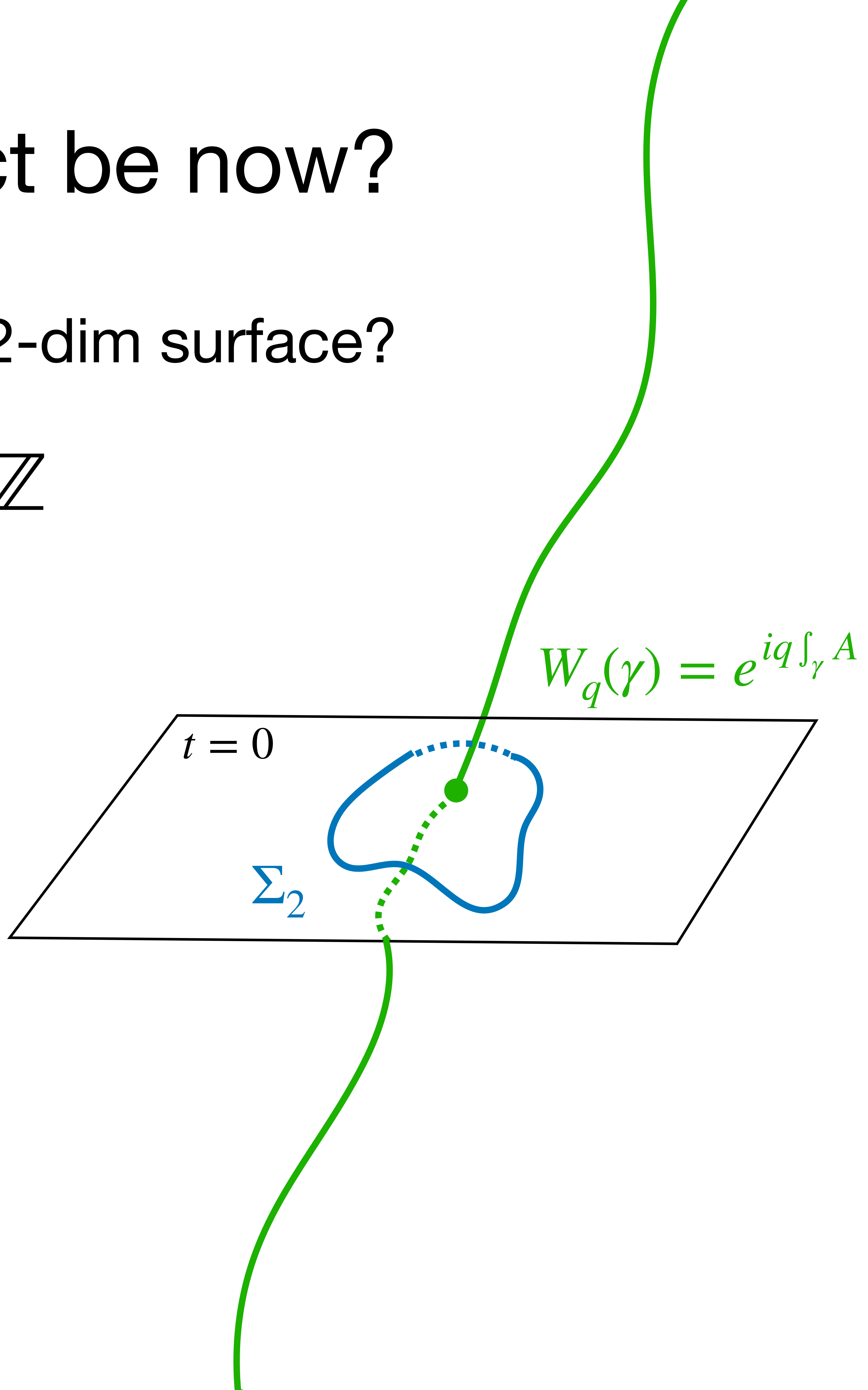
It's a Wilson line!

$$W_q(\gamma) = e^{iq\int_\gamma A}$$

(worldline γ of charge q
object in limit $m \to \infty$)



Core conceptual point of generalized symmetries

Symmetries can be understood as the existence of some surface operators that are topologically invariant.

∃ topological surfaces

Noether charge

Gauss' law

AB effect

Discrete symmetries

New notions of symmetry you can't understand from the Lagrangian

Some basic intros: Gomes '23, Brennan & Hong '23, Iqbal '24, my papers
More formal: Schäfer-Nameki '23, Bhardwaj et al. '23, Simons Lectures '24

Generalized Symmetry Breaking

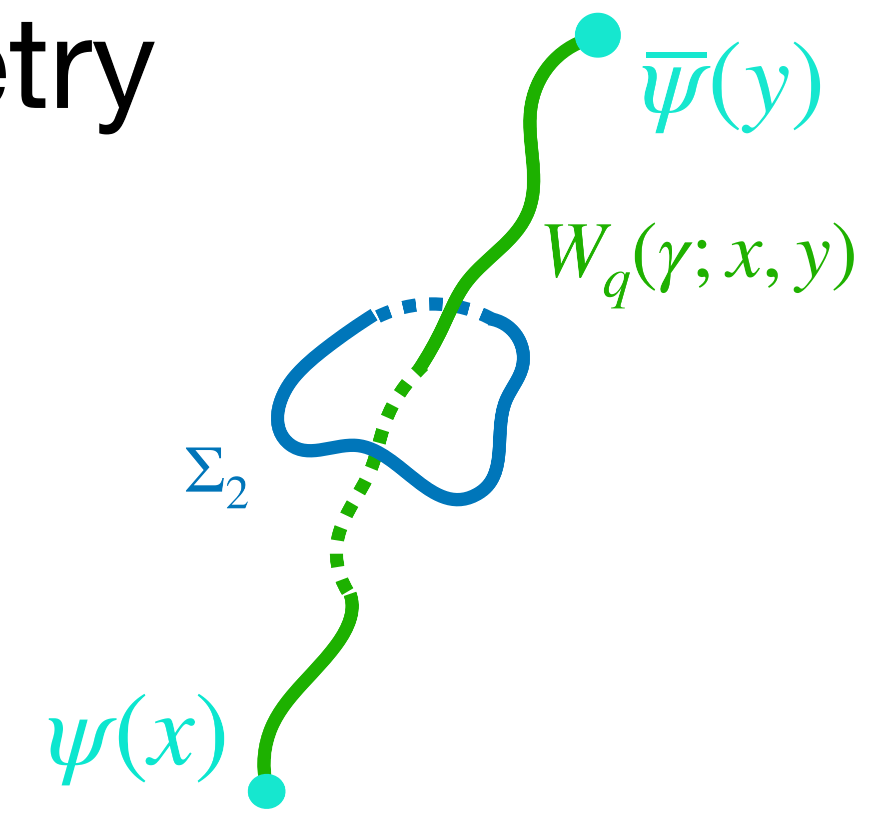
Extended operator symmetry breaking is qualitatively different from local operator symmetry breaking!

Local operator symmetry

Explicit breaking from charged local operators in $\mathscr{L}$
E.g. $\mathscr{L} \supset c_{ij}(\tilde{H}L_i)(\tilde{H}L_j)$

Extended operator symmetry

Explicit breaking when new dynamical degrees of freedom in UV



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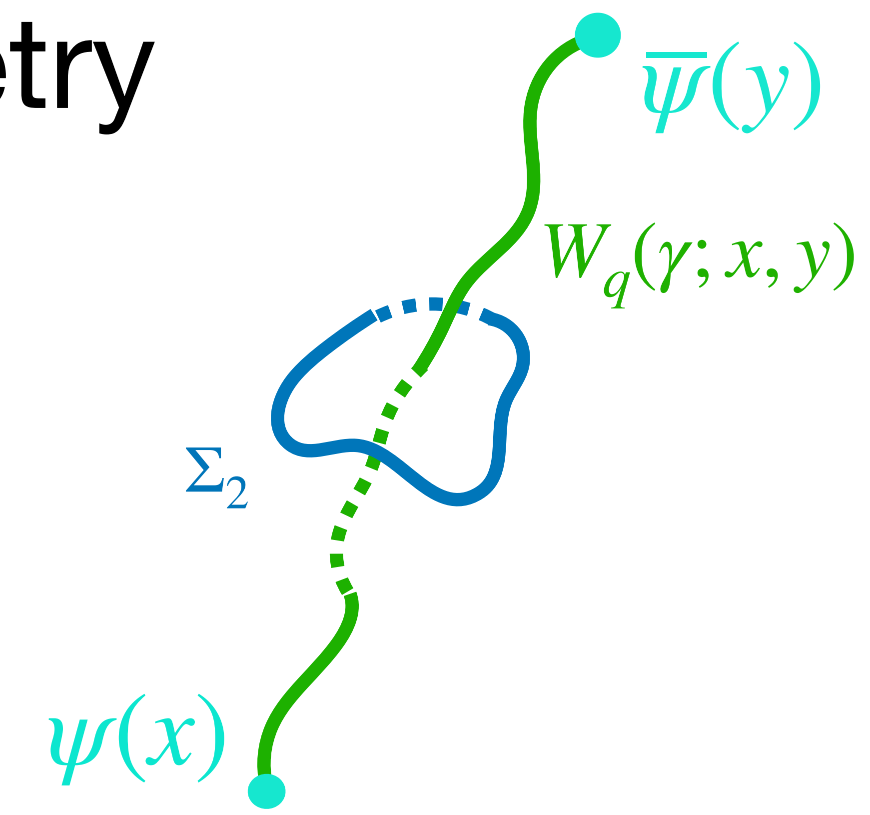

See Córdova, Ohmori, Rudelius '22

Gauss' law breaks with dynamical electron!

$$V(r) = \frac{-q^2}{4\pi r}\left(1 + \frac{q^2}{16\pi^{3/2}}\frac{e^{-2m_e r}}{(m_e r)^{3/2}} + \dots\right), \qquad r \gg m_e$$

Uehling, '35

Magnetic charges

Recall Maxwell's equations $\nabla \cdot E = 4\pi\rho$ $\nabla \cdot B = 0$

Low-energy theory of electromagnetism is asymmetric

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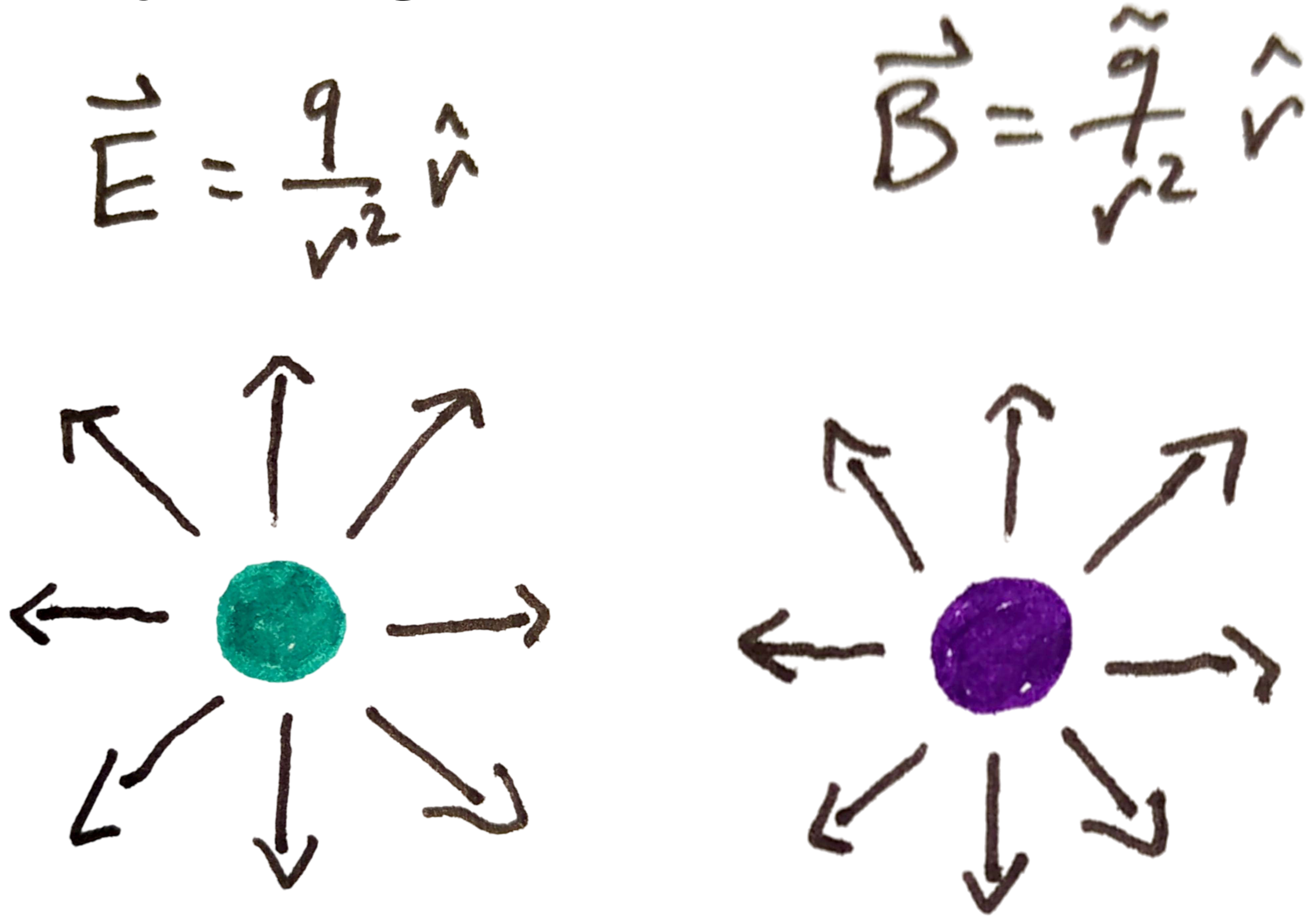

Dirac '31; 't Hooft '74; Polyakov '74

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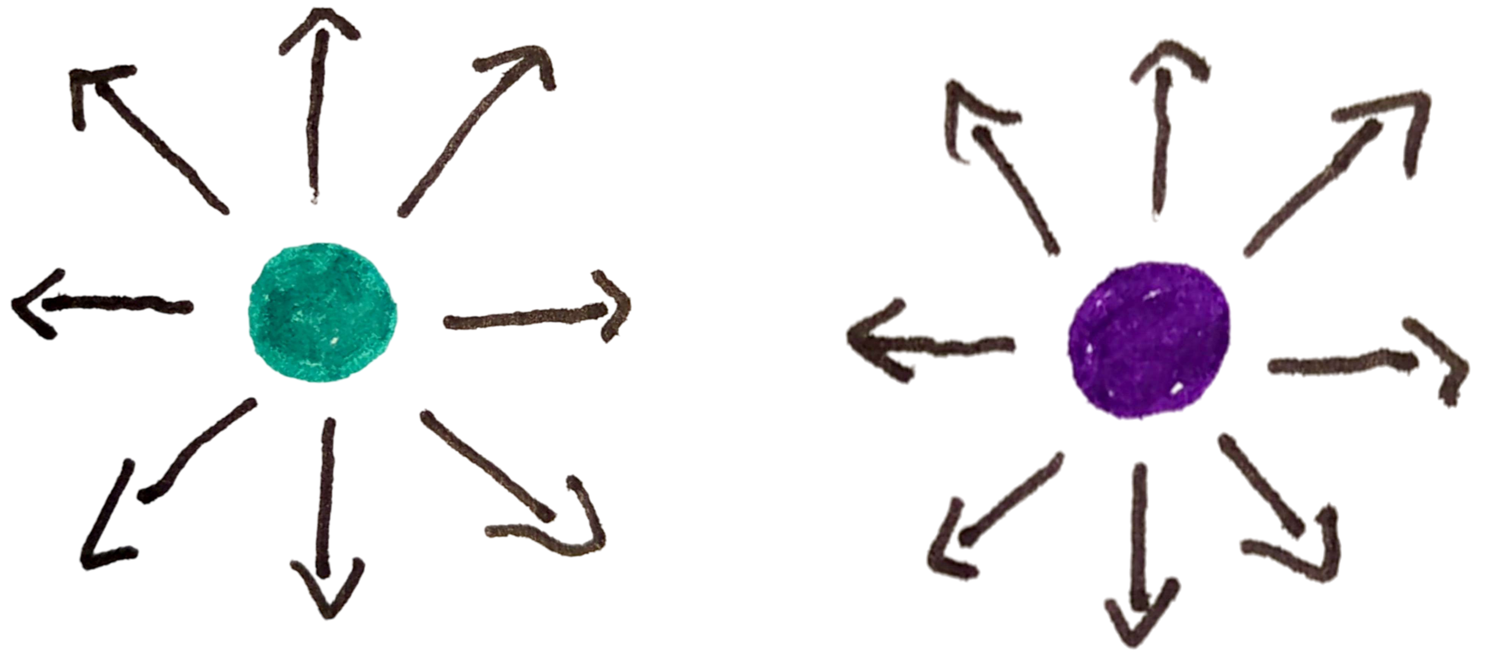
$\vec{E} = \frac{q}{r^2}\hat{r}$ $\vec{B} = \frac{\tilde{q}}{r^2}\hat{r}$



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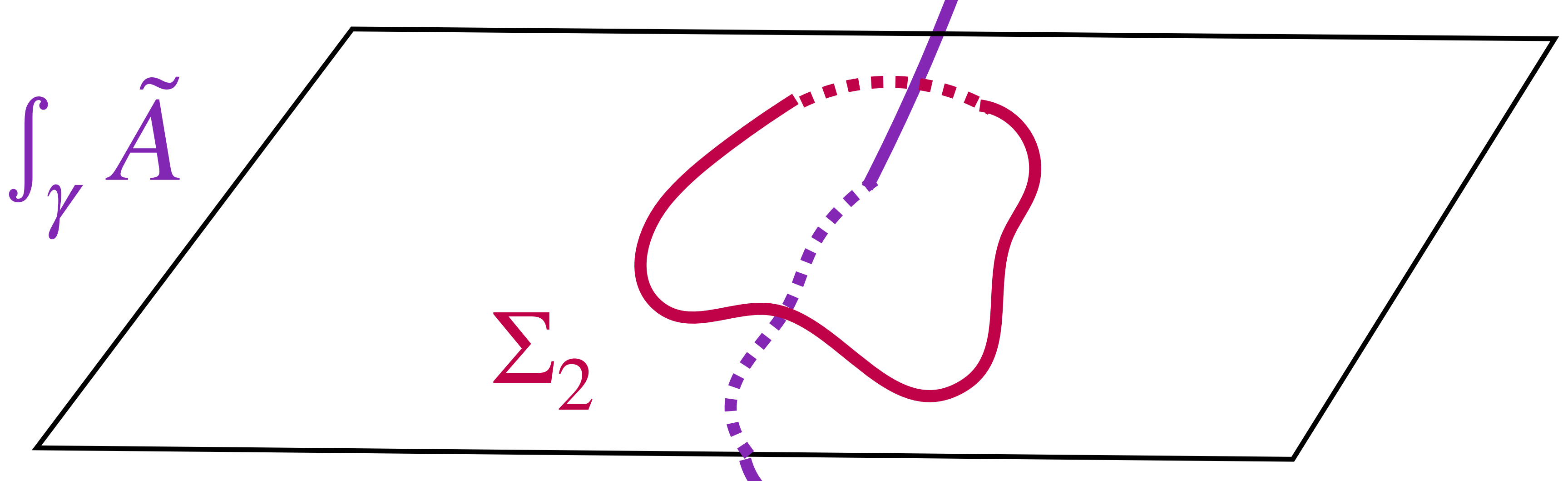
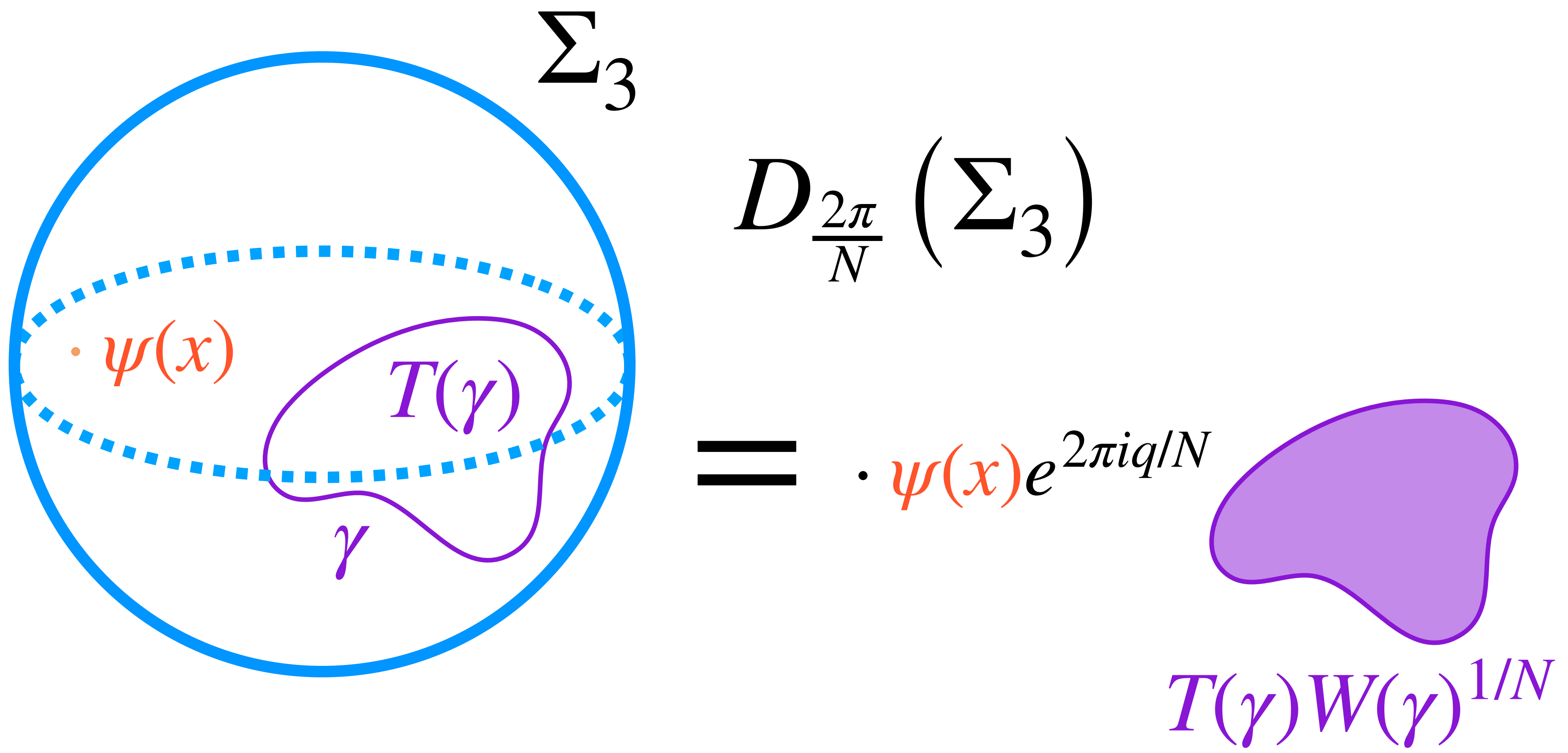
Dual magnetic line symmetry

$$T_{\tilde{q}}(\gamma) = e^{i\tilde{q}\int_\gamma \tilde{A}}$$



Magnetic symmetry is only broken in an ultraviolet theory where degrees of freedom are rearranged!

Non-invertible Naturalness



This is a symmetry structure which acts on *both* local operators and magnetic worldlines.

Córdova, Ohmori ’22
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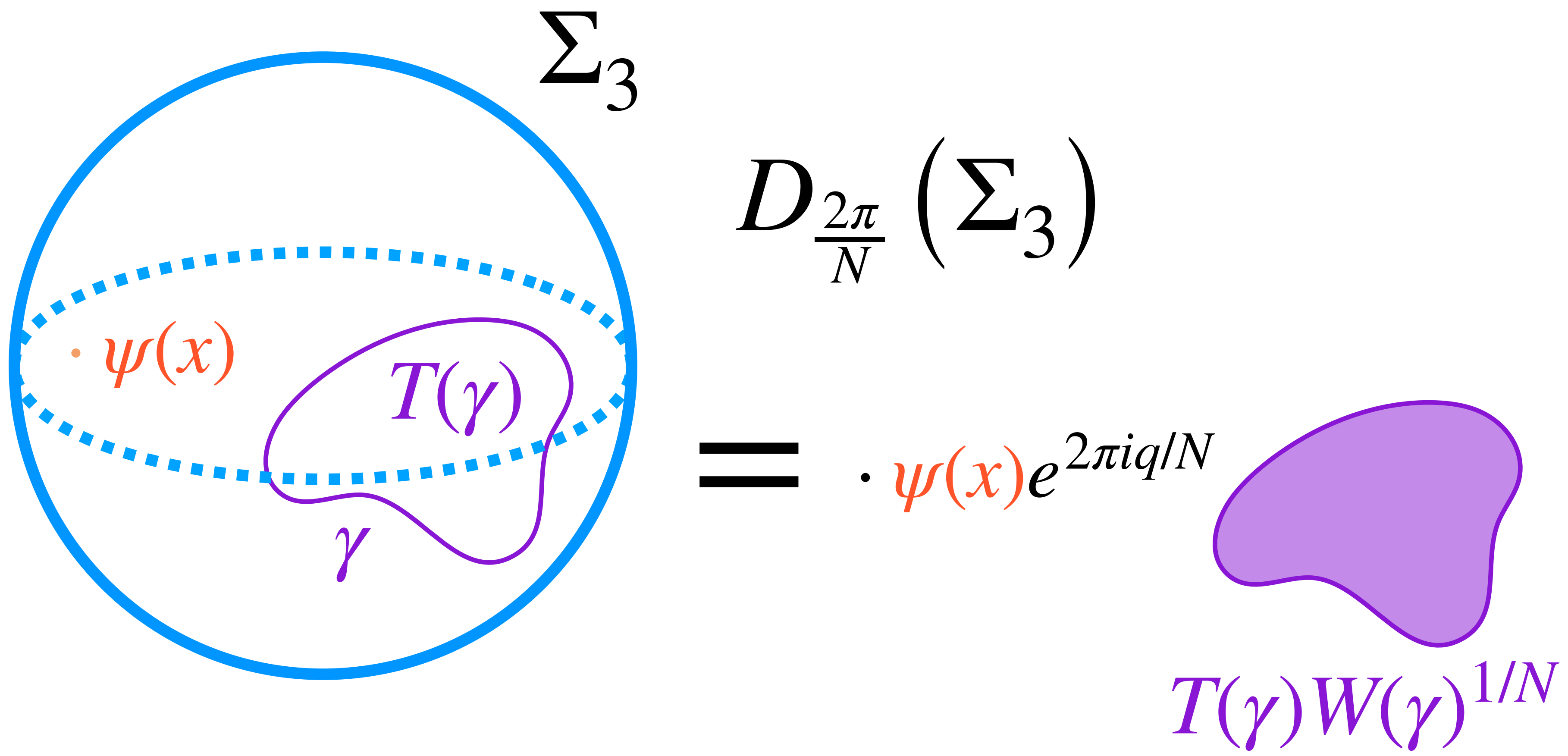

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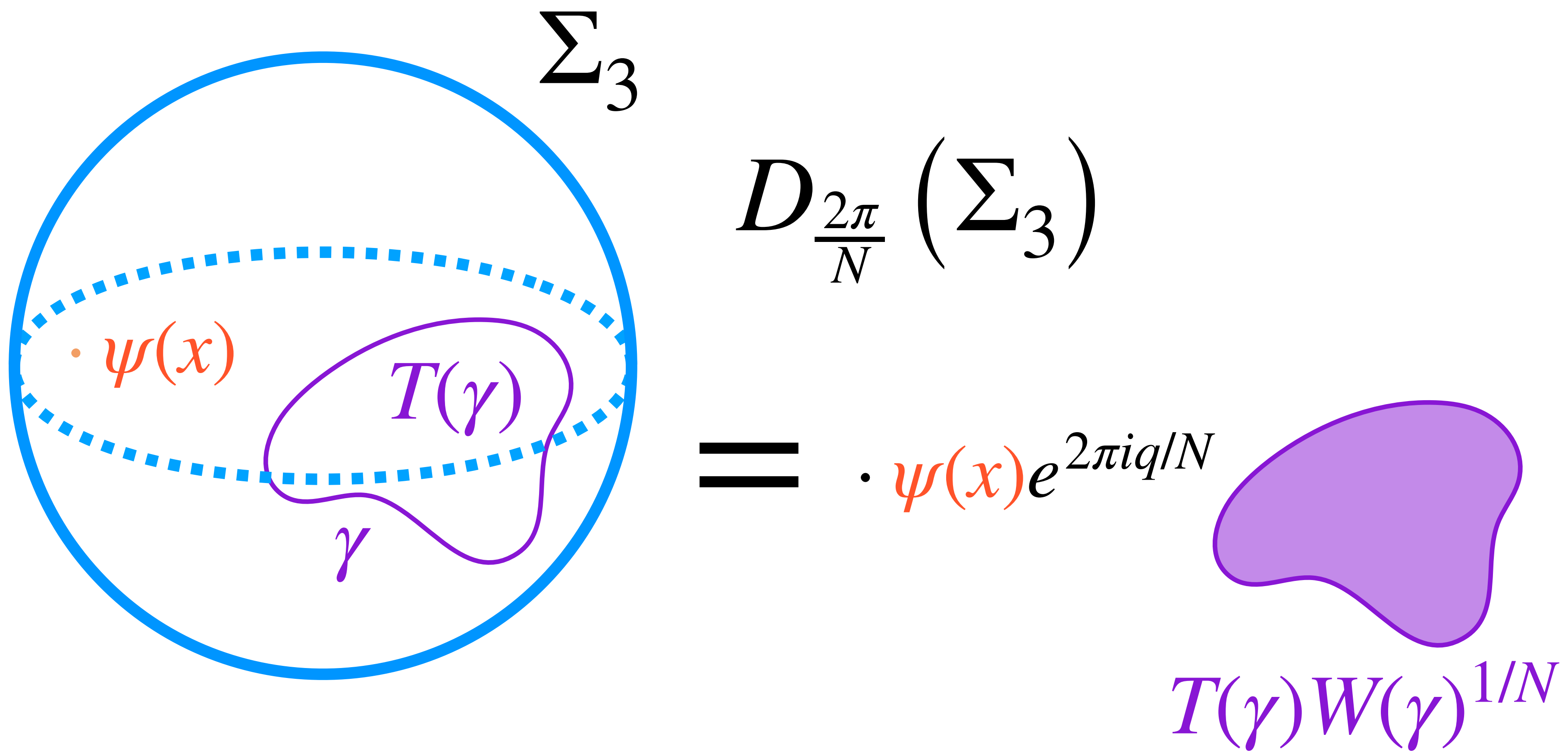

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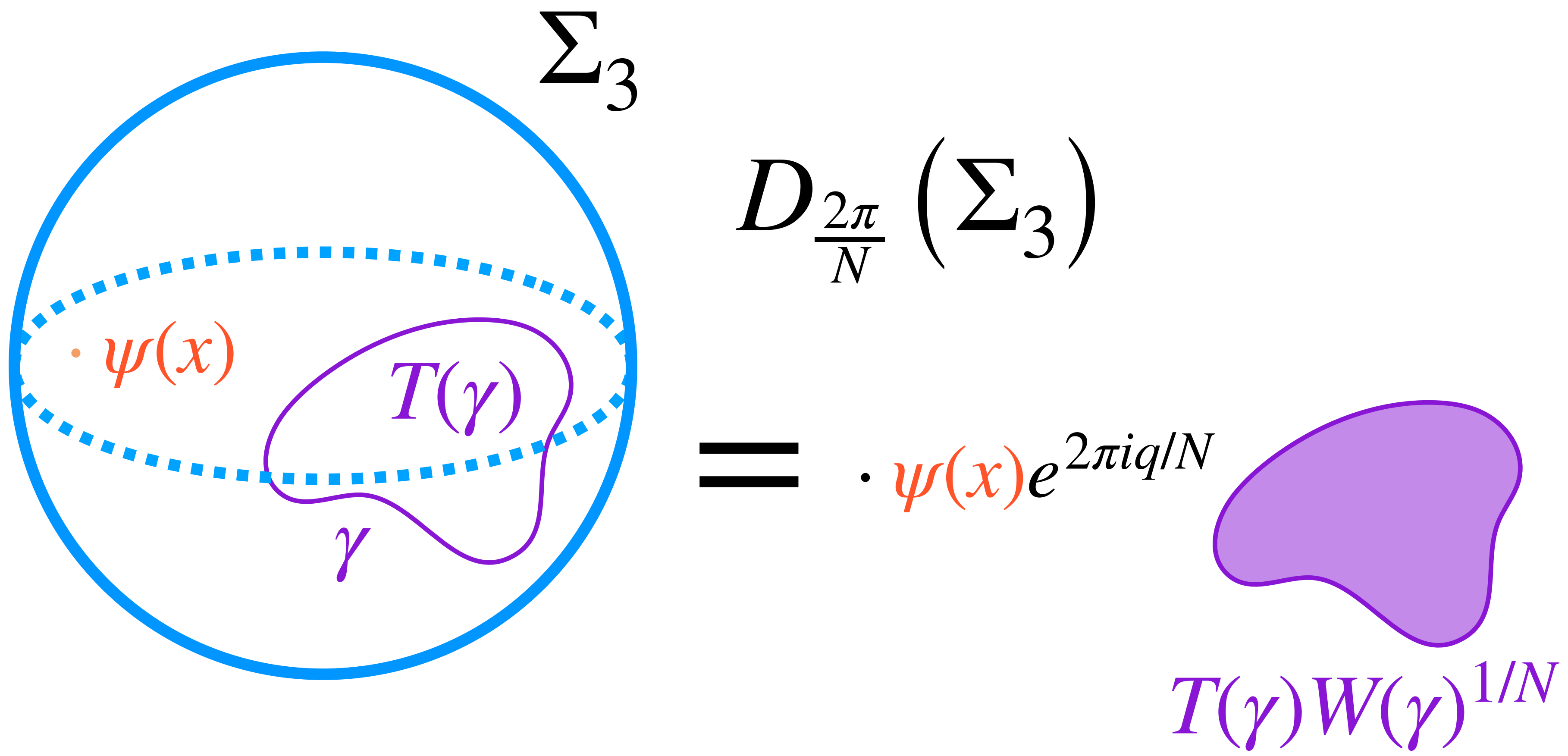

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Symmetry guidance *past* technical naturalness!

Non-invertible symmetry in lepton flavor gauge theory

2211.07639/**PRX** Clay Córdova, Sungwoo Hong, SK, Kantaro Ohmori

Gauged $U(1)_{L_\mu - L_\tau}$ gives non-invertible symmetry protecting neutrino masses!

$$\xrightarrow{U(1)_{L_\mu - L_\tau} \subset SU(2) \times U(1)}$$

Instantons produce Majorana neutrinos

$$\mathscr{L} \sim \frac{y_\mu y_\tau}{v_\Phi} e^{-\frac{8\pi^2}{g_H^2}} (\tilde{H}L)(\tilde{H}L)$$

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$3\bar{\nu} + U(1)_{L_\mu - L_\tau} \subset SU(3)$

	$SU(3)$
$\mathbf{L}$	$\square$
$\bar{\mathrm{e}}$	$\overline{\square}$
$\bar{\nu}$	$\overline{\square}$

Instantons produce Dirac neutrinos

$$\mathscr{L} \sim y_\tau e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}L\bar{\nu}$$

Non-invertible PQ Symmetry in quark flavor gauge theory

Since $N_c = N_g$, can gauge $\left(SU(3)_C \times U(1)_{B_1+B_2-2B_3}\right)/\mathbb{Z}_3$ and get non-invertible symmetry!

Breaking in e.g. $SU(9)$ quark color-flavor unification.

	$SU(9)$
$\mathbf{Q}$	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$

SM matter

Massless quark solution from $y_b \sim y_t e^{-8\pi^2/g_9^2}$

2402.12453/**PRX** Córdova, Hong, SK

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SM+H_u

2HDM Alignment with Visible Axion

$m_{12}^2 \sim y_t y_b v_9^2 e^{-8\pi^2/g_9^2}$

2412.05362/**JHEP** Antonio Delgado, SK
(see talk from Antonio on Tuesday)

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$\xrightarrow{\text{SM}+H_u}$ 2HDM Alignment with Visible Axion
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$\xrightarrow{\text{SM} + H_u + \phi}$ Solve DFSZ DW prob
$\delta V(a) \sim f_a v_9^3 e^{-8\pi^2/g_9^2} \cos a$

25XX Gongjun Choi, Hong, SK

Model building applications of non-invertible chiral symmetry

	Technically Natural	Unnatural
Dim. 0	y_ν (CHKO 2211.)	$\bar{\theta}$ (CHK 2402.)
Dim. 2	m_{12}^2 (DK 2412.)	*The* Hierarchy Problem
Dim. 4	$\delta V(a)$ (CHK 25XX.)	*The* CC Problem

Seth’s conclusions

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BSM Non-invertible Naturalness:

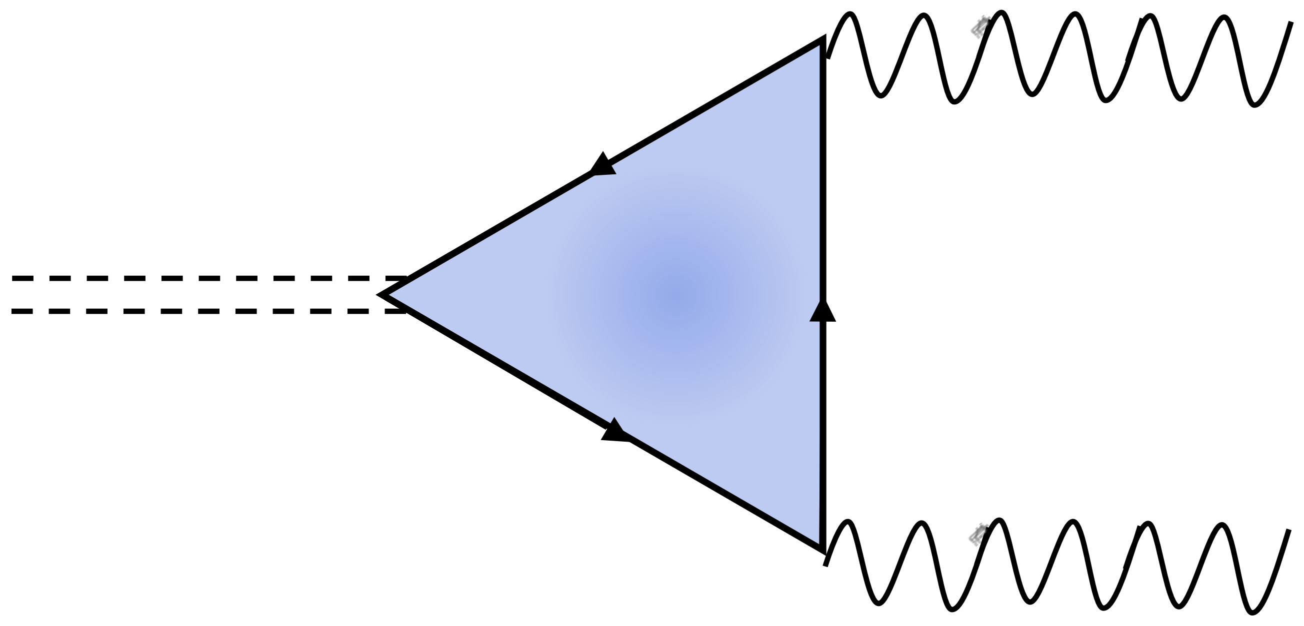
Neutrino Mass: 2211.07639 with Clay Córdova, Sungwoo Hong, Kantaro Ohmori
Strong CP à la Massless Quark: 2402.12453 with Clay & Sungwoo
Strong CP à la PQWW Visible Axion: 2412.05362 with Antonio Delgado
Strong CP à la DFSZ Invisible Axion: 25XX with Sungwoo & Gongjun Choi

Generalized Symmetries in Standard Model:

Discrete point symmetry: 2204.01741
Line symmetry possibilities & collider pheno: 2406.17850, 25XX with Adam Martin
Flavor-hypercharge point-line intertwining: 2212.13193 with Clay Córdova
Surface symmetry cosmo pheno: 2204.01750
Surface symmetry possibilities: 25XX with Sungwoo Hong & Daniel Brennan

Motivating non-invertible symmetries: The mystery of the missing instantons

Recall a classical zero-form global symmetry $U(1)_X$ can be anomalous in quantum theory with G gauge group



$$\partial_\mu J^\mu_X = 0 \quad \longrightarrow \quad \partial_\mu J^\mu_X = \frac{\mathscr{A}_X}{8\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}$$

Instanton configurations have $\displaystyle\int_{\mathscr{M}} F\tilde{F} \neq 0$ so 'activate' the anomaly

But what about when they don't?

E.g. famously $\pi_3\big(U(1)\big) = 1$ and *there are no Abelian instantons in* $\mathbb{R}^4$, so $\displaystyle\int_{\mathbb{R}^4} F\tilde{F} = 0$

Old lesson: X is anomalous but S-matrix preserves X anyway

EFT philosophy: If there is ever a zero, there should be a symmetry!

Somehow despite X being anomalous there must remain a subtle sort of symmetry that demands the S-matrix preserves X

A hint: X *can* be violated around magnetic monopoles

c.f. Callan-Rubakov

Dirac ’31
Callan, Rubakov ‘80s
Ongoing…

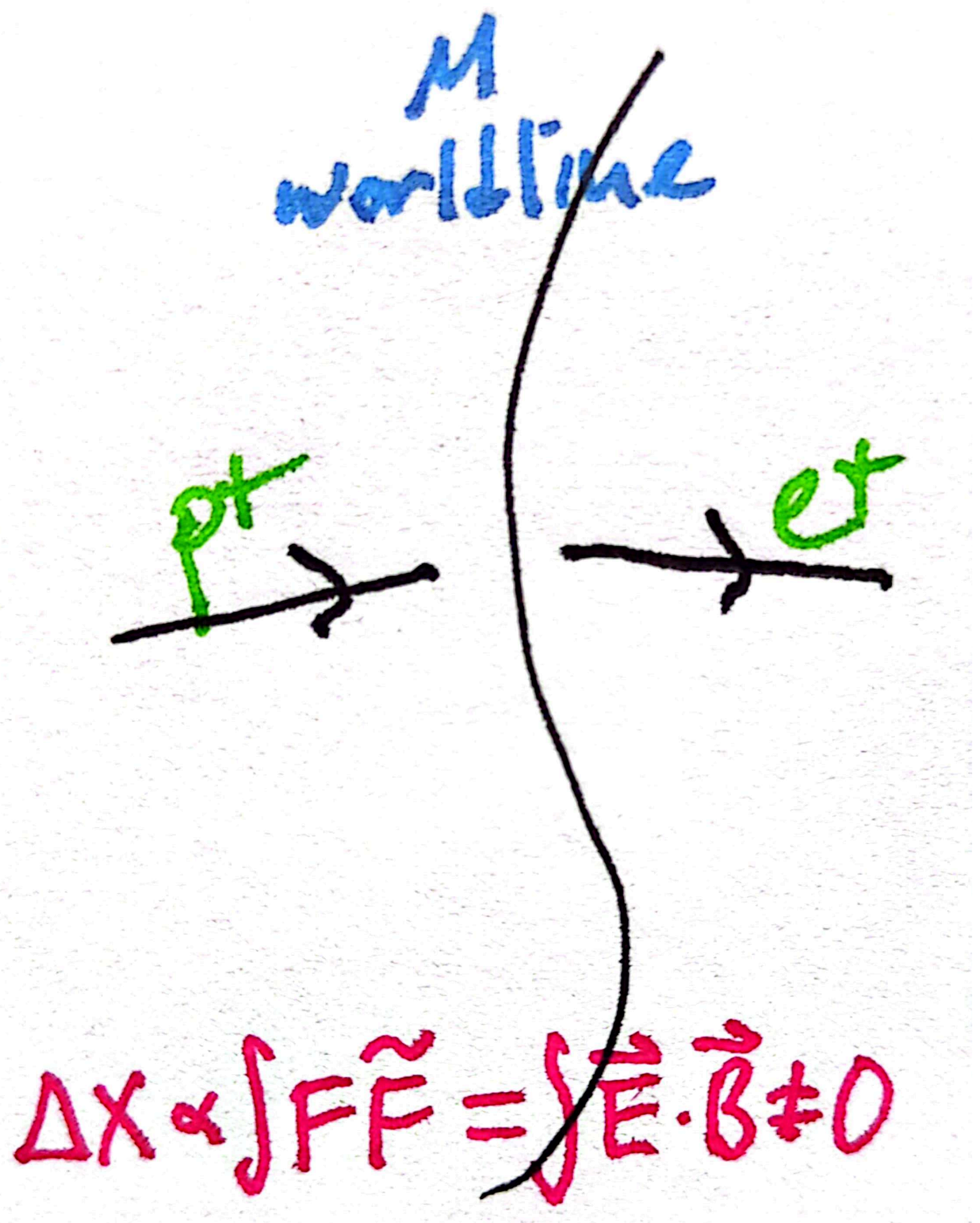


Fig. 1: A confused effective field theorist

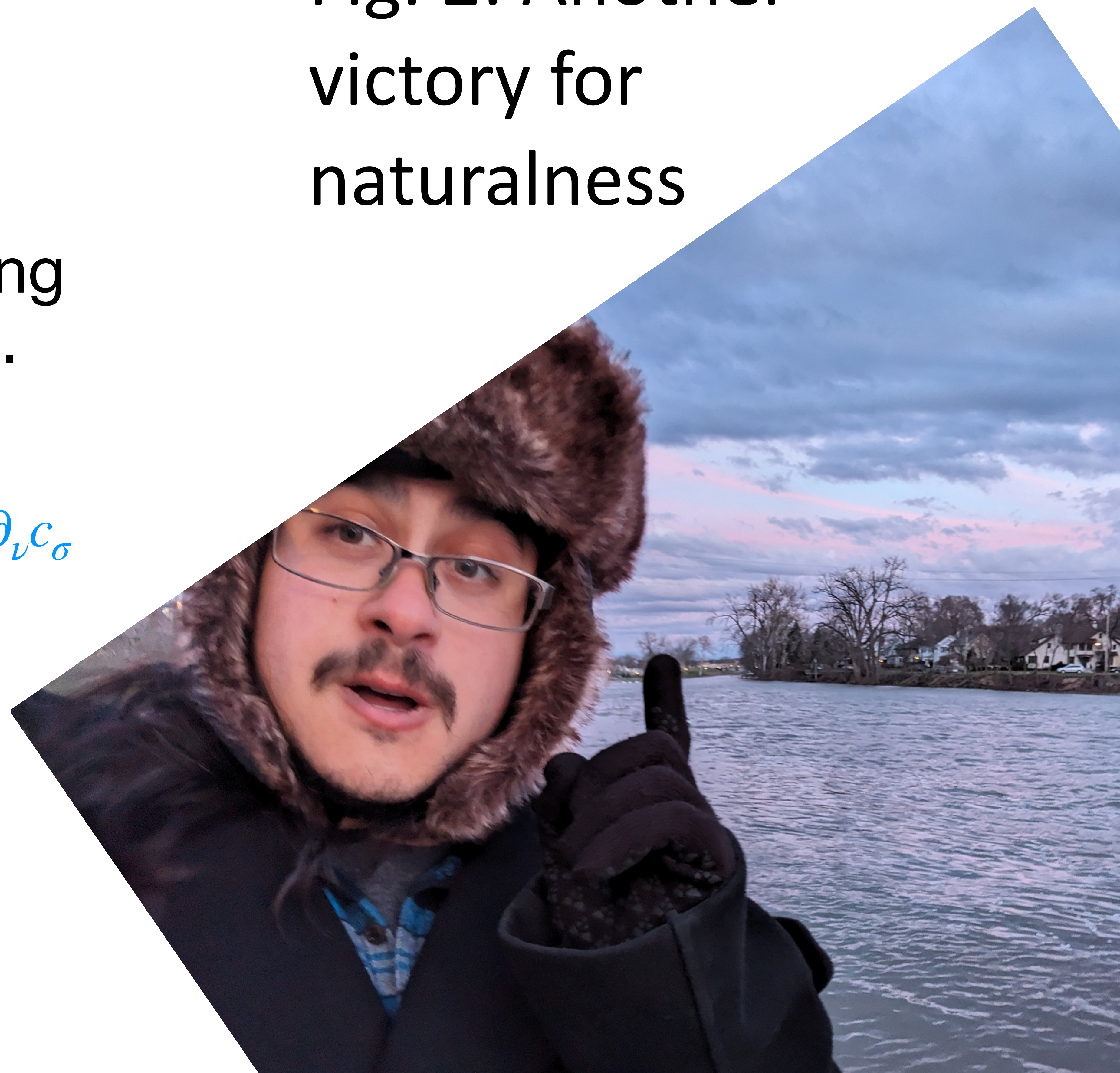
There's a subtler notion of symmetry!

Choi, Lam, Shao
2205.05086
Córdova, Ohmori
2205.06243

Can't do $U_\alpha[\Sigma_3] = e^{i\alpha Q[\Sigma_3]} = e^{i\alpha \int_{\Sigma_3} J^\mu_X \hat{n}_\mu d^3x}$, not conserved

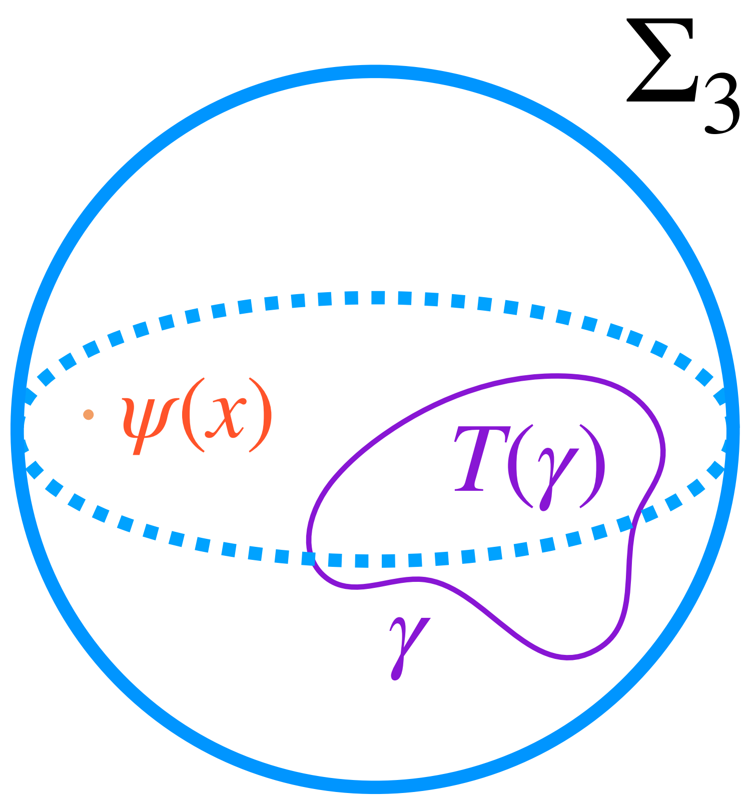
Can't do $\hat{J}^\mu_X = J^\mu_X - \dfrac{\mathscr{A}}{4\pi^2}\epsilon^{\mu\nu\rho\sigma}A_\nu \partial_\rho A_\sigma$, not gauge invariant

Fig. 2: Another victory for naturalness

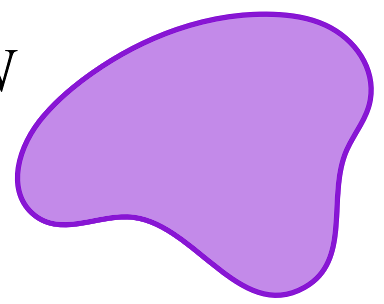


Can construct a topological, gauge invariant operator by including a Chern-Simons theory which talks to the bulk magnetic current.

$$D_{\frac{2\pi}{N}}\left(\Sigma_3\right) = \int Dc \; e^{i\int_{\Sigma_3} \frac{2\pi}{N} J_\mu \hat{n}^\mu + \frac{1}{2\pi}\epsilon^{\mu\nu\sigma}c_\mu \partial_\nu A_\sigma + \frac{N}{4\pi}\epsilon^{\mu\nu\sigma}c_\mu \partial_\nu c_\sigma}$$



$$= \cdot\, \psi(x) e^{2\pi i q/N}$$



$T(\gamma)W(\gamma)^{1/N}$

Quality control

All solutions rely on good quality Peccei-Quinn symmetries, but only the invisible axion has a quality ‘problem’

Invisible axion admits PQ-violating $\mathscr{L} \supset c_n \phi^n / M_{\text{Pl}}^{n-4}$ and has the normal quality problem $f_a^4 \left(f_a / M_{\text{pl}} \right)^{n-4} \lesssim \bar{\theta} \Lambda_{\text{QCD}}^4$

Heavy visible axion admits PQ-violating $\mathscr{L} \supset c_H (H_u H_d) |\Xi|^4 / M_{\text{Pl}}^2$ but does not perturb minimum as long as $v_9^4 \lesssim \bar{\theta} v_{\text{EW}}^2 M_{\text{pl}}^2 \to v_9 \lesssim 10^{-11} M_{\text{pl}}$

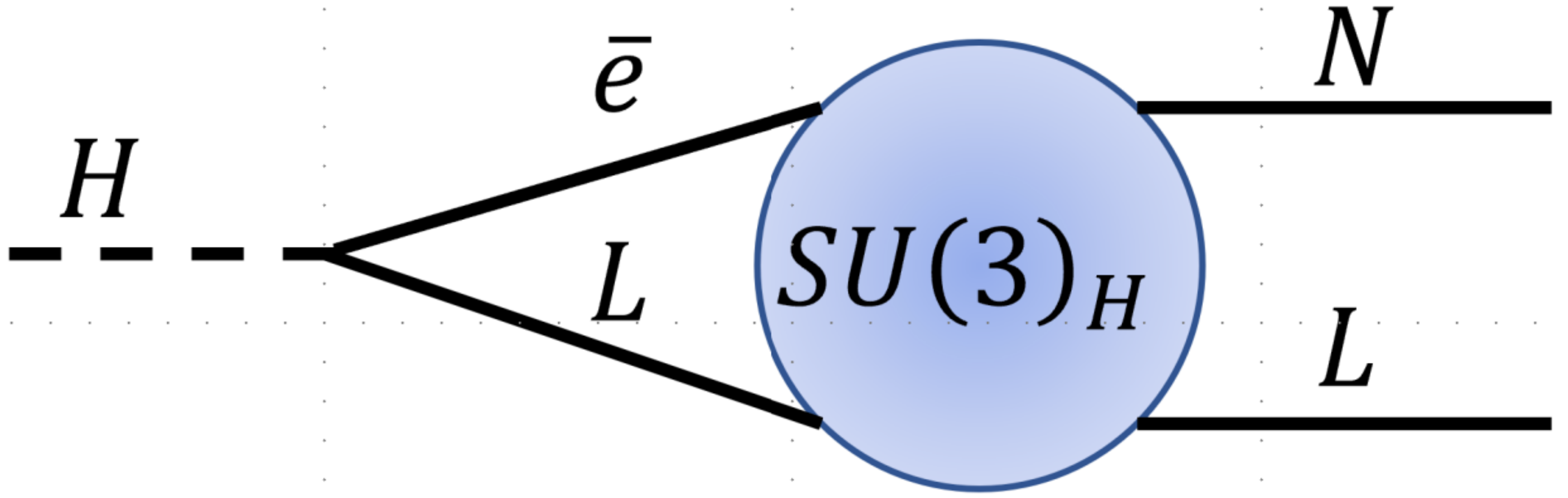
Massless quark admits PQ-violating $\mathscr{L} \supset c_\Sigma \tilde{H} Q \Sigma \bar{d} / M_{\text{Pl}}$ but as long as $\langle \Sigma \rangle / M_{\text{pl}} \lesssim \bar{\theta}$ you’re guaranteed Im$(y) \lesssim \bar{\theta}$ Re(y). Quark flavor physics is not too far away!

Dirac masses:

Write down charged lepton mass $\mathscr{L} \sim y_\tau H\mathbf{L}\bar{\mathbf{e}}$

	$SU(3)_H$	$U(1)_{\mu-\tau}$	$U(1)_L$	$U(1)_N$
$\mathbf{L}$	3	$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$	$+1$	0
$\bar{\mathbf{e}}$	$\bar{3}$	$\begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \bar{\tau} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	-1	0
$\mathbf{N}$	$\bar{3}$	$\begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	-1	$+1$

Classical $U(1)_N$ symmetry protects the Dirac neutrino mass $\tilde{H}\mathbf{L}\mathbf{N}$



$$\mathscr{L} \sim y_\tau^\star e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}\mathbf{L}\mathbf{N}$$

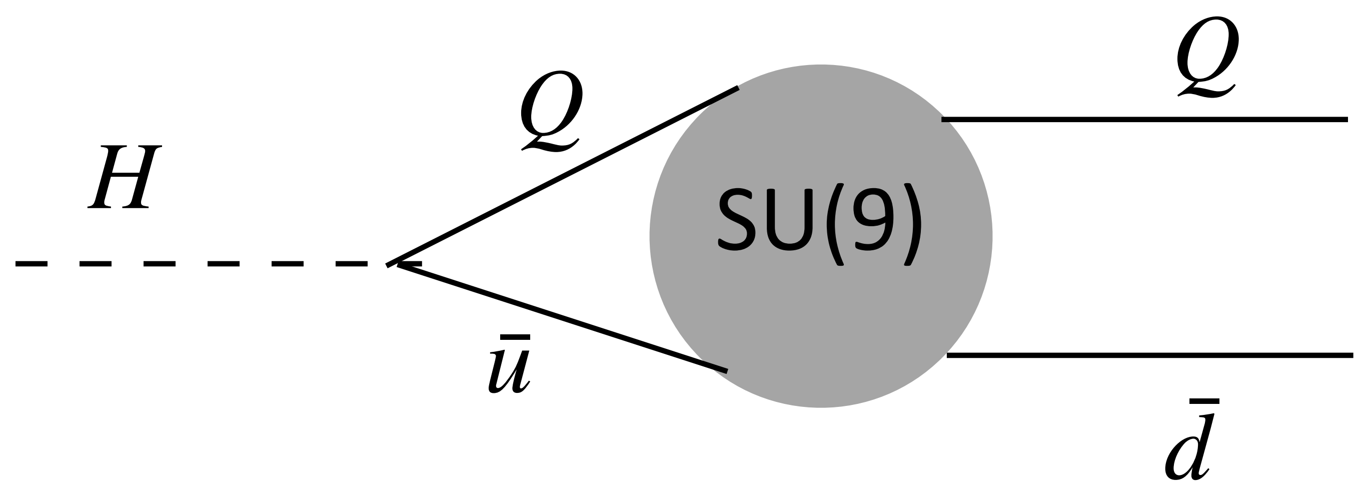
Color-flavor unification!

This all points to a beautiful $SU(9)$ unified theory in which the colors and flavors of the quarks are placed together into the fundamental

	$SU(9)$
$\mathbf{Q}$	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$

$$\mathscr{L}_0 = y_t \tilde{H} \mathbf{Q}\bar{\mathbf{u}} + \text{h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

Again start with good $U(1)_{\text{PQ}}$ and no strong CP violation, then



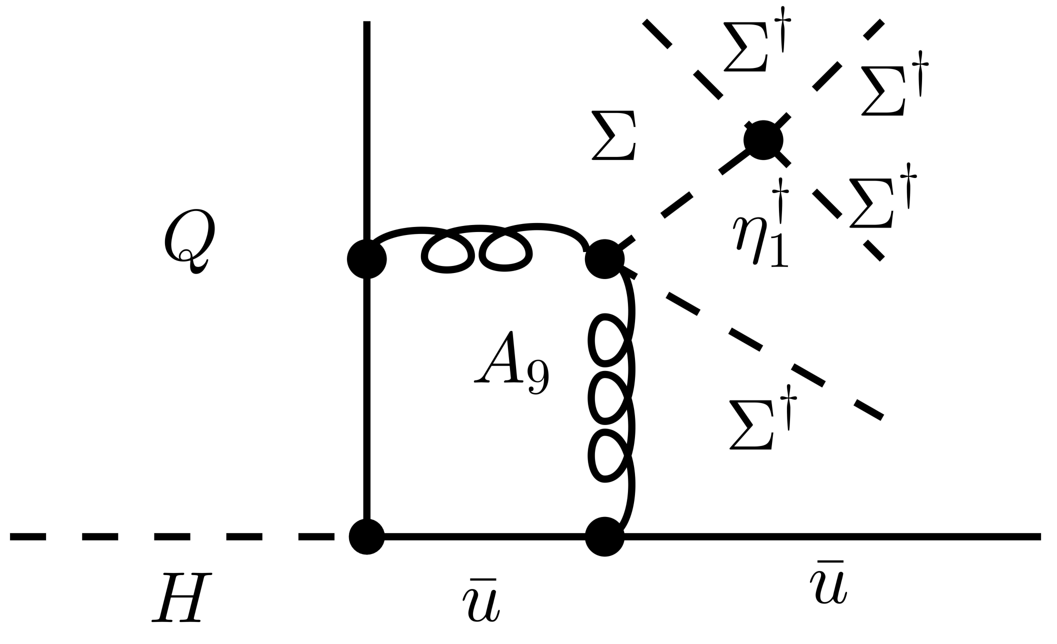
$$\mathscr{L}(\Lambda) \sim y_t H \mathbf{Q}\bar{\mathbf{u}} + y_t^{\star} e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda)}} H \mathbf{Q}\bar{\mathbf{d}} + \text{ h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

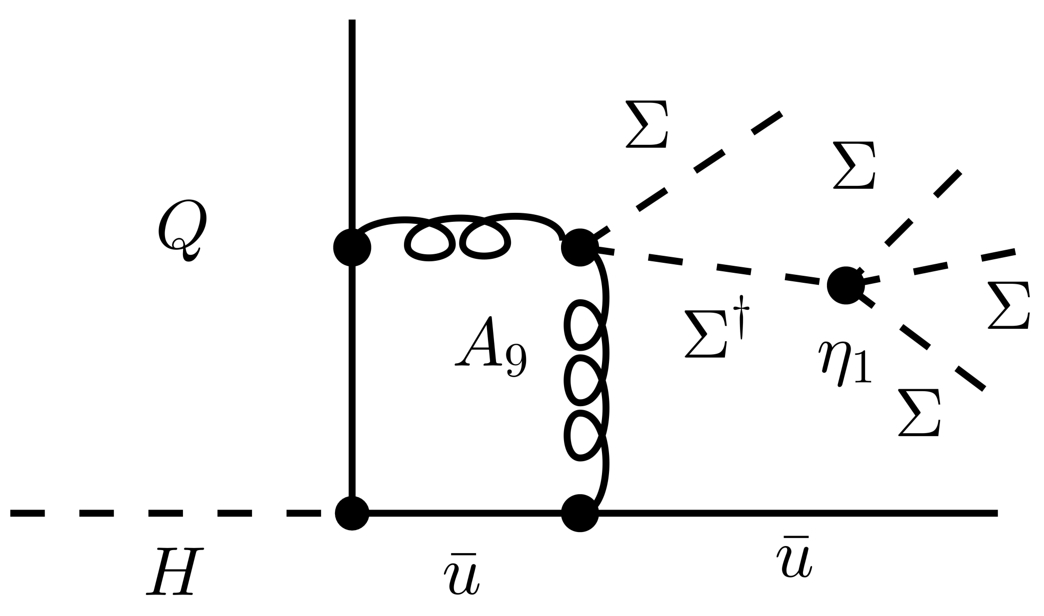
Generating CKM very briefly

Idea: Communicating flavor-breaking $\langle \Sigma^a{}_b \rangle$ through gauged flavor symmetry lets you generate *hermitian* yukawas

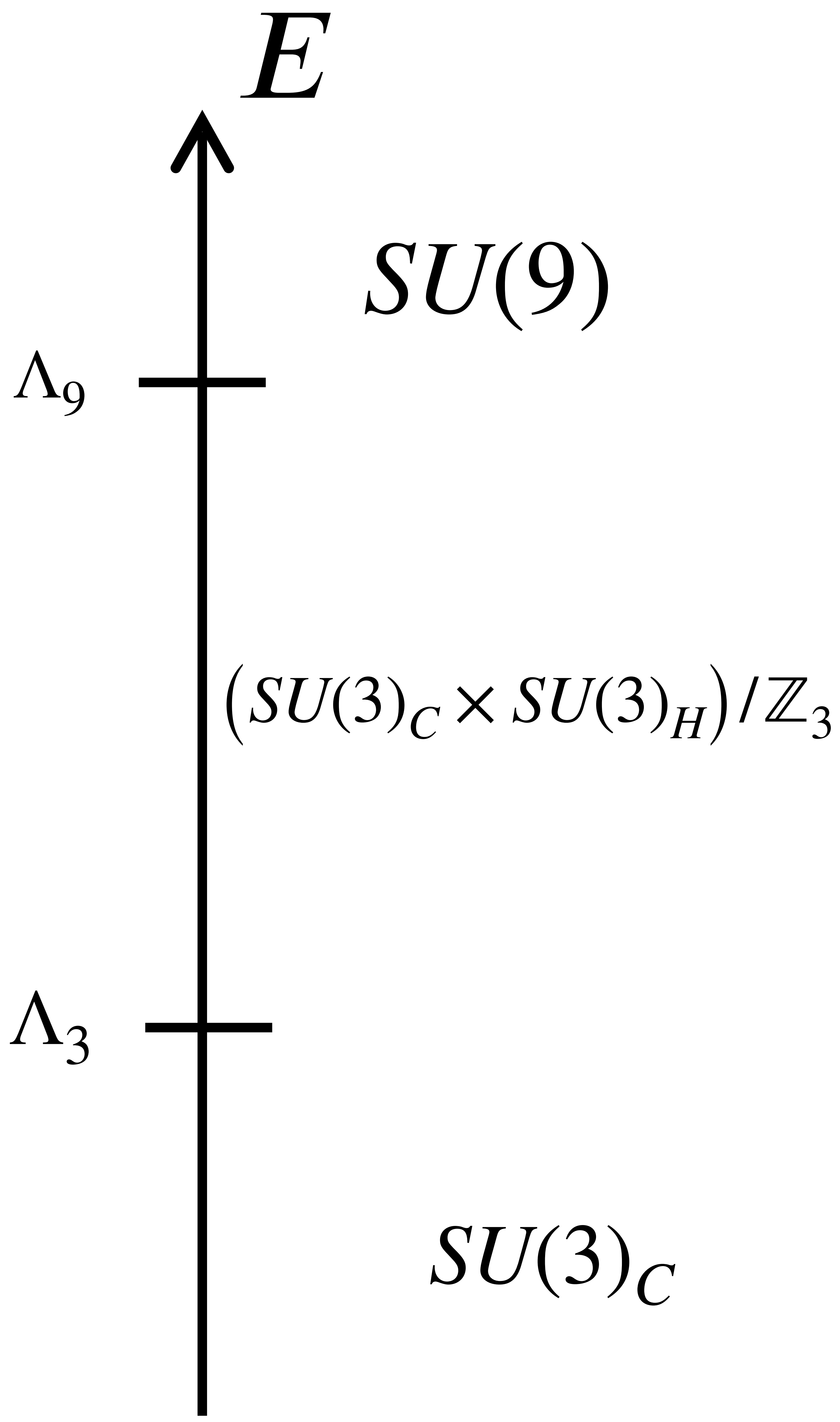
$M = \det(y_u y_d)$ automatically real

$$V_{\mathbb{Z}_4}(\Sigma) = \eta_1 \mathrm{Tr}\left(\Sigma^4\right) + \eta_2 \mathrm{Tr}\left(\Sigma^2\right)^2 + \text{ h.c.}$$





$$(y_u)^a{}_b \sim y_t \left(\mathbb{1}^a{}_b + \frac{\alpha_9}{(4\pi)} \frac{\eta_1^\dagger (\Sigma^{\dagger 4})^a{}_b + \eta_2^\dagger \mathrm{Tr}(\Sigma^{\dagger 2})(\Sigma^{\dagger 2})^a{}_b}{\Lambda_9^4} + \frac{\alpha_9}{(4\pi)} \frac{\eta_1 (\Sigma^4)^a{}_b + \eta_2 \mathrm{Tr}(\Sigma^2)(\Sigma^2)^a{}_b}{\Lambda_9^4} + \dots \right)$$

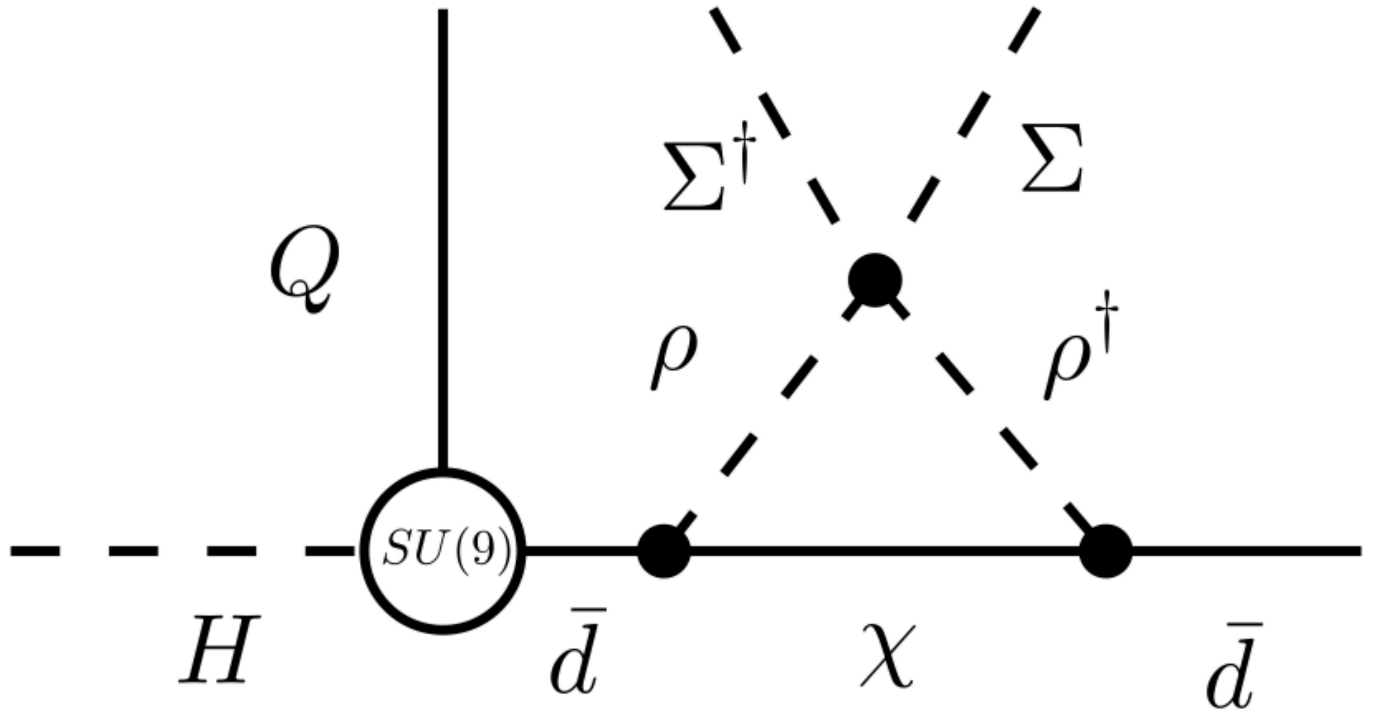


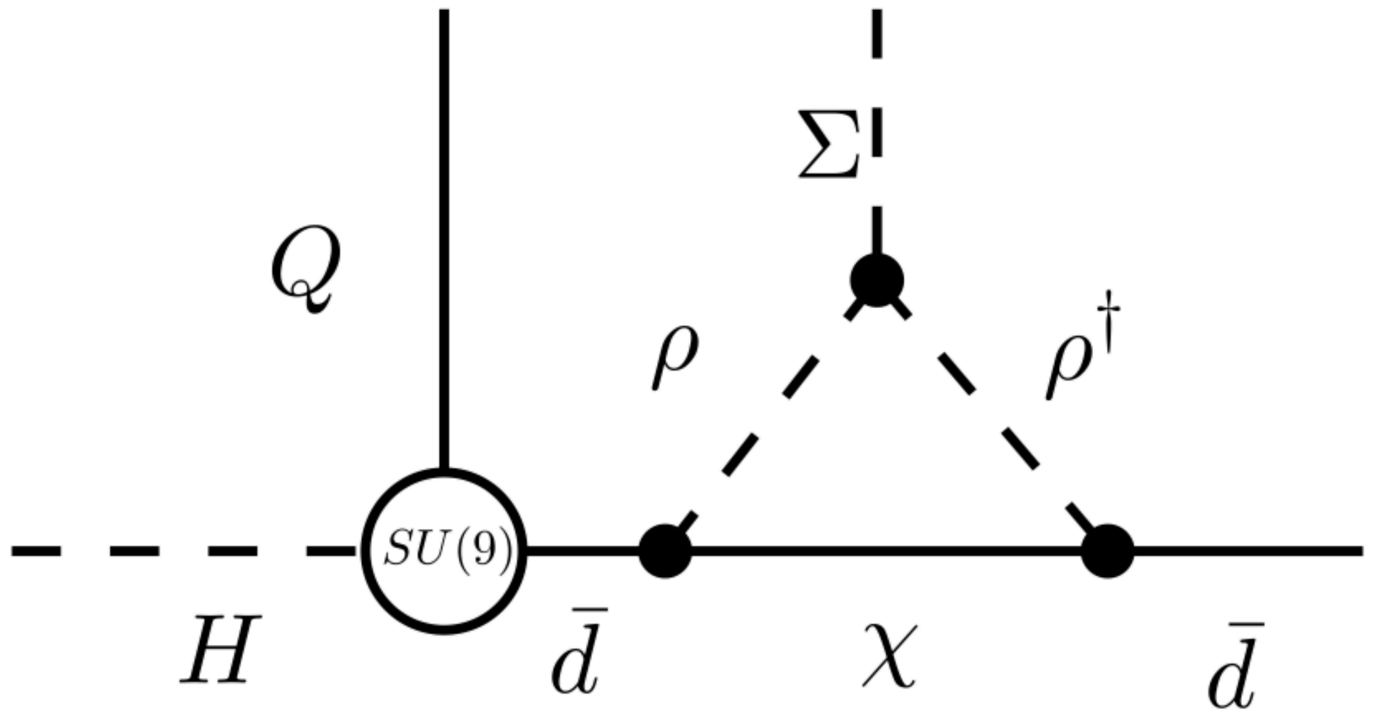
Generating CKM

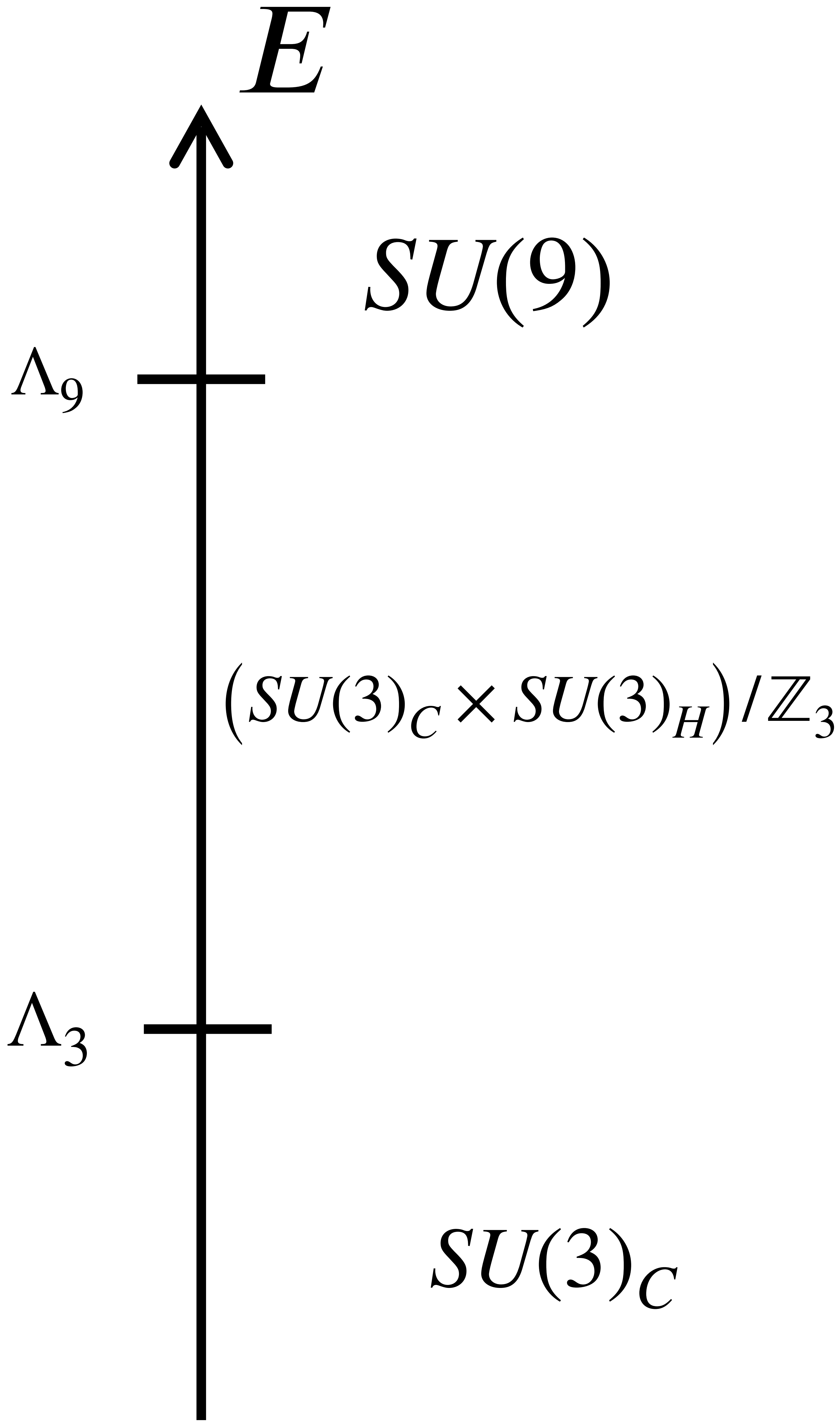
Yukawas stay hermitian yet $V(\Sigma)$ breaks CP explicitly and/or spontaneously so can generate

$$\delta_{CKM} \propto \arg\det\left(\left[y_u^\dagger y_u, y_d^\dagger y_d\right]\right) \neq 0$$

Another wrinkle: Must treat $\bar{u}, \bar{d}$ differently so they don't commute in flavor space.







Peccei-Quinn Weinberg Wilczek

From the 1HDM to the 2HDM we now have an extra pseudoscalar which can be the axion a

	$U(1)_{\mathrm{PQ}}$
Q_i	0
$\bar{u}^j$	+1
$\bar{d}^j$	0
H_u	-1
H_d	0

$$\mathscr{L} \supset (y_u)^i_j H_u Q_i \bar{u}^j + (y_d)^i_j H_d Q_i \bar{d}^j$$

$$\to i\frac{\mathscr{A}_{\mathrm{PQ}}}{32\pi^2}(a+\bar{\theta})G\tilde{G}$$

$\mathscr{A}_{\mathrm{PQ}} = N_g, \int \frac{G\tilde{G}}{32\pi^2} \equiv \mathscr{N}_C \in \mathbb{Z}$ breaks $U(1)_{\mathrm{PQ}} \to \mathbb{Z}_3$

Confinement generates a potential $V(a) \sim \Lambda^4_{\mathrm{QCD}} \cos(3a/v_{\mathrm{EW}})$

But $m_a \sim 100\ \mathrm{keV}$ pseudoscalar with large couplings quickly ruled out

Once more with flavor

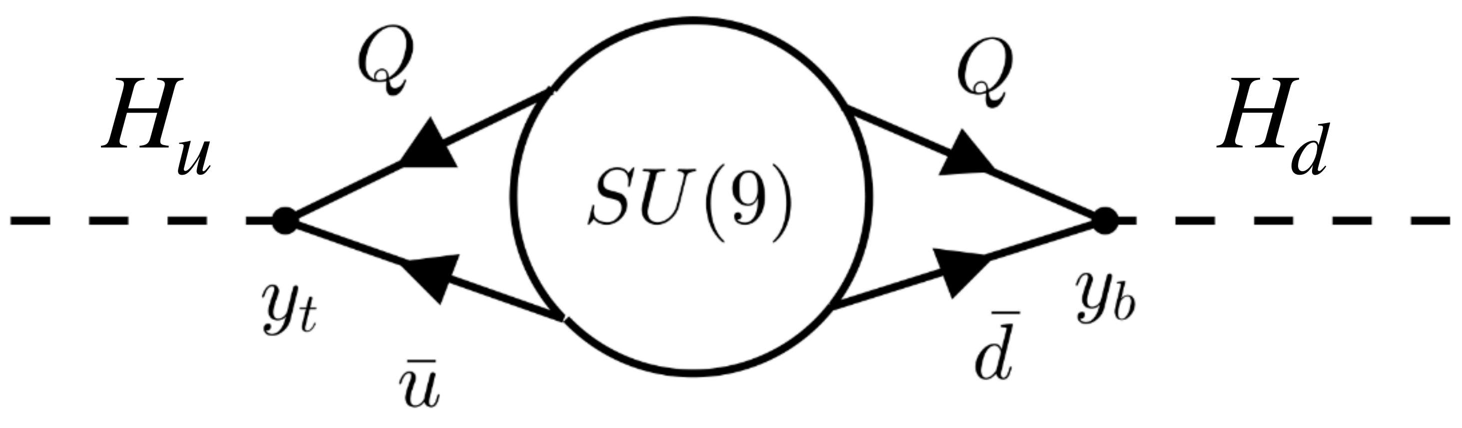
Now we have a gauge group $(SU(3)_C \times SU(3)_H)/\mathbb{Z}_3$

$$\mathscr{L} \supset y_t H_u Q\bar{u} + y_b H_d Q\bar{d}$$

Here $\Delta Q_{\text{PQ}} = N_g \mathcal{N}_C + N_c \mathcal{N}_H$ and now
$\mathcal{N}_C, \mathcal{N}_H \in \mathbb{Z}/3$ satisfying $\mathcal{N}_C = \mathcal{N}_H \pmod 1$

	$SU(3)_C$	$SU(3)_H$	$U(1)_{\text{PQ}}$
Q	3	3	0
$\bar{u}$	$\bar{3}$	$\bar{3}$	+1
$\bar{d}$	$\bar{3}$	$\bar{3}$	0
H_u	0	0	-1
H_d	0	0	0

$U(1)_{\text{PQ}} \to \mathbb{Z}_3$ non-invertible!



Again the $SU(9)$ color-flavor theory breaks the $\mathbb{Z}^{(1)}_{3,\text{mag}}$ now generating
$V \supset m_{12}^2 H_u H_d$ with $m_{12}^2 \propto y_t y_b v_9^2 e^{-2\pi/\alpha_H(v_9)}$

Dine-Fischler-Srednicki Zhitnitsky

Make the axion 'invisible' by adding a complex scalar singlet ϕ

$$\mathscr{L} \supset (y_u)^i_j H_u Q_i \bar{u}^j + (y_d)^i_j H_d Q_i \bar{d}^j + \lambda H_u H_d \phi$$

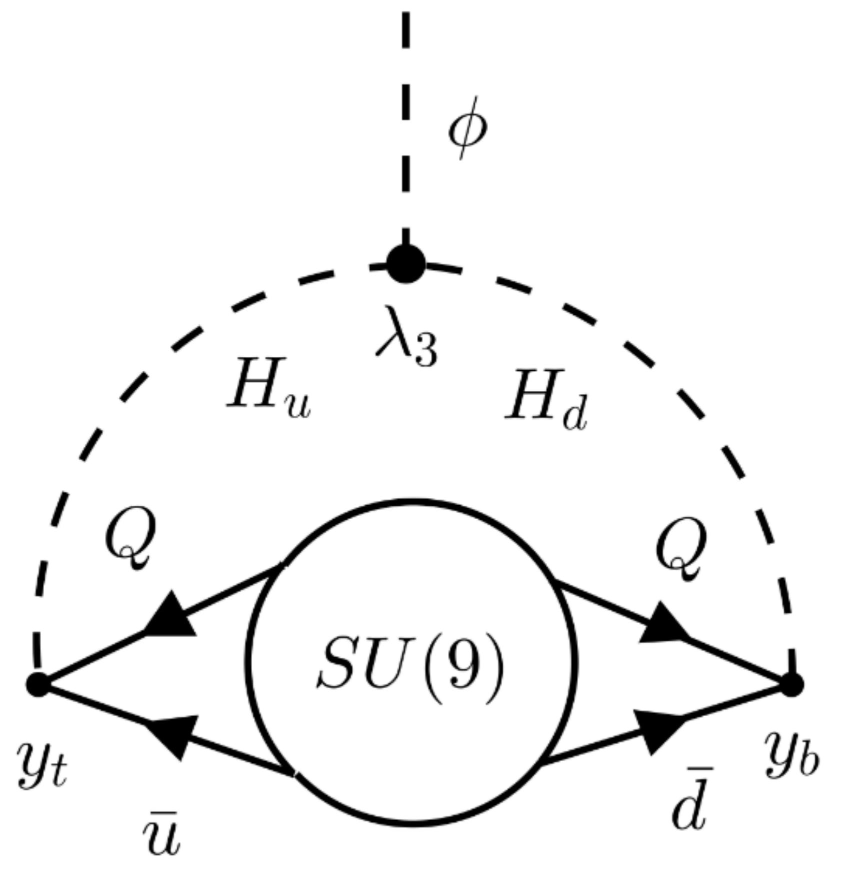
	$U(1)_{\mathrm{PQ}}$
Q_i	0
$\bar{u}^j$	+1
$\bar{d}^j$	0
H_u	-1
H_d	0
ϕ	+1

Same anomaly story $U(1)_{\mathrm{PQ}} \to \mathbb{Z}_3$ but now axion lives dominantly in ϕ. Axion background value spontaneously breaks this $\mathbb{Z}_3$.

Great, *but* faces the domain wall problem.

With flavor again this becomes non-invertible, and now tiny breaking destabilizes domain walls.

$$\delta V(a) \sim \lambda y_t y_b v_9^3 f_a e^{-2\pi/\alpha(v_9)} \cos(a)$$



	E-series e.g. $SO(10)$	Electroweak Flavor $SU(4) \times Sp(6)_L \times Sp(6)_R$	Color Flavor $SU(12) \times SU(2)_L \times SU(2)_R$
Gauge coupling unification	Yes	No	No
Irrep. unification	No, N_g	Yes	Yes

	E-series x horizontal e.g. $SO(10)_V \times SU(3)_H$	Electroweak Flavor $SU(4) \times Sp(6)_L \times Sp(6)_R$	Color Flavor $SU(12) \times SU(2)_L \times SU(2)_R$
Gauge coupling unification	No	No	No
Irrep. unification	Yes	Yes	Yes
Proton decay	Perturbative	Not perturbatively Davighi & Tooby-Smith Yes non-perturbatively SK	Not necessarily Davighi, Greljo Thomsen Preserve Z_9 subgroup combining (B-L) and lepton flavor
Gauged flavor	Same for Q & L	Same for Q & L	Separate for Q & L
Nearby flavor physics	Probably not?	Could be Davighi & Tooby-Smith	Probably not quarks? But maybe leptons
Pheno beaten to death	Yes	No	No

Naturalness ≈ Robustness

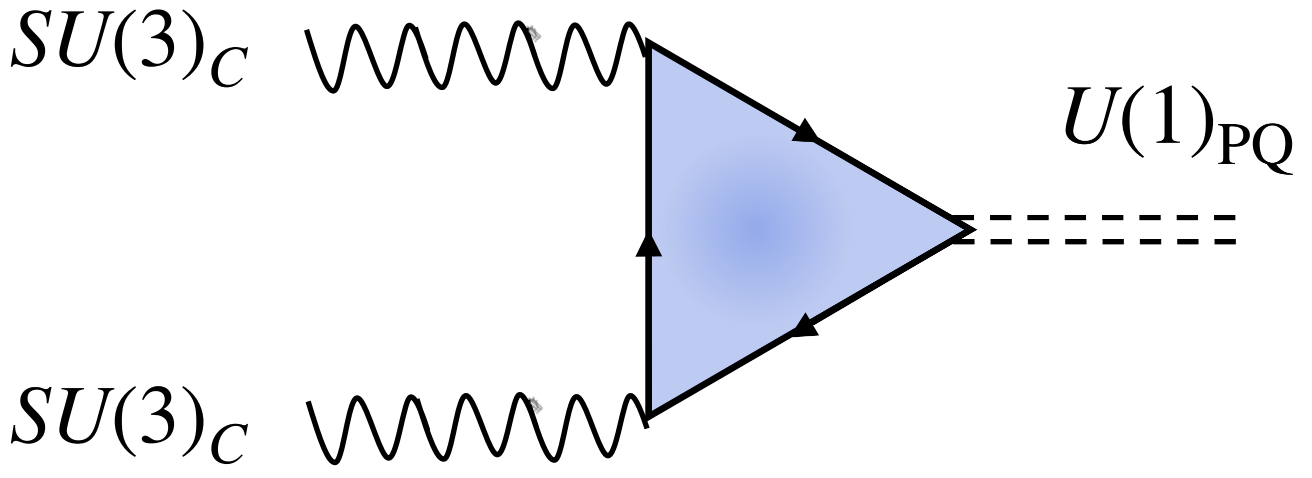
Structures which rely on some **integer invariants** of the SM particles are among the most robust.

$$\beta_3^{(1)} = \frac{1}{3}\left(4N_g - 11N_c\right) < 0 \quad \Rightarrow \quad \text{Explanation for } m_p \ll M_{\rm pl}$$

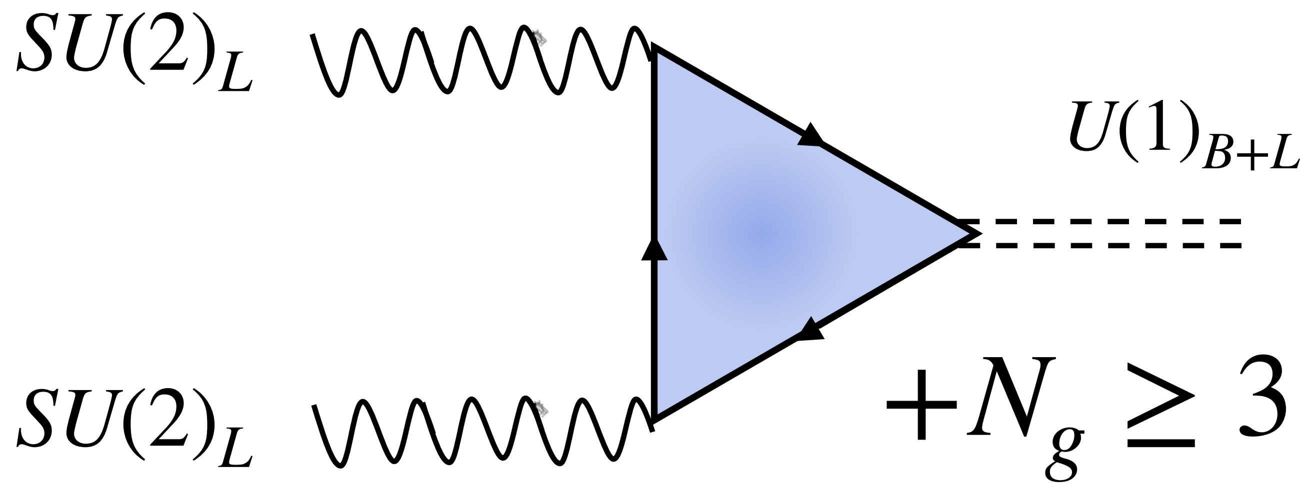
$$\beta_2^{(1)} = \frac{1}{3}\left(4N_g + \frac{1}{2} - 22\right)$$

$$\beta_2^{(1)} = \frac{1}{3}\left(\frac{20}{3}N_g + \frac{1}{2}\right)$$

$\Rightarrow$ Vertical unification possible



$\Rightarrow$ Existence of Peccei-Quinn-based explanations for strong CP



$+N_g \geq 3$

$\Rightarrow$ Existence of electroweak baryogenesis models

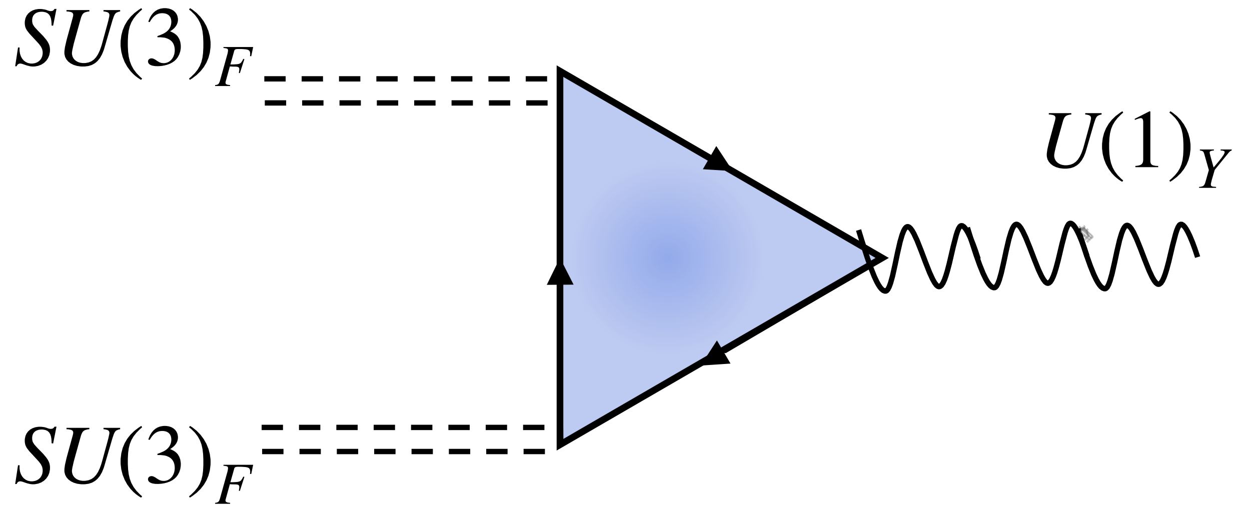
But $\mathbb{Z}_{2N_g}^{B+L}$ preserved!

See my note on proton stability 2204.01741/**Universe**

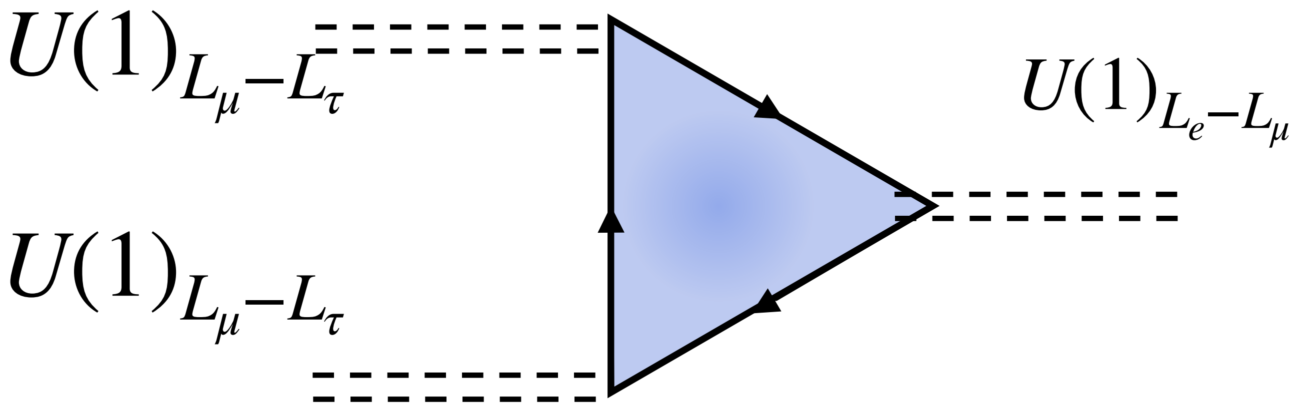
$N_g = 3$ allows symmetry-based solution to lithium problem 2204.01750/**PRL**

There's more there to understand!

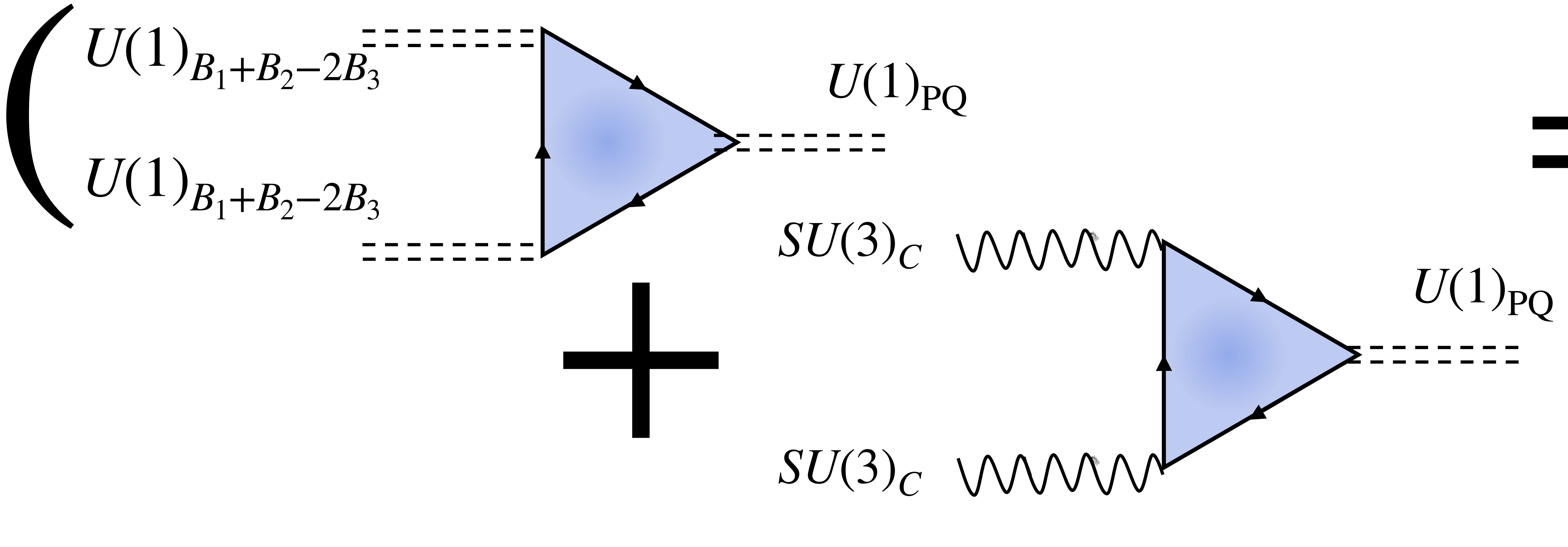
SM $\in$ Rep $\left(\left(SU(3)_C \times SU(2)_L \times U(1)_Y \right) / \mathbb{Z}_{1,2,3,6} \right) \Rightarrow$ One-form symmetry probed by searching for $e/6$ fractionally charged species
2406.17850/**SciPost Phys.** w/A. Martin



$\Rightarrow$ Flavor symmetries intertwined with hypercharge $U(1)^{(1)}_m$ in 2-group
2212.13193/**Annalen Phys.** w/C. Córdova



$\Rightarrow$ Automatically exponentially suppressed neutrino masses



$\big)/\mathbb{Z}_{N_c = N_g}$

$\Rightarrow$ Revive the simplest PQ-based solutions to strong CP

Seth's conclusions

The simple data of various integer invariants associated to the SM fermions has already lead us to new unified theories to solve SM naturalness issues based on the rigid data of the SM.

At least any place nonperturbative effects might be phenomenologically relevant, I expect paradigm of generalized global symmetries will offer better understanding.

As particle physicists we are not yet done learning about the role of symmetries!

Fig. 3: A primate pleased they newly uncovered some simple, reductionist BSM models