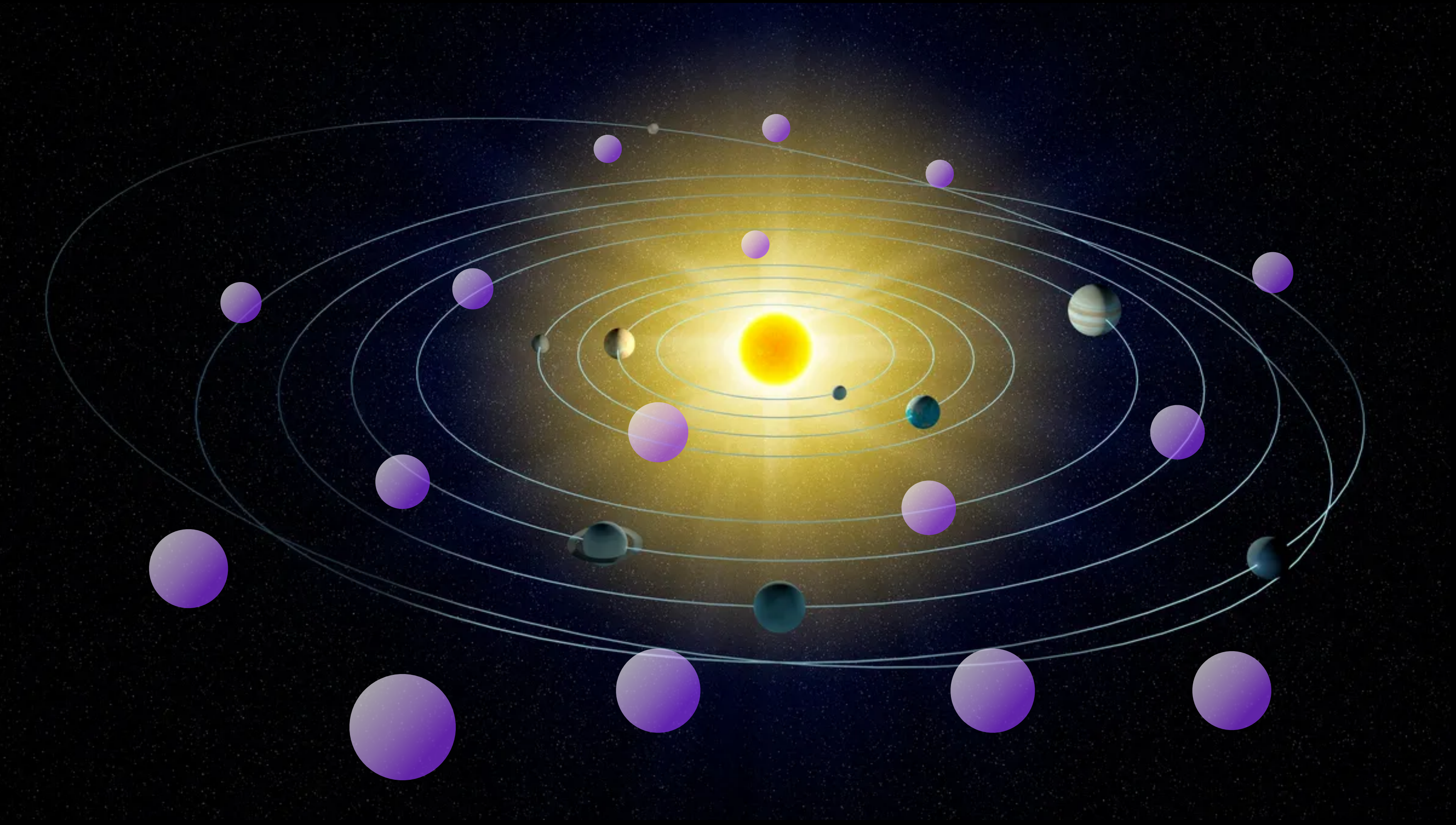
Light dark matter bound states:
News on axion stars

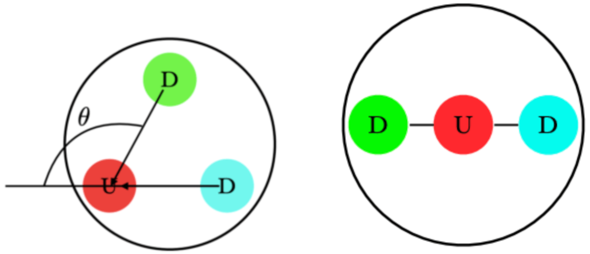


Marco Gorghetto

- MG, E.Hardy, G.Villadoro [2405.19389]
- (D.Budker, J.Eby, MG, M.Jiang, G.Perez [2306.12477])

QCD axion:

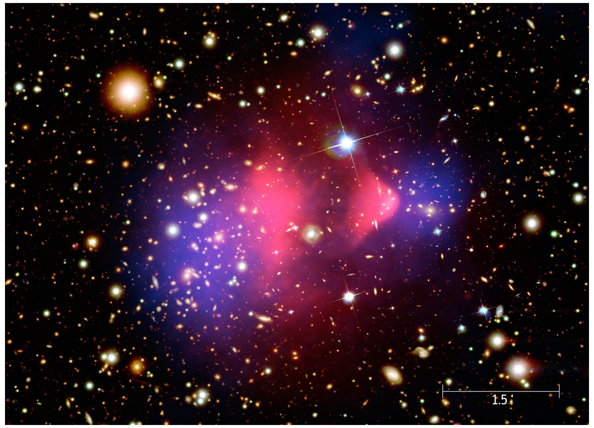
$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial a)^2 + \frac{a}{\boxed{f_a}}\frac{\alpha_s}{8\pi}G\tilde{G} + \dots \qquad \Rightarrow \qquad m = \frac{\chi_{\rm top}^{1/2}}{f_a} \simeq 0.57\,{\rm meV}\left(\frac{10^{10}\ {\rm GeV}}{f_a}\right)$$



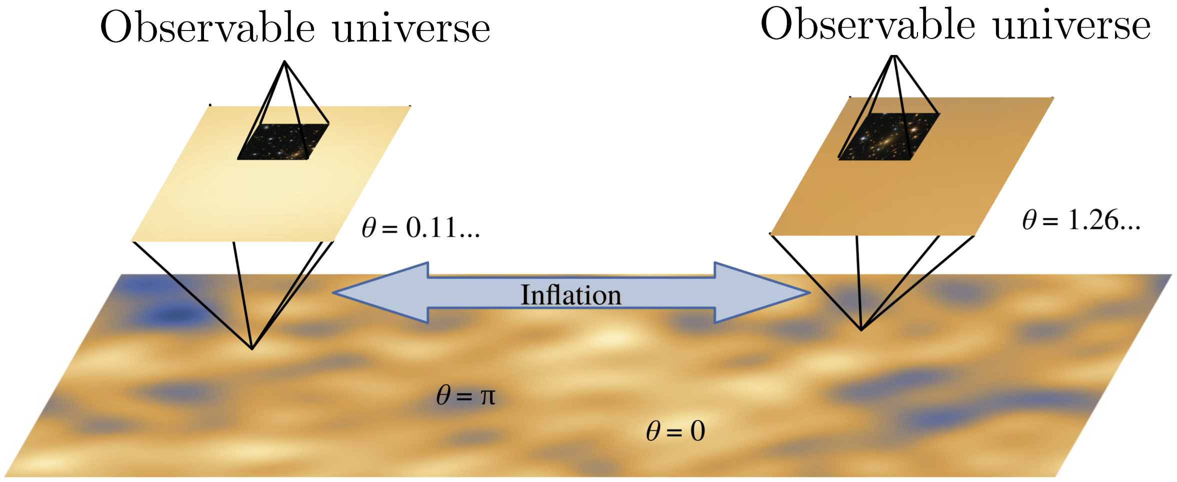
[picture from A. Hook]

- Dynamically explains no neutron EdM

- Contributes to all/part of the dark matter



Pre-inflationary

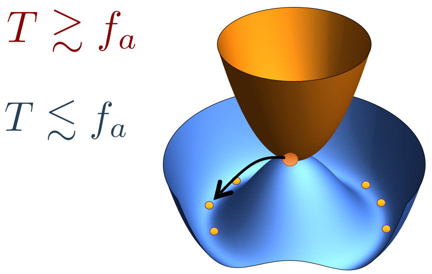


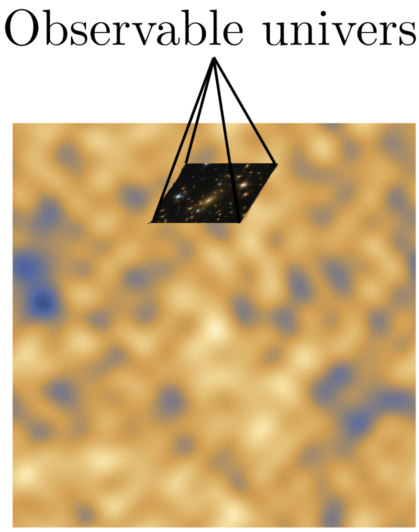
$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi]$

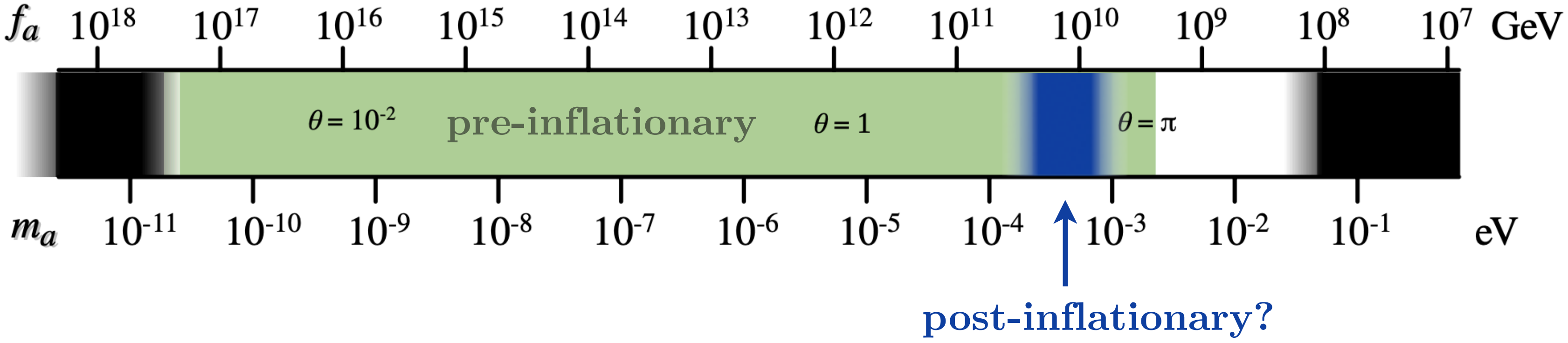
$$\Omega_a \simeq \theta_0^2 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{1.2} \Omega_{\mathrm{DM}}$$

misalignment

Post-inflationary

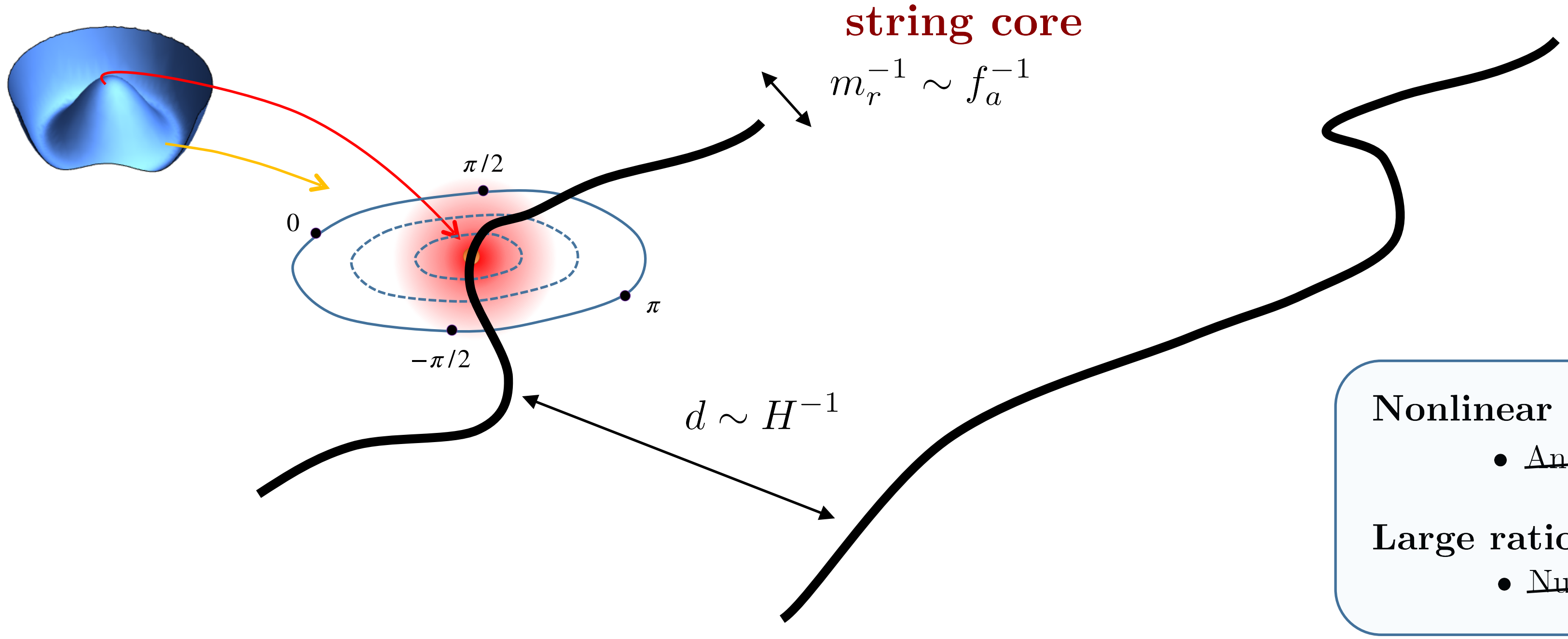






@ $T \simeq f_a$ (or $H \simeq f_a$)

Kibble mechanism $\Longrightarrow$ **Axion strings**



string core

$m_r^{-1} \sim f_a^{-1}$

$d \sim H^{-1}$

Nonlinear dynamics:

- ~~Analytical approach~~ ☹

Large ratio of scales:

- ~~Numerical approach~~ ☹

string tension

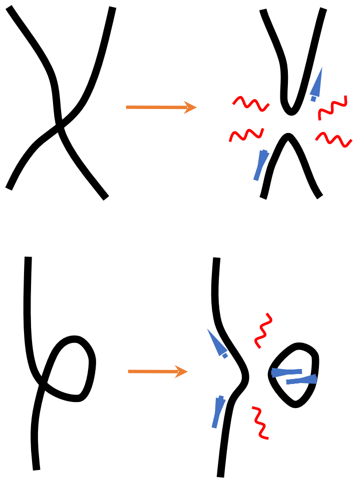
$$\mu = \frac{E}{L} \sim \underbrace{\boxed{\pi f_a^2}}_{\text{core}} \underbrace{\boxed{\log \frac{d}{m_r^{-1}}}}_{\text{axion gradient}} \sim \pi f_a^2 \log \frac{m_r}{\boxed{H}}$$

$H \rightarrow T^2/M_p$

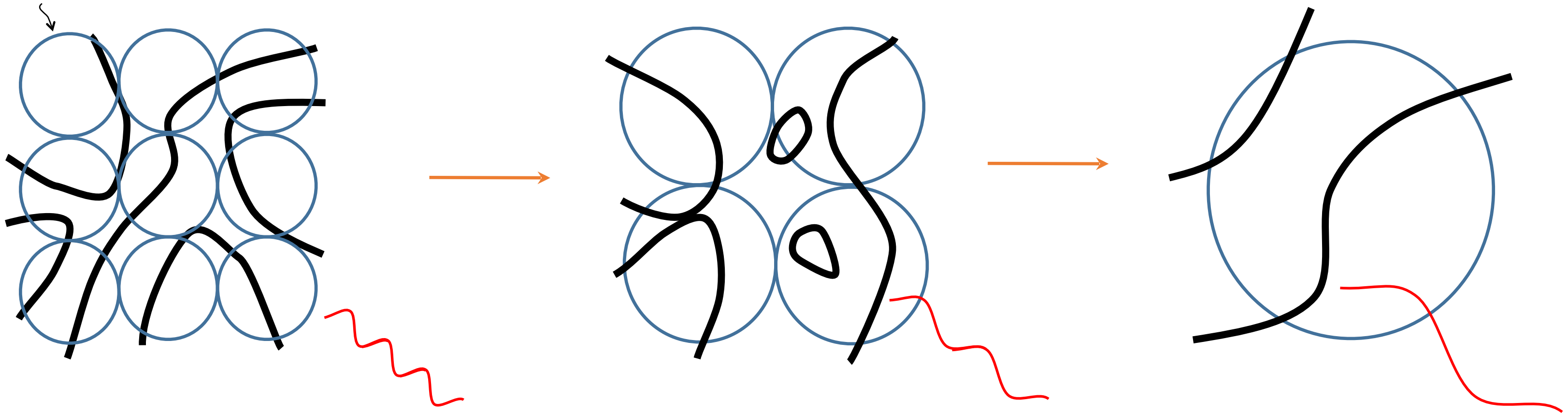
↑

grows logarithmically in time

The Scaling Regime



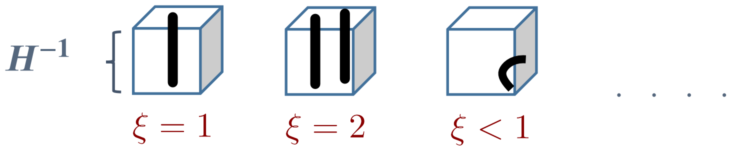
causal patch $\propto 1/H = 2t$

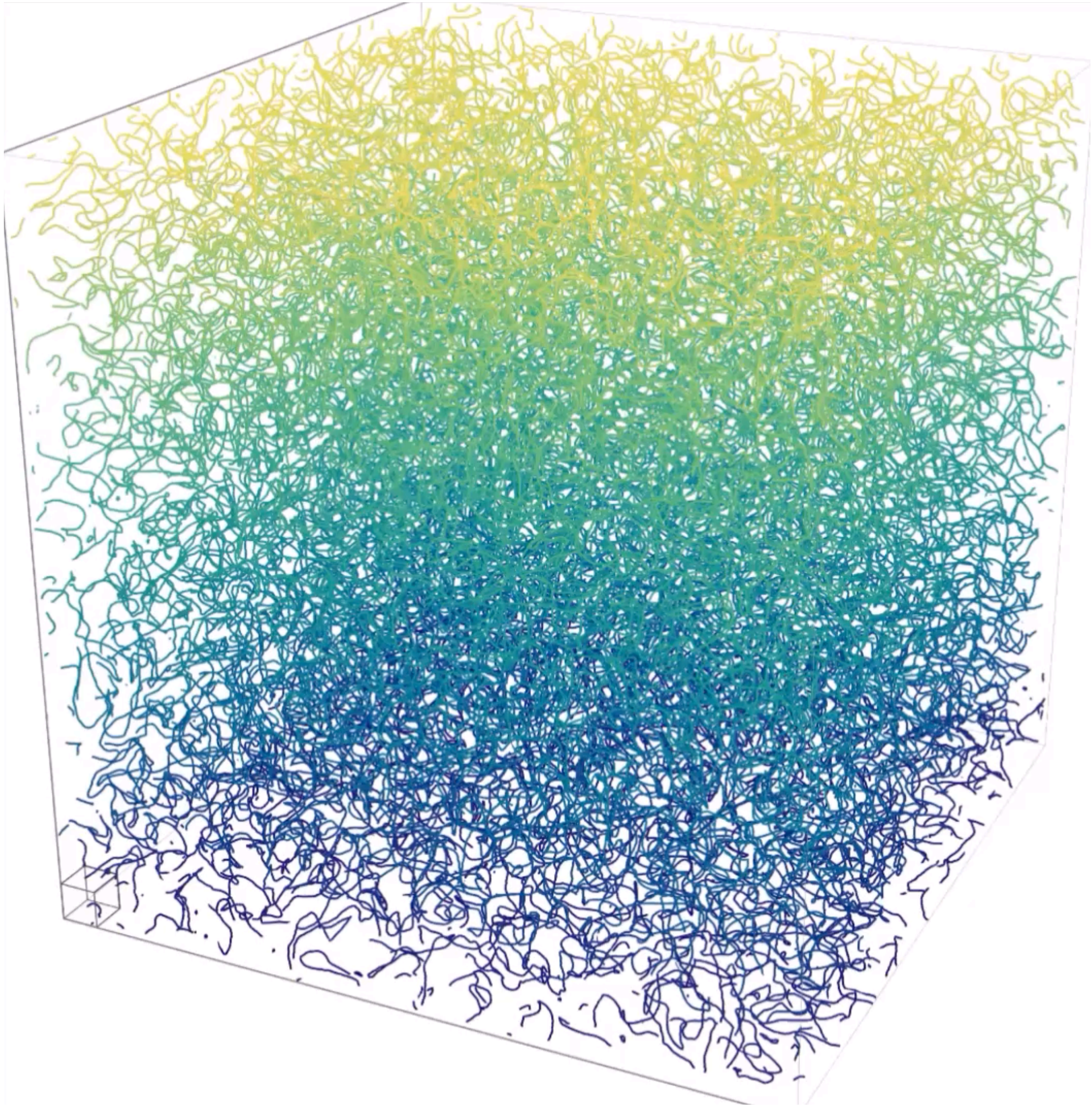


rate of energy loss:

$$\Gamma \simeq \frac{\boxed{\xi}\mu}{t^3}$$

number of strings per Hubble patch

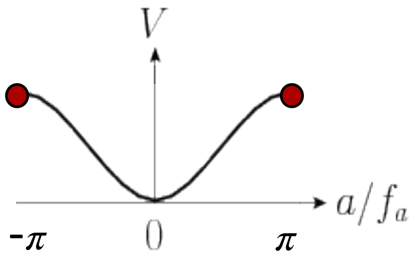


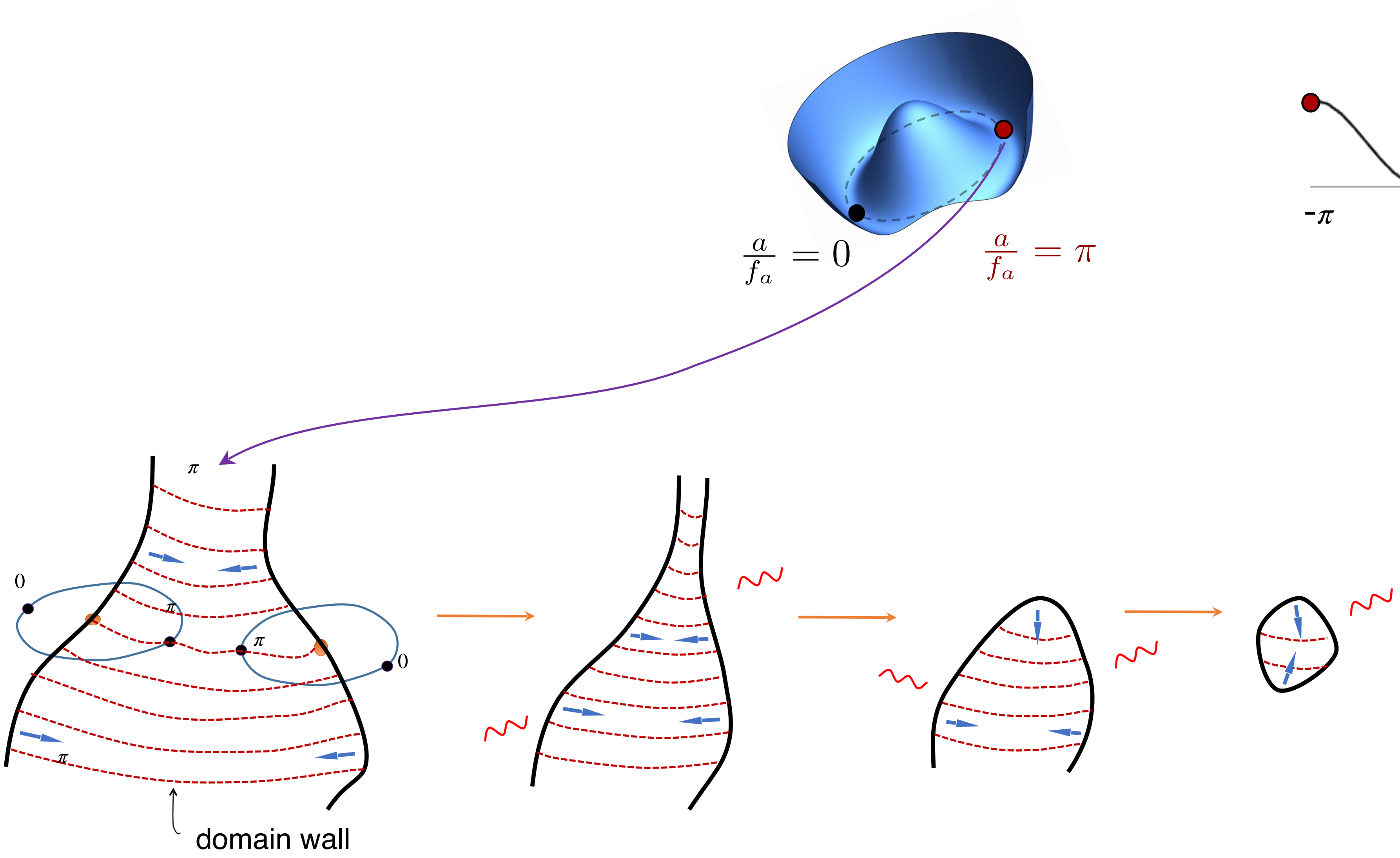


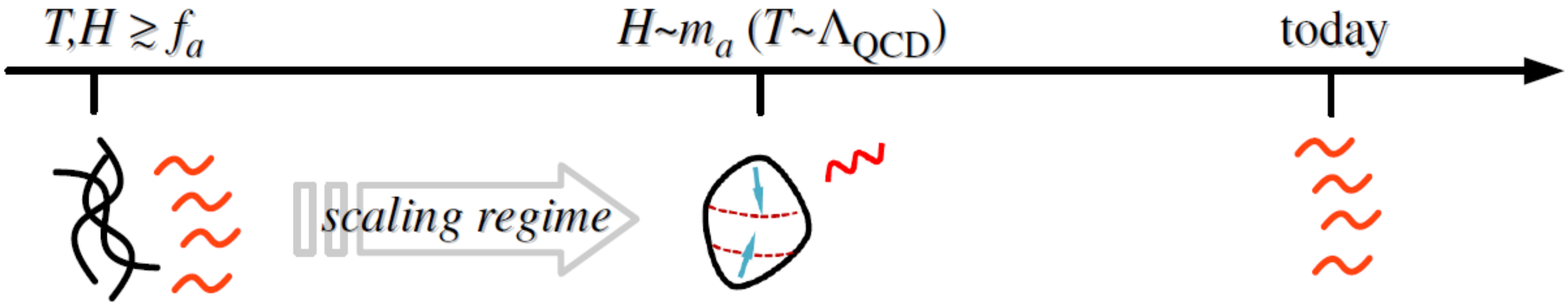
Domain Walls

$@\, T \simeq 1\,\mathrm{GeV} \quad (\, m = H \equiv H_\star \,)$

Axion potential from QCD:







$T,H \gtrsim f_a$ $H \sim m_a\ (T \sim \Lambda_{QCD})$ today

$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ $\sim \mathbf{70}$

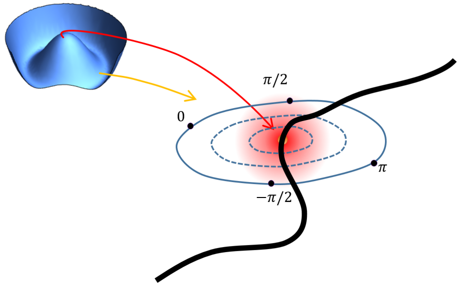
strings form

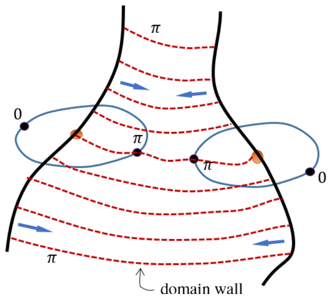
domain walls form and annihilate

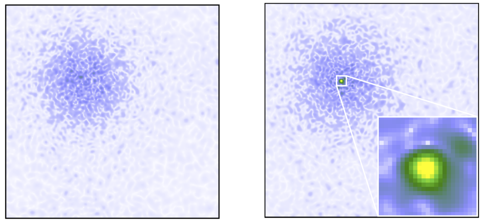
relic axions

strings **domain walls**

axion stars miniclusters

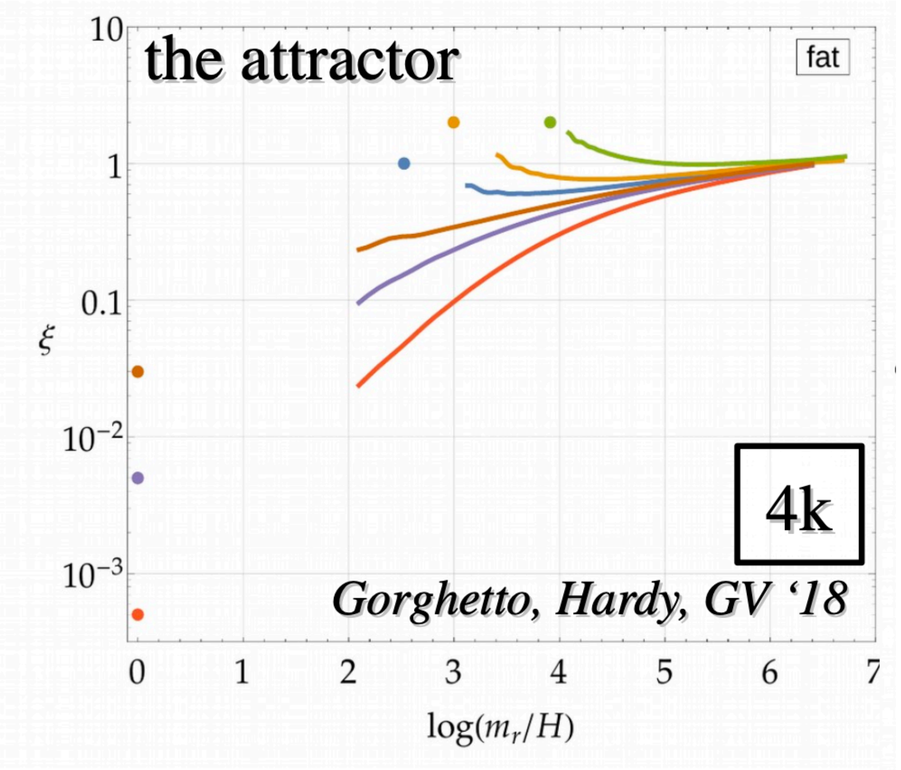


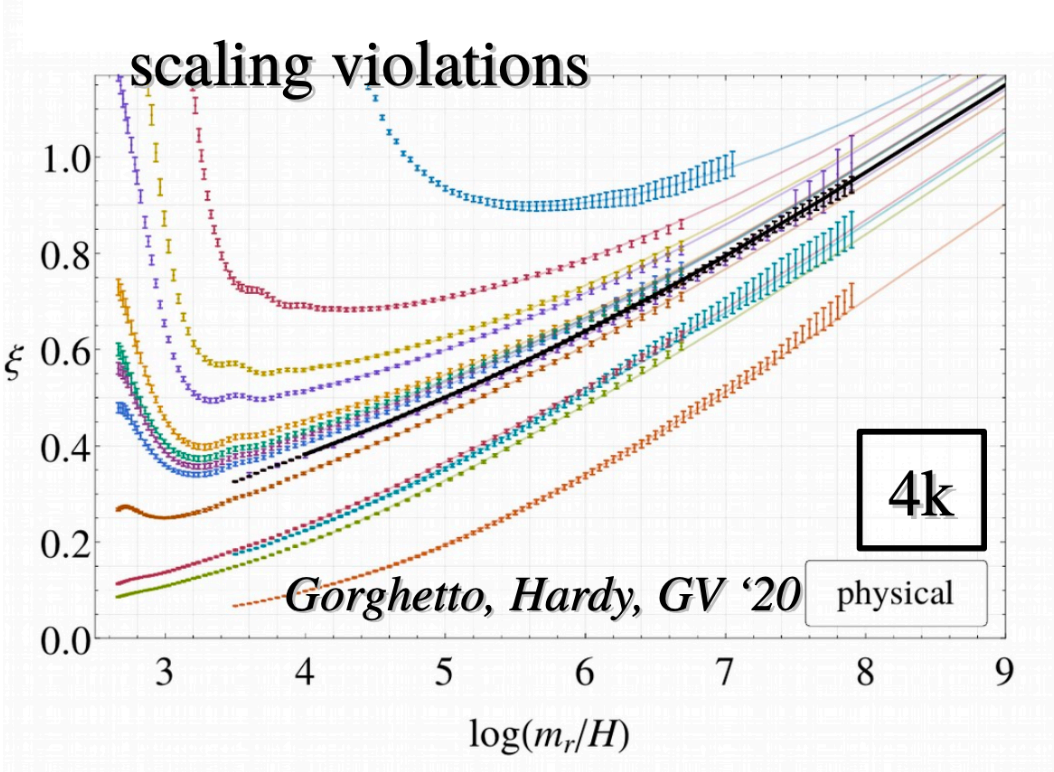


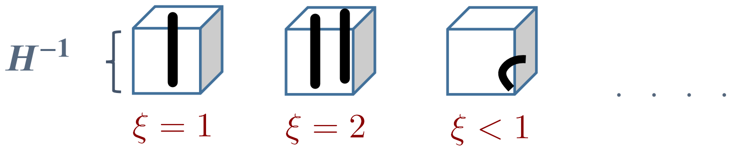


[from 1804.05857]

The Scaling Regime



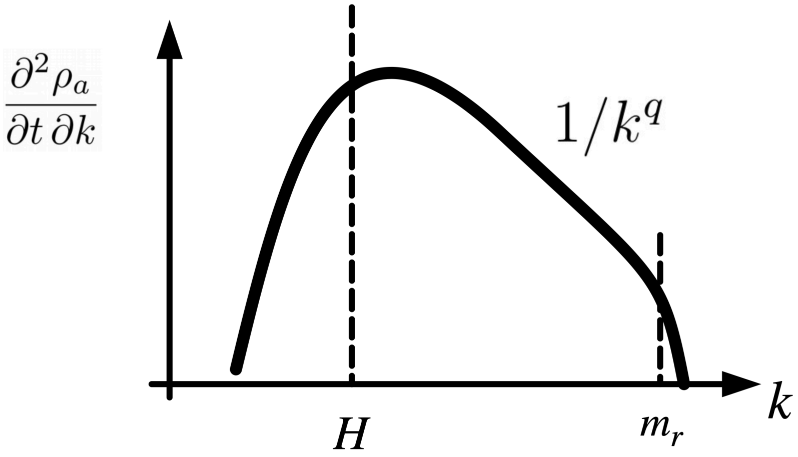




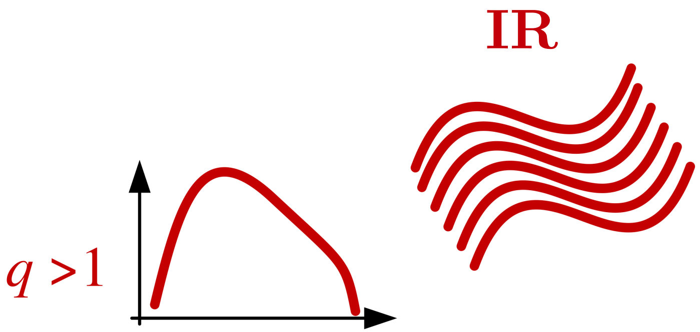
$$\xi \to \frac{\log(m_r/H)}{4 \div 5}$$

$$\boxed{\xrightarrow{\log\,\to\,70} 15(2)}$$

The Spectrum



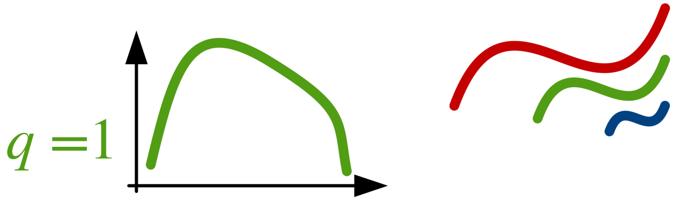
$$n \sim \frac{\rho}{\langle k \rangle}$$



Davies, Shellard, ...

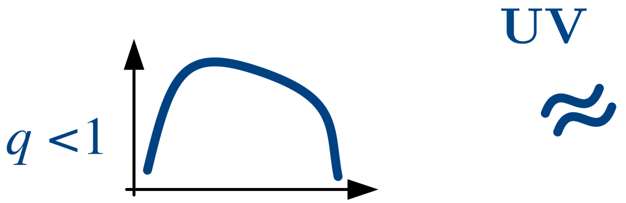
$$n \sim \frac{\rho}{H} \sim \xi \log f^2 H \sim \boxed{\xi \log} n^{mis}$$

$\sim 10^3$



Sikivie, ...

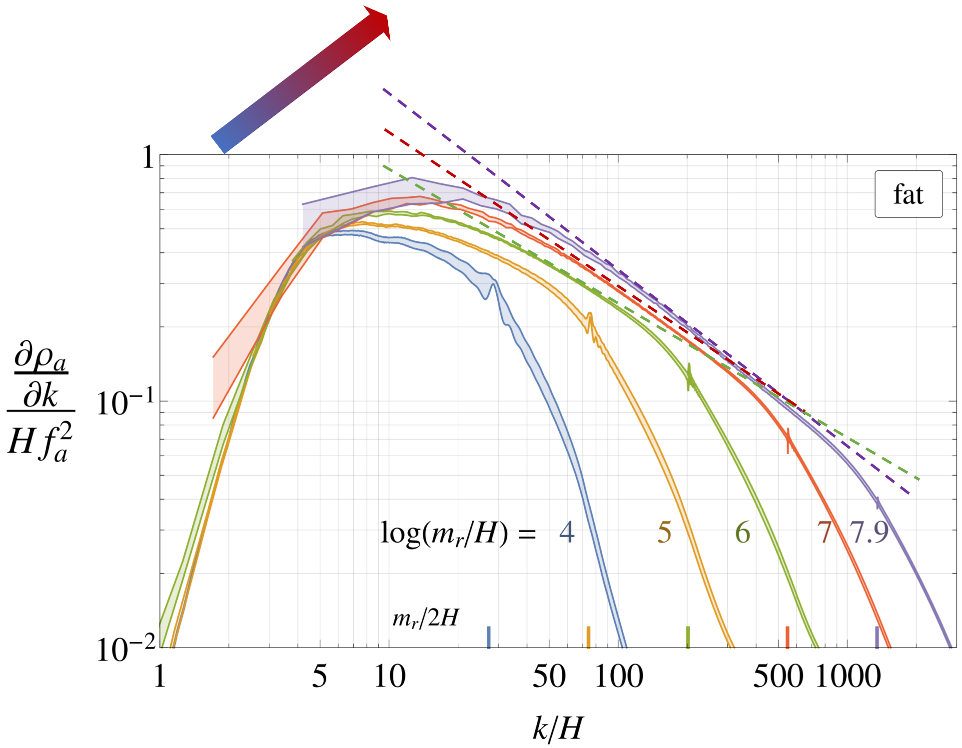
$$n \sim \frac{\rho}{H \log} \sim \xi f^2 H \sim \boxed{\xi} n^{mis}$$

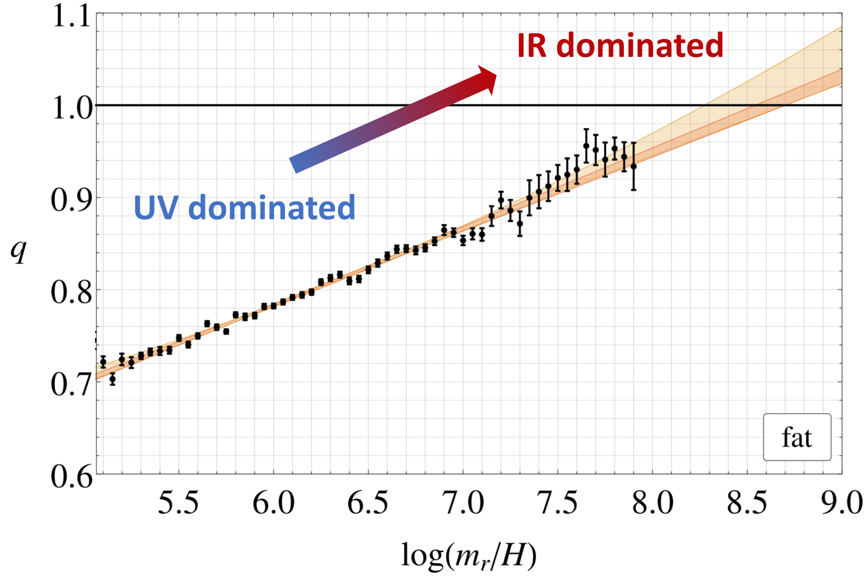


$$n \sim \frac{\rho}{H}\left(\frac{H}{m_r}\right)^{1-q} \sim n^{mis} \boxed{\left(\frac{H}{m_r}\right)^{1-q}}$$

$\lll 1$

The Spectral Index

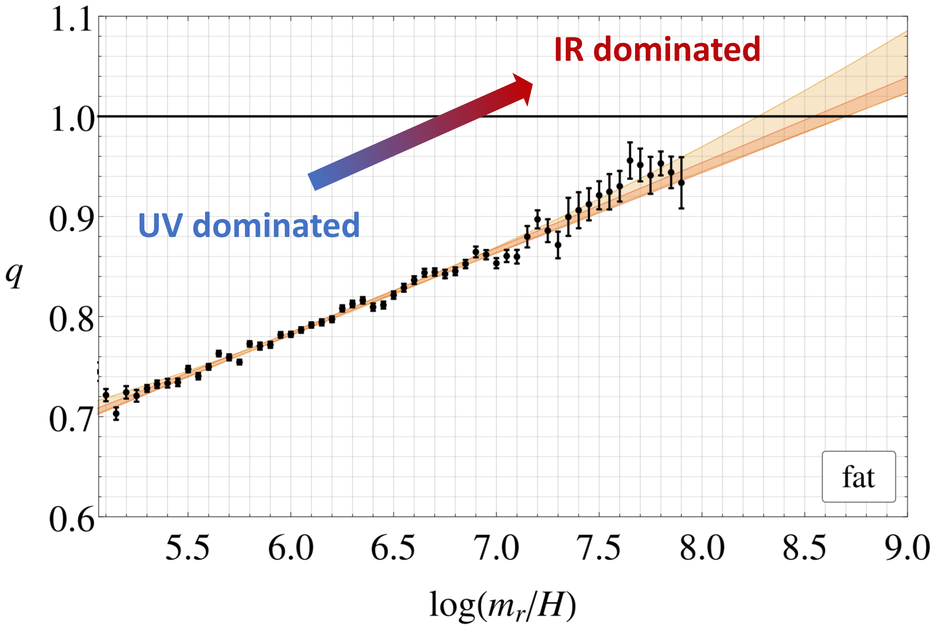


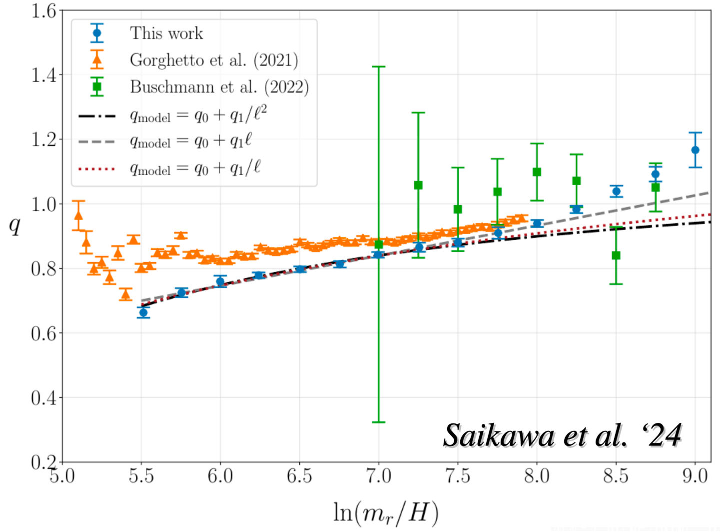


Running of *q* ➡ $\boxed{q \overset{\log\to 70}{>} 1}$

$$f_a \simeq (\overset{q>1}{1} \div \overset{q=1}{6}) \cdot 10^{10}\ \text{GeV} \quad + \text{DW?}$$

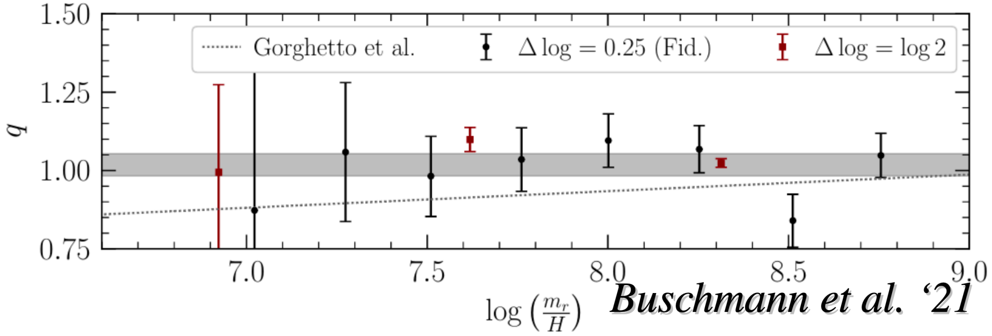
Comparisons



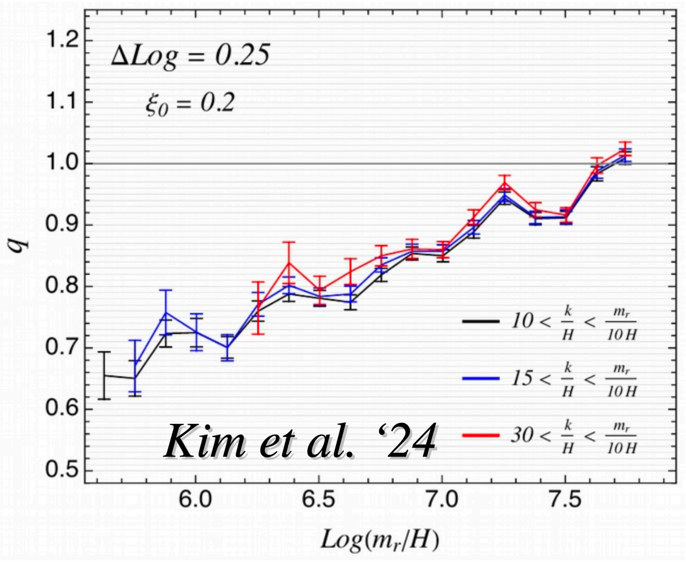


Saikawa et al. '24

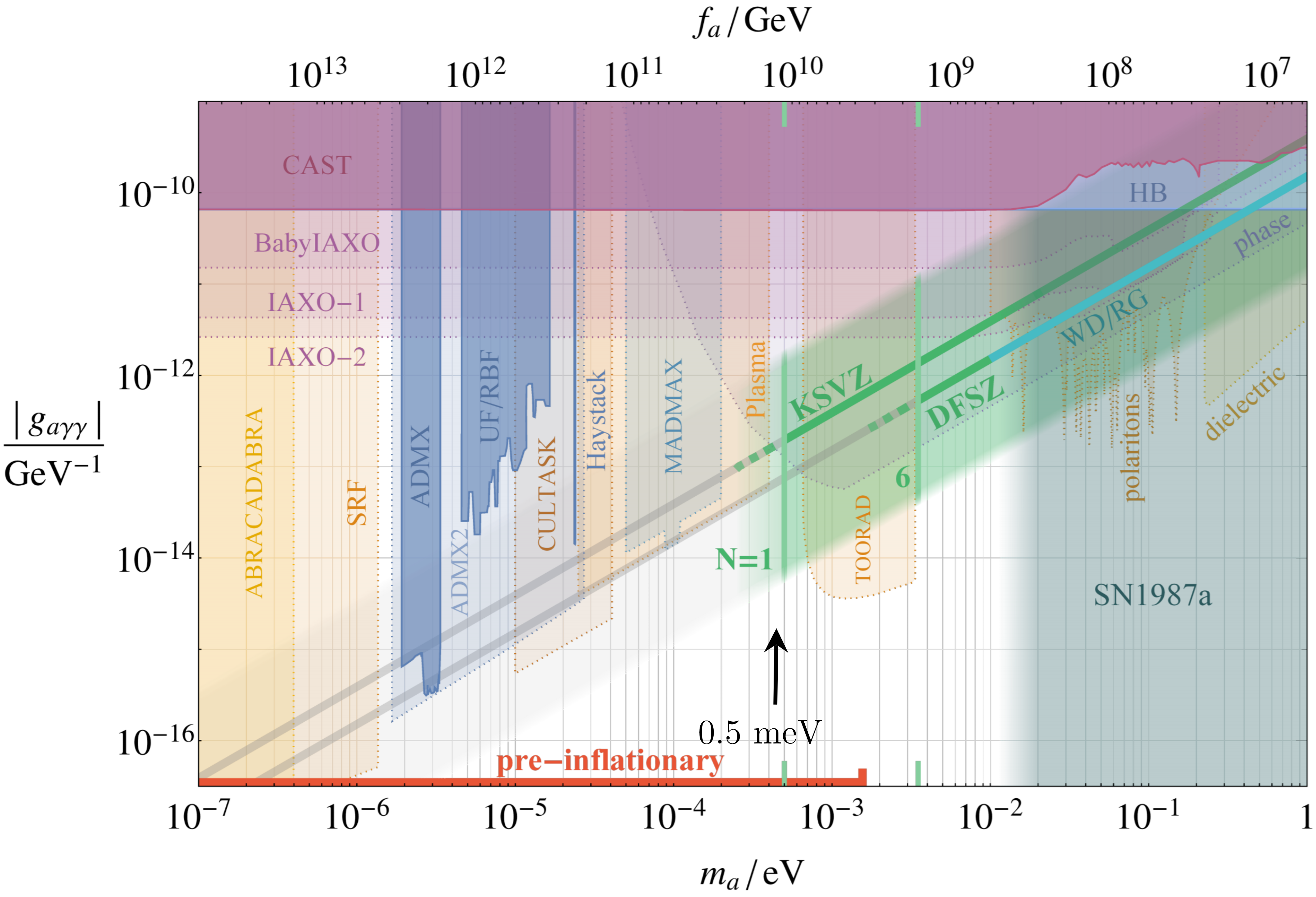
$q \simeq 1$?



Buschmann et al. '21



Kim et al. '24



$\underbrace{\qquad}_{q \simeq 1}$ $\underbrace{\qquad}_{q > 1}$

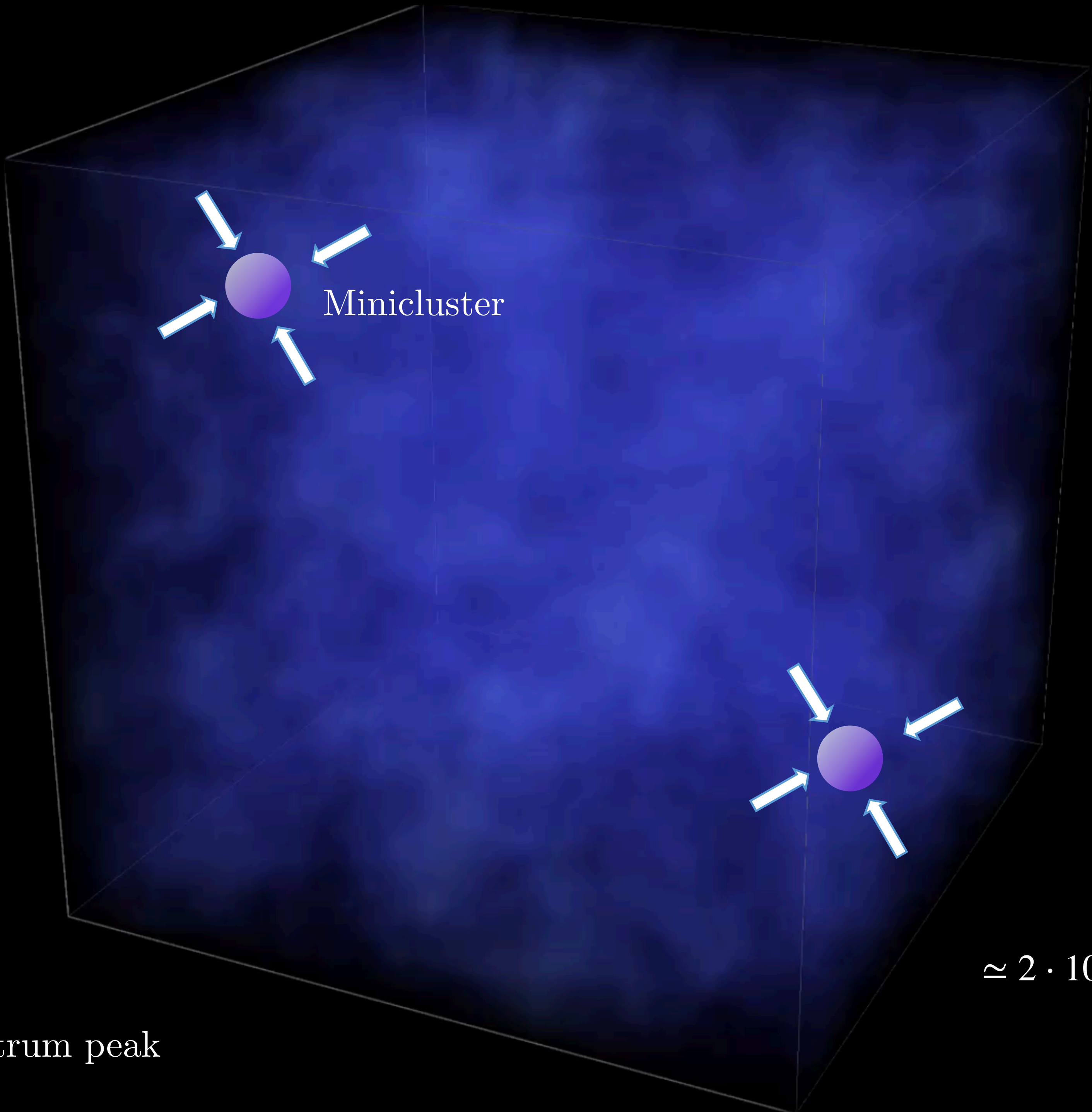
$f_a \lesssim 10^{10}$ GeV from DM overproduction

$\simeq$ if domain walls negligible

Formation of structures

after wall decay, $T \ll \Lambda_{\mathrm{QCD}}$

$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$



$\sim H_\star^{-1}$

$\sim k_p^{-1}$ spectrum peak

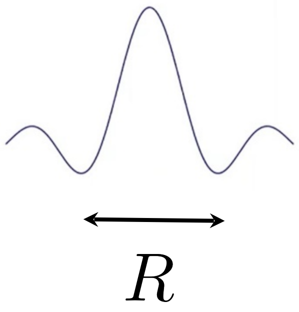
$$M_C \simeq \frac{4\pi}{3}\bar{\rho}\left(\frac{\pi}{H_\star}\right)^3$$

$$\simeq 2 \cdot 10^{-12} M_\odot \left(\frac{0.1\,\mathrm{meV}}{m_a}\right)^{1/2}$$

(naive)

Gravitational collapse *vs* quantum Jeans scale

overdensity ρ



spatial size of the overdensity $\longleftrightarrow$ de Broglie wavelength of the particles in the resulting clump

$$R \qquad \frac{1}{mv}$$

$$\simeq \frac{1}{m(\frac{GM}{R})^{1/2}} \simeq \frac{1}{R(4\pi G\rho m^2)^{1/2}}$$

$$4\pi\rho R^3/3$$

$$\downarrow$$

$$R_{\rm crit} \simeq \lambda_J \simeq (16\pi G\rho m^2)^{-1/4}$$

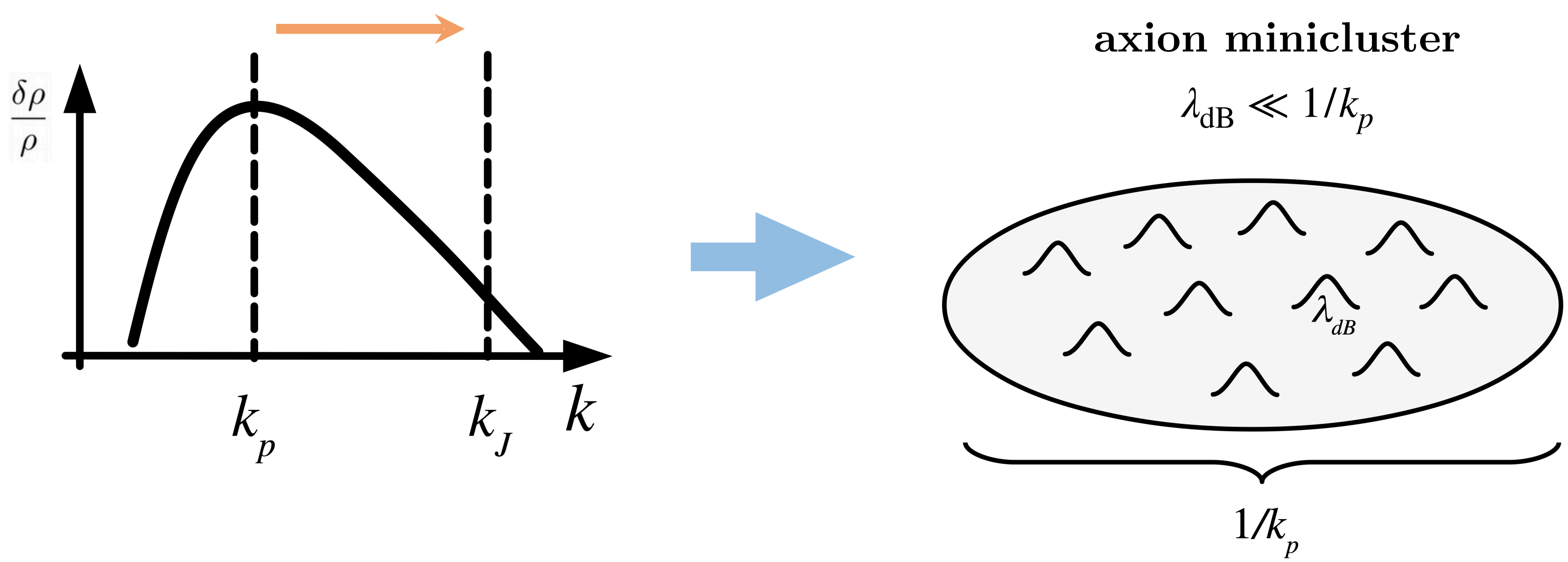
quantum Jeans length $\lambda_J = 2\pi/k_J$ $\equiv$ smallest scale an overdensity can have before wave effects (quantum pressure) have to be considered

The standard lore after DW decay

quantum Jeans scale

$k_J \equiv (16\pi G\rho m^2)^{\frac{1}{4}}$

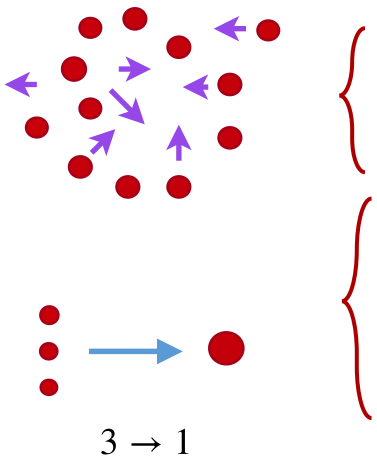
@MRE

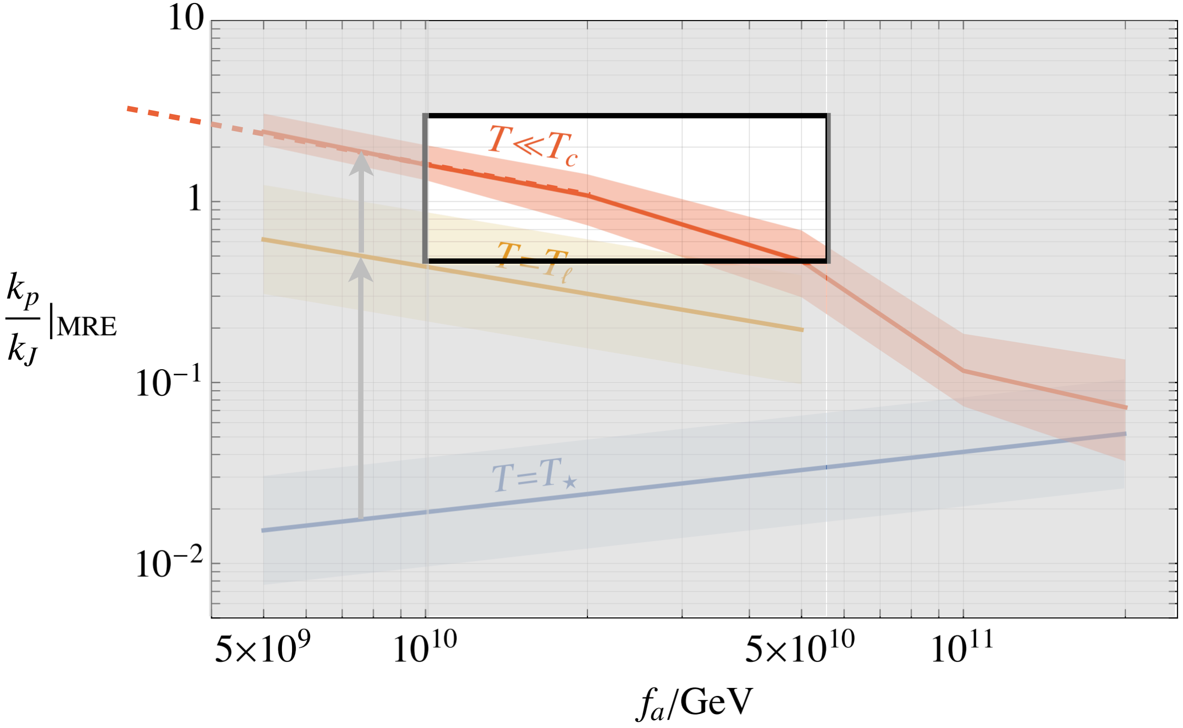


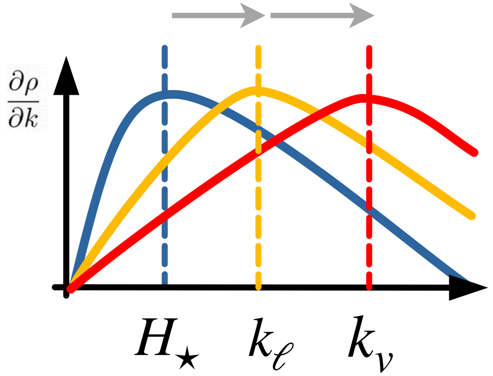
$$\left.\frac{k_p}{k_J}\right|_{\rm MRE} \simeq \left(\frac{f_a}{M_p}\right)^{1/3} \frac{k_{p\star}}{H_\star} \sim 10^{-3}\, \frac{k_{p\star}}{H_\star}$$

Naive because k_p increases due to the self-interactions and becomes of order k_J

The remarkable coincidence







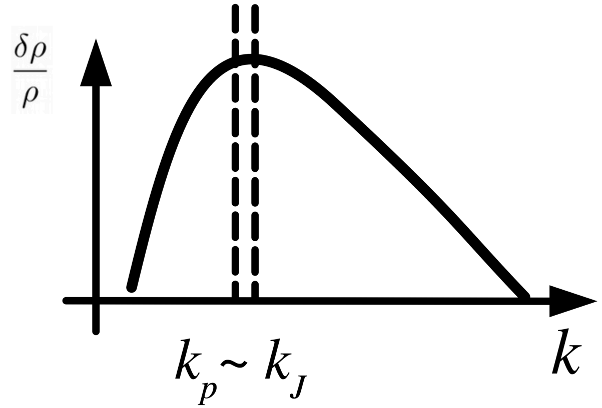
grows $\propto T^{-8}$

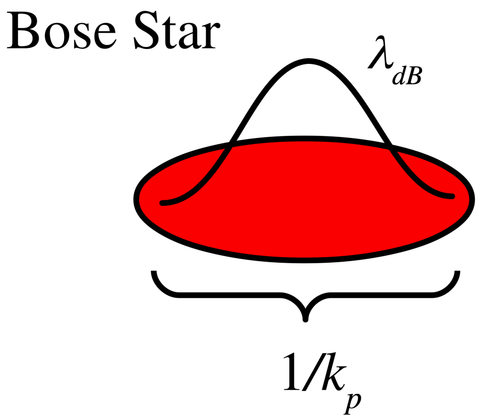
until $T \simeq 150\,\mathrm{MeV}$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\boxed{\lambda}|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

$$k_p \to k_v = \sqrt{\lambda\langle\phi^2\rangle} \simeq \sqrt{\rho}/f_a$$

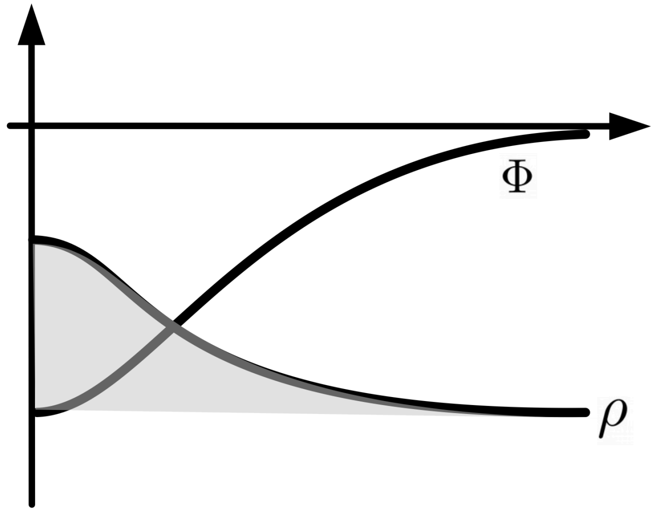
Axion stars:

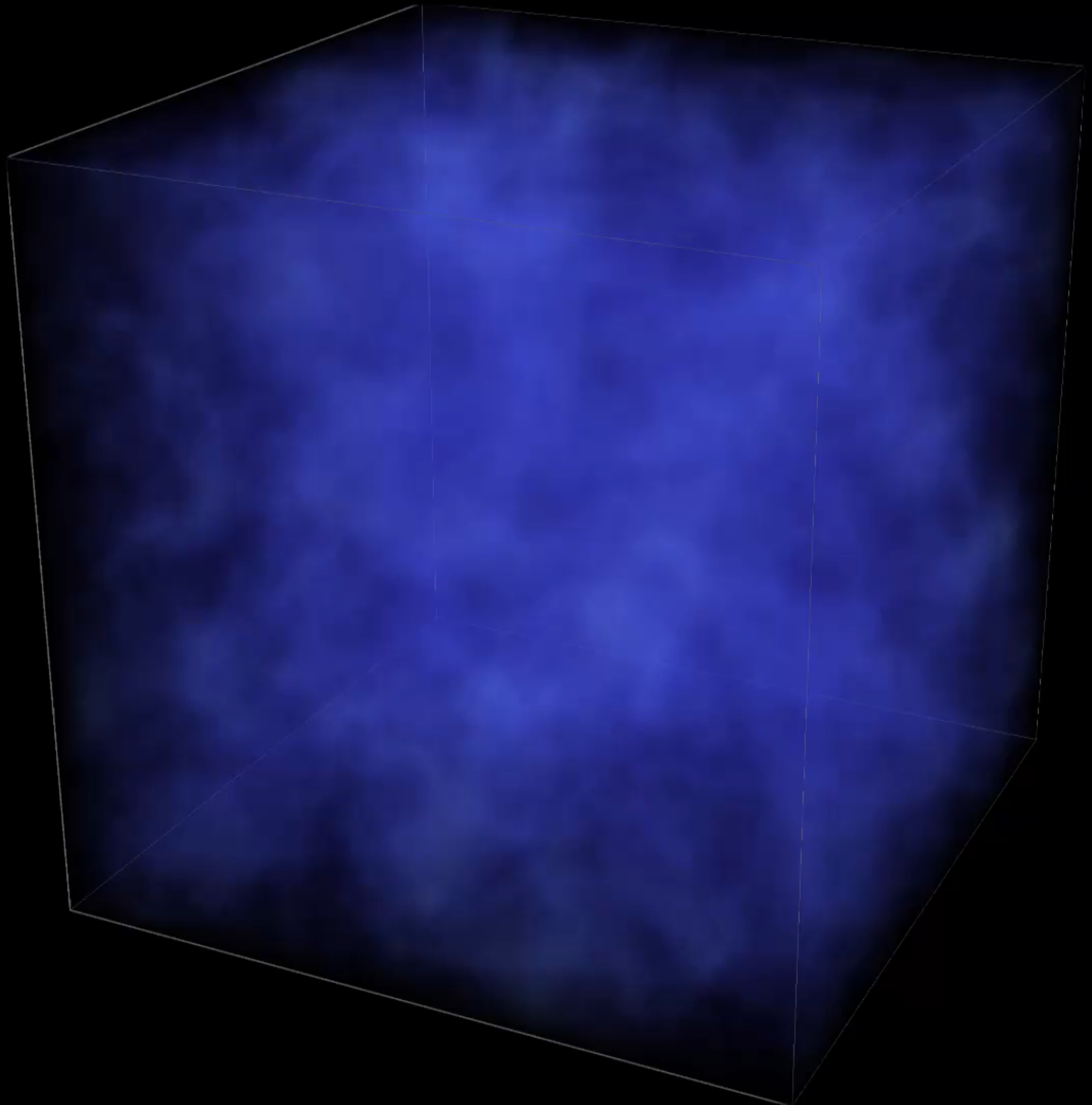




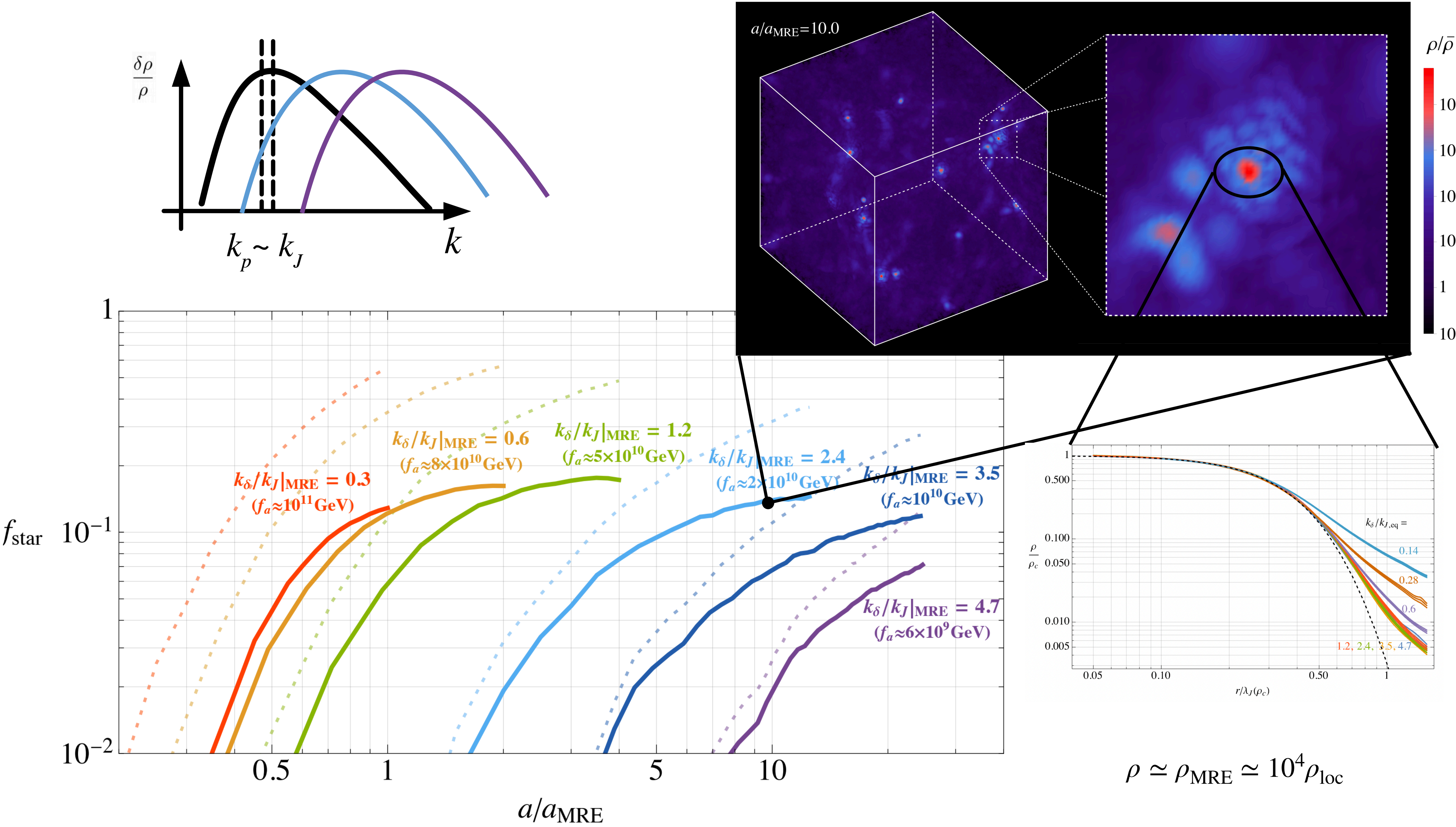
$$\begin{cases} \dot{\psi} + \frac{\nabla^2}{2m}\psi + m\Phi\psi = 0 \\ \nabla^2\Phi = 4\pi G|\psi|^2 \end{cases} \to \begin{cases} \nabla^2\sqrt{\rho} = 2m^2\Phi\sqrt{\rho} \\ \nabla^2\Phi = 4\pi G\rho \end{cases}$$

$$\rho = |\psi|^2$$



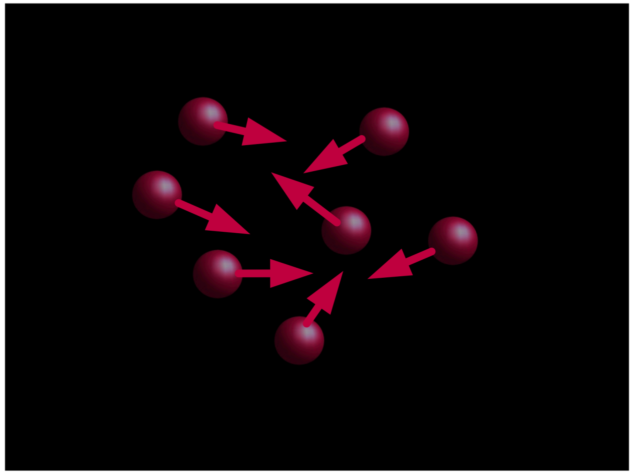


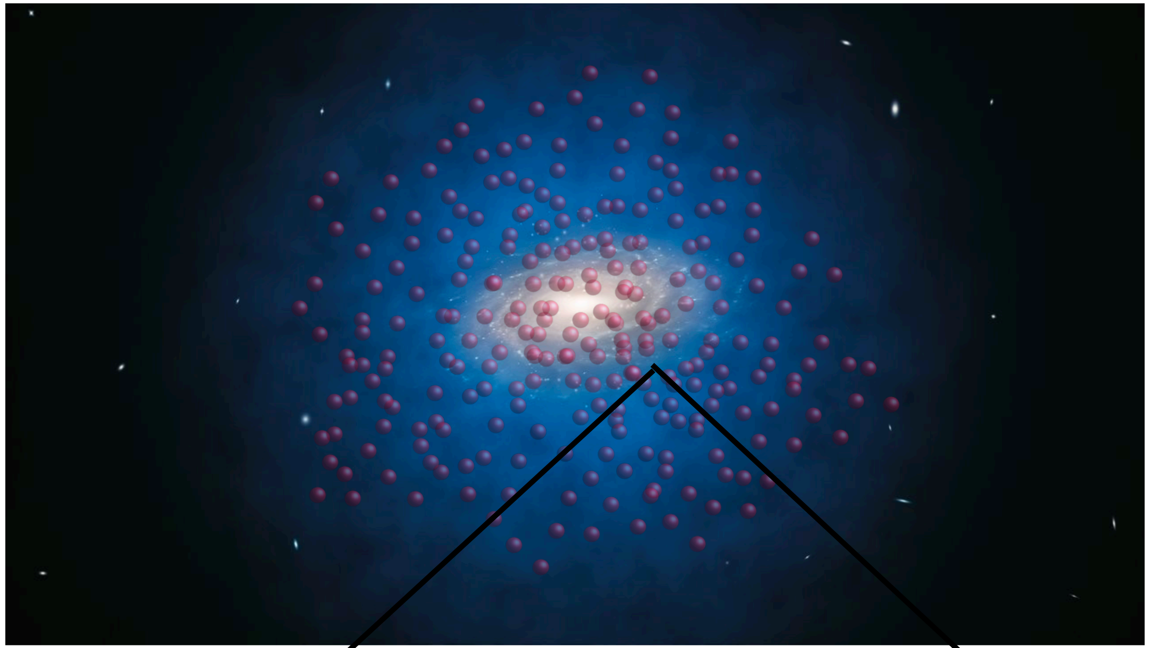
$$0.5 < \frac{a}{a_{\mathrm{eq}}} < 7$$



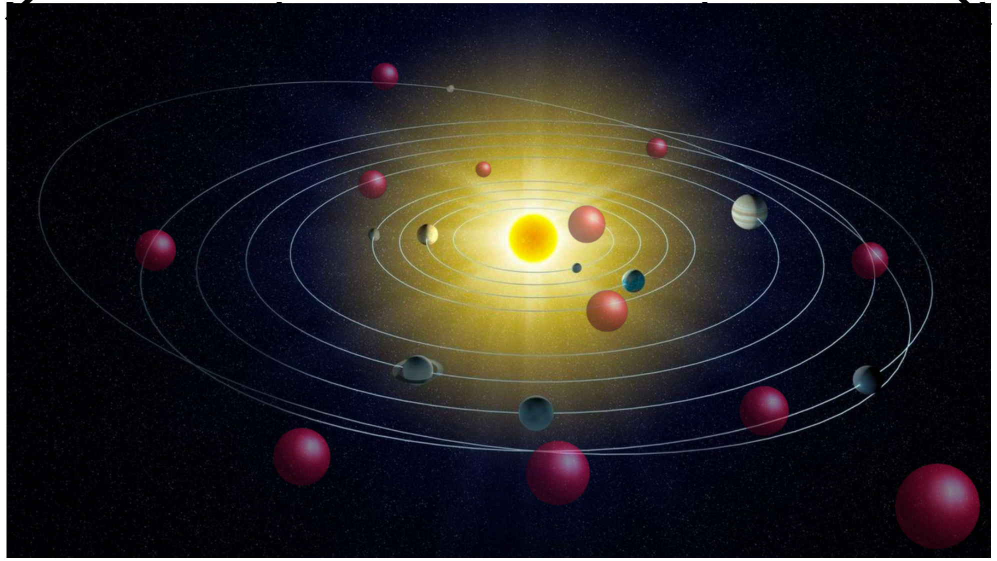
$$\rho \simeq \rho_{\rm MRE} \simeq 10^4 \rho_{\rm loc}$$

Axion stars (after MRE):





e.g. for $\begin{cases} M_s = 10^{-19} M_\odot \\ f_a = 10^{10}\ \text{GeV} \\ f_s = 0.1 \end{cases} \longrightarrow \begin{cases} n_s^{-1/3} = 1.4 \cdot 10^8\,\text{km} \\ \tau_\oplus = 5\ \text{yrs} \\ \Delta t \simeq 8\ \text{hrs} \end{cases}$



Conclusions

- Post-inflationary abundance **uncertain**, despite progress

$$f_a \lesssim 10^{10}\,\mathrm{GeV} \quad \text{or} \quad m_a \gtrsim 0.5\,\mathrm{meV} \quad \text{from dark matter over-production}$$

- **Axion star** formation enhanced at MRE

➡ Potential for new observational opportunities

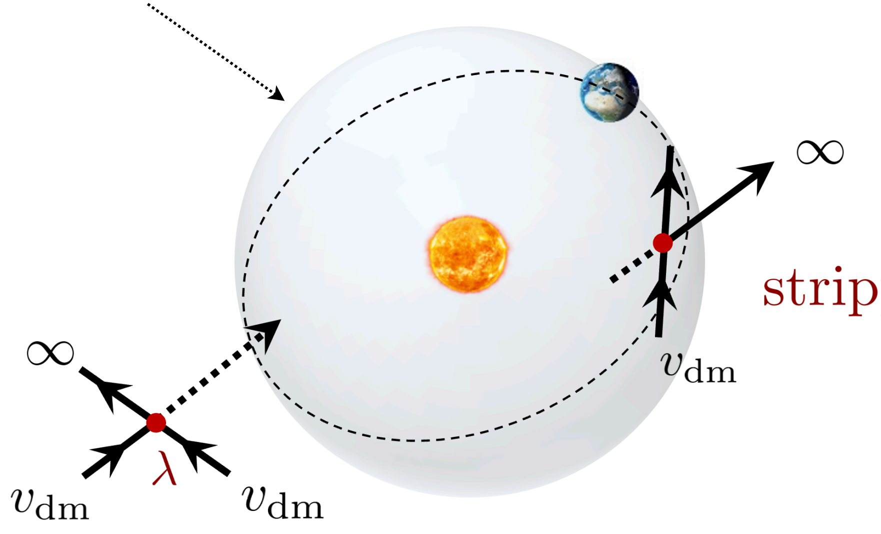
Thanks!

Backup

Advert:

Solar halos of ultra-light dark matter

- DM is ϕ with $V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \underbrace{\frac{m^2}{f_a^2}}_{-\lambda} \phi^4 + \dots$



Solar halo

stripping

capture

$$R_\star = \frac{1}{m\underset{\searrow\, GM_\odot m}{\alpha}} = 1\,\mathrm{AU}\left[\frac{1.3\cdot 10^{-14}\,\mathrm{eV}}{m}\right]^2$$

Bohr radius

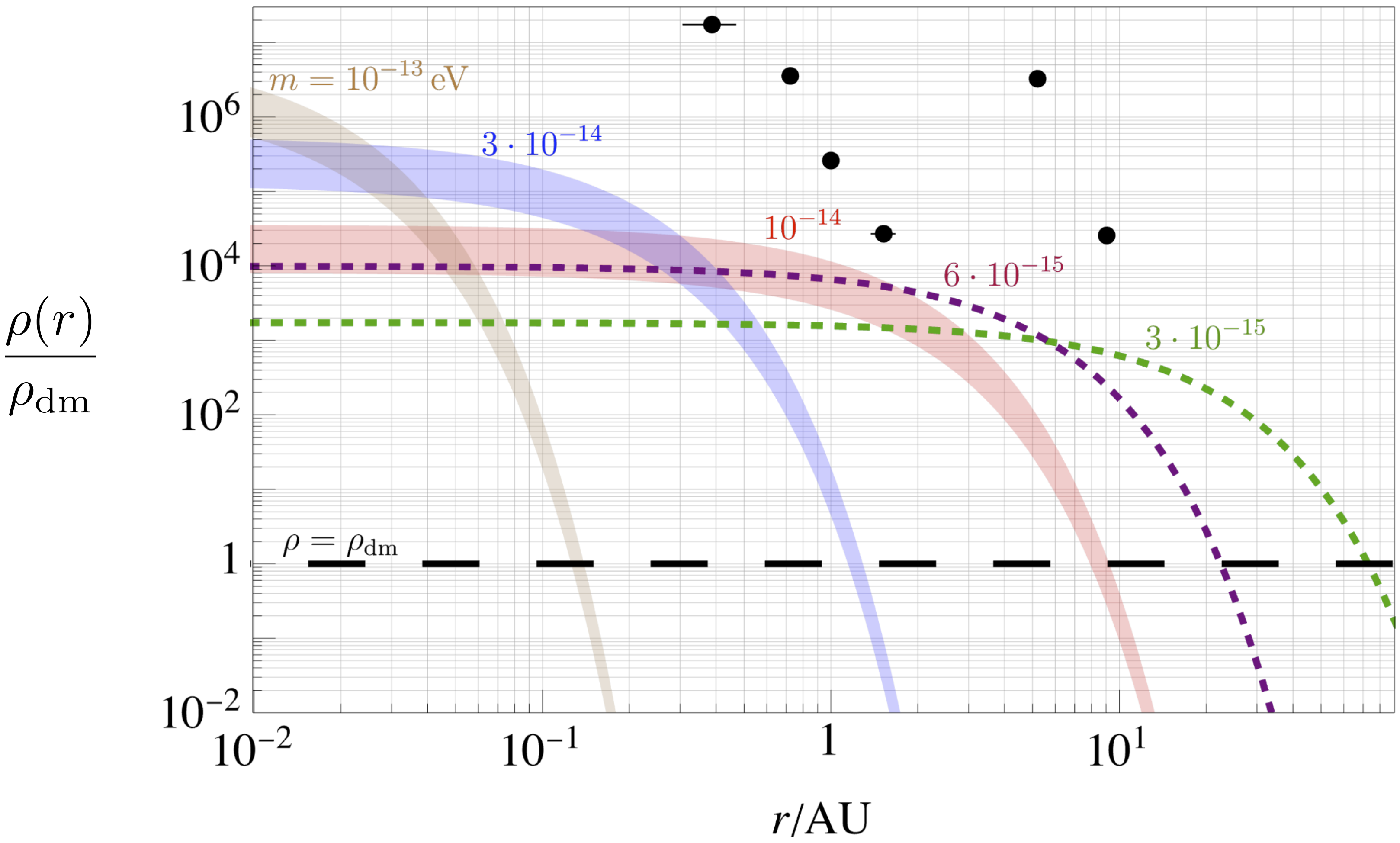
Capture dominates over stripping when:

$$v_{\rm dm} \simeq 10^{-3} \lesssim 2\pi\,\alpha$$

i.e. if $\quad m \gtrsim 1.5 \cdot 10^{-14}\,\mathrm{eV}$

$$\dot{N}_{\rm bound} \sim \Gamma(m, f_a) \cdot N_{\rm bound} \quad \rightarrow \quad N_{\rm bound} \propto e^{\Gamma t}$$

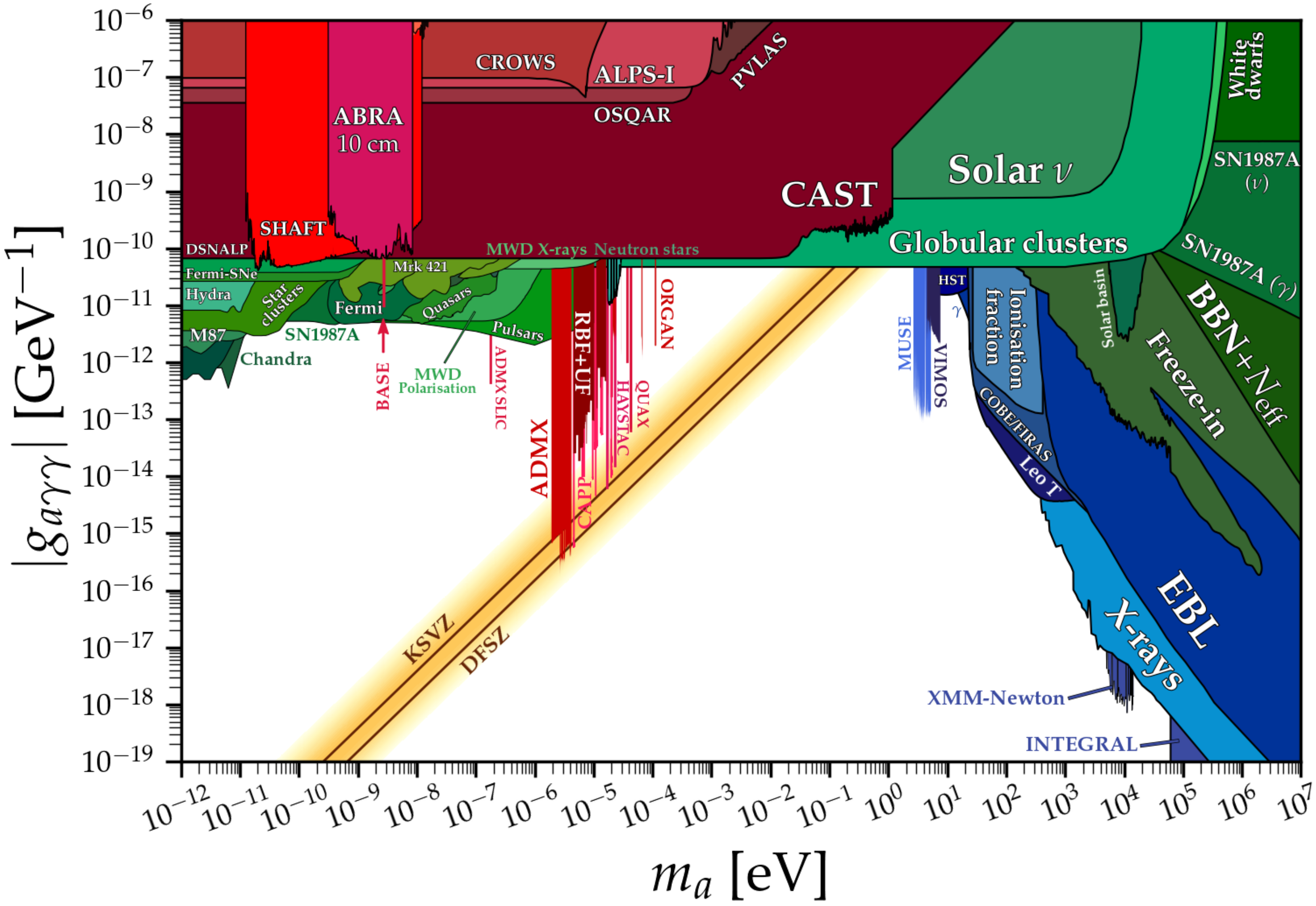
density profile after 5 Gyr



- bands have $v_{\mathrm{dm}} = 50 \div 240\,\mathrm{km/s}$
- f_a (or λ) fixed in $10^7 \div 10^8\,\mathrm{GeV}$

Axion searches

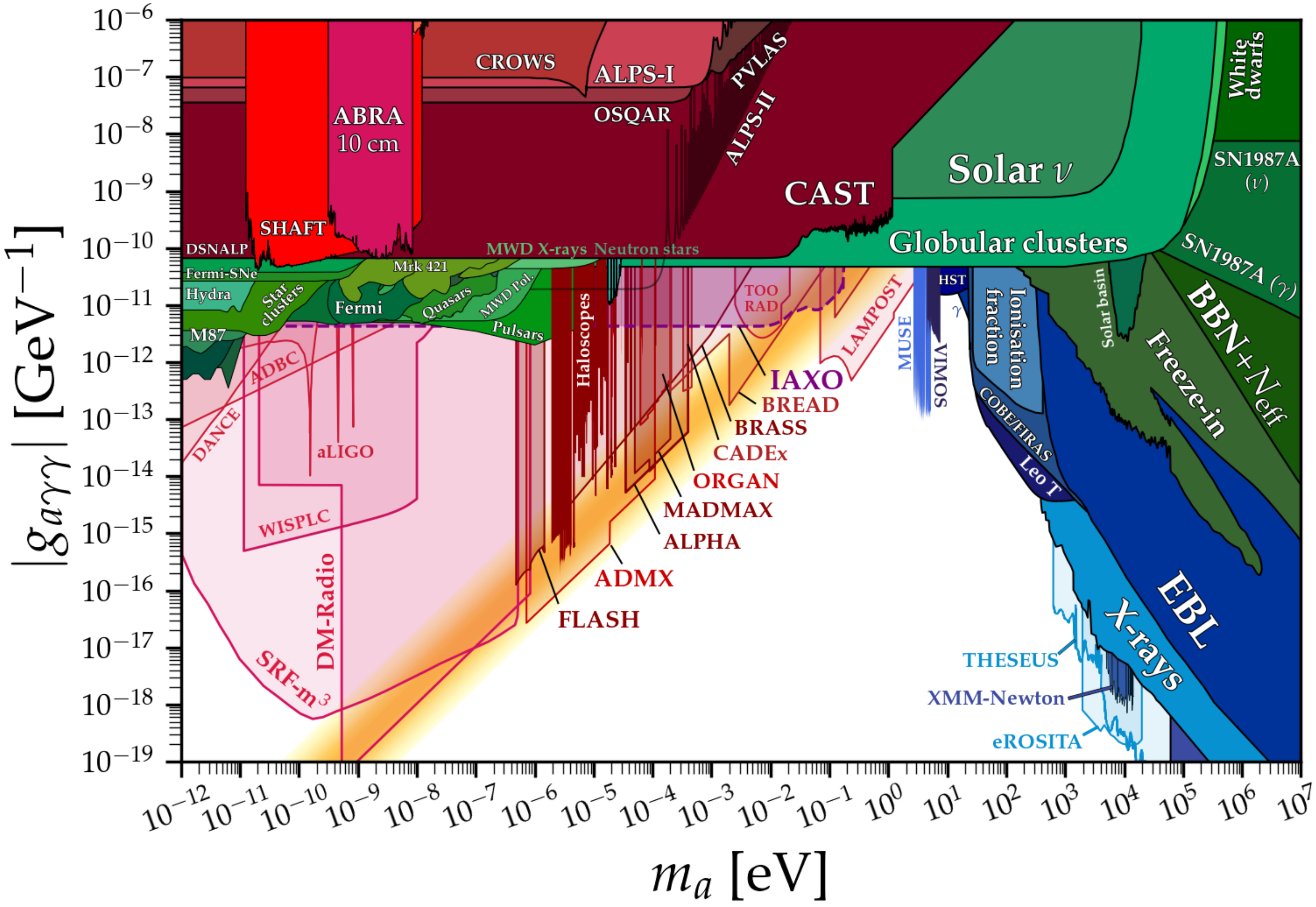
$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$



https://cajohare.github.io/AxionLimits

Axion searches

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

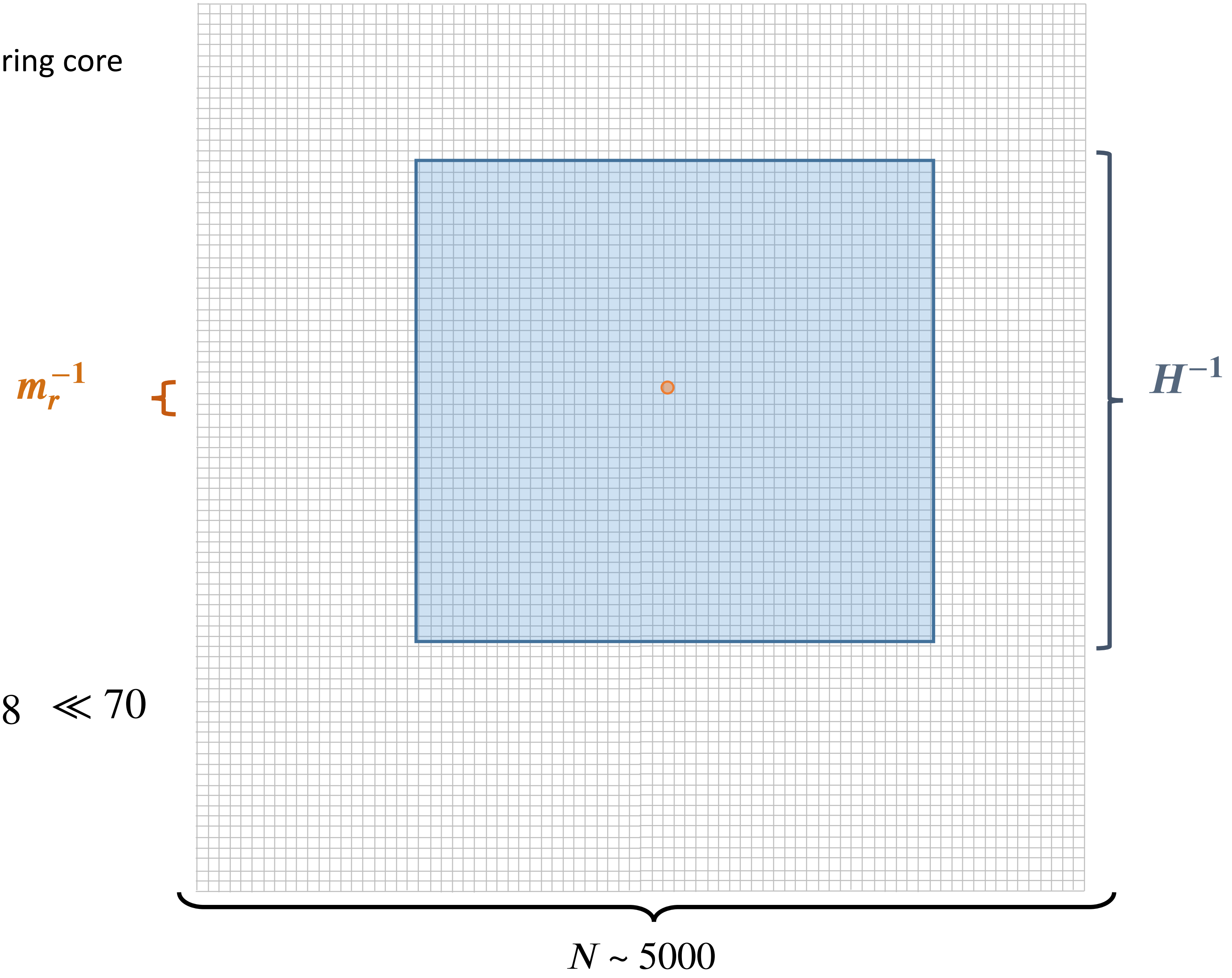


https://cajohare.github.io/AxionLimits

Axion dark matter

Y

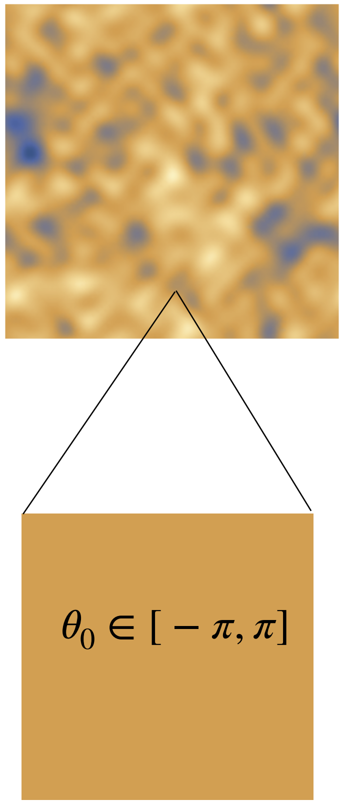
- a few lattice points per string core
- a few Hubble patches



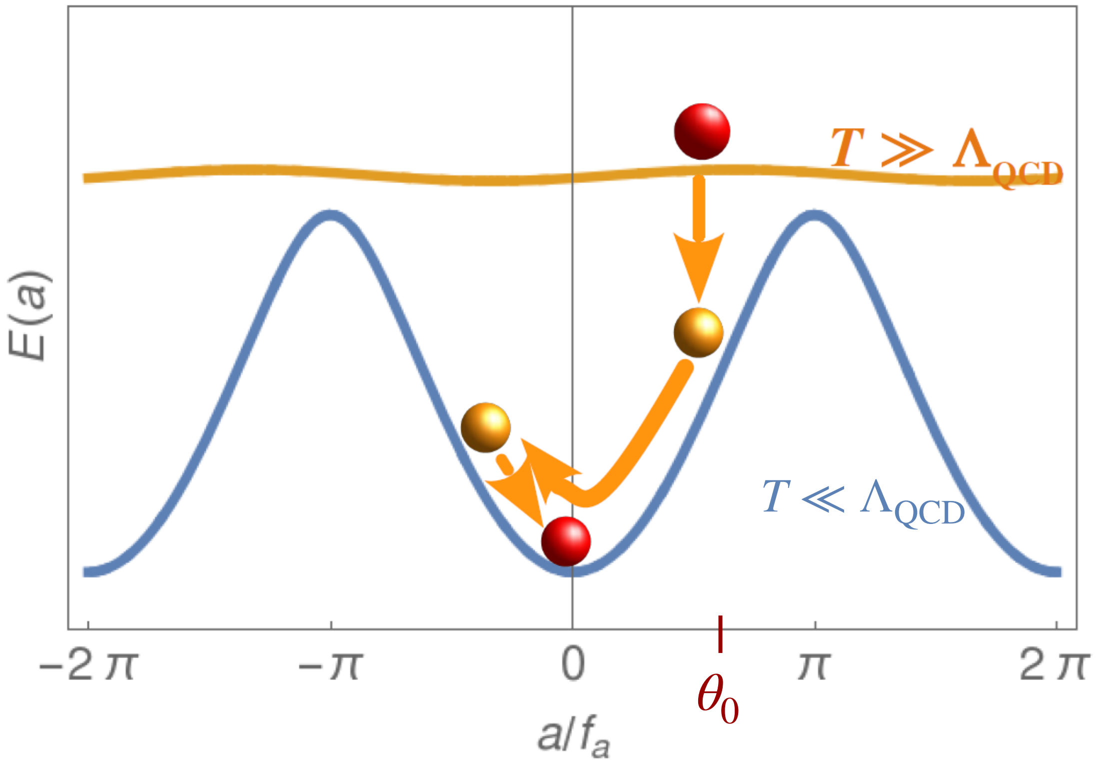
$$\log\frac{m_r}{H} \leq \log\left(\frac{\square}{\bigcirc}\right) \sim 8 \ll 70$$

Pre-inflationary scenario

$$\ddot{a} + 3H(T)\dot{a} + m_a^2(T)\, a = 0$$



$a(t) =$ const in space



$$a(t) \simeq \frac{1}{R(t)^{\frac{3}{2}}} \cos m_a t$$

scale factor

Energy density:

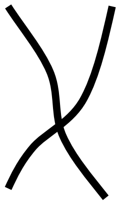
$$\rho_a(t) \propto R(t)^{-3}$$

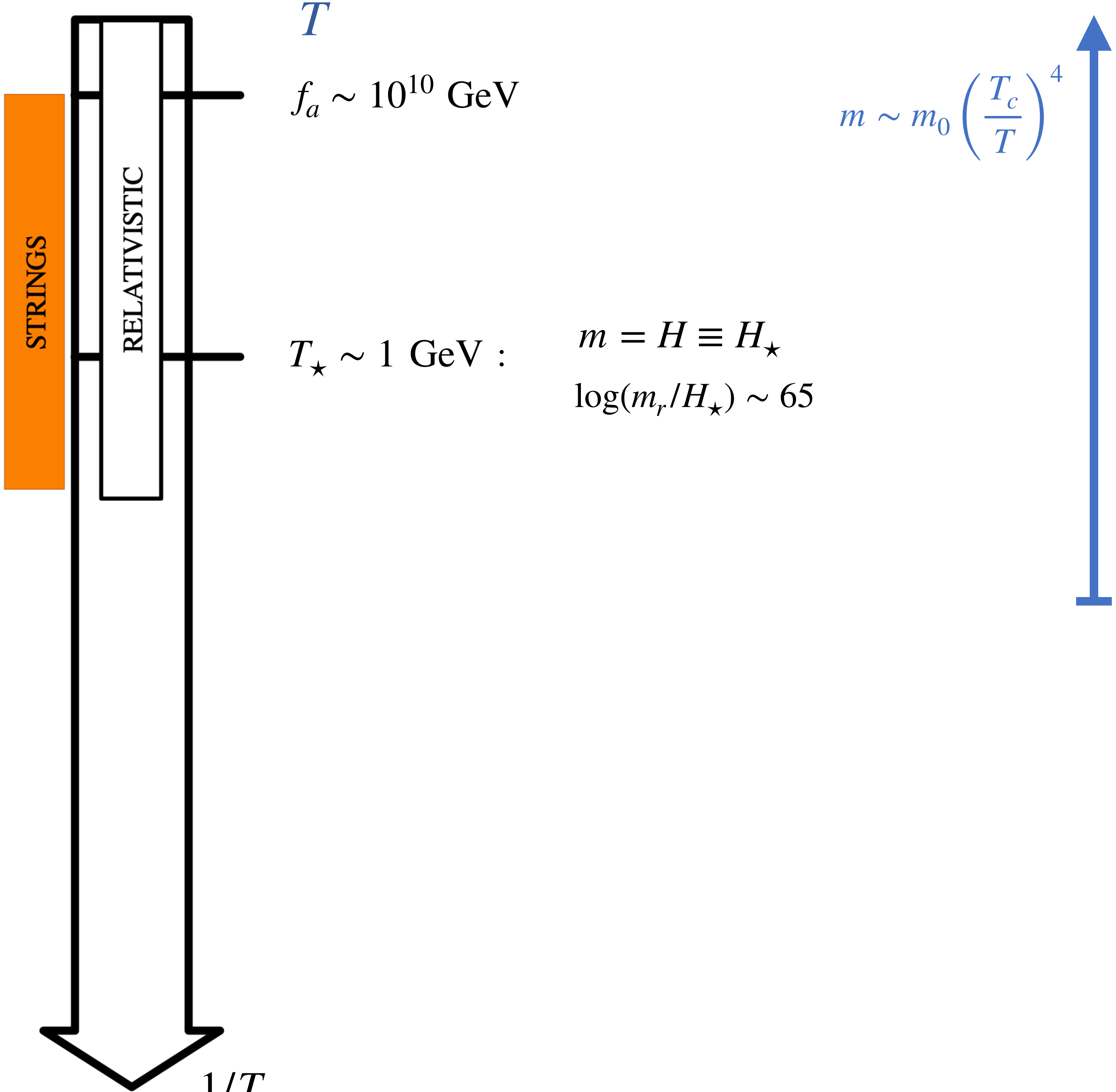
$$\boxed{\Omega_a \approx 0.1\theta_0^2 \left[\frac{f_a}{10^{12}\mathrm{GeV}}\right]^{1+\epsilon}} \rightarrow$$

$$0 \leq |\theta_0| < \pi$$

$$10^{18} \gtrsim f_a/\mathrm{GeV} \gtrsim 4 \cdot 10^9$$

$$10^{-11} \lesssim m_a/\mathrm{eV} \lesssim 1.5 \cdot 10^{-3}$$





T

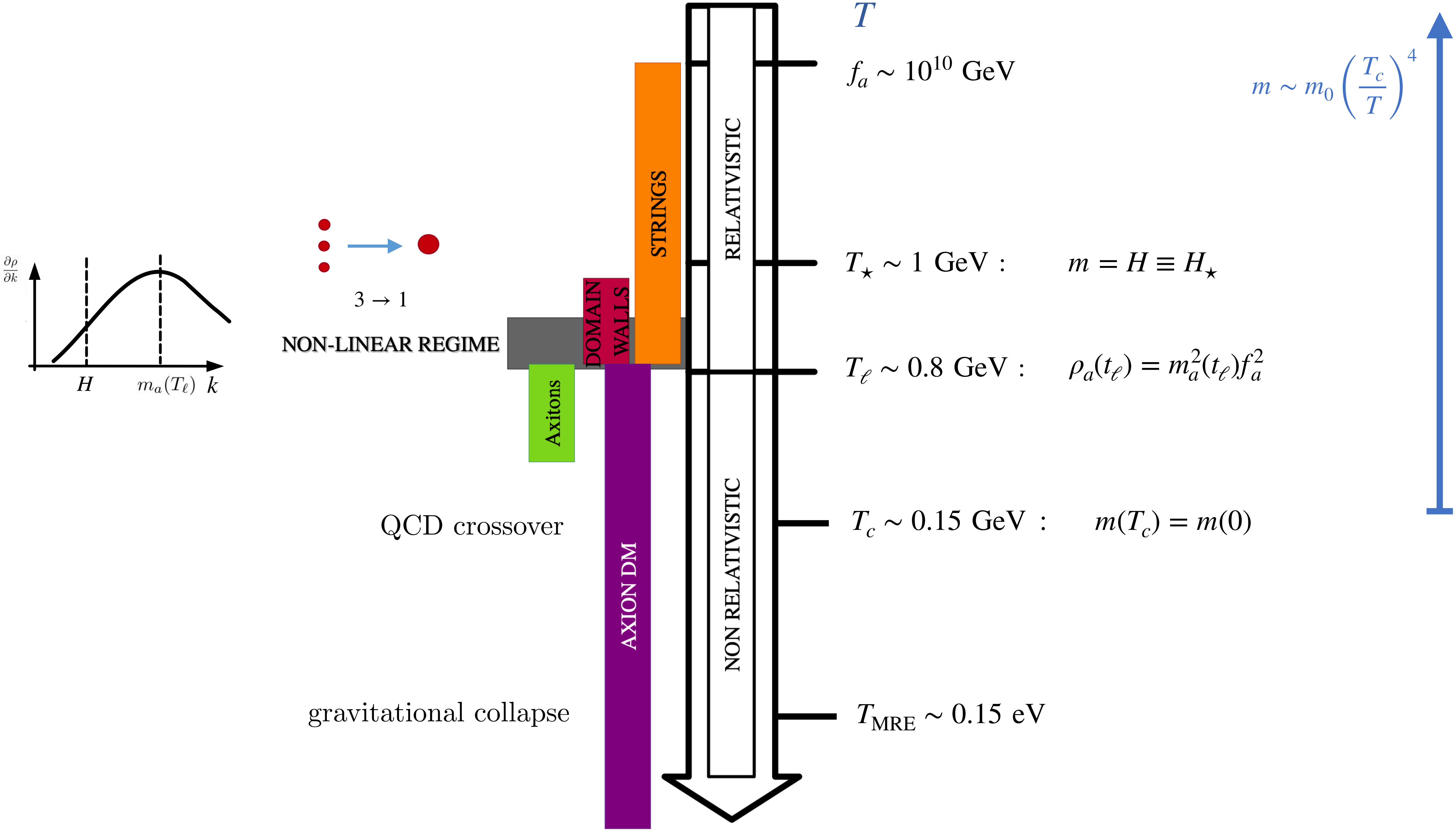
$f_a \sim 10^{10}$ GeV

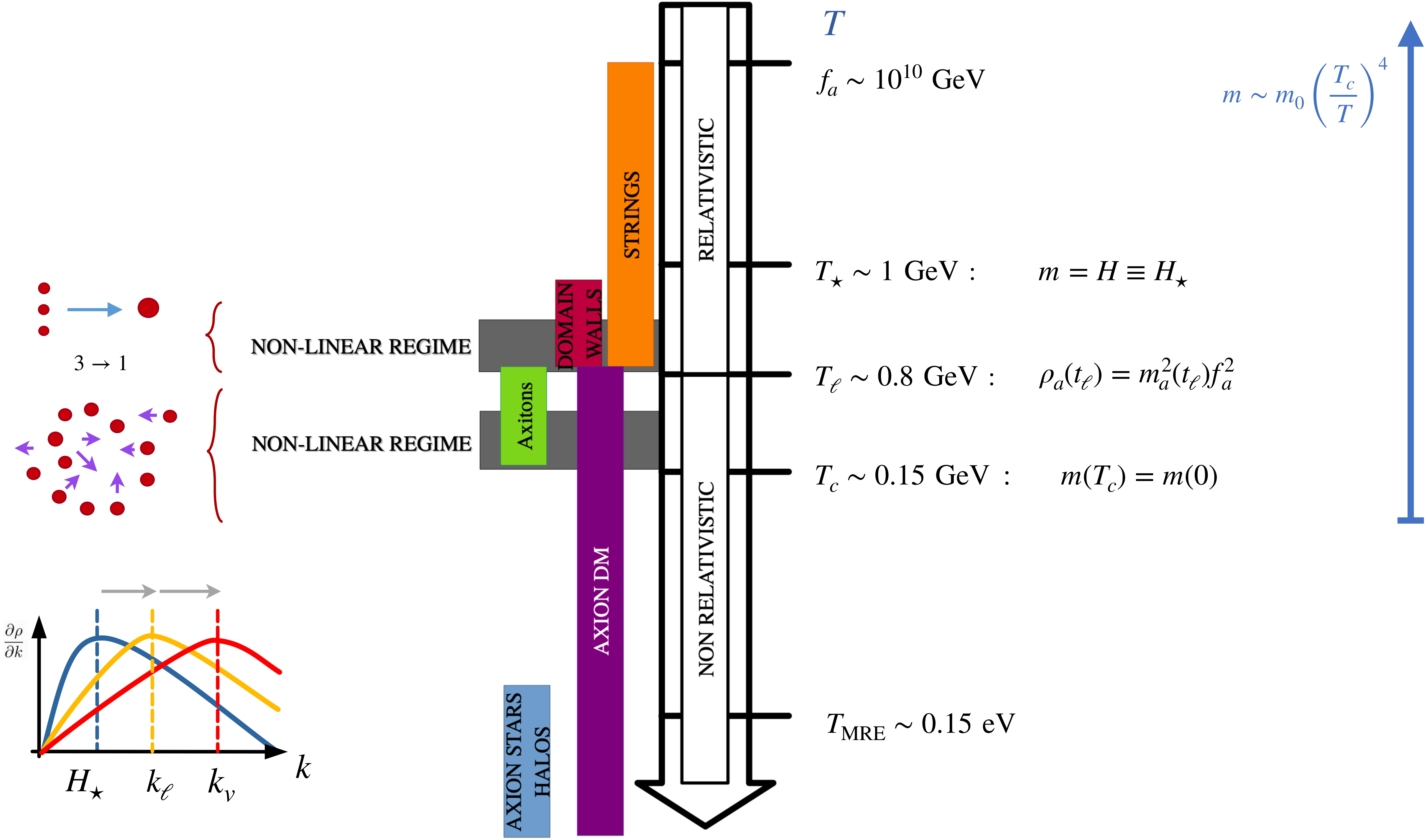
$m \sim m_0 \left(\frac{T_c}{T}\right)^4$

$T_\star \sim 1$ GeV : $m = H \equiv H_\star$

$\log(m_r/H_\star) \sim 65$

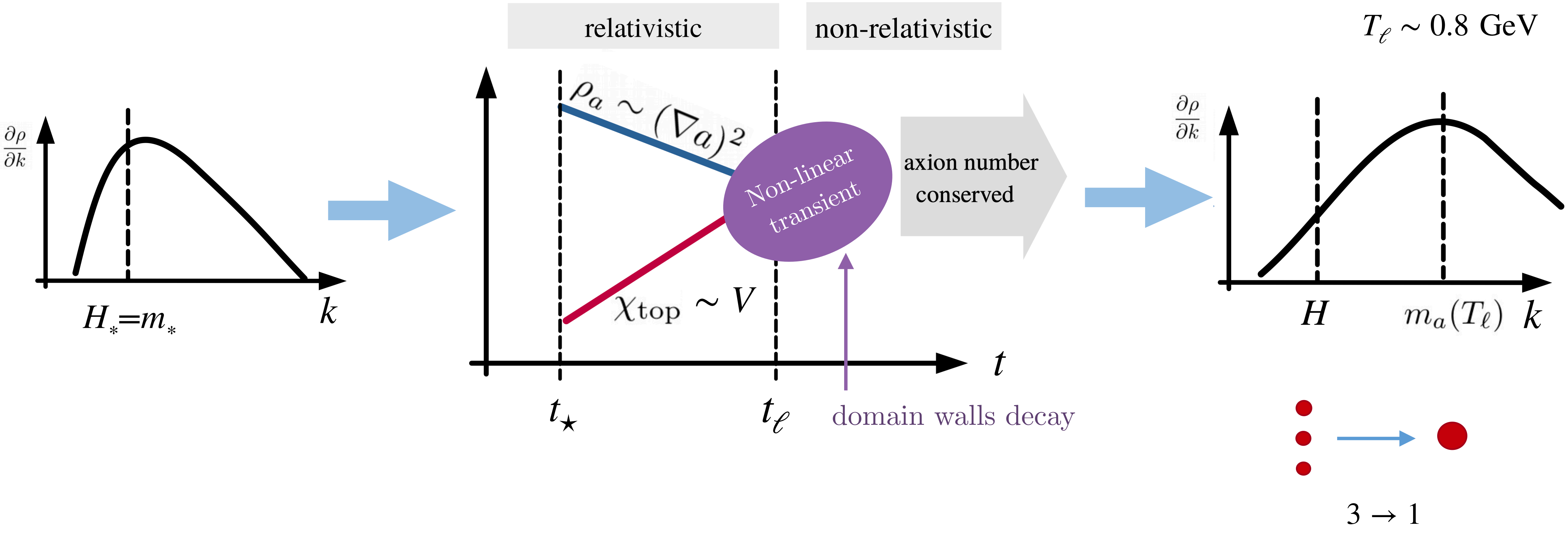
$1/T$


T
$f_a \sim 10^{10}$ GeV
$m \sim m_0 \left(\frac{T_c}{T}\right)^4$
RELATIVISTIC
STRINGS
$3 \rightarrow 1$
$T_\star \sim 1$ GeV : $m = H \equiv H_\star$
DOMAIN WALLS
NON-LINEAR REGIME
$\frac{\partial \rho}{\partial k}$
H
$m_a(T_\ell)$
k
$T_\ell \sim 0.8$ GeV : $\rho_a(t_\ell) = m_a^2(t_\ell) f_a^2$
Axitons
QCD crossover
AXION DM
NON RELATIVISTIC
$T_c \sim 0.15$ GeV : $m(T_c) = m(0)$
gravitational collapse
$T_{\text{MRE}} \sim 0.15$ eV


T
$m \sim m_0 \left(\frac{T_c}{T}\right)^4$
$f_a \sim 10^{10}$ GeV
$T_\star \sim 1$ GeV : $m = H \equiv H_\star$
$T_\ell \sim 0.8$ GeV : $\rho_a(t_\ell) = m_a^2(t_\ell) f_a^2$
$T_c \sim 0.15$ GeV : $m(T_c) = m(0)$
$T_{\mathrm{MRE}} \sim 0.15$ eV
RELATIVISTIC
NON RELATIVISTIC
STRINGS
DOMAIN WALLS
Axitons
AXION DM
AXION STARS HALOS
NON-LINEAR REGIME
NON-LINEAR REGIME
$3 \to 1$
$\frac{\partial \rho}{\partial k}$
k
$H_\star$
k_ℓ
k_v

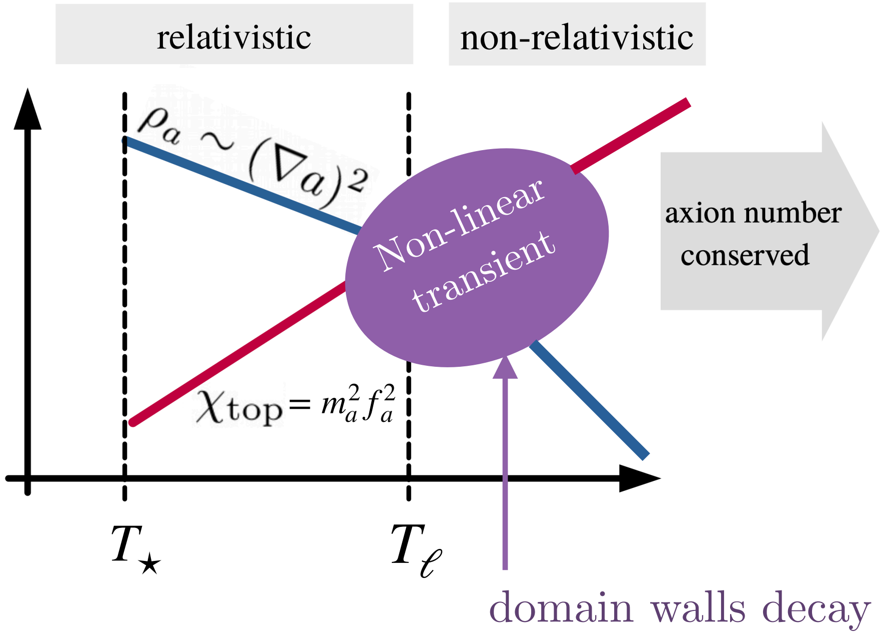
Effect of non-linearities (I)

If $q \geq 1$: $\rho_a(t_\star) \gg \rho^{\text{mis}} \sim m_\star^2 f_a^2 = \chi_{\text{top}}(T_\star)$

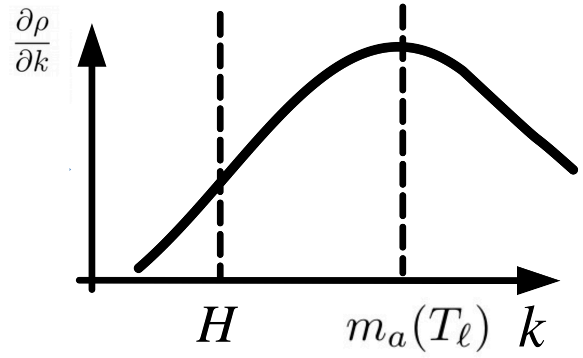


$$f_a \simeq (\overset{q>1}{1} \div \overset{q=1}{6}) \cdot 10^{10}\ \text{GeV} \quad + \text{DW?}$$

After DW decay: the standard lore



$m_a(T) \gg k_p$ → axions become non-relativistic



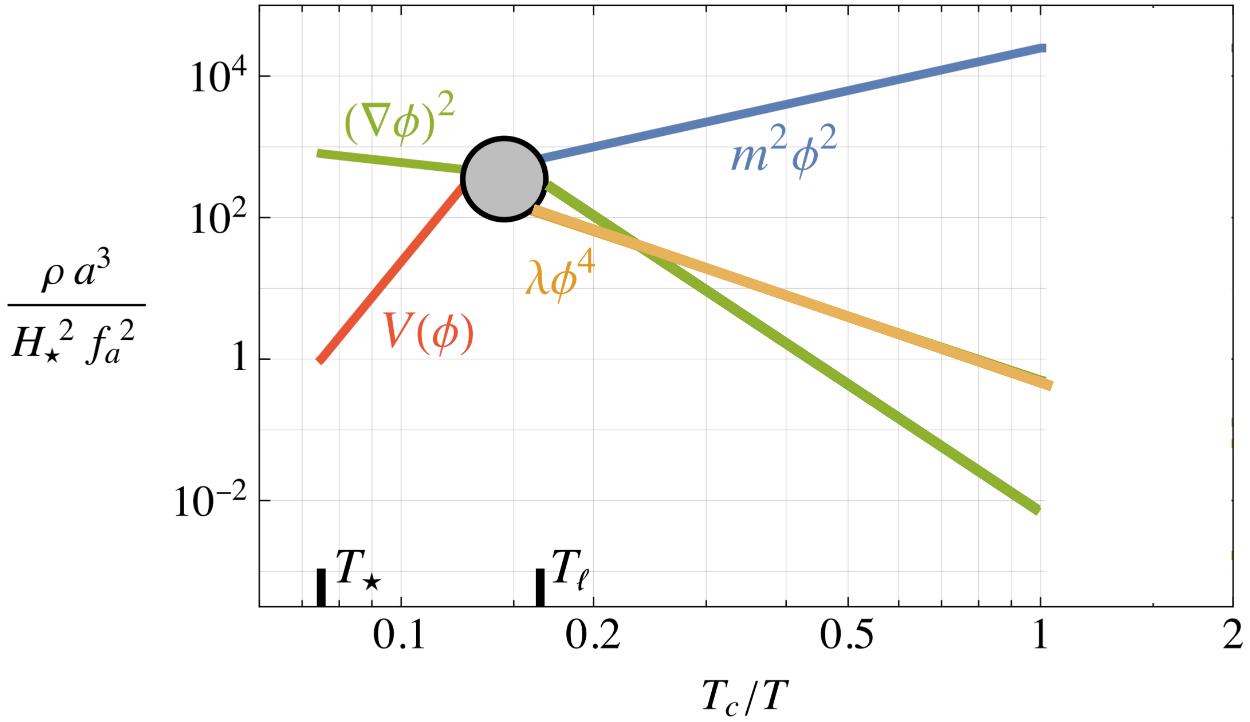
$$m \sim m_0 \left(\frac{T_c}{T}\right)^4$$

$$V(a) \simeq \frac{1}{2} m^2 a^2$$ axions become free

$\Longrightarrow$ the field redshifts like CDM until MRE

@ MRE, fluctuations $\delta\rho/\rho \sim 1$ gravitationally collapse in objects of size $\sim 1/k_p$

However: effect of non-linearities (II)



$$\rho \sim \dot{\phi}^2 + m^2\phi^2 +$$

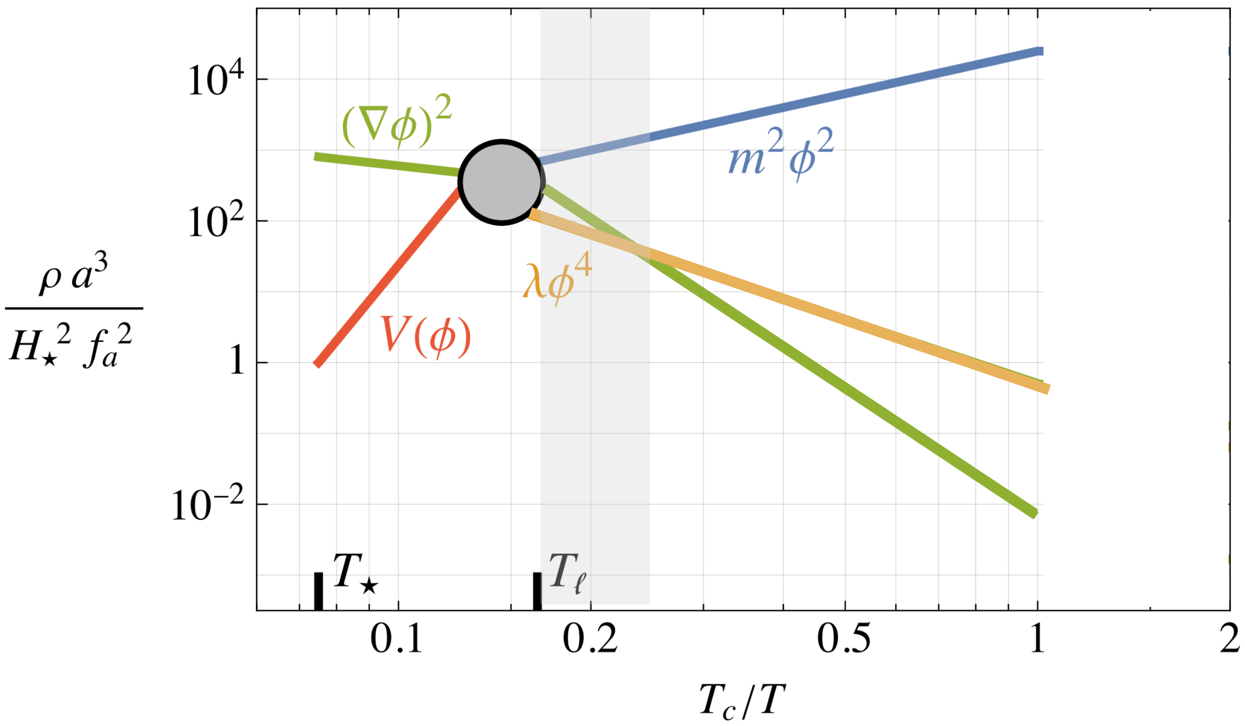
$$(\nabla\phi)^2 \quad + \quad \lambda\phi^4$$

$$\phi \sim \psi e^{-imt}$$

$$T_c \sim 0.15\,\mathrm{GeV}$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} - \xcancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

However: effect of non-linearities (II)

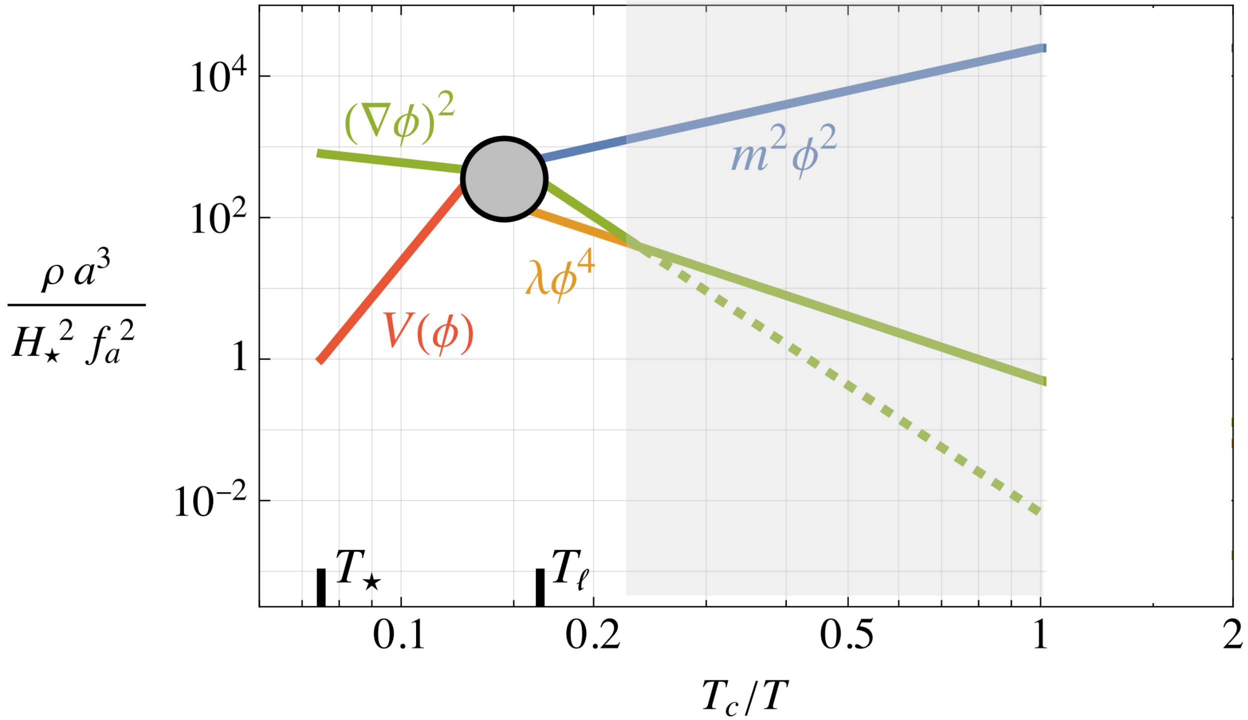


$$\rho \sim \dot\phi^2 + m^2\phi^2 + (\nabla\phi)^2 + \boxed{\lambda}\phi^4$$

grows

$$\left(i\partial_t + \frac{\nabla^2}{2m} \xcancel{- m\Phi} + \underbrace{\frac{\lambda|\psi|^2}{8a^3 m_0 m^2}}_{\text{perturbation}} \right)\psi = 0$$

However: effect of non-linearities (II)

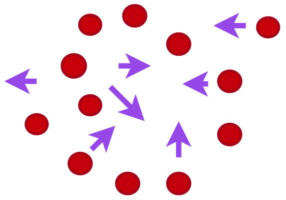


$$\rho \sim \dot{\phi}^2 + m^2\phi^2 +$$

$$(\nabla\phi)^2 + \boxed{\lambda}\phi^4$$

↓

grows



$$k_p \to k_v = \sqrt{\lambda\langle\phi^2\rangle} \simeq \sqrt{\rho}/f_a$$

⤡

$$(\nabla\phi)^2 \sim \lambda\phi^4$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} - \cancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

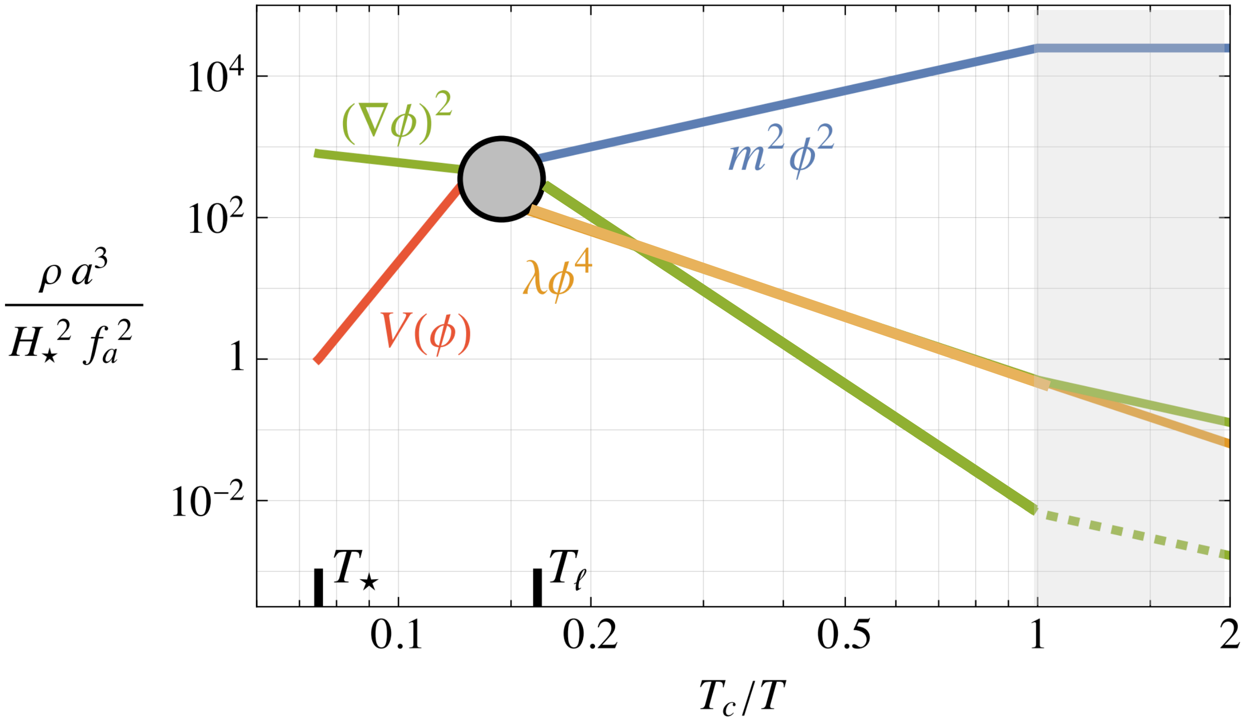
↓

same order as the others

$$\left.\frac{k_p}{k_J}\right|_{\mathrm{MRE}} \sim \left(\frac{10^{10}\ \mathrm{GeV}}{f_a}\right)^{1/2}$$

$$\tau_v = 8m/(\lambda\phi^2) \simeq 0.5\left(\frac{f_a}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{T_c}{T}\right)$$

However: effect of non-linearities (II)



$$\rho \sim \dot{\phi}^2 + m^2\phi^2 +$$

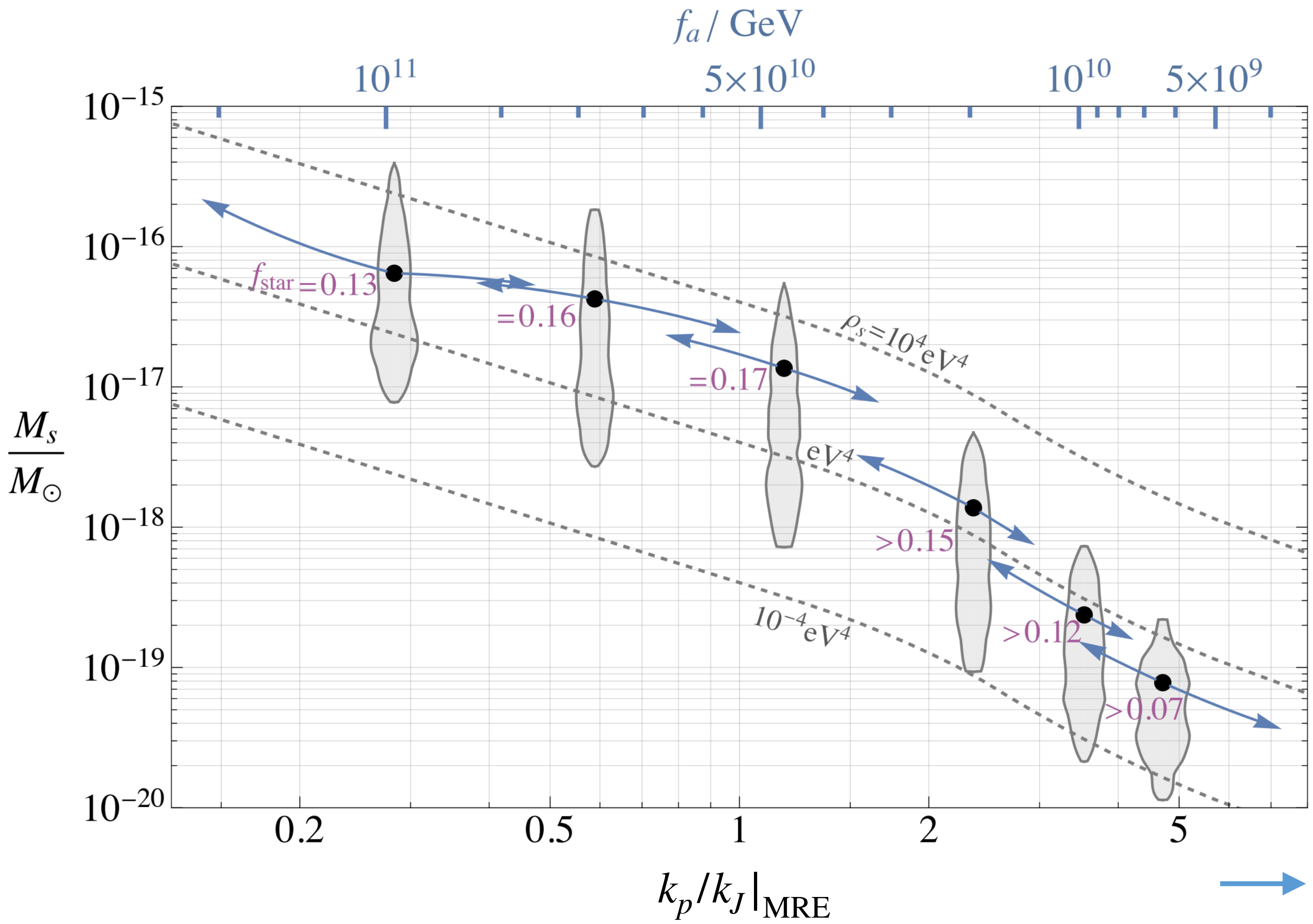
$$(\nabla\phi)^2 + \boxed{\lambda}\phi^4$$

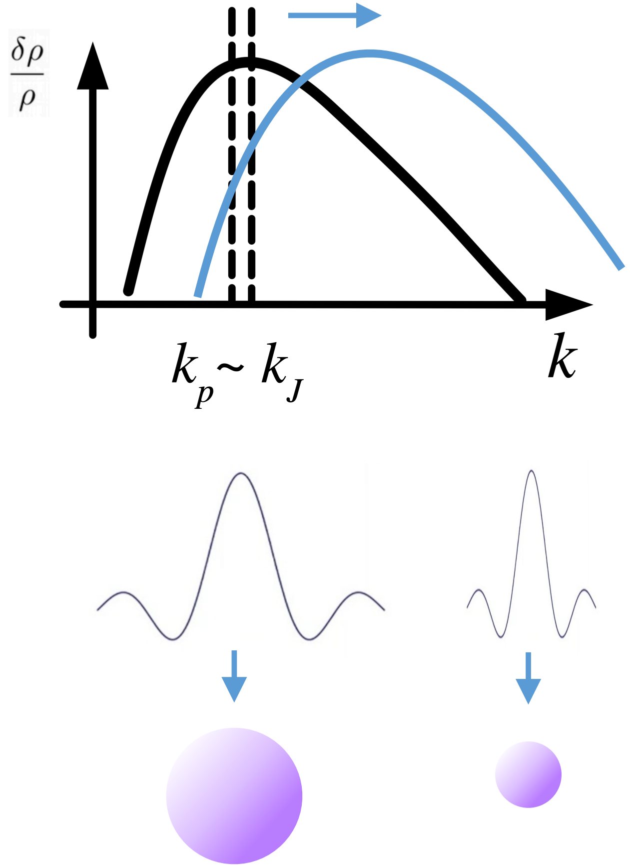
constant

$$\left(i\partial_t + \frac{\nabla^2}{2m} - \xcancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2} \right)\psi = 0$$

perturbation

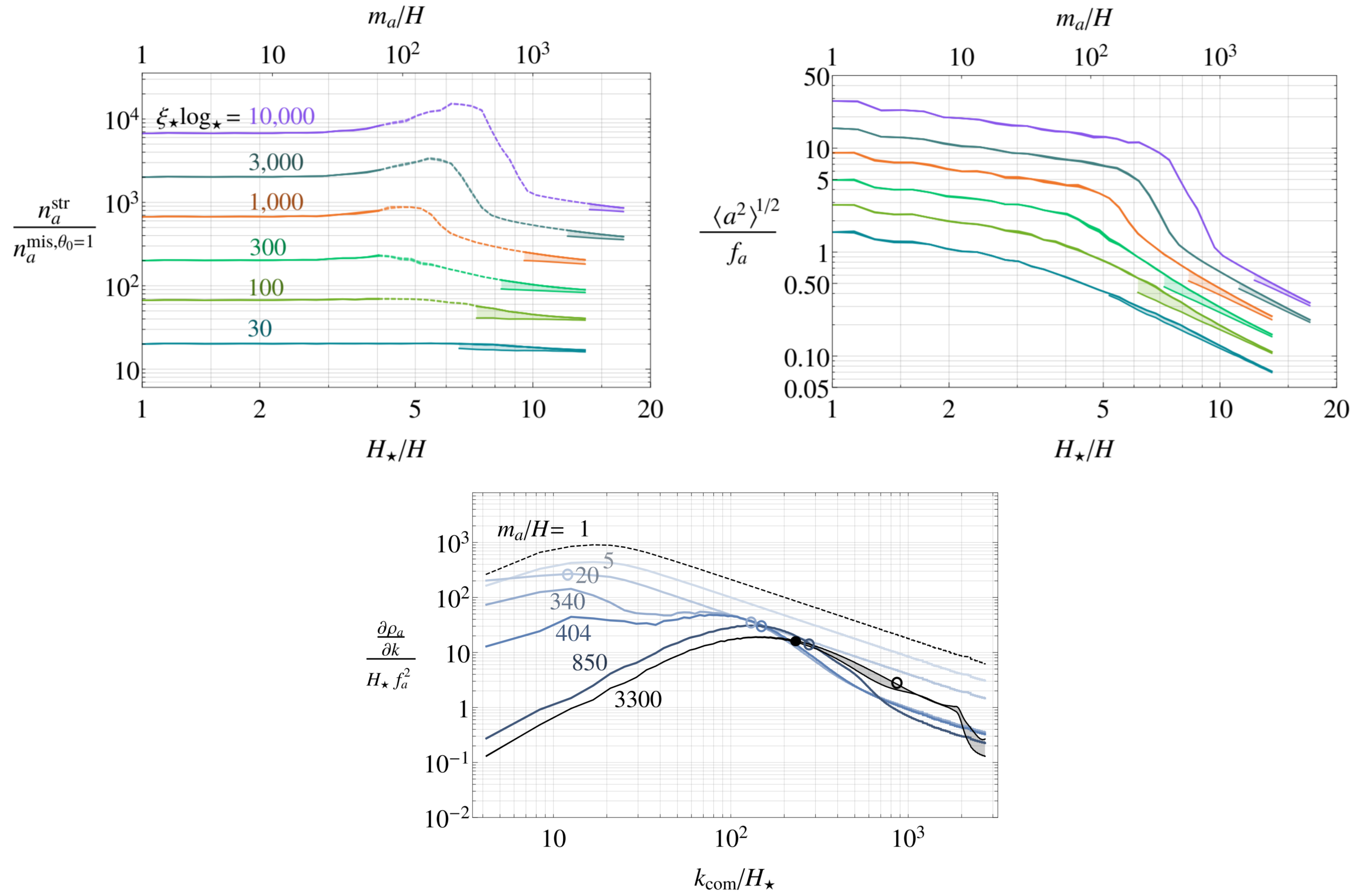
Axion stars properties:

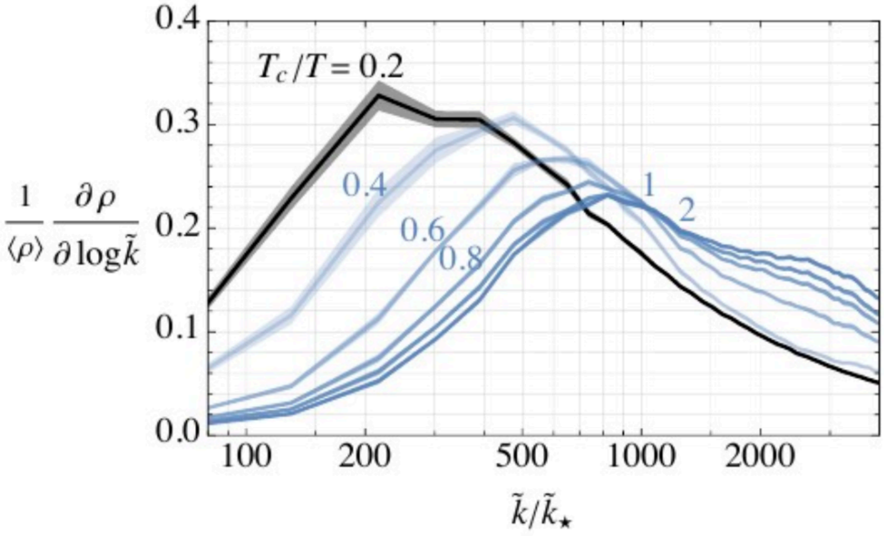


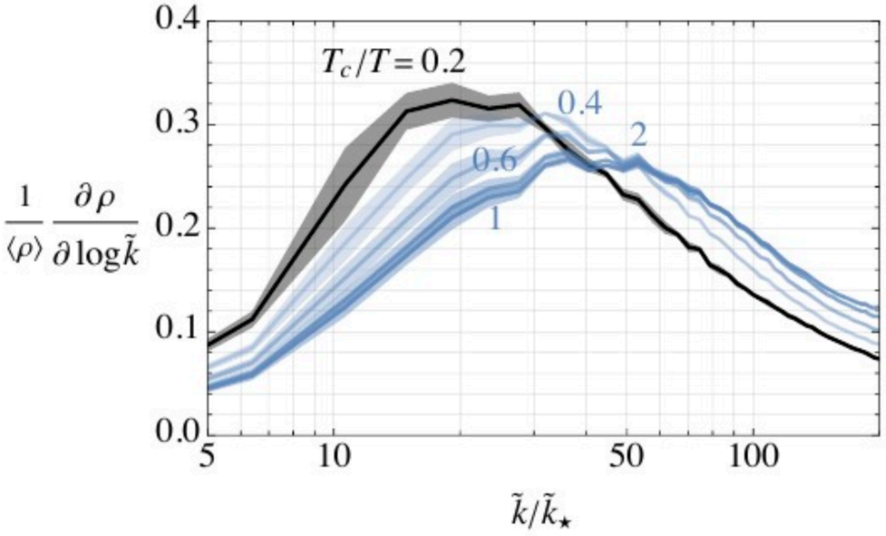


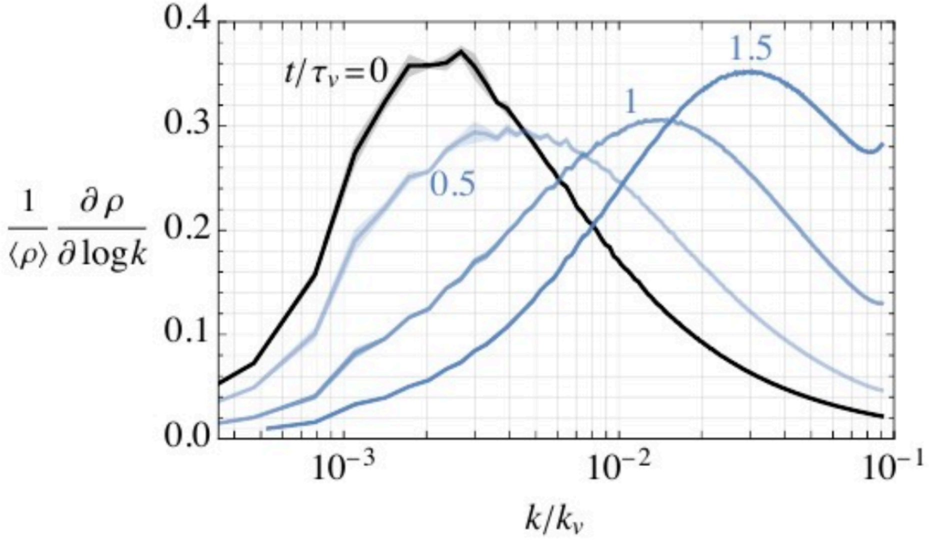
$$\bar{R}_{0.1} \approx 2.1 \cdot 10^6 \ \text{km} \ \left(\frac{10^{10}\ \text{GeV}}{f_a}\right)^{\frac{1}{2}} \qquad v_a \approx \text{mm/s}$$

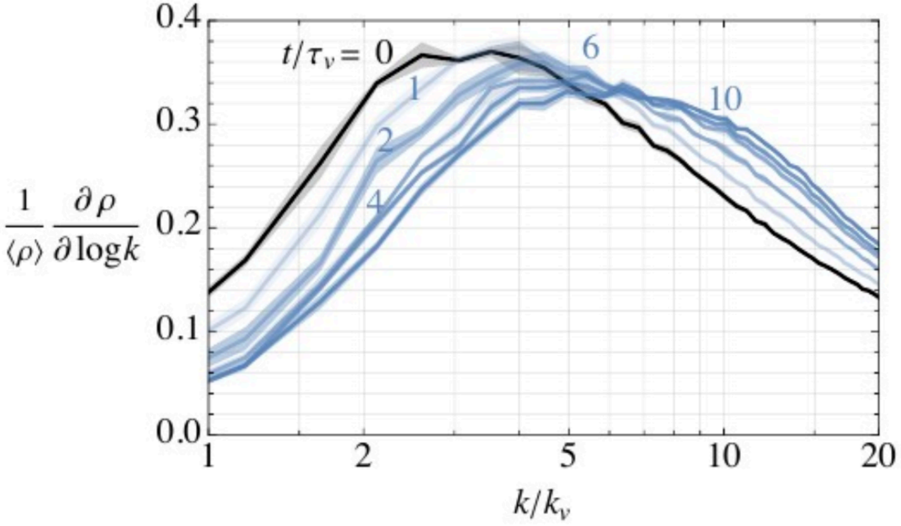
Relativistic Regime and Nonlinear Transient

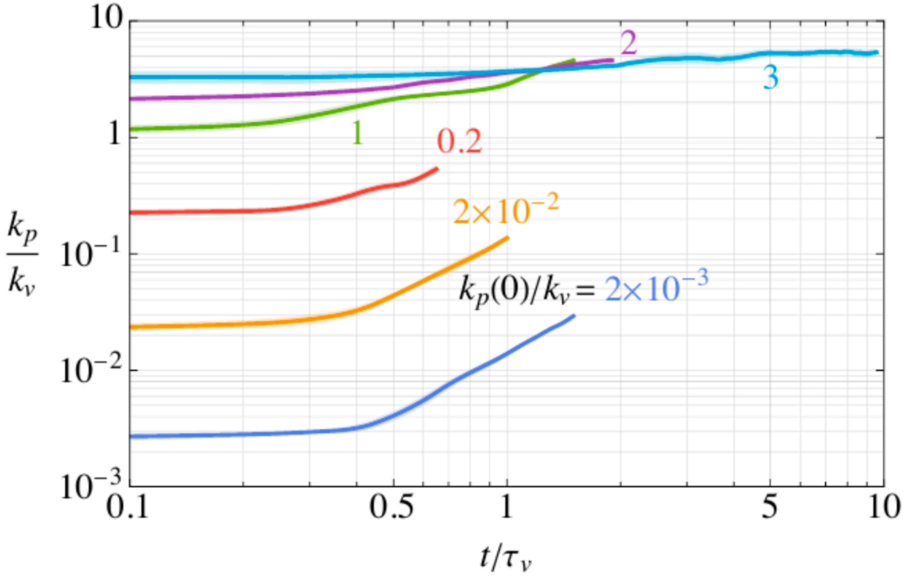


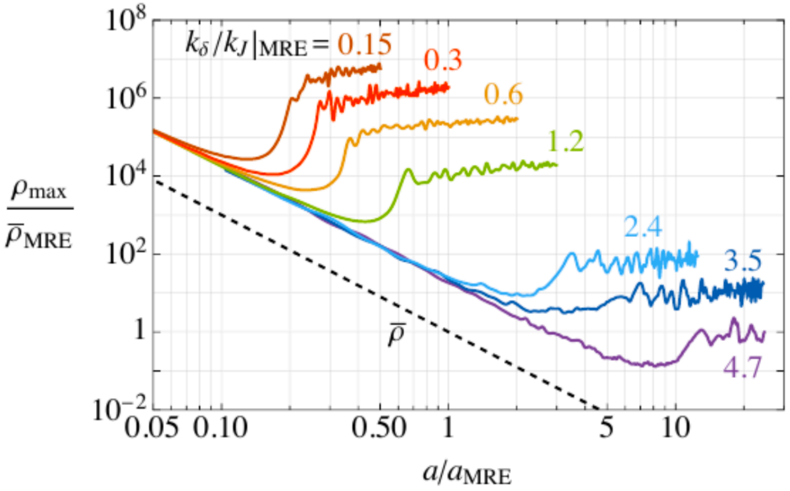

0.4
0.3
0.2
0.1
0.0
$\frac{1}{\langle\rho\rangle}\frac{\partial\rho}{\partial\log\tilde{k}}$
$T_c/T = 0.2$
0.4
0.6
0.8
1
2
100
200
500
1000
2000
$\tilde{k}/\tilde{k}_\star$

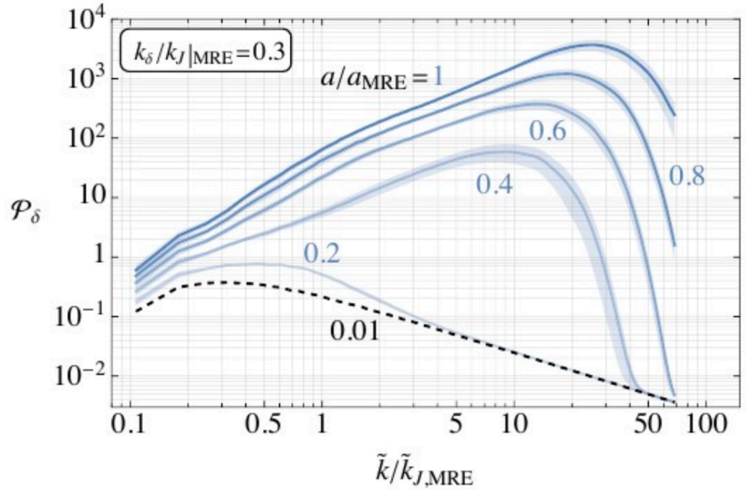

0.4
0.3
0.2
0.1
0.0
$\frac{1}{\langle\rho\rangle}\frac{\partial\rho}{\partial\log\tilde{k}}$
$T_c/T = 0.2$
0.4
2
0.6
1
5
10
50
100
$\tilde{k}/\tilde{k}_\star$

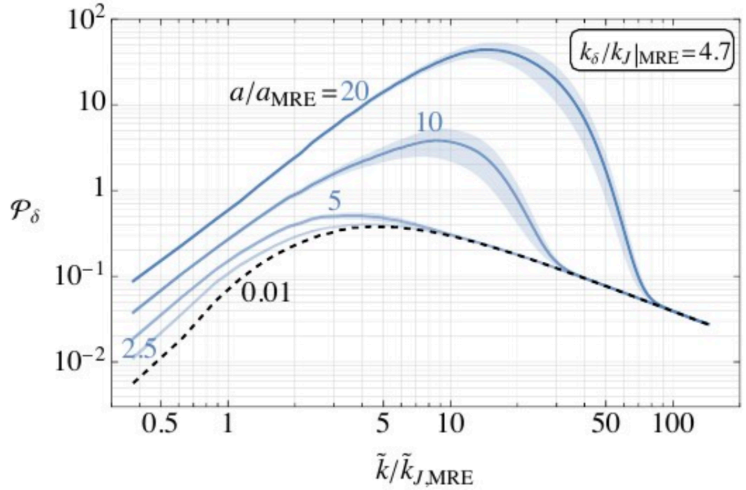

0.4
0.3
0.2
0.1
0.0
$\frac{1}{\langle\rho\rangle}\frac{\partial\rho}{\partial\log k}$
$t/\tau_v=0$
0.5
1
1.5
10^{-3}
10^{-2}
10^{-1}
k/k_v

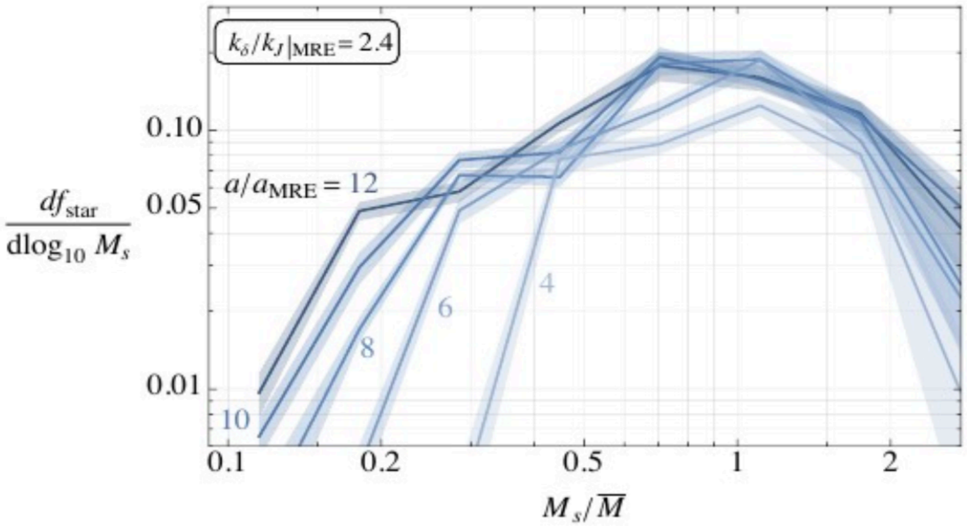

0.4
0.3
0.2
0.1
0.0
$\frac{1}{\langle\rho\rangle}\frac{\partial\rho}{\partial\log k}$
$t/\tau_v=$ 0
1
2
4
6
10
1
2
5
10
20
k/k_v


10
1
10^{-1}
10^{-2}
10^{-3}
$\frac{k_p}{k_v}$
2
3
1
0.2
2×10^{-2}
$k_p(0)/k_v=2\times10^{-3}$
0.1
0.5
1
5
10
t/τ_v

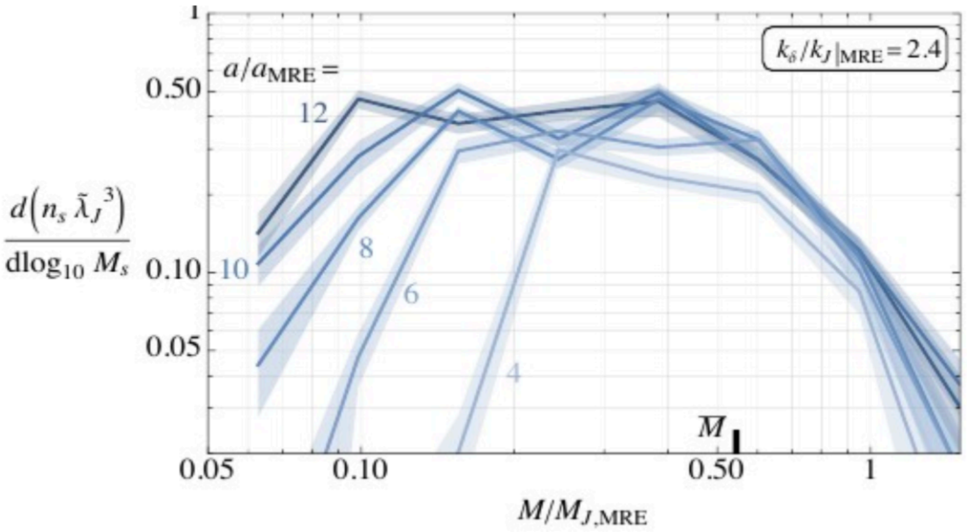

10^8
10^6
10^4
10^2
1
10^{-2}
$\frac{\rho_{\rm max}}{\bar\rho_{\rm MRE}}$
$k_\delta/k_J|_{\rm MRE}=$ 0.15
0.3
0.6
1.2
2.4
3.5
4.7
$\bar\rho$
0.05
0.10
0.50
1
5
10
$a/a_{\rm MRE}$


$k_\delta/k_J|_{\rm MRE}=0.3$
$a/a_{\rm MRE}=1$
0.8
0.6
0.4
0.2
0.01
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
$\mathcal{P}_\delta$
0.1
0.5
1
5
10
50
100
$\tilde k/\tilde k_{J,\rm MRE}$

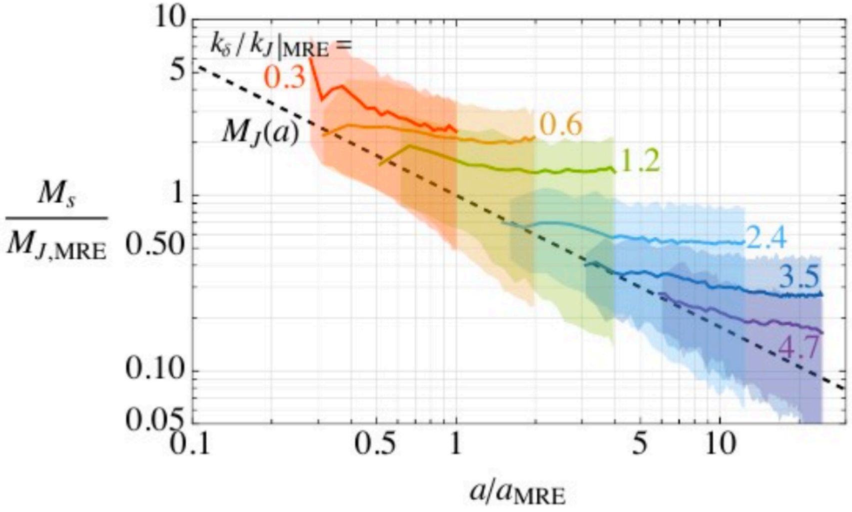

$k_\delta/k_J|_{\rm MRE}=4.7$
$a/a_{\rm MRE}=20$
10
5
2.5
0.01
10^2
10
1
10^{-1}
10^{-2}
$\mathcal{P}_\delta$
0.5
1
5
10
50
100
$\tilde k/\tilde k_{J,\rm MRE}$



$k_\delta/k_J|_{\rm MRE} = 2.4$
$a/a_{\rm MRE} = 12$
10
8
6
4
$\frac{df_{\rm star}}{d\log_{10} M_s}$
0.10
0.05
0.01
0.1
0.2
0.5
1
2
$M_s/\overline{M}$



$k_\delta/k_J|_{\rm MRE} = 2.4$
$a/a_{\rm MRE} =$
12
10
8
6
4
$\overline{M}$
$\frac{d\left(n_s\,\tilde{\lambda}_J^{\,3}\right)}{d\log_{10} M_s}$
1
0.50
0.10
0.05
0.05
0.10
0.50
1
$M/M_{J,\rm MRE}$



$k_\delta/k_J|_{\rm MRE} =$
0.3
0.6
1.2
2.4
3.5
4.7
$M_J(a)$
$\frac{M_s}{M_{J,\rm MRE}}$
10
5
1
0.50
0.10
0.05
0.1
0.5
1
5
10
$a/a_{\rm MRE}$

,

The collapsed objects should be captured by a stationary solution of the Schroedinger–Poisson eq. (for $a = 1$)

$$\psi_i = \sqrt{\rho_i} e^{i\theta_i} \qquad \vec{v}_i = \frac{1}{m}\nabla\theta_i \longrightarrow$$

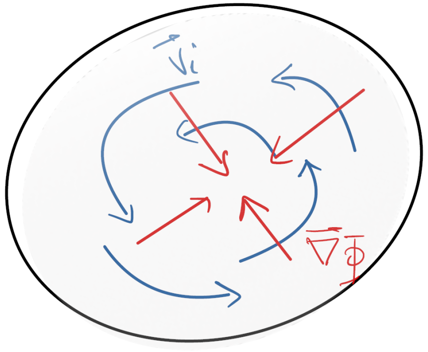
$$\partial_t\rho_i + 3H\rho_i + a^{-1}\nabla\cdot(\rho_i\vec{v}_i) = 0$$
$$\boxed{\partial_t\vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i\cdot\nabla)\vec{v}_i} = -a^{-1}(\boxed{\nabla\Phi} + \boxed{\nabla\Phi_{Qi}})$$
$$\nabla^2\Phi = 4\pi G a^2(\rho - \bar{\rho}) \ ,$$

halos **solitons**

Halos

$$\Phi_Q = 0$$

→ gravitational potential balanced by the velocity terms

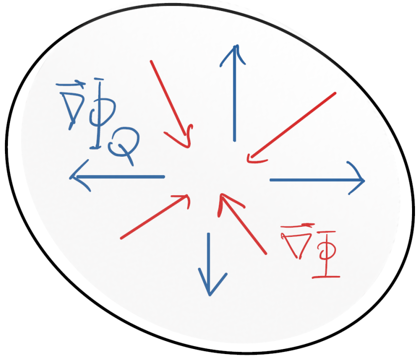


angular momentum ‘supports’ the gravitational potential

Solitons

$$\Phi_Q = -\Phi \qquad \longleftrightarrow \qquad \vec{v}_i = 0$$

→ gravitational potential balanced by the quantum pressure

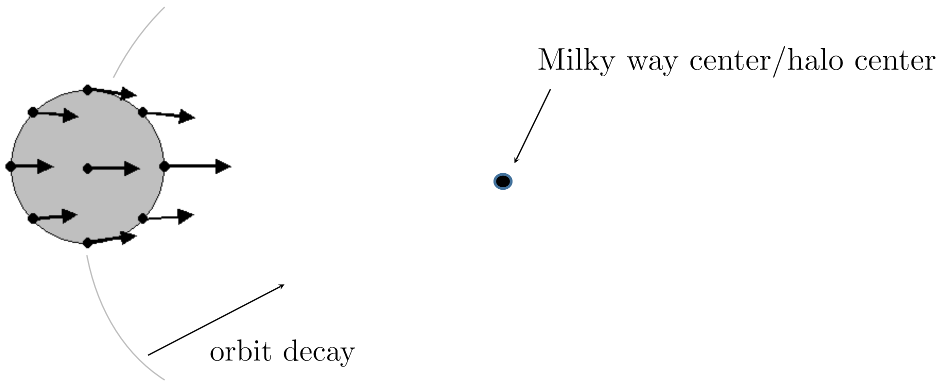


quantum pressure ‘supports’ the gravitational potential

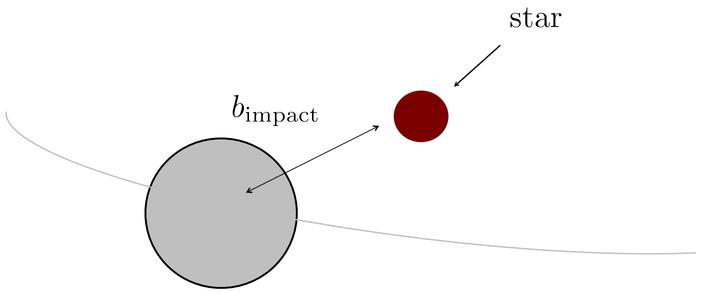
Late time dynamics

Survival of the bound objects

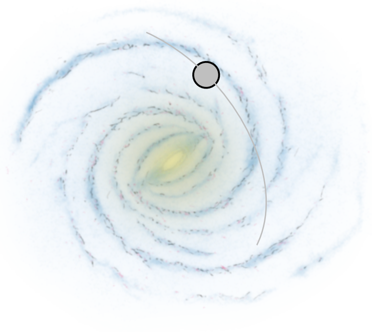
- tidal forces/disruption by a central potential
- dynamical friction $\Longrightarrow$ orbit decay

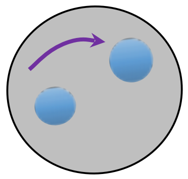


- collisions with stars

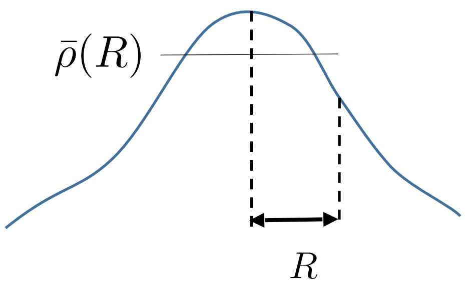


- tidal shocks by the galactic disk
- tidal shocks during formation of halos/merging?





Disruption probability related to the average density of the objects within R



$$\bar{\rho}(R) = \frac{1}{4\pi R^3/3} \int_0^R d^3r \rho(r)$$

→ if the mean density of an object $> 0.05\text{eV}^4 \simeq 10^5 \rho_{\text{loc}} = O(10)\bar{\rho}_{\text{gal}}$, the parts with $r < R$ survive

- **solitons**: $\rho_s = (0.1 \div 100)\text{eV}^4$, and $\bar{\rho}(R_{\text{edge}}) \simeq 0.2\rho_s$

  → most of the solitons and the fuzzy halo around them survive undirsupted

- **compact halos**: those with mass $(10^2 \div 10^4)M_J^{\text{eq}}$ have average density $(10^{-6} \div 10^{-3}\text{eV}^4)$

  → likely to be dirsupted except at their core

$T-$independent mass :

$$\left.\frac{k_p}{k_J}\right|_{\rm MRE} = \frac{k_{p\star}a_\star/a_{\rm MRE}}{(16\pi G\rho_{\rm MRE}m^2)^{1/4}} \simeq \frac{k_{p\star}}{H_\star}$$

$$\rho^{\rm (SM)}_{\rm MRE} \simeq \rho^{\rm (SM)}_\star (a_\star/a_{\rm MRE})^4$$

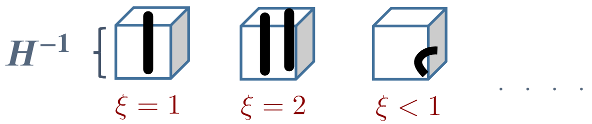
$$3H_\star^2/(8\pi G)$$

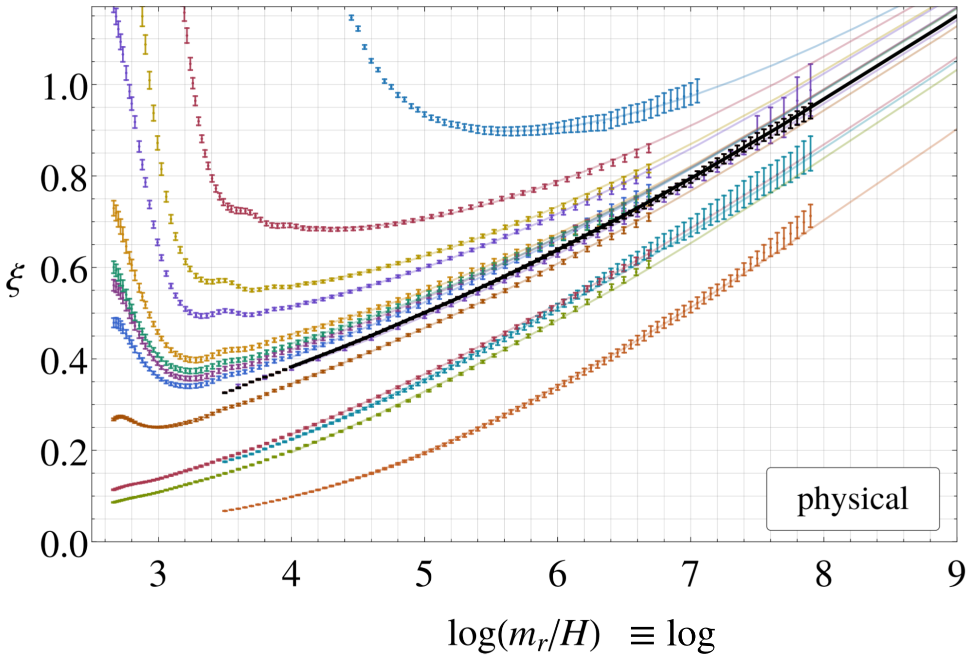
QCD axion case :

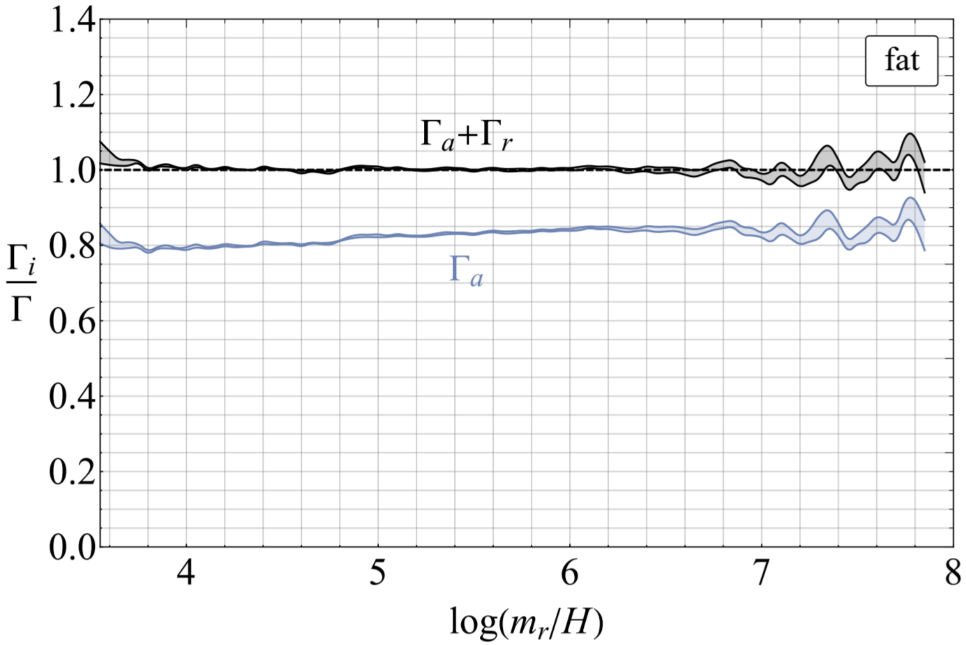
$$\left.\frac{k_p}{k_J}\right|_{\rm MRE} = \frac{k_{p\star}a_\star/a_{\rm MRE}}{(16\pi G\rho_{\rm MRE}m^2)^{1/4}} \simeq \frac{k_{p\star}}{H_\star}\boxed{\left(\frac{m_\star}{m}\right)}^{1/2} \simeq \left(\frac{f_a}{M_p}\right)^{1/3}\frac{k_{p\star}}{H_\star} \sim 10^{-3}\,\frac{k_{p\star}}{H_\star}$$

$$m_\star/m \simeq (T_c/T_\star)^4 \sim (f_a/M_p)^{2/3}$$

Attractor and Energy Emission





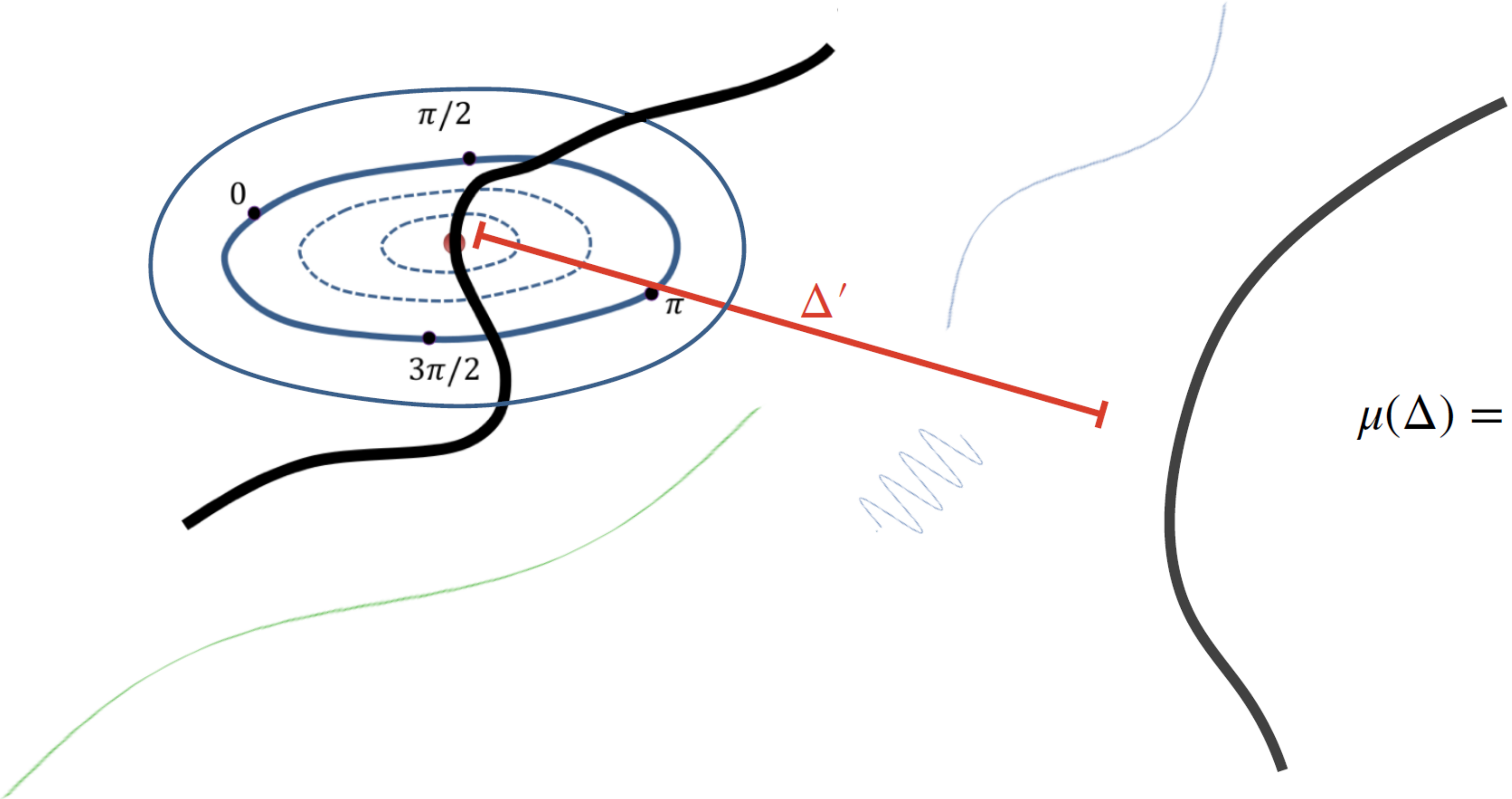


Scaling Violation

$$\boxed{\xi = c_1 \log} + c_0 + \frac{c_{-1}}{\log} + \frac{c_{-2}}{\log^2}$$

→

$$\Gamma_a = \frac{\xi \mu}{t^3} \propto \frac{f_a^2 \log^2}{t^3}$$



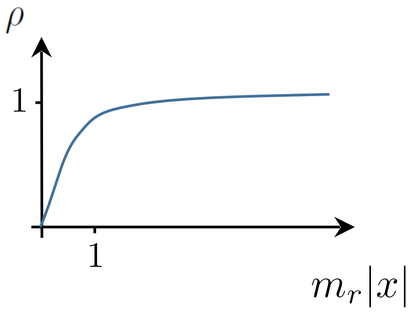
$$\mu(\Delta) = \pi f_a^2 \log\left(\Delta/m_r^{-1}\right)$$

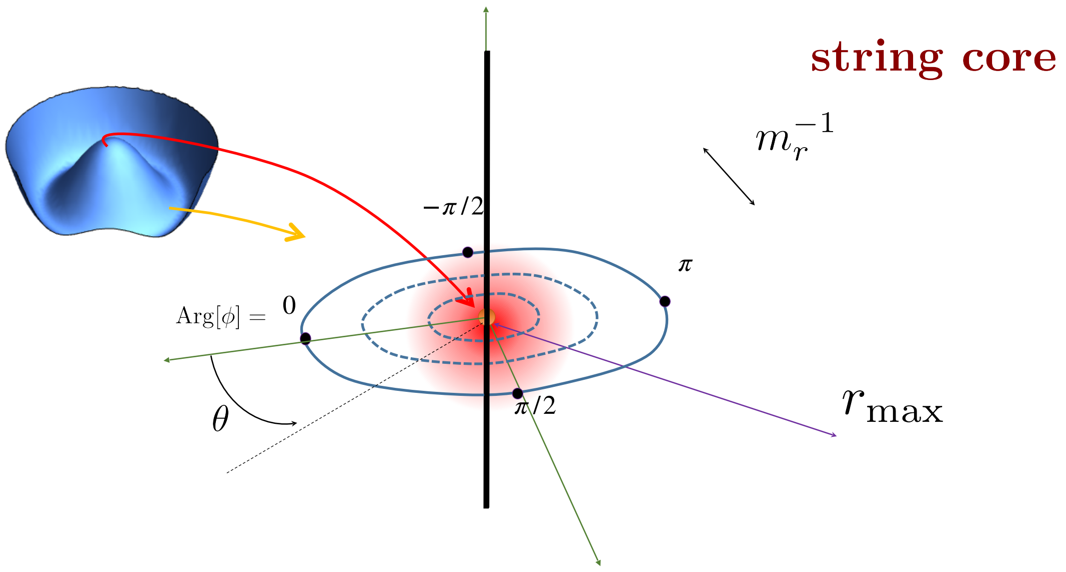
$$\mu(\Delta') = \mu(\Delta) + (g^2/2\pi)\log(\Delta'/\Delta) = \mu(\Delta) + \pi f_a^2 \log(\Delta'/\Delta)$$

[Lund & Regge, 1976]
also [Horn, Nicolis, Penco] in the context of superfluids

Axion Strings

$$\phi = \frac{f_a}{\sqrt{2}}\rho(m_r|x|)e^{i\theta}$$



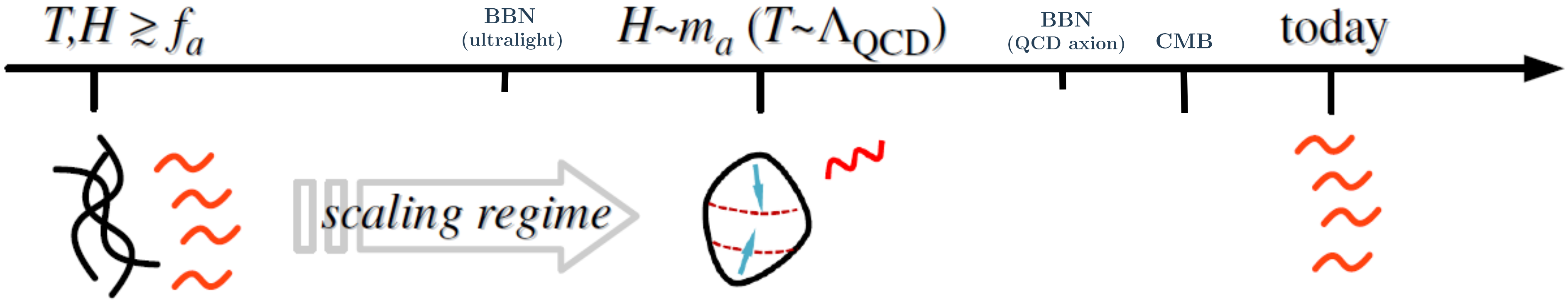


(IR) logarithmically divergent energy!

string tension

$$\frac{E}{L} \equiv \mu = \int_0^{\infty} dr\, r \int_0^{2\pi} d\theta\, \left[|\nabla\phi|^2 + V(\phi)\right]$$

$$= \underset{\textbf{core}}{\boxed{\pi f_a^2}}\,\underset{\textbf{axion gradient}}{\boxed{\log(r_{\max}m_r)}}$$



$\Gamma_a = \Gamma$

$\log(m_r/H)$ $\sim$ **1 ÷ 15** $\sim$ **70 ÷ 100**

strings form

domain walls form and annihilate

relic axions and gravitational waves

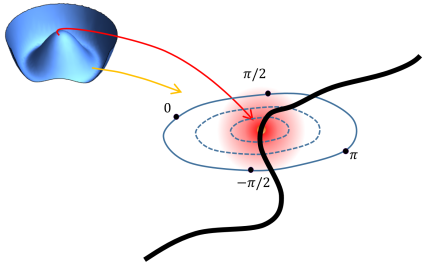
strings

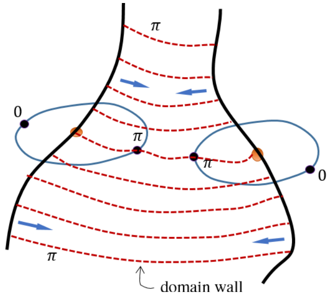
domain walls

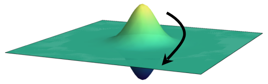
oscillons

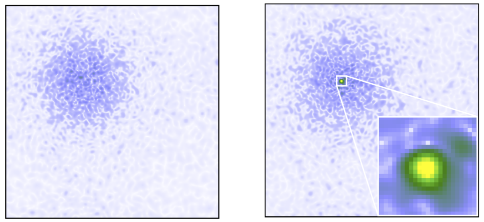
miniclusters

axion stars?

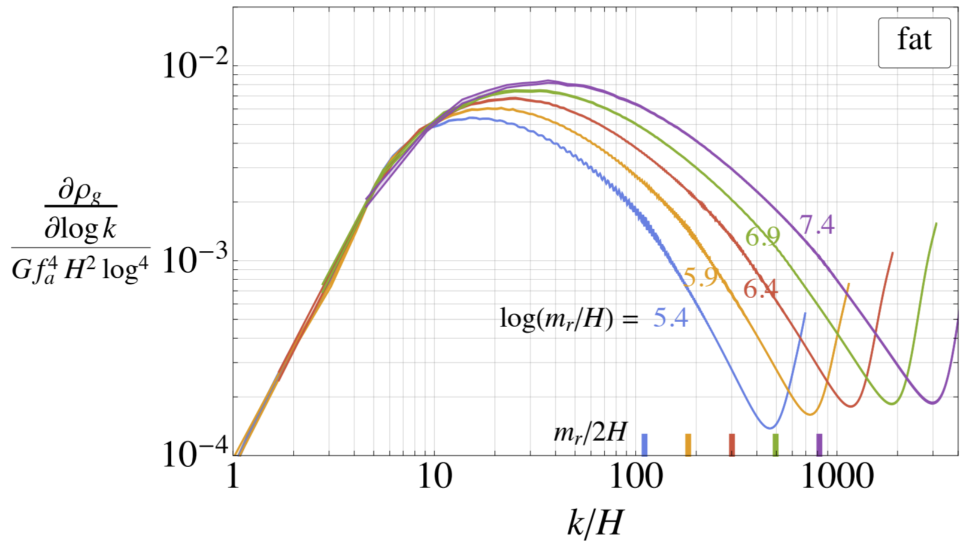


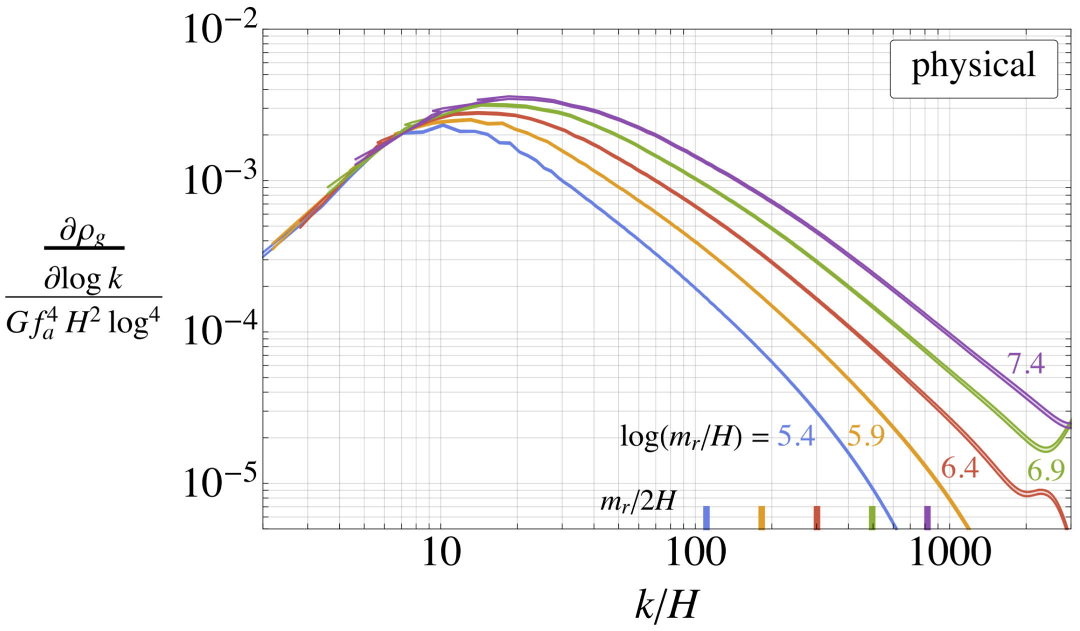


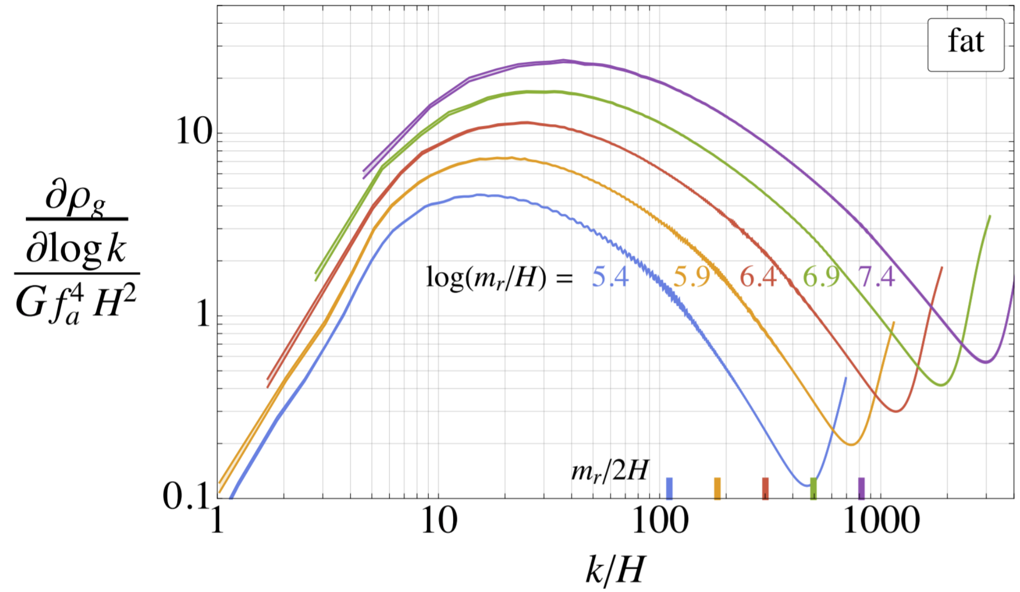


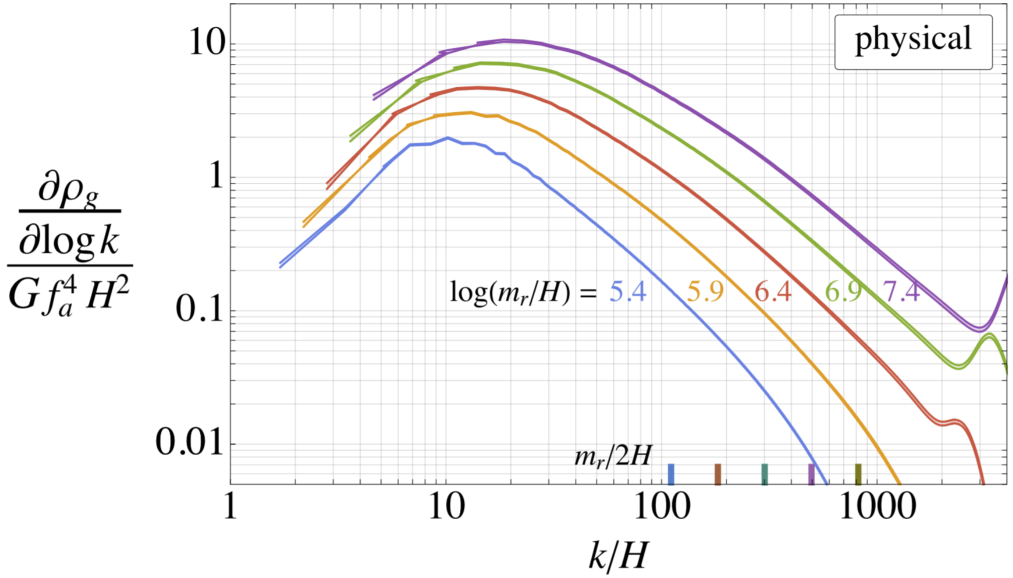


[from 1804.05857]


fat
$\frac{\frac{\partial \rho_g}{\partial \log k}}{G f_a^4 H^2 \log^4}$
10^{-2}
10^{-3}
10^{-4}
1
10
100
1000
k/H
$\log(m_r/H) =$ 5.4
5.9
6.4
6.9
7.4
$m_r/2H$


physical
$\frac{\frac{\partial \rho_g}{\partial \log k}}{G f_a^4 H^2 \log^4}$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10
100
1000
k/H
$\log(m_r/H) =$ 5.4
5.9
6.4
6.9
7.4
$m_r/2H$

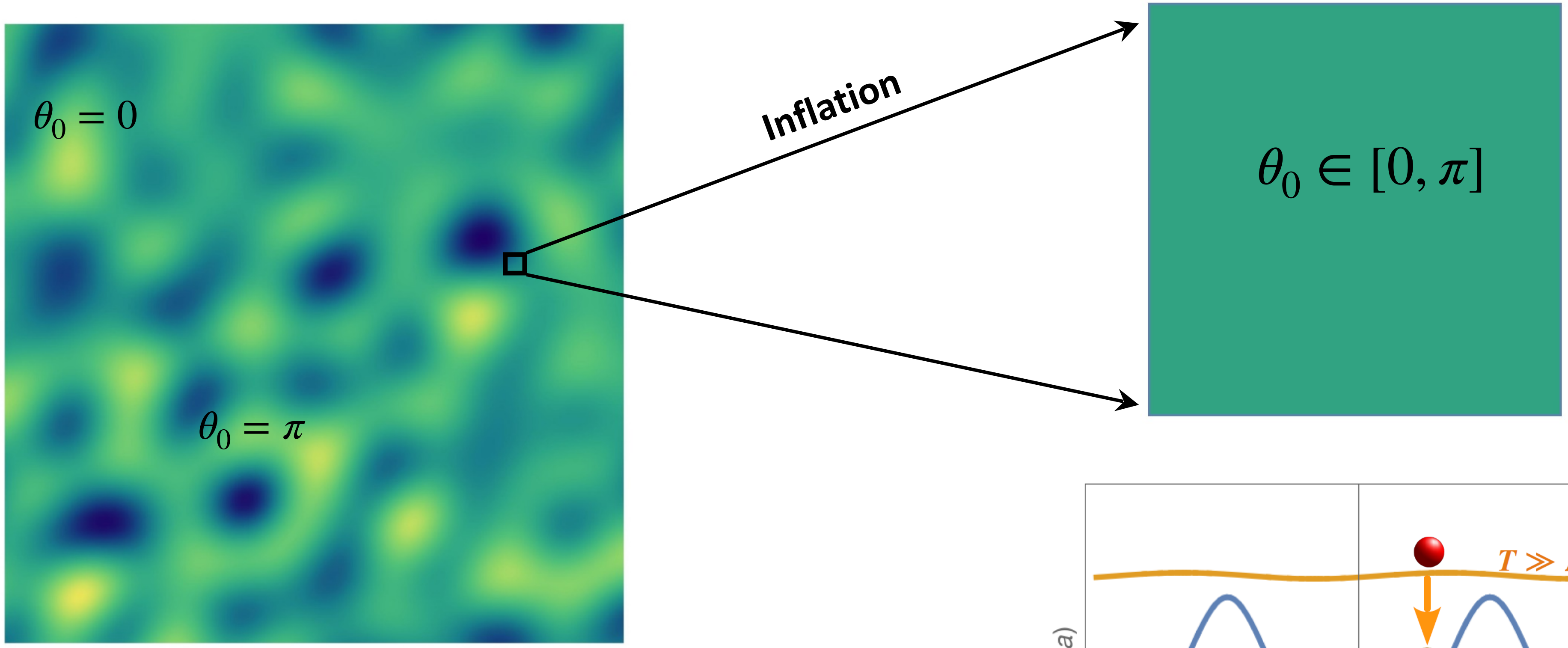

fat
$\frac{\frac{\partial \rho_g}{\partial \log k}}{G f_a^4 H^2}$
10
1
0.1
1
10
100
1000
k/H
$\log(m_r/H) =$ 5.4
5.9
6.4
6.9
7.4
$m_r/2H$

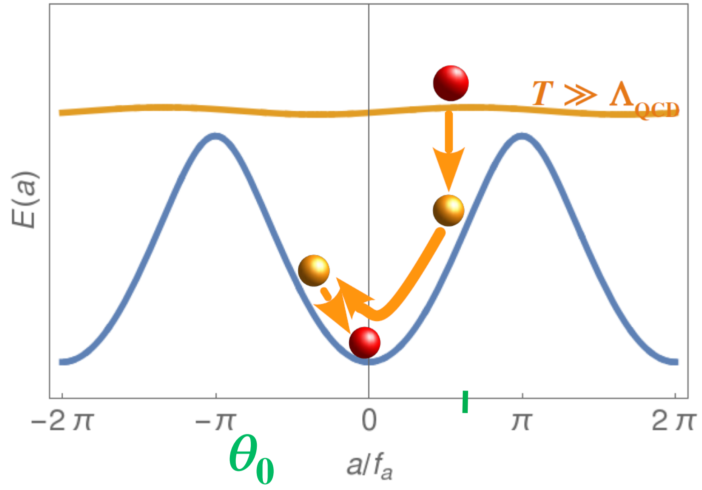

physical
$\frac{\frac{\partial \rho_g}{\partial \log k}}{G f_a^4 H^2}$
10
1
0.1
0.01
1
10
100
1000
k/H
$\log(m_r/H) =$ 5.4
5.9
6.4
6.9
7.4
$m_r/2H$

Axion Cosmology

Pre-inflationary:

$f_a \gtrsim \max(H_I, T_R)$

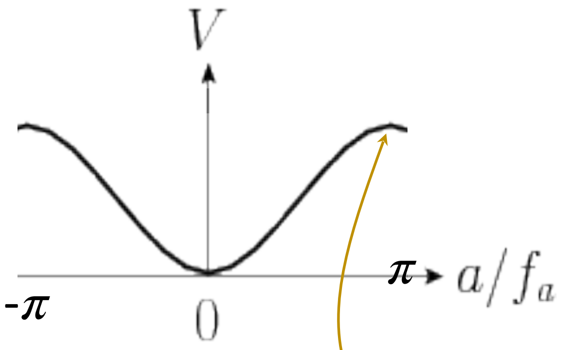


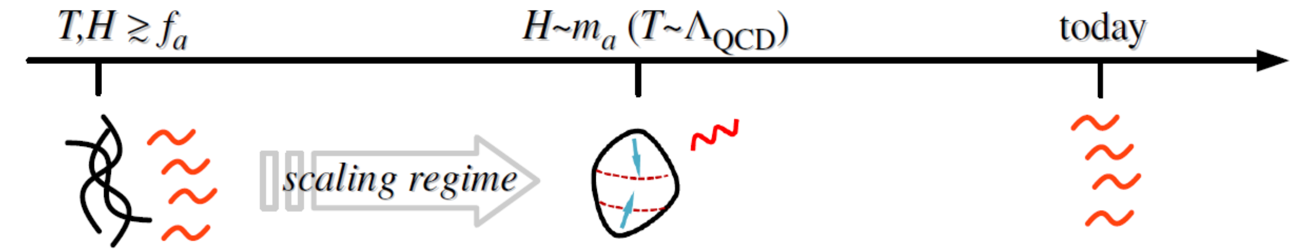


$$\Omega_a \approx 0.1\theta_0^2 \left[\frac{f_a}{10^{12}\mathrm{GeV}}\right]^{1+\epsilon}$$

Axion Domain Walls

@ $H \sim m_a$ ($T \sim \Lambda_{QCD}$)





$T,H \gtrsim f_a$ | $H \sim m_a$ ($T \sim \Lambda_{QCD}$) | today

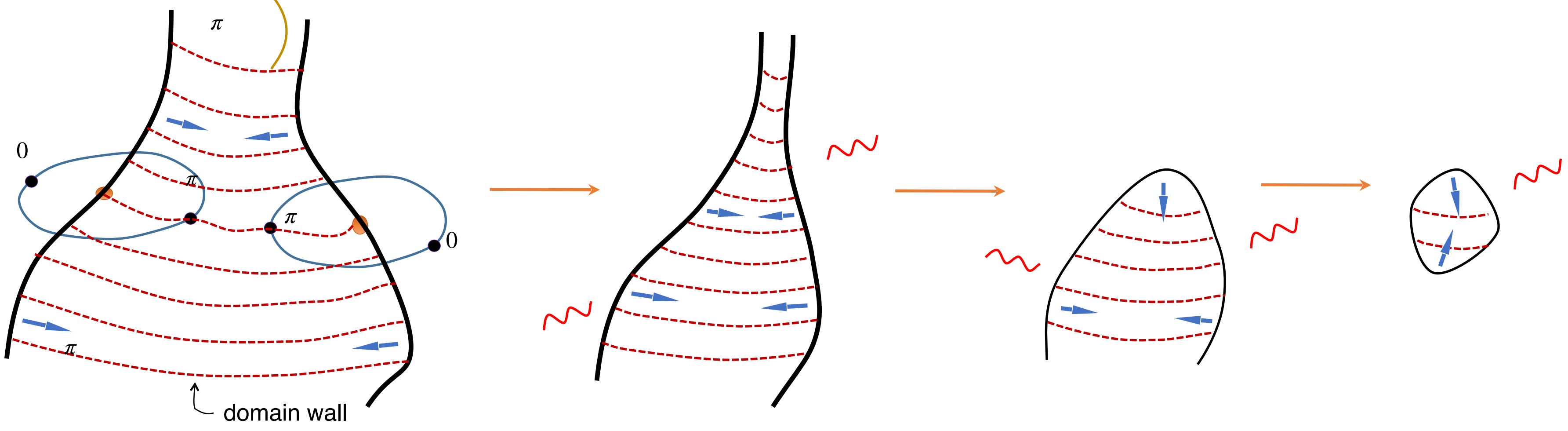
scaling regime

$\log(m_r/H)$ 1÷15 | ~70

strings form

domain walls form and annihilate

relic axions

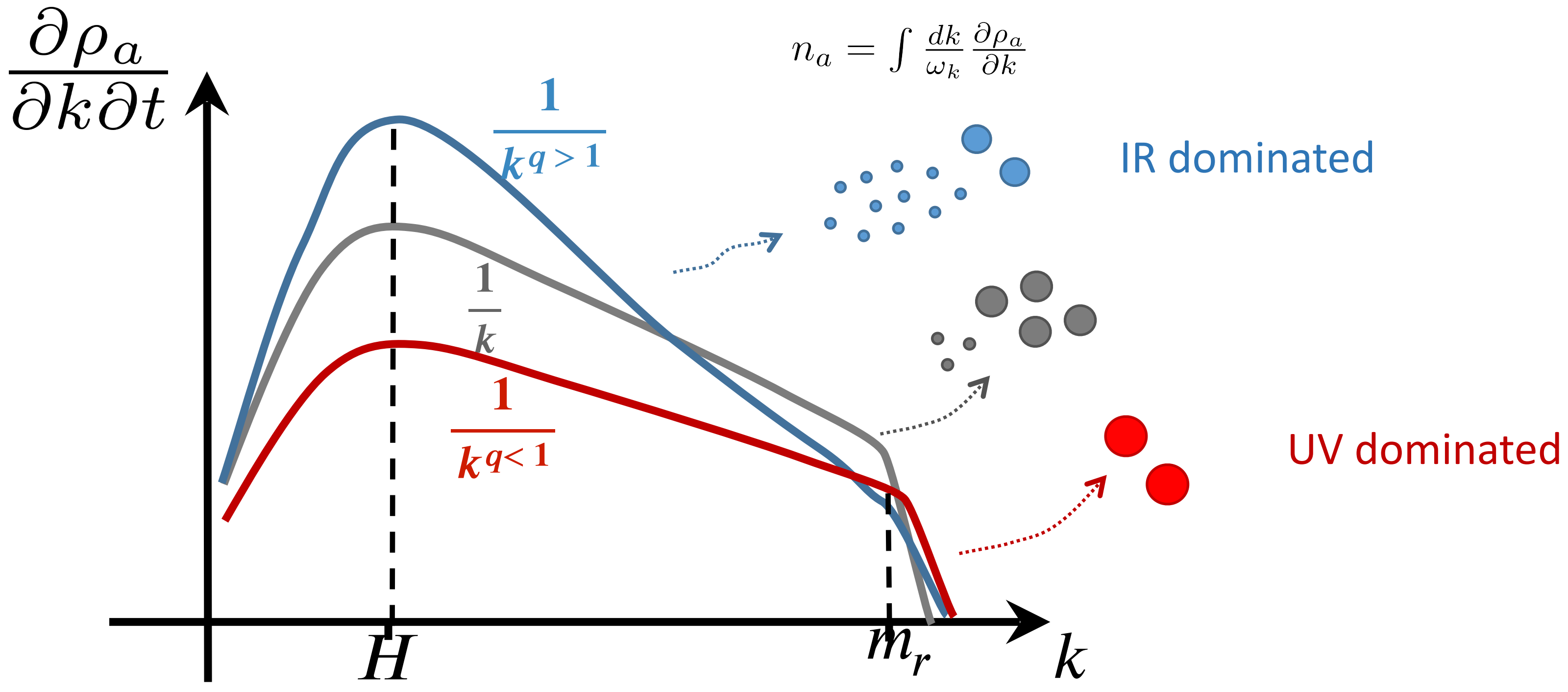


2) The Axion Spectrum

$$\boxed{\frac{\partial \rho_a}{\partial k \partial t} \equiv \text{energy spectrum of axions emitted}}$$

Theoretical expectation

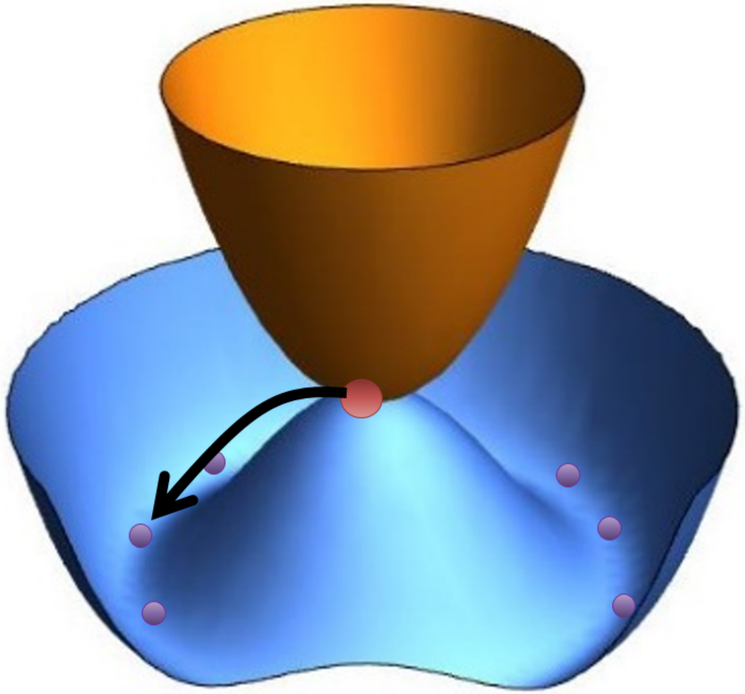
- natural cut-offs at H and m_r
- peak at H because strings have curvature of $O(H)$
- in between an approximate power law: $\boxed{\frac{\partial \rho_a}{\partial k \partial t} \propto \frac{1}{k^q}}$
- in principle q could be time-dependent, $q = q(\log)$



$$n_a = \int \frac{dk}{\omega_k} \frac{\partial \rho_a}{\partial k}$$

The PQ Phase Transition

$T \gtrsim f_a$

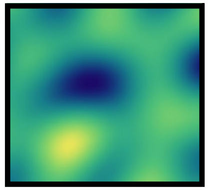


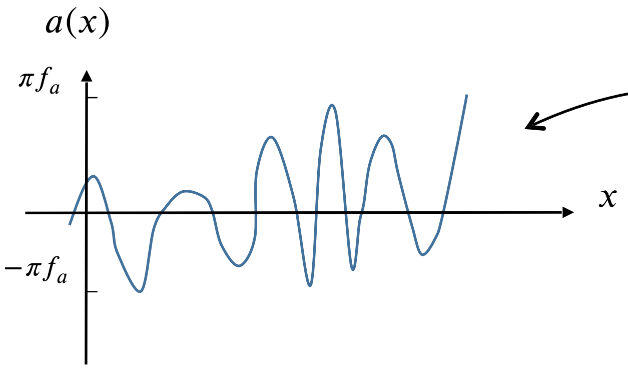
$$\phi = \left|\phi\right| e^{i\frac{a}{f_a}}$$

$T \lesssim f_a$

Similarly if:

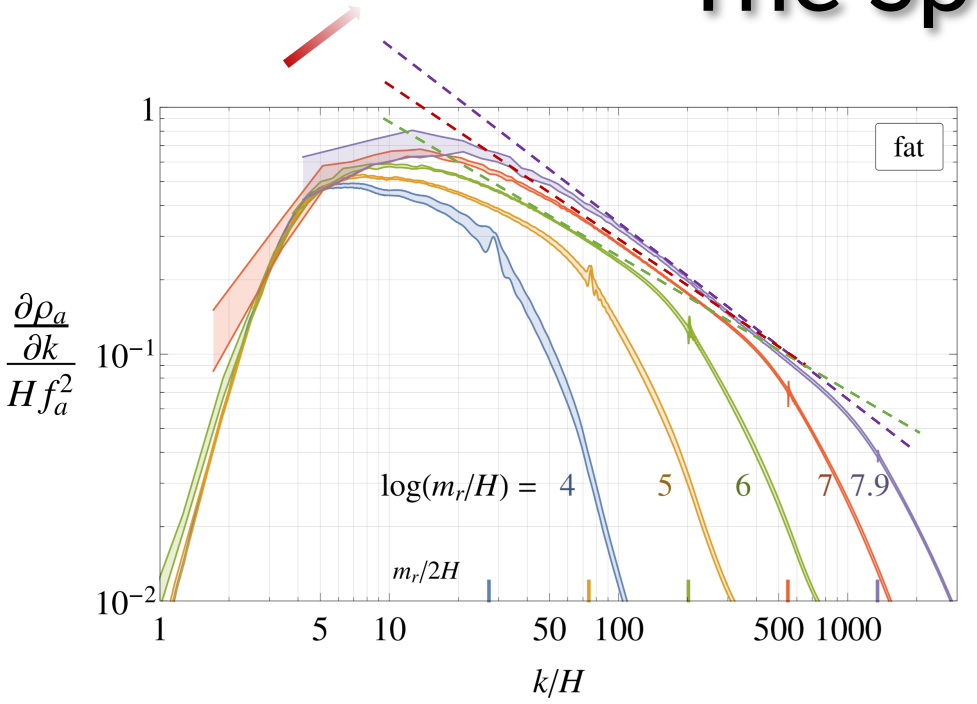
$H_I \gtrsim f_a$

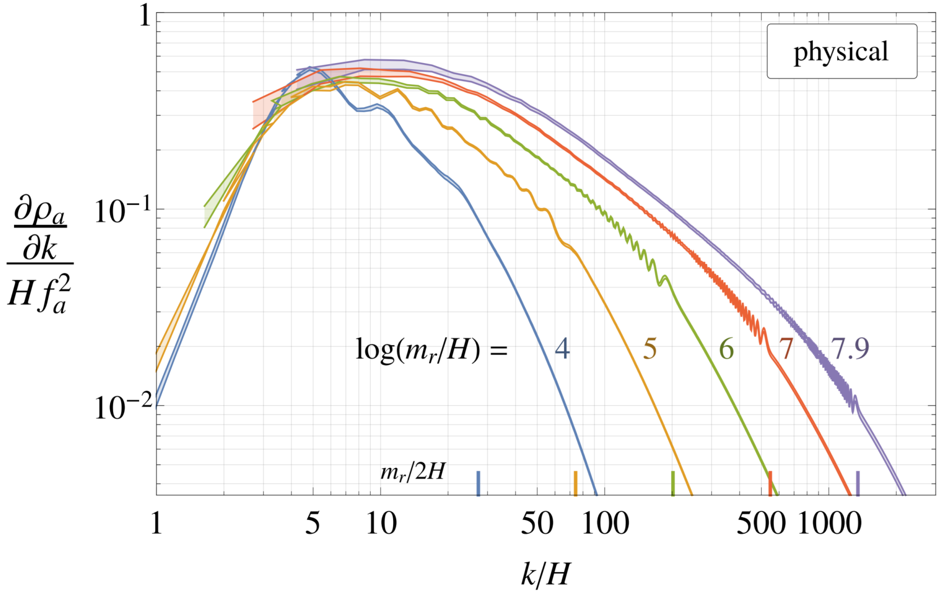




after PQ breaking axion field has random fluctuations within the Hubble horizon

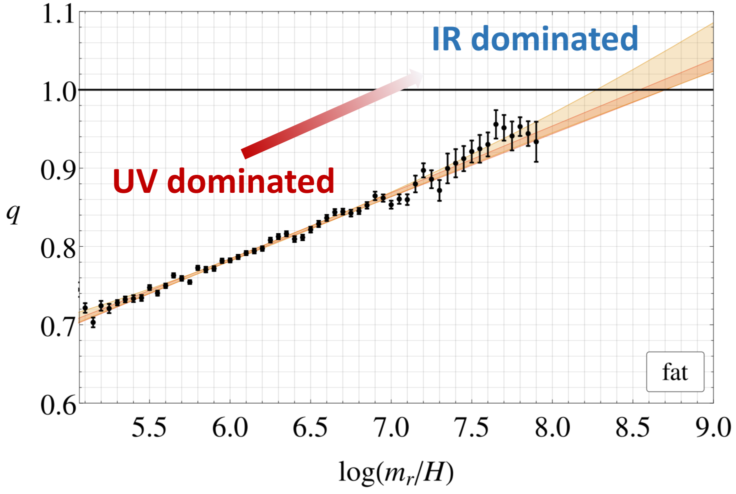
The Spectral Index

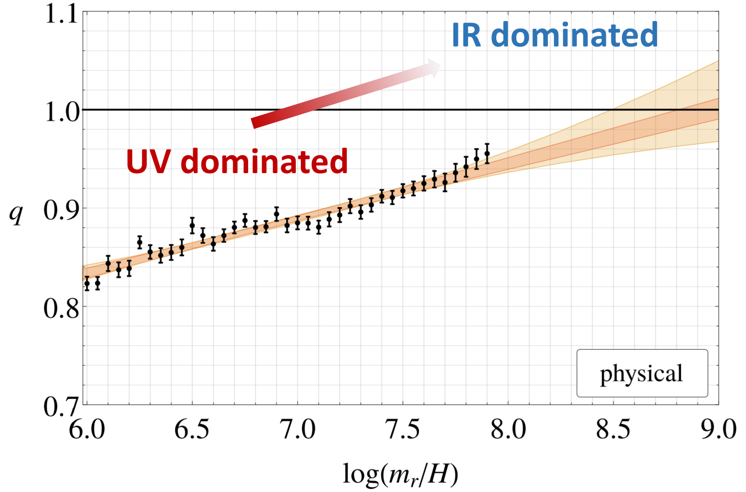




Running of *q*

$\log \to 70$
$q > 1$





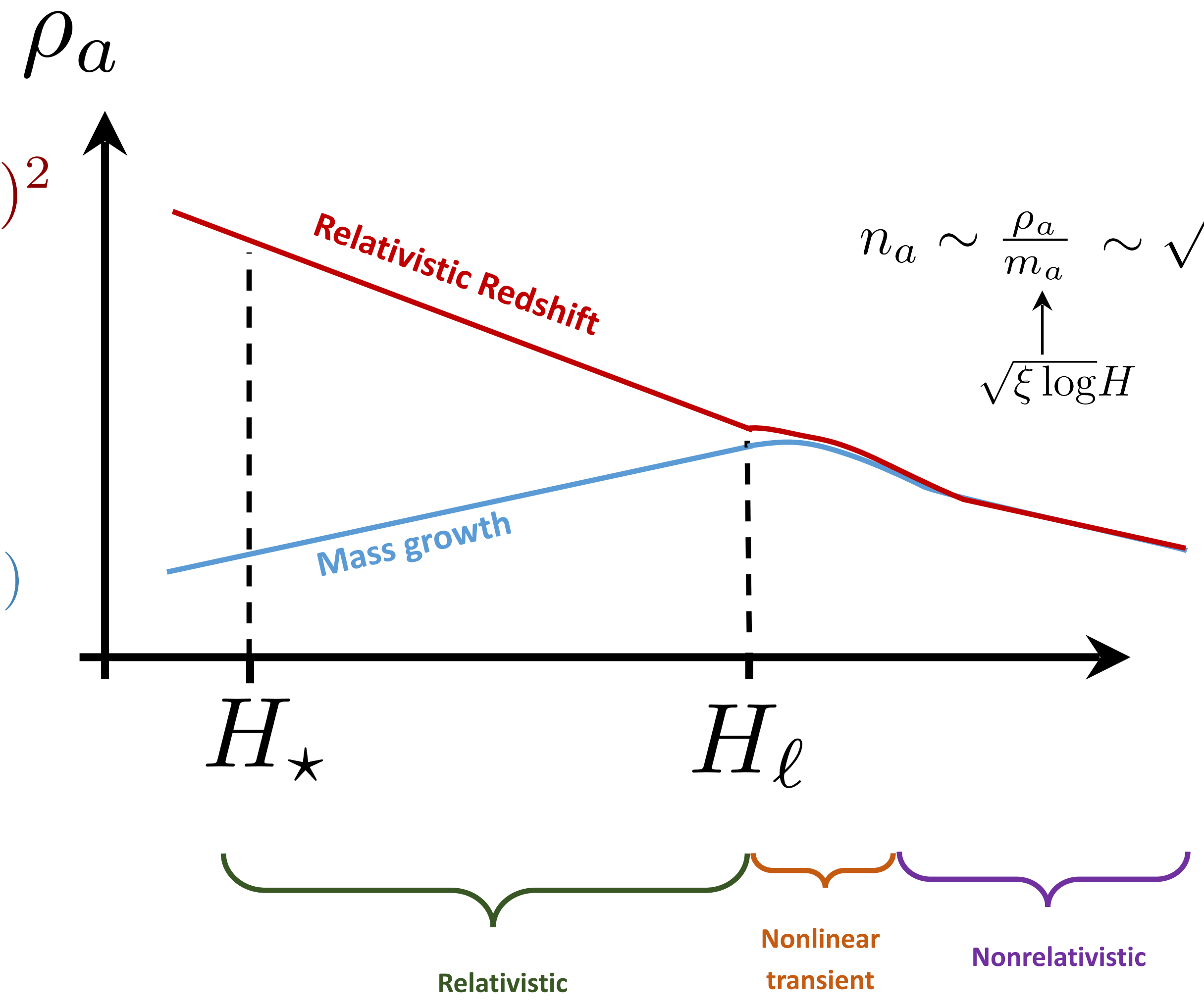
3) Number density after the nonlinear regime

@ $H = m_a \equiv H_\star$

$$\frac{\xi\mu}{t^2} \sim 10^3 \left[\frac{\xi \log}{10^3}\right] H^2 f_a^2 \sim \quad (\nabla a)^2$$

@ $H_\star$ $\gg$

$$m_a^2 f_a^2 \sim \quad V(a)$$

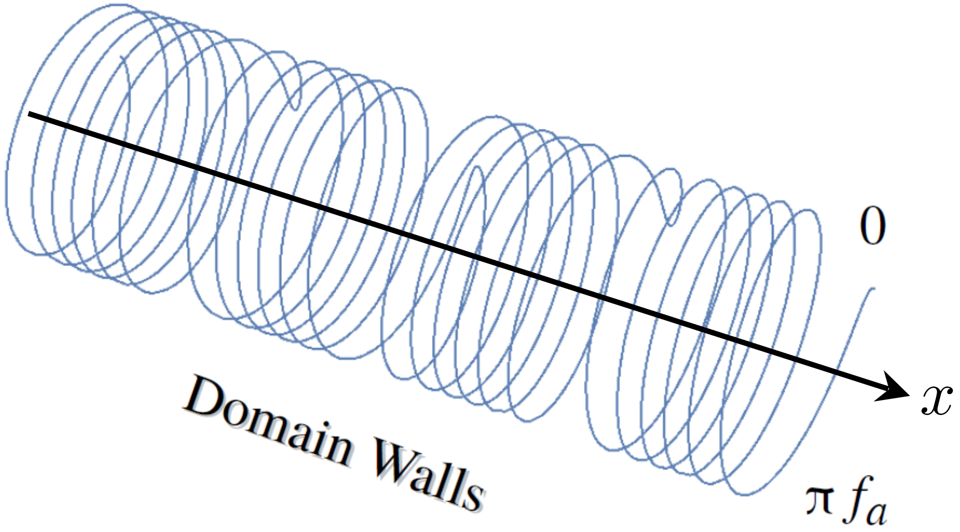


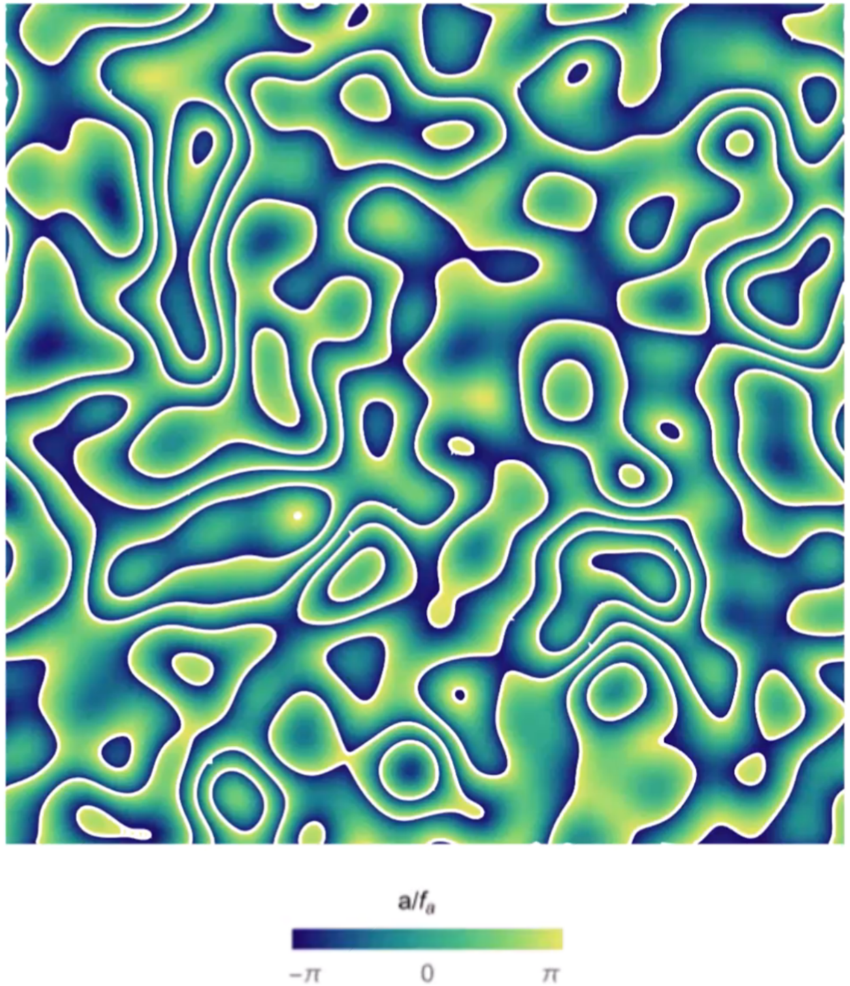
$$n_a \sim \underset{\substack{\uparrow \\ \sqrt{\xi \log} H}}{\frac{\rho_a}{m_a}} \sim \sqrt{\xi \log} H f_a^2$$

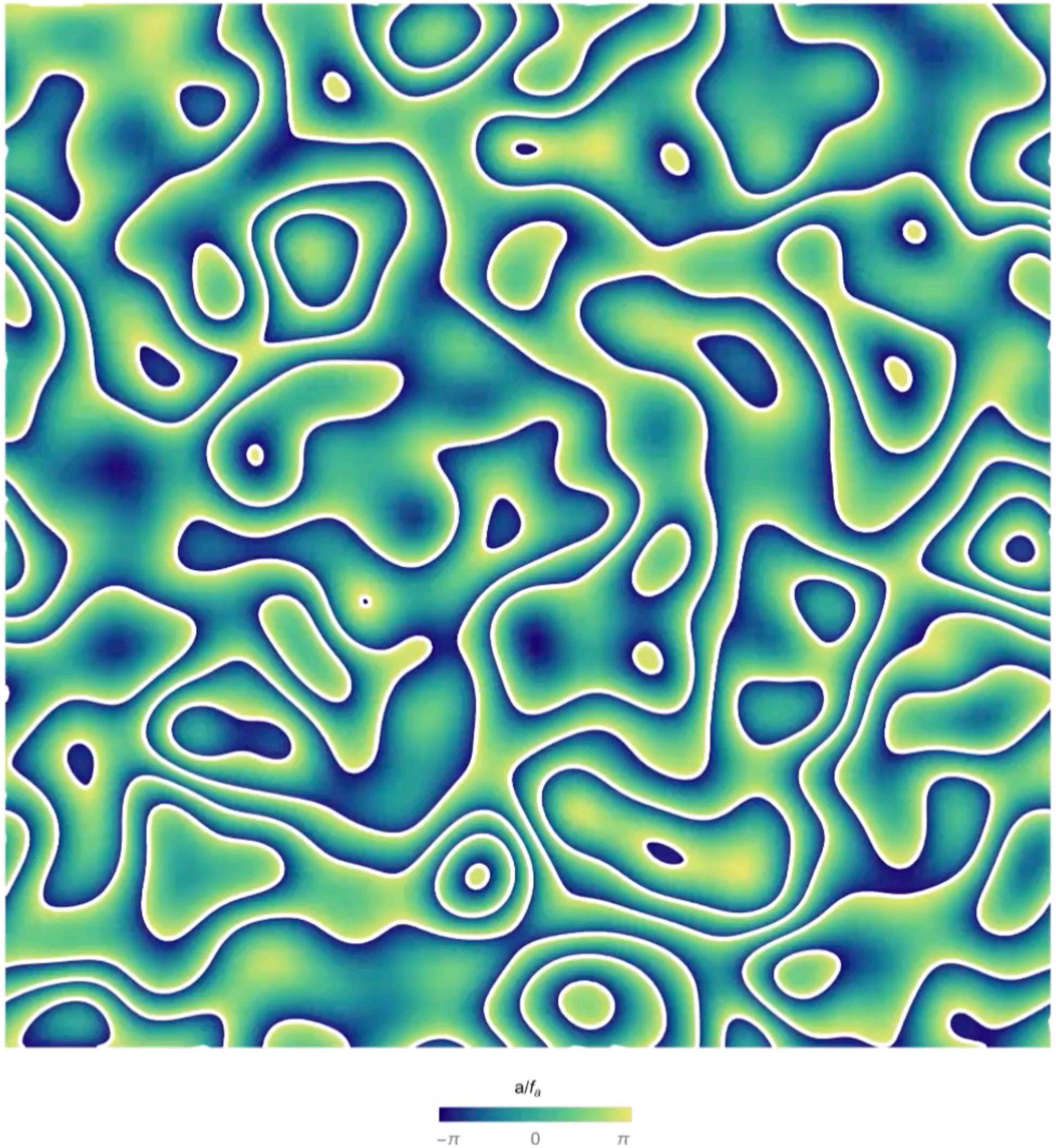
More Domain Walls

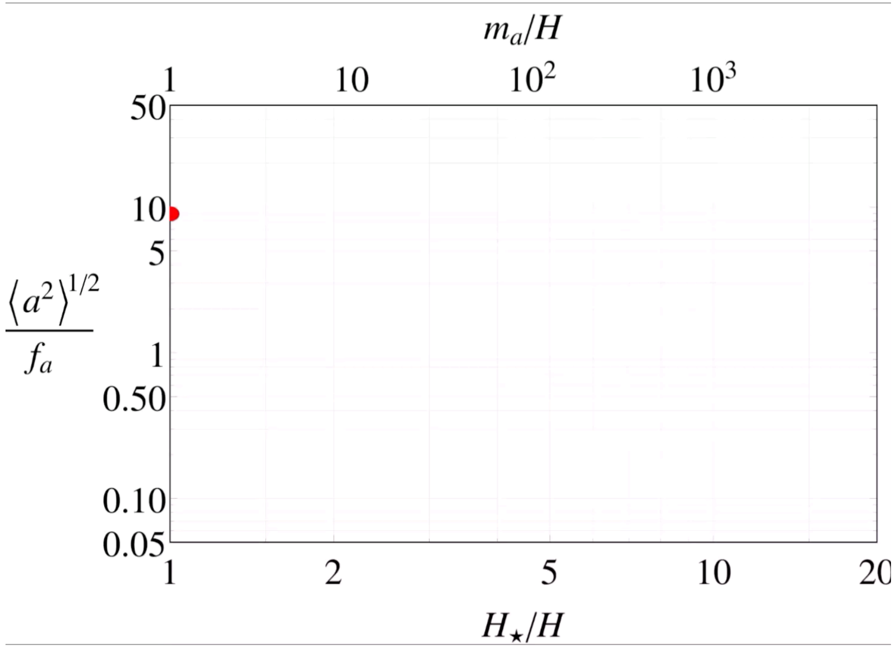
$\boxed{@ \; H_\star}$

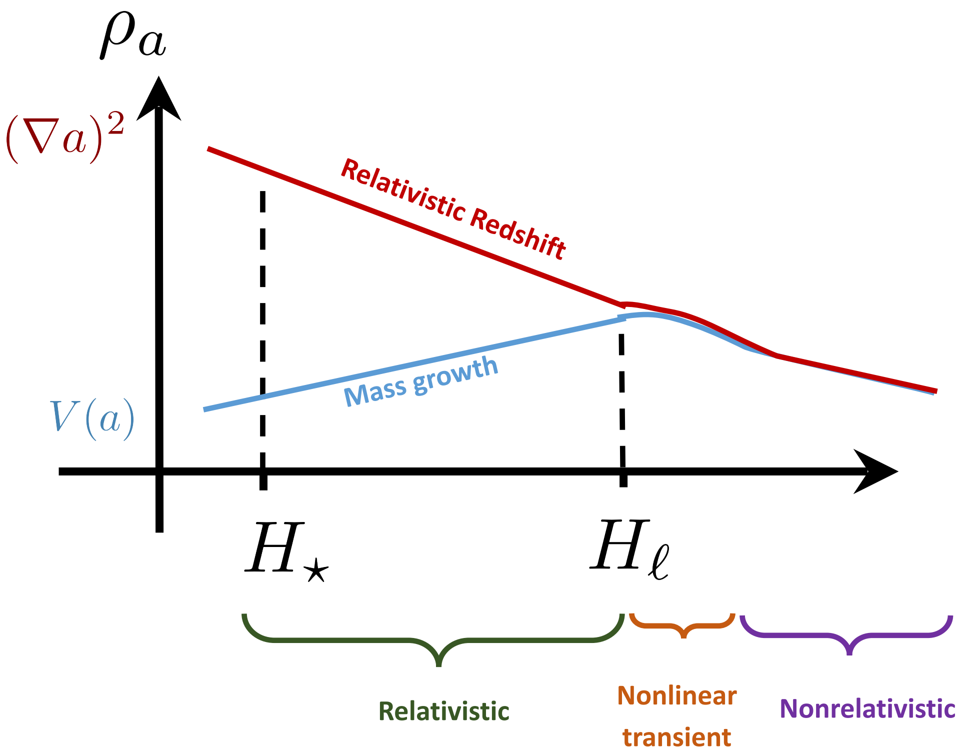
$$\frac{a}{2\pi f_a} \sim \sqrt{\xi \log} = O(10)$$






a/f_a
$-\pi$
0
π


m_a/H
1
10
10^2
10^3
50
10
5
1
0.50
0.10
0.05
$\frac{\langle a^2 \rangle^{1/2}}{f_a}$
1
2
5
10
20
$H_\star/H$

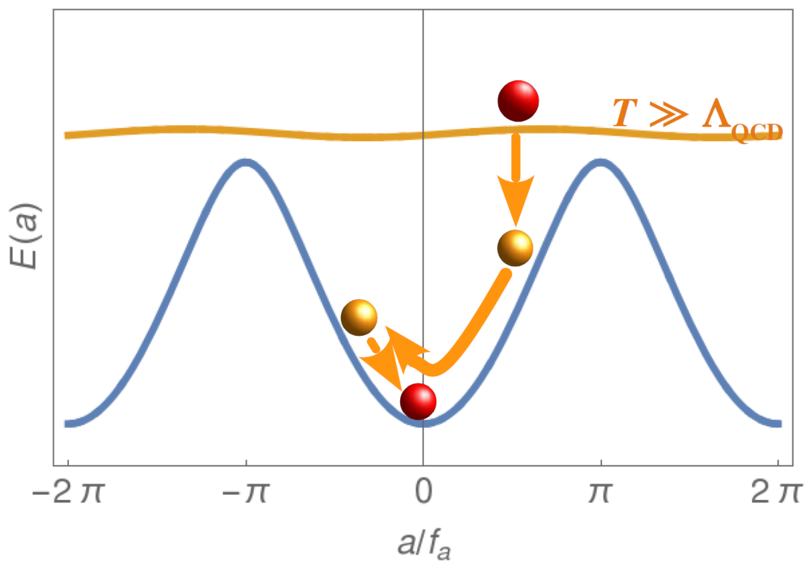

ρ_a
$(\nabla a)^2$
Relativistic Redshift
Mass growth
$V(a)$
$H_\star$
H_ℓ
Relativistic
Nonlinear transient
Nonrelativistic

Scenario #1: $(T, H) < f_a$

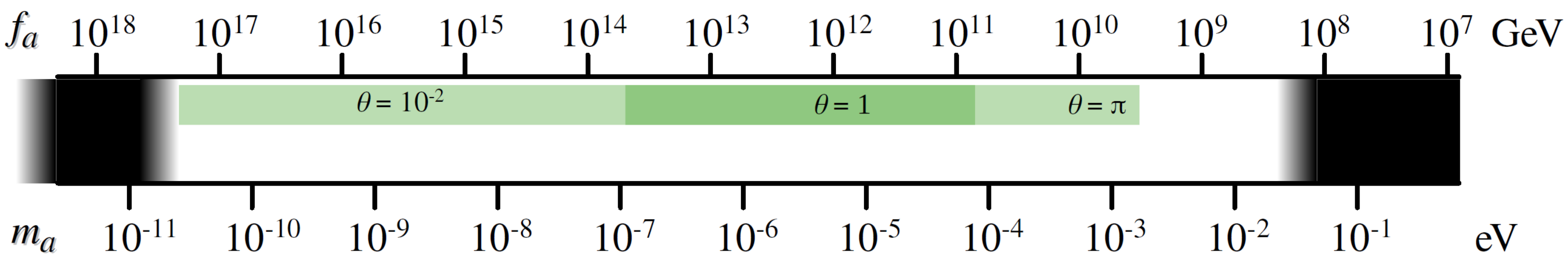
$\theta = 0 \div \pi$

$a(t_0) = \text{const} \equiv \theta_0 f_a$

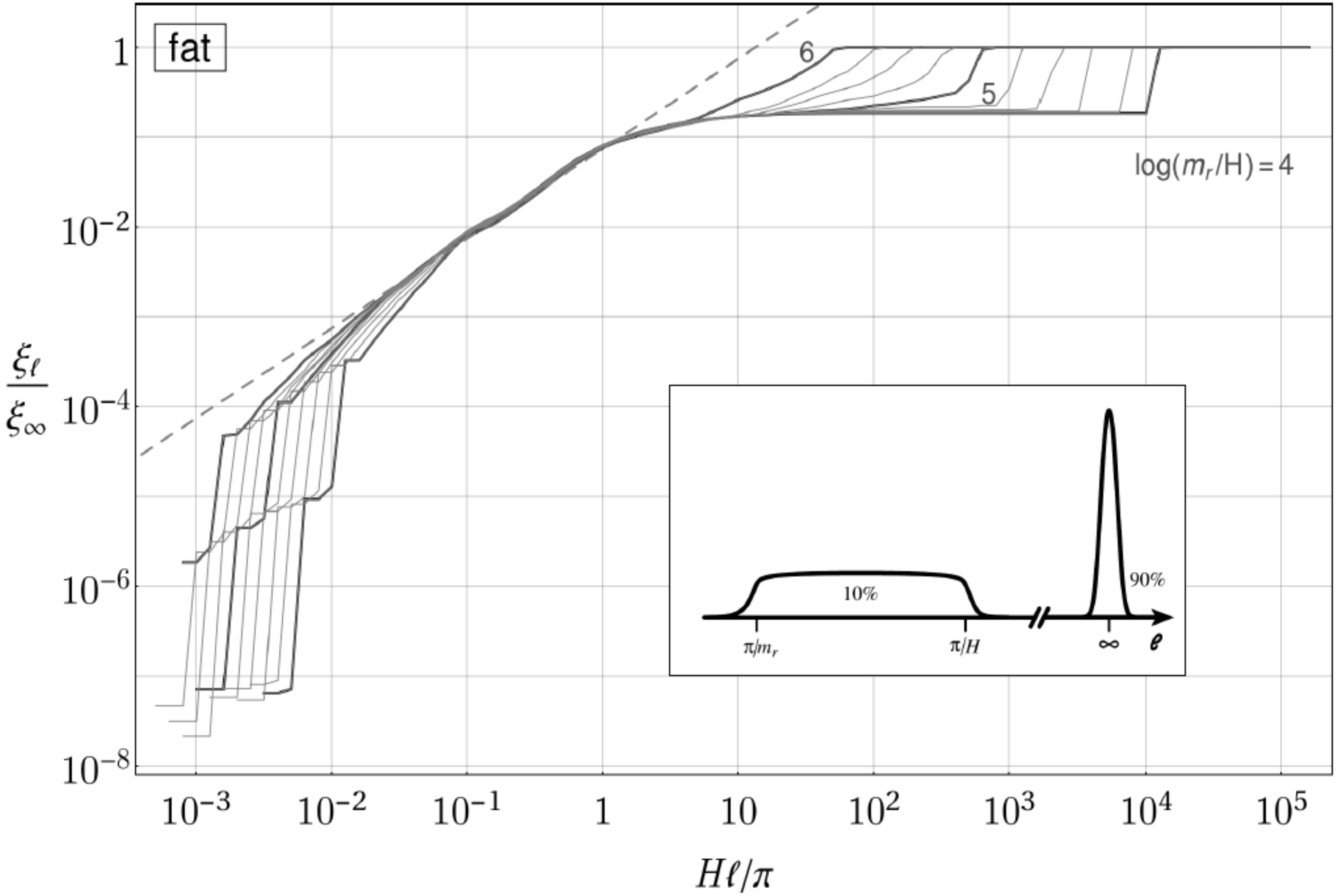
$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$



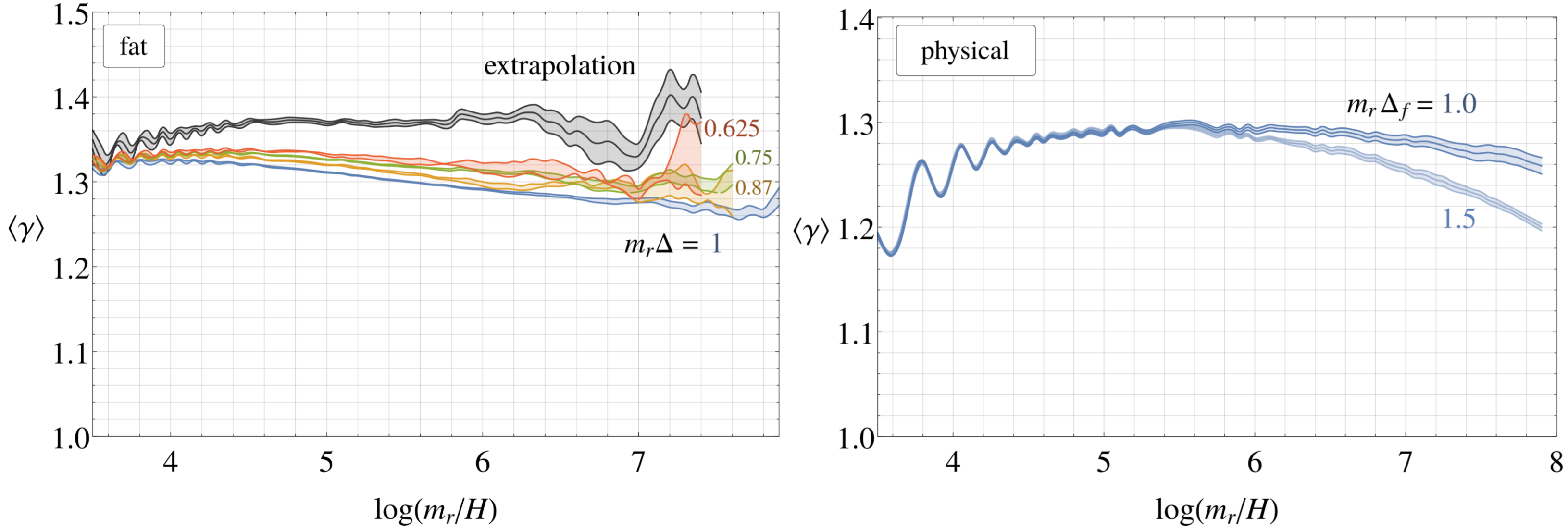
$$\Omega_a \approx 0.1\theta_0^2 \left[\frac{f_a}{10^{12}\text{GeV}}\right]^{1+\epsilon} = \Omega_{DM}$$



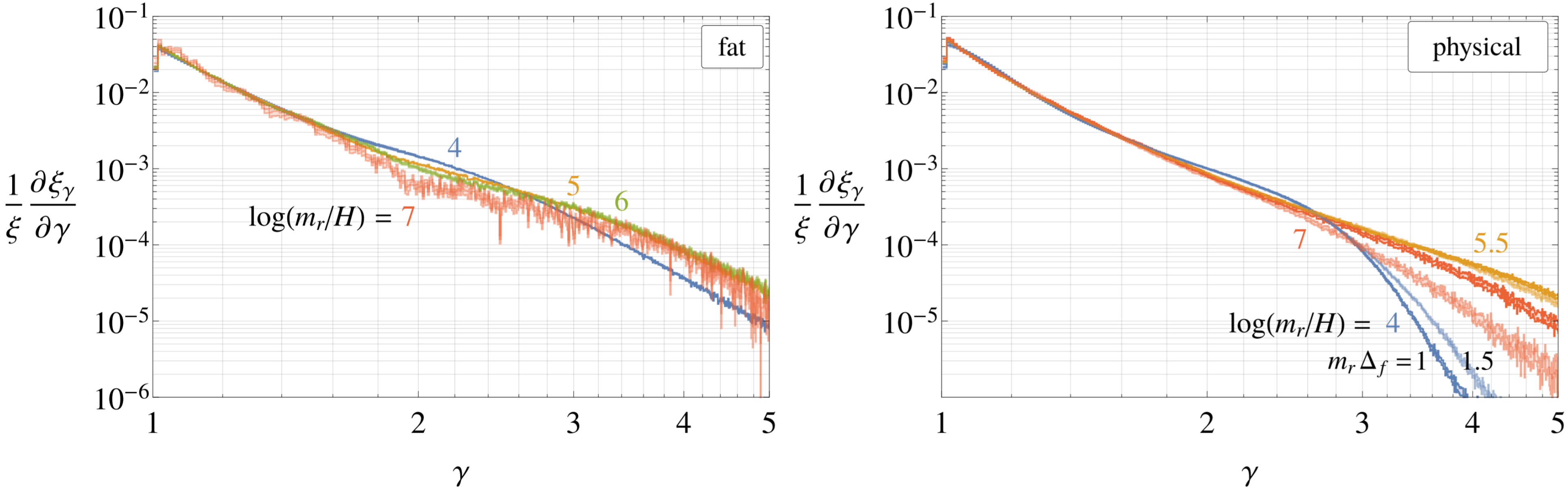
Loop Distribution



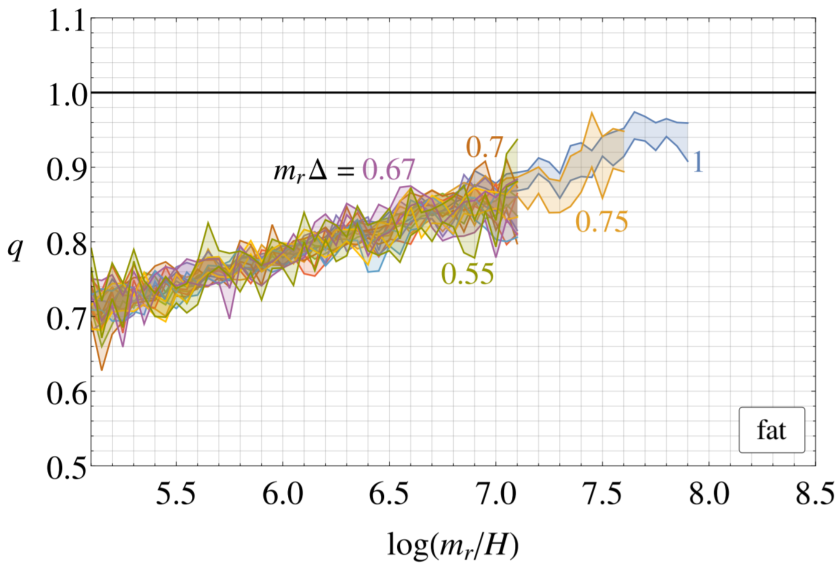
Boost Factors

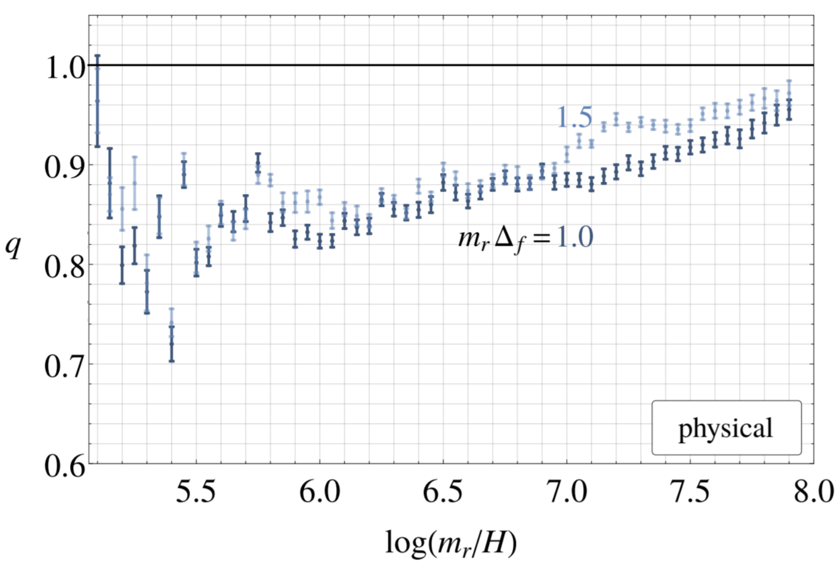


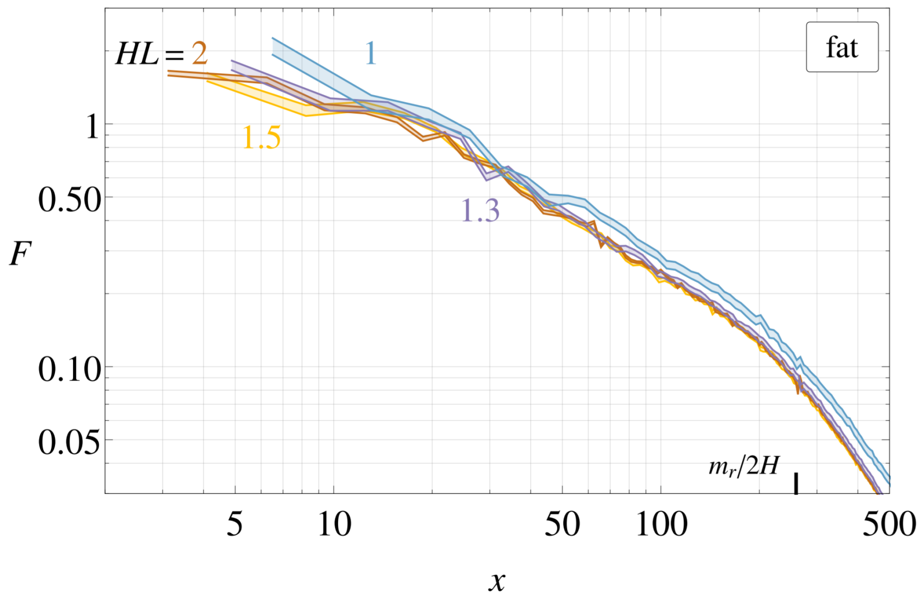
Increase in Higher Boosts:



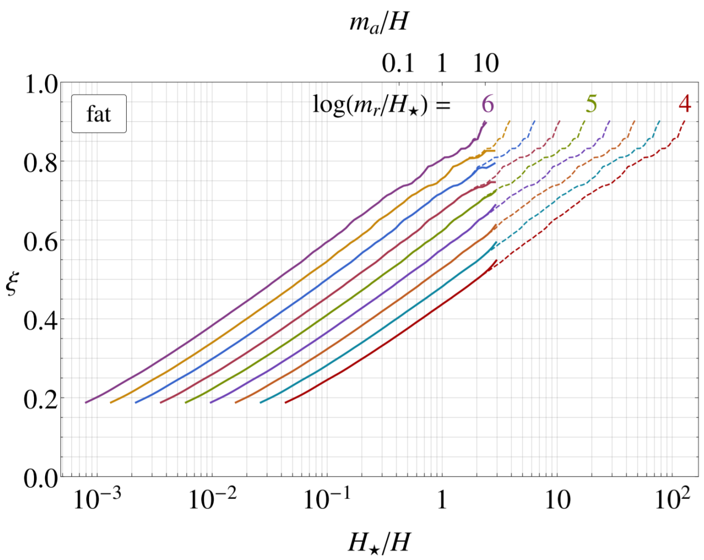
Lattice Spacing and Finite Volume Effects on q

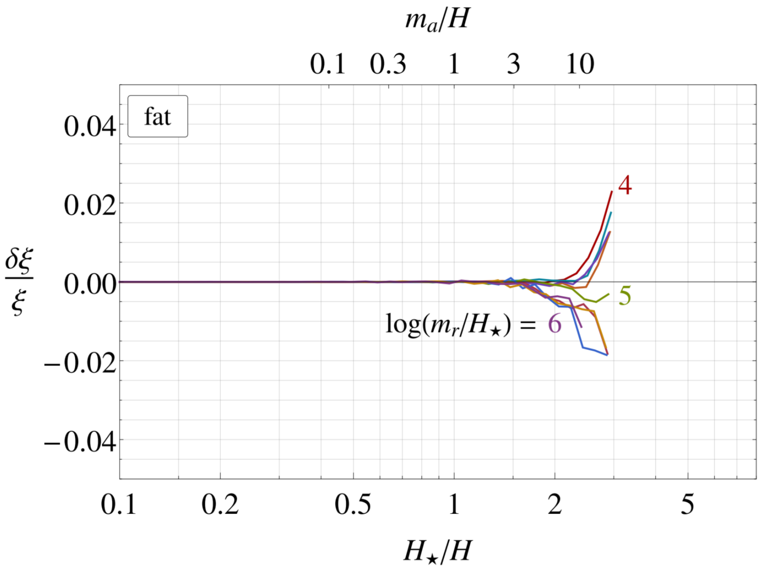


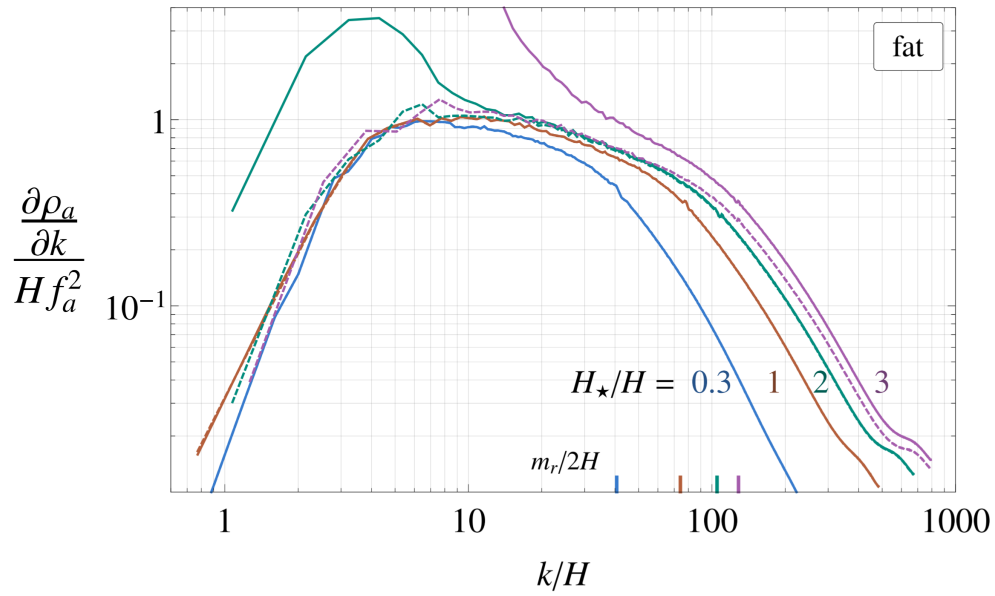




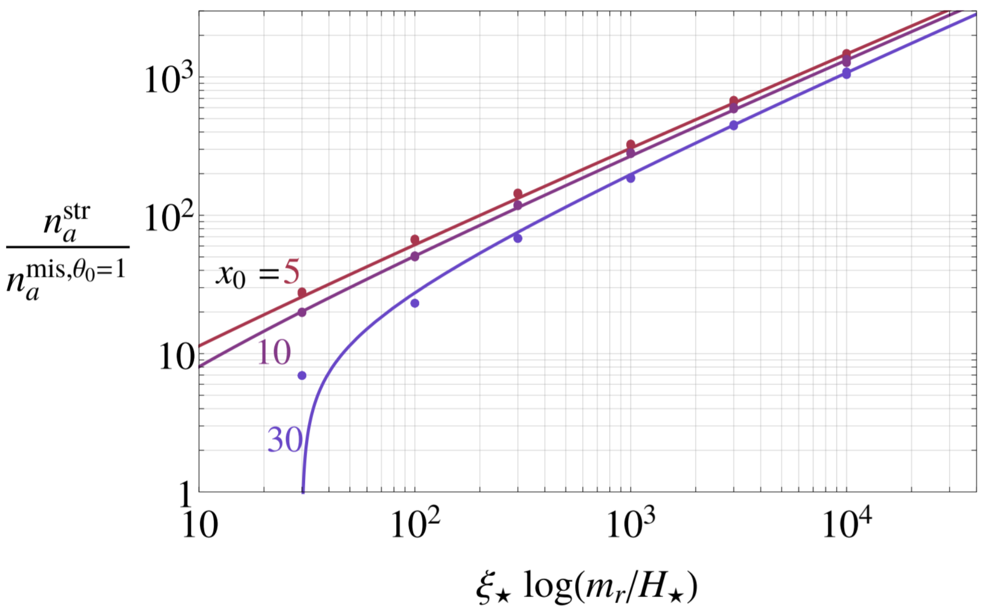
End of the Scaling regime: $H = m_a \equiv H_*$

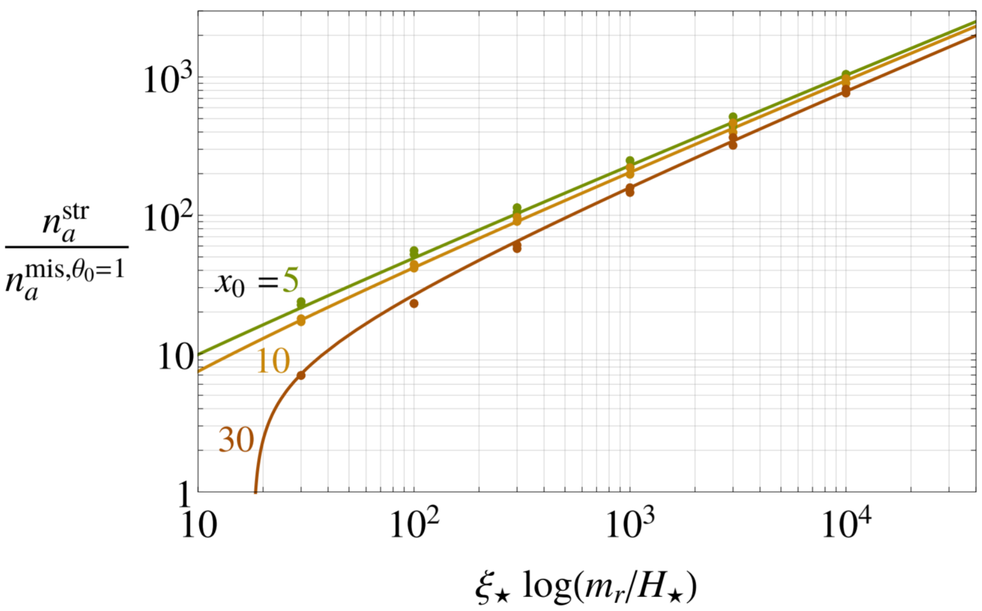






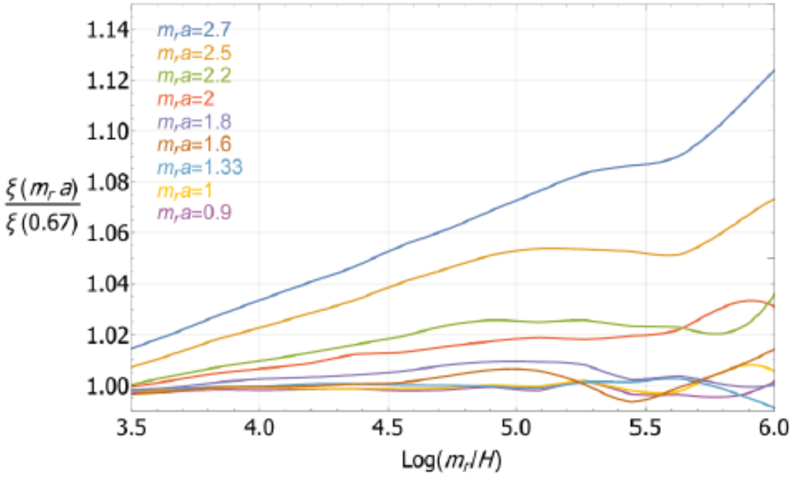
Axion Number Density after the transient

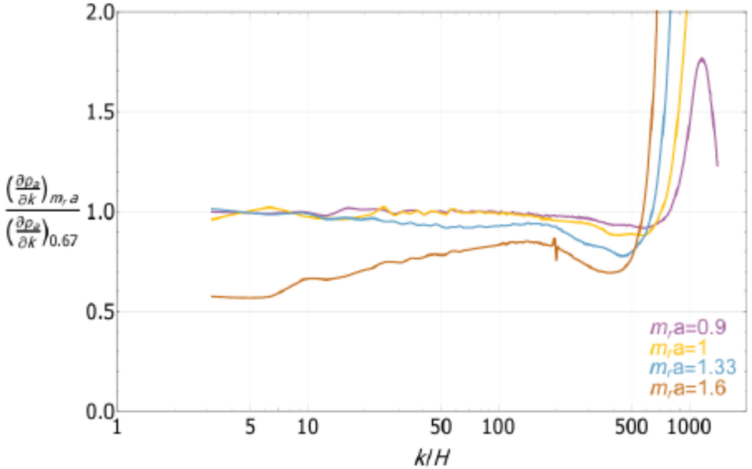


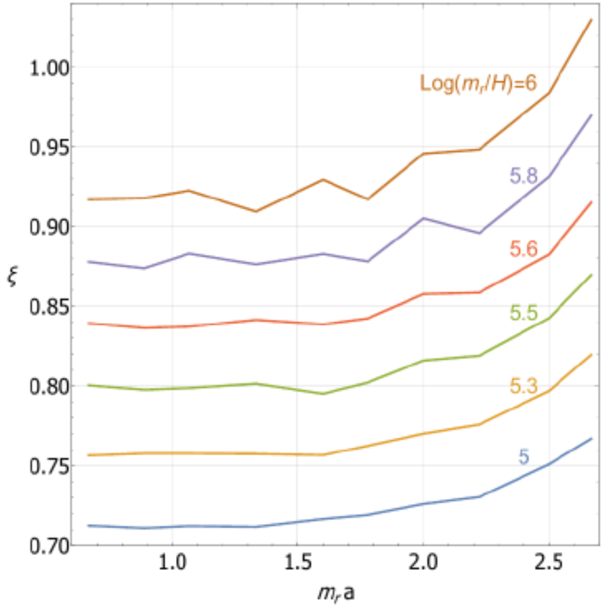


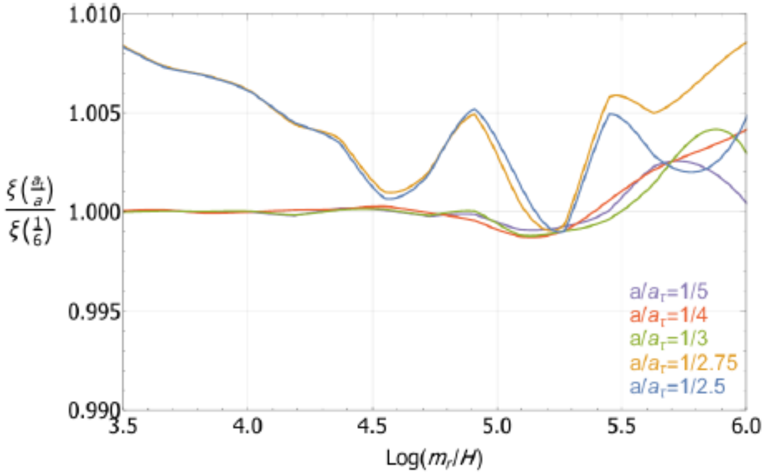
$$\begin{aligned} Q(t_\ell) &\equiv \frac{n_a^{\rm str}(t_\ell)}{n_a^{{\rm mis},\theta_0=1}(t_\ell)} \\ &= \frac{c_n}{c_n'} c_V \left[\frac{W_{-1}\left(-\frac{c_V\left(1+\frac{2}{\alpha+2}\right)}{4\pi\xi_\star \log_\star} \left(\frac{x_0}{c_m}\right)^{2\left(1+\frac{2}{\alpha+2}\right)} \right)}{-\frac{c_V\left(1+\frac{2}{\alpha+2}\right)}{4\pi\xi_\star \log_\star}} \right]^{\frac{1}{2}\left(1+\frac{2}{\alpha+4}\right)} \\ &= \frac{c_n}{c_n'} c_V \left[\frac{4\pi\xi_\star \log_\star}{c_V} \left[1 - \frac{2}{\alpha+4}\right] \log\left(\frac{4\pi\xi_\star \log_\star}{c_V} \left[1 - \frac{2}{\alpha+4}\right] \left[\frac{c_m}{x_0}\right]^{2(1+\frac{2}{\alpha+2})} \log(\ldots) \right) \right]^{\frac{1}{2}(1+\frac{2}{\alpha+4})} . \end{aligned} \tag{36}$$

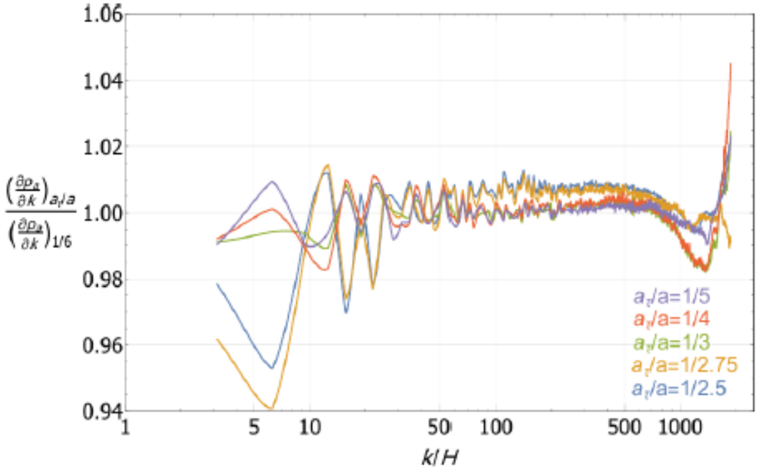
Lattice Spacing



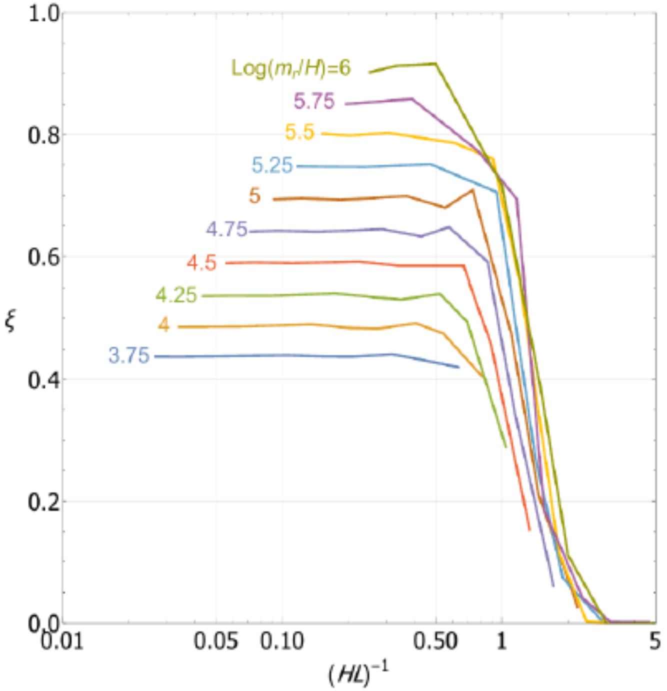


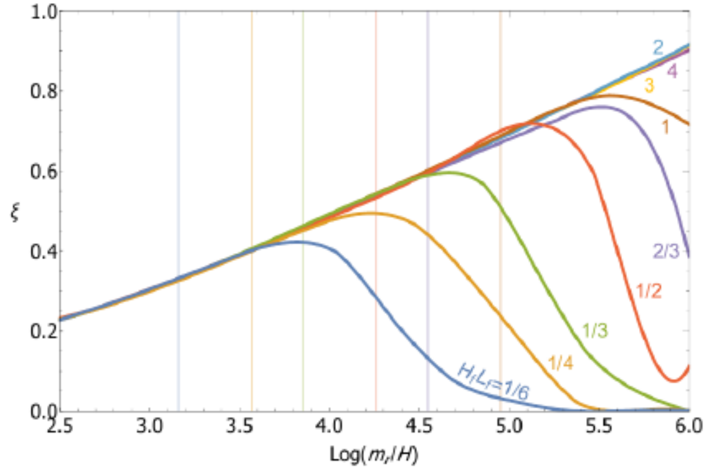


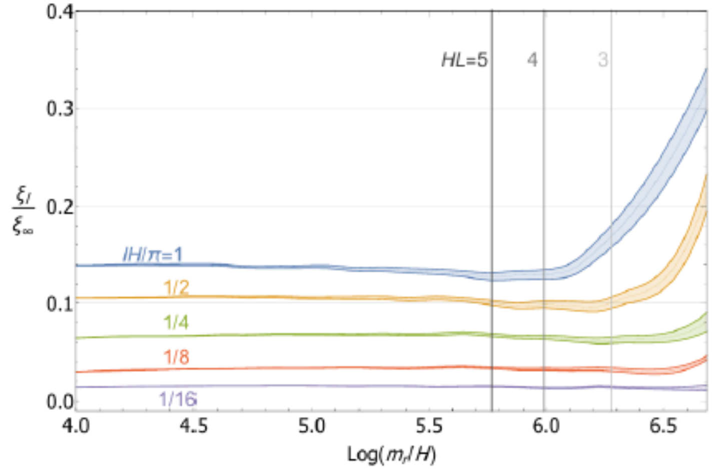


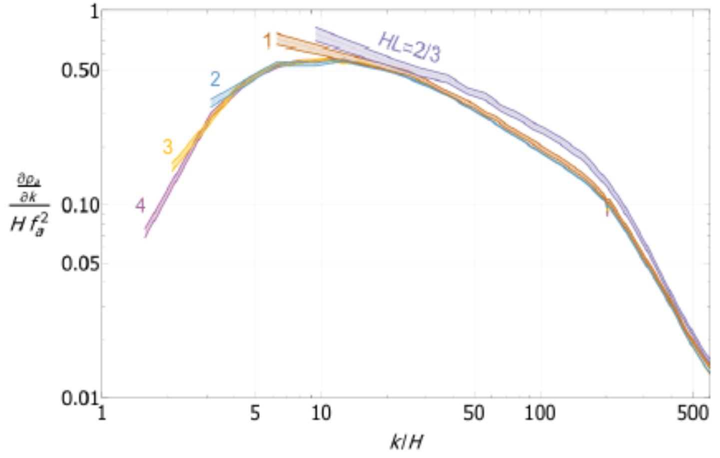


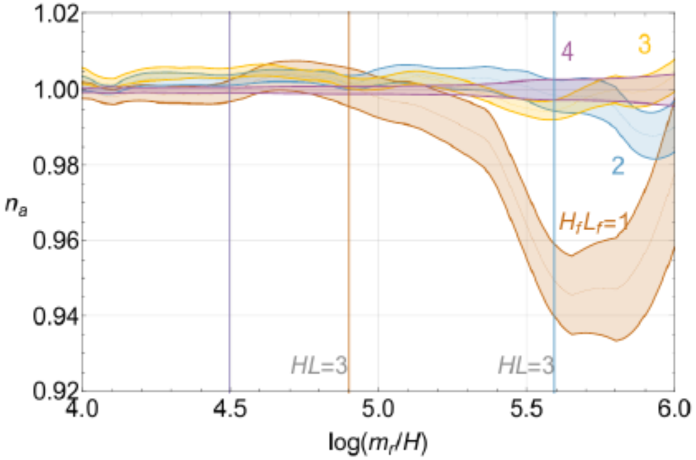
Finite Volume





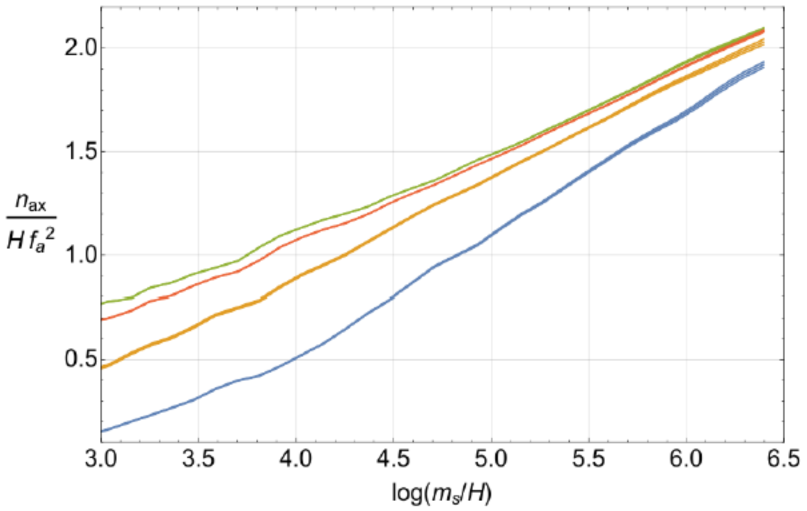


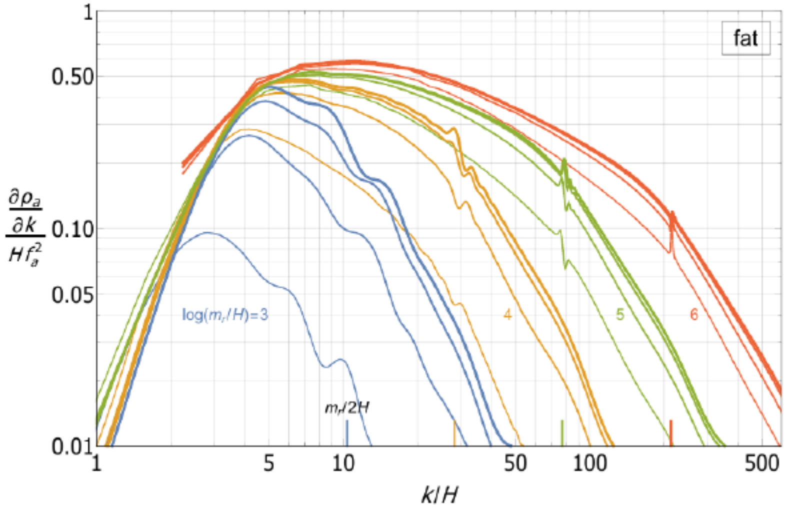


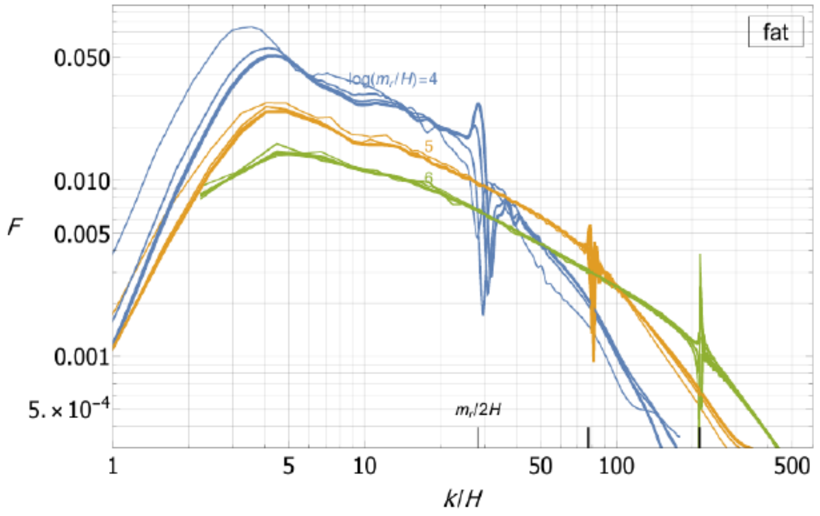


Dependence on the Initial Conditions

Scaling Solution *vs* Initial Conditions







Domain wall tension

$$\sigma = \beta m_a f_a^2$$

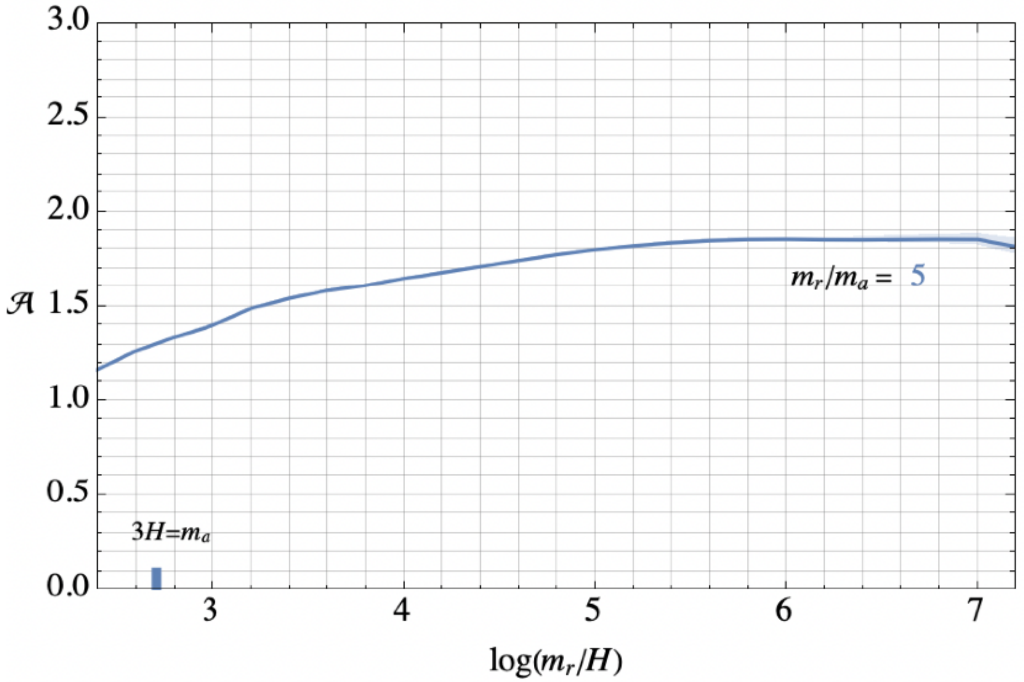
Number of domain walls per Hubble patch

$$\mathcal{A} = \lim_{V\to\infty} At/V$$

Energy density

$$\rho_{\rm w} = \sigma\mathcal{A}/t = 2\sigma\mathcal{A}H$$

$$\Gamma_a^{\rm w} \simeq \rho_{\rm w}/(2t)$$



$$n_a^{\rm w}(t) \simeq \int_{\log t_\star}^{\log t_d} \mathcal{A}\frac{\beta f_a^2}{2t'}\left(\frac{R(t')}{R(t)}\right)^3 \, d\log t'$$
$$\simeq \int_{\log t_\star}^{\log t_d} \mathcal{A}\beta f_a^2 \frac{t'^{1/2}}{2t^{3/2}} \, d\log t'$$

$$\frac{\Omega_{\rm a}}{\Omega_{\rm DM}} \simeq \frac{\beta\mathcal{A}_d m_a f_a^2}{T_{\rm eq}H_d^{1/2}M_{\rm Pl}^{3/2}}$$
$$\simeq 2\left(\frac{\mathcal{A}_d}{20}\right)\left(\frac{m_a}{H_d}\right)^{1/2}\left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^2\left(\frac{m_a}{10^{-6}\ {\rm eV}}\right)^{1/2}$$