MasieroFest

I have known Antonio since the university years, back in 1987

He has been my Master advisor, my Ph.D. advisor, my advisor

He is responsible for indicating me the road of
astroparticle physics,
with care, kindness and wisdom

Antonio is the demonstration of a simple, but fundamental principle of life:

The best way to do physics is to be friends

Antonio is the demonstration of a simple, but fundamental principle of life:

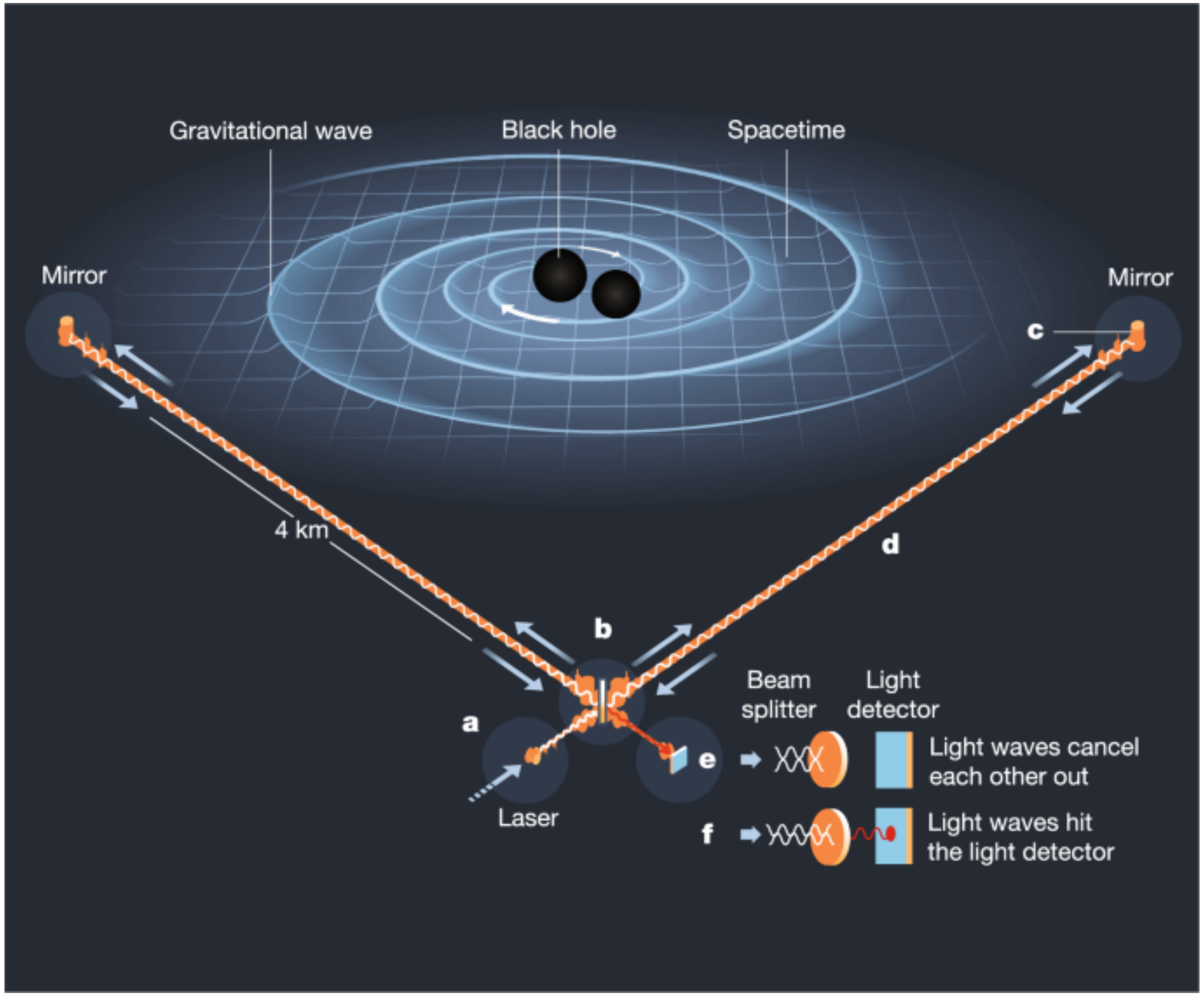
The best way to do physics is to be friends

Sometimes it is better to be *roughly* right than *precisely* wrong

Black Holes and Symmetries

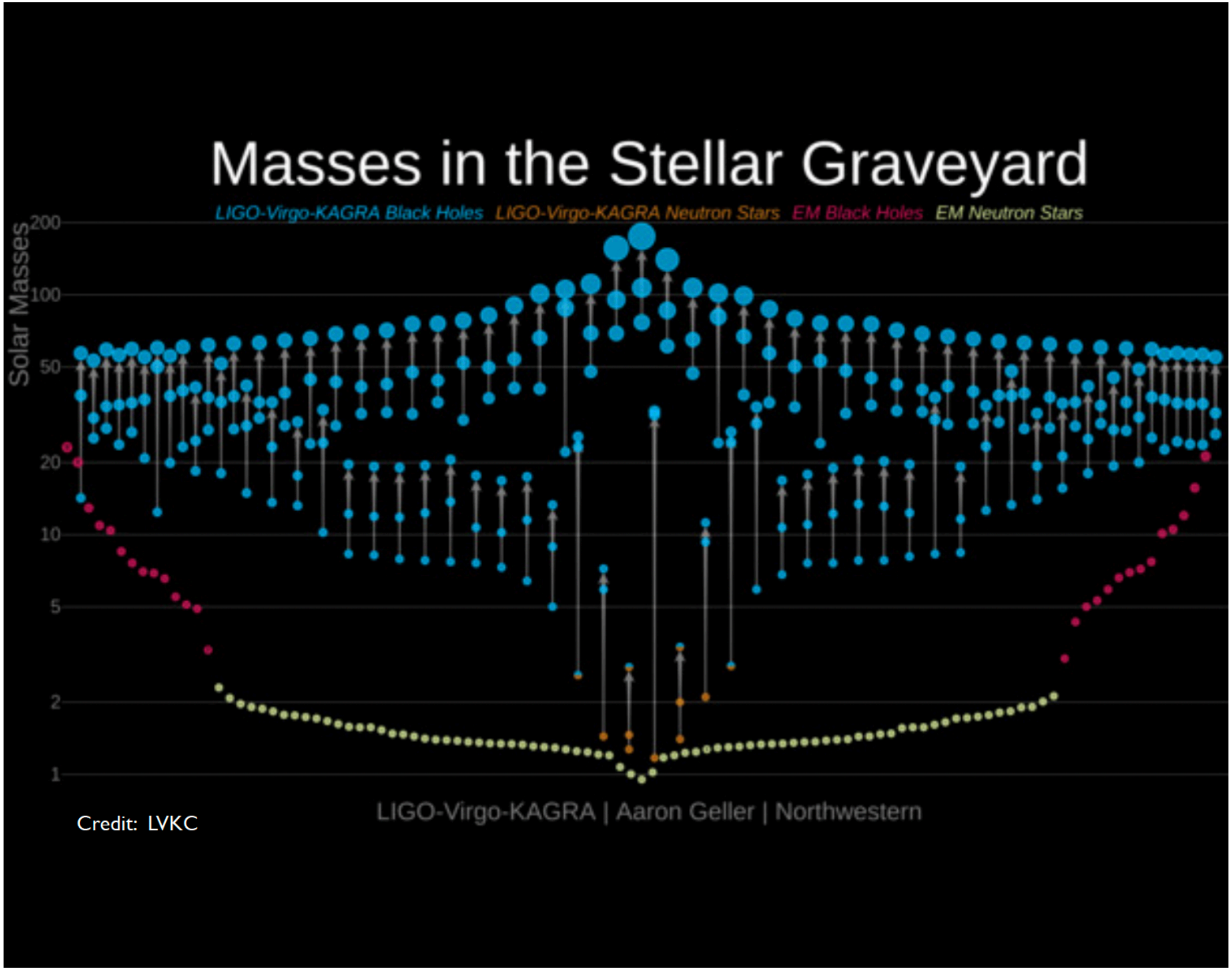
Antonio Riotto
University of Geneva and GWSC

Gravitational waves and Black Holes are key predictions of General Relativity

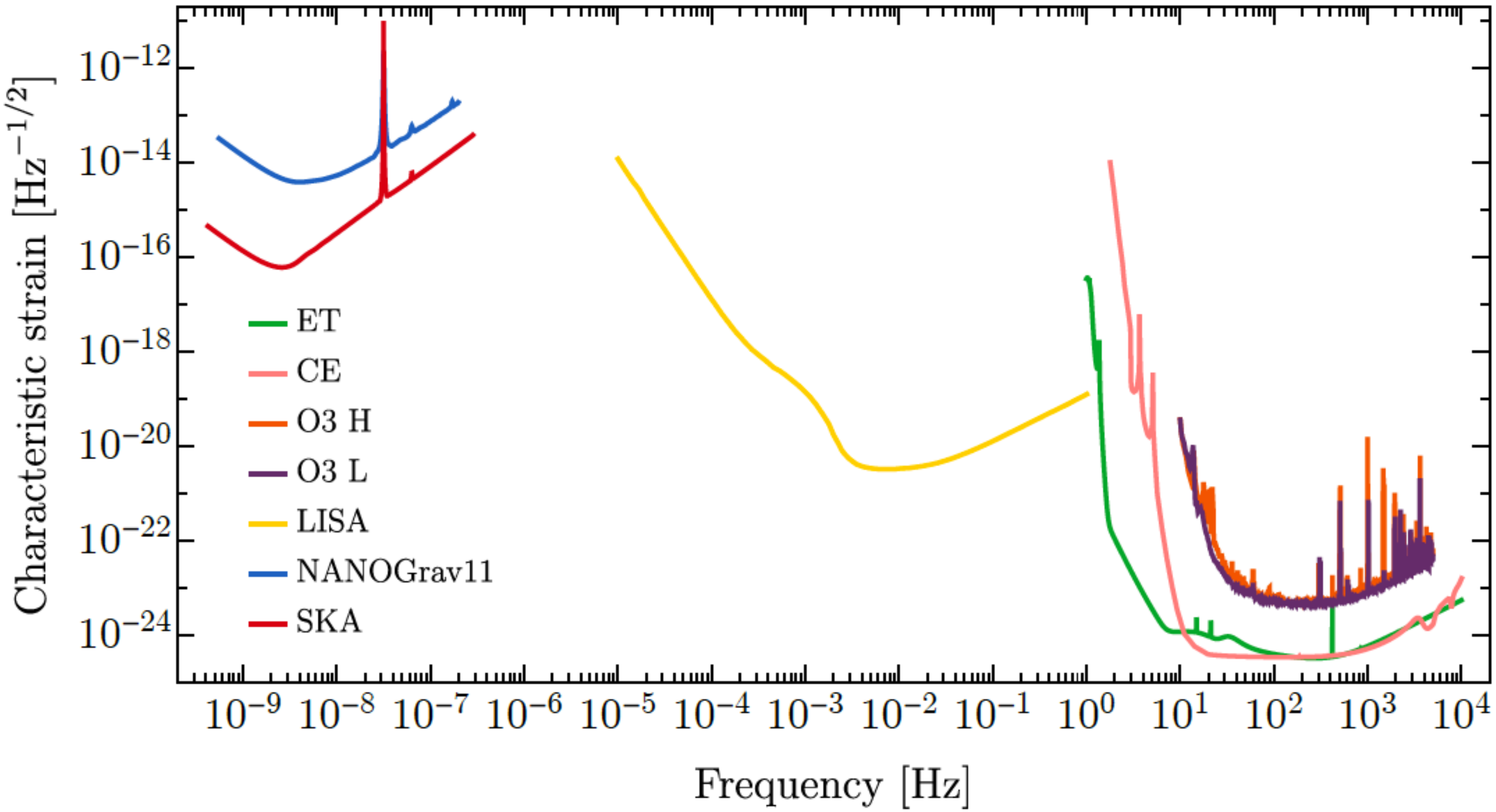


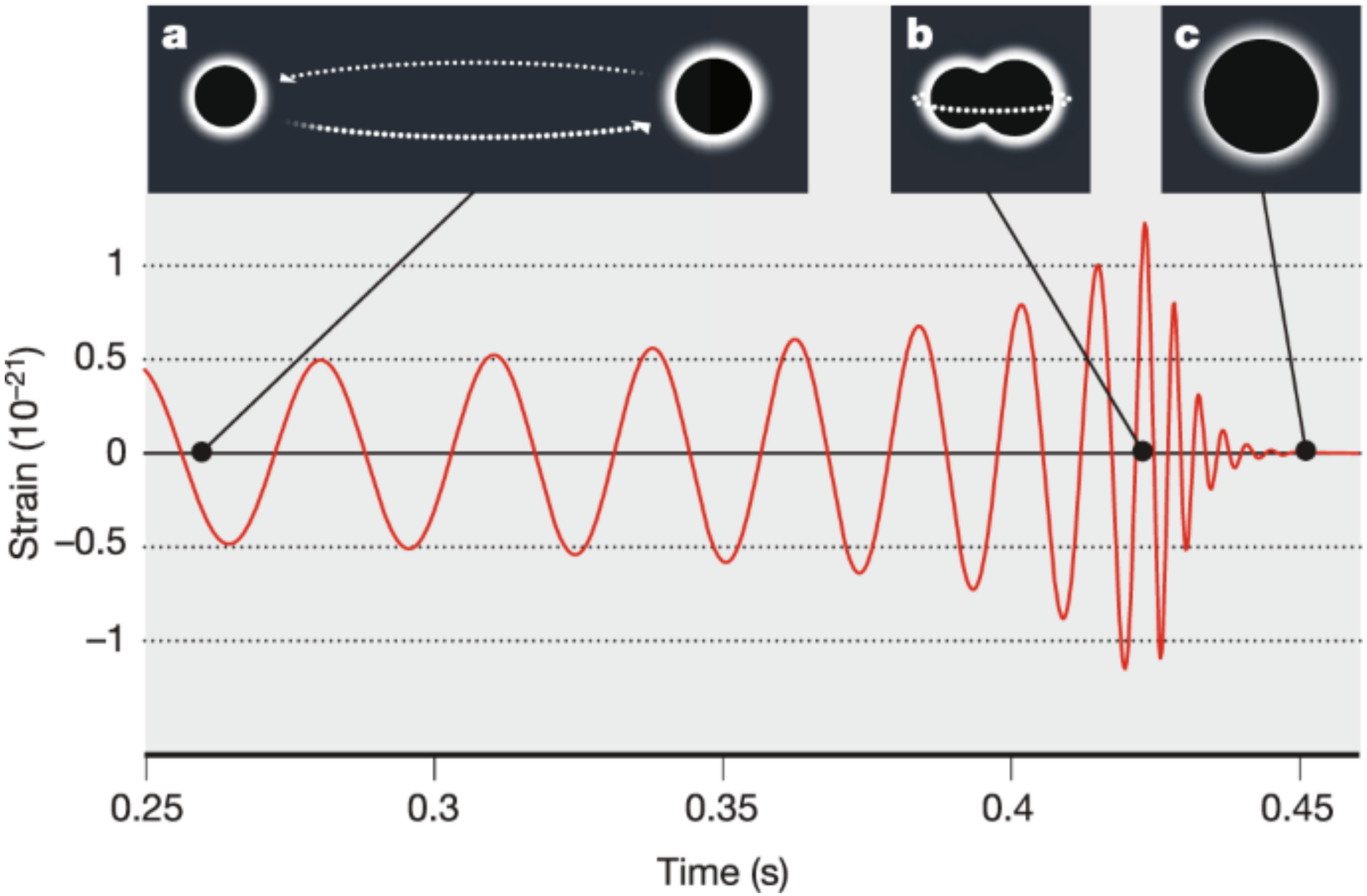
Nature 24/4/2019

Era of Gravitational Wave Astronomy



Current and future sensitivities

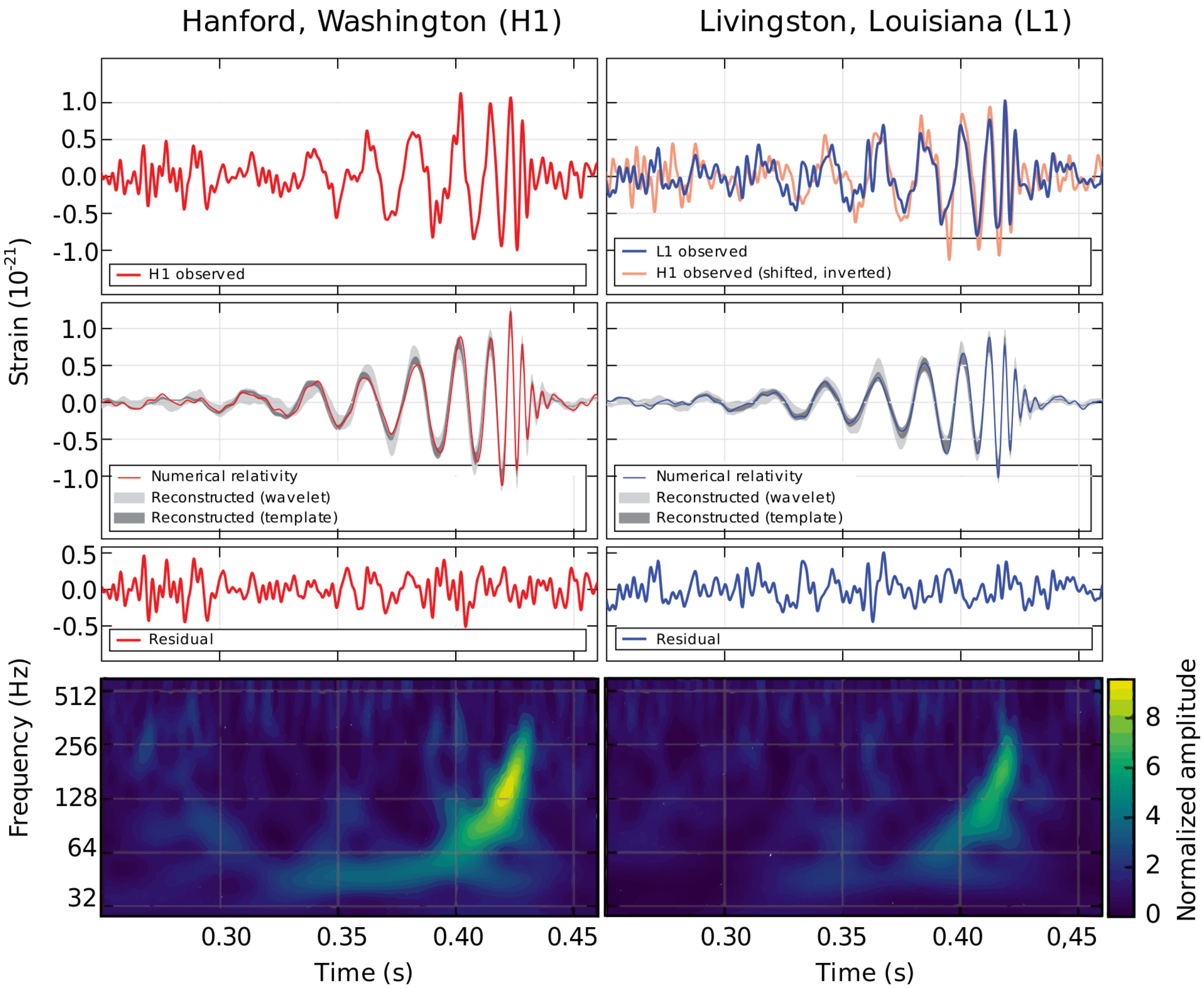




$$\text{Strain} \sim \frac{\delta L}{L} \sim h$$

Nature 24/4/2019

GW150914



Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
0.30
0.35
0.40
0.45
0,45
Time (s)
Normalized amplitude
0
2
4
6
8

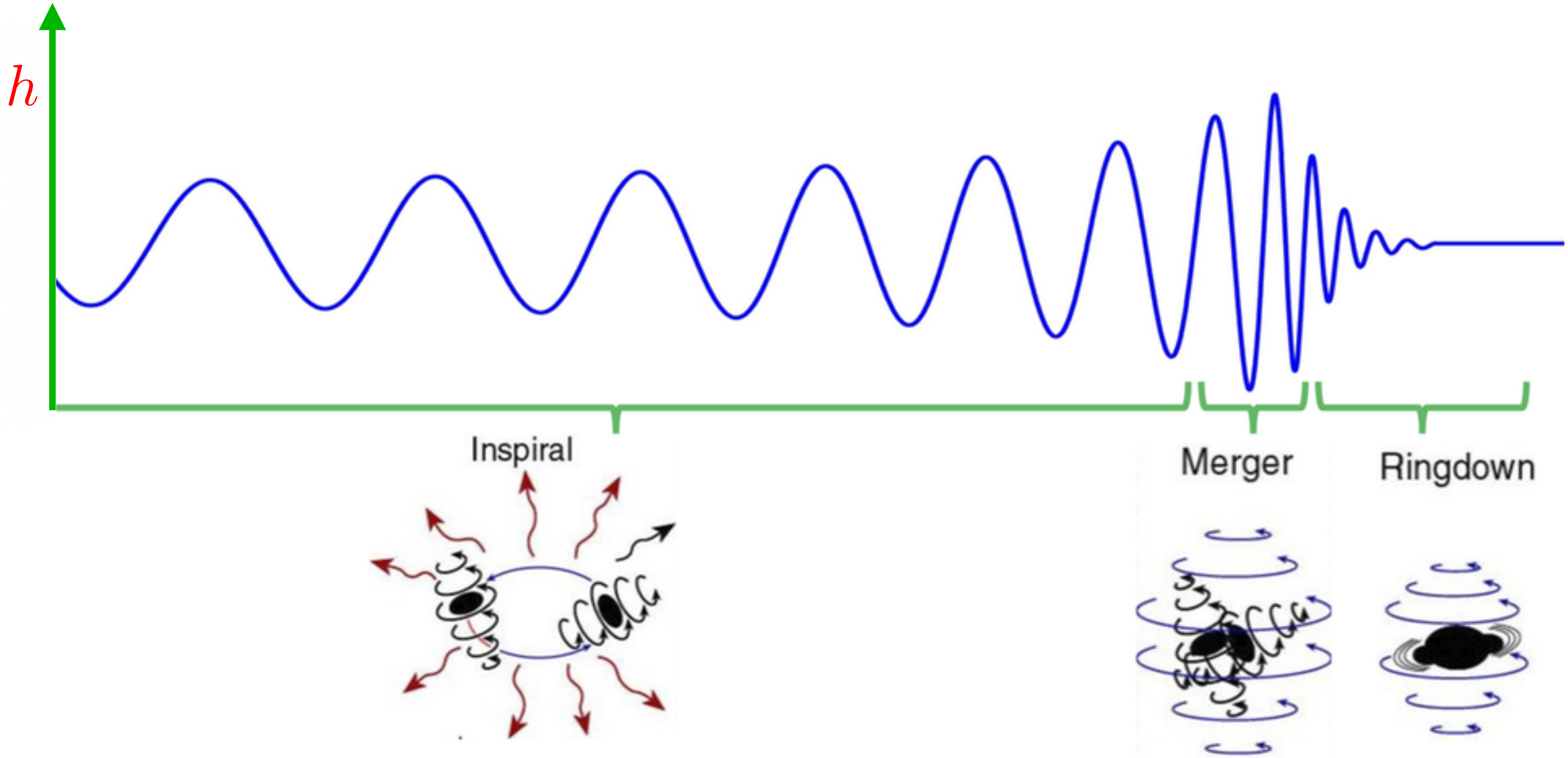
Role of Symmetries in BH observables

Two examples:

- Non-linearities in the BH ringdowns
- Static Love number

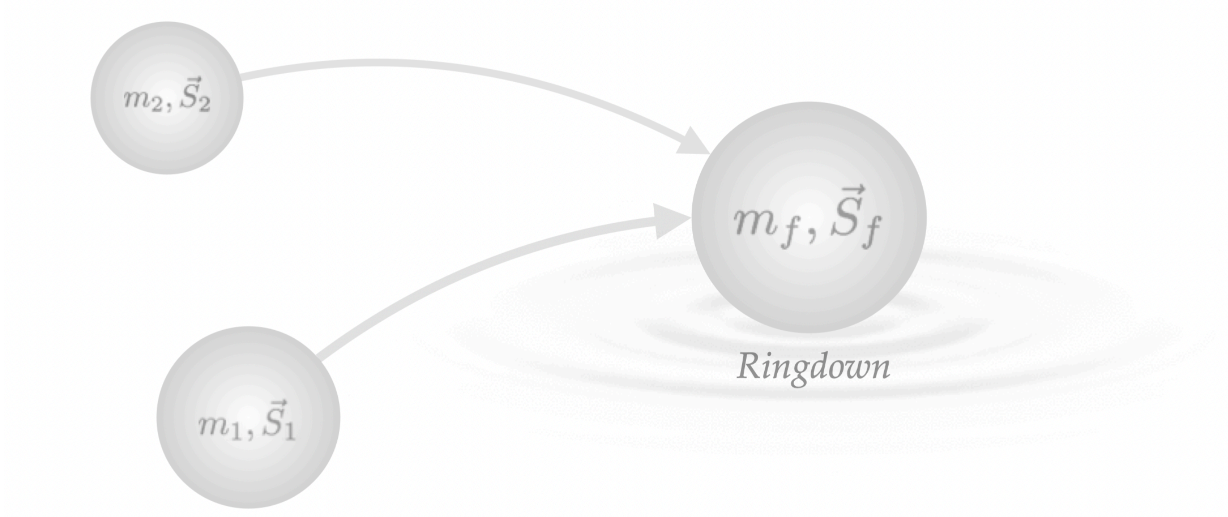
Non-linearities in the BH Ringdown

BH Quasi-Normal Modes



The BH ringdown is described by the QNMs

Black Hole Spectroscopy



QNMs dominate the BH response to any kind of disturbances
Frequencies determined by the BH mass, spin and charge

$$\omega_{(n,\ell,m)} = \mathrm{Re}\,\omega_{(n,\ell,m)} + i\,\mathrm{Im}\,\omega_{(n,\ell,m)}$$

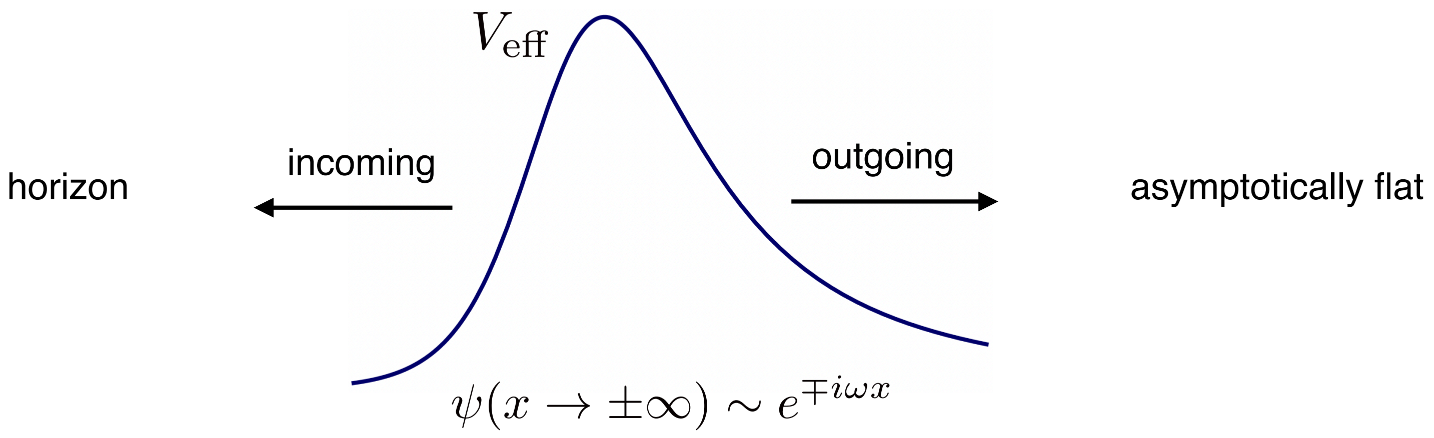
Solutions of the perturbation equations for purely outgoing GWs at infinity and purely ingoing waves at the horizon

Black Hole QNMs

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0, \;\; g_{\mu\nu} = g^{\rm bg}_{\mu\nu} + \delta g_{\mu\nu}$$

After separation of variables, decoupled master radial equation(s)

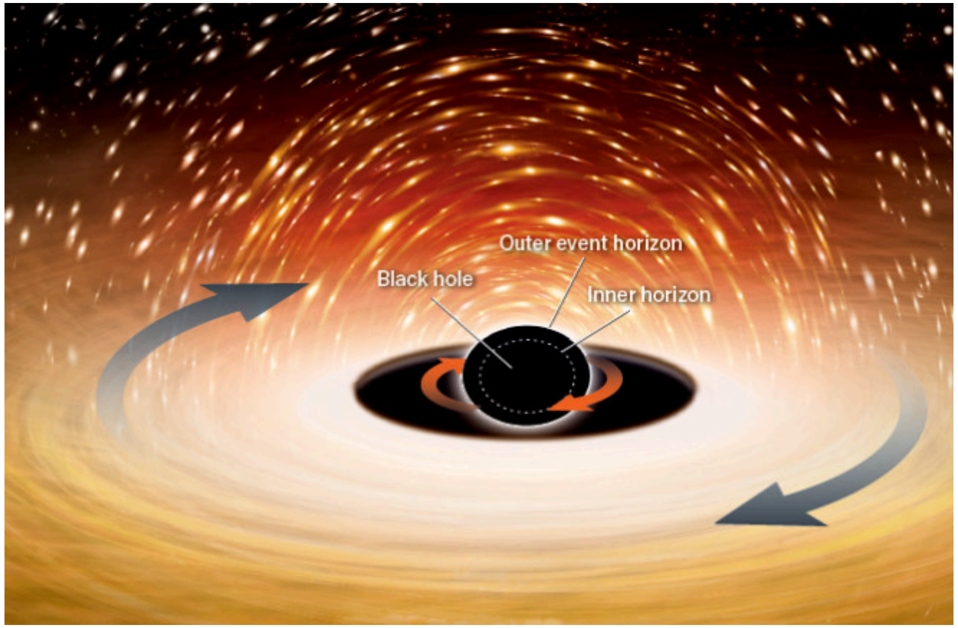
$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + (\omega^2 - V_{\rm eff})\right)\psi(x) = 0$$



Kerr Black Hole

$$ds^2 = -\frac{\Delta}{\rho^2}\left(d\hat{t} - a\sin^2\theta\right)^2 + \frac{\rho^2}{\Delta}d\hat{r}^2 + \frac{\sin^2\theta}{\rho^2}\left[(\hat{r}^2 + a^2)d\hat{\phi} - a d\hat{t}\right]^2 + \rho^2 d\theta^2$$

$$\Delta = \hat{r}^2 - 2M\hat{r} + a^2, \quad \rho^2 = \hat{r}^2 + a^2\cos^2\theta$$



$$\hat{r}_\pm = M \pm \sqrt{M^2 - a^2}$$

$$T_H = \frac{\hat{r}_+ - \hat{r}_-}{8\pi M \hat{r}_+}$$

QNMs for Kerr Black Holes

$$\Psi_s(t, r, \theta, \phi) = e^{-i\omega t} e^{im\phi} S(\theta) R(r)$$

Teukolsky separable equations

$$\Delta^{-s} \frac{\mathrm{d}}{\mathrm{d}r} \left(\Delta^{s+1} \frac{\mathrm{d}R}{\mathrm{d}r} \right) + \left(\frac{K^2 - is(\mathrm{d}\Delta/\mathrm{d}r)K}{\Delta} + 4is\omega r - \lambda_\omega \right) R = 0$$

$$\frac{1}{\sin\theta} \frac{\mathrm{d}}{\mathrm{d}\theta} \left(\sin\theta \frac{\mathrm{d}S}{\mathrm{d}\theta} \right) + \left(a^2\omega^2 \cos^2\theta - \frac{m^2}{\sin^2\theta} - 2a\omega s \cos\theta - \frac{2m\, s\cos\theta}{\sin^2\theta} - s^2 \cot^2\theta + s + A \right) S = 0$$

$$\begin{aligned} K &= (r^2 + a^2)\omega - am \\ \lambda_\omega &= A + a^2\omega^2 - 2am\omega \end{aligned}$$

Black Hole QNMs

$$h(u,\theta,\phi) = \sum_{\ell\geq 2}\sum_{|m|\leq\ell} h_{(\ell,m)}(u)\,{}_{-2}Y_{(\ell,m)}(\theta,\phi)$$

$$h_{(\ell,m)}(u) = \sum_{n\geq 0} A_{(\ell,m,n)} e^{-i\omega_{(\ell,m,n)}(u-u_{\mathrm{pk}})}$$

$$u = t - r$$

Black Hole QNMs

$M\omega$

$\ell = 2,\ n = 0$					
j	$m = 2$	$m = 1$	$m = 0$	$m = -1$	$m = -2$
0.00	.3737,.0890	.3737,.0890	.3737,.0890	.3737,.0890	.3737,.0890
0.10	.3870,.0887	.3804,.0888	.3740,.0889	.3678,.0890	.3618,.0891
0.20	.4021,.0883	.3882,.0885	.3751,.0887	.3627,.0889	.3511,.0892
0.30	.4195,.0877	.3973,.0880	.3770,.0884	.3584,.0888	.3413,.0892
0.40	.4398,.0869	.4080,.0873	.3797,.0878	.3546,.0885	.3325,.0891
0.50	.4641,.0856	.4206,.0862	.3833,.0871	.3515,.0881	.3243,.0890
0.60	.4940,.0838	.4360,.0846	.3881,.0860	.3489,.0876	.3168,.0890
0.70	.5326,.0808	.4551,.0821	.3941,.0845	.3469,.0869	.3098,.0887
0.80	.5860,.0756	.4802,.0780	.4019,.0822	.3454,.0860	.3033,.0885
0.90	.6716,.0649	.5163,.0698	.4120,.0785	.3444,.0849	.2972,.0883
0.98	.8254,.0386	.5642,.0516	.4223,.0735	.3439,.0837	.2927-.0881

E. Berti, V. Cardoso and C.M. Will (2006)

Common lore: linear perturbation theory suffices

However GR is nonlinear

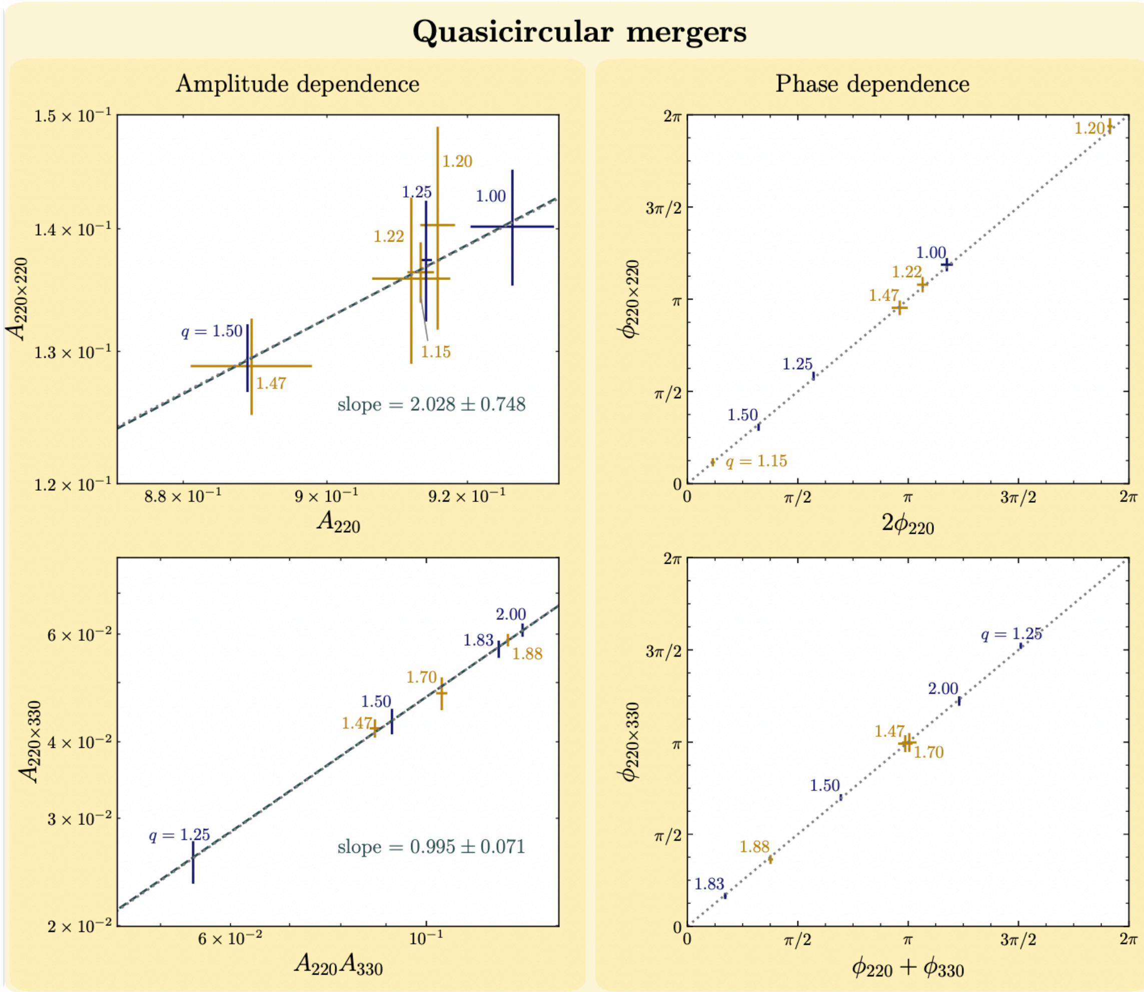
$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + (\omega^2 - V_{\mathrm{eff}})\right)\psi_2 = \mathcal{O}\left(\psi_1^2\right)$$

Black Hole QNMs

$$h(u,\theta,\phi) = \sum_{\ell\geq 2}\sum_{|m|\leq\ell} h_{(\ell,m)}(u)\,{}_{-2}Y_{(\ell,m)}(\theta,\phi)$$

$$h_{(\ell,m)}(u) = \sum_{n\geq 0} A_{(\ell,m,n)} e^{-i\omega_{(\ell,m,n)}(u-u_{\rm pk})}$$

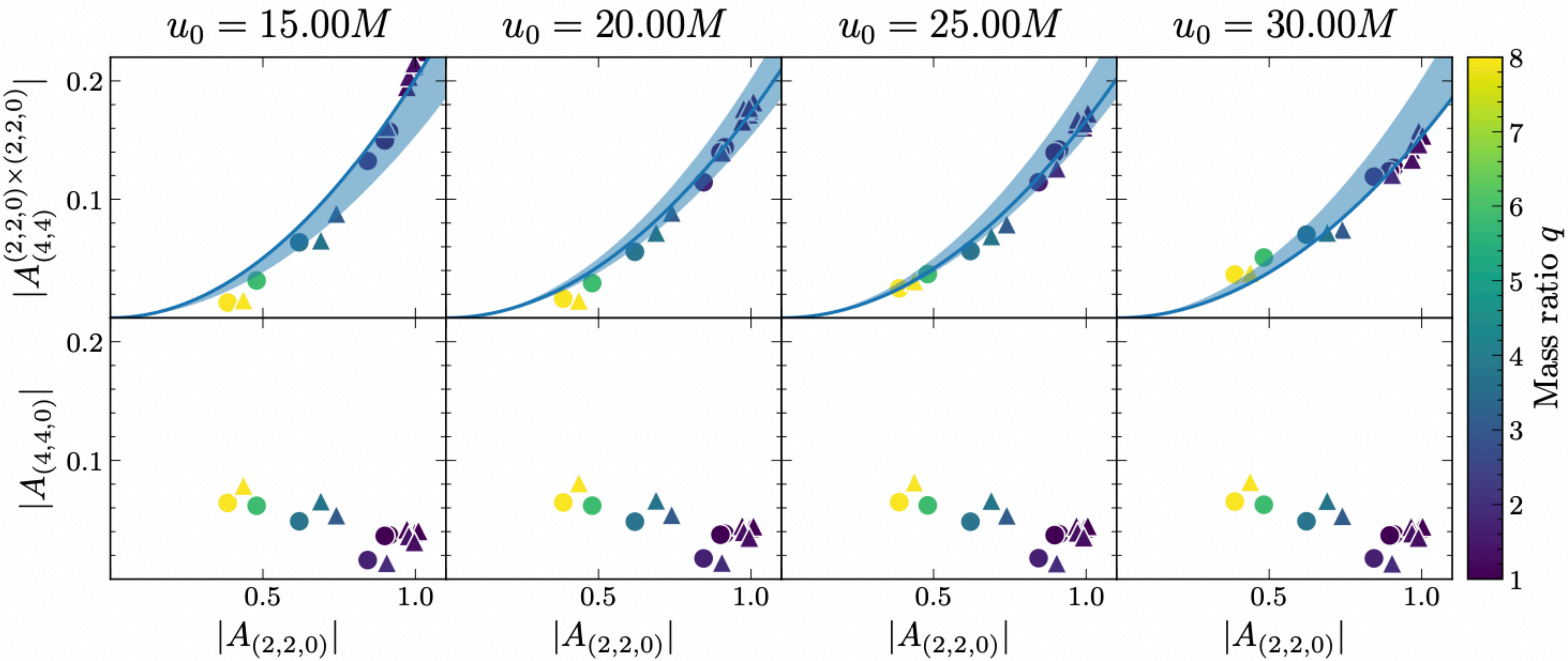
$$u = t - r$$



M. Cheung et al. (2022)

$$\frac{\left|A_{(4,4)}^{(2,2,0)\times(2,2,0)}\right|}{\left|A_{(2,2,0)}\right|^2} = 0.1637 \pm 0.0018 \qquad \frac{\left|A_{(5,5)}^{(2,2,0)\times(3,3,0)}\right|}{\left|A_{(2,2,0)}\right|\left|A_{(3,3,0)}\right|} = 0.4735 \pm 0.0062$$

Second-order amplitudes from linear amplitudes are sizeable



K. Mitman et al. (2022)

Second-order amplitudes from linear amplitudes are sizeable

A symmetry argument

QNMs are generated near the horizon

Go to the near horizon limit and extremal BH limit

$$a = M \qquad r_+ = r_- = M \qquad T_H = 0$$

A. Kehagias, D. Perrone, A.R. and F. Riva, Phys. Rev. D108 (2023), 2

Kerr/CFT Correspondence

Near Horizon geometry of Extremal Kerr (NHEK)

$$ds^2 = 2M^2\Gamma(\theta)\left[-r^2\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2} + \mathrm{d}\theta^2 + \Lambda^2(\theta)(\mathrm{d}\phi + r\,\mathrm{d}t)^2\right]$$

$$\Gamma(\theta) = \frac{1+\cos^2\theta}{2}, \quad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}$$

NHEK is a warped geometry AdS_3

Isometry group $\mathrm{SL}(2,\mathbb{R}) \otimes U(1)$

$$H_{-1} = \partial_t,\ H_0 = t\partial_t - r\partial_r,\ H_1 = \left(\frac{1}{2r^2} + \frac{t^2}{2}\right)\partial_t - tr\partial_r - \frac{1}{r}\partial_\phi,\ Q_0 = -\partial_\phi$$

M. Guica et al. (2008)

Kerr/CFT Correspondence

Asymptotic symmetry in bulk gravity
=
global symmetry of boundary field theory

Asymptotic Symmetry Group

Boundary conditions

$$h_{\mu\nu} = \begin{pmatrix} h_{tt} = \mathcal{O}(r^2) & h_{t\phi} = \mathcal{O}(1) & h_{t\theta} = \mathcal{O}(r^{-1}) & h_{tr} = \mathcal{O}(r^{-2}) \\ h_{\phi t} = h_{t\phi} & h_{\phi\phi} = \mathcal{O}(1) & h_{\phi\theta} = \mathcal{O}(r^{-1}) & h_{\phi r} = \mathcal{O}(r^{-1}) \\ h_{\theta t} = h_{t\theta} & h_{\theta\phi} = h_{\phi\theta} & h_{\theta\theta} = \mathcal{O}(r^{-1}) & h_{\theta r} = \mathcal{O}(r^{-2}) \\ h_{rt} = h_{tr} & h_{r\phi} = h_{\phi r} & h_{r\theta} = h_{\theta r} & h_{rr} = \mathcal{O}(r^{-3}) \end{pmatrix}$$

The asymptotic symmetry group contains
one copy of the conformal group of the circle generated by

$$\xi_\epsilon = \epsilon(\phi)\partial_\phi - r\epsilon'(\phi)\partial_r$$

$$\epsilon_m = -e^{-im\phi}$$

ξ_0 generates the $U(1)$ isometry
enhanced to a Virasoro algebra

Kerr/CFT Correspondence

Excitations around the NHEK depend on
the Asymptotic Symmetric Group = global symmetry of the dual theory

$$\xi_m = -e^{-im\phi}\left(imr\partial_r + \partial_\phi\right)$$

$$i[\xi_m, \xi_n]_{\text{L.B.}} = (m-n)\xi_{m+n}$$

The symmetry $U(1)$ is enhanced to a Virasoro with central charge $12J$

States in the NHEK form a representation of one copy of the Virasoro

Duality with a chiral half of a 2D finite temperature CFT

Left-movers with temperature

$$T_L = \frac{1}{2\pi}$$

M. Guica et al. (2008)

Kerr/CFT Correspondence

2D chiral CFT

$r \to \infty$

The NHEK black hole can be described by a dual chiral 2D CFT

$$Z_{\text{AdS, eff}}[\Phi] = e^{iS_{\text{eff}}[\Phi]} = \left\langle \mathrm{T}\, e^{\int_{\partial \text{AdS}} \Phi_b \mathcal{O}} \right\rangle_{\text{CFT}}$$

Boundary operator of the gravitational strain = energy-momentum tensor

Correlators fixed by the central charge

Kerr/CFT Correspondence

2D chiral CFT

$z \to w = x^1 + i\tau \to x^1 - x^0 = \phi$

$r \to \infty$

$$\langle T(z_1)T(z_2)\rangle = \frac{1}{(2\pi)^2}\frac{c/2}{z_{21}^4} \qquad \langle T(z_1)T(z_2)T(z_3)\rangle = \frac{1}{(2\pi)^3}\frac{c}{z_{21}^2\, z_{32}^2\, z_{13}^2}$$

$$z \to e^{2\pi T_L w} \quad \underset{\text{temperature}}{\overset{\text{finite}}{\Longrightarrow}} \quad T(z) \to T(w) = (\partial_z w)^2 \left(T(z) - \frac{c}{12}\{w; z\}\right)$$

Prescription

1. Calculate correlators of the 2D energy momentum tensor at finite temperature

$$\langle T_{m_1} T_{m_2} \rangle' = \frac{c}{24} \frac{(2\pi T_L)^3}{(2\pi)^2} e^{m_1/(2T_L)} \left| \Gamma\left(2 + i\frac{m_1}{2\pi T_L}\right) \right|^2$$

$$\langle T_{m_1} T_{m_2} T_{m_3} \rangle' = -\frac{c}{2\sqrt{2}} \frac{(2\pi T_L)^4}{(2\pi)^3} e^{-(m_1+m_2)/(2T_L)} G^{3,3}_{3,3}\left(\begin{array}{ccc|} -i\frac{m_1}{2\pi T_L}, & 0, & i\frac{m_2}{2\pi T_L} \\ 1 - i\frac{m_1}{2\pi T_L}, & 1, & 1 + i\frac{m_2}{2\pi T_L} \end{array} \; e^{i\pi} \right)$$

2. Find find the gravitational strain correlators on the boundary

$$\langle h_m h_{-m} \rangle' = -\frac{1}{2\,\mathrm{Re}\langle T_m T_{-m} \rangle'}, \quad \langle h_{m_1} h_{m_2} h_{m_3} \rangle' = \frac{2\,\mathrm{Re}\langle T_{m_1} T_{m_2} T_{m_3} \rangle'}{\prod_{i=1}^{3}(-2\,\mathrm{Re}\langle T_{m_i} T_{-m_i} \rangle')}$$

3. Integrate over the remaining part of the spin-with spherical harmonics along the polar angle θ

A. Kehagias, D. Perrone, A.R. and F. Riva, Phys. Rev. D108 (2023), 2

Results

$$\frac{\langle h_{(\ell_1,m_1)} h_{(\ell_2,m_2)} h_{(\ell_1+\ell_2,m_1+m_2)} \rangle}{\langle h^2_{(\ell_1,m_1)}\rangle \langle h^2_{(\ell_2,m_2)}\rangle} = \frac{6\sqrt{2}}{2\pi} \, {}_{-2}C^{m_1,m_2,m_1+m_2}_{\ell_1,\ell_2,\ell_1+\ell_2} \frac{G^{3,3}_{3,3}\left(\begin{matrix} -im_1, & 0, & im_2 \\ 1-im_1, & 1, & 1+im_2 \end{matrix} \middle| e^{i\pi} \right)}{\left|\Gamma\left(2-i(m_1+m_2)\right)\right|^2}$$

$$ {}_{-2}C^{m_1,m_2,m_3}_{\ell_1,\ell_2,\ell_3} = 2\pi \int_0^\pi \mathrm{d}\theta \, \sin\theta \, {}_{-2}Y_{(\ell_1,m_1)} {}_{-2}Y_{(\ell_2,m_2)} {}_2\overline{Y}_{(\ell_3,m_3)} \qquad T_L = 1/2\pi$$

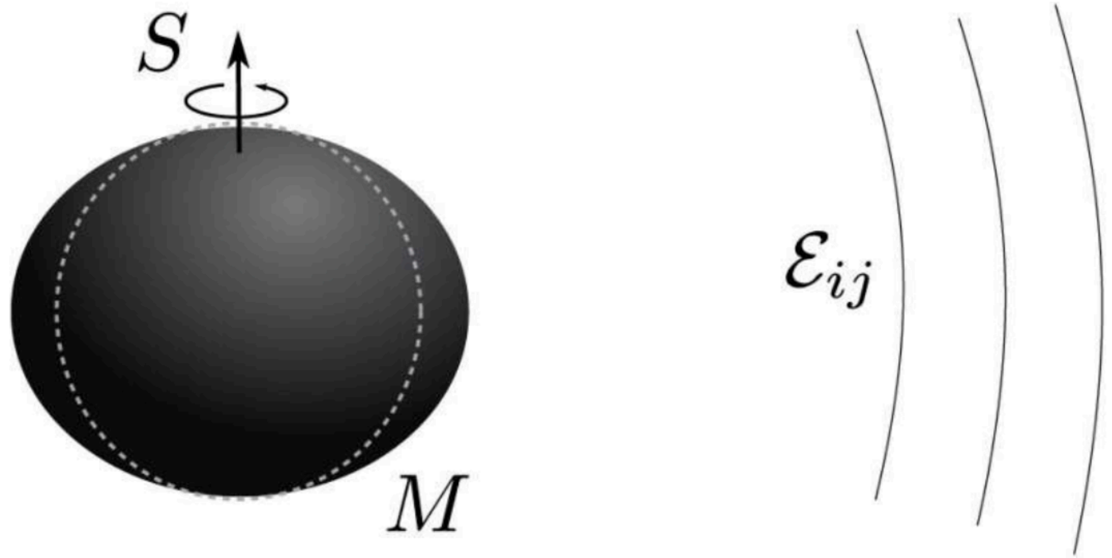
$$= \frac{\Gamma\left(-2+\sum_{i=1}^{3}\frac{|m_i|}{2}\right)\Gamma\left(4+\sum_{i=1}^{3}\frac{|m_i|}{2}\right)}{2\sqrt{\pi}\,\Gamma\left(2+\sum_{i=1}^{3}|m_i|\right)} \left(\prod_{i=1}^{3}\frac{(2|m_i|+1)!}{(|m_i|+2)!\,(|m_i|-2)!}\right)^{1/2}$$

$$\frac{\langle h_{(2,2)} h_{(2,2)} h_{(4,4)}\rangle}{\langle h^2_{(2,2)}\rangle^2} \simeq 0.62 \cdot \frac{5}{24}\sqrt{\frac{7}{\pi}} \simeq 0.17 \quad \text{vs} \quad \frac{\left|A^{(2,2,0)\times(2,2,0)}_{(4,4)}\right|}{\left|A_{(2,2,0)}\right|^2} = 0.1637 \pm 0.0018$$

$$\frac{\langle h_{(2,2)} h_{(3,3)} h_{(5,5)}\rangle}{\langle h^2_{(2,2)}\rangle\langle h^2_{(3,3)}\rangle} \simeq 1.57 \cdot \frac{2}{3}\sqrt{\frac{7}{11\pi}} \simeq 0.47 \quad \text{vs} \quad \frac{\left|A^{(2,2,0)\times(3,3,0)}_{(5,5)}\right|}{\left|A_{(2,2,0)}\right|\left|A_{(3,3,0)}\right|} = 0.4735 \pm 0.0062$$

A. Kehagias, D. Perrone, A.R. and F. Riva, Phys. Rev. D108 (2023), 2

BH Static Love Number



$$\mathcal{E}_{ij} \sim \mathcal{E}_\ell \, r^\ell \Rightarrow h_{ij} \sim \mathcal{E}_\ell \, r^\ell + \frac{\mathcal{L}_\ell}{r^{\ell+1}}$$ (= static Love number)

The Love number enters at the fifth PN order order

At the linear order the Love number of the Schwarzschild BH vanishes because of ladder symmetries

L. Hui et al. (2021), P. Charalambous (2021), M. Ivanov et al. (2022)

The static Love number of the Schwarzschild BH vanishes at *any order* in perturbation theory due to a *non-linear ladder symmetry*

The static axisymmetric vacuum spacetime in Weyl coordinates

$$\mathrm{d}s^2 = -e^{2U}\mathrm{d}t^2 + e^{-2U}e^{2k}\left(\mathrm{d}\rho^2 + \mathrm{d}z^2\right) + \rho^2 e^{-2U}\mathrm{d}\phi^2$$

$$\nabla^2 U = 0$$

$$\partial_\rho k = \rho\left[(\partial_\rho U)^2 + (\partial_z U)^2\right],\ \ \partial_z k = 2\rho\,\partial_\rho U \partial_z U.$$

A. Kehagias and A.R., gr-qc/2410.11014

In prolate spheroidal coordinates

$$\rho = \rho_0 (x^2 - 1)^{1/2} (1 - y^2)^{1/2}$$

$$z = \rho_0 x y$$

$$U(x, y) = \sum_{\ell} U_\ell(x) Y_\ell(y)$$

Ladder operators

$$L_\ell^+ = -(x^2 - 1)\frac{\mathrm{d}}{\mathrm{d}x} - (\ell + 1)x$$

$$L_\ell^- = (x^2 - 1)\frac{\mathrm{d}}{\mathrm{d}x} - \ell x,$$

$$P_\ell = (x^2 - 1)\frac{\mathrm{d}}{\mathrm{d}x} \left(L_1^- L_2^- \cdots L_\ell^-\right) U_\ell$$

$$\frac{\mathrm{d}P_\ell}{\mathrm{d}x} = 0$$

Very far from and very close to the BH

$$U_\ell(x \to \infty) \sim \frac{\mathcal{L}_\ell}{x^{\ell+1}} \Rightarrow P_\ell \neq 0 \Rightarrow U_\ell(x \to 1) \sim \ln(x-1)$$

The decaying mode diverges at the horizon and is unphysical

The full non-linear Love number vanishes because of symmetry arguments:

The ladder operators are part of the generators of the Geroch group

Conclusions

- Symmetries play a fundamental role in BH physics
- Related observables will be measured in the next future