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Padova

A unique coupling of the massive spin-2 field to supergravity

based on **2502.09599** with G. Bossard, G. Casagrande and A. Loty

- Massive gravity and bigravity theories have cutoffs $\Lambda_n = (m^{n-1} M_P)^{\frac{1}{n}}$

- The cutoff can be raised for KK theories and string theory: infinite set of massive spin-2 fields

- SUSY massive graviton multiplet $(h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu)$: couple it to SUGRA $(g_{\mu\nu}, \psi_\mu)$

- Usually SUGRA solves problems, ex. causality in gravitino propagation. We find a result superficially orthogonal to this lore

Is the corresponding action related to a KK truncated action ? Or string theory effective action ?
Until which energy scale it is valid ?

Summary & Outline

- massive spin-2 supermultiplet $+$ $D = 4, \mathcal{N} = 1$ SUGRA

→ *Unique* at leading order in $M_{\rm Pl}$, *consistency* with local SUSY

→ *Supercurrent superfield*

→ Properties of the coupling
  - 4-derivatives coupling to $R_{\mu\sigma\nu\rho}$
  - connection with string theory

→ Unitarity and cutoff

→ Relation to Kaluza–Klein theory*

The massive spin-2 multiplet

- Massive spin-2 *supermultiplet*: $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$

[Zinoviev '02]
[Buchbinder, Gates, Linch, Phillips '02]

$$\delta A_\mu = \frac{1}{2}\bar{\epsilon}\lambda_\mu$$

$$\delta \lambda_\mu = -\frac{i}{4}\left(m h_{\mu\rho} + \tilde{F}_{\mu\rho} + 2i\partial_\rho A_\mu \gamma_5\right)\gamma_5\gamma^\rho \epsilon$$

$$\delta h_{\mu\nu} = \frac{i}{m}\bar{\epsilon}\gamma_5\left(\partial_{(\mu}\lambda_{\nu)} - m\gamma_{(\mu}\chi_{\nu)}\right)$$

$$\delta \chi_\mu = \frac{i}{4}\partial_\rho h_{\mu\sigma}\gamma^{\rho\sigma}\gamma_5\epsilon + \frac{1}{8m}\partial_\mu F_{\rho\sigma}\gamma^{\rho\sigma}\epsilon + \frac{m}{4}A_\rho\left(2\delta^\rho_\mu + \gamma^\rho{}_\mu\right)\epsilon$$

The massive spin-2 multiplet

- Massive spin-2 *supermultiplet*: $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$

[ZINOVIEV '02]
[BUCHBINDER, GATES, LINCH, PHILLIPS '02]

- Supersymmetric Fierz–Pauli lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{SFP}} =& \frac{1}{4}\partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} - \frac{1}{8}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} - \frac{1}{4}\partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{8}\partial_\mu h\partial^\mu h - \frac{m^2}{8}\left(h^{\mu\nu}h_{\mu\nu} - h^2\right) \\ &- \frac{3}{8}F^{\mu\nu}F_{\mu\nu} - \frac{3}{4}m^2 A^\mu A_\mu - \frac{1}{2}\bar{\lambda}_\mu\gamma^{\mu\nu\rho}\partial_\nu\lambda_\rho - \frac{1}{2}\bar{\chi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\chi_\rho - m\bar{\lambda}_\mu\gamma^{\mu\nu}\chi_\nu\end{aligned}$$

The massive spin-2 multiplet

- Massive spin-2 *supermultiplet*: $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$

[Zinoviev '02]
[Buchbinder, Gates, Linch, Phillips '02]

- Coupling to supergravity:

$$\{g_{\mu\nu}, \psi_\mu\} + \{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\} \longleftrightarrow T^{\mu\nu} \; \& \; \Xi^\mu$$

$\longrightarrow$ *Supercurrent superfield* S^μ

The supercurrent superfield

- Multiplets of currents: $\mathcal{S}^\mu = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X\}$

[Ferrara, Zumino '74] [Sohnius, West '81]
[Komargodski, Seiberg '10]

$$\mathcal{S}^\mu = \bar{\mathcal{S}}^\mu \quad \& \quad D^2\mathcal{S}^\mu = 0 \qquad\qquad \bar{D}^{\dot\alpha}\mathcal{S}_{\alpha\dot\alpha} = C_\alpha \sim \gamma^\mu\Xi_\mu,\ T^\mu{}_\mu,\ X$$

linear multiplet

conservation-like equation

The supercurrent superfield

- Multiplets of currents: $\mathcal{S}^\mu = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X\}$

[FERRARA, ZUMINO '74] [SOHNIUS, WEST '81]
[KOMARGODSKI, SEIBERG '10]

- Local SUSY $\longrightarrow$ $\mathcal{S}^\mu$ is the current of the SUGRA gauge fields:

$$\mathcal{L} \sim -\frac{1}{2}(g_{\mu\nu} - \eta_{\mu\nu})T^{\mu\nu} - \frac{1}{2}\bar{\psi}_\mu \Xi^\mu - \frac{1}{2}V_\mu J^\mu + C_\mu X^\mu$$

off-shell SUGRA

[FERRARA, VAN NIEUWENHUIZEN '78] [STELLE, WEST '78]
[SIEGEL, GATES '79] [SOHNIUS, WEST '81]

The supercurrent superfield

- Currents supermultiplets & off-shell SUGRA formulations:

(1) Ferrara–Zumino multiplet: $\mathcal{S}^\mu_{\rm FZ} = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X\}$ $\longleftrightarrow$ *old-minimal* off-shell SUGRA

[Ferrara, Zumino '74]

[Ferrara, van Nieuwenhuizen '78]
[Stelle, West '78]

$\partial_\mu J^\mu \neq 0$ complex scalar

(2) R-symmetry multiplet: $\mathcal{S}^\mu_{\rm R} = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X^{\mu\nu}\}$ $\longleftrightarrow$ *new-minimal* off-shell SUGRA

[Sohnius, West '81]

[Sohnius, West '81]

$\partial_\mu J^\mu_{\rm R} = 0$ antisymmetric field

The massive spin-2 supercurrent & the coupling to supergravity

The massive spin-2 supercurrent

- Supercurrent for the multiplet $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$:

⟶ R-symmetry: $\lambda_\mu \to e^{i\alpha\gamma_5}\lambda_\mu \ , \quad \chi_\mu \to e^{-i\alpha\gamma_5}\chi_\mu \ , \quad \epsilon \to e^{-i\alpha\gamma_5}\epsilon$

The massive spin-2 supercurrent

- Supercurrent for the multiplet $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$:

$\longrightarrow$ R-symmetry current: $J_0^\mu = \varepsilon^{\mu\nu\rho\sigma}\left(\bar{\lambda}_\rho\gamma_\nu\lambda_\sigma - \bar{\chi}_\rho\gamma_\nu\chi_\sigma\right)$ $\partial_\mu J_0^\mu = 0$

$\Longrightarrow$ *R-symmetry multiplet*:

[SOHNIUS, WEST '81]

$$\delta J_\mu = -i\bar{\epsilon}\gamma_5\Xi_\mu$$

$$\delta\Xi_\mu = T_{\mu\nu}\gamma^\nu\epsilon + X_{\mu\nu}\gamma^\nu\epsilon + \frac{1}{8}\varepsilon_{\mu\nu\rho\sigma}\partial^\rho J^\sigma\gamma^\nu\epsilon + \frac{i}{4}\partial_\rho J_\mu\gamma_5\gamma^\rho\epsilon$$

$$\delta T_{\mu\nu} = -\frac{1}{4}\bar{\epsilon}\gamma_{(\mu}{}^\lambda\partial_\lambda\Xi_{\nu)}$$

$$\delta X_{\mu\nu} = -\frac{i}{8}\varepsilon_{\mu\nu\rho\sigma}\bar{\epsilon}\gamma_5\gamma^\rho\gamma^\lambda\partial^\sigma\Xi_\lambda$$

new-minimal
off-shell SUGRA

The massive spin-2 supercurrent

- Supercurrent for the multiplet $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$:

$\longrightarrow$ R-symmetry current: $J_0^\mu = \varepsilon^{\mu\nu\rho\sigma}\left(\bar{\lambda}_\rho \gamma_\nu \lambda_\sigma - \bar{\chi}_\rho \gamma_\nu \chi_\sigma\right)$ $\quad \partial_\mu J_0^\mu = 0$

$\xrightarrow{\delta_{\text{SUSY}}}$ stress-energy tensor: $T_0^{\mu\nu} = T_{\mathrm{K}}^{\mu\nu} + \partial_\rho \partial_\sigma G_0^{\mu\sigma\nu\rho} + \partial_\rho B_0^{\rho\mu\nu}$ $\quad$ improvement terms $\longleftrightarrow$

[Callan, Coleman, Jackiw '69]

$\longleftrightarrow$ non-minimal couplings

$$g_{\mu\nu}\partial_\rho\partial_\sigma G_0^{\mu\sigma\nu\rho} \sim R_{\mu\sigma\nu\rho} G_0^{\mu\sigma\nu\rho} \quad \text{diff-invariant}$$

$$g_{\mu\nu}\partial_\rho B_0^{\rho\mu\nu} \sim \Gamma_{\rho\mu\nu} B_0^{\rho\mu\nu} \quad \textit{not}\ \text{diff-invariant}$$

$\Longrightarrow$ *$B^{\rho\mu\nu}$ terms do not define consistent couplings to gravity*

The unique consistent coupling

- *Improvements* to the supercurrent: [Ferrara, Zumino '74] [Komargodski, Seiberg '10]

$$\mathcal{S}_0^\mu \longrightarrow \mathcal{S}^\mu = \mathcal{S}_0^\mu + \Delta\mathcal{S}^\mu$$

multiplet of improvements

$$\longrightarrow \quad \Delta\mathcal{S}^\mu = \{\Delta J^\mu, \Delta\Xi^\mu, \Delta T^{\mu\nu}, \Delta X^{\mu\nu}\} = \frac{1}{m}\partial_\nu \mathcal{L}^{\mu\nu} + \sigma^{\mu\,\alpha\dot\beta}[D_\alpha, \bar{D}_{\dot\beta}]\mathcal{U}$$

linear superfield improvement

$$D^2\mathcal{L}^{\mu\nu} = 0$$

Parity + linear multiplet + four-derivatives

The unique consistent coupling

- Unique consistent improvements :

$$\Longrightarrow \quad J^\mu = J_0^\mu + \frac{4i}{m}\partial_\nu \left(\bar{\chi}^{[\mu}\gamma_5\lambda^{\nu]}\right) \Longrightarrow \quad \boxed{T^{\mu\nu} = T_{\mathrm{K}}^{\mu\nu} + \partial_\rho\partial_\sigma\mathcal{G}^{\mu\sigma\nu\rho}}$$

unique coupling to gravity at 4-derivatives

$$\longrightarrow \quad \mathcal{G}^{\mu\sigma\nu\rho} = \mathcal{G}_{hh}^{\mu\sigma\nu\rho} + \mathcal{G}_{AA}^{\mu\sigma\nu\rho} + \mathcal{G}_{Ah}^{\mu\sigma\nu\rho} + \mathcal{G}_{\mathrm{F}}^{\mu\sigma\nu\rho}$$

non-trivial coupling to the Riemann tensor

$$-\frac{3}{16m^2}R_{\mu\sigma\nu\rho}F^{\mu\sigma}F^{\nu\rho}$$

The full action quadratic in massive fields is

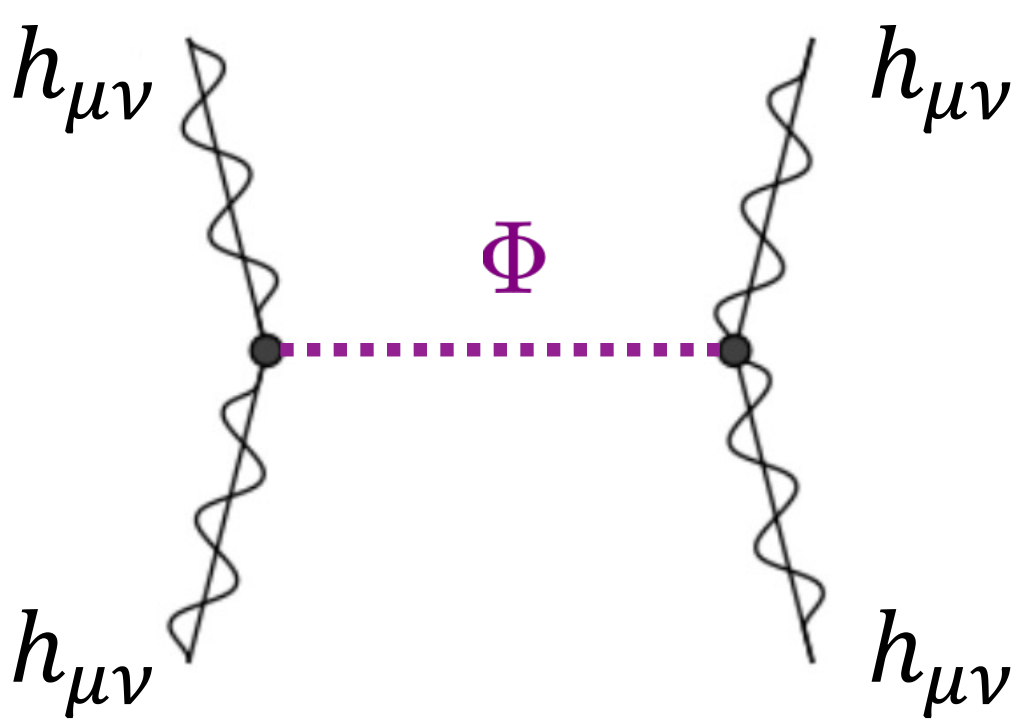
$$
\begin{aligned}
e^{-1}\mathcal{L} = {} & \frac{1}{2\kappa^2}R - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho \\
& -\frac{3}{8}F^{\mu\nu}F_{\mu\nu} - \frac{3}{4}m^2A^\mu A_\mu - \frac{1}{2}\bar{\lambda}_\mu\gamma^{\mu\nu\rho}D_\nu\lambda_\rho - \frac{1}{2}\bar{\chi}_\mu\gamma^{\mu\nu\rho}D_\nu\chi_\rho - m\bar{\lambda}_\mu\gamma^{\mu\nu}\chi_\nu \\
& +\frac{1}{4}\nabla_\mu h_{\nu\rho}\nabla^\nu h^{\mu\rho} - \frac{1}{8}\nabla_\rho h_{\mu\nu}\nabla^\rho h^{\mu\nu} - \frac{1}{4}\nabla_\mu h^{\mu\nu}\nabla_\nu h + \frac{1}{8}\nabla_\mu h\nabla^\mu h - \frac{m^2}{8}\left(h^{\mu\nu}h_{\mu\nu} - h^2\right) \\
& -\frac{e^{-1}}{4}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\Big[\left(mh_{\rho\tau} + \tilde{F}_{\rho\tau} - 2i\nabla_\tau A_\rho\gamma_5\right)\gamma^\tau\gamma_\nu\lambda_\sigma \\
& \quad + \left(\nabla_\lambda h_{\tau\rho}\gamma^{\lambda\tau}\gamma_5 - \frac{i}{2m}\nabla_\rho F_{\lambda\tau}\gamma^{\lambda\tau} + imA_\tau\left(2\delta^\tau_\rho + \gamma_\rho{}^\tau\right)\right)\gamma_\nu\gamma_5\chi_\sigma\Big] \\
& +\frac{i}{2m}\bar{\rho}_{\mu\nu}\Big[\left(mh^\nu{}_\lambda + \tilde{F}^\nu{}_\lambda - 2i\nabla_\lambda A^\nu\gamma_5\right)\gamma^\lambda\gamma_5\chi^\mu \\
& \qquad + \left(\nabla_\rho h^\mu{}_\sigma\gamma^{\rho\sigma}\gamma_5 - \frac{i}{2m}\nabla^\mu F_{\rho\sigma}\gamma^{\rho\sigma} + imA^\mu\right)\lambda^\nu\Big] \\
& +\frac{1}{4}R_{\mu\sigma\nu\rho}\left[-\frac{1}{2}h^{\rho\mu}h^{\sigma\nu} - \frac{3}{4m^2}F^{\mu\sigma}F^{\nu\rho} + \frac{e^{-1}}{m}\varepsilon^{\mu\sigma\lambda\tau}A_\lambda\nabla^\nu h^\rho{}_\tau + \frac{2}{m}\bar{\lambda}^\sigma\gamma^{\mu\rho}\chi^\nu\right] + \dots ,
\end{aligned} \tag{3.31}
$$

where $\rho_{\mu\nu} = D_\mu\Psi_\nu - D_\nu\Psi_\mu$

Higher-derivative terms, potentially dangerous !

$2 \to 2$ massive spin-2 scattering amplitude

(1) High-energy limit: $s \to \infty, \ \theta$ fixed



$\Phi =$ spin-2, spin-1, spin-0

$$\mathcal{A}(s) \sim s^5 \qquad \& \qquad \mathcal{A}(s) \geq s^3$$

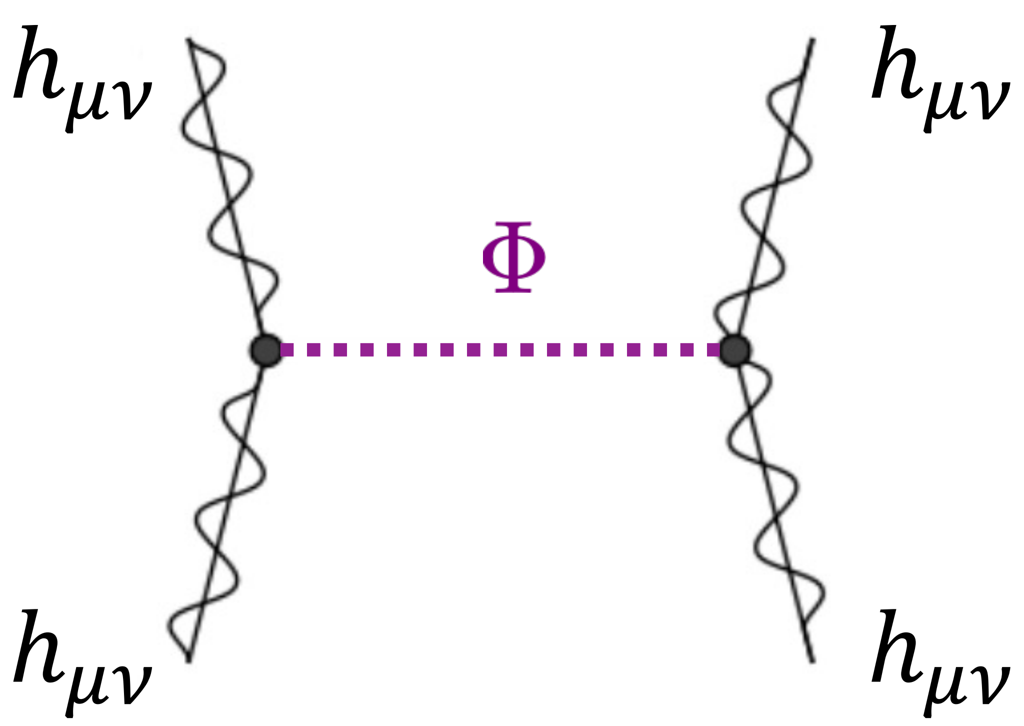
[Arkani-Hamed, Georgi, Schwartz '03]

[Bonifacio, Hinterbichler '18]
[Bonifacio, Hinterbichler, Rosen '19]

$\mathcal{A}(s) = s^3 \longleftrightarrow$ dRGT massive gravity & Hassan-Rosen bigravity

[de Rham, Gabadadze, Tolley '11] [Hassan, Rosen '12]

$2 \to 2$ massive spin-2 scattering amplitude



Φ = spin-2, spin-1, spin-0

(1) High-energy limit: $s \to \infty, \ \theta$ fixed

$$\mathcal{A}(s) \sim s^5 \qquad \& \qquad \mathcal{A}(s) \geq s^3$$

[Arkani-Hamed, Georgi, Schwartz '03]

[Bonifacio, Hinterbichler '18]
[Bonifacio, Hinterbichler, Rosen '19]

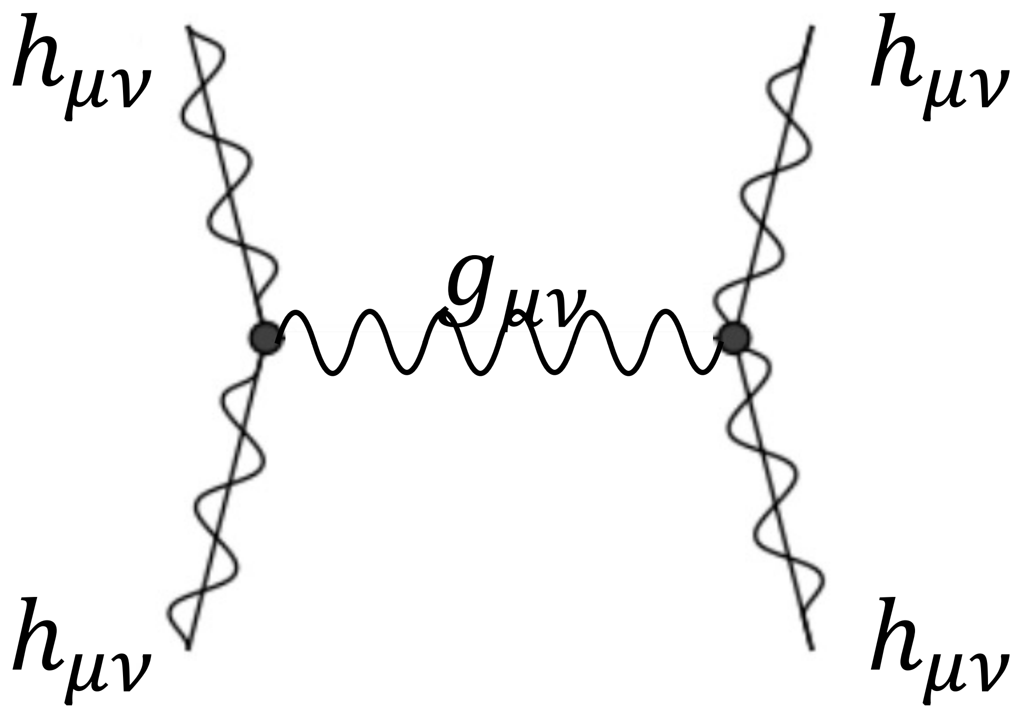
(2) Regge limit: $s \to \infty, \ t$ fixed

$$\mathcal{A}(s) \geq s^3$$

[Kundu, Palti, Quirant '23]

$2 \to 2$ massive spin-2 scattering amplitude

- Massive spin-2 scattering from the unique coupling:



$$T_h^{\mu\nu} = T_{\rm FP}^{\mu\nu} + \partial_\rho\partial_\sigma\left(-\frac{1}{2}h^{\rho[\mu}h^{\sigma]\nu} + \frac{1}{2}\eta^{\rho][\mu}h^{\sigma]}{}_\lambda h^{\lambda[\nu}\right)$$

$$\implies \quad \mathcal{A}(s) \sim s^4$$ both high-energy & Regge

$\longrightarrow$ agreement with [Arkani-Hamed, Georgi, Schwartz '03] [Bonifacio, Hinterbichler '18] [Bonifacio, Hinterbichler, Rosen '19] [Kundu, Palti, Quirant '23]

$\longrightarrow$ better behaviour than the generic s^5

$\longrightarrow$ non-minimal improvements worsen the amplitude

The unique coupling & open string theory

- Connection with *superstring theory*:

(1) $T_h^{\mu\nu} = \frac{1}{4} T_{\rm FP}^{\mu\nu} + \partial_\rho \partial_\sigma \left(-\frac{1}{2} h^{\rho[\mu} h^{\sigma]\nu} + \frac{1}{2} \eta^{\rho][\mu} h^{\sigma]}{}_\lambda h^{\lambda[\nu} \right) \equiv$ coupling of a massive *open string* state of mass $m = 1/\sqrt{\alpha'}$

[Lüst, Markou, Mazloumi, Stieberger '21]

(2) $R_{\mu\sigma\nu\rho} \mathcal{G}^{\mu\sigma\nu\rho} \supset R_{\mu\sigma\nu\rho} F^{\mu\sigma} F^{\nu\rho} \quad \longleftrightarrow$ potential violations of causality $\longleftrightarrow$ tower of higher-spin fields

[Camanho, Edelstein, Maldacena, Zhiboedov '14]

$\Longrightarrow$ Evidence for the string lamppost principle for $D = 4$ and $\mathcal{N} = 1$

[Adams, DeWolfe, Taylor '10] [Kim, Tarazi, Vafa '19] [Montero, Vafa '20] [Bedroya, Hamada, Montero, Vafa '21]

The Kaluza–Klein coupling

The Kaluza–Klein coupling

- Kaluza–Klein reduction of *5D pure supergravity on the orbifold* $S^1/\mathbb{Z}_2$:

$$\{E_M{}^A, \Psi^i_M, \mathcal{A}_M\}$$

$$\Big\downarrow \quad S^1/\mathbb{Z}_2 \quad \mathcal{N}=2 \longrightarrow \mathcal{N}=1$$

$$\{g_{\mu\nu}, \psi_\mu\} \quad + \quad \{T, \zeta\} \quad + \quad \sum_n \{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}^{(n)}$$

SUGRA multiplet — chiral multiplet — tower of massive spin-2 supermultiplets

The Kaluza–Klein coupling

- Kaluza–Klein reduction $\longleftrightarrow$ *Stückelberg organisation* of the massive degrees of freedom

$$h^{\mathrm{S}}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m}\partial_{(\mu}B_{\nu)} - \frac{1}{m^2}\partial_\mu\partial_\nu\phi \quad \longleftrightarrow \quad \delta_{\mathrm{S}} g_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}, \;\; \delta_{\mathrm{S}} B_\mu = -m\xi_\mu + \partial_\mu\Lambda, \;\; \delta_{\mathrm{S}}\phi = m\Lambda$$

5D gauge invariance

$$\longrightarrow \quad \{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\} \quad \longleftrightarrow \quad \textit{unitary gauge:} \quad B_\mu = 0 \quad \& \quad \phi = 0$$

$\longrightarrow$ unitary gauge + massive SUSY $\delta_\epsilon B_\mu = 0, \;\; \delta_\epsilon\phi = 0 \quad \Longrightarrow$ *deformation of the massless SUSY*

$\Longrightarrow$ the Kaluza–Klein coupling cannot be captured by the undeformed supercurrent superfield setup

Infinite distance paradigm

String oscillator

- long multiplet in $\mathcal{N} \geq 2$ SUSY
- unique coupling to gravity $\mathcal{S}^{\mu}$ [Bossard, Casagrande, , E.D, Loty '25]
- higher-spin tower becoming light for $m \to 0$

$\updownarrow$

tensionless limit of the string

Kaluza–Klein mode

- 1/2-BPS short multiplet in $\mathcal{N} \geq 2$ SUSY
- KK coupling to gravity deformation expected to be unique
- massive spin-2 tower becoming light for $m \to 0$

$\updownarrow$

decompactification limit

$\Longrightarrow$ realisation of the infinite distance paradigm of the swampland program
[Ooguri, Vafa '06] [Grimm, Palti, Valenzuela '18] [Lee, Lerche, Weigand '19]

Conclusions & future directions

The coupling of $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$ to *undeformed* $D = 4, \mathcal{N} = 1$ *SUGRA is unique*

but it contains higher-derivative terms.

$\longrightarrow$ **Main (conservative interpretation) result : eliminating causality issues needs higher spins with no mass gap**

- extension to a fully non-linear lagrangian $\longleftrightarrow$ propagation of ghosts ? causality violation?
- extend the improvement analysis to higher-derivative orders
- prove that the KK is (not) the *only* deformed coupling.
- Massive spin-2 field could be a DM candidate (freeze-in production)

Thank you!

Non diff-invariant improvements

$$B_0^{\rho\mu\nu} = \frac{1}{4}\left(h^{(\nu}{}_{\sigma}\partial^{\mu)}h^{\rho\sigma} - \partial^{(\mu}h^{\nu)}{}_{\sigma}h^{\rho\sigma}\right) + \frac{5}{4}\left(A^{(\nu}\partial^{\mu)}A^{\rho} - A^{\rho}\partial^{(\mu}A^{\nu)}\right)$$
$$+ \frac{1}{4m}\left(\partial^{(\mu}\tilde{F}^{\rho\sigma}h^{\nu)}{}_{\sigma} - \tilde{F}^{\rho\sigma}\partial^{(\mu}h^{\nu)}{}_{\sigma} + \tilde{F}^{(\nu}{}_{\sigma}\partial^{\mu)}h^{\rho\sigma} - \partial^{(\mu}\tilde{F}^{\nu)}{}_{\sigma}h^{\rho\sigma}\right)$$
$$+ \frac{1}{2m}\left(\partial^{(\mu}\bar{\chi}^{\nu)}\lambda^{\rho} - \bar{\chi}^{(\nu}\partial^{\mu)}\lambda^{\rho} + \partial^{(\mu}\bar{\lambda}^{\nu)}\chi^{\rho} - \bar{\lambda}^{(\nu}\partial^{\mu)}\chi^{\rho}\right) ,$$

$$\Delta B_{\mathrm{B}}^{\rho\mu\nu} = \frac{1}{8m^2}\partial_{\sigma}\left(A^{(\mu}\partial_{\sigma}\partial^{\nu)}A^{\rho} - A^{\rho}\partial^{\sigma}\partial^{(\mu}A^{\nu)}\right)$$

$$\Delta B_{\mathrm{F1}}^{\rho\mu\nu} = \frac{1}{2m}\left(\partial^{(\mu}\bar{\chi}^{\nu)}\lambda^{\rho} - \bar{\chi}^{(\nu}\partial^{\mu)}\lambda^{\rho} + \partial^{(\mu}\bar{\lambda}^{\nu)}\chi^{\rho} - \bar{\lambda}^{(\nu}\partial^{\mu)}\chi^{\rho}\right)$$
$$+ \frac{1}{2m}\left(\partial^{(\mu|}\bar{\lambda}_{\sigma}\gamma^{\rho}\gamma^{|\nu)}\chi^{\sigma} - \partial^{\rho}\bar{\lambda}_{\sigma}\gamma^{(\mu}\gamma^{\nu)}\chi^{\sigma}\right)$$

Non-minimal coupling to the Riemann tensor

$$\begin{aligned}
\mathcal{G}^{\mu\sigma\nu\rho} =& -\frac{1}{2}h^{\rho[\mu}h^{\sigma]\nu} + \frac{1}{2}\eta^{\rho][\mu}h^{\sigma]}{}_{\lambda}h^{\lambda[\nu} + 2\eta^{\rho][\mu}A^{\sigma]}A^{[\nu} - \frac{3}{4m^2}F^{\mu\sigma}F^{\nu\rho} \\
&+ \frac{1}{2m^2}\eta^{\rho][\mu}F^{\sigma]}{}_{\lambda}F^{\lambda[\nu} + \frac{1}{2m}\eta^{\rho][\mu}\left(\tilde{F}^{\sigma]}{}_{\lambda}h^{\lambda[\nu} - h^{\sigma]}{}_{\lambda}\tilde{F}^{\lambda[\nu}\right) \\
&+ \frac{1}{m^2}\left(\eta^{\sigma][\nu}F^{\rho]}{}_{\lambda}\partial^{\lambda}A^{[\mu} + \eta^{\rho][\mu}F^{\sigma]}{}_{\lambda}\partial^{\lambda}A^{[\nu}\right) \\
&+ \frac{1}{2m}\left(\varepsilon^{\mu\sigma\lambda\tau}A_{\lambda}\partial^{[\nu}h^{\rho]}{}_{\tau} + \varepsilon^{\nu\rho\lambda\tau}A_{\lambda}\partial^{[\mu}h^{\sigma]}{}_{\tau}\right) \\
&+ \frac{1}{m}\left(\bar{\lambda}^{[\sigma}\gamma^{\mu][\rho}\chi^{\nu]} + \bar{\lambda}^{[\rho}\gamma^{\nu][\sigma}\chi^{\mu]}\right) \\
&- \frac{1}{2}RU + \bar{\rho}_{\mu\nu}\gamma^{\mu\nu}\Upsilon
\end{aligned}$$

The Kaluza–Klein reduction

- 5D pure supergravity on the orbifold $S^1/\mathbb{Z}_2$:

1) 5D supergravity algebra: [Günaydin, Sierra, Townsend '83] [Ceresole, Dall'Agata '00]

$$\delta E_M{}^A = \frac{1}{2}\bar{\varepsilon}\gamma^A\Psi_M$$

$$\delta \mathcal{A}_M = -\frac{i}{2}\bar{\varepsilon}\Psi_M$$

$$\delta \Psi_M = D_M\varepsilon + \frac{i}{2}E_M{}^A\mathcal{F}_{BC}\left(\frac{1}{4}\gamma_A{}^{BC} - \delta_A{}^B\gamma^C\right)\varepsilon$$

The Kaluza–Klein reduction

- 5D pure supergravity on the orbifold $S^1/\mathbb{Z}_2$:

1) 5D supergravity algebra [Günaydin, Sierra, Townsend '83] [Ceresole, Dall'Agata '00]

2) compactification ansatz:

$$E^a = \phi^{-\frac{1}{2}} e^a$$

$$E^4 = \phi \, (dy + B)$$

$$\mathcal{A} = a \, (dy + B) + A$$

$$\Psi = \phi^{\frac{5}{4}} \lambda \, (dy + B) + \phi^{-\frac{1}{4}} \left(\psi - \frac{1}{2} e^a \gamma_a \gamma_5 \lambda \right)$$

The Kaluza–Klein reduction

- 5D pure supergravity on the orbifold $S^1/\mathbb{Z}_2$:

1) 5D supergravity algebra [GÜNAYDIN, SIERRA, TOWNSEND '83] [CERESOLE, DALL'AGATA '00]

2) compactification ansatz

3) mode decomposition of the fields along. $S^1/\mathbb{Z}_2$:

$$A_\mu(y) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \sin\left(\frac{ny}{R}\right) \qquad a(y) = a_0 + \sum_{n=1}^{\infty} a_n(x) \cos\left(\frac{ny}{R}\right)$$

The Kaluza–Klein reduction

- 5D pure supergravity on the orbifold $S^1/\mathbb{Z}_2$:

1) 5D supergravity algebra [GÜNAYDIN, SIERRA, TOWNSEND '83] [CERESOLE, DALL'AGATA '00]

2) compactification ansatz

3) mode decomposition of the fields along $S^1/\mathbb{Z}_2$

4) Stückelberg organisation of the massive degrees of freedom:

$$A^{\mathrm{S}}_\mu = A_\mu - \frac{1}{m}\partial_\mu a \quad \longleftrightarrow \quad \delta_{\mathrm{s}} A_\mu = \partial_\mu \lambda \quad \& \quad \delta_{\mathrm{s}} a = m\lambda$$

Stückelberg combinations

$$A_\mu^{(n)\,\mathrm{S}} = \phi^{(0)\,\frac{1}{2}} A_\mu^{(n)} - \frac{\phi^{(0)\,-1}}{m_n}\partial_\mu a^{(n)}$$

$$\lambda_\mu^{(n)\,\mathrm{S}} = \left(\psi_{1\mu\mathrm{L}}^{(n)} + \psi_{2\mu\mathrm{R}}^{(n)}\right) - \frac{1}{2}\gamma_\mu\left(\zeta_{1\mathrm{R}}^{(n)} - \zeta_{2\mathrm{L}}^{(n)}\right) + \frac{i}{m_n}\partial_\mu\left(\zeta_{1\mathrm{L}}^{(n)} + \zeta_{2\mathrm{R}}^{(n)}\right)$$

$$\chi_\mu^{(n)\,\mathrm{S}} = -i\left[\left(\psi_{1\mu\mathrm{R}}^{(n)} - \psi_{2\mu\mathrm{L}}^{(n)}\right) - \frac{1}{2}\gamma_\mu\left(\zeta_{1\mathrm{L}}^{(n)} + \zeta_{2\mathrm{R}}^{(n)}\right) + \frac{i}{m_n}\partial_\mu\left(\zeta_{1\mathrm{R}}^{(n)} - \zeta_{2\mathrm{L}}^{(n)}\right)\right]$$

$$g_{\mu\nu}^{(n)\,\mathrm{S}} = 2e_{(\mu}^{(0)\,a}e_{\nu)\,a}^{(n)} - \phi^{(n)}g_{\mu\nu}^{(0)} + \frac{\phi^{(0)\,\frac{3}{2}}}{m_n}\left(\partial_\mu B_\nu^{(n)} + \partial_\nu B_\mu^{(n)}\right) - \frac{2}{m_n^2}\partial_\mu\partial_\nu\phi^{(n)}$$