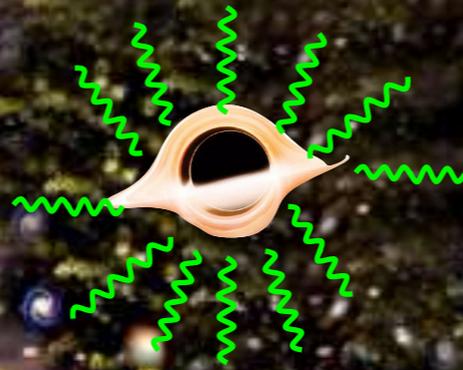


Gravitational Waves Signatures in presence of Primordial Black Holes



ϕ



Yann Mambrini, IJCLab, Université Paris-Saclay

Planck Conference, Padova

May 27th 2025



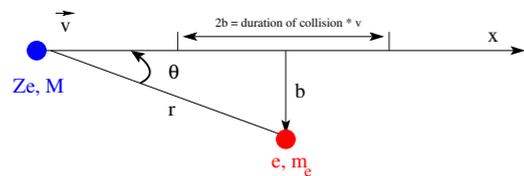


Fig. 5.9 Interaction of a high energy particle of charge Ze with an electron at rest.

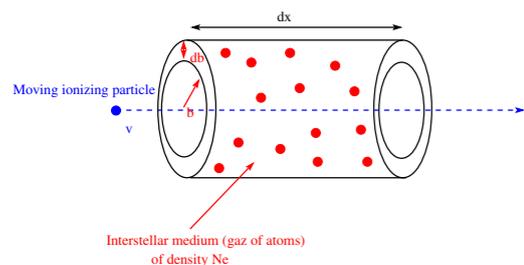


Fig. 5.10 Moving particle in an interstellar medium of density N_e .

distance at which the influence of the traveling particle on the electron is negligible. It corresponds roughly to the time when the orbital period is lower than the typical interaction time. In other words, if the electron takes more time to move around the nucleus than to interact with the moving particle, the electromagnetic influence of the later becomes weak. If one write τ the interacting time and v_0 the frequency of the rotating electron in the atom ($v_0 = \omega_0/2\pi$), it corresponds to

$$\tau \approx \frac{2b}{v} < \frac{1}{v_0} \Rightarrow b < \frac{v}{2v_0} = b_{max} \quad (5.37)$$

The lower limit b_{min} can be obtained if we suppose, by a quantum treatment and the application of the uncertainty principle, that the maximum energy transfer is $\Delta p_{max} = 2m_e v$ (because as we discussed earlier, the maximum velocity transferred to the electron is $2v$) from $\Delta p \Delta x \geq \hbar$ (Heisenberg principle) we have $\Delta x \geq \hbar/2m_e v$. We can then write

The two parts of the Lagrangian one needs to compute the scalar annihilation of Dark Matter $SS \rightarrow h \rightarrow f\bar{f}$ are (see B.235)

$$\begin{aligned} \mathcal{L}_{HSS} &= -\lambda_{HS} \frac{M_W}{2g} hSS \rightarrow C_{HSS} = -i \frac{\lambda_{HS} M_W}{g} \\ \text{and } \mathcal{L}_{Hff} &= -\frac{gm_f}{2M_W} h\bar{f}f \rightarrow C_{Hff} = -i \frac{gm_f}{2M_W} \end{aligned} \quad (B.145)$$

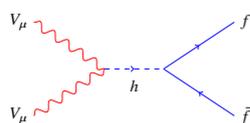
which gives

$$|\mathcal{M}|^2 = \frac{\lambda_{HS}^2 m_f^2 (s/2 - 2m_f^2)}{(s - M_H^2)^2 + \Gamma_H^2 M_H^2} \quad (B.146)$$

Γ_H being the width of the Higgs boson (including its own decay into SS , see next section). When one implements this value of $|\mathcal{M}|^2$ into Eq.(B.111) one obtains after simplification

$$\langle \sigma v \rangle_{f\bar{f}}^S = \frac{|\mathcal{M}|^2}{8\pi s} \sqrt{1 - \frac{m_f^2}{M_S^2}} = \frac{\lambda_{HS}^2 (M_S^2 - m_f^2) m_f^2}{16\pi M_S^2 (4M_S^2 - M_H^2)^2} \sqrt{1 - \frac{m_f^2}{M_S^2}} \quad (B.147)$$

B.4.4.11 Annihilation in the case of vectorial Dark Matter to pairs of fermions



One can compute this annihilation cross section by the normal procedure or noticing that a neutral vectorial dark matter of spin 1 corresponds to 3 degrees of freedom. After averaging on the spin one can then write $\langle \sigma v \rangle^V = \frac{3}{3 \times 3} \langle \sigma v \rangle^S = \frac{1}{3} \langle \sigma v \rangle^S$. The academical computation for $V_\mu(p_1) V_\mu(p_2) \rightarrow f\bar{f}$ gives

Yann Mambrini

Particles in the Dark Universe

A Student's Guide to Particle Physics and Cosmology

Second Edition



650+ pages, from inflation to dark matter detection.

2nd edition (+ PBH + unification + history of cosmological models...)

All what is needed to compute cross-sections, relic abundance, and retrace the history of a Dark Universe.

Preface and forewords by K. Olive, J. Peebles and J. Silk

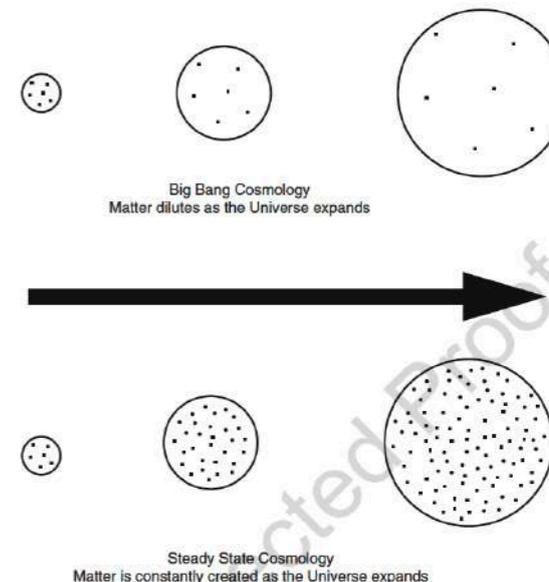


Fig. 2.9 Difference between the Steady State and the Big Bang cosmology

with

1189

$$H_0^2 = \frac{\rho_m}{3M_p^2} \quad (2.123)$$

and z the redshift being defined by Eq. (2.23). Evolution is therefore similar in every way to a de Sitter type Universe, but with a constant density of matter. It is therefore not possible to distinguish these two models by the flow of a source L_0 at $r = R_0 \chi$, which will be redshifted in the same way in both cases

1190

1191

1192

1193

$$L = \frac{L_0}{4\pi r^2 (1+z)^2} \quad (2.124)$$

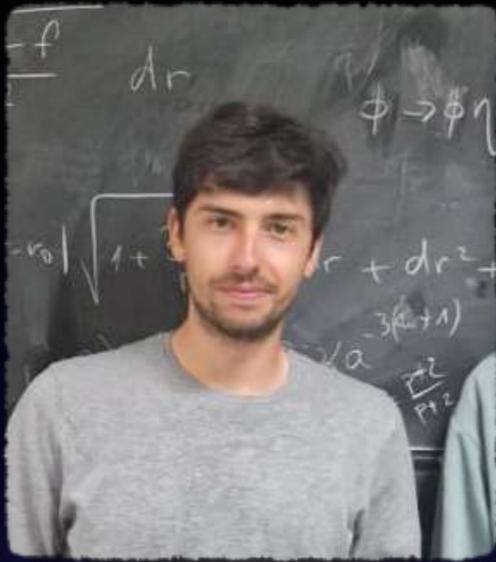
On the other hand, the number of sources is completely different, since in the case of the steady state, the density remaining constant, the number of sources decreases in the past (for a smaller volume, fewer sources) whereas for Big Bang type models,

1194

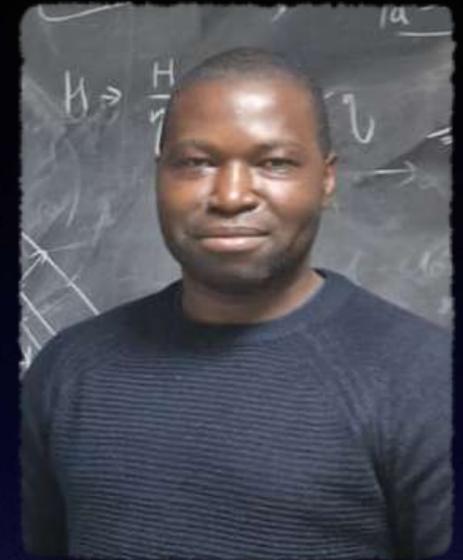
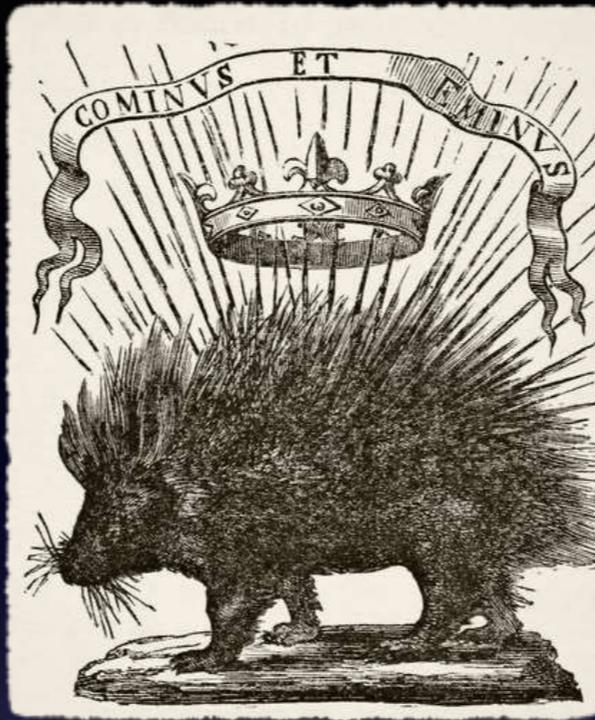
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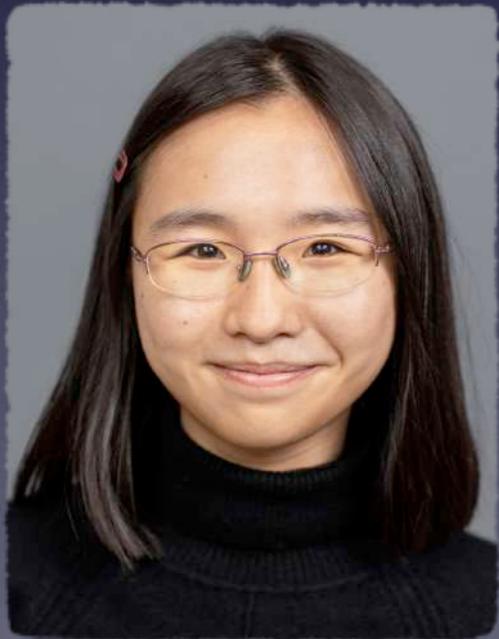
Students/postdocs :



Simon Clery
(TUM, Munchen)



Essodjolo Kpatcha
(IJCLab, Paris-Saclay)



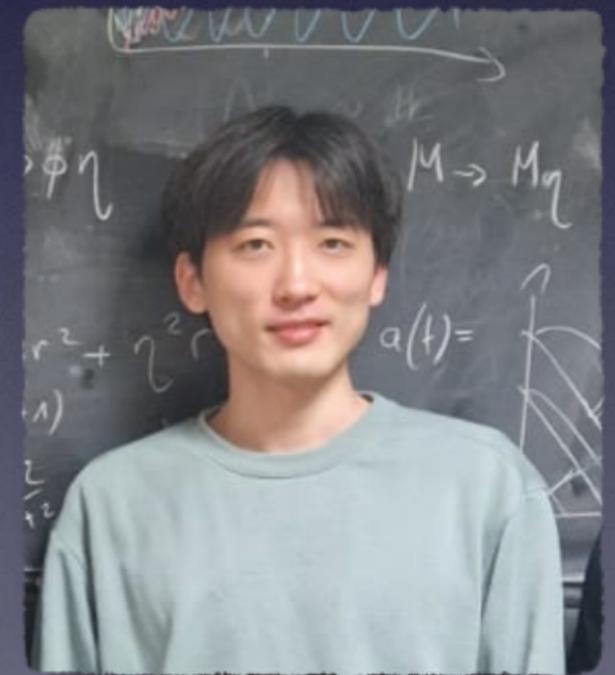
Wenqi Ke
(FI, Minneapolis)



Mathieu Gross
(Paris-Saclay)



Mathias Pierre
(DESY)



Jong-Hyun Yoon
(CNU, Daejeon)

Based (mainly) on

M. R. Haque, E. Kpatcha, D. Maity and Y.M., ``Primordial black hole versus inflaton,"
Phys. Rev. D **109** (2024) no.2, 023521 ; [arXiv:2309.06505 [hep-ph]].

M. Riajul Haque, E. Kpatcha, D. Maity and Y.M., ``Primordial black hole reheating,"
Phys. Rev. D **108** (2023) no.6, 063523 ; [arXiv:2305.10518 [hep-ph]].

B. Barman, S. Jyoti Das, M. R. Haque and Y.M., ``Leptogenesis, primordial gravitational waves, and PBH-induced reheating,"
[arXiv:2403.05626 [hep-ph]].

M. Gross, E. Kpatcha, Y.M., M. Olea-Romacho and R. Roshan ``Gravitational Wave Production During Reheating: From the Inflaton to Primordial Black Holes" ; [arXiv:2411.04189 [hep-ph]].

M. R. Haque, S. Maity, D. Maity and Y.M. ``Quantum effects on the evaporation of PBHs: contributions to dark matter,"
JCAP **07** (2024), 002 ; [arXiv:2404.16815 [hep-ph]].

And

G. Choi, M. A. G. Garcia, W. Ke, Y. Mambrini, K. A. Olive and S. Verner,
``Inflaton Production of Scalar Dark Matter through Fluctuations and Scattering," [arXiv:2406.06696 [hep-ph]].

M. A. G. Garcia, M. Gross, Y. Mambrini, K. A. Olive, M. Pierre and J. H. Yoon,
``Effects of fragmentation on post-inflationary reheating," JCAP **12** (2023), 028

Y. Mambrini and K. A. Olive, Phys. Rev. D **103** (2021) no.11, 115009 [arXiv:2102.06214]

S. Clery, Y. Mambrini, K. A. Olive and S. Verner, Phys. Rev. D **105** (2022) no.7, 075005 ; [arXiv:2112.15214]

R. T. Co, Y. Mambrini and K. A. Olive, ``Inflationary Gravitational Leptogenesis," ; [arXiv:2205.01689 [hep-ph]].

Gravitational wave spectrum
in the presence of inflaton
and primordial black holes

$$1 \text{ g} \lesssim M_{\text{BH}} \lesssim 10^8 \text{ g}$$

$$10^{-23} \text{ s} \lesssim t_{\text{BH}} \lesssim 1 \text{ s}$$

BH energy density can then dominate over inflaton energy density and lead to reheating, DM production, lepto-baryogenesis...

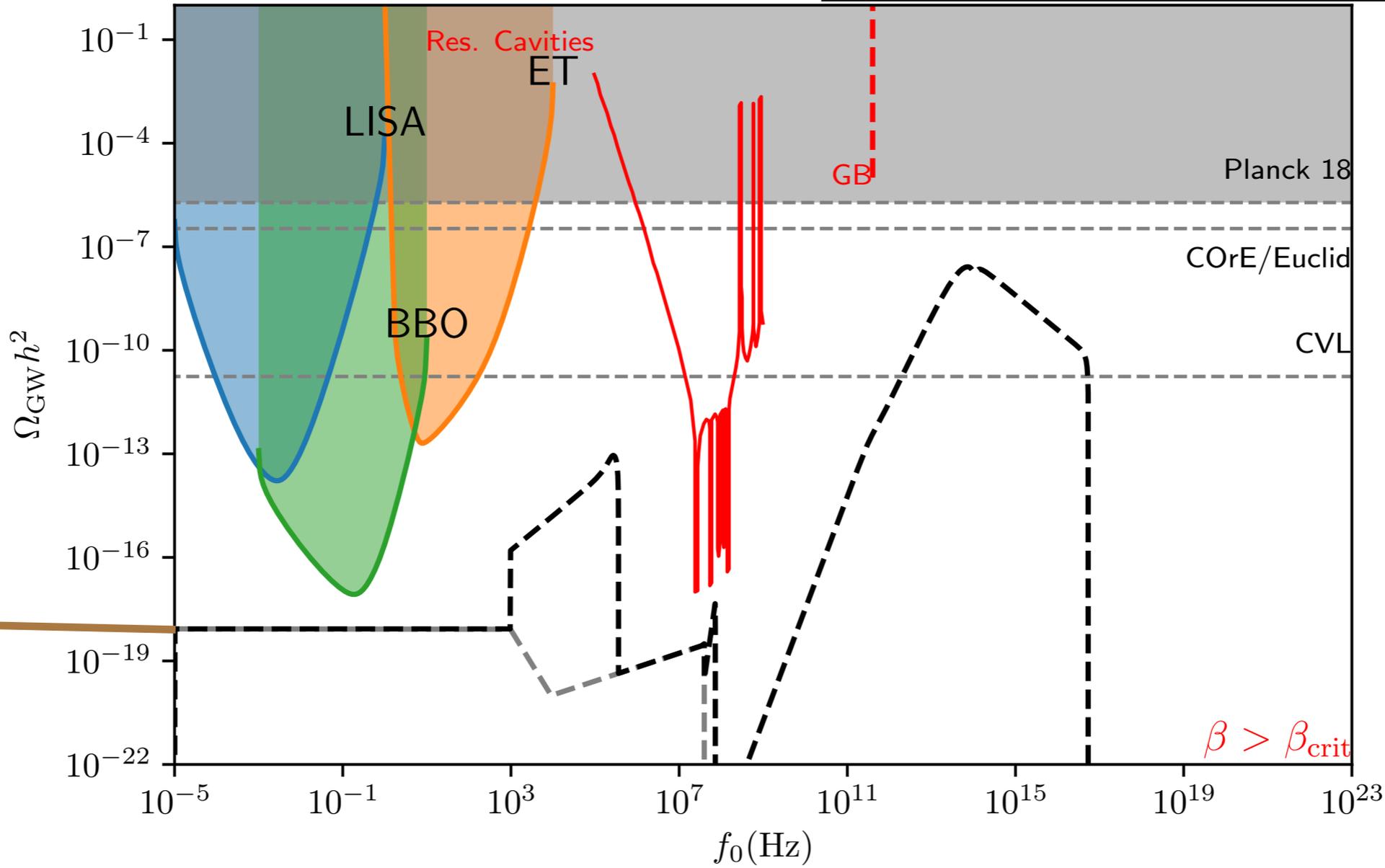
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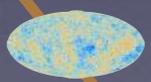
BH energy density can then dominate over inflaton energy density and lead to reheating, DM production, lepto-baryogenesis...

But the talk can be applied to any early matter domination era

Benchmark point :
 $M_{BH} = 1 \text{ g}, \beta = 10^{-5}, w_\phi = 0.5 (k = 6)$



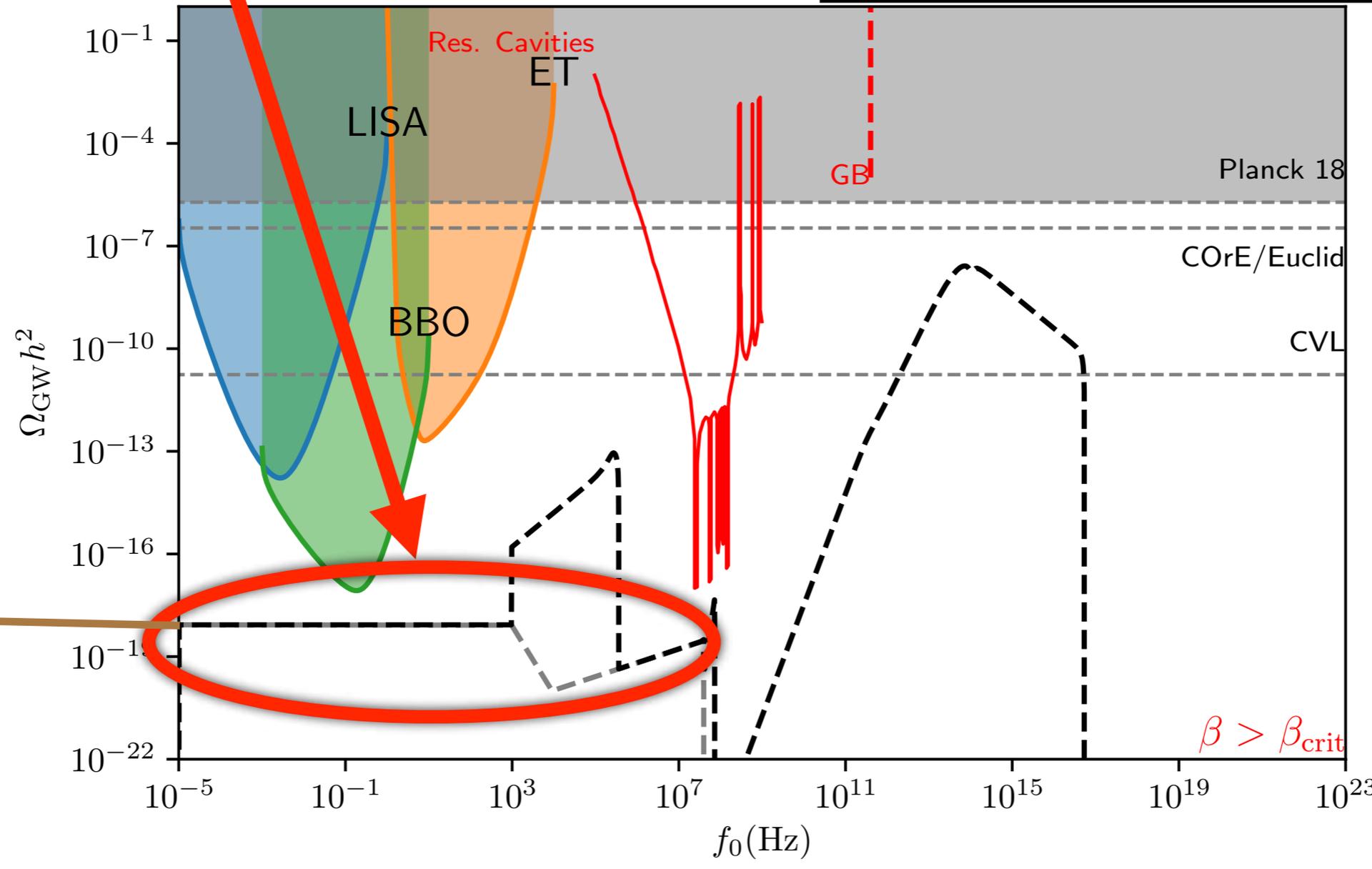
10^{-18} Hz
 10^{-14} Hz



Primordial GW

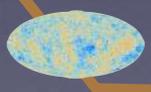


Benchmark point :
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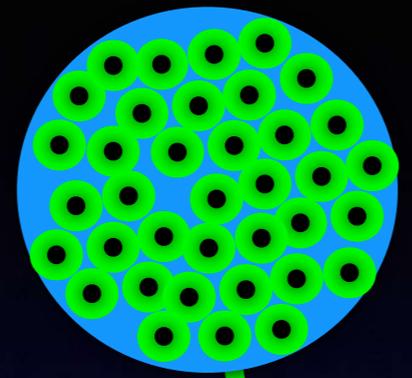
10^{-18} Hz

10^{-14} Hz

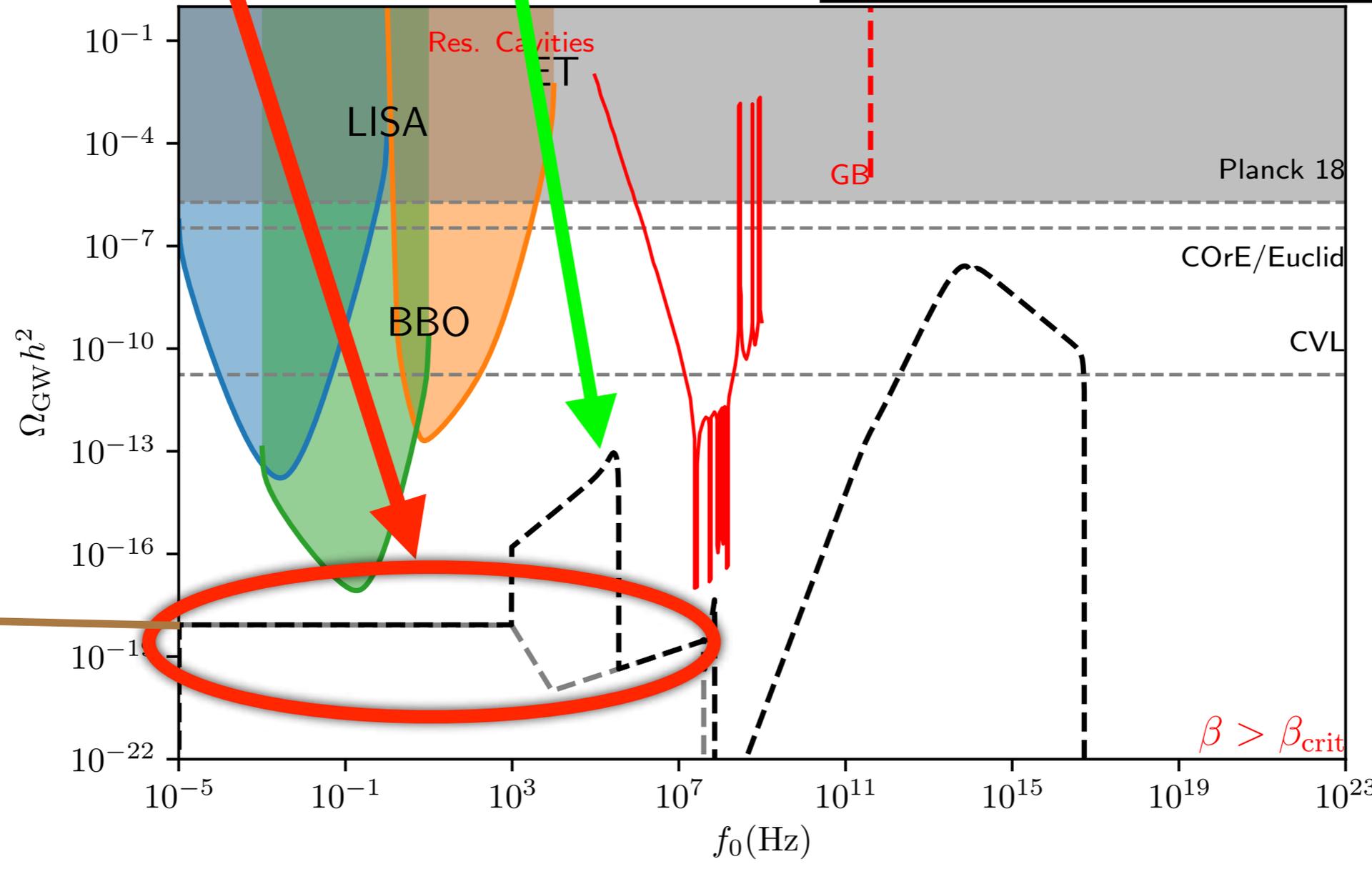


Primordial GW

PBHs density fluctuations



Benchmark point :
 $M_{BH} = 1 \text{ g}, \beta = 10^{-5}, w_\phi = 0.5 (k = 6)$



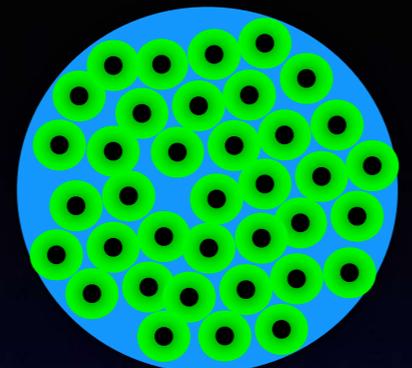
10^{-18} Hz

10^{-14} Hz

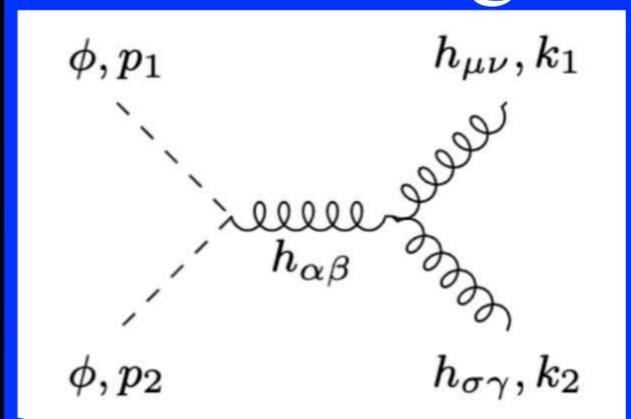
Primordial GW



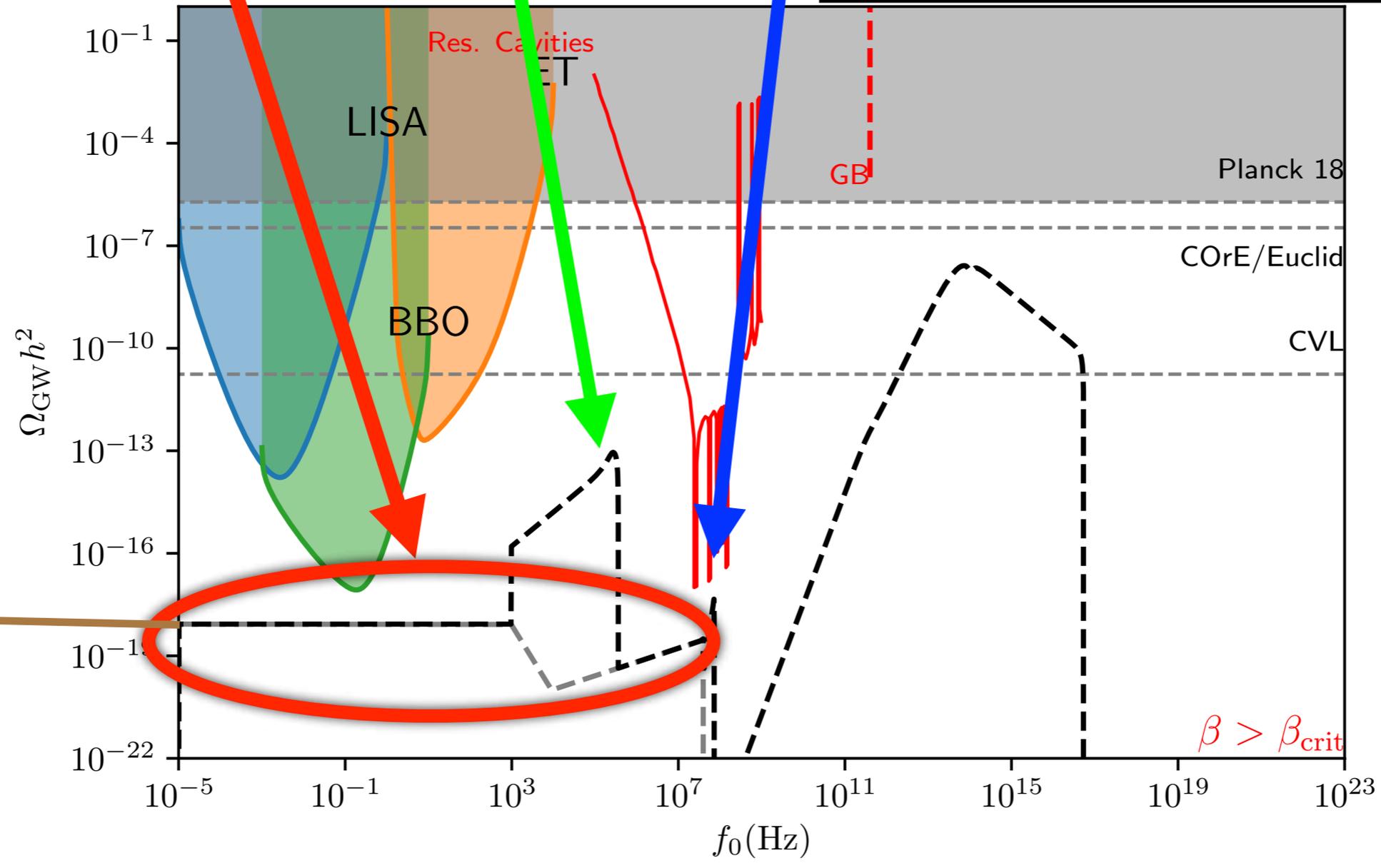
PBHs density fluctuations



Inflaton scattering



Benchmark point :
 $M_{BH} = 1 \text{ g}, \beta = 10^{-5}, w_\phi = 0.5 (k = 6)$



10^{-18} Hz

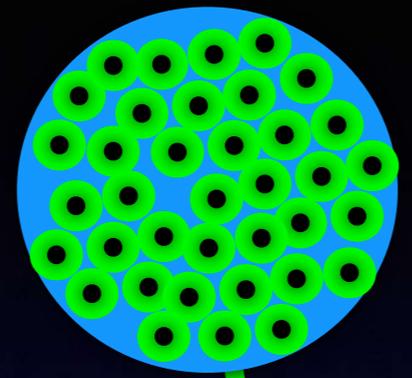
10^{-14} Hz

$\beta > \beta_{crit}$

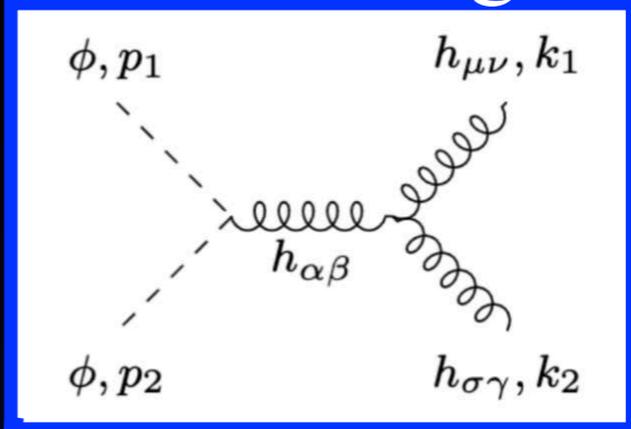
Primordial GW



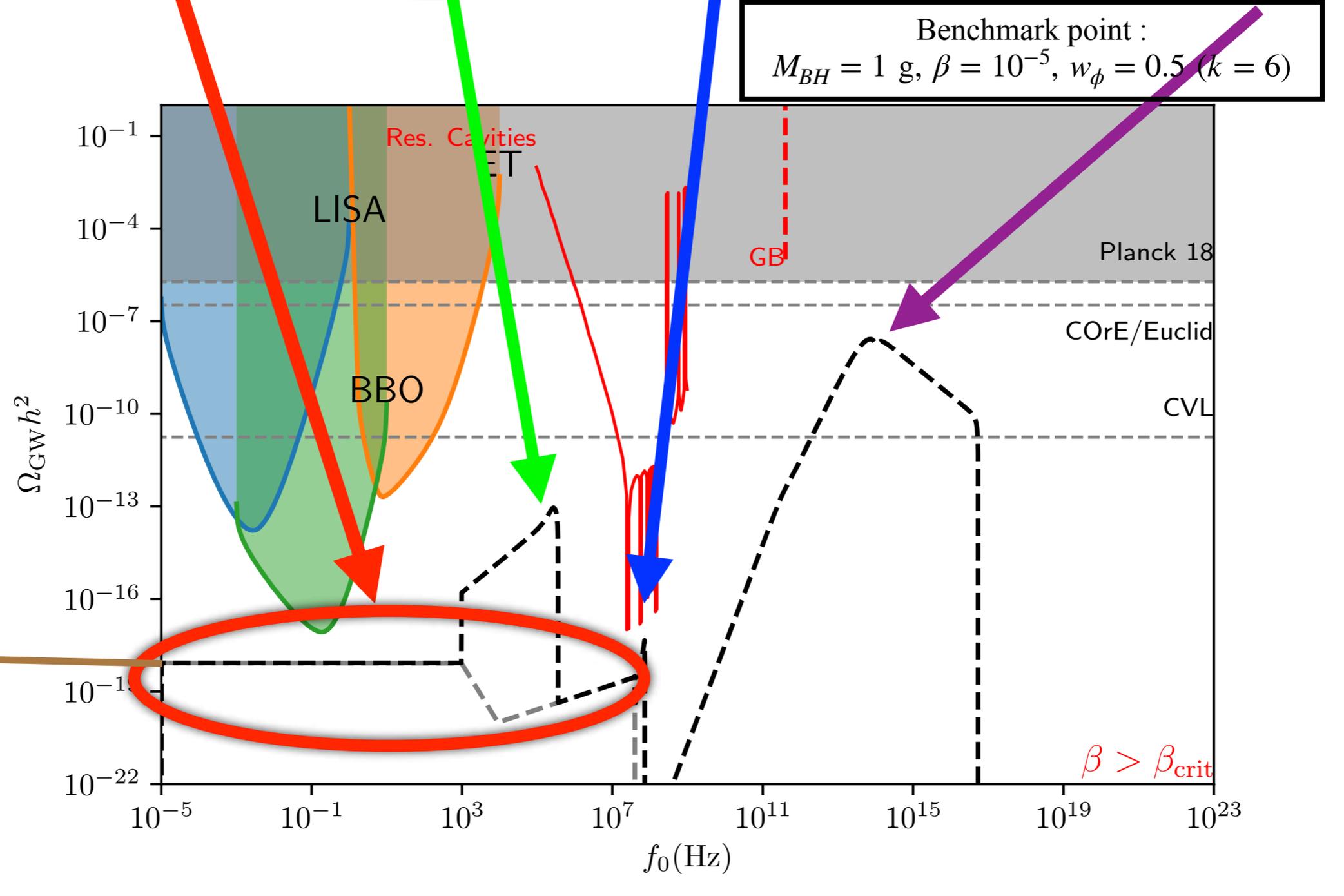
PBHs density fluctuations



Inflaton scattering



PBHs decay



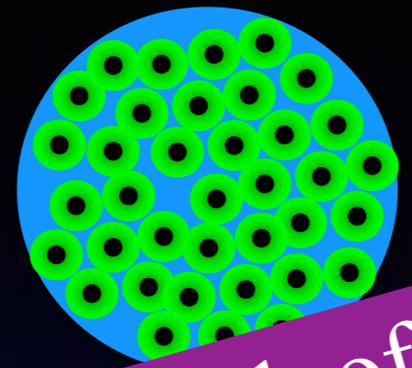
10^{-18} Hz

10^{-14} Hz

Primordial GW



PBHs density fluctuations

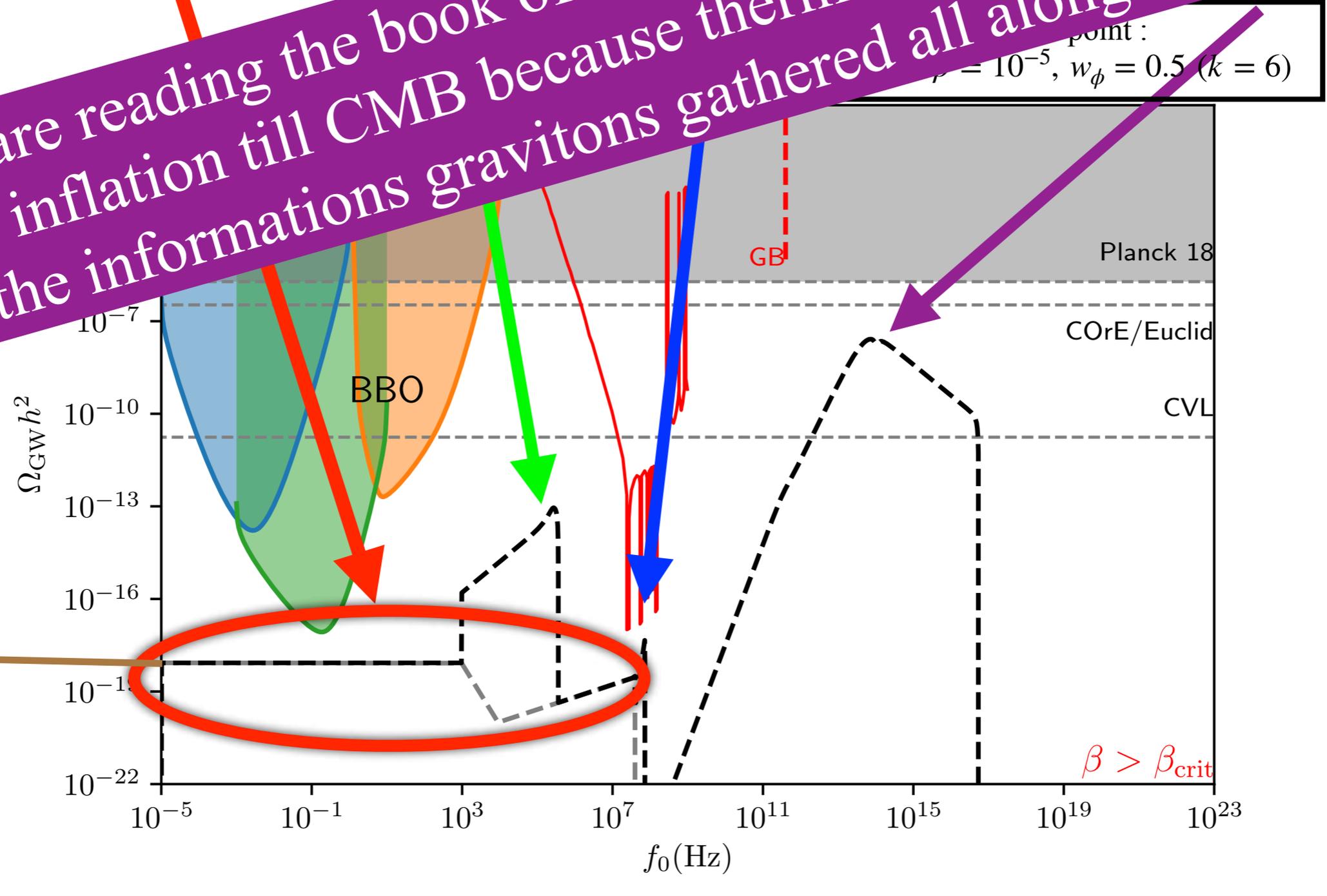


Inflaton scattering



PBHs decay

We are reading the book of the Universe (from right to left), from inflation till CMB because thermalization did not destroy the informations gravitons gathered all along their way

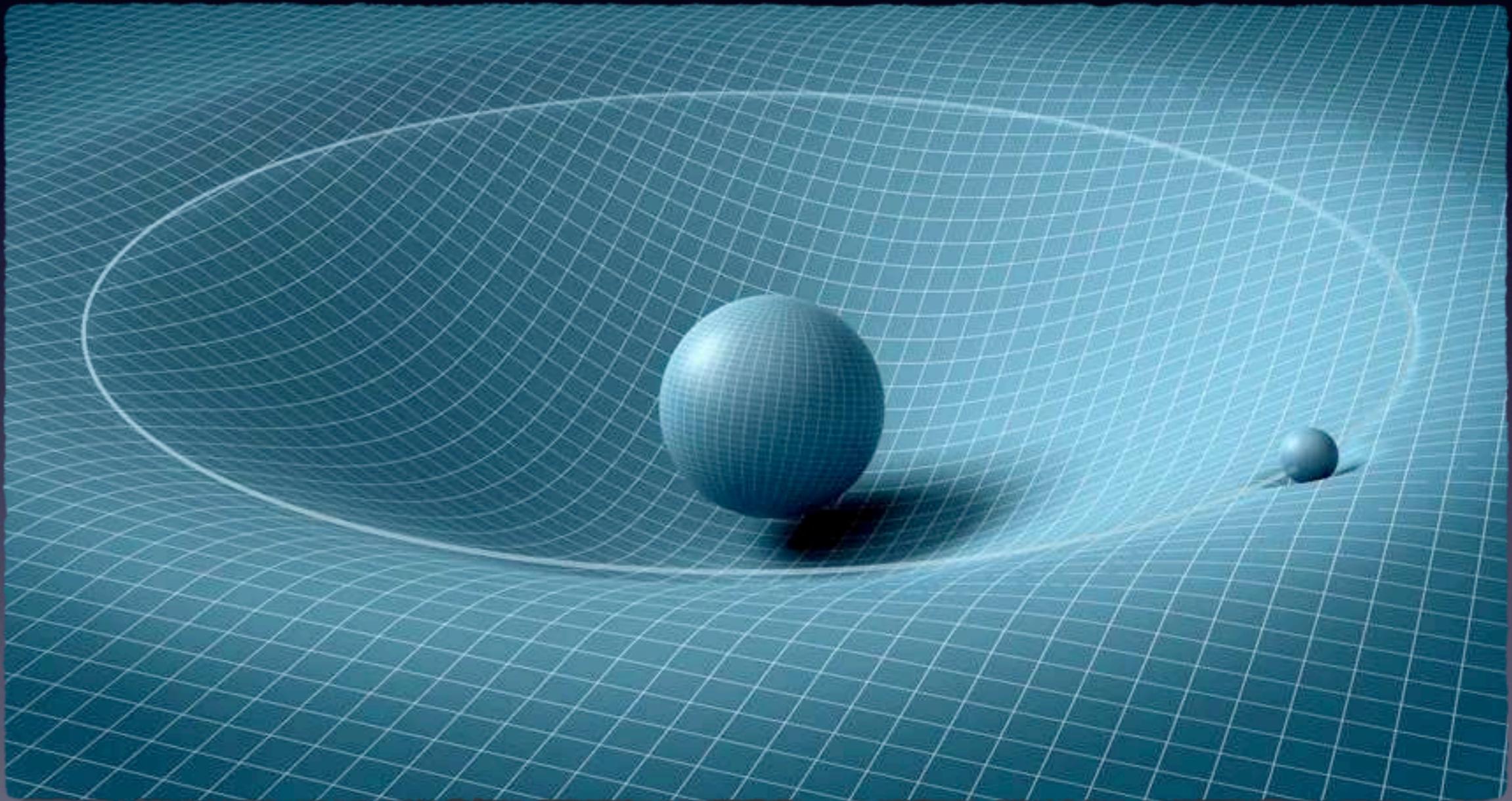


10^{-18} Hz

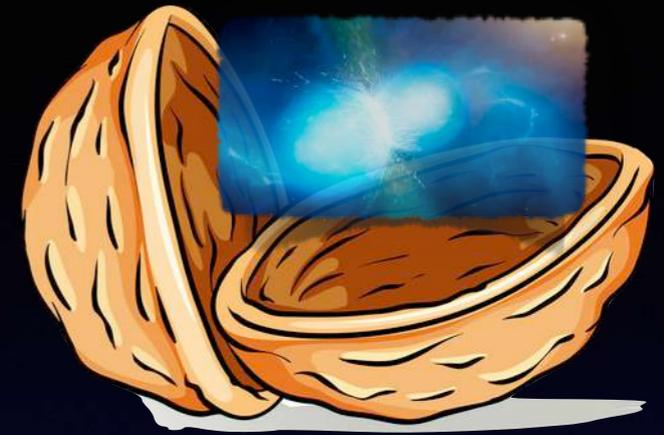
10^{-14} Hz

I) Much ado about gravity

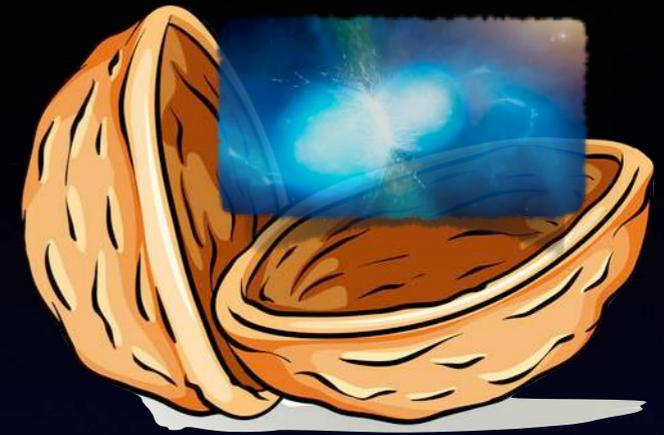
When horizon(s) source the matter



Gravitational production in a nutshell



Gravitational production in a nutshell



$$G_{\mu\nu} = \frac{1}{M_{\text{P}}^2} T_{\mu\nu} = \frac{1}{M_{\text{P}}^2} \times \frac{1}{2} \langle \rho_{\chi} \rangle g_{\mu\nu}$$

Gravitational production in a nutshell

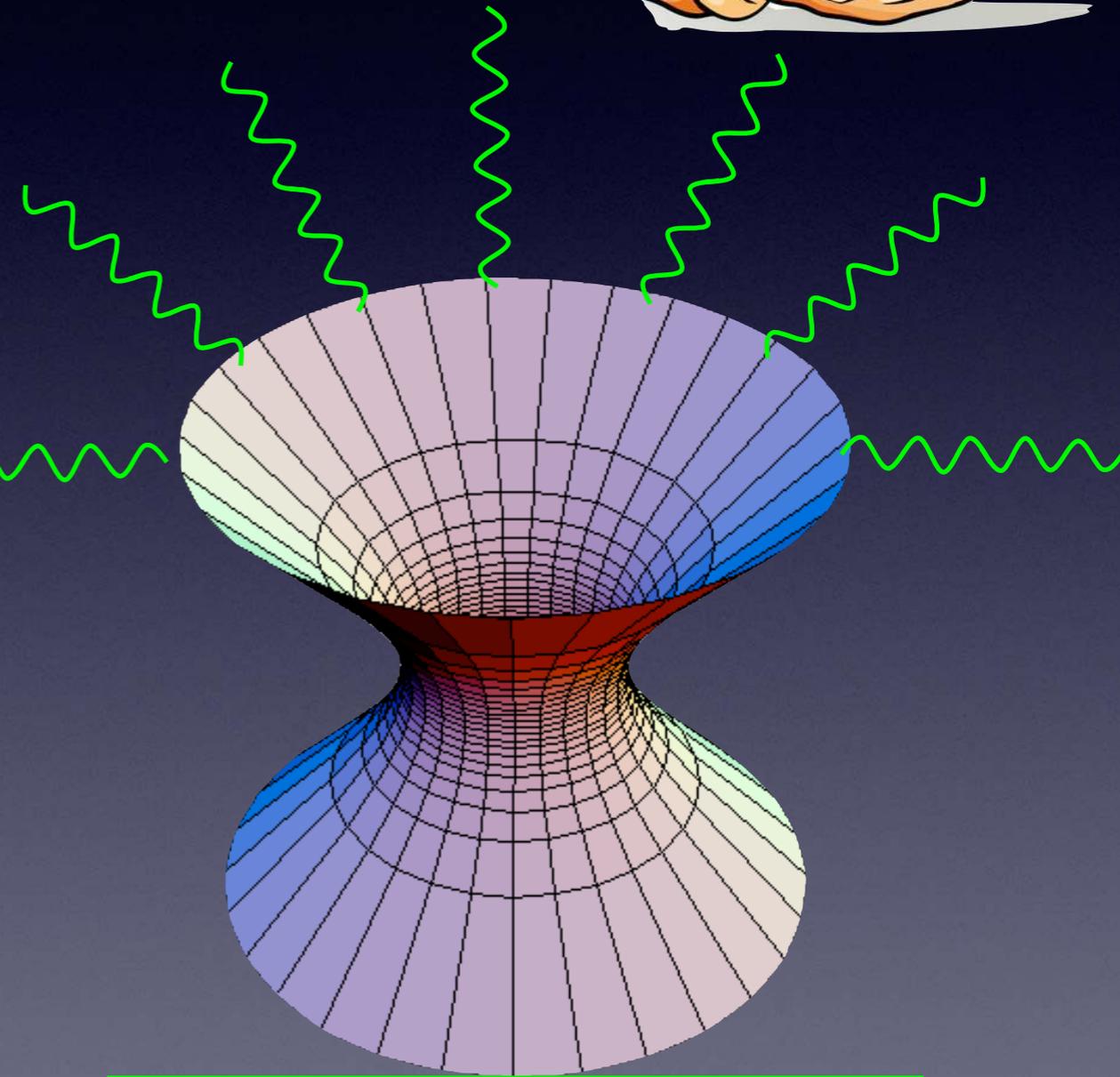
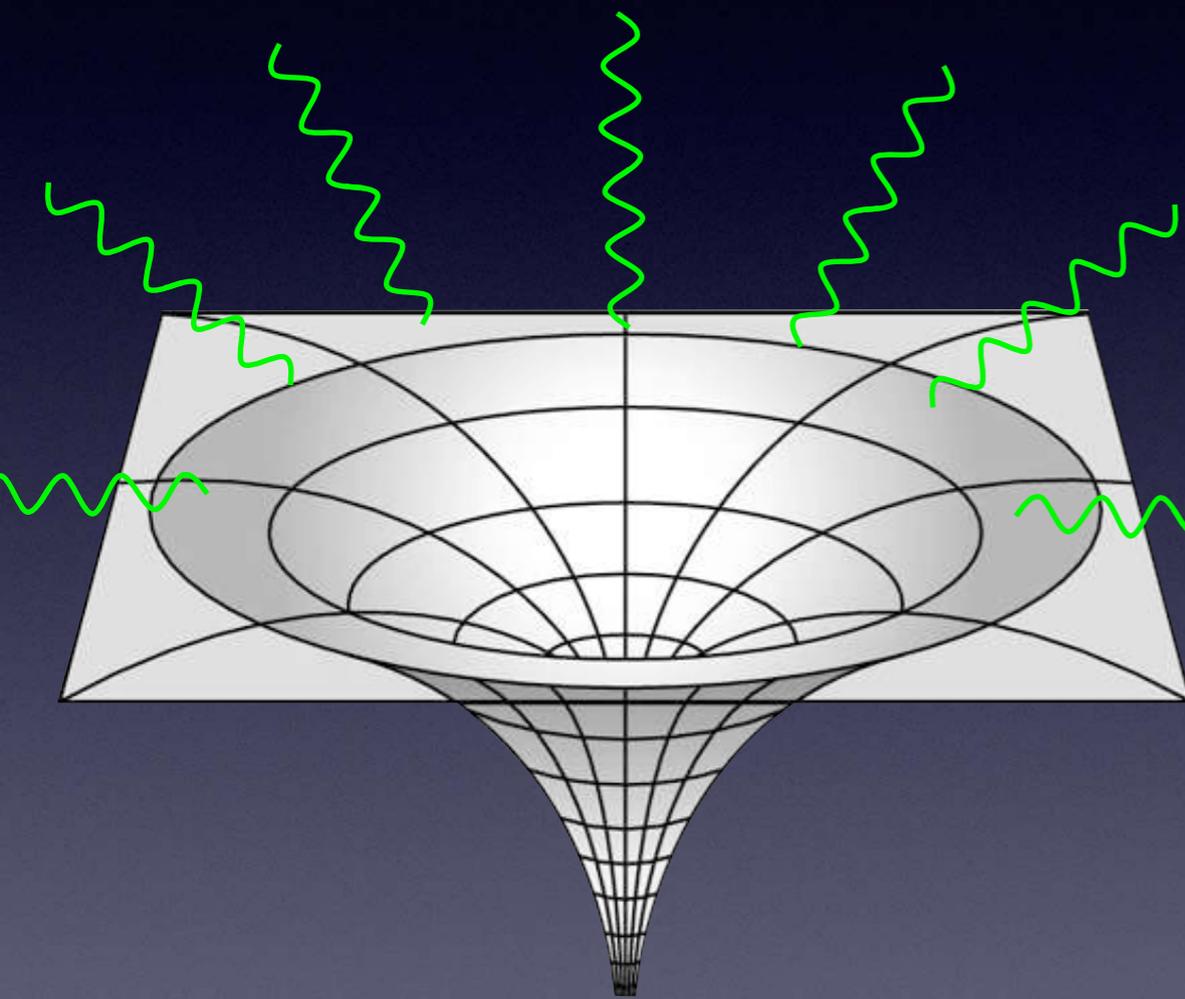


$$G_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu} = \frac{1}{M_P^2} \times \frac{1}{2} \langle \rho_\chi \rangle g_{\mu\nu}$$

$$\Rightarrow \langle \rho_\chi \rangle \sim \mathcal{R} M_P^2 \sim H^2 M_P^2$$

Gravitational production in a nutshell

$$\langle \rho_\chi \rangle \sim \mathcal{R} M_P^2 \sim H^2 M_P^2$$



Schwarzschild horizon, $T \sim \frac{1}{R_S} = \frac{M_P^2}{M_{BH}}$

De Sitter horizon, $T = \frac{H_{dS}}{2\pi}$

Gravitational production in a nutshell

$$\langle \rho_\chi \rangle \sim \mathcal{R} M_P^2 \sim H^2 M_P^2$$



Gravitational
waves

Black hole

Inflaton

$$\text{Schwarzschild horizon, } T \sim \frac{1}{R_S} = \frac{M_P^2}{M_{BH}}$$

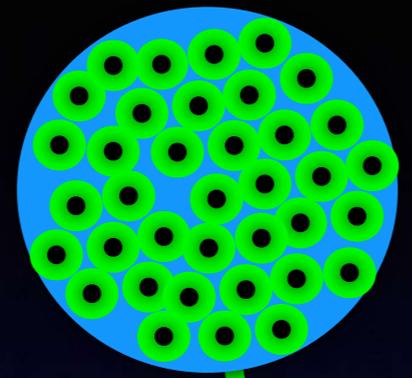
$$\text{De Sitter horizon, } T = \frac{H_{dS}}{2\pi}$$

All the rest is just poetry...

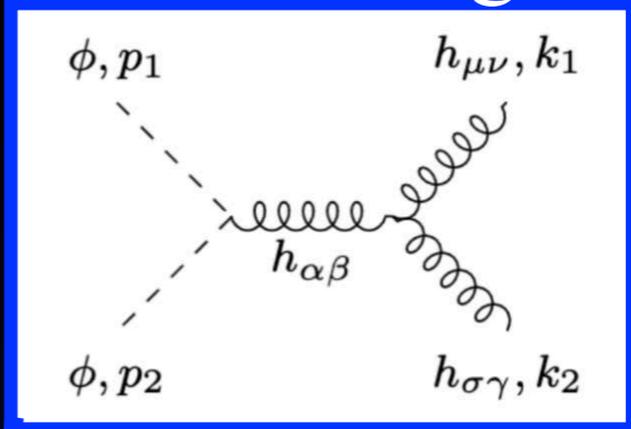
Primordial GW



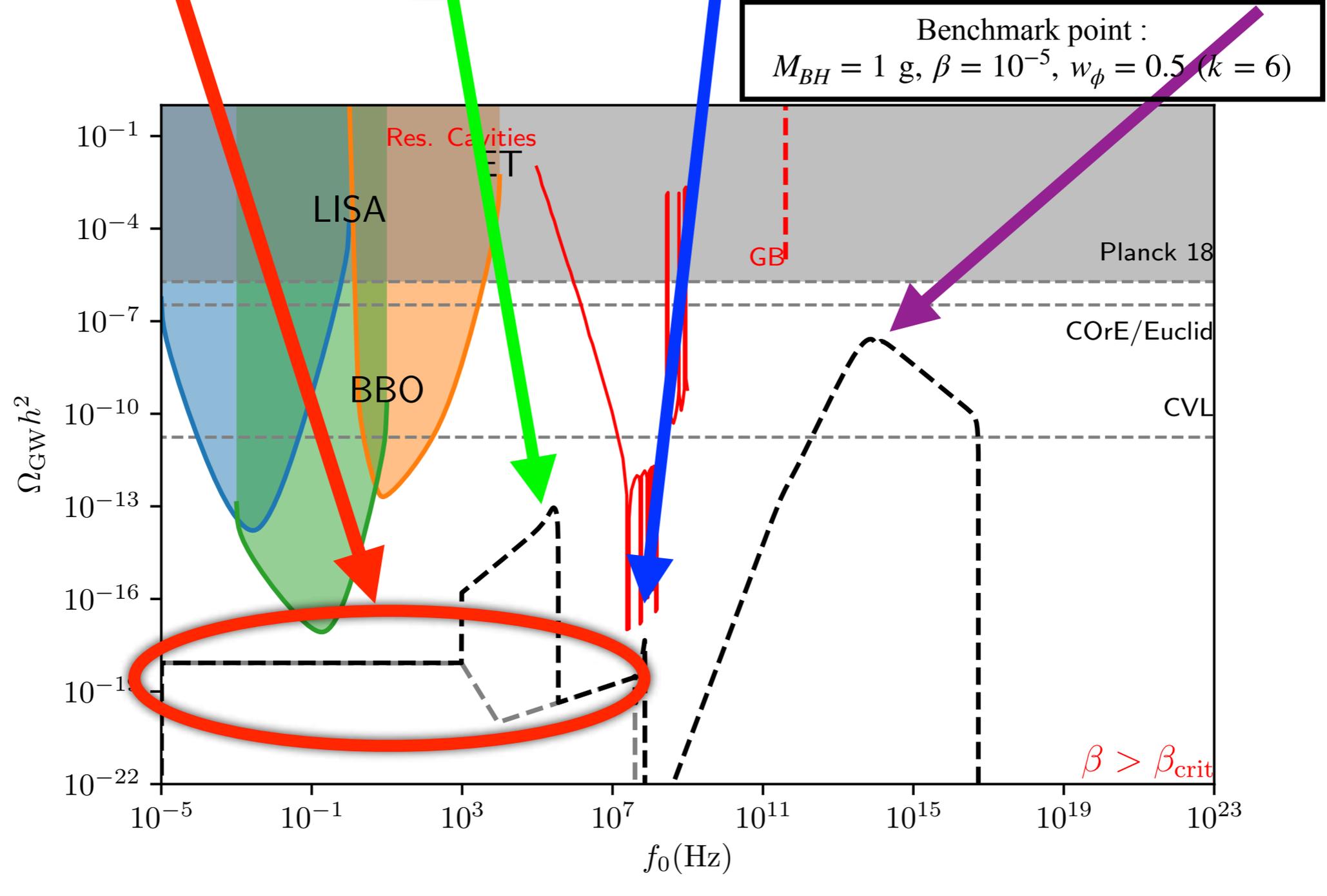
Density fluctuations



Inflaton scattering



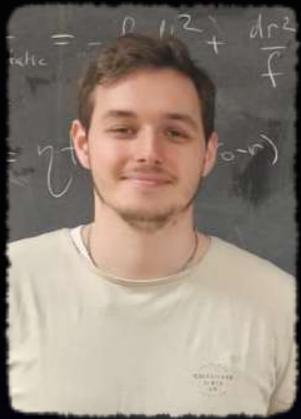
PBH decay



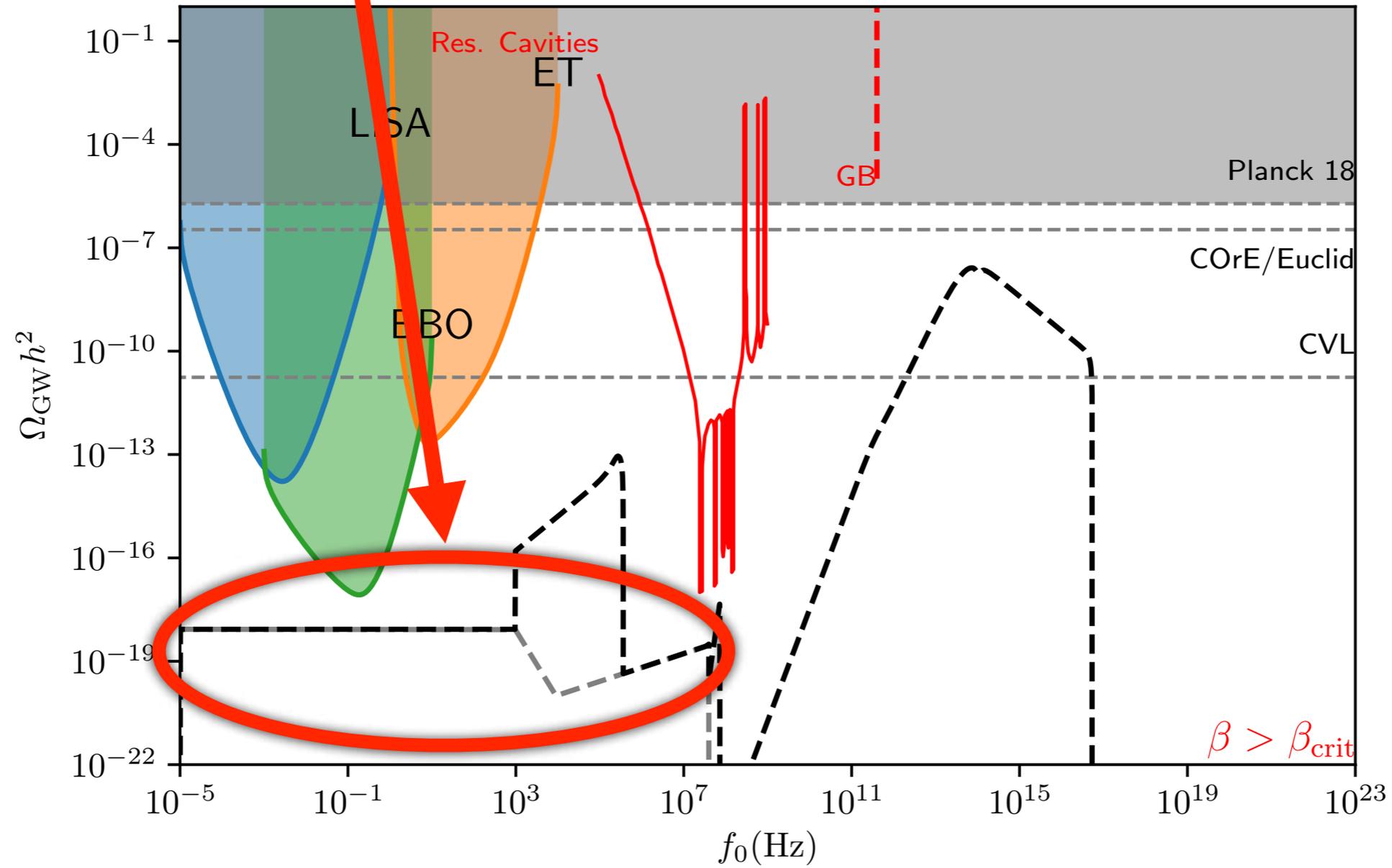
Primordial GW



Primordial GW



*Mathieu
Gross*



Primordial GW : a primer

In the case of a scalar field, $\chi = \int \frac{d^3k}{a(2\pi)^3} [\chi_k(\tau) e^{-ikx} a_k^\dagger + \chi_k(\tau)^\dagger e^{ikx} a_k]$

within the gravitational background, the equation of motion for $\chi_k(\tau)$ (τ being the *conformal* time) is

$$\chi_k'' + \left(k^2 - \frac{a''}{a}\right) \chi_k = 0 \quad \Rightarrow \quad \chi_k'' + \left(k^2 - \frac{2}{\tau^2}\right) \chi_k = 0, \quad a = -\frac{1}{H\tau},$$

whose solution is $\chi_k(a) \sim \frac{e^{-ik\tau}}{a\sqrt{2k}} \left(1 + i\frac{H}{k/a}\right)$.

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Needs acceleration (Schwarzschild or de Sitter)

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Needs acceleration (Schwarzschild or de Sitter)

whose solution is $\chi_k(a) \sim \frac{e^{-ik\tau}}{a\sqrt{2k}} \left(1 + i\frac{H}{k/a}\right)$.

One then has on super horizon scale ($\frac{k}{a} \ll H$),

$$|\chi_k(\tau)|^2 \sim \frac{H^2}{2k^3} \quad \Rightarrow \quad \delta\chi^2 = |\chi_k|^2 d^3k = |\chi_k|^2 \frac{k^3}{2\pi^2} \frac{dk}{k} = \left(\frac{H}{2\pi}\right)^2 d \ln k$$

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We then obtain $\frac{d\rho_\chi}{d \ln k} = \frac{\nabla^2 |\delta\chi|^2}{d \ln k} = k^2 \left(\frac{H}{2\pi}\right)^2$

$$|\chi_k(\tau)|^2 \sim \frac{H^2}{2k^3} \Rightarrow \delta\chi^2 = |\chi_k|^2 d^3k = |\chi_k|^2 \frac{k^3}{2\pi^2} \frac{dk}{k} = \left(\frac{H}{2\pi}\right)^2 d \ln k$$

$$\text{We then obtain } \frac{d\rho_\chi}{d \ln k} = \frac{\nabla^2 |\delta\chi|^2}{d \ln k} = k^2 \left(\frac{H}{2\pi}\right)^2$$

The treatment for the graviton is the same, considering it as \sim two massless degrees of freedom. We then obtain

$$\Omega_{GW} = \frac{\frac{d\rho_{GW}}{d \ln k}}{\rho_{tot}} = \frac{k^2 H_{\text{end}}^2}{12\pi^2 M_p^2 H^2} = \frac{1}{12\pi^2} \left(\frac{H_{\text{end}}}{M_p}\right)^2 \frac{k^2}{H^2}$$

$$|\chi_k(\tau)|^2 \sim \frac{H^2}{2k^3} \Rightarrow \delta\chi^2 = |\chi_k|^2 d^3k = |\chi_k|^2 \frac{k^3}{2\pi^2} \frac{dk}{k} = \left(\frac{H}{2\pi}\right)^2 d \ln k$$

$$\text{We then obtain } \frac{d\rho_\chi}{d \ln k} = \frac{\nabla^2 |\delta\chi|^2}{d \ln k} = k^2 \left(\frac{H}{2\pi}\right)^2$$

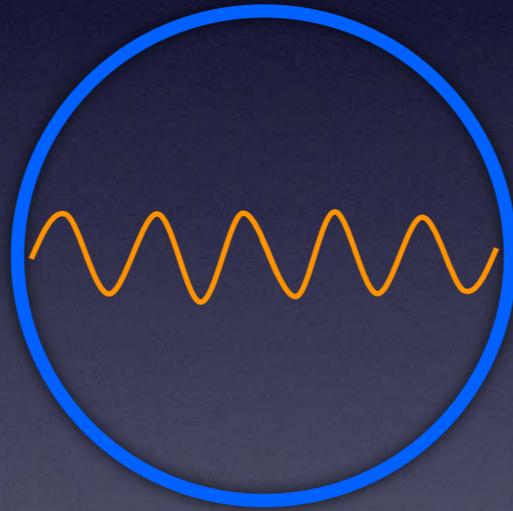
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$$\text{At horizon crossing, } k = H \Rightarrow \frac{d\rho_{GW}}{d \ln k} = \frac{1}{12\pi^2} \left(\frac{H_{\text{end}}}{M_p}\right)^2 \rho_{tot} \simeq 10^{-14} \rho_{tot}$$

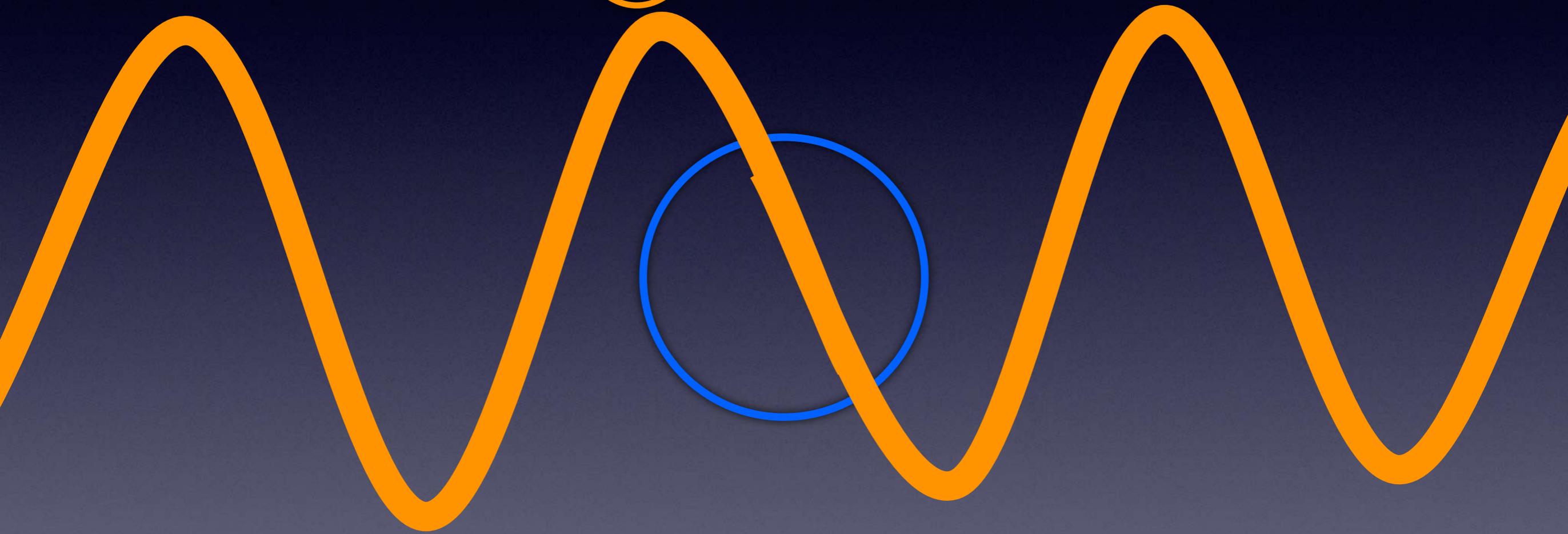
PGW are modes which *exited the horizon* during inflation :

$$\chi_k(a) \sim \frac{e^{-ik\tau}}{a\sqrt{2k}} \left(1 + i \frac{H}{k} \frac{k}{a} \right)$$



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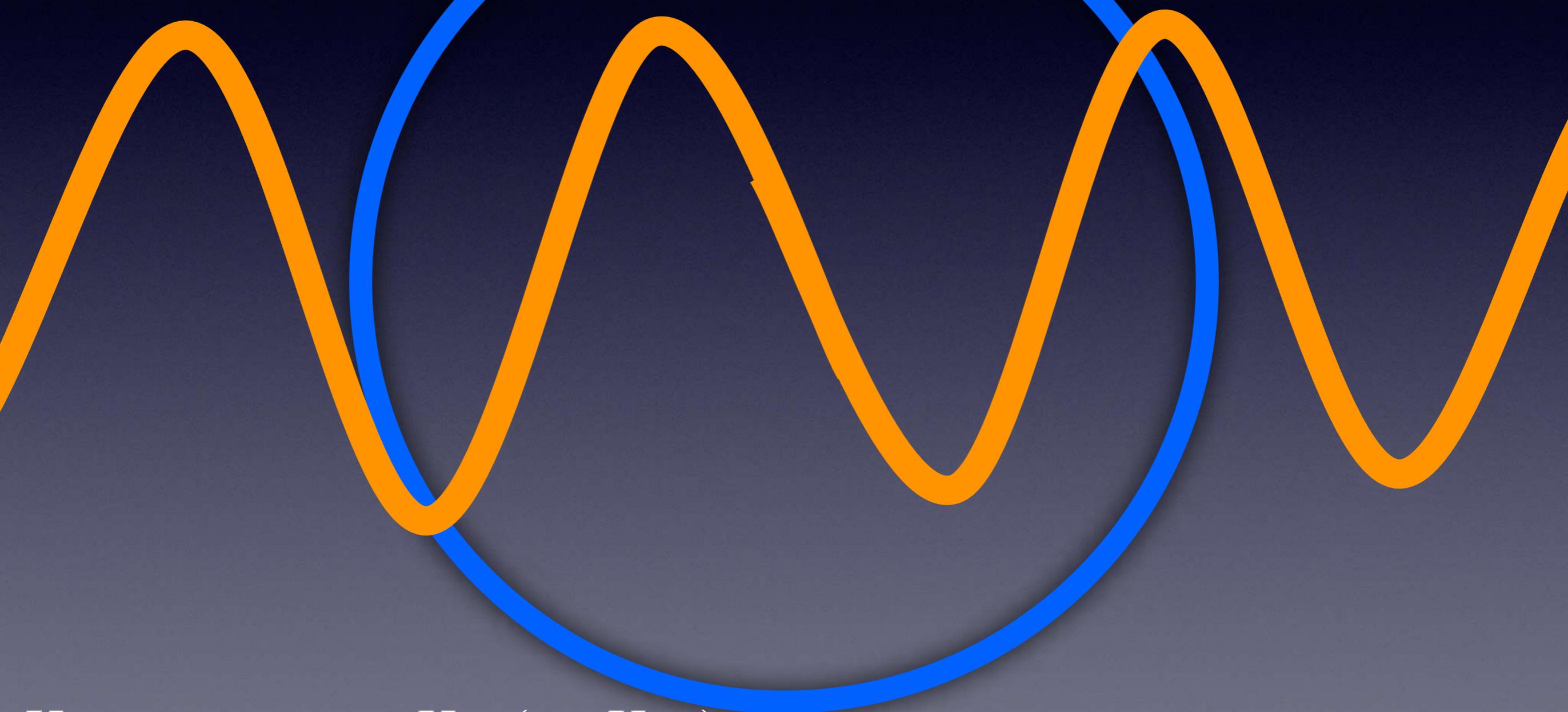
$$\chi_k(a) \sim \frac{e^{-ik\tau}}{a\sqrt{2k}} \left(1 + i \frac{H}{\frac{k}{a}} \right)$$



$$H_{\text{horizon crossing}} = H_{hc} (< H_{\text{end}})$$

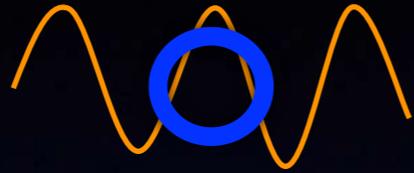
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$H_{\text{horizon crossing}} = H_{hc} (< H_{\text{end}})$ Frozen fluctuations \rightarrow oscillating particles

Primordial GW with PBH and ϕ



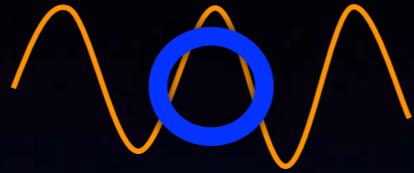
$$a < a_{\text{hc}}$$



$$a = a_{\text{hc}}$$



Primordial GW with PBH and ϕ



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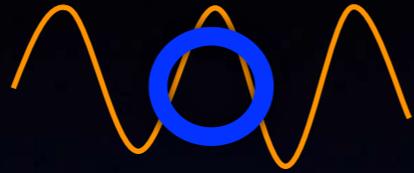


$$a = a_{\text{hc}}$$

$$\Omega_{GW} \sim \frac{H_{\text{end}}^2}{2\pi^2} \frac{H_{\text{hc}}^2}{\rho_{\text{hc}}} \frac{\left(\frac{a_{\text{hc}}}{a_0}\right)^4}{\left(\frac{a_{\text{hc}}}{a_0}\right)^{3(1+w_\phi)}} \sim \frac{H_{\text{end}}^2}{12\pi^2 M_{\text{P}}^2} \left(\frac{3M_{\text{P}}^2}{\rho_0}\right)^{\frac{1-3w_\phi}{3(1+w_\phi)}} \int_0^{\frac{6w_\phi-2}{1+3w_\phi}}$$



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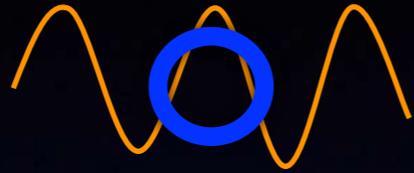
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Radiation domination ($w_\phi = \frac{1}{3}$)

$$\Omega_{GW} \sim \frac{H_{\text{end}}^2}{12\pi^2 M_{\text{P}}^2} \simeq 10^{-18}$$

Primordial GW with PBH and ϕ



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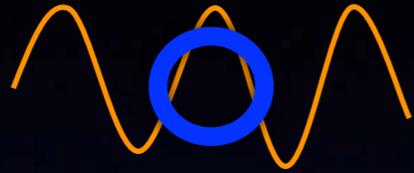
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Matter (PBH) domination ($w_\phi = 0$)

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Primordial GW with PBH and ϕ



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$$\Omega_{GW} \sim 10^{-18} \left(\frac{10^3 \text{ Hz}}{f_0}\right)^2$$

Inflaton domination ($w_\phi = \frac{1}{2}$, $V(\phi) \sim \phi^6$)

$$\Omega_{GW} \sim 10^{-18} \left(\frac{f_0}{10^5 \text{ Hz}}\right)^{\frac{2}{5}}$$

Primordial GW with PBH and ϕ

PBH domination



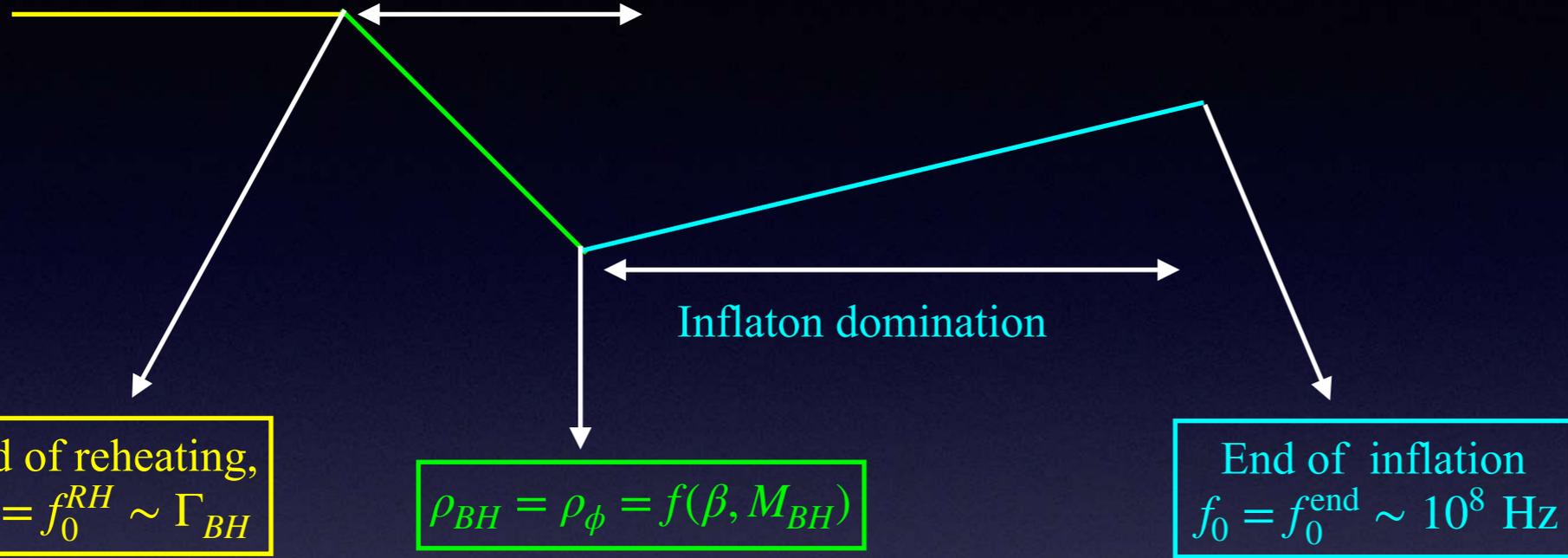
End of reheating,
 $f_0 = f_0^{RH} \sim \Gamma_{BH}$

$$\rho_{BH} = \rho_\phi = f(\beta, M_{BH})$$

Inflaton domination



End of inflation
 $f_0 = f_0^{\text{end}} \sim 10^8 \text{ Hz}$



Primordial GW with PBH and ϕ

Having access to PGW spectrum is literally reading line by line the pre-BBN history

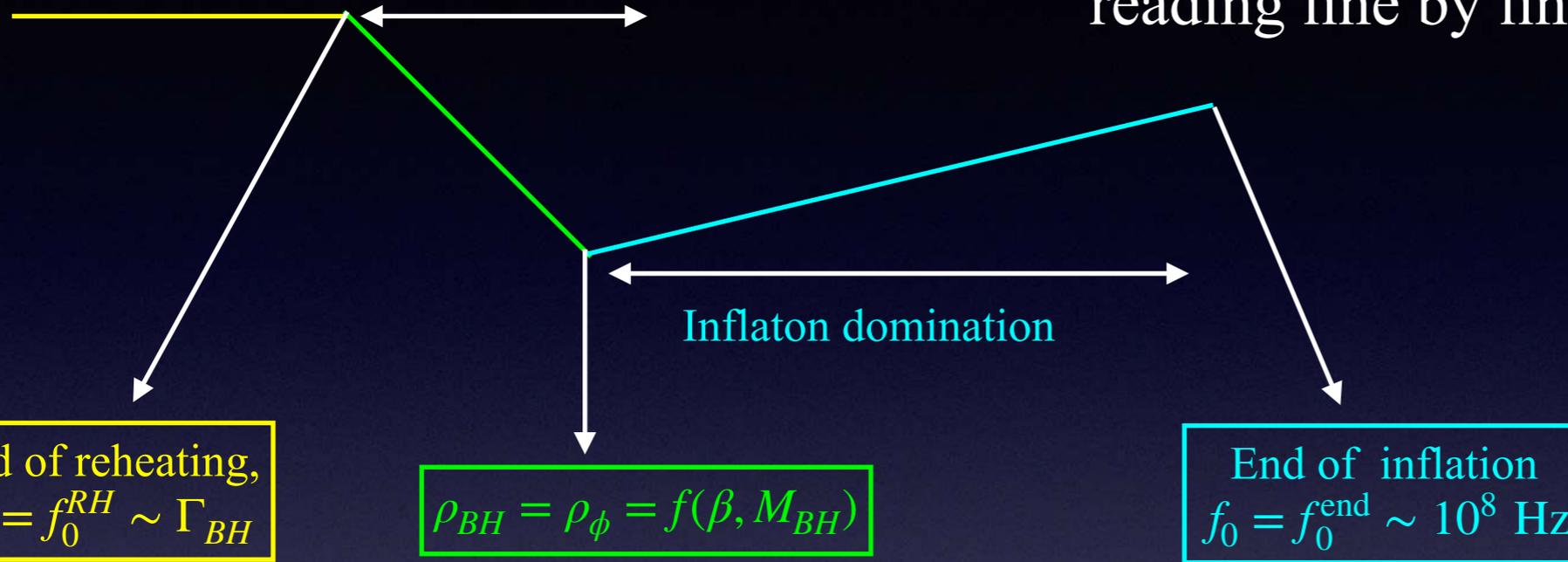
PBH domination

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Primordial GW with PBH and ϕ

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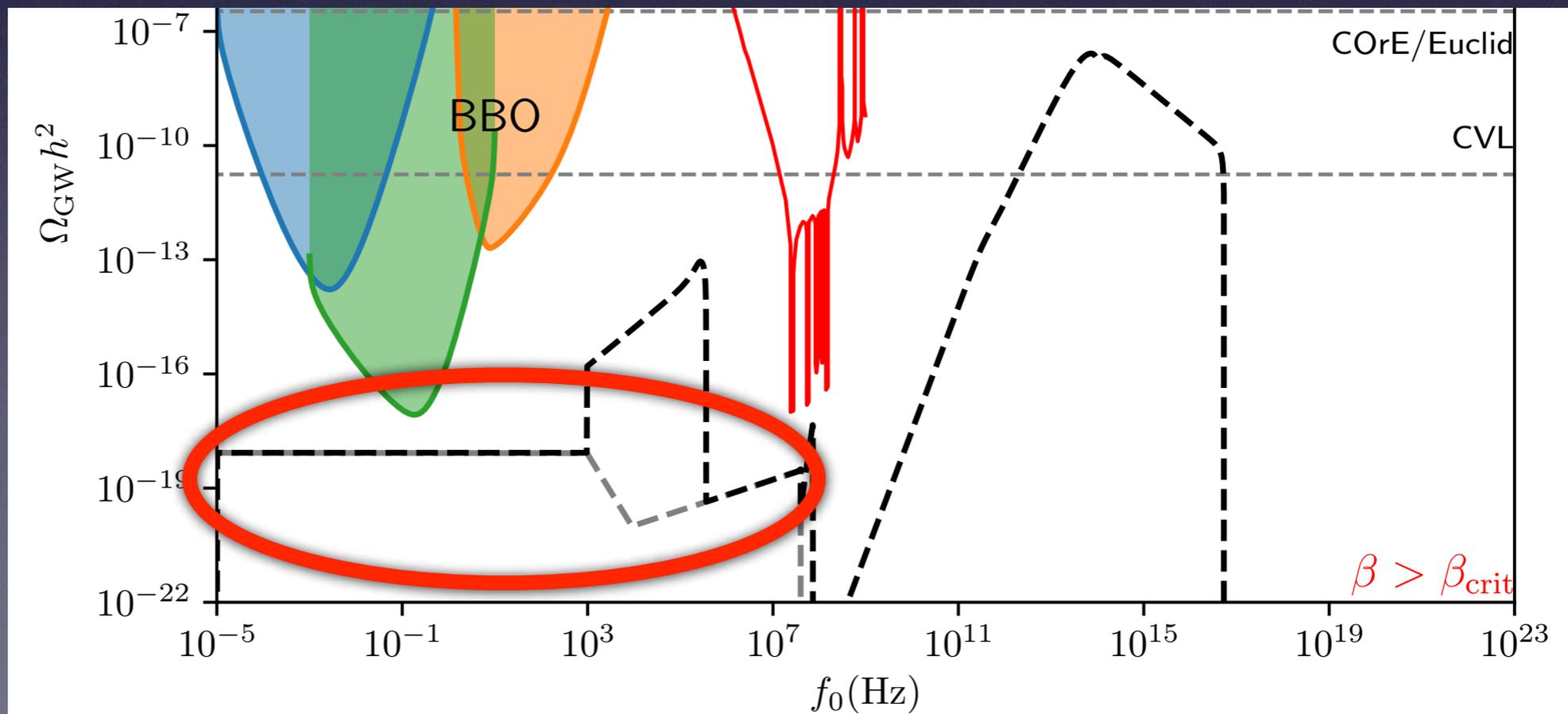
PBH domination

Inflaton domination

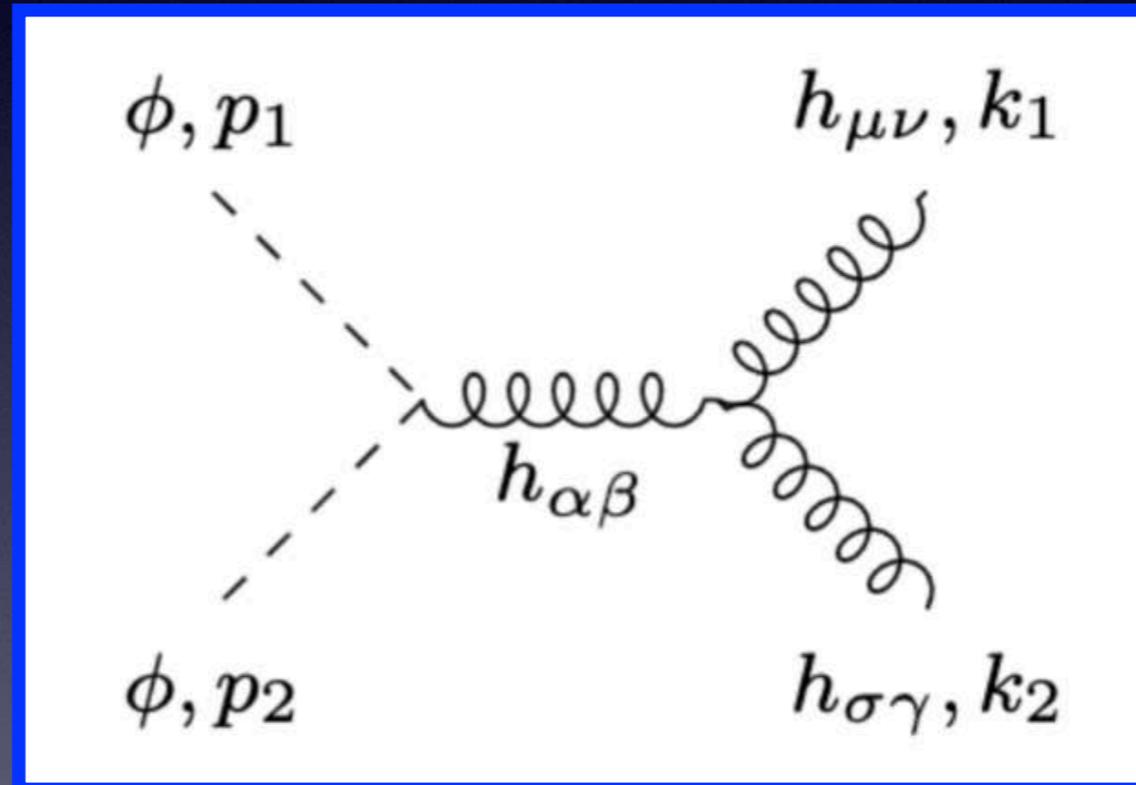
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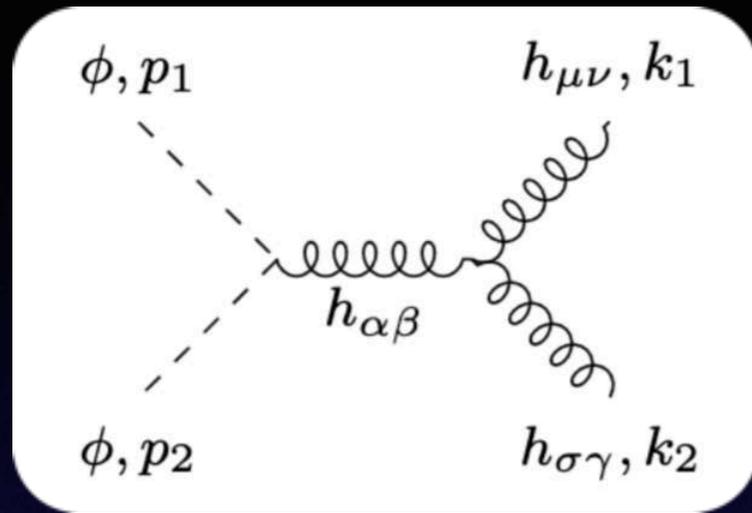
End of inflation
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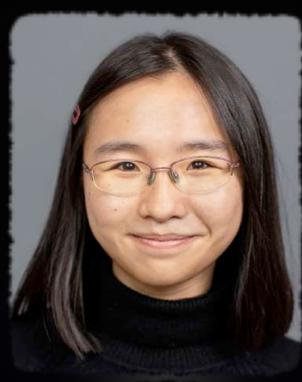
GW from inflaton scattering



GW from inflaton scattering

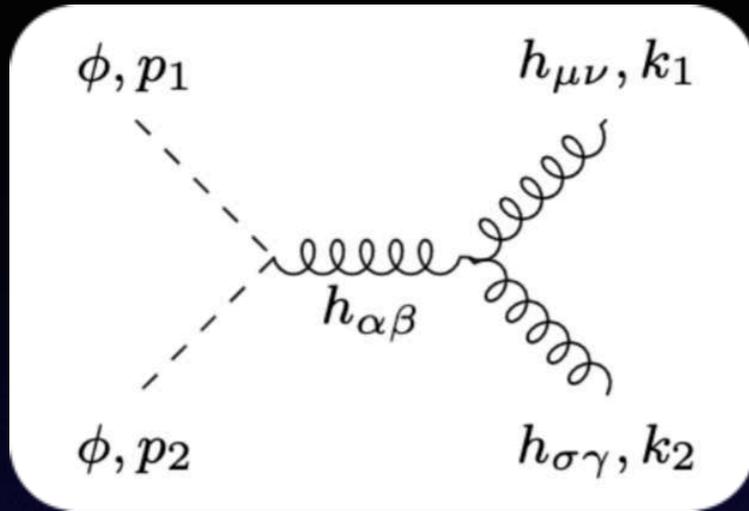


*Gyonju
Choi*

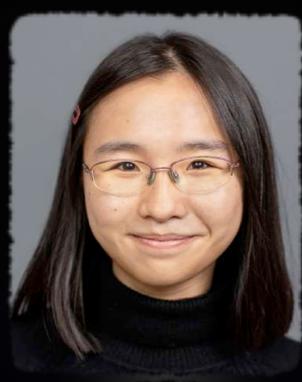


*Wenqi
Ke*

GW from inflaton scattering



*Gyonju
Choi*



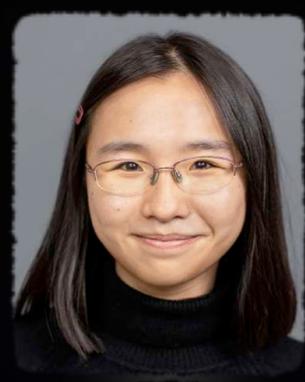
*Wenqi
Ke*

$$\Omega_{\text{GW}}^{\phi} = \frac{1}{\rho_c^0} \frac{d\rho_{\text{GW}}^{\phi}}{d \ln f_0} = \frac{M_{\text{P}}^2}{4\sqrt{3}H_0^2} \frac{k+2}{|k-4|} \left(\frac{2\pi}{\gamma_k} \right)^{\frac{3k-3}{k-4}} \frac{1}{\alpha^{\frac{3k+6}{2k(k-4)}}} \times \left(\frac{M_{\text{P}}}{T_{\text{RH}}} \right)^{\frac{6k+12}{k(k-4)}} \left(\frac{g_{\text{RH}}^{\frac{1}{3}} T_{\text{RH}}}{g_0^{\frac{1}{3}} T_0} \right)^{\frac{9}{k-4}} \Sigma^k \left(\frac{f_0}{M_{\text{P}}} \right)^{\frac{4k-7}{k-4}},$$

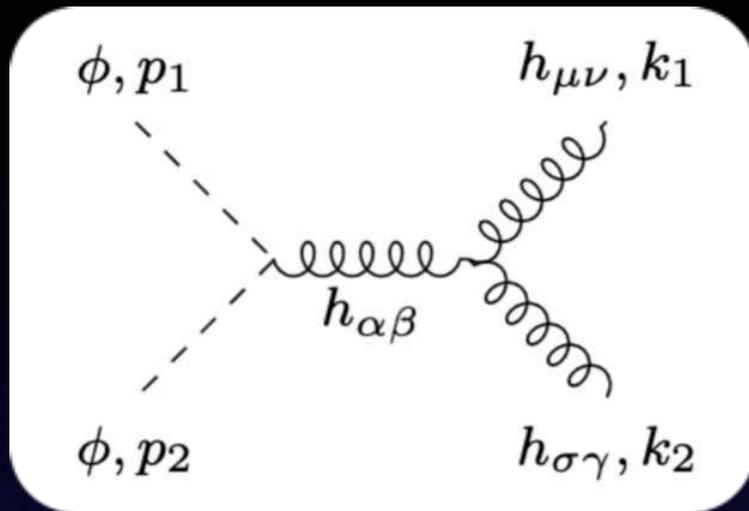
GW from inflaton scattering



*Gyonju
Choi*



*Wenqi
Ke*



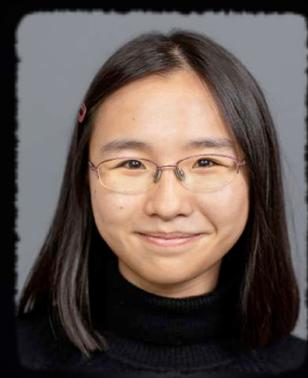
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$$\Omega_{\text{GW}} \sim T_{\text{RH}}^{\frac{3}{2} \frac{1-w_{\phi}}{1+w_{\phi}}} f_0^{\frac{1-15w_{\phi}}{6w_{\phi}-2}}$$

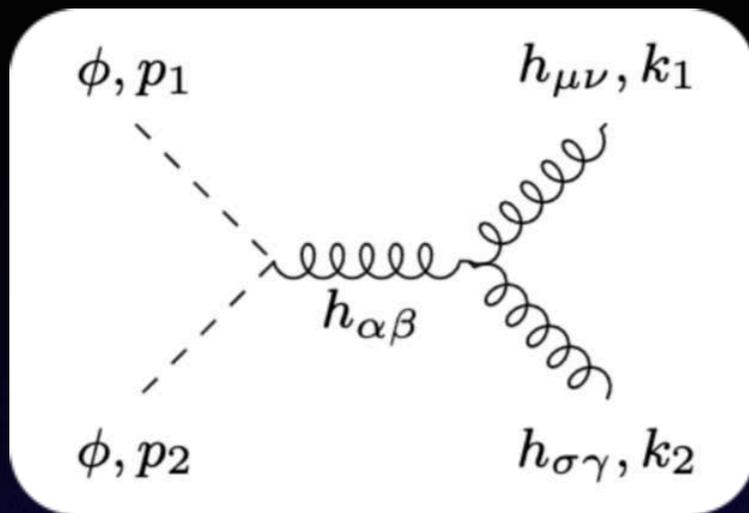
GW from inflaton scattering



Gyonju
Choi

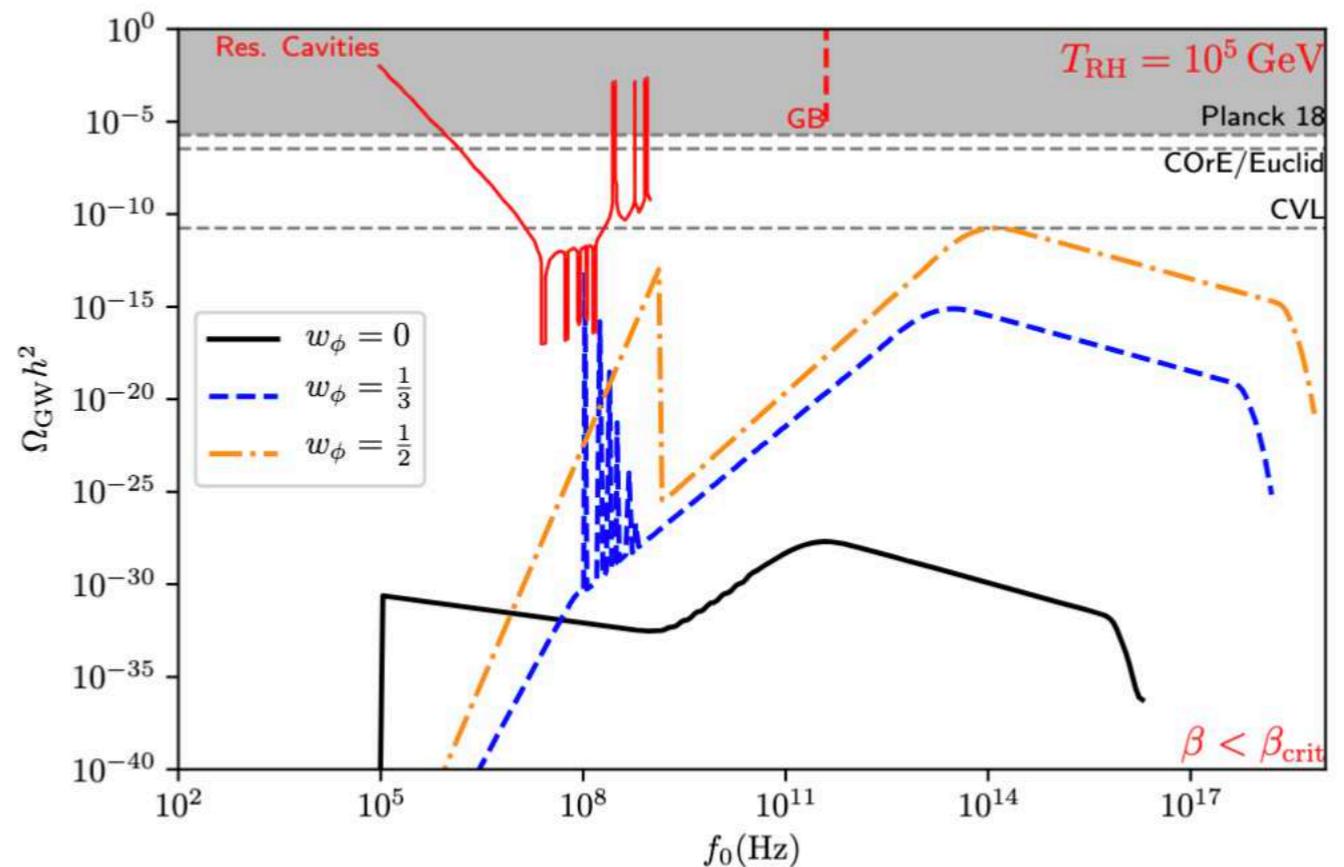


Wenqi
Ke



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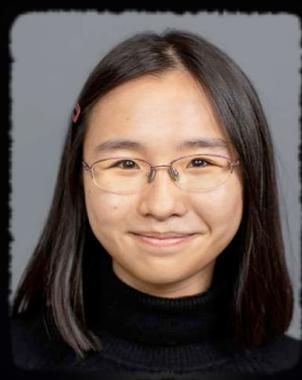
$$\Omega_{\text{GW}} \sim T_{RH}^{\frac{3}{2} \frac{1-w_{\phi}}{1+w_{\phi}}} f_0^{\frac{1-15w_{\phi}}{6w_{\phi}-2}}$$



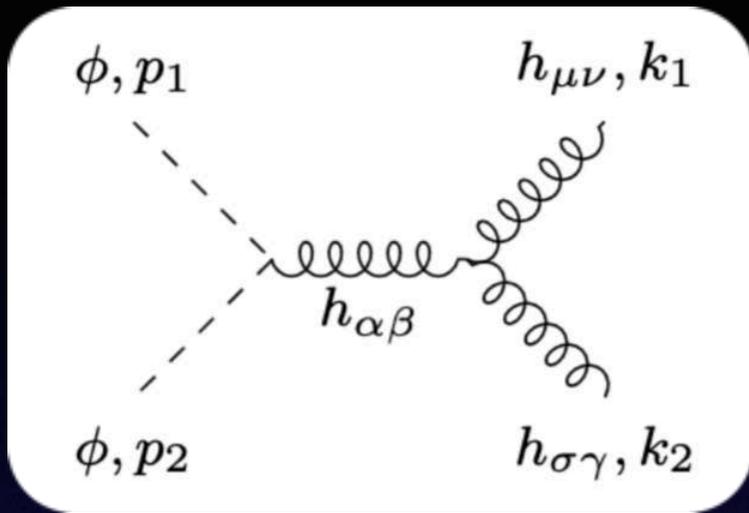
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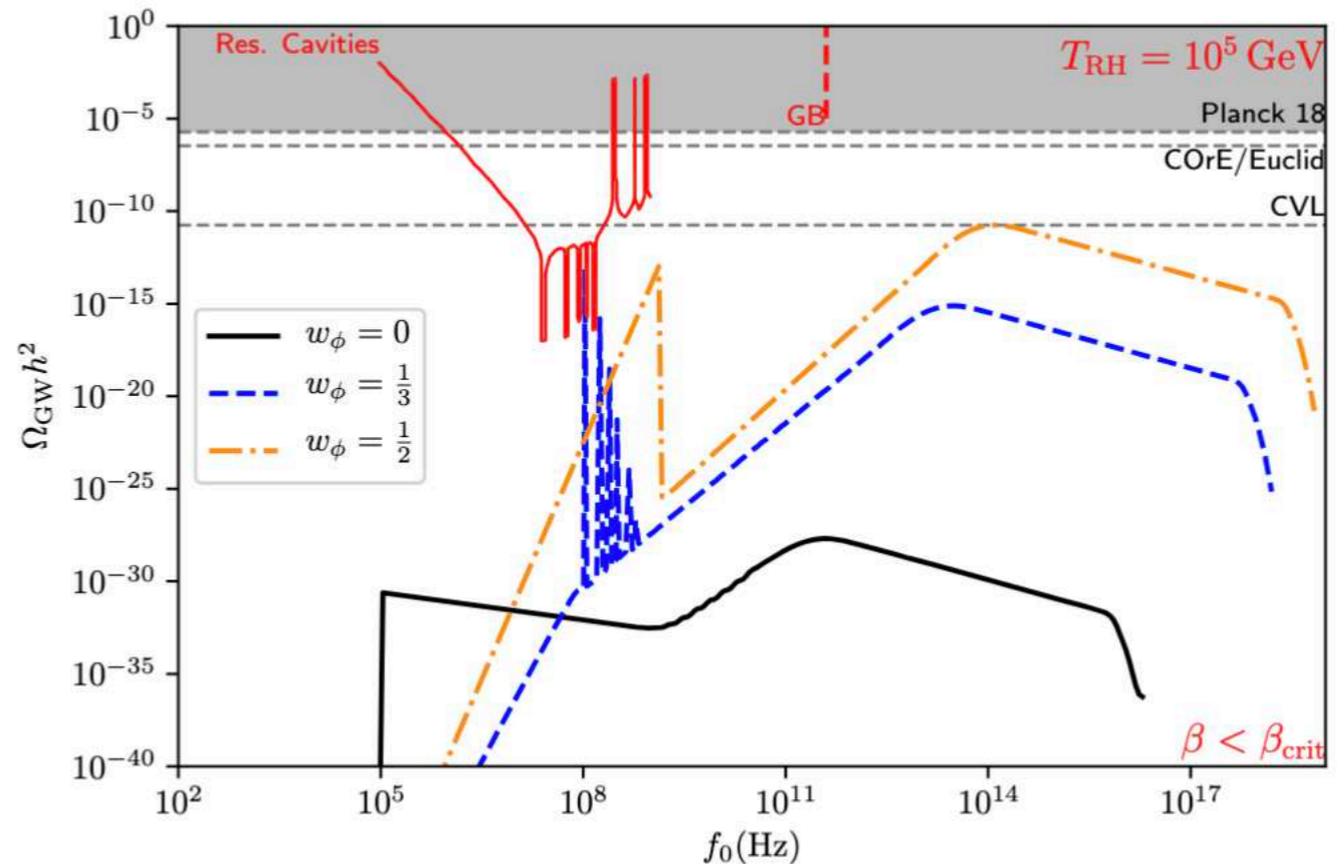


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The PBHs would influence the spectrum from

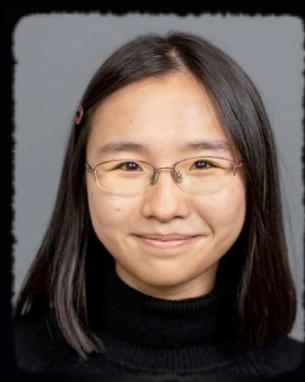
$$\rho_{\text{RH}} = \alpha T_{\text{RH}}^4 = \frac{4}{3} \Gamma_{\text{BH}}^2 M_P^2$$



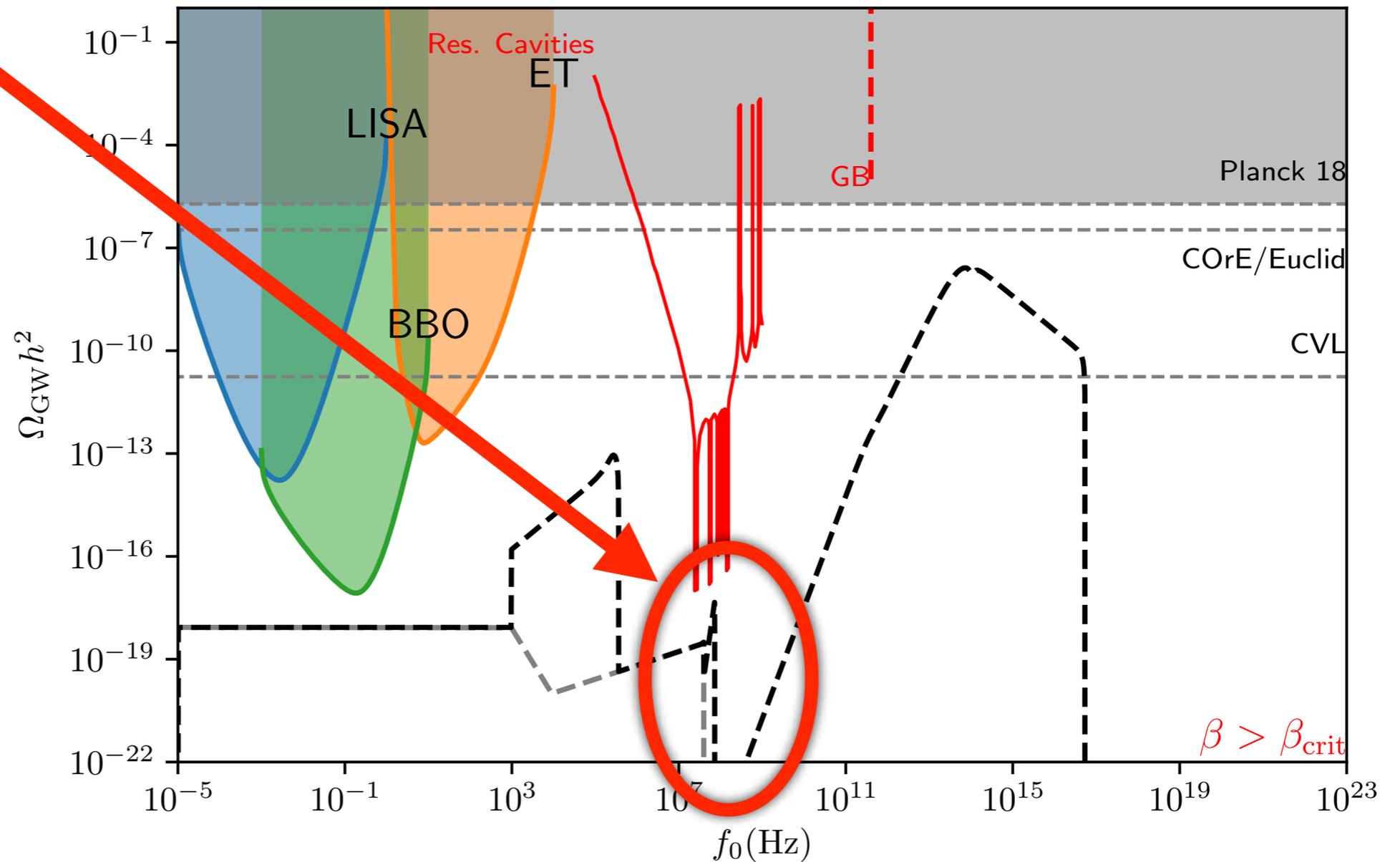
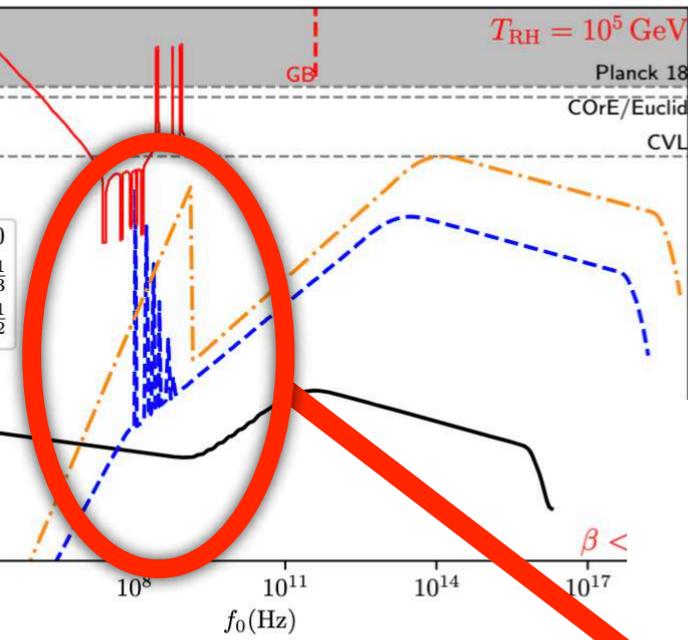
GW from inflaton scattering



*Gyonju
Choi*



*Wenqi
Ke*



Aparté : the Boltzmann-Bogoliubov fight



*Simon
Clery*



The Bogoliubov approach



$$\chi(\eta, \vec{x}) = \int \frac{d^3 \vec{k}}{(2\pi)^{3/2} a} X_{\vec{k}}(\eta) e^{i \vec{k} \cdot \vec{x}}, \quad X_{\vec{k}}'' + \left[\underbrace{k^2 + a^2 m_\chi^2 + \frac{a^2 R}{6}}_{\omega_k^2} \right] X_{\vec{k}} = 0$$

ω_k^2
can be negative if the
Ricci $R = -6 \frac{a''}{a^3}$ is negative

The Bogoliubov approach



$$\chi(\eta, \vec{x}) = \int \frac{d^3 \vec{k}}{(2\pi)^{3/2} a} X_{\vec{k}}(\eta) e^{i \vec{k} \cdot \vec{x}}, \quad X''_{\vec{k}} + \left[\underbrace{k^2 + a^2 m_\chi^2 + \frac{a^2 R}{6}}_{\omega_k^2} \right] X_{\vec{k}} = 0$$

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$$n_\chi = \frac{1}{a^3} \int \frac{d^3 k}{(2\pi)^3} n_k,$$

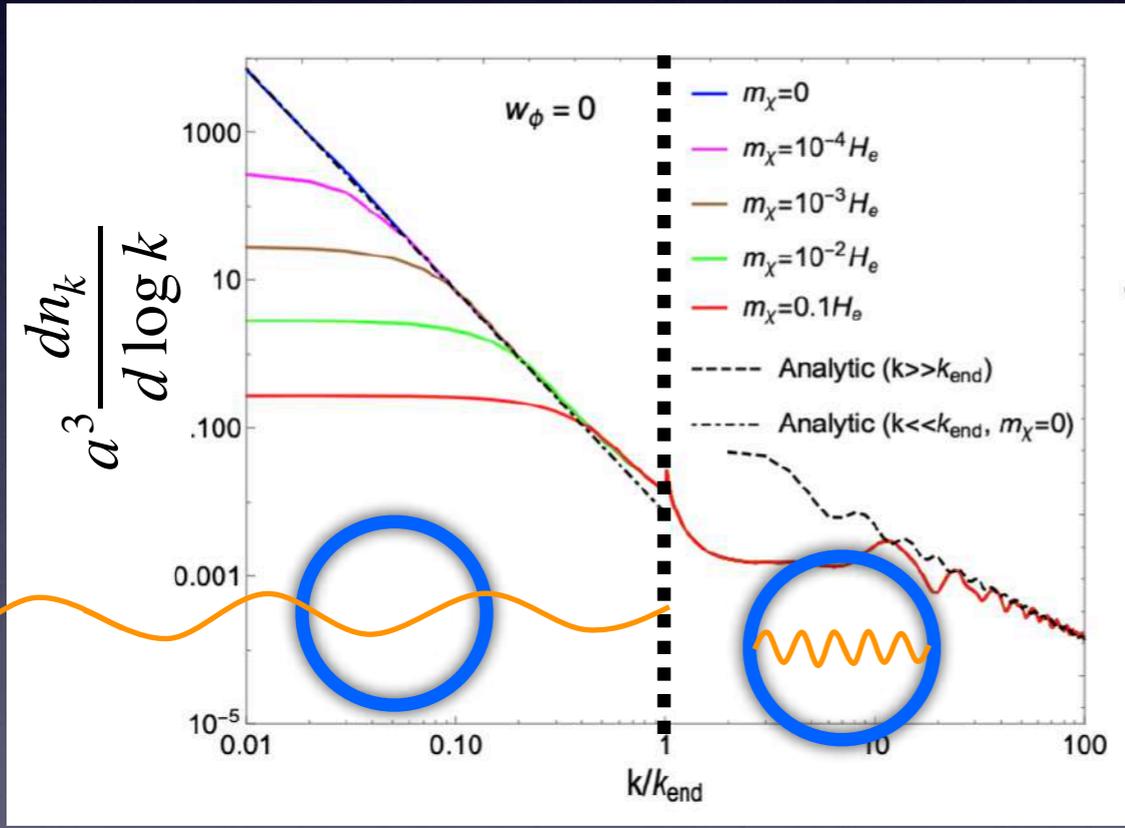
$$n_k = |\beta_{\vec{k}}|^2 = \frac{1}{2\omega_k} |\omega_k X_{\vec{k}} - i X'_{\vec{k}}|^2$$



The Bogoliubov approach

$$\chi(\eta, \vec{x}) = \int \frac{d^3 \vec{k}}{(2\pi)^{3/2} a} X_{\vec{k}}(\eta) e^{i \vec{k} \cdot \vec{x}},$$

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The Bogoliubov approach

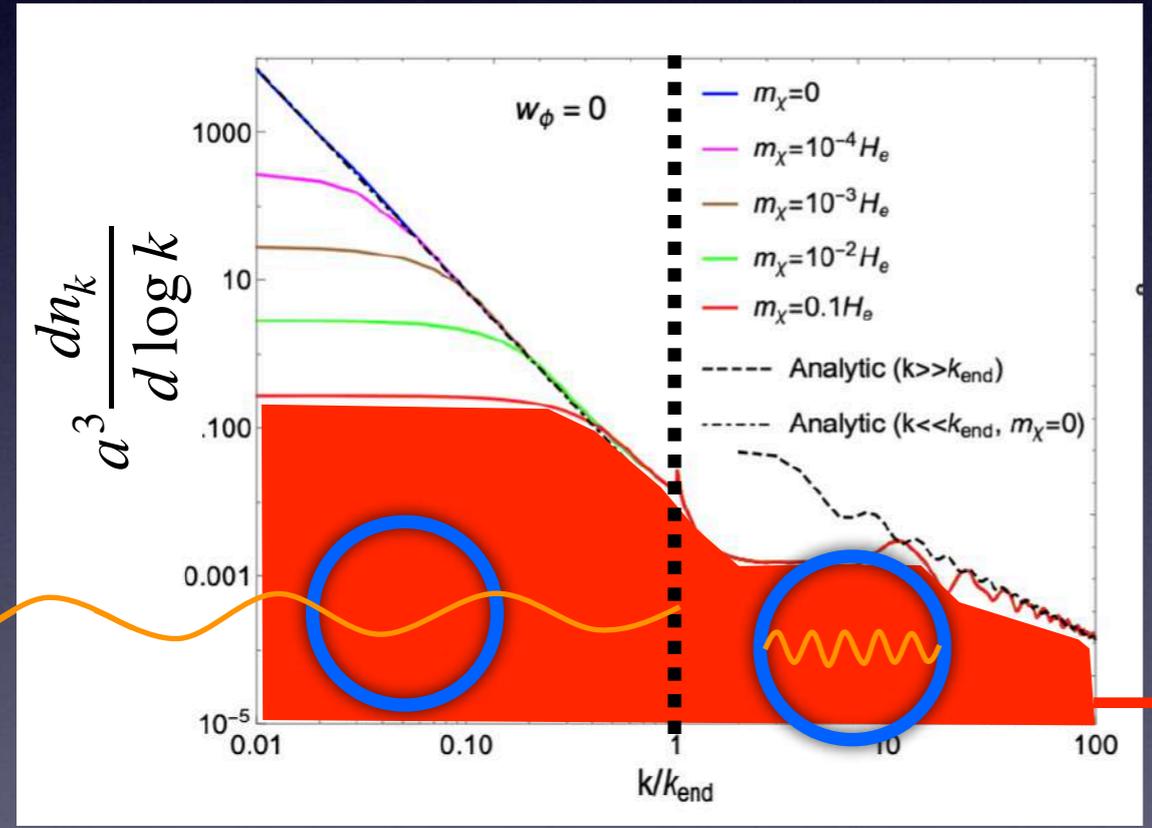
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$$\omega_k^2$$

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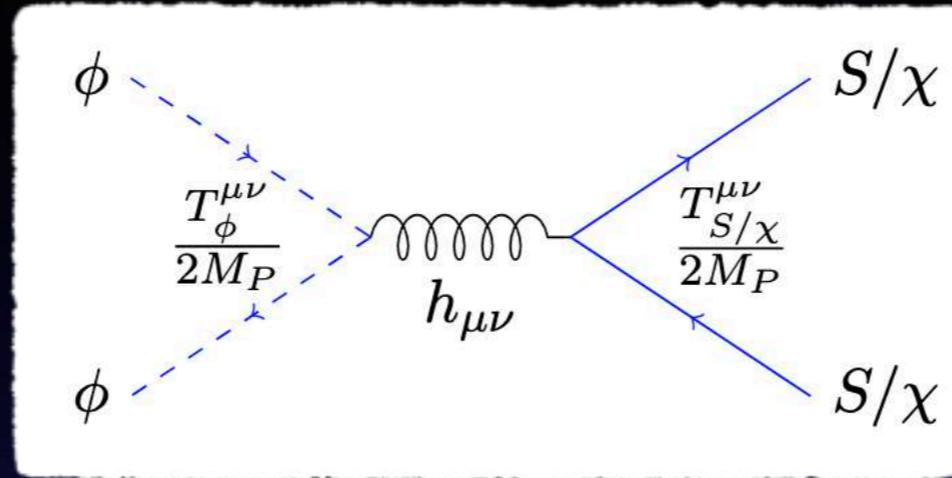


$$\rho \sim T_{dS}^4 \sim \frac{H^4}{16\pi^4}$$

$$n_\chi = \frac{1}{a^3} \int \frac{d^3 k}{(2\pi)^3} n_k,$$

$$n_k = |\beta_{\vec{k}}|^2 = \frac{1}{2\omega_k} |\omega_k X_{\vec{k}} - i X'_{\vec{k}}|^2$$

The Boltzmann approach



Boltzmann would have calculated the number density, solving the equation

$$\frac{dn_\chi}{dt} + 3H(t) n_\chi = R(t)$$

with

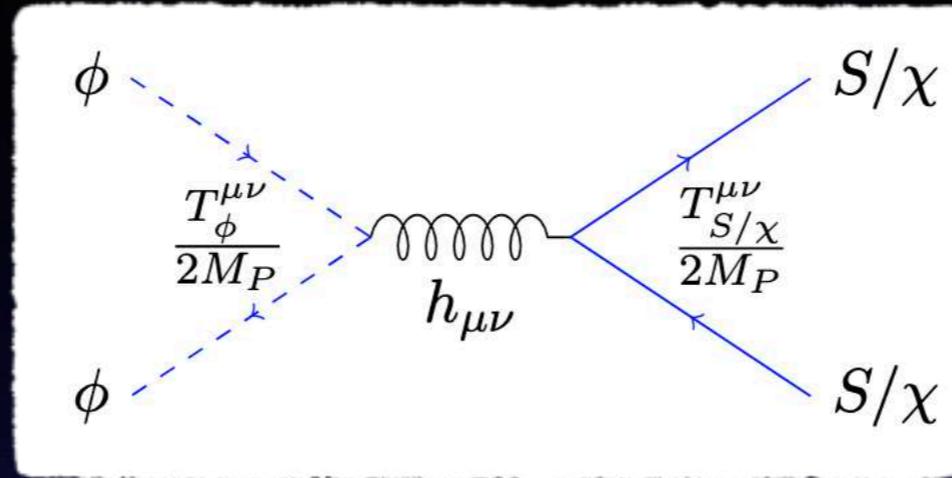
$$R(t) \sim n_\phi^2 \times \sigma_{\phi\phi \rightarrow \chi\chi} v \sim \left(\frac{\rho_\phi}{m_\phi} \right)^2 \times \frac{|\mathcal{M}_{\phi\phi \rightarrow \chi\chi}|^2}{8\pi m_\phi^2},$$

ρ_ϕ obtained from the Friedmann equation

$$H^2 = \frac{\rho_\phi^2}{3M_P^2}$$



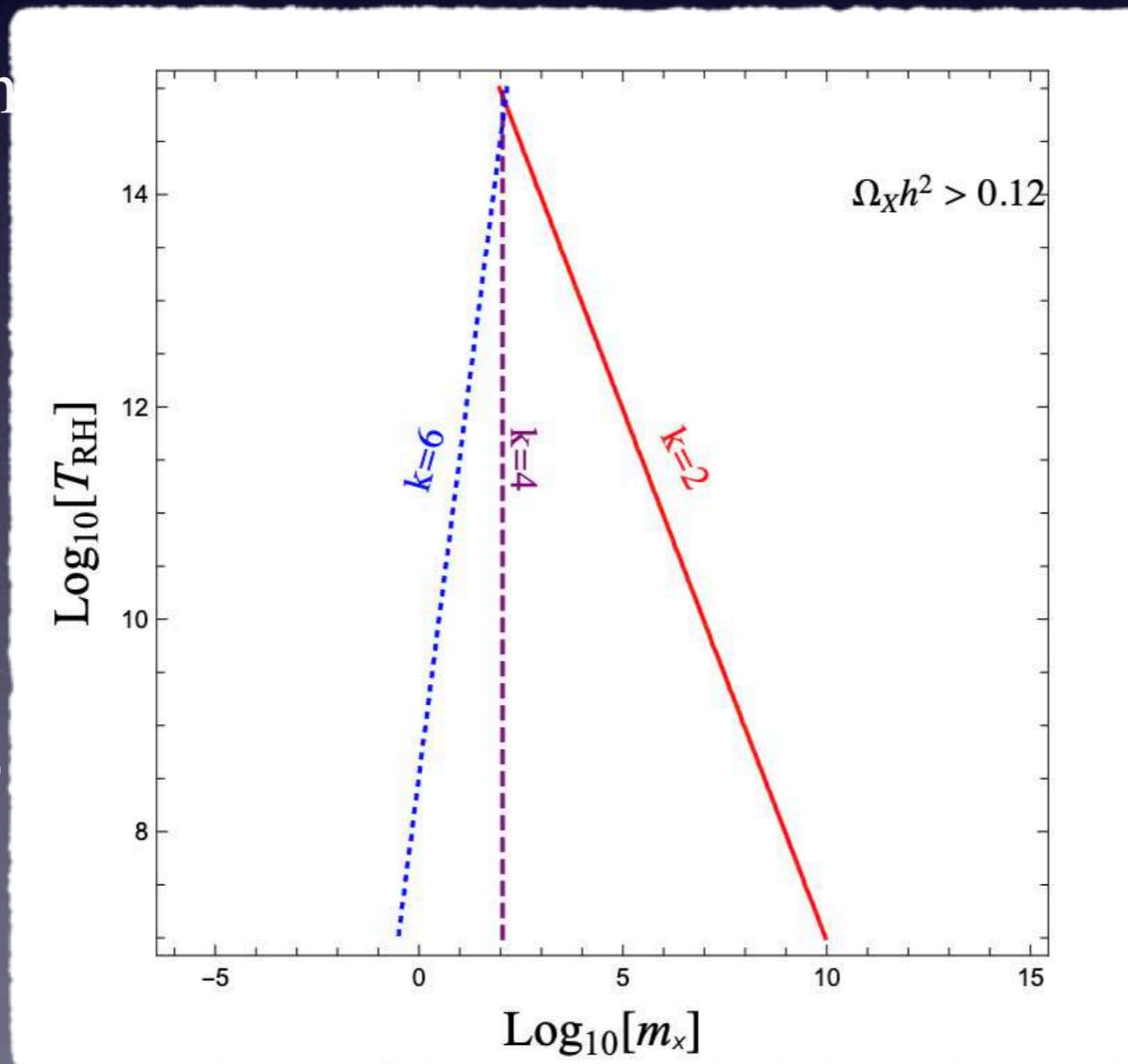
The Boltzmann approach



Boltzmann would have

solved the equation

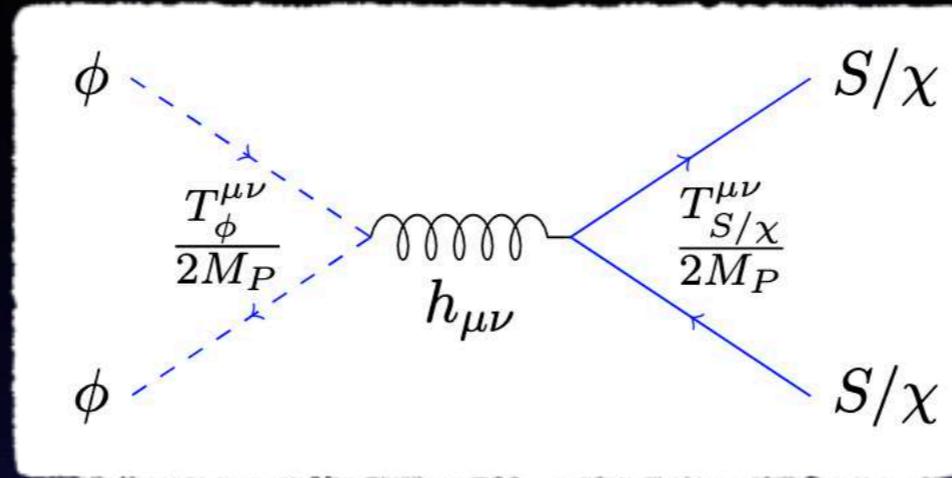
$$R(t) \sim \rho_{\phi}$$



$$\frac{\chi \chi^{\dagger 2}}{m_{\phi}^2},$$

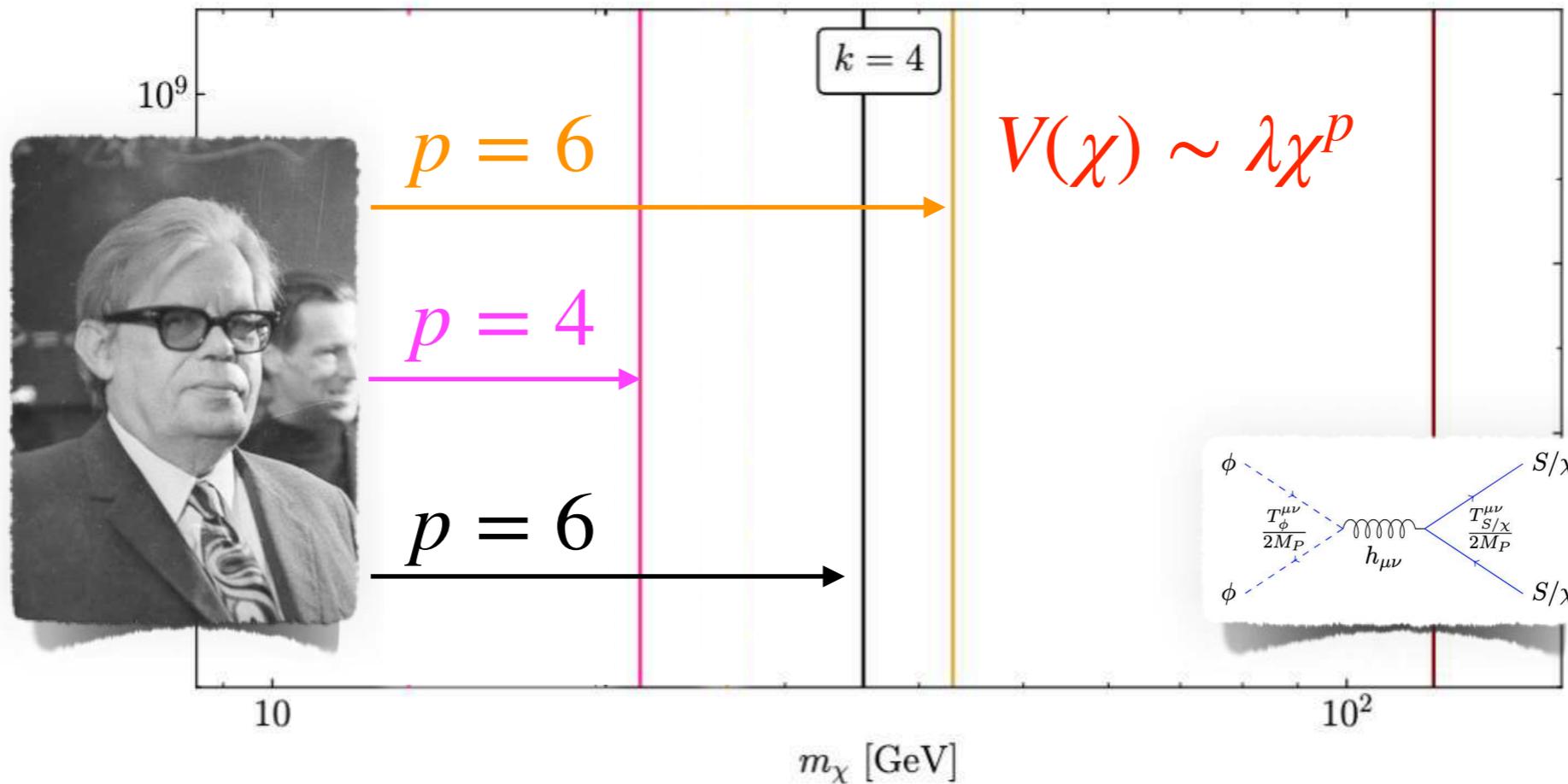
on

The Boltzmann approach



Boltzman

uation



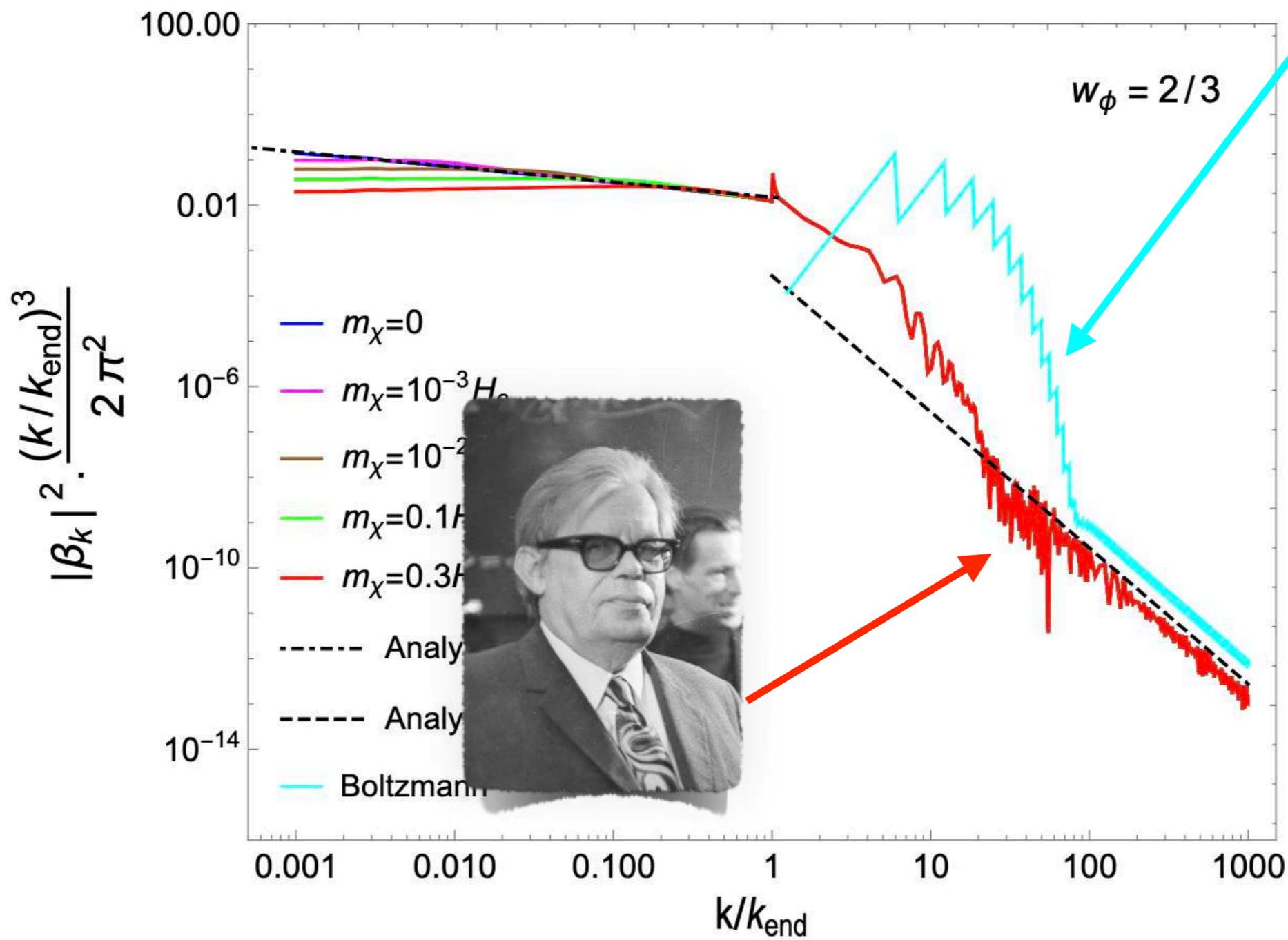
— $p = 4$ (ana.)	— $p = 6$ (ana.)	— $p = 8$ (ana.)	— CMOV
- - $p = 4$ (num.)	- - $p = 6$ (num.)	- - $p = 8$ (num.)	

The Boltzmann approach



Boltz

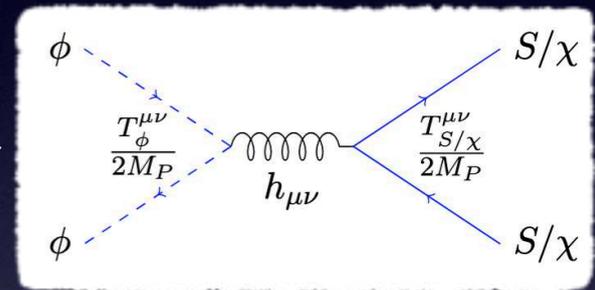
uation



Equivalence for modes *inside* the horizon



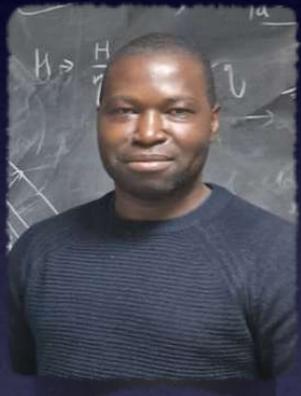
$$X''_{\vec{k}} + \left[k^2 + a^2 m_\chi^2 + \frac{a^2 R}{6} \right] X_{\vec{k}} = 0$$



Primordial Black Holes as *source* of GW

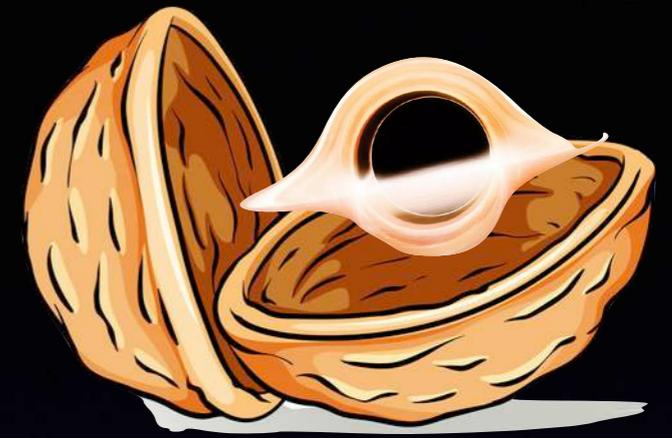


*Mathieu
Gross*



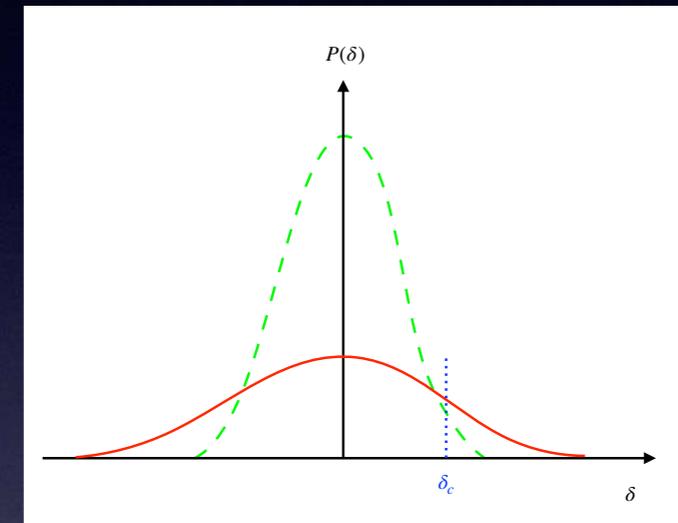
*Donald
Kpatcha*

PBH in a nutshell

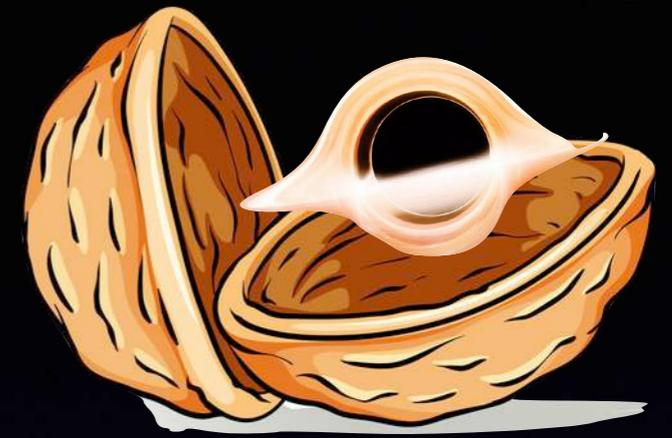


A Black hole can be formed in regions where there is an over density

$$\frac{\delta\rho}{\rho} \gtrsim \delta_c, \text{ with } \delta_c \simeq 1.$$

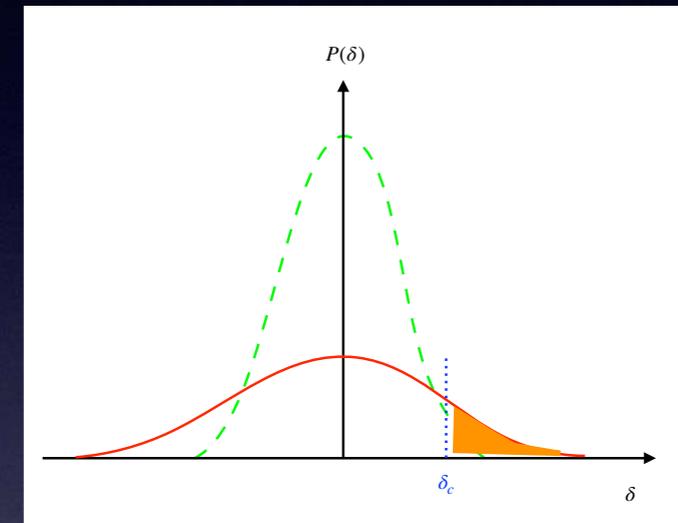


PBH in a nutshell



A Black hole can be formed in regions where there is an over density

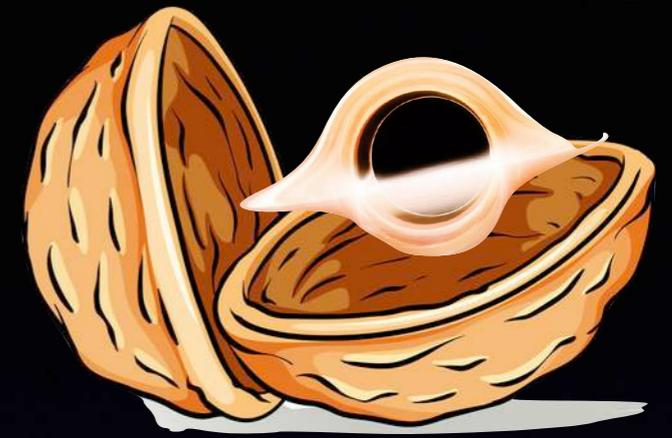
$$\frac{\delta\rho}{\rho} \gtrsim \delta_c, \text{ with } \delta_c \simeq 1.$$



The fraction

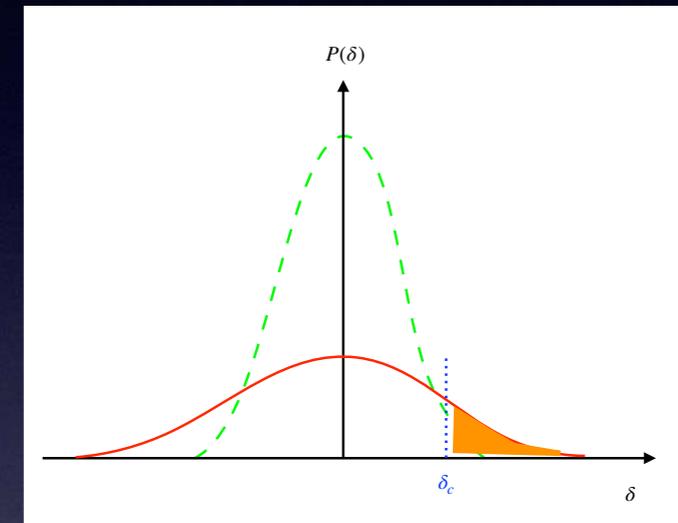
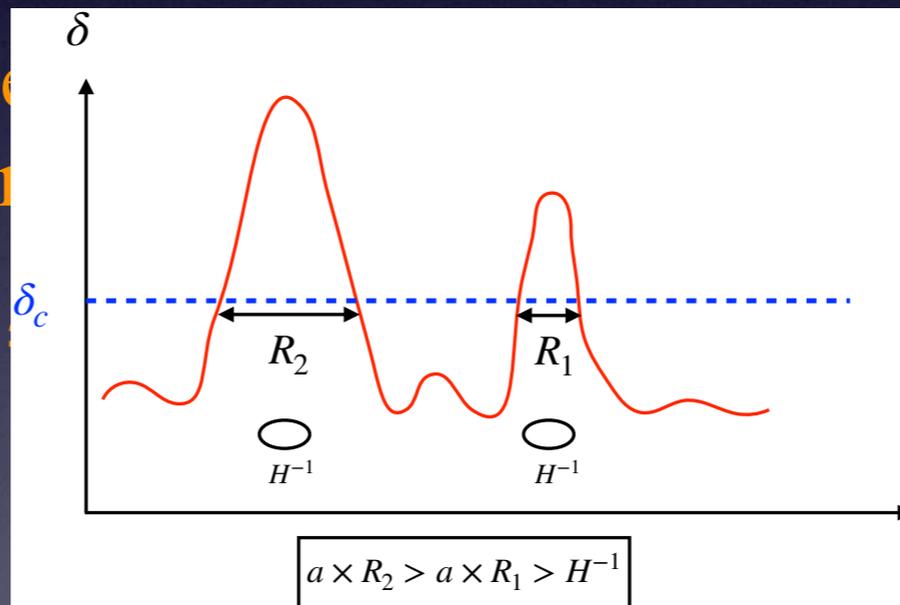
$$\beta = \frac{\rho_{\text{BH}}}{\rho_{\text{tot}}} \sim \int_{\delta_c}^{\infty} P(\delta) d\delta$$

PBH in a nutshell



A Black hole can be formed
where there is an overdensity

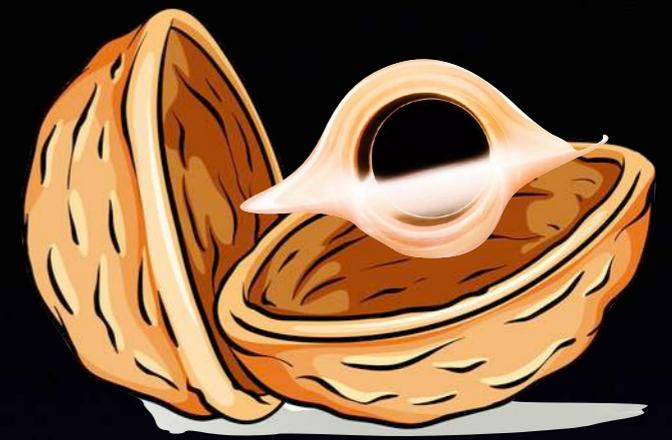
$$\frac{\delta\rho}{\rho} \gtrsim \delta_c, \text{ with } \delta_c \approx 0.3$$



The fraction

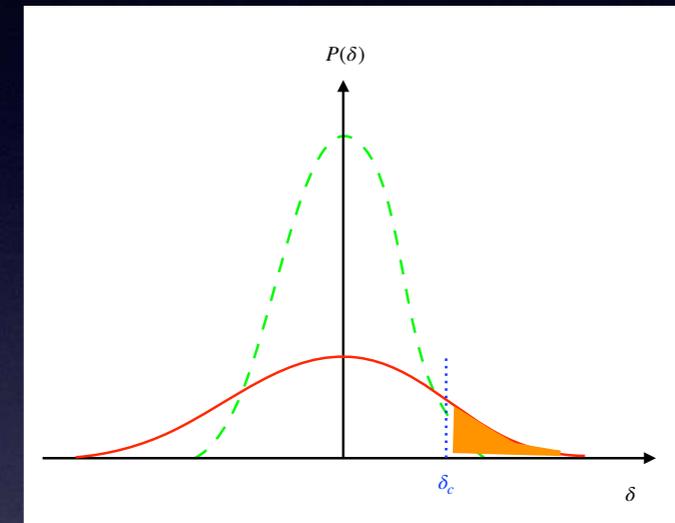
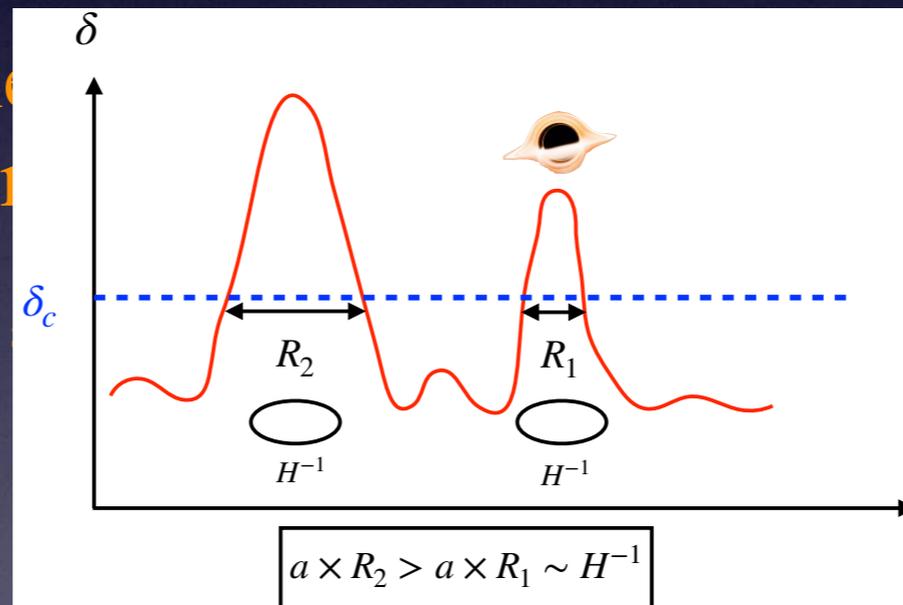
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PBH in a nutshell



A Black hole can be formed
where there is an over

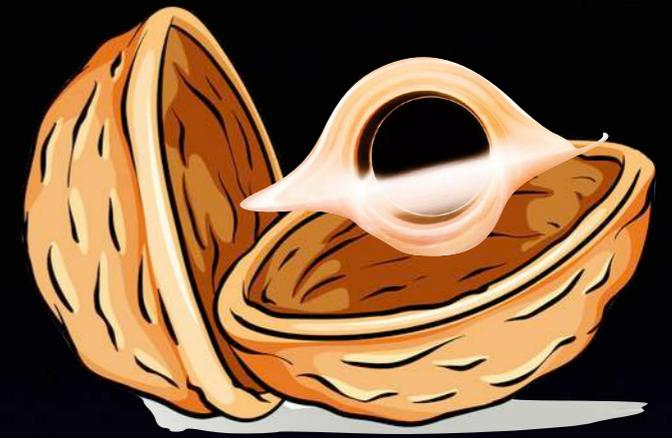
$$\frac{\delta\rho}{\rho} \gtrsim \delta_c, \text{ with } \delta_c$$



The fraction

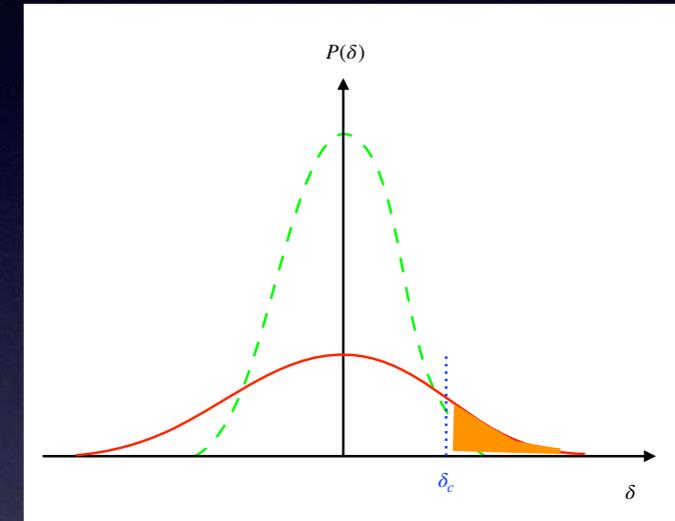
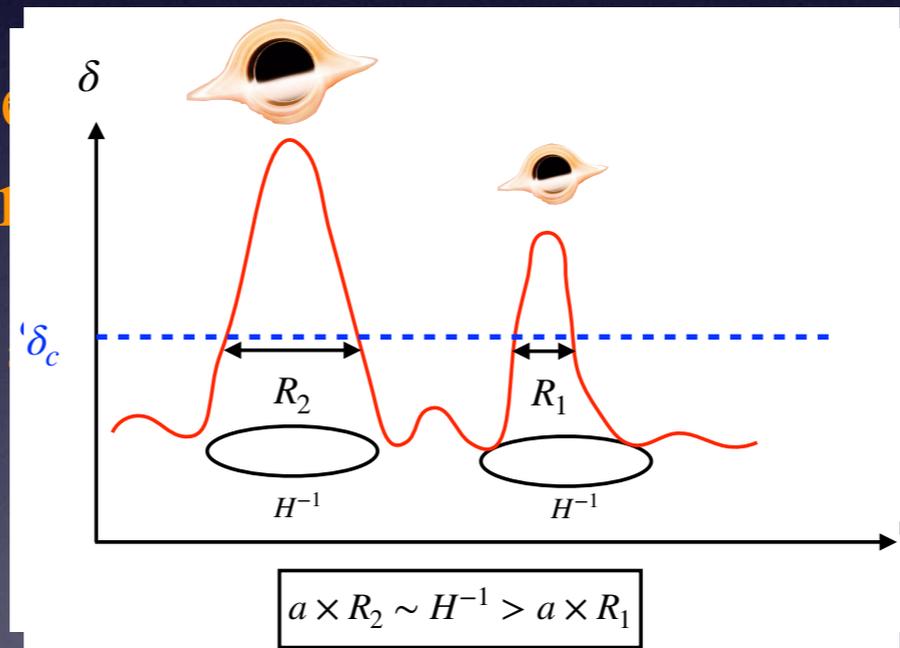
$$\beta = \frac{\rho_{\text{BH}}}{\rho_{\text{tot}}} \sim \int_{\delta_c}^{\infty} P(\delta) d\delta$$

PBH in a nutshell



A Black hole can be formed
where there is an over

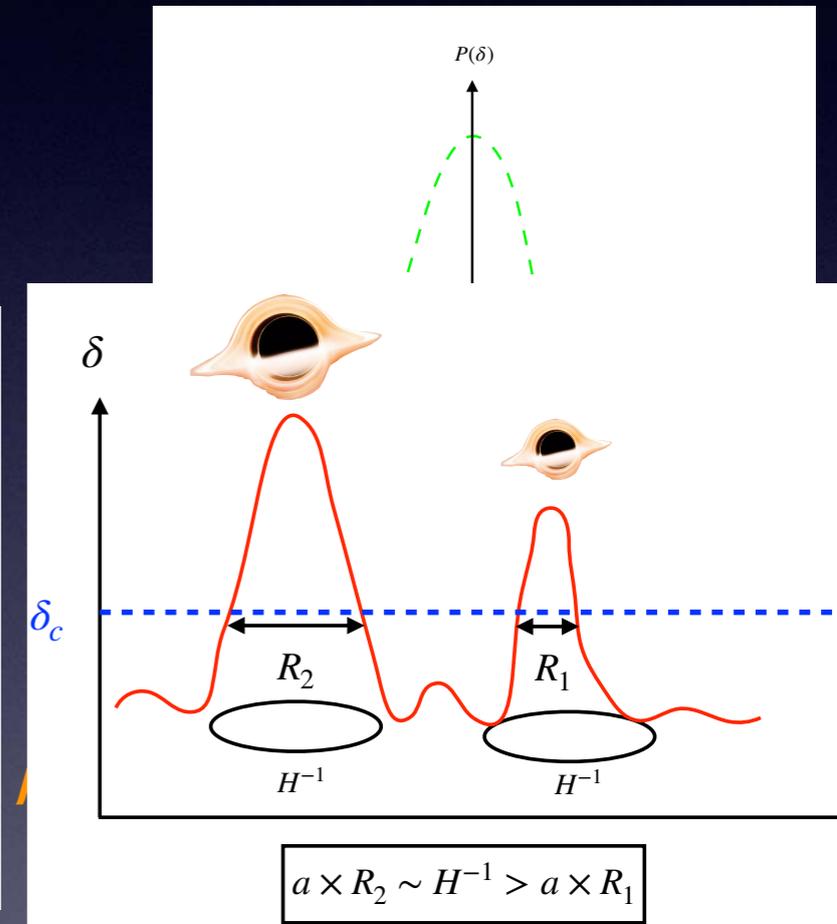
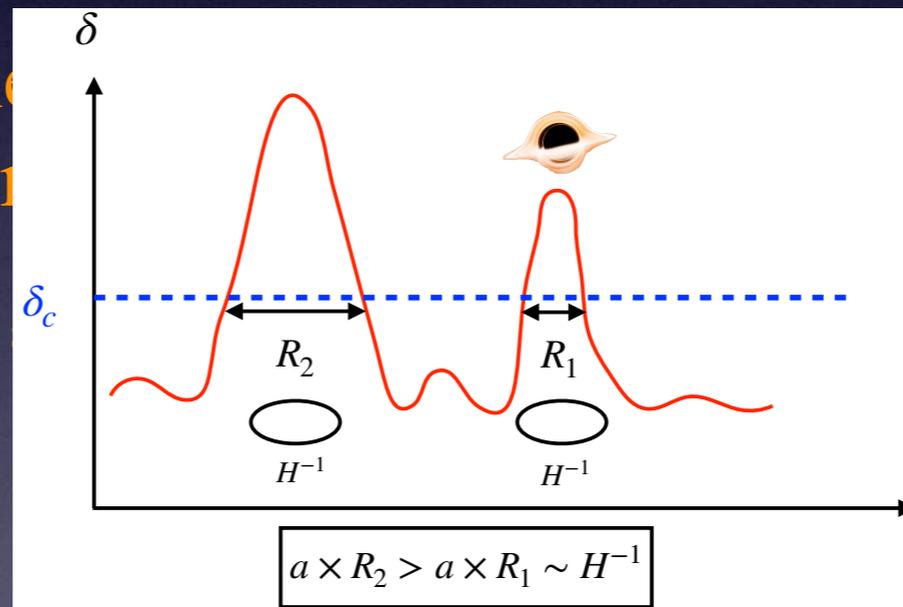
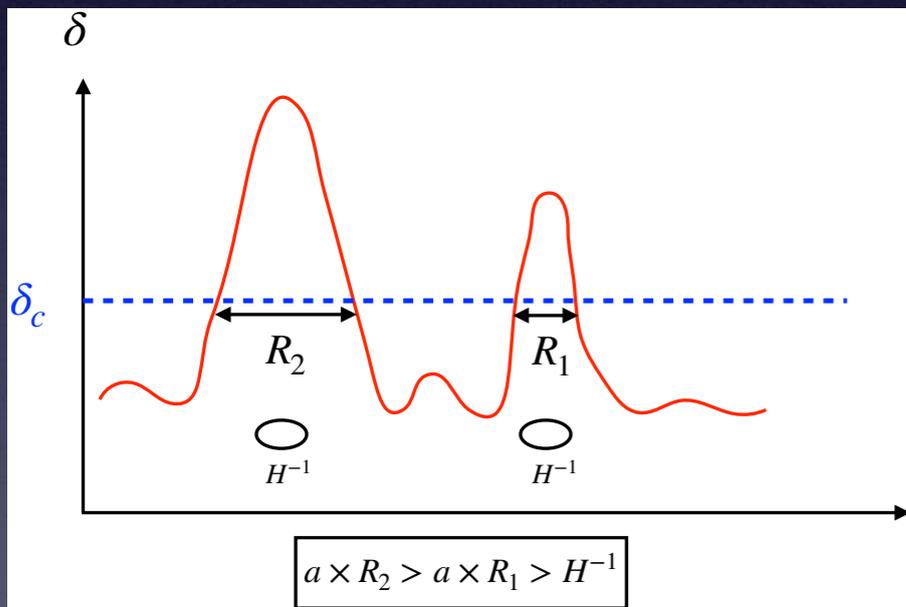
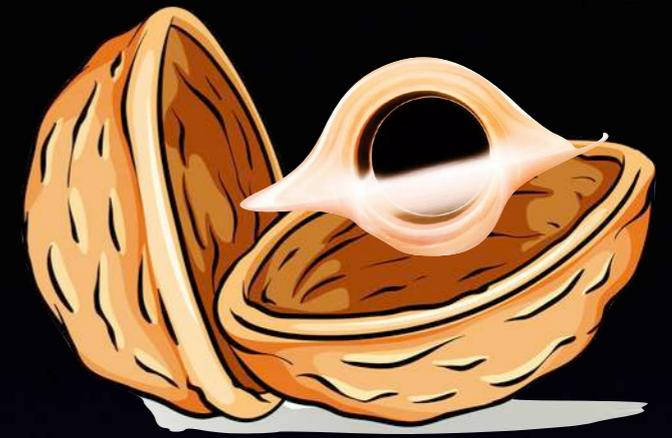
$$\frac{\delta\rho}{\rho} \gtrsim \delta_c, \text{ with } \delta_c \ll 1$$



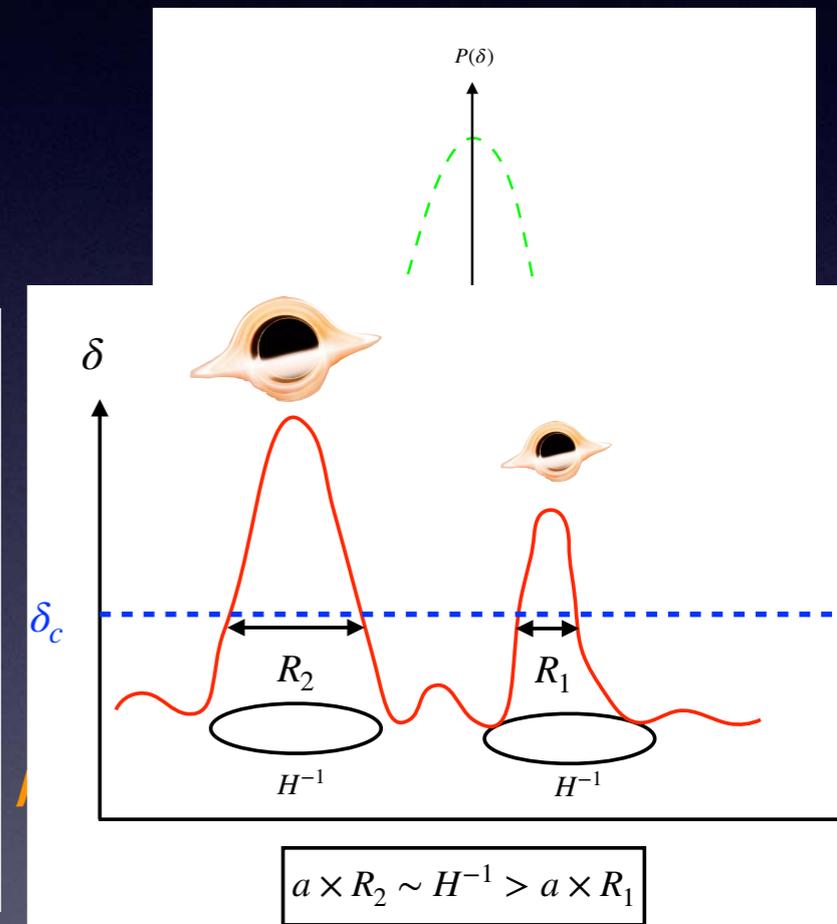
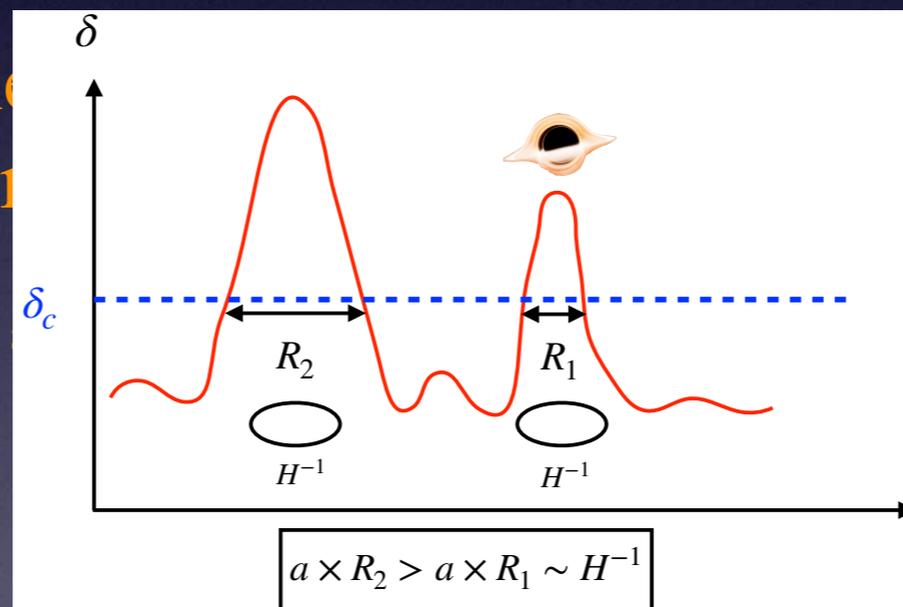
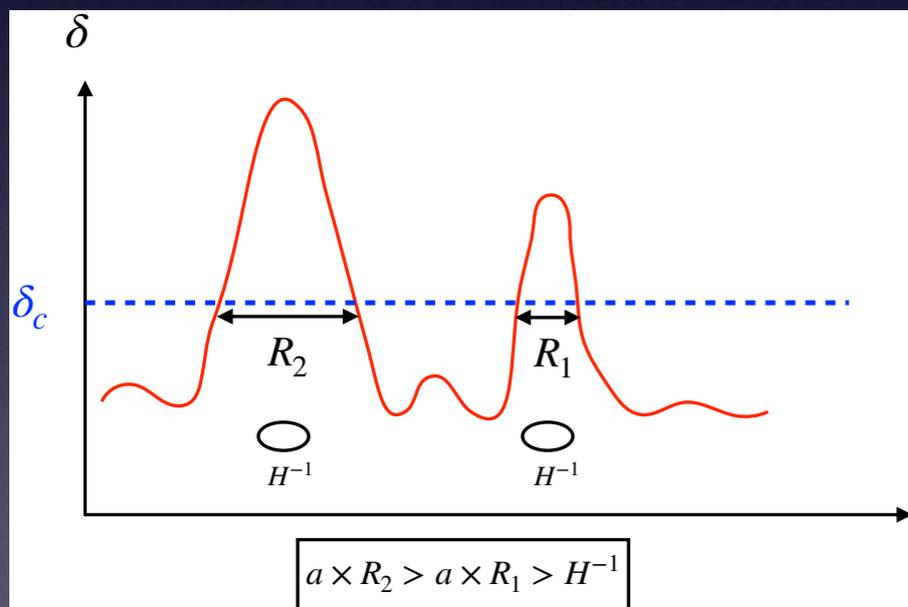
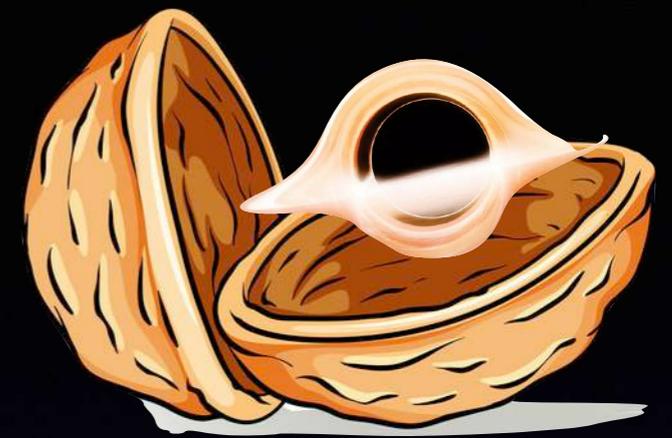
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PBH in a nutshell



PBH in a nutshell



The mass in the horizon is : $M_{\text{BH}} \sim \frac{4\pi}{3} (cH^{-1})^3 \rho = 8\pi t M_p^2$

$$M_{\text{BH}} \sim 3.6 \times 1 \text{ g} \left(\frac{t}{10^{-38} \text{ s}} \right) = 10^{16} \text{ g} \left(\frac{t}{10^{-23} \text{ s}} \right) = M_{\odot} \left(\frac{t}{10^{-6} \text{ s}} \right)$$

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! PBHs are unstable : Hawking radiation !

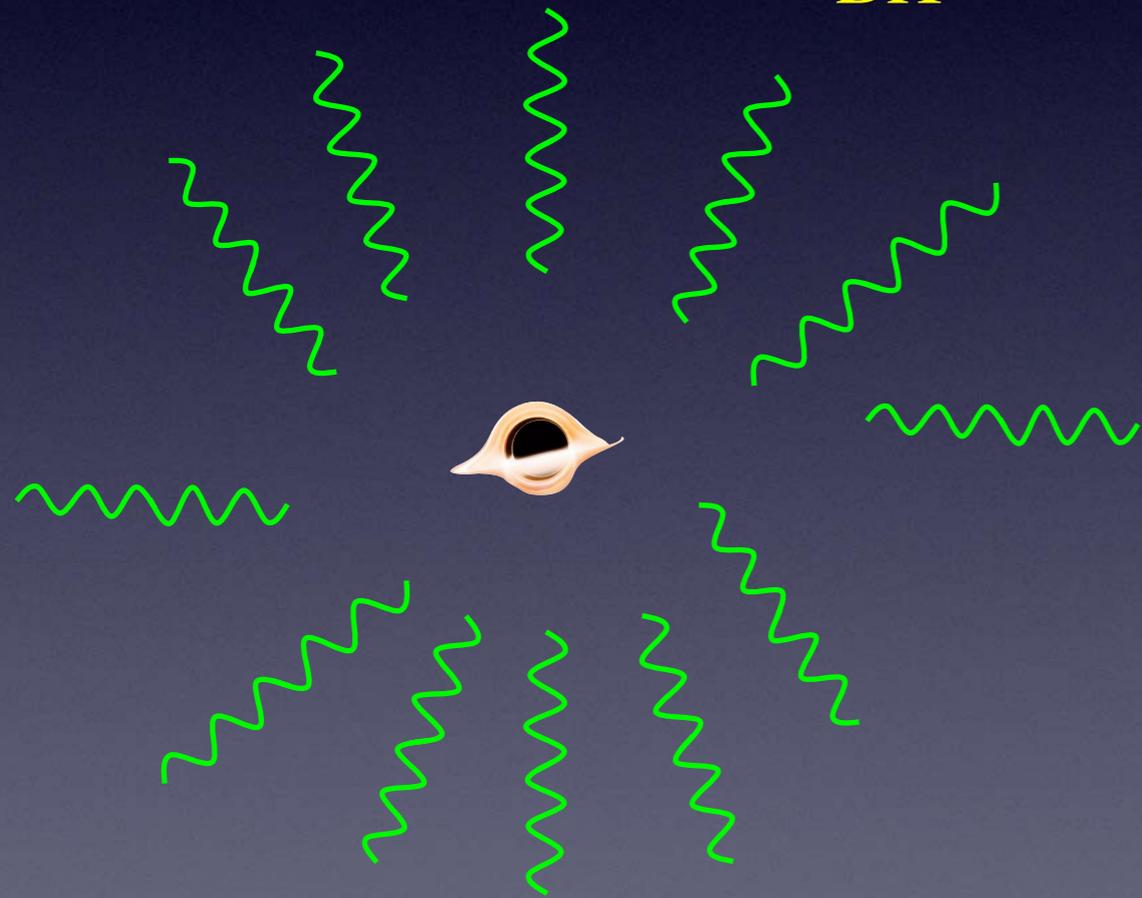
$$T_{\text{BH}} = \frac{M_{\text{P}}^2}{M_{\text{BH}}} \simeq 10^{13} \text{ GeV} \left(\frac{1 \text{ g}}{M_{\text{BH}}} \right)$$



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⚠ PBHs are unstable : Hawking radiation ⚠

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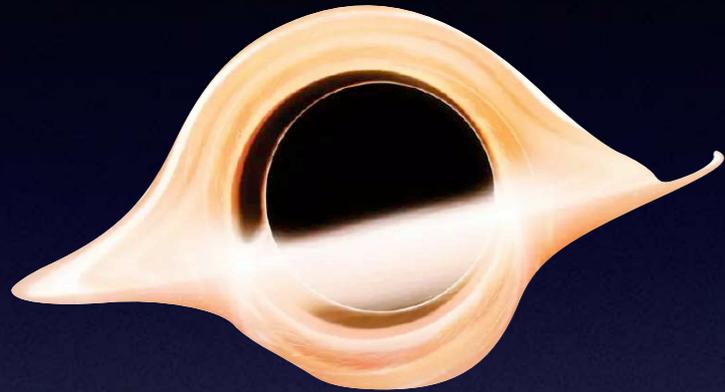


$$\frac{dM_{\text{BH}}}{dt} = - \frac{g_* \pi}{480} \frac{M_P^4}{M_{\text{BH}}^2}$$

Black Hole lifetime : $\tau_{\text{BH}} = \frac{M_{\text{BH}}^3}{M_P^4} \simeq 10^{17} \text{ s} \left(\frac{M_{\text{BH}}}{10^{15} \text{ g}} \right)^3$

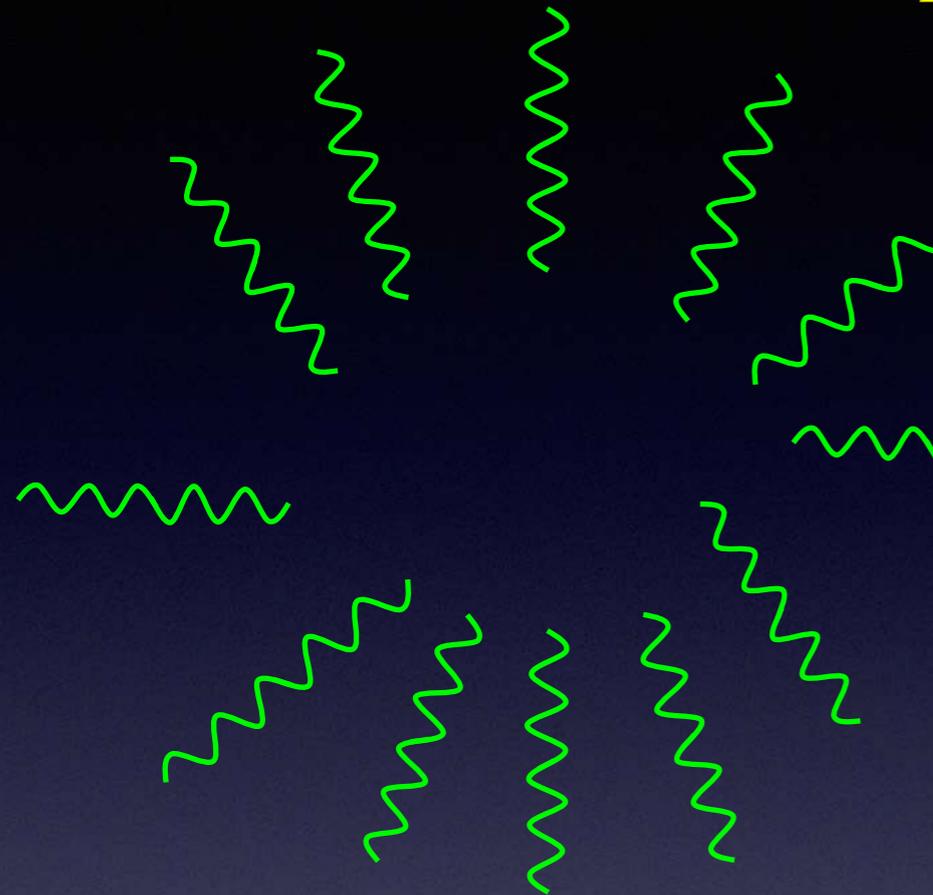
GW generated by PBH

1) from direct decay



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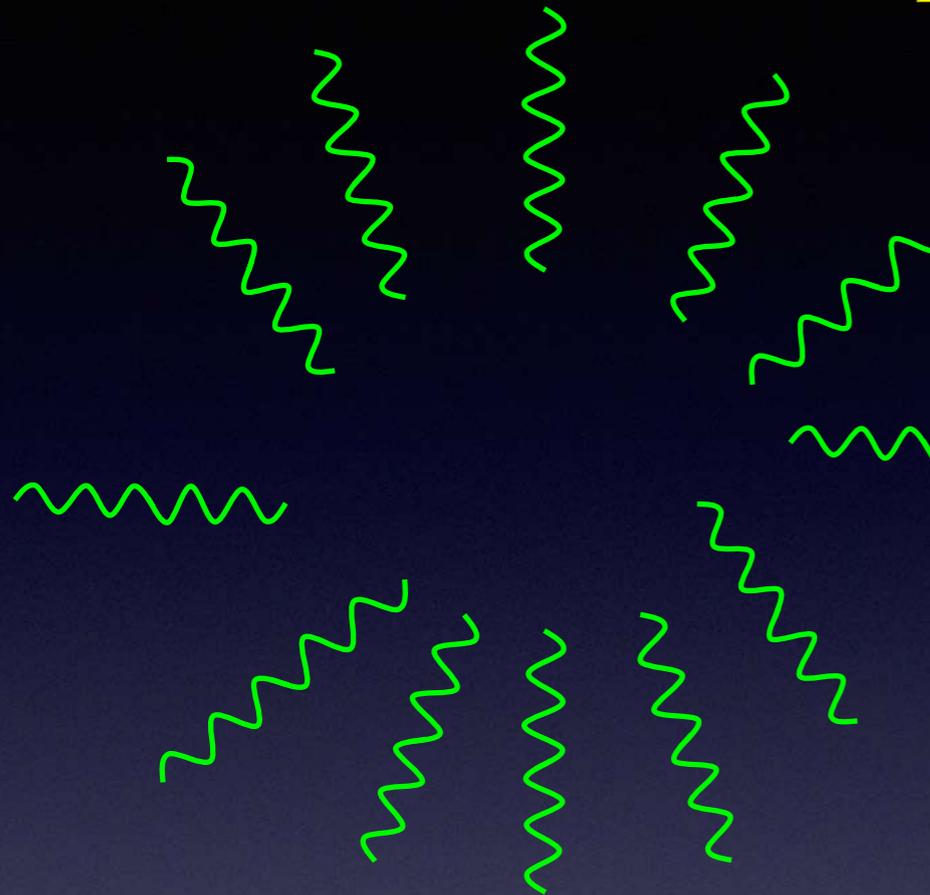


A diagram illustrating the direct decay of a Primordial Black Hole (PBH). It shows several green wavy lines representing gravitational waves radiating outwards from a central point, symbolizing the decay process.

$$f_{peak}^{PBH} \sim T_{BH} \frac{a_{RH}}{a_0} \sim T_{BH} \frac{T_0}{T_{RH}} \sim \frac{T_{BH} T_0}{\sqrt{\Gamma_{BH} M_P}}$$
$$\sim \sqrt{M_{BH}} \simeq 3 \times 10^{13} \sqrt{\frac{M_{BH}}{1 \text{ g}}} \text{ Hz}$$

GW generated by PBH

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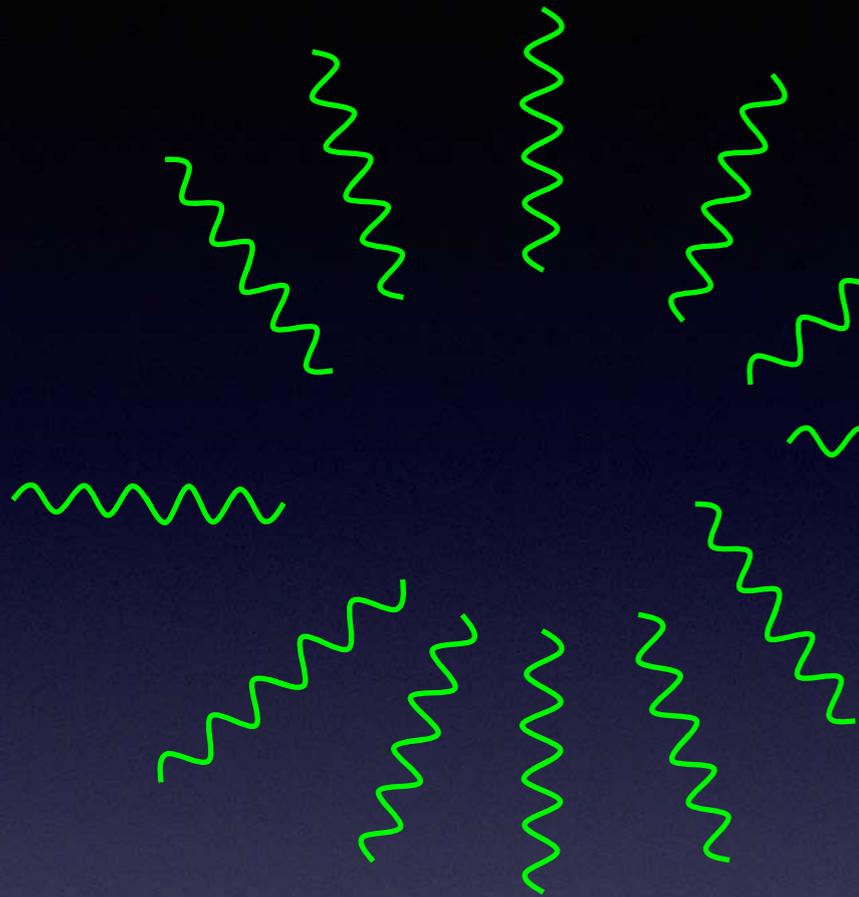

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During the whole decay
process, $T_{BH} = \frac{M_P^2}{M_{BH}} \sim cst$

The spectrum is thus a
~ **thermal spectrum**,
redshifted by the expansion.

GW generated by PBH

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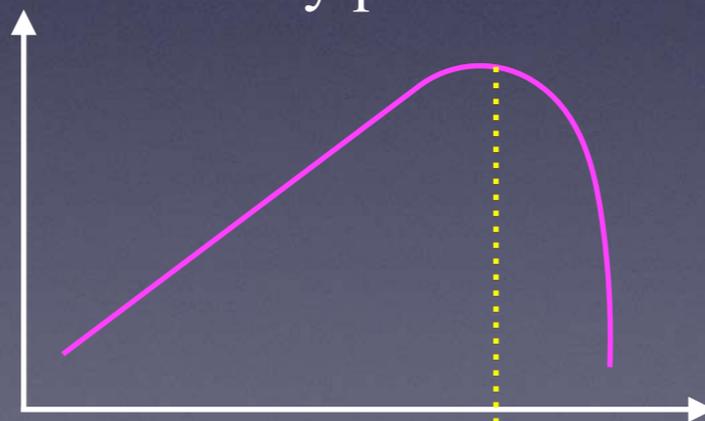
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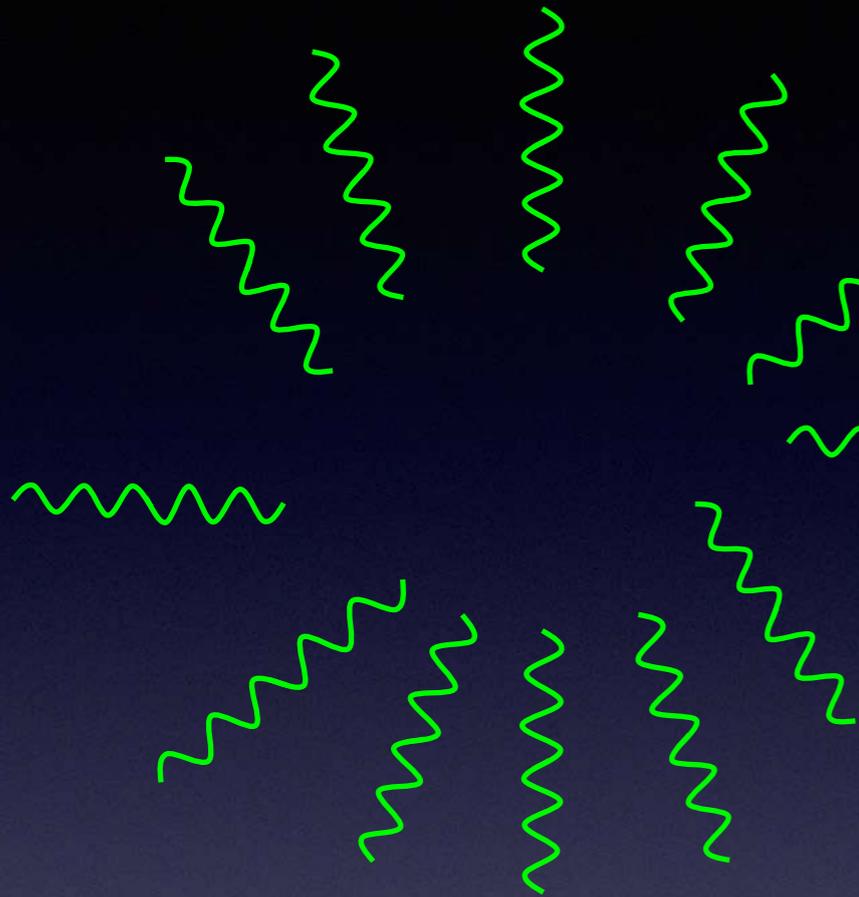
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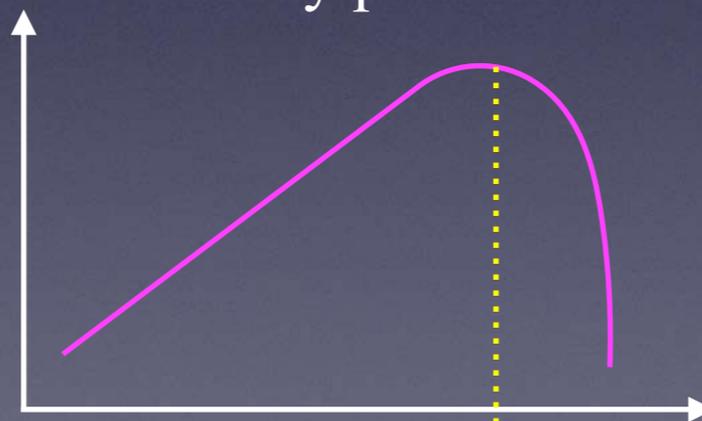
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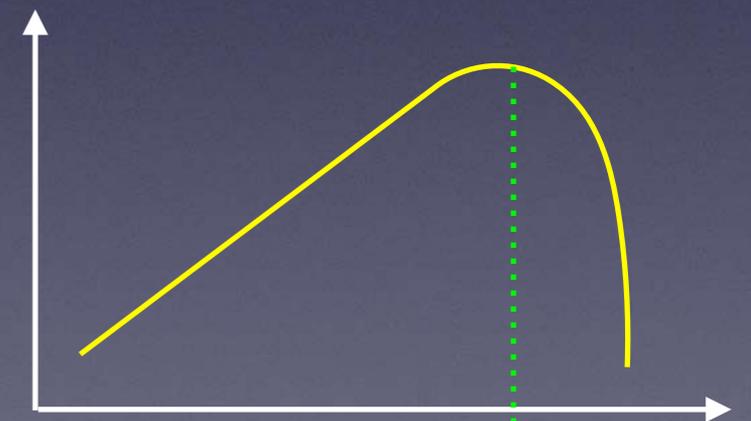
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Nowadays

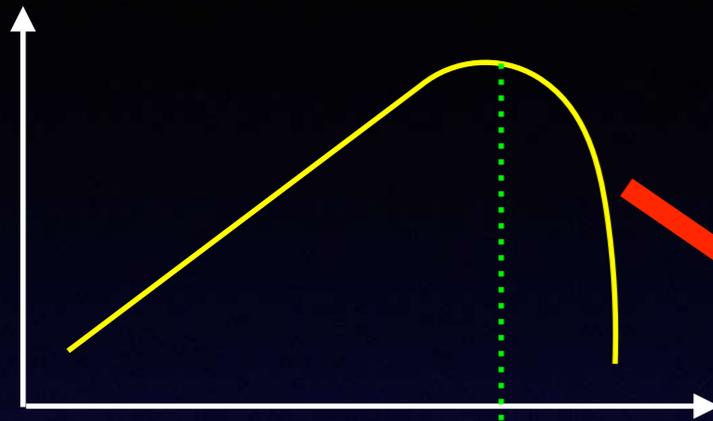


$$f_0^{BH} \sim 10^{13} \sqrt{\frac{M_{BH}}{1 \text{ g}}} \text{ Hz}$$

GW generated by PBH

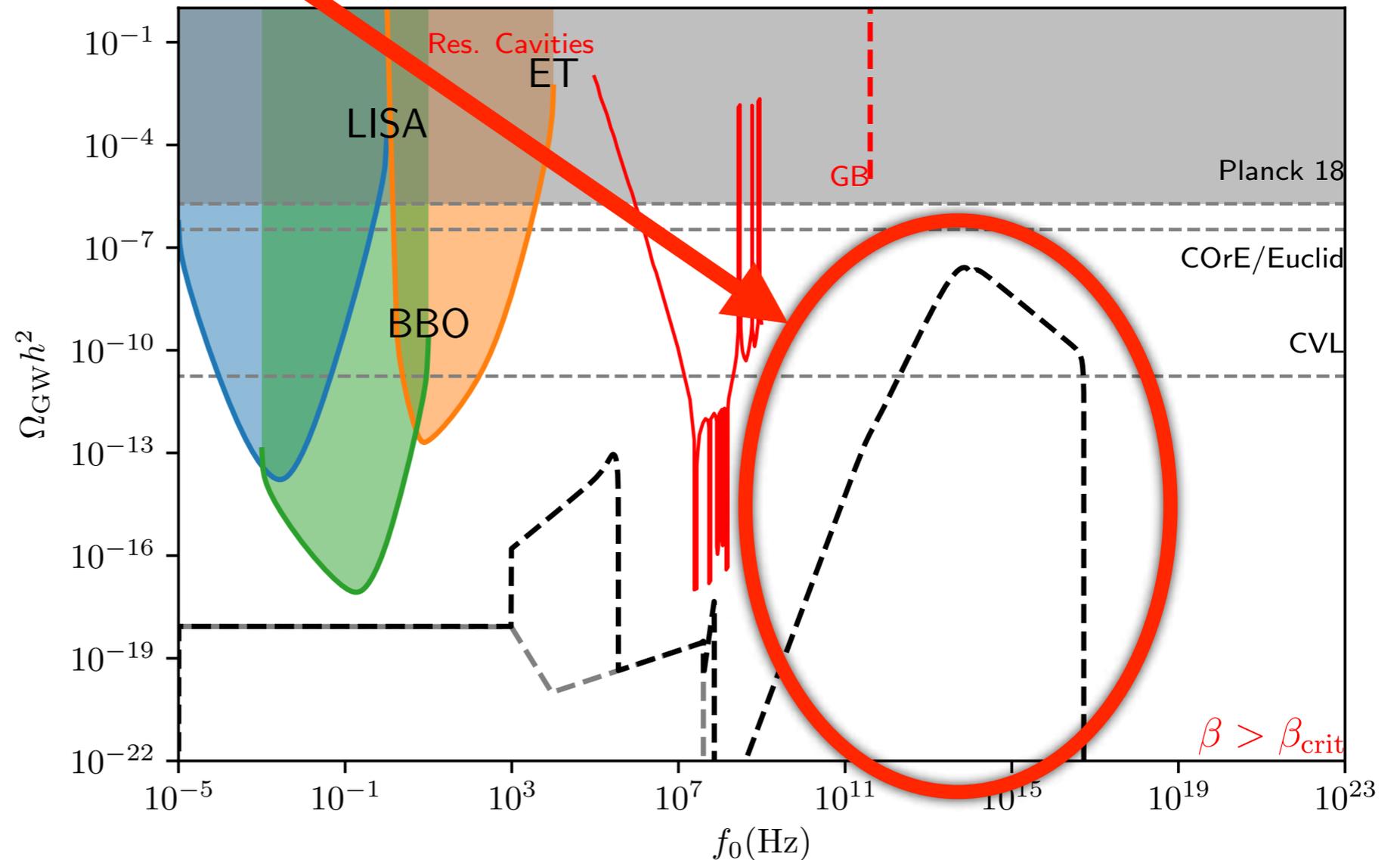
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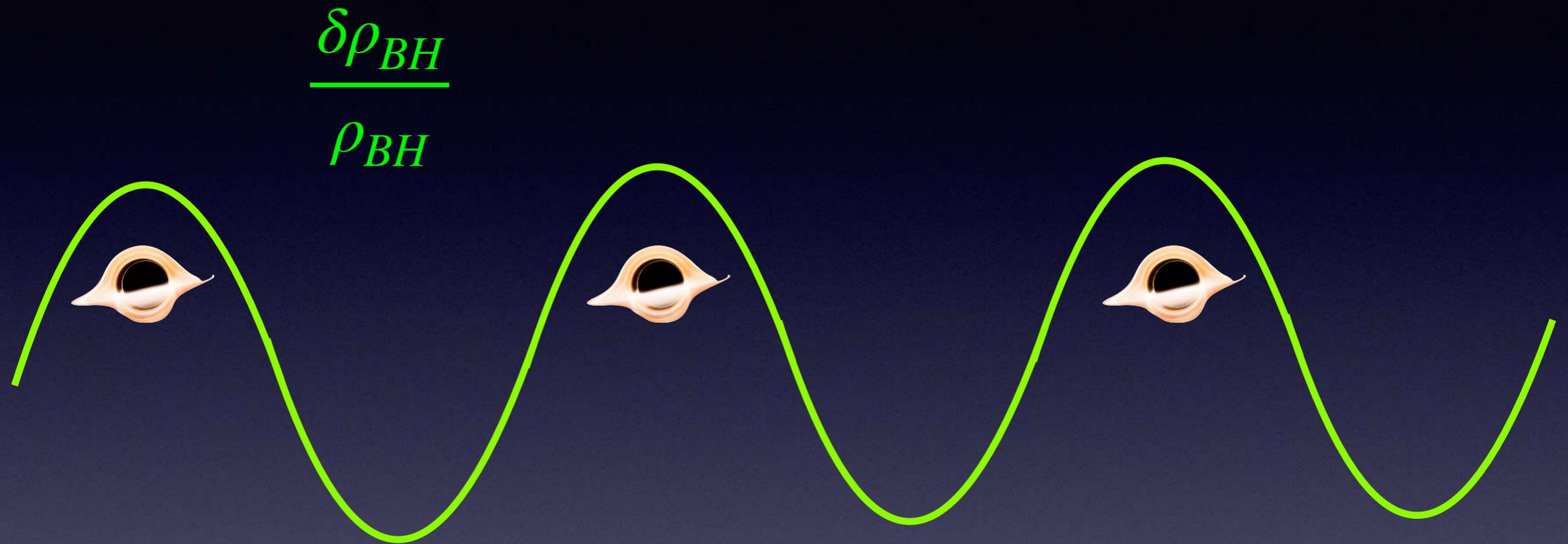
Taking into account grey-body factor, finite lifetime, exact evolution of $M_{BH}(t)$, using *BlackHawk* software

$$f_0^{BH} \sim 10^{13} \sqrt{\frac{M_{BH}}{1 \text{ g}}} \text{ Hz}$$



GW generated by PBH

2) *from density fluctuations*



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$$\frac{\delta\rho_{BH}}{\rho_{BH}}$$

$$\rho_{BH}$$



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GW generated by PBH



2 steps :

1) Poisson equation transfers *density fluctuations* $\frac{\delta\rho_{BH}}{\rho_{BH}}$ into *potential* (scalar) Φ -fluctuations (isocurvature \rightarrow curvature) :

$$\frac{\delta\rho_{BH}}{\rho_{BH}}$$

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GW generated by PBH



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$$\nabla^2 \Phi = \frac{\delta\rho_{BH}}{2M_p^2}, \quad \delta\rho_{BH} = \rho_{BH} - \bar{\rho}_{BH}$$

$$\Rightarrow \Phi_k = - \frac{\delta_k}{2M_p^2 k^2}$$

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$$ds^2 = (1 - 2\Phi)dt^2 - a^2(1 + 2\Phi + h_{ij}) dx^i dx^j$$

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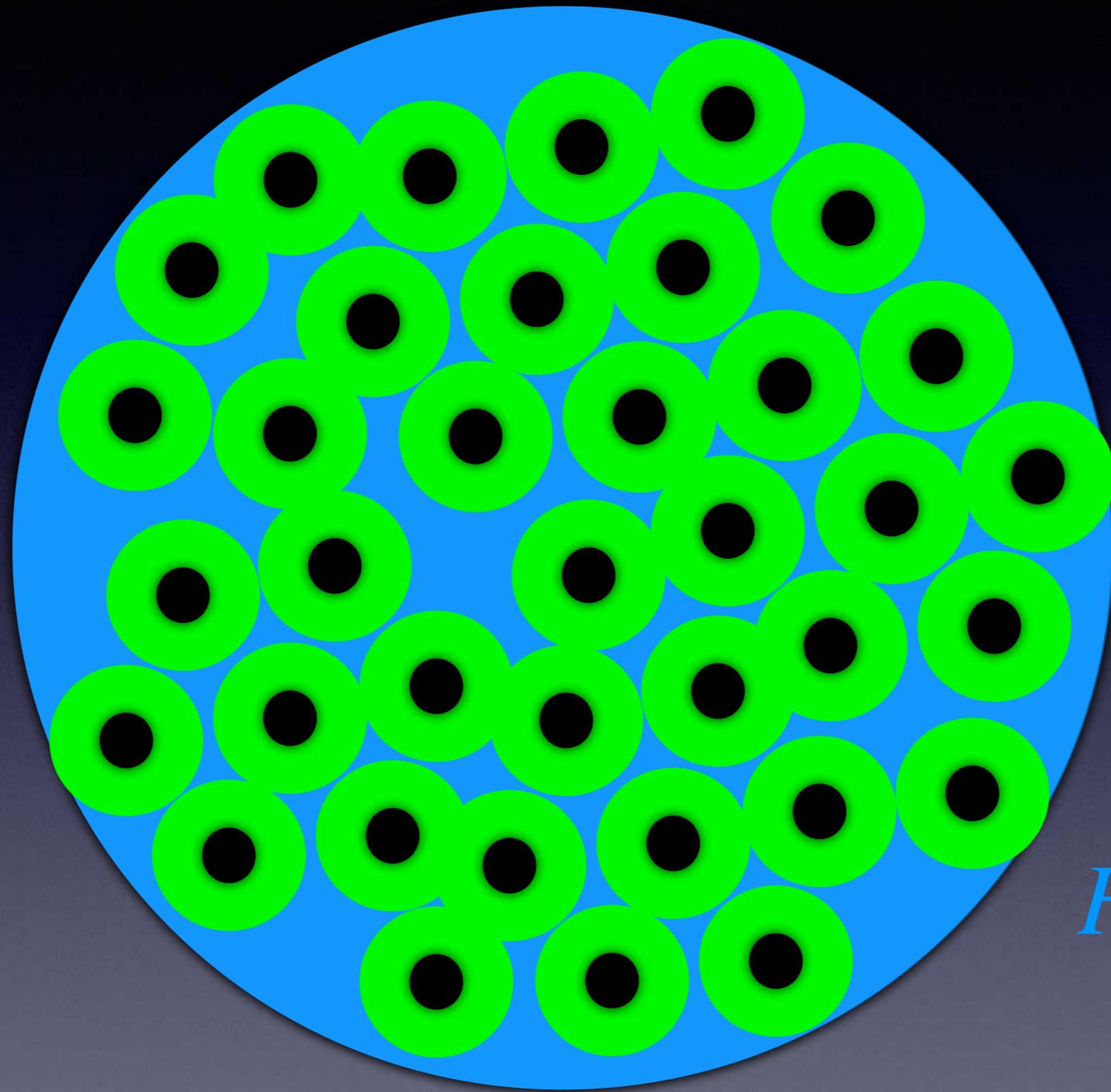
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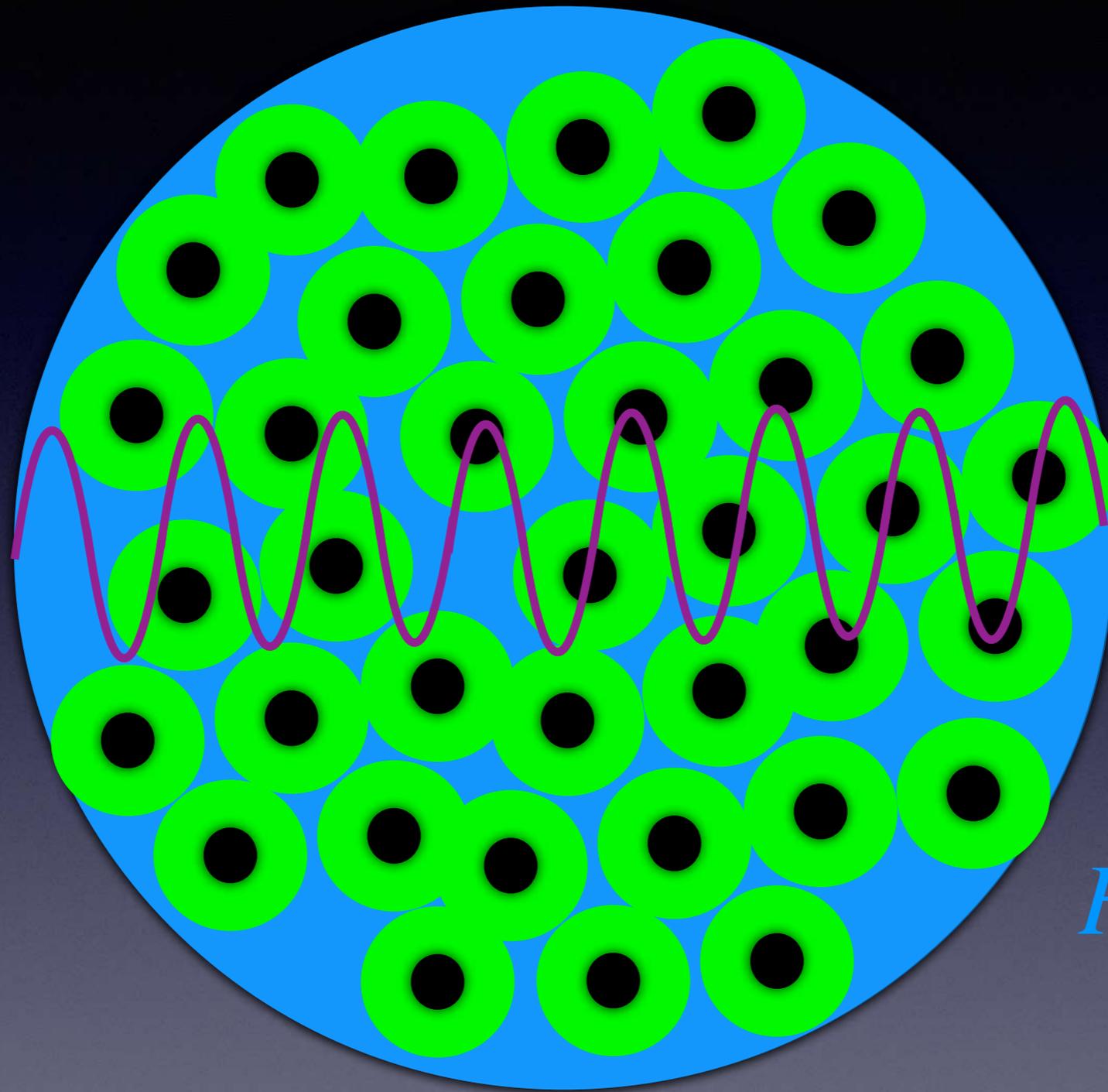
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Computation of transfer functions \mathcal{T}_{ij}



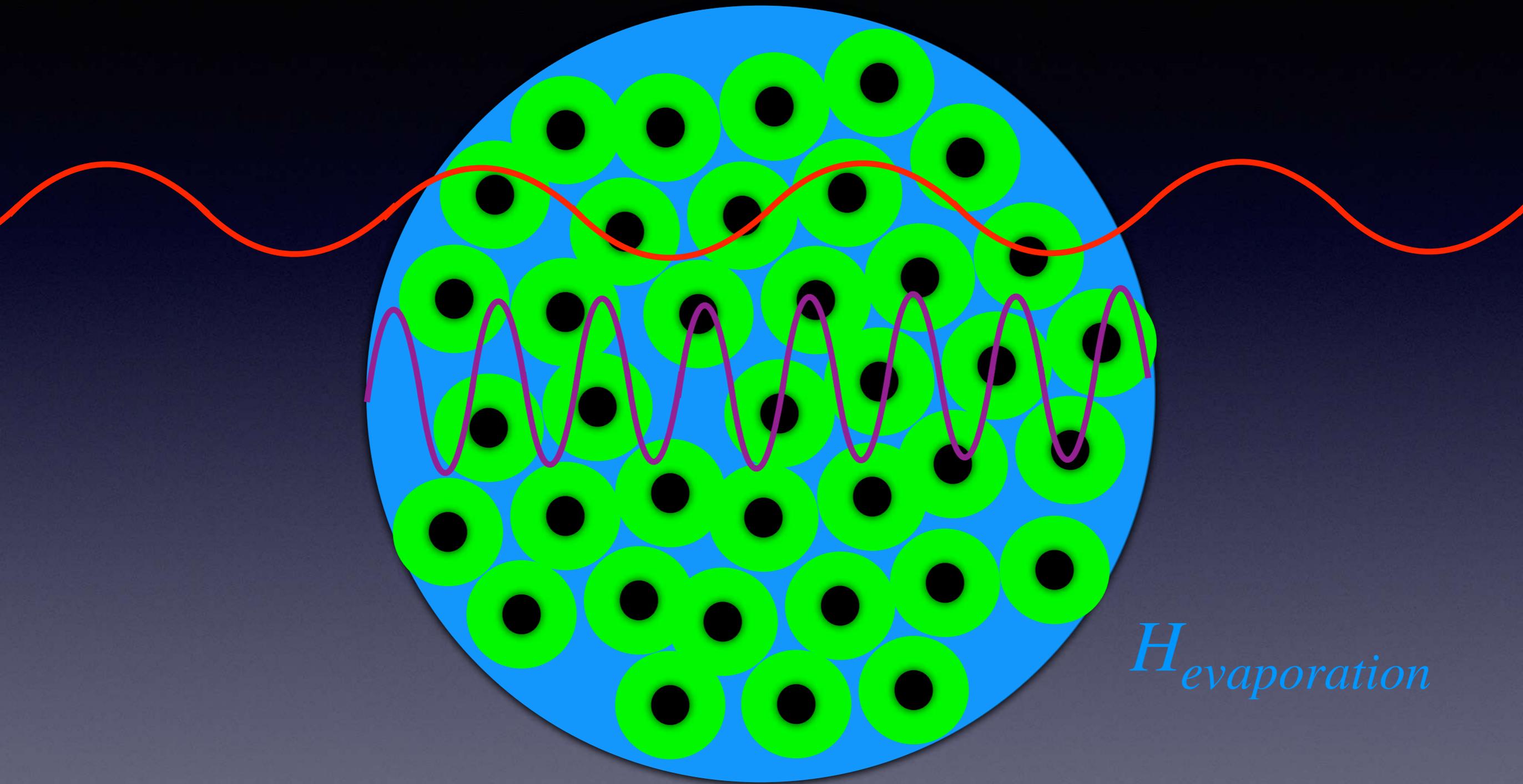
$H_{evaporation}$



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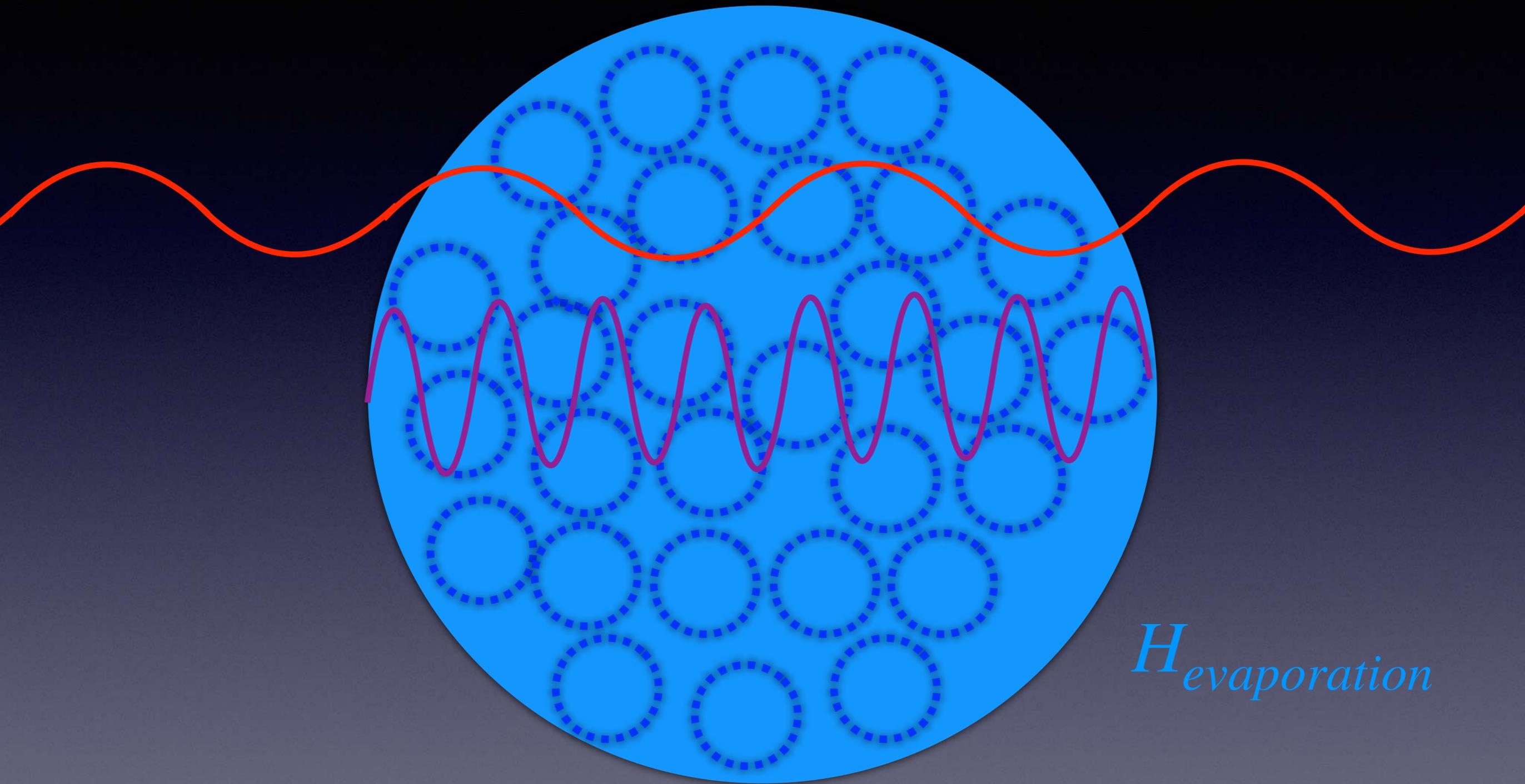
Minimum wavelength (maximum frequency) corresponds to the distance between 2 black holes

Maximum wavelength corresponds to the size of the horizon at evaporation time



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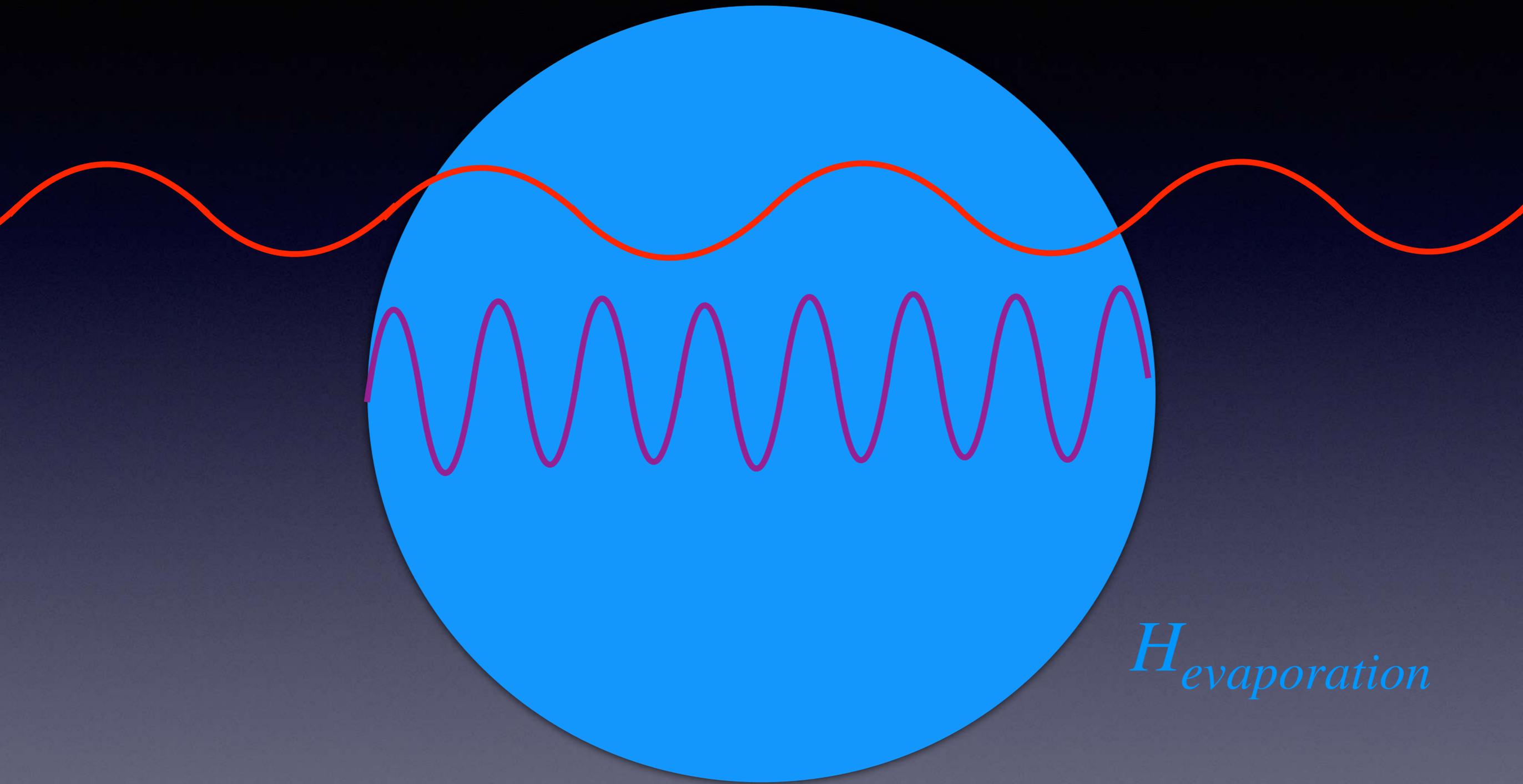
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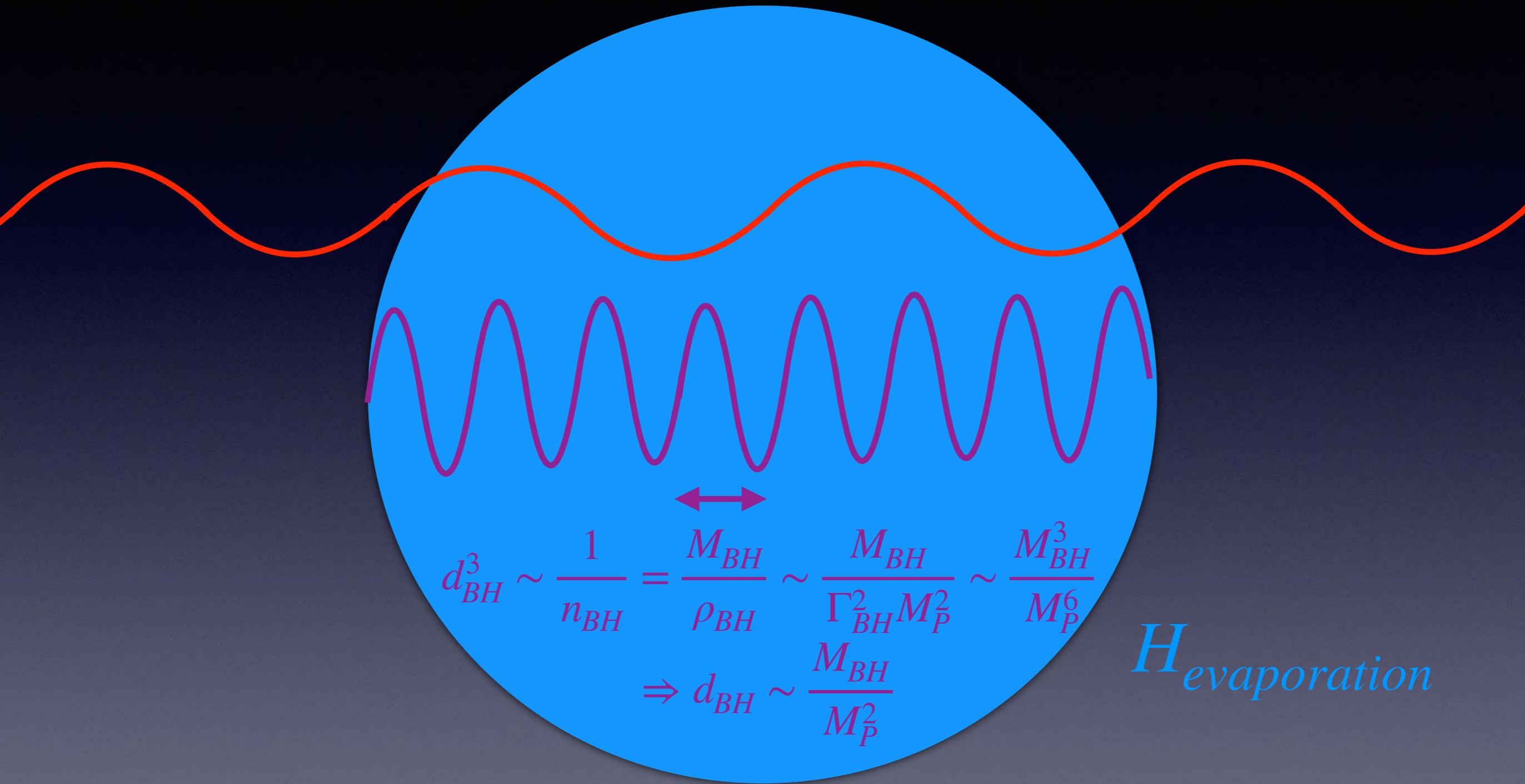
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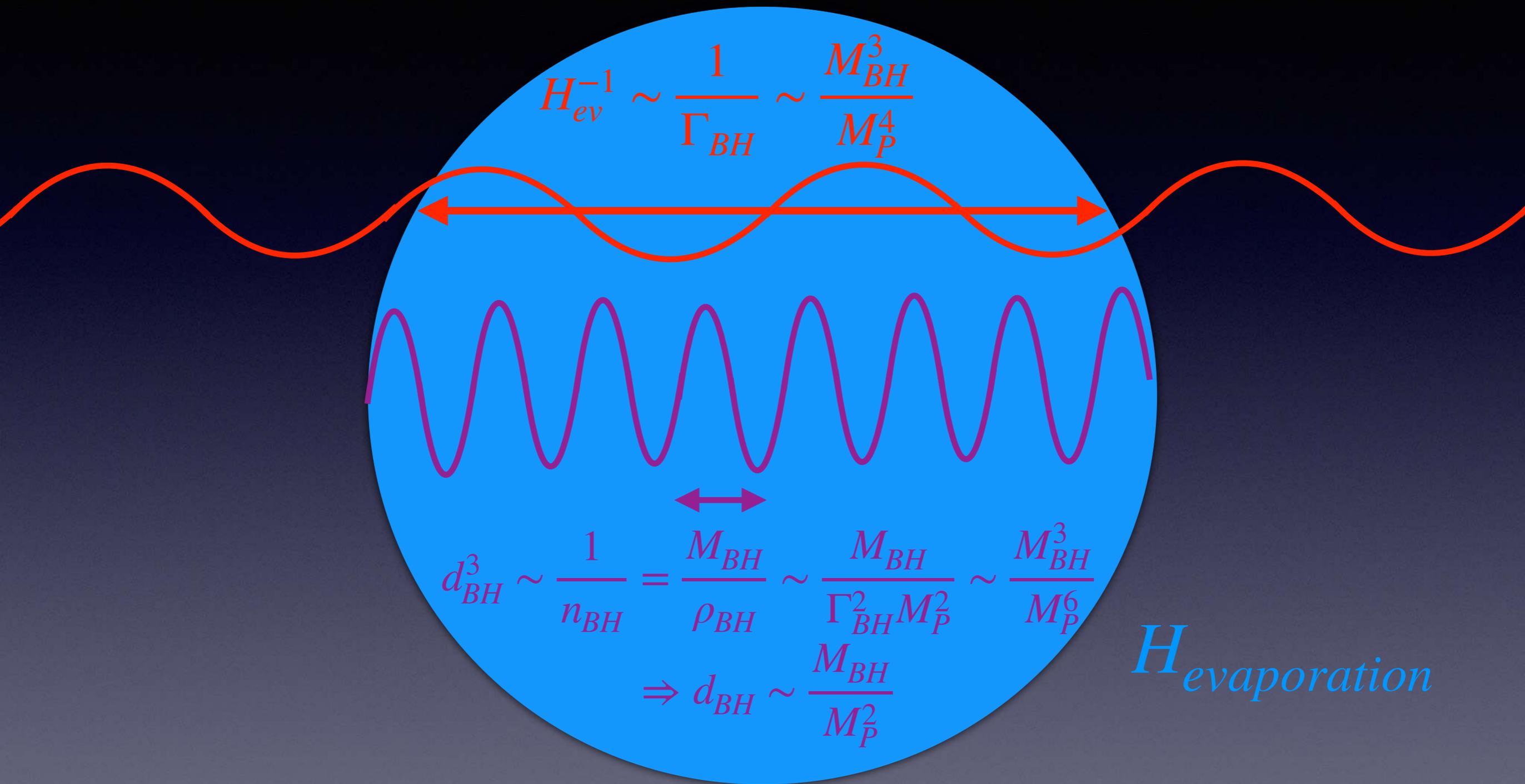
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$$H_{ev}^{-1} \sim \frac{1}{\Gamma_{BH}} \sim \frac{M_{BH}^3}{M_P^4}$$

$$d_{BH}^3 \sim \frac{1}{n_{BH}} = \frac{M_{BH}}{\rho_{BH}} \sim \frac{M_{BH}}{\Gamma_{BH}^2 M_P^2} \sim \frac{M_{BH}^3}{M_P^6}$$

$$\Rightarrow d_{BH} \sim \frac{M_{BH}}{M_P^2}$$

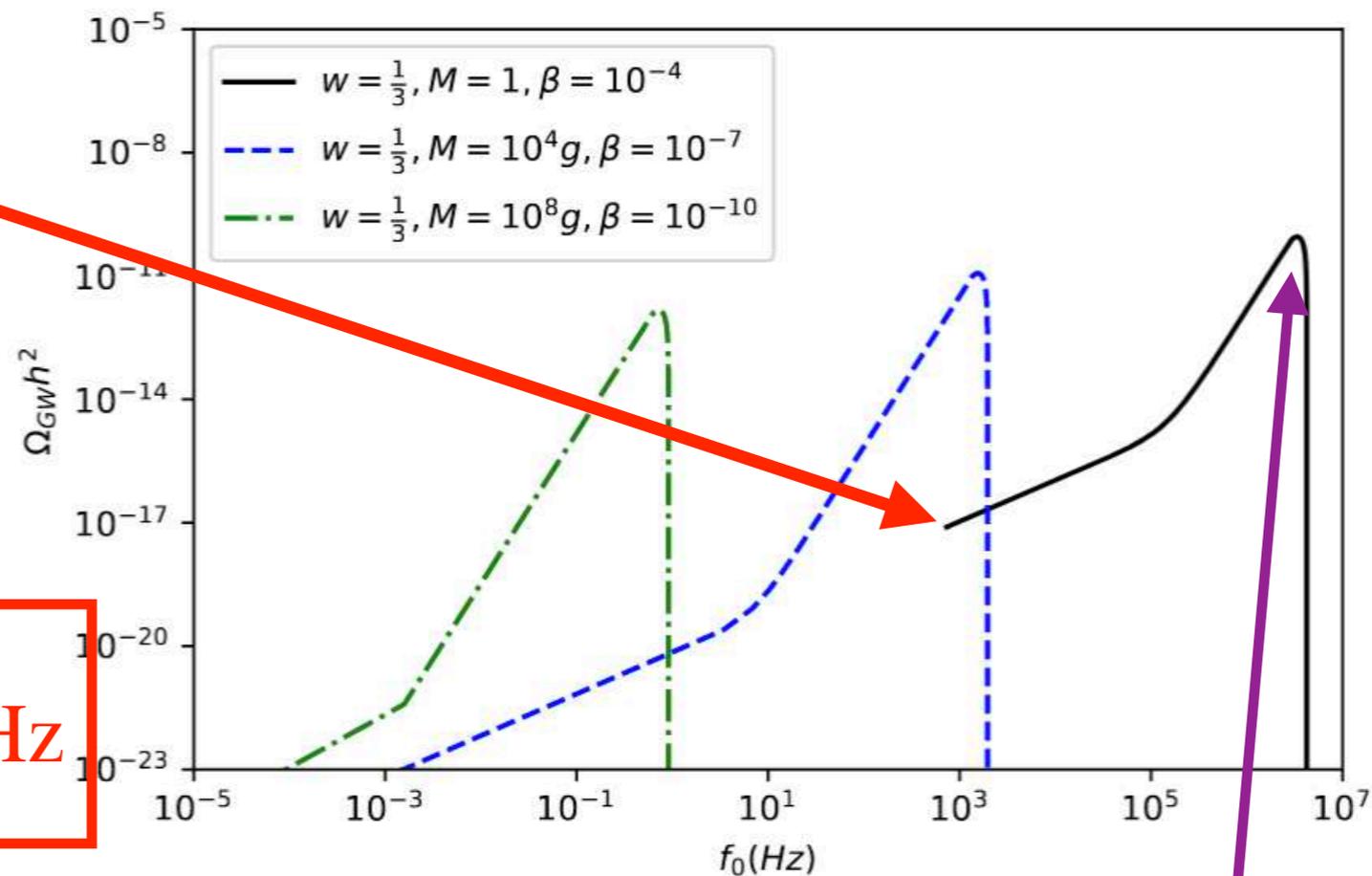
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Horizon at evaporation

$$H_{ev}^{-1} \sim \frac{1}{\Gamma_{BH}} \sim \frac{M_{BH}^3}{M_P^4}$$

$$f_0^{ev} \simeq 1.4 \times 10^3 \left(\frac{1 \text{ g}}{M_{BH}} \right)^{\frac{3}{2}} \text{ Hz}$$

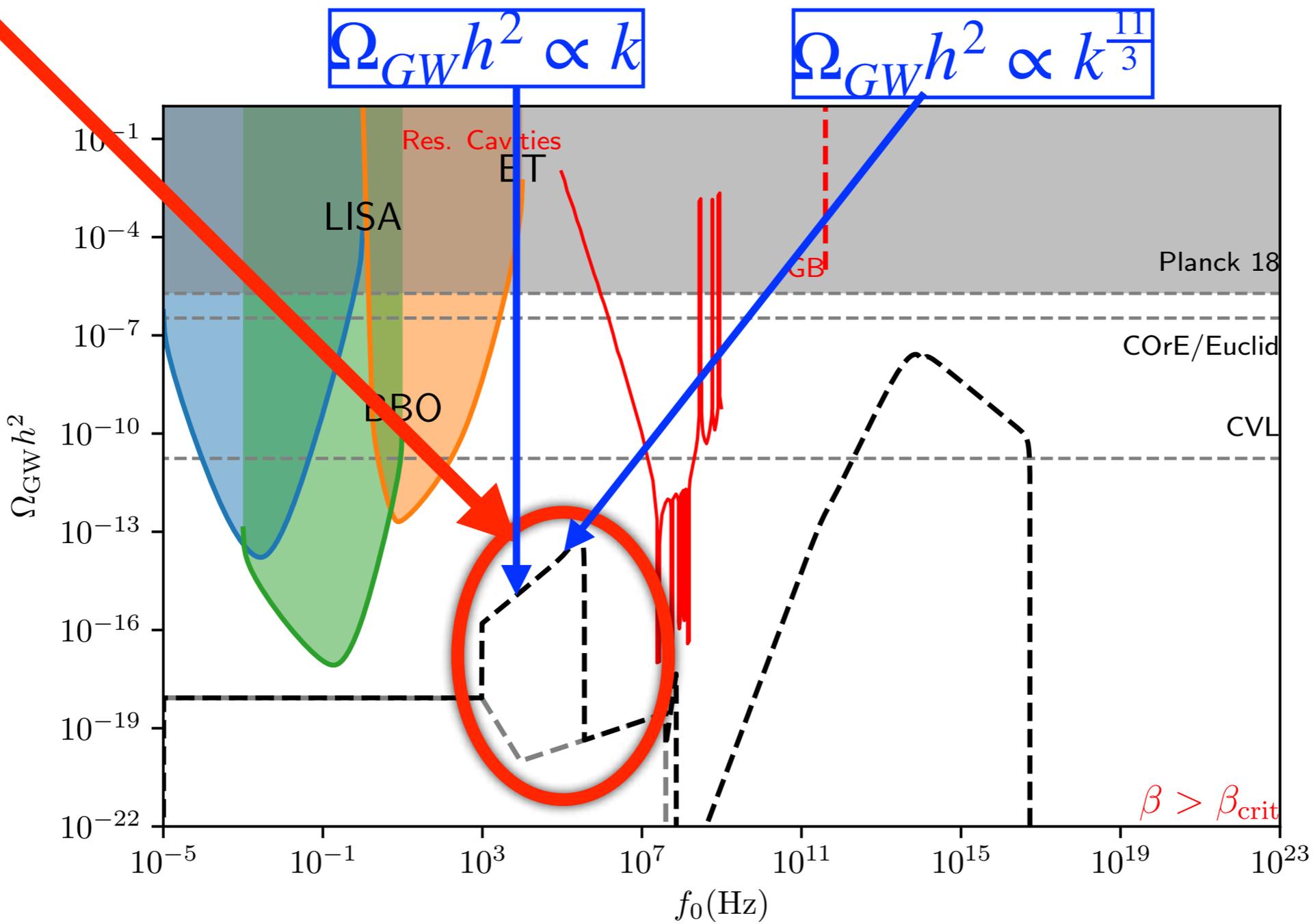
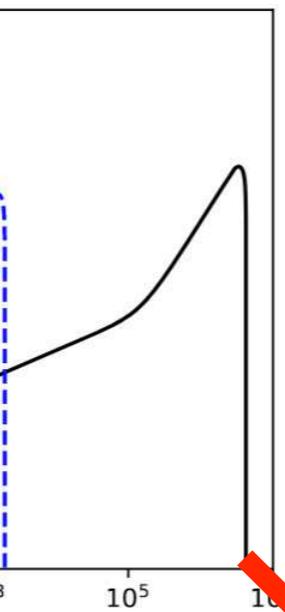


Distance between PBH

$$d_{BH}^3 \sim \frac{1}{n_{BH}} = \frac{M_{BH}}{\rho_{BH}} \sim \frac{M_{BH}}{\Gamma_{BH}^2 M_P^2} \sim \frac{M_{BH}^3}{M_P^6}$$

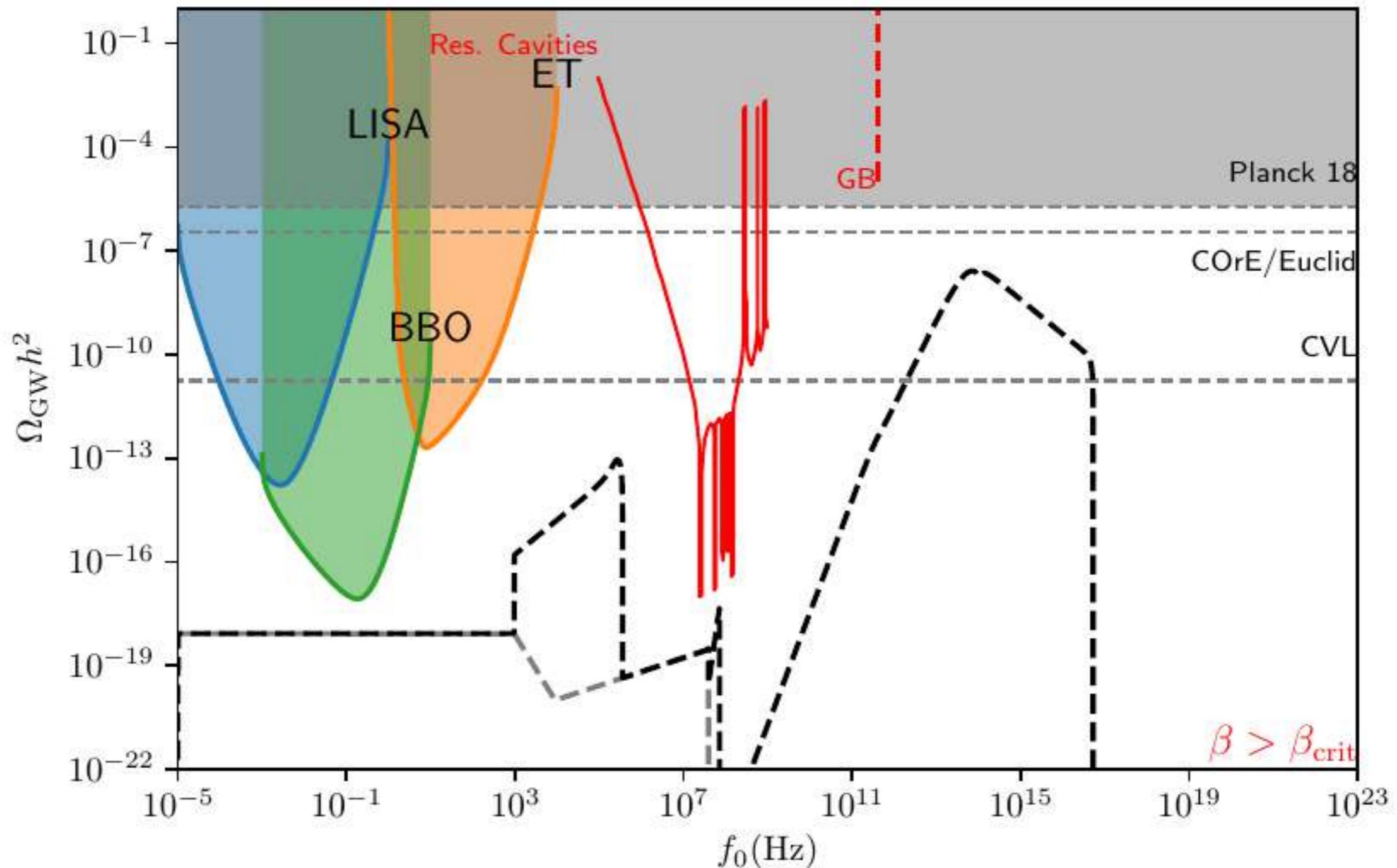
$$\Rightarrow d_{BH} \sim \frac{M_{BH}}{M_P^2}$$

$$f_0^{BH} \simeq 3.7 \times 10^6 \left(\frac{1 \text{ g}}{M_{BH}} \right)^{\frac{5}{6}} \text{ Hz}$$



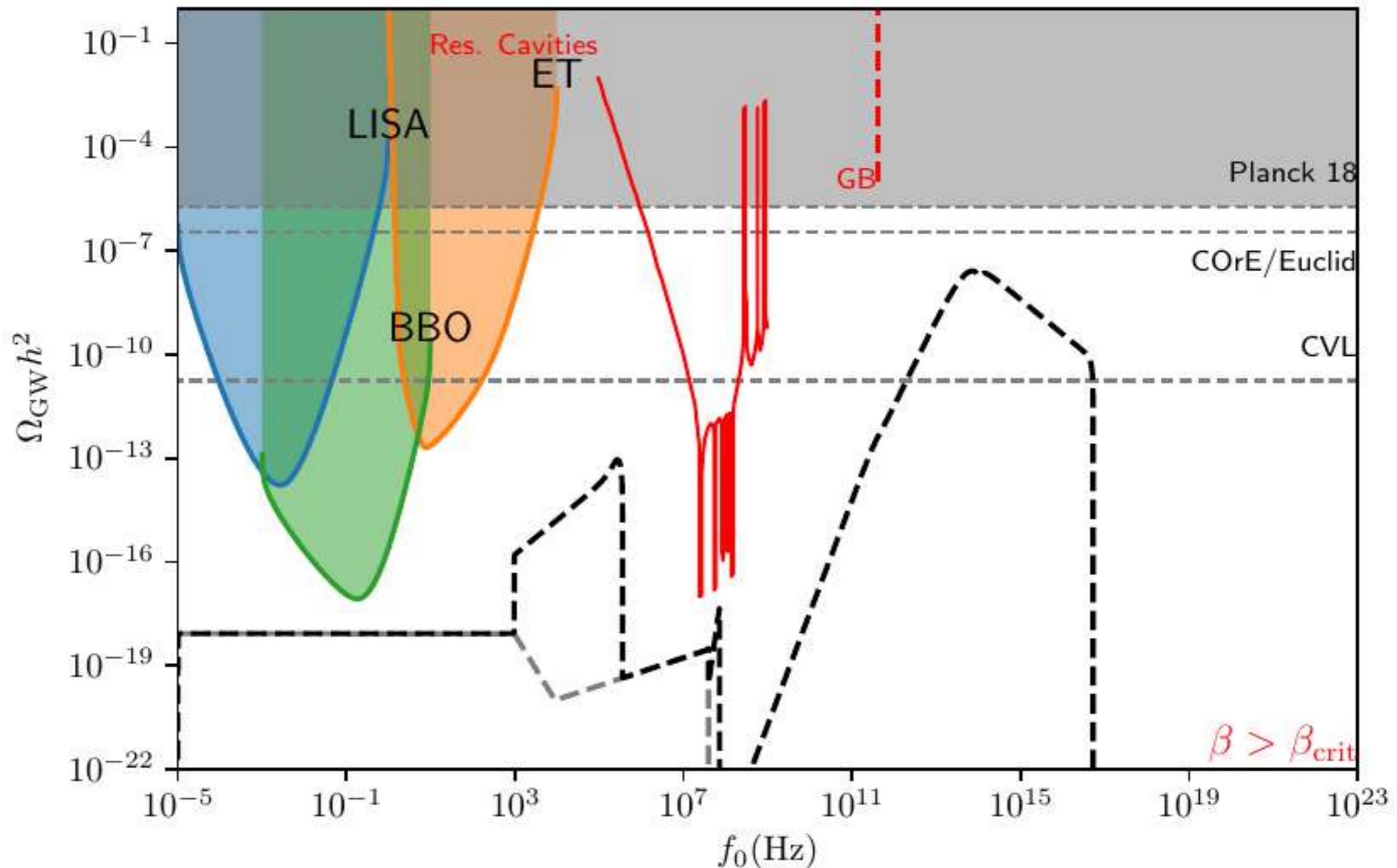
For other PBH masses?

From 1 g to 10^8 g



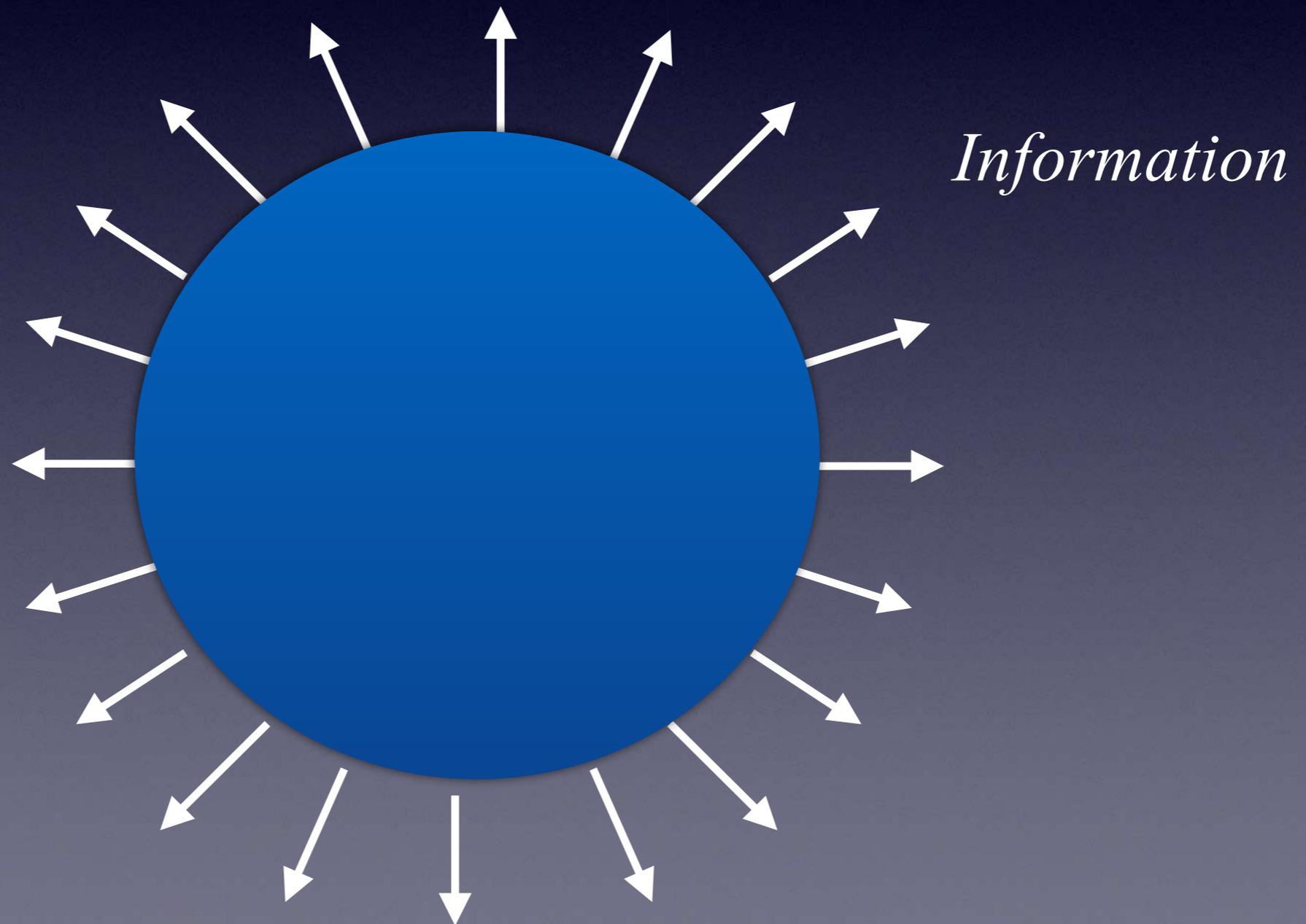
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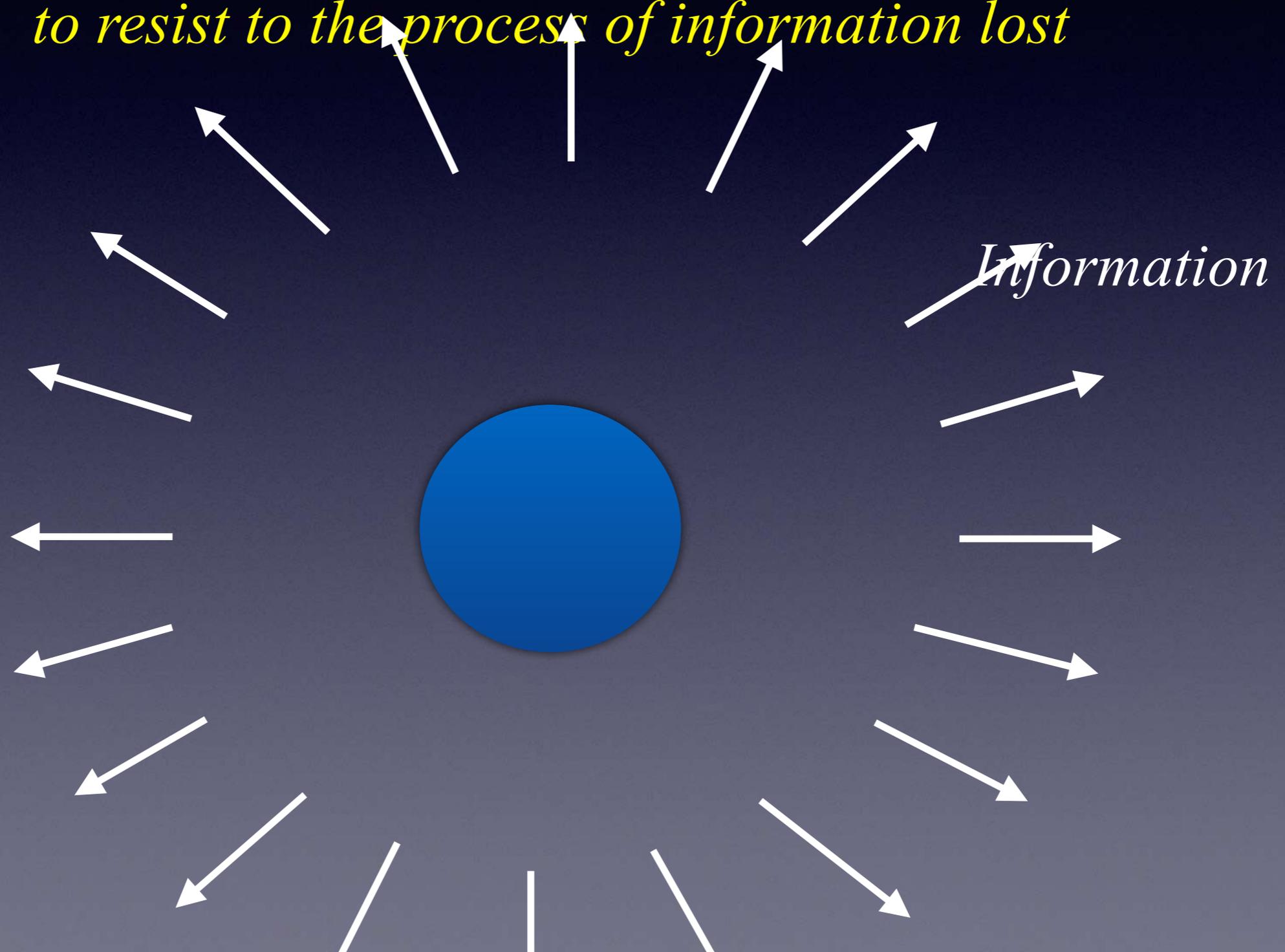
Memory burdening (P)BH

*In a system of high storage capacity,
the information stored in the system tends to backreact
to resist to the porcess of information lost*



Memory burdening (P)BH

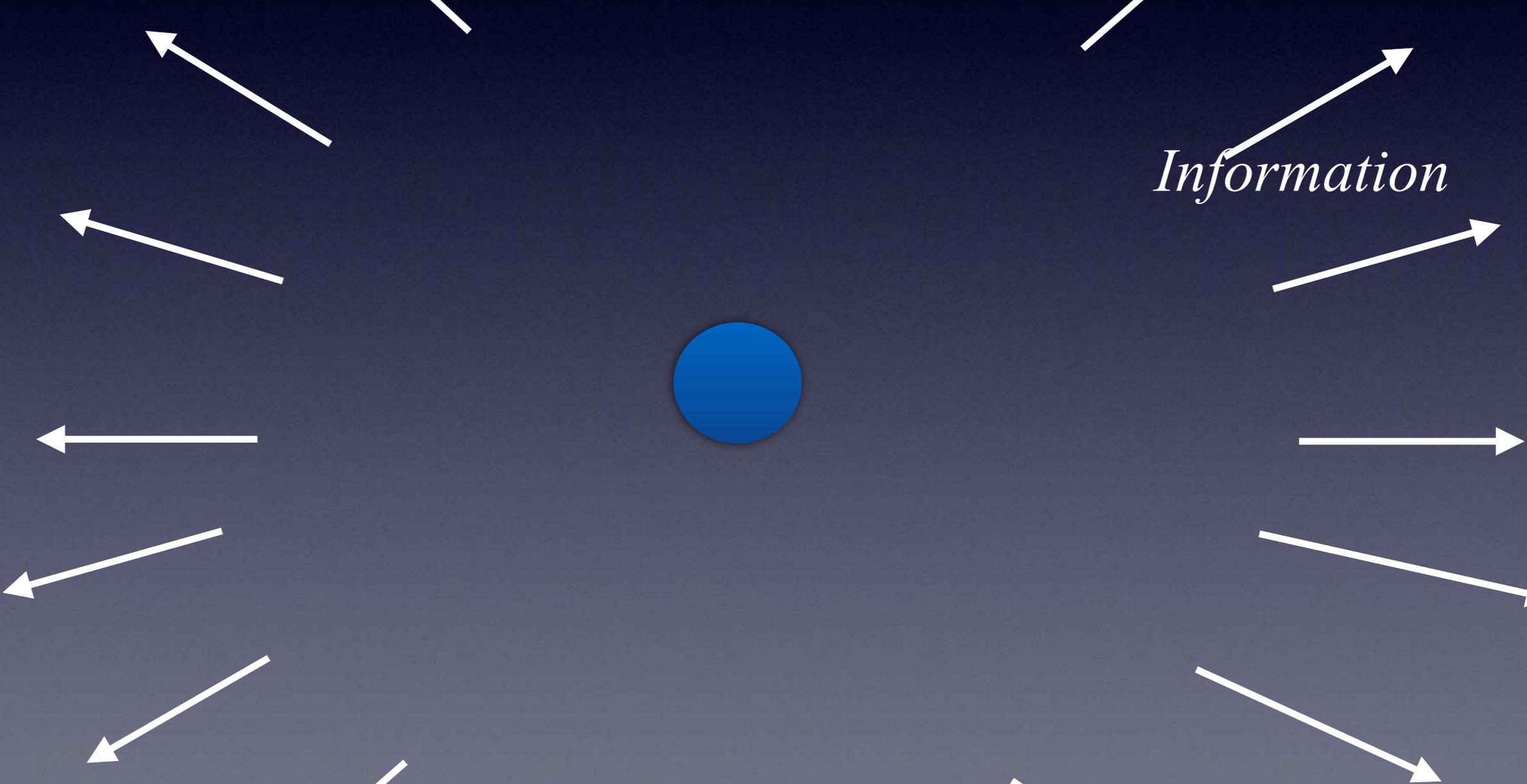
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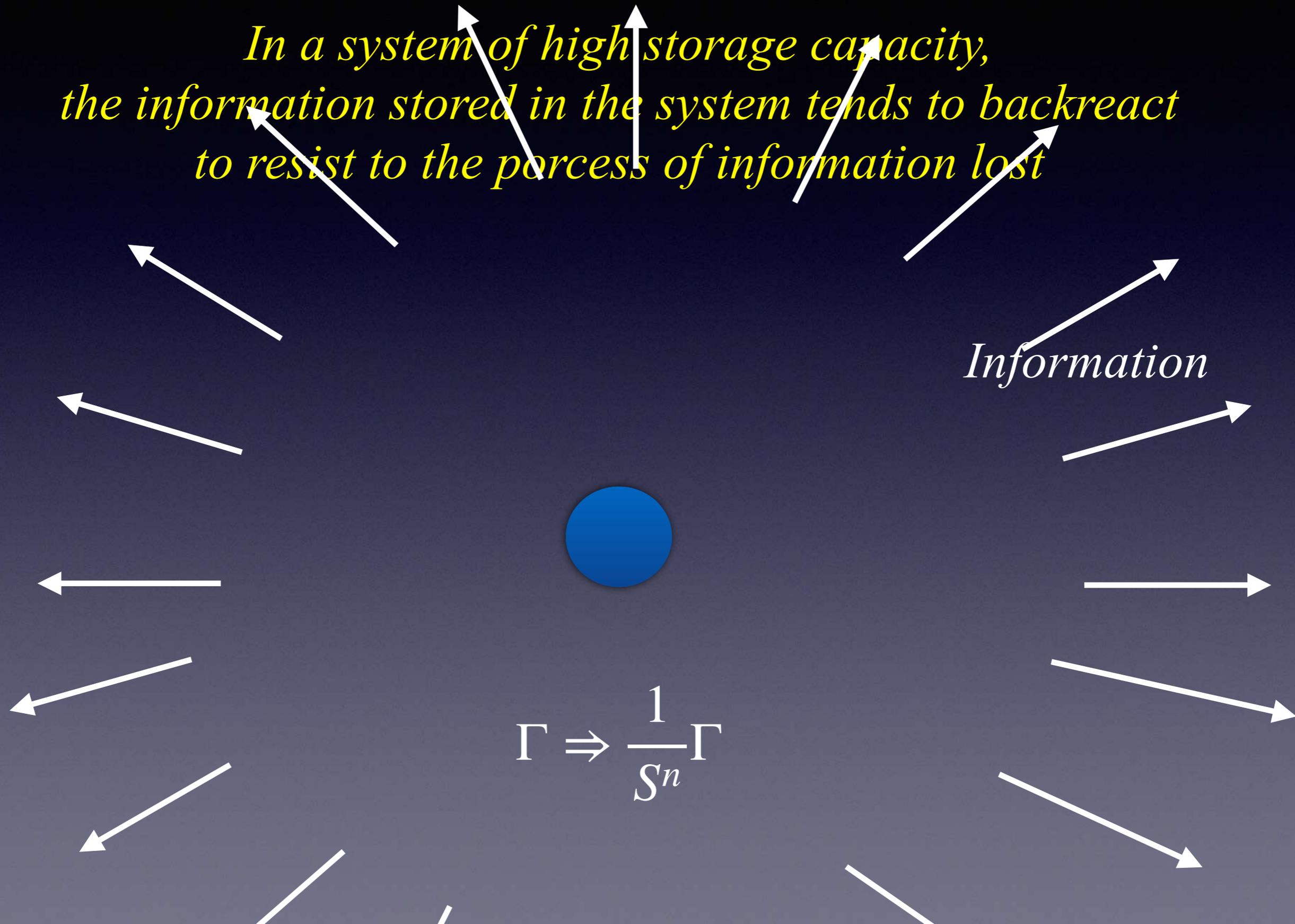


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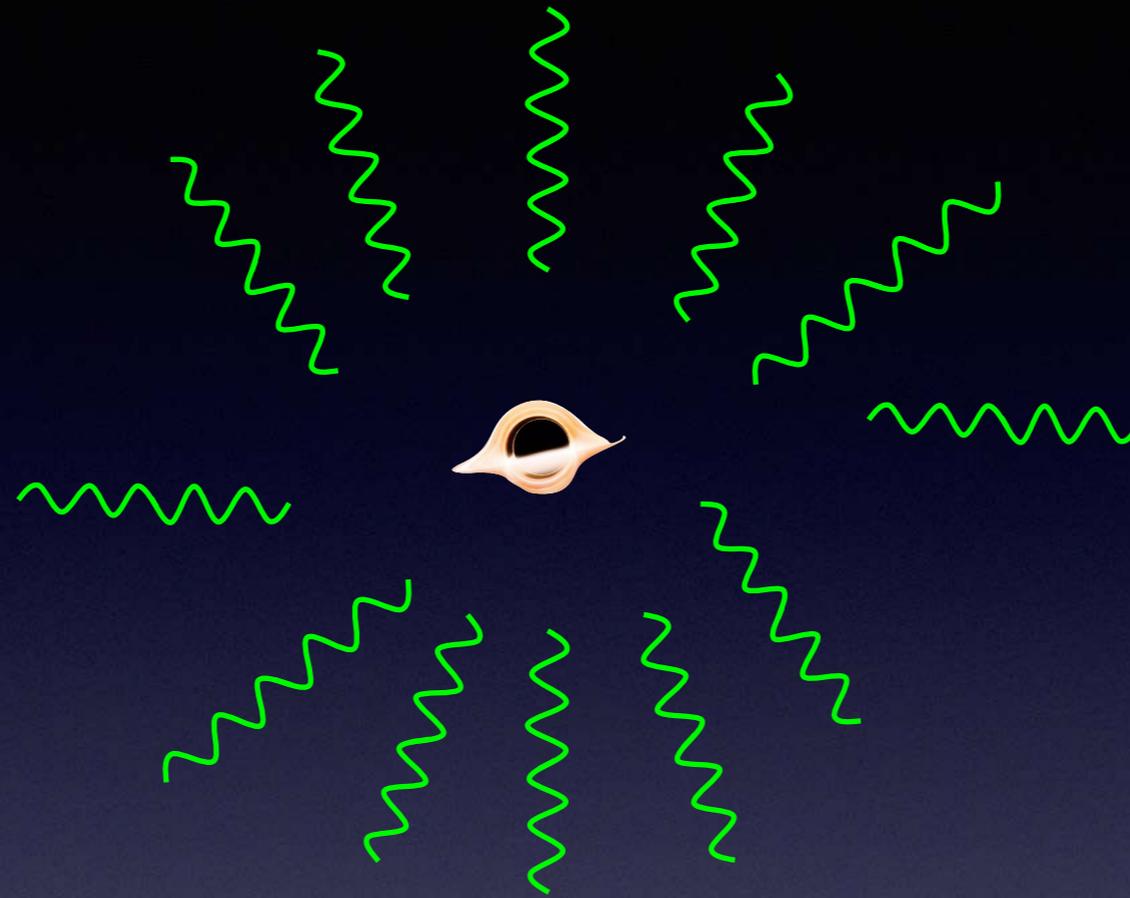
$$\Gamma \Rightarrow \frac{1}{S^n} \Gamma$$



Memory burdening (P)BH

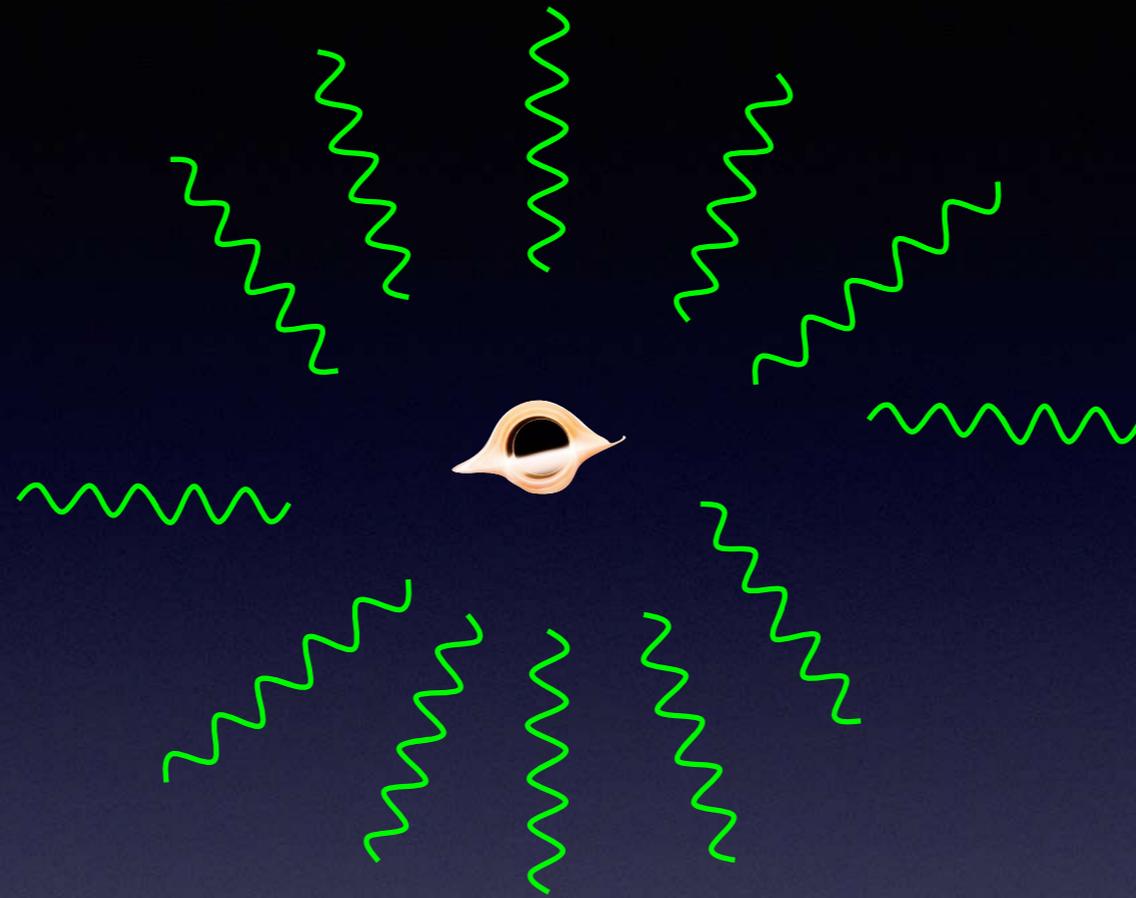


Memory burdening (P)BH

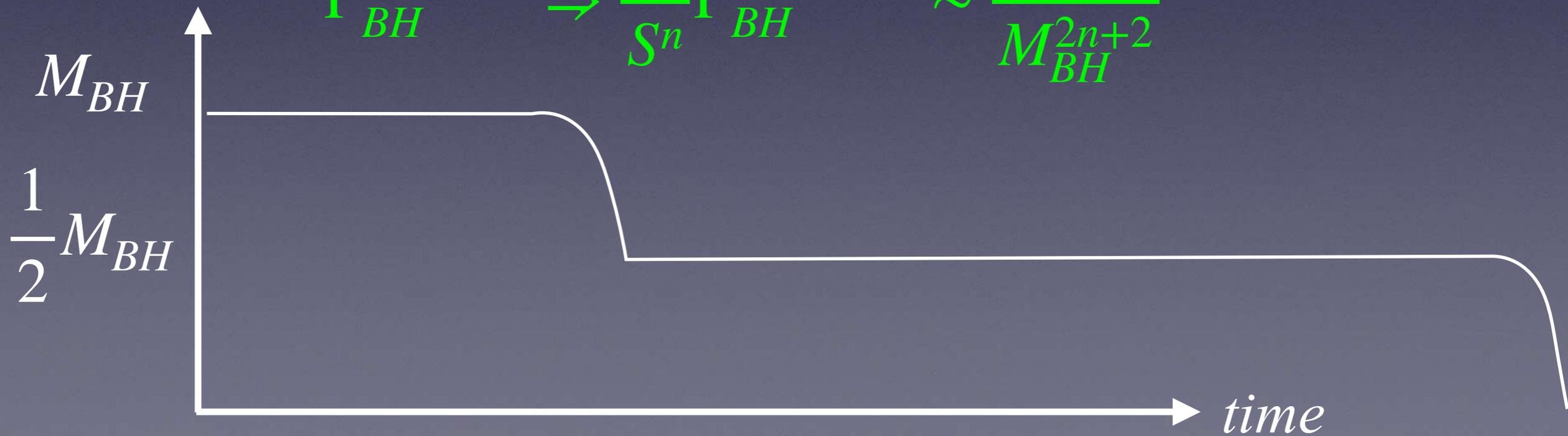


$$\Gamma_{BH}^{sem.clas.} \Rightarrow \frac{1}{S^n} \Gamma_{BH}^{sem.class.} \sim \frac{M_P^{2n+3}}{M_{BH}^{2n+2}}$$

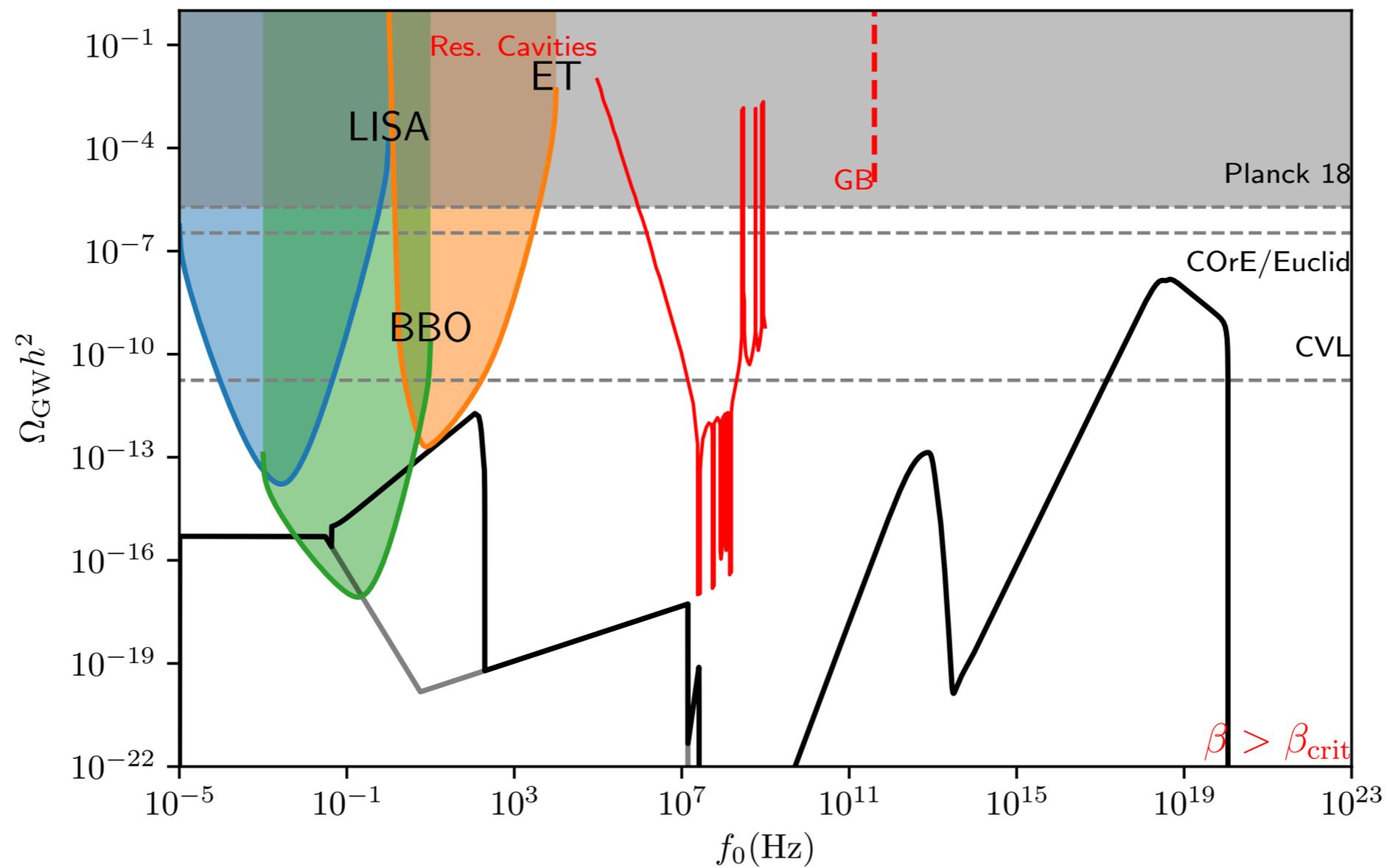
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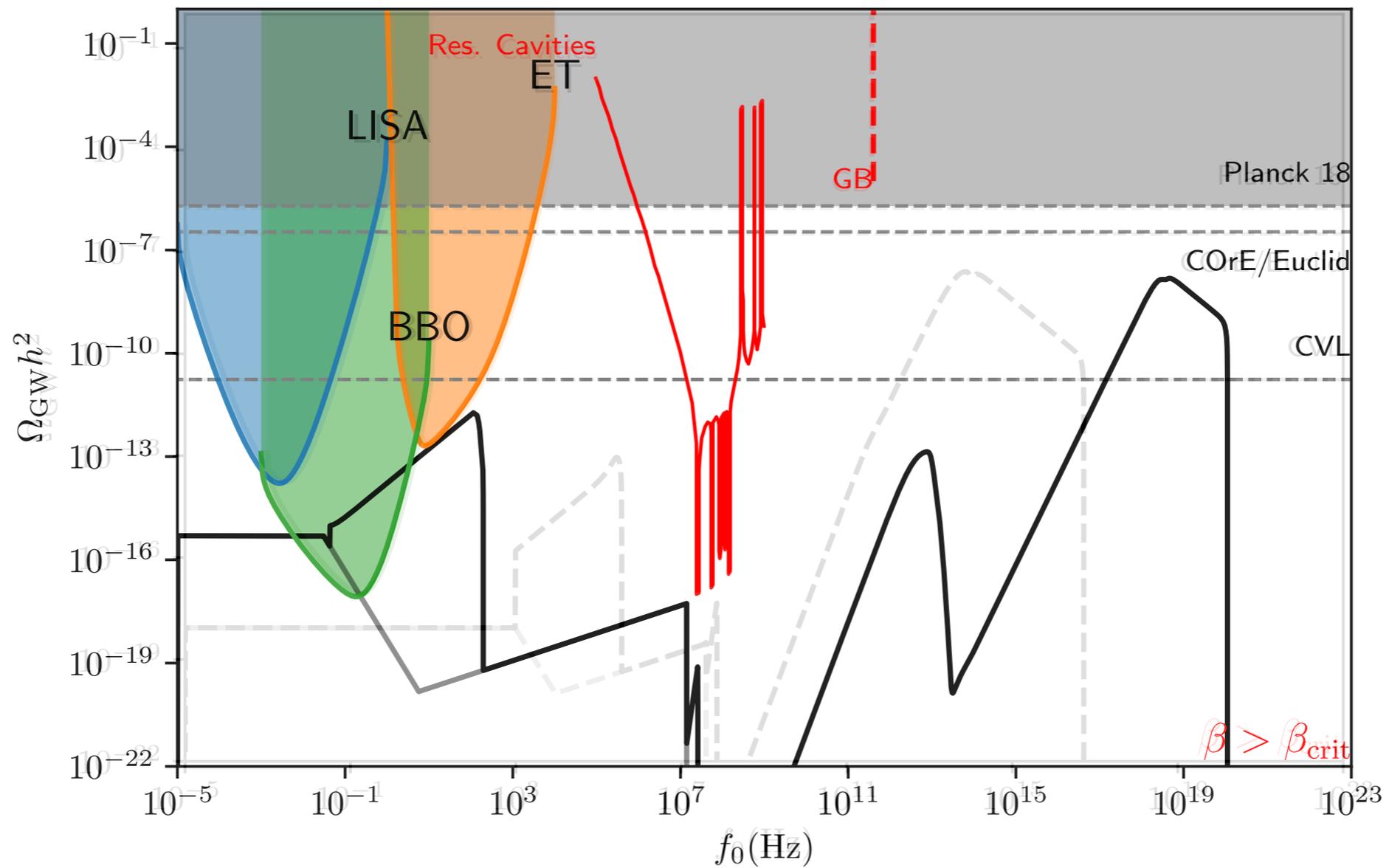
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Memory burdening the PBH



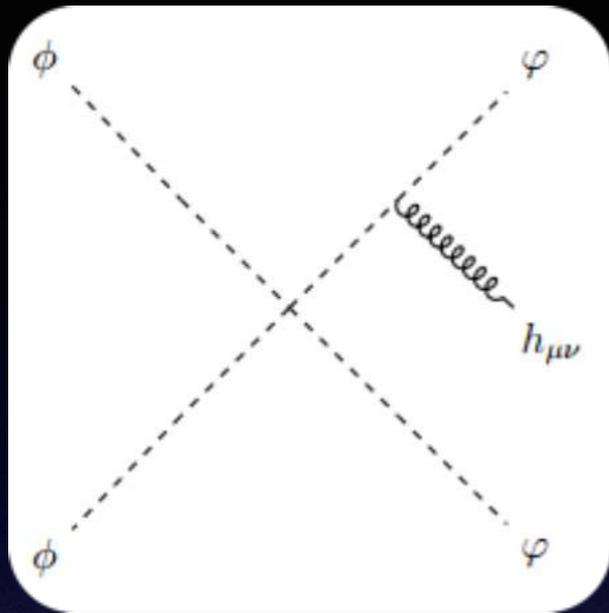
Memory burdening the PBH



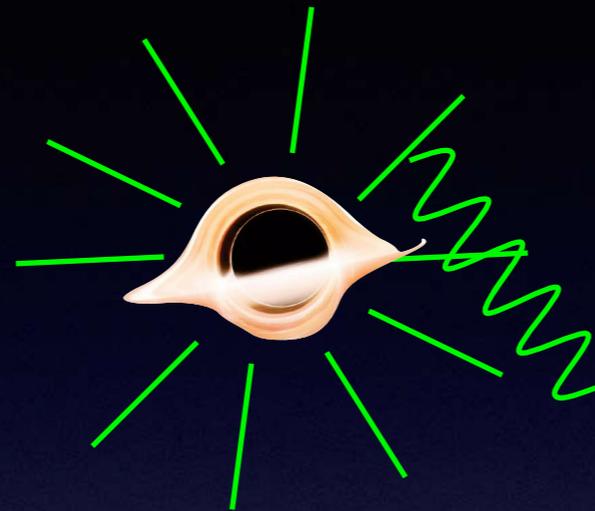
Graviton bremsstrahlung



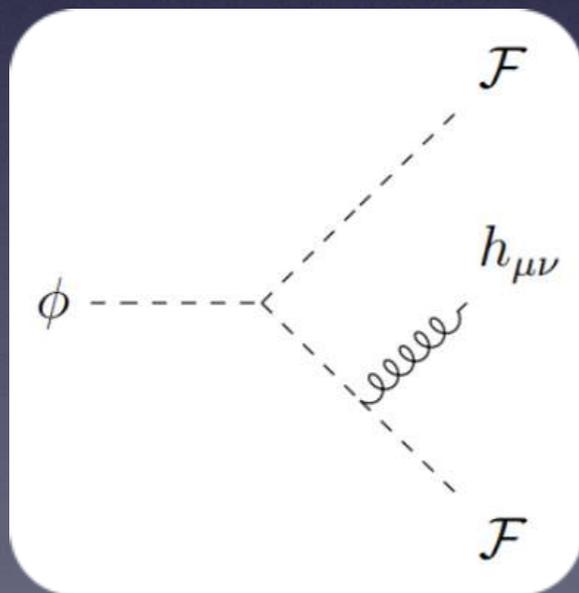
*Simon
Clery*



N. Bernal, S. Clery, Y. Mambrini
and Y. Xu,
JCAP **01** (2024), 065
[arXiv:2311.12694 [hep-ph]].



K. Y. Choi, E. Lkhagvadorj
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JCAP **07** (2024), 064
[arXiv:2403.15269 [hep-ph]].

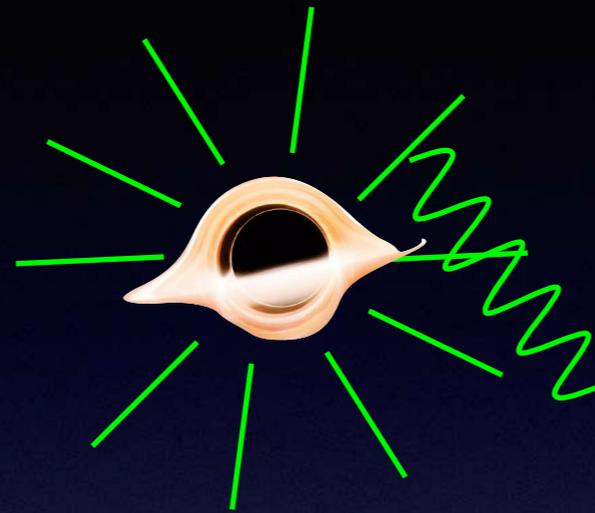
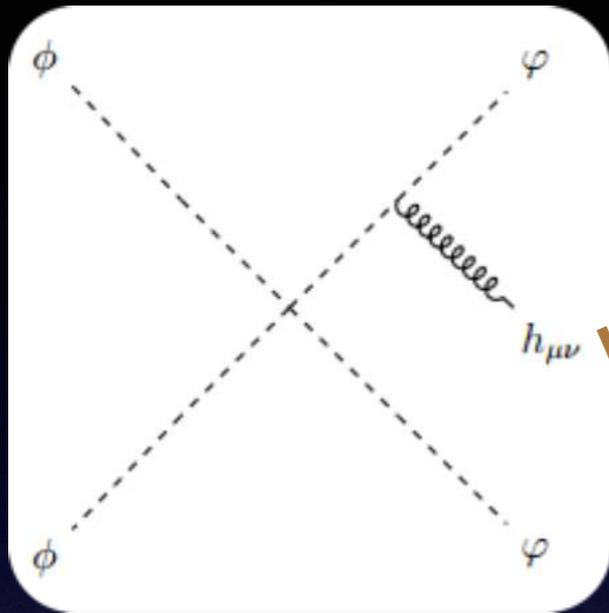


B. Barman, N. Bernal, Y. Xu
and O. Zapata,
JCAP **05** (2023), 019
[arXiv:2301.11345 [hep-ph]].

Graviton bremsstrahlung

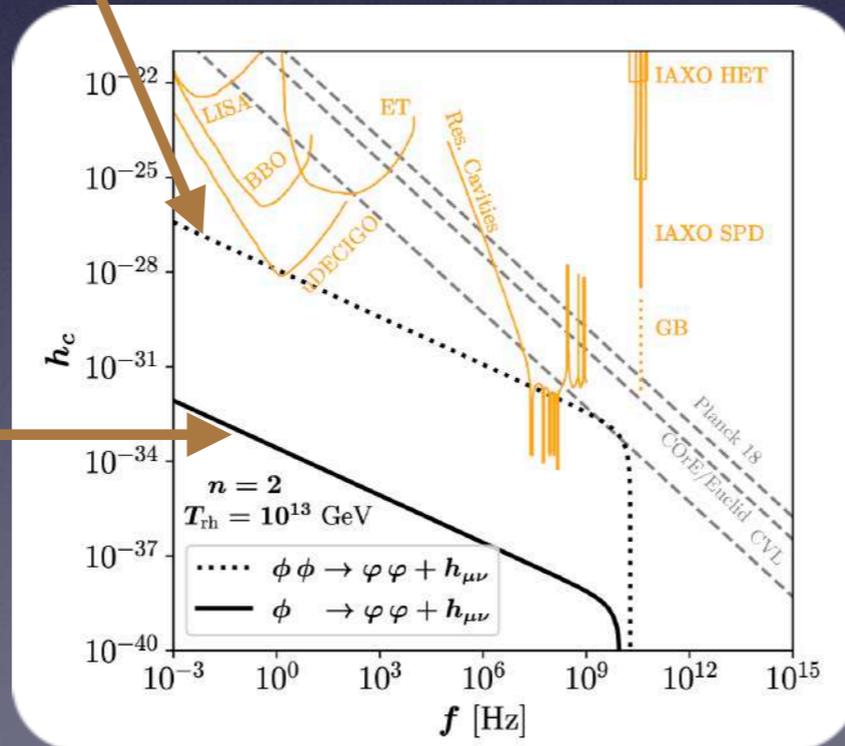
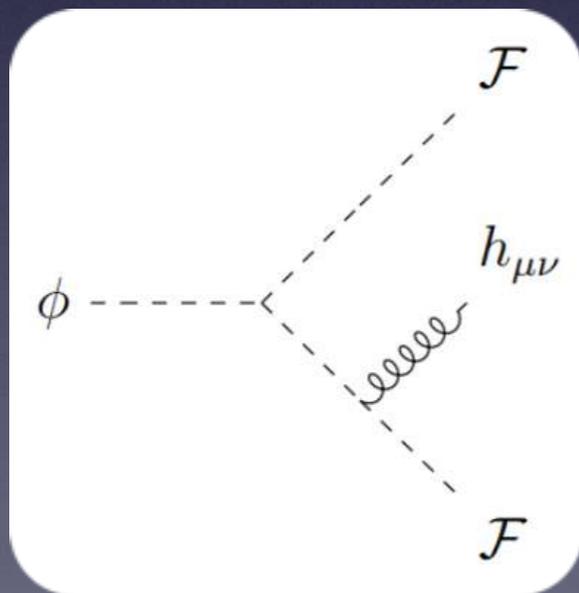


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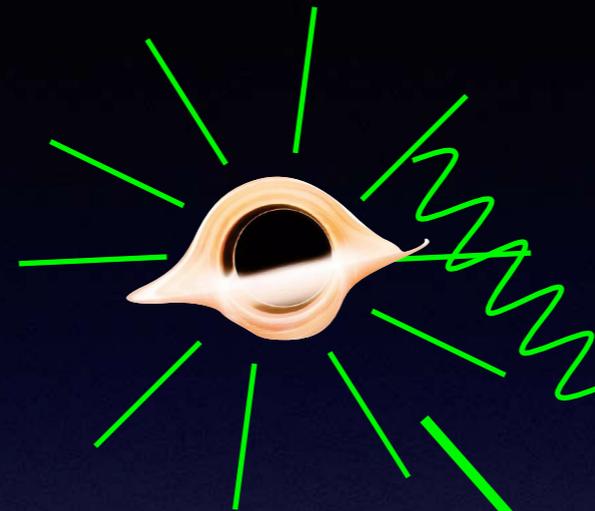
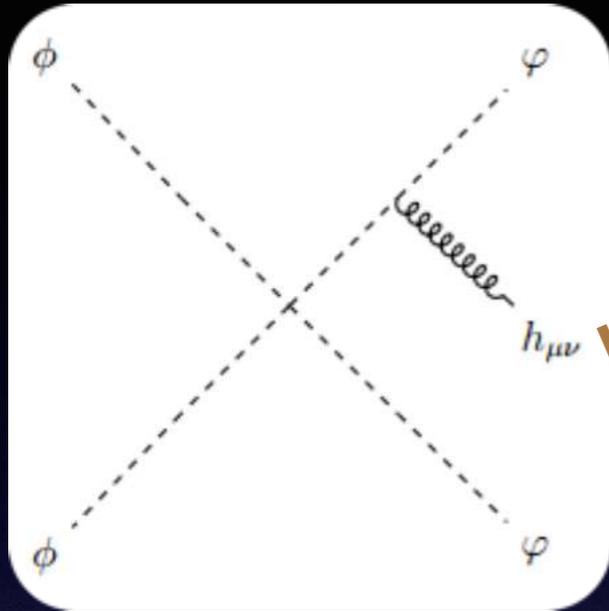
B. Barman, N. Bernal, Y. Xu
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Graviton bremsstrahlung

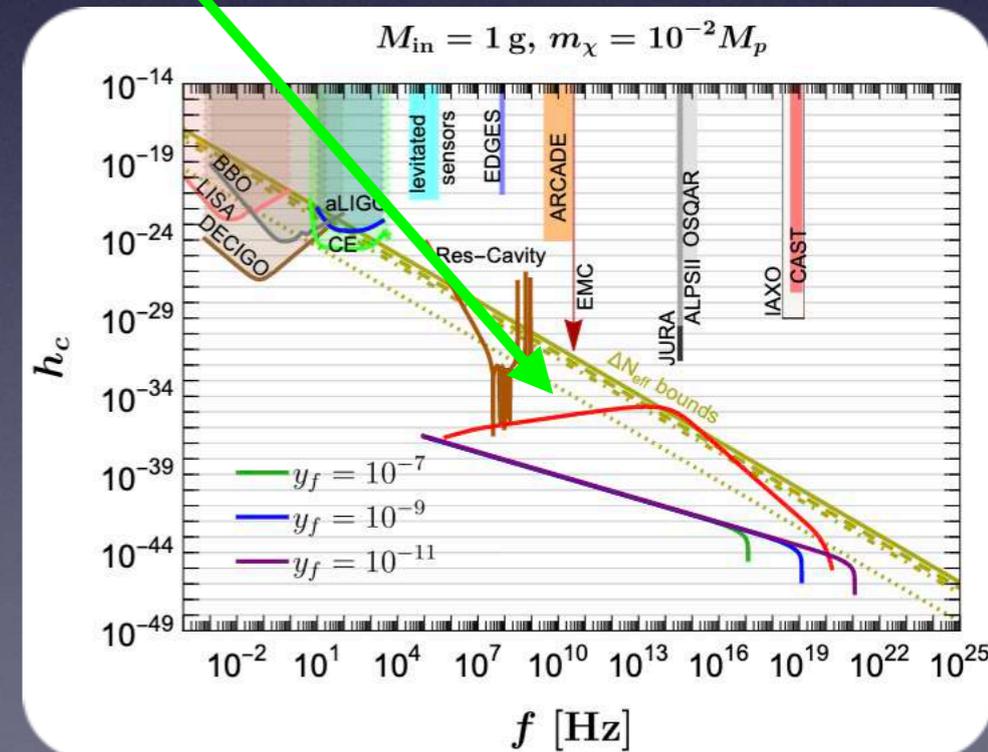
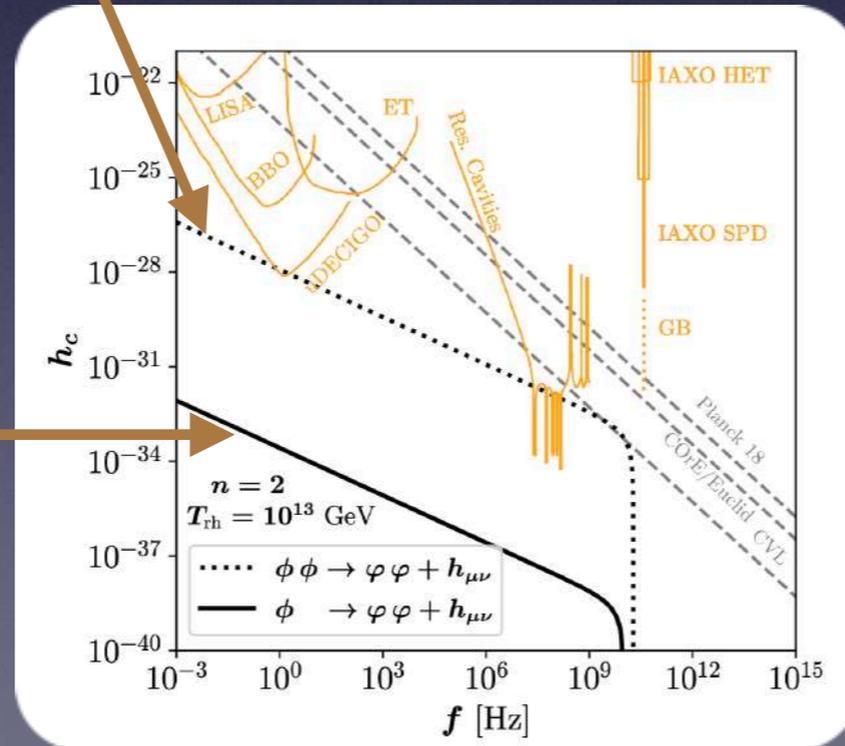
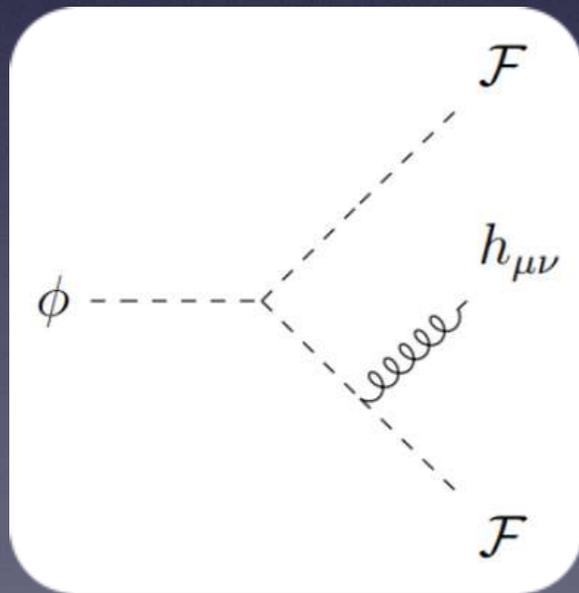


Simon Clery

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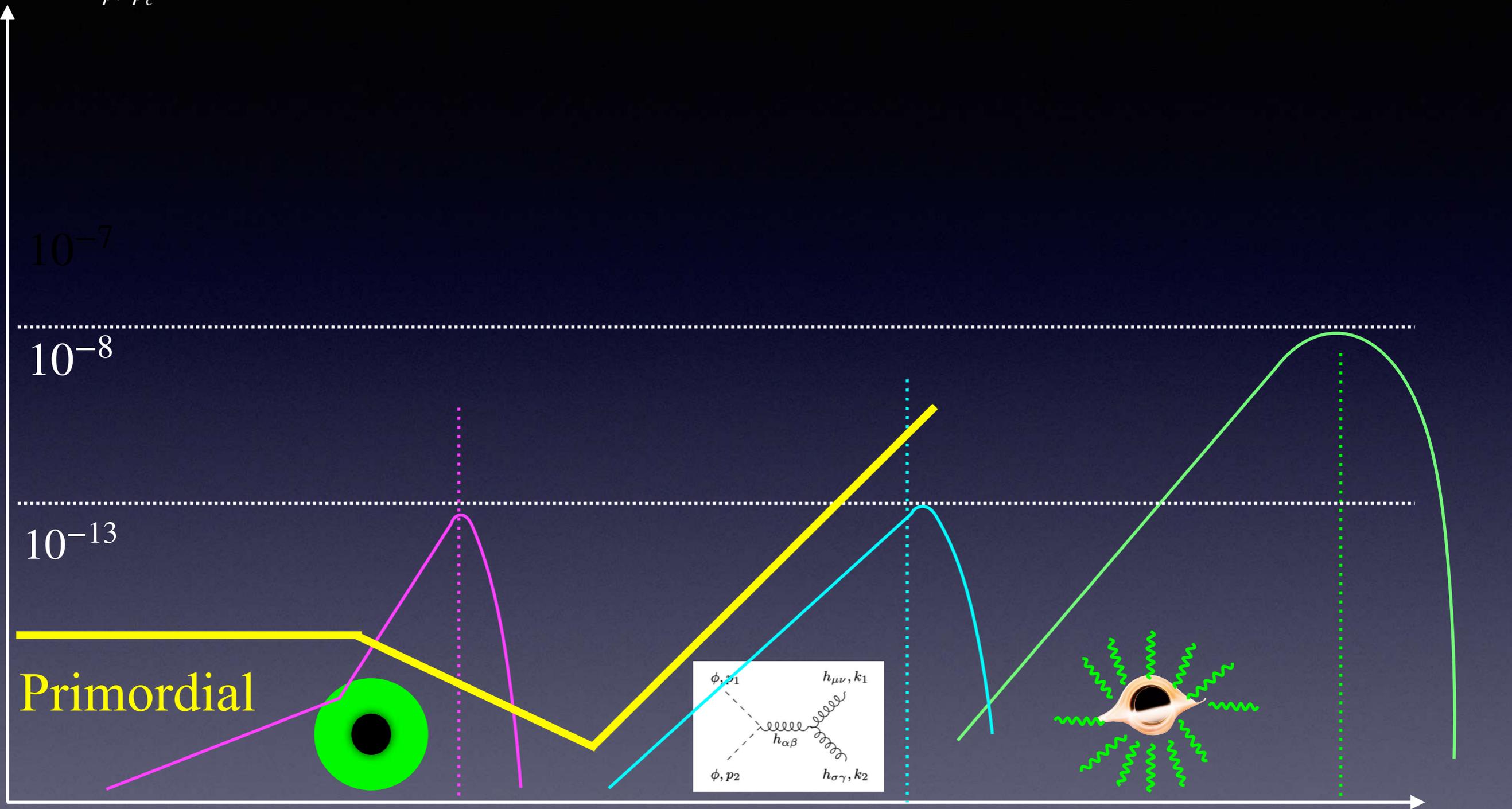
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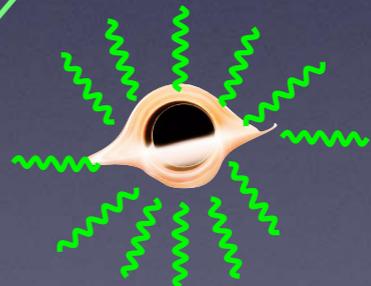
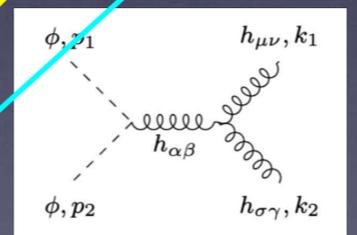
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Conclusion : PGW spectrum summary

$$\left. \frac{d\Omega_{GW}^{BH}}{d \ln k_0} \right|_{\beta > \beta_c}$$



Primordial



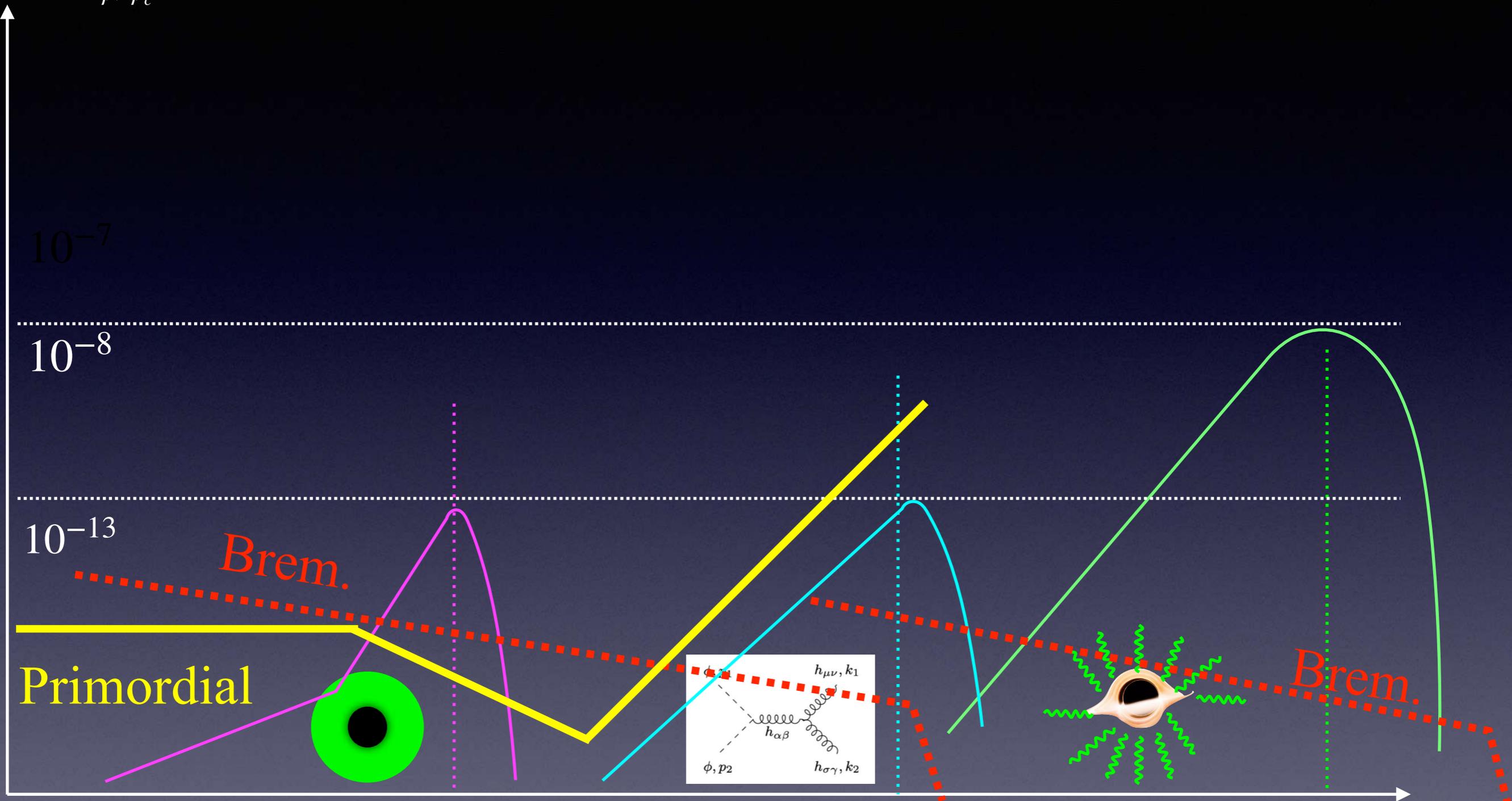
$$f_0^\phi \sim 10^3 \left(\frac{10^4 \text{ g}}{M_{BH}} \right)^{\frac{5}{6}} \text{ Hz}$$

$$f_0^\phi \sim 10^9 \left(\frac{10^{10}}{T_{RH}} \right)^{\frac{5k-2}{6k}} \text{ Hz}$$

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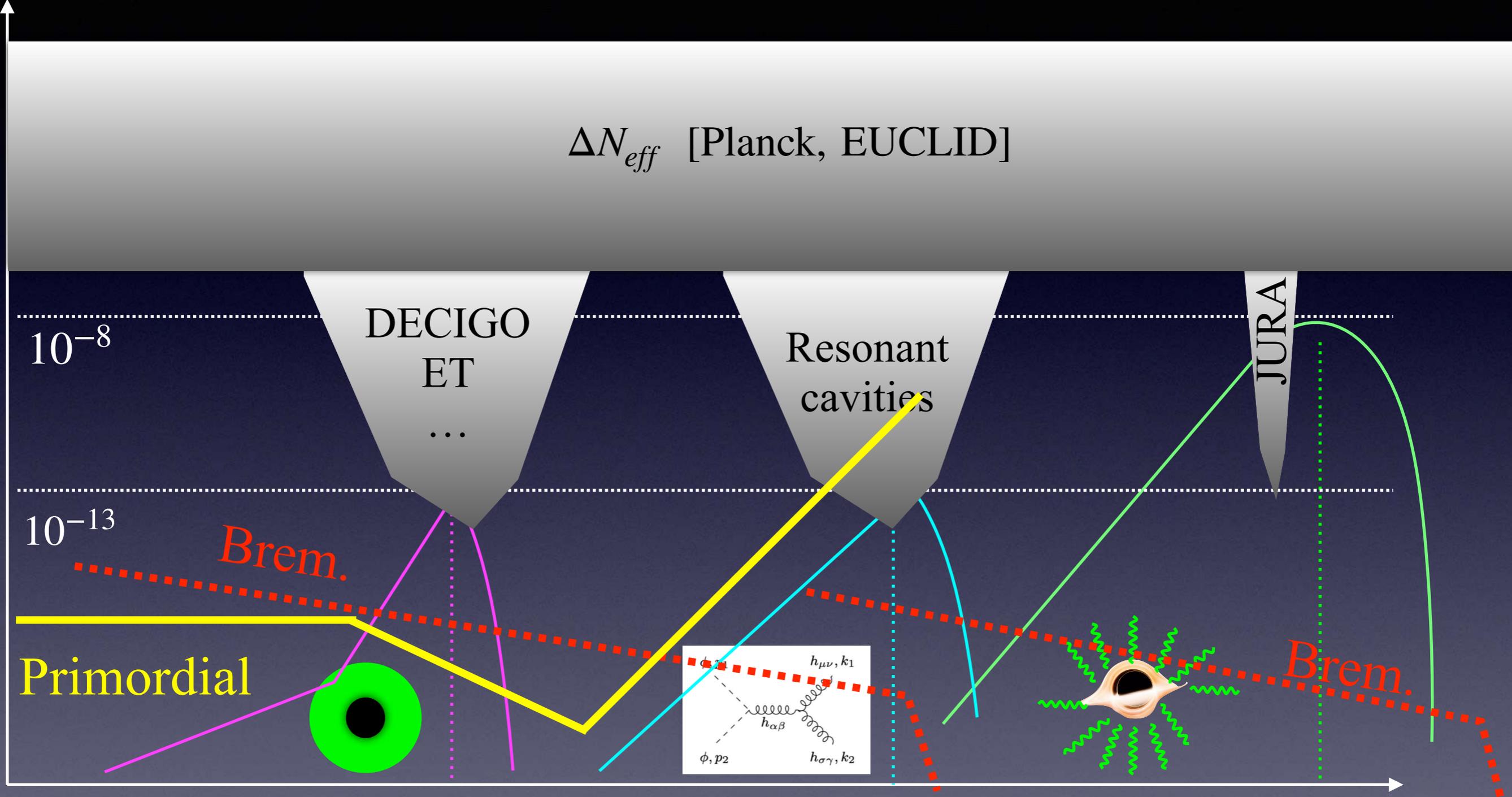
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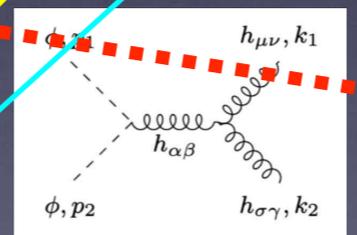
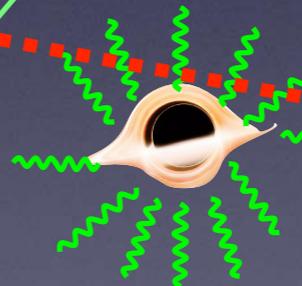
ΔN_{eff} [Planck, EUCLID]



Primordial

Brem.

Brem.



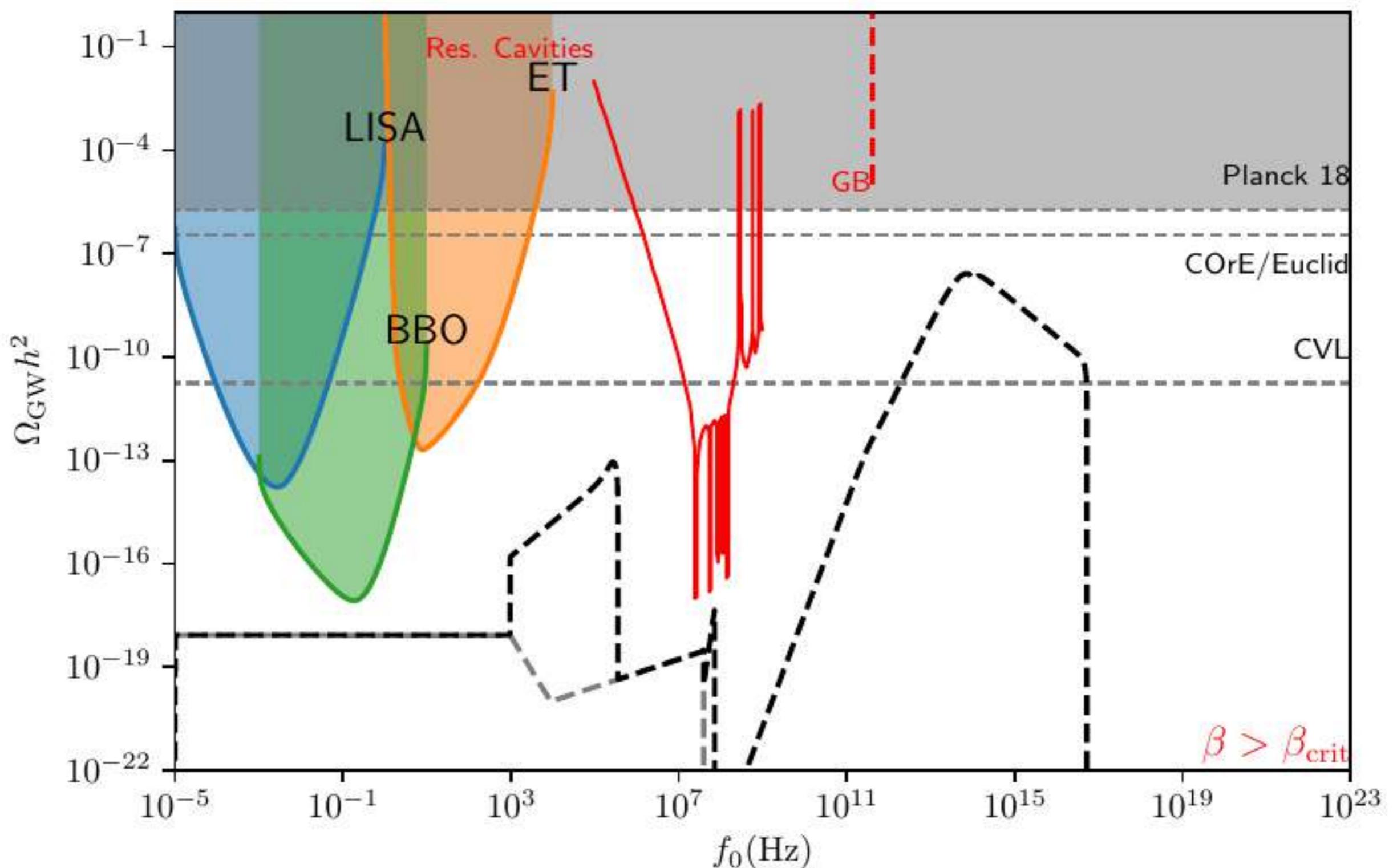
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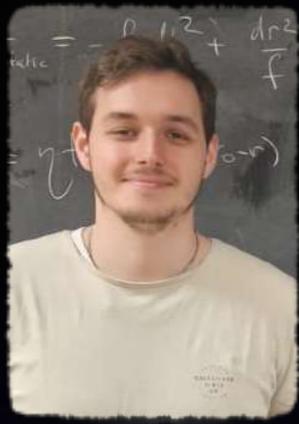
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Thank you!



*Mathieu
Gross*



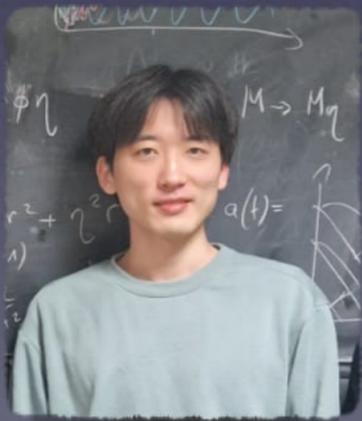
*Wenqi
Ke*



*Simon
Clery*



*Gyonju
Choi*



*Jong-Hyun
Yoon*

$$ds^2 = \left(1 - \frac{R_S}{r} - \frac{\Lambda}{3}r^2\right) dt^2 - \frac{dr^2}{1 - \frac{R_S}{r} - \frac{\Lambda}{3}r^2}$$

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Two horizons, the Schwarzschild one

$$r = R_S = \frac{M_{BH}}{4\pi M_P^2} \Rightarrow ds^2 = d\tau^2 - e^{2\frac{\rho}{R_S}} d\rho^2 \Rightarrow H_S = 4\pi \frac{M_P^2}{M_{BH}}$$

and the de Sitter one

$$r = \sqrt{\frac{3}{\Lambda}} \Rightarrow ds^2 = d\tau^2 - e^{2\sqrt{\frac{\Lambda}{3}}\tau} d\rho^2 \Rightarrow H_\Lambda = \sqrt{\frac{\Lambda}{3}}$$

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$$\phi = \int \alpha_\omega e^{i\omega\tau} + \beta_\omega e^{-i\omega\tau} \quad \text{followed by a Bogoliubov transformation}$$

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$$|\beta_\omega|^2 : 0 \rightarrow \frac{1}{e^{\frac{2\pi\omega}{H}} - 1} \quad \text{which implies} \quad T_H = \frac{H}{2\pi} = \begin{cases} \frac{M_P^2}{M_{BH}} \\ \frac{H_\Lambda}{2\pi} \end{cases}$$

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The vacuum state of a field depends on the motion of the observer

or

The number of particles is observer dependent.

$$T_H = \frac{H}{2\pi} = \begin{cases} \frac{M_P^2}{M_{BH}} \\ \frac{H_\Lambda}{2\pi} \end{cases} \Rightarrow \rho_\chi \sim \begin{cases} n_{BH} \times T_{BH} \\ \frac{3H_\Lambda^4}{16\pi^2} \end{cases}$$

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A second remark about the new concept of proper vibration is, that it is not always invariantly determined by the form of the universe. The separation of time from the spatial coordinates may succeed in a number of different space-time-frames. For De Sitter's universe I know three of them. Besides which P. O. Müller (l.c.) has recently given two more. In addition, there is an expanding form with infinite R (the form with finite R^*). A proper vibration of one form may be transformed into a proper vibration of the other frame, if the time variable is destroyed by the transformation:

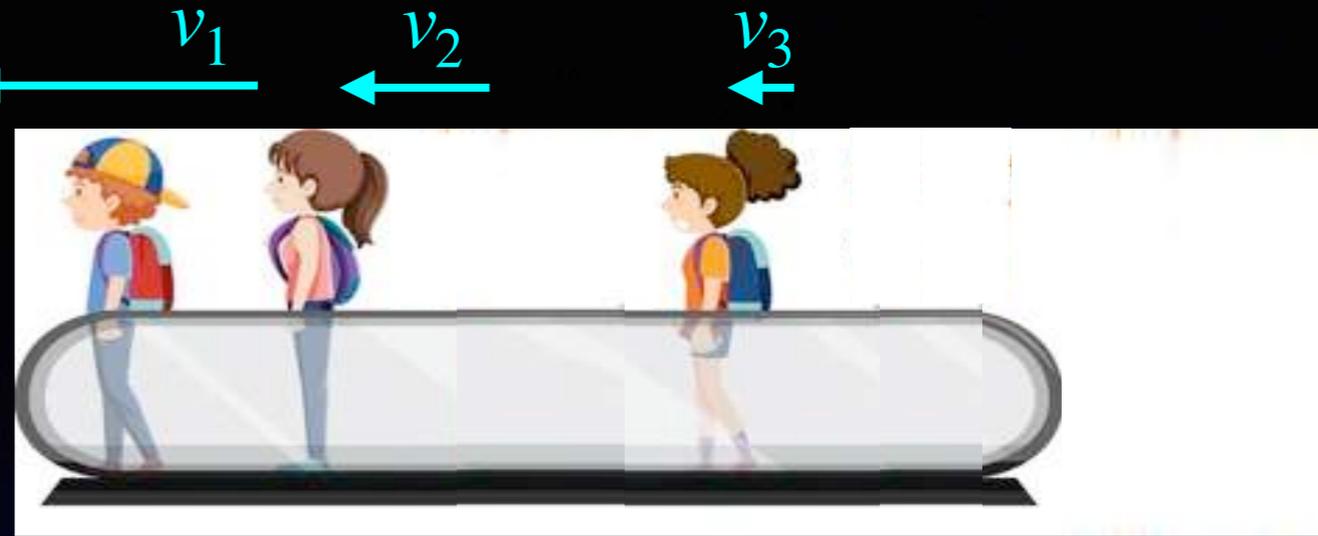


For all I have found hitherto I would conclude, that the alarming phenomena (i.e. pair production and reflexion of light in space) are not connected with the *velocity* of expansion, but would probably be caused by accelerated expansion. They may play an important part in the critical periods of cosmology, when expansion changes to contraction or vice-versa.

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“The proper vibrations of the expanding universe,”
 PHYSICA, vol. 6, no. 7–12, pp. 899–912, 1939

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$$\vec{V}_{walkway} \rightarrow$$

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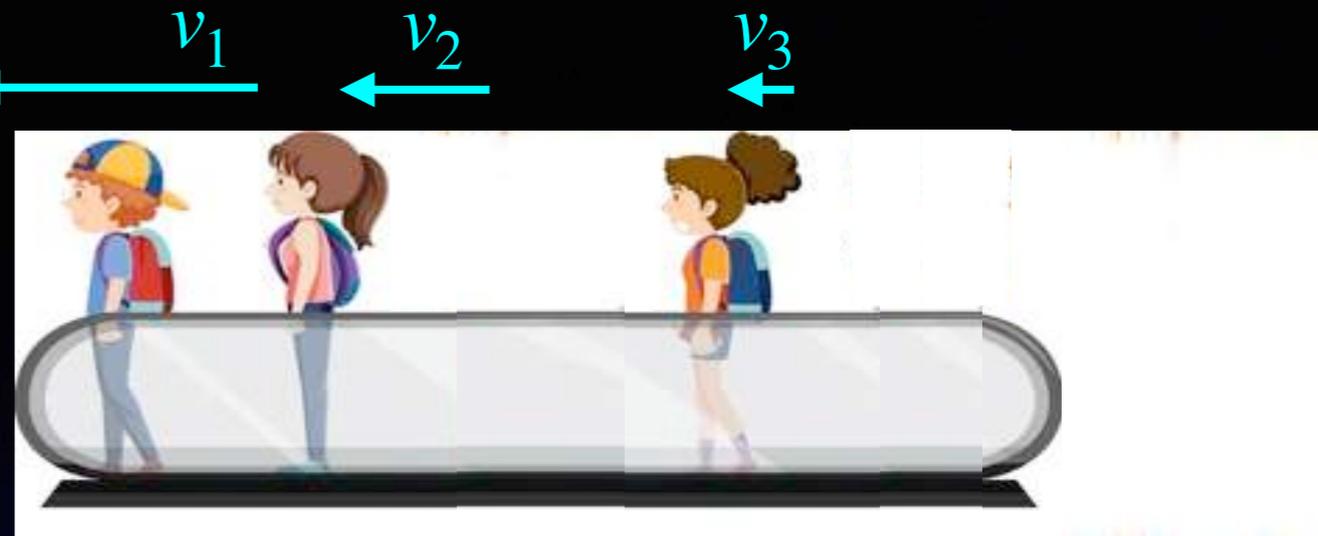


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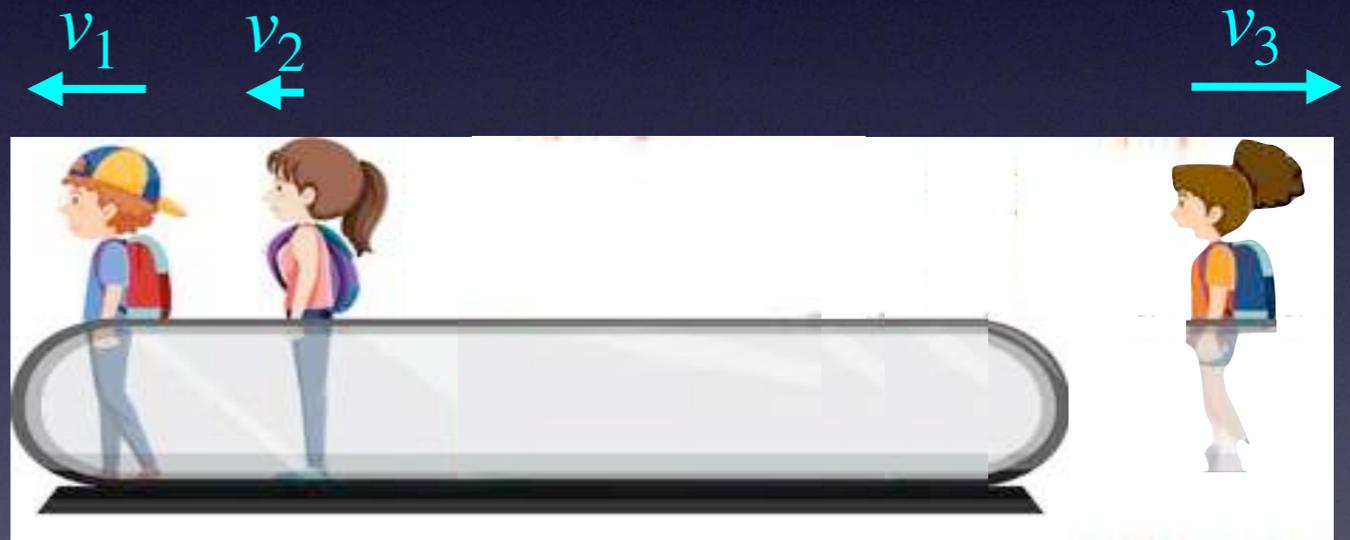
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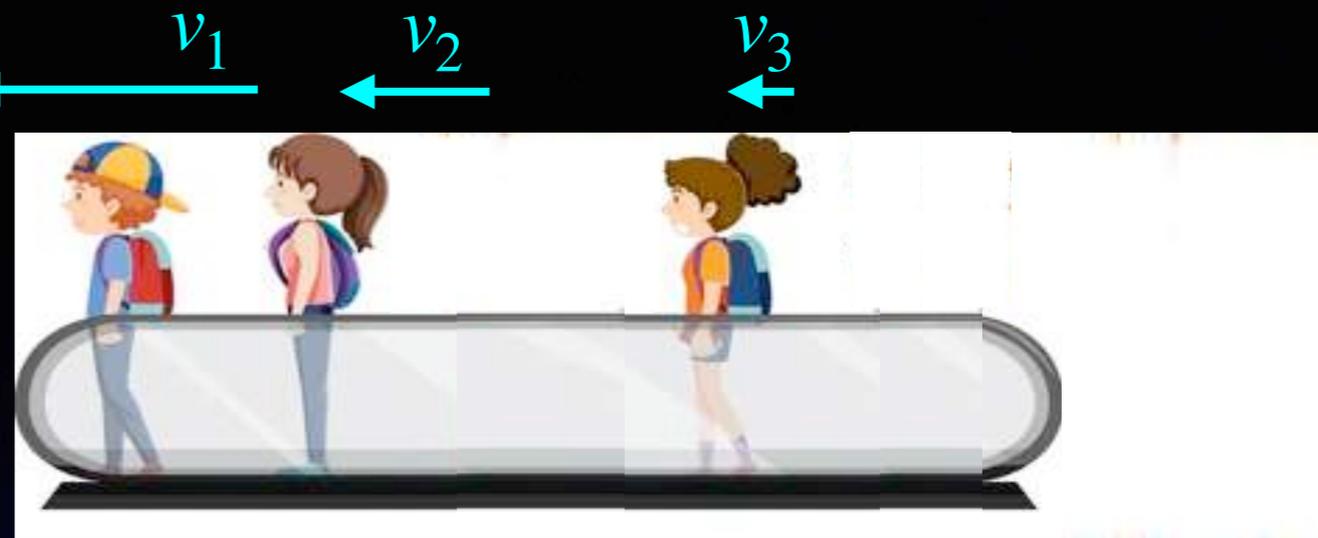


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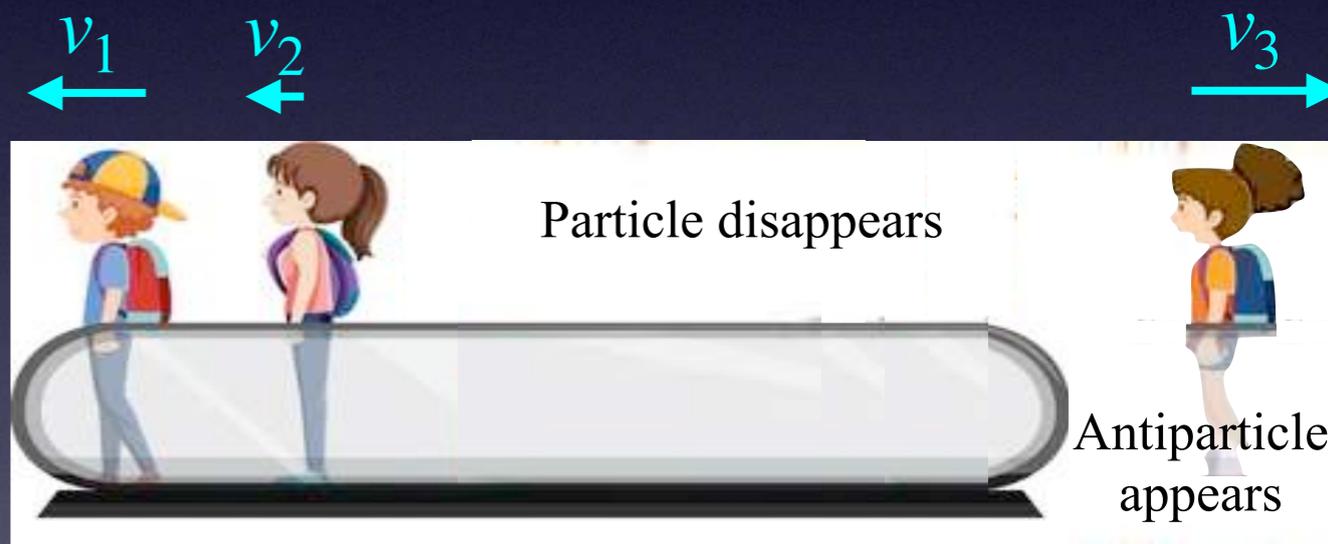
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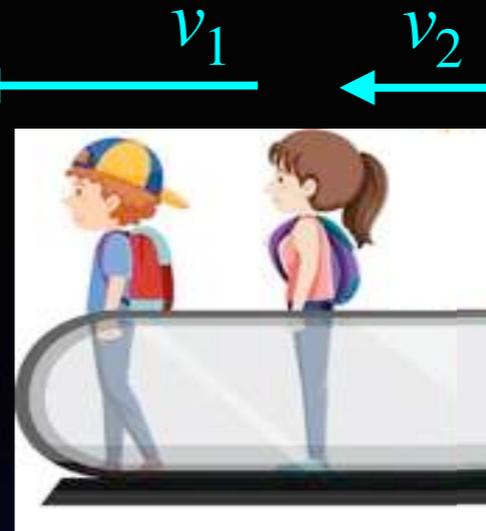
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Positive frequencies

$$a_\omega^\dagger e^{-i\omega t}$$



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$$V_{walkway} < v_1, v_2, v_3$$

$$\text{Pos. + neg. frequencies} \sim a_\omega^\dagger e^{-i\omega t} + \beta b_\omega^\dagger e^{+i\omega t}$$



Particle disappears

$$g_{ii}(t) = e^{Ht}$$

Antiparticle appears

$$V_{walkway} > v_3$$

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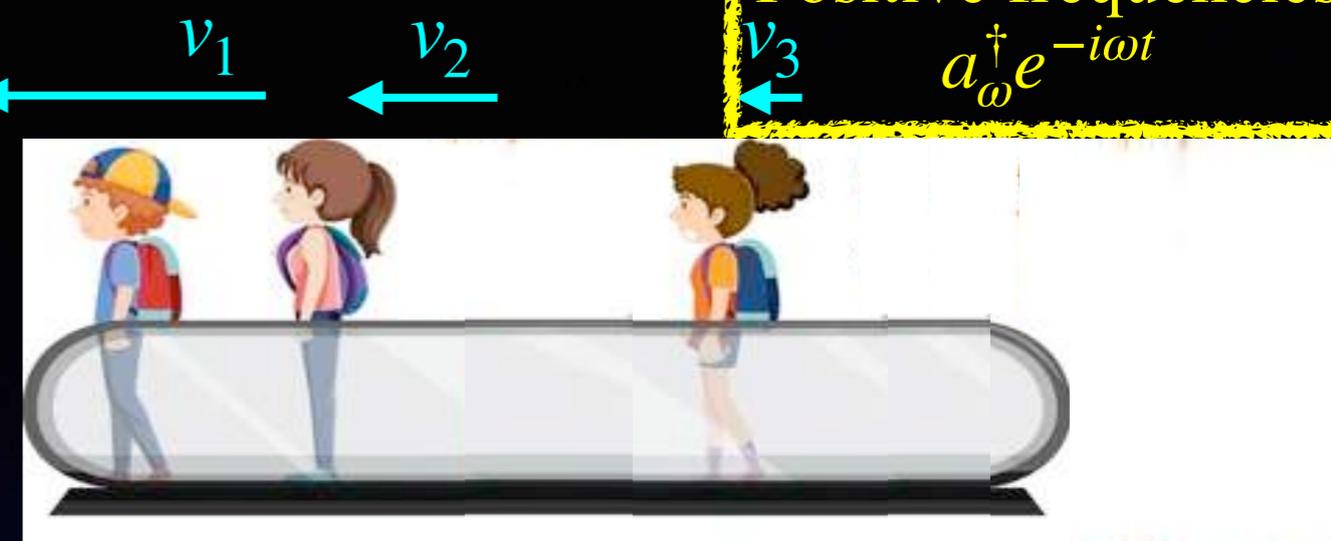
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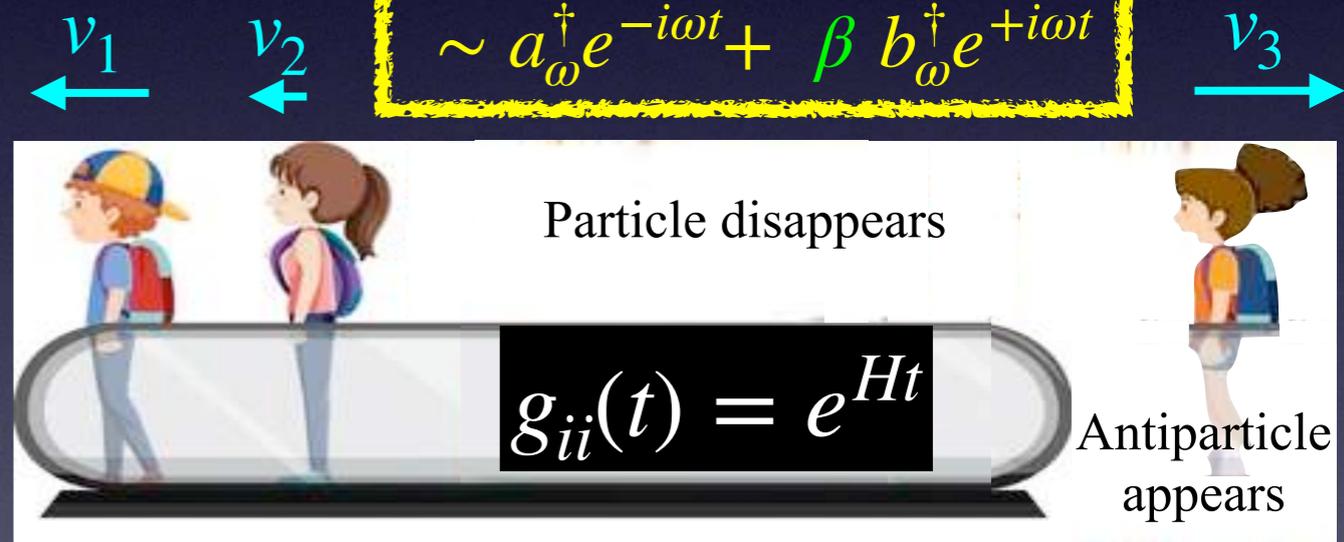
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$$V_{walkway} > v_3$$

See also Schwinger effect (1951), Unruh effect (1976)

Bunch, Davies, Ford and Parker works...

E. Schrödinger,

“The proper vibrations of the expanding universe,” PHYSICA, vol. 6, no. 7–12, pp. 899–912, 1939

Early history of gravitational production

- 1939 : in *The Proper Vibrations of the Expanding Universe*, Schrodinger propose to treat his equation in a de Sitter metric

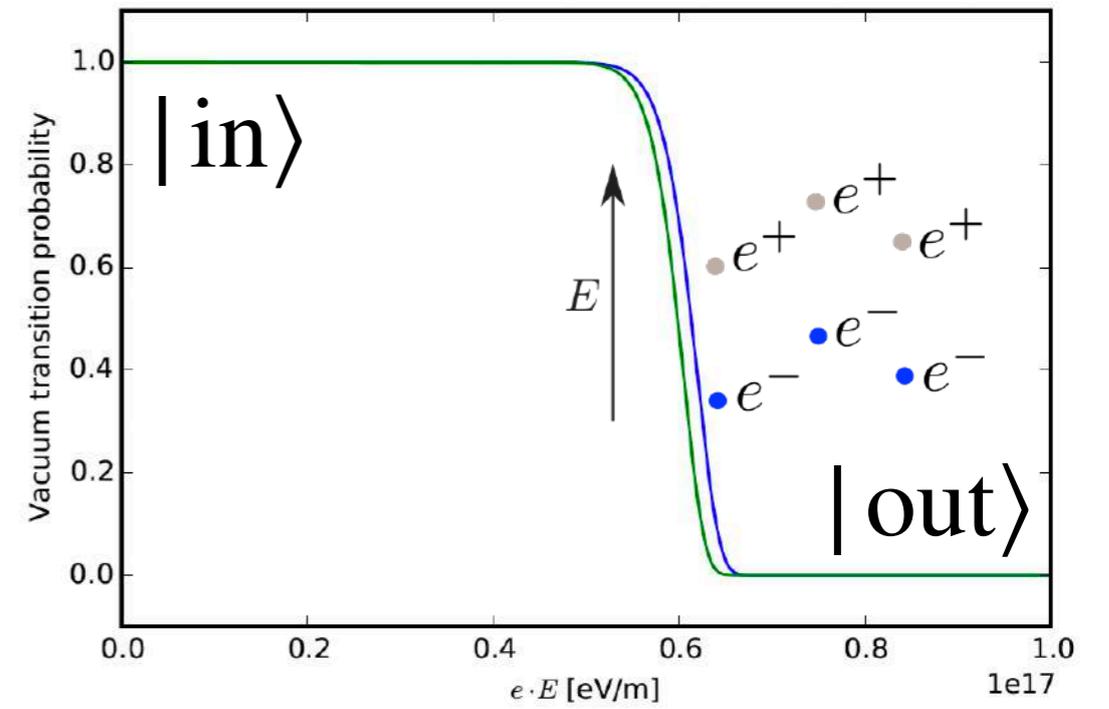
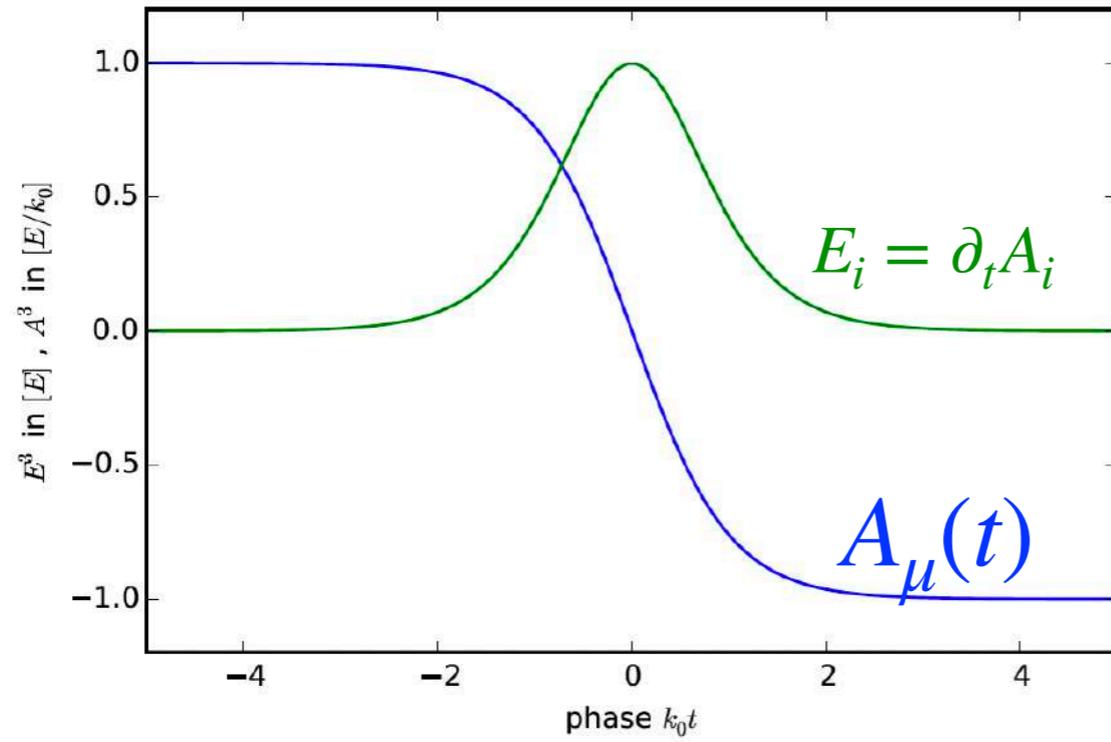
The expansion of the Universe can mix positive- and negative-frequency mode solutions of the wave equation. He calls it « mutual adulteration », and considered it as an « alarming phenomena » of « outstanding importance », which can produce matter « merely by the expansion of the Universe ».
- 1965 : Parker's thesis

« ... gravitational production seems inescapable if one accepts quantum field theory and general relativity »
- 1971 : Zeldovich + Starobinsky

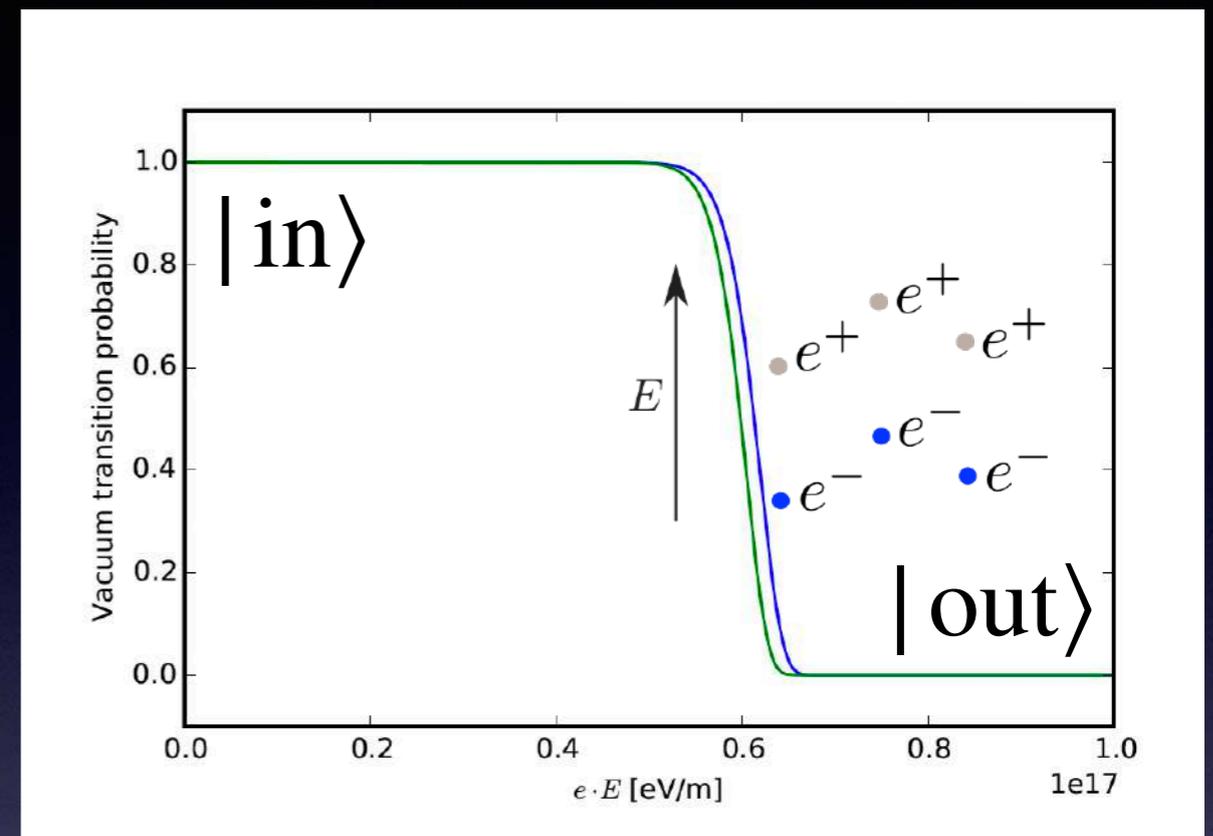
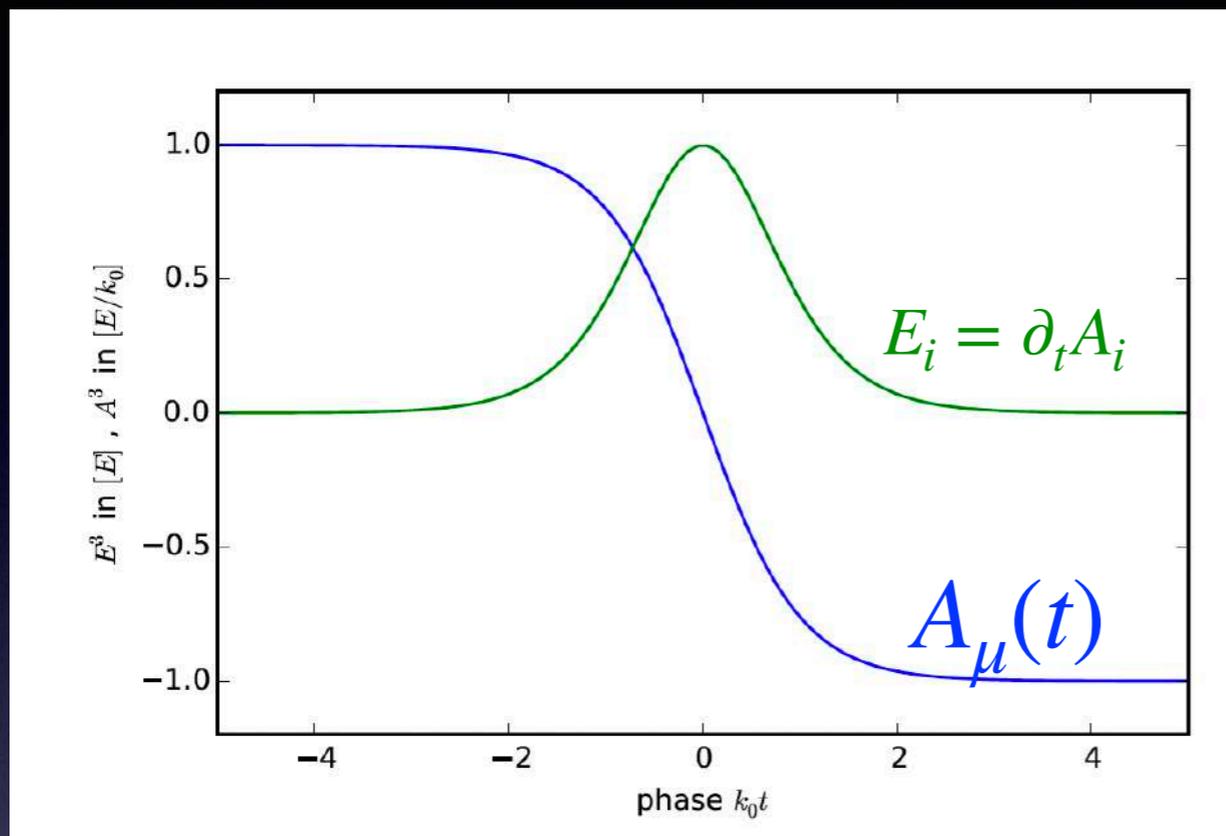
Gravitational production to render the Universe homogeneous (due to the homogeneity of the curvature)
- 1981 : Mukhanov, Chibisov

Quantum fluctuations in the inflaton field leads to predictions for density perturbations explaining anisotropies of the CMB
- 1987 : Ford *Applying GP to reheating, lepto and dark matter (axion) production*

The Schwinger effect



The Schwinger effect

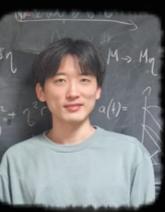
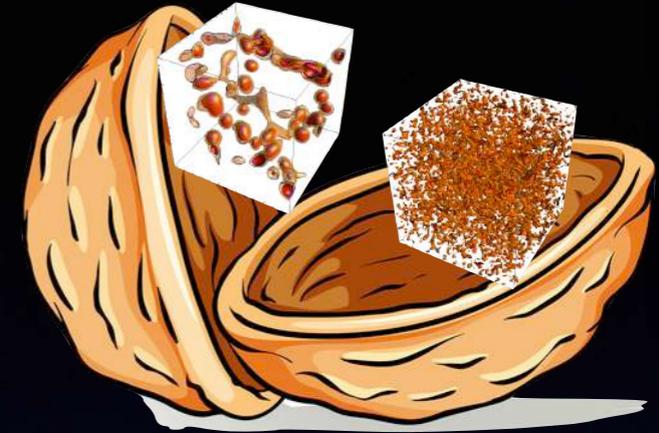


$$\mathcal{L} \supset S(t) \chi^\dagger \chi$$

$$S(t) = \phi(t), A_\mu(t), g_{\mu\nu}(t)$$

	Schwinger effect	Unruh effect	Hawking radiation	de Sitter temperature
Mean number	$e^{-\frac{m}{T_S}}$	$\frac{1}{e^{\frac{F}{T_U}} \pm 1}$	$\frac{1}{e^{\frac{F}{T_{BH}}} \pm 1}$	$\frac{1}{e^{\frac{F}{T_{dS}}} \pm 1}$
Temperature	$T_S = \frac{1}{2\pi} \frac{qE}{m}$	$T_U = \frac{a}{2\pi}$	$T_{BH} = \frac{M_P^2}{8\pi M_{BH}}$	$T_{dS} = \frac{H_{dS}}{2\pi}$

Reheating in a nutshell

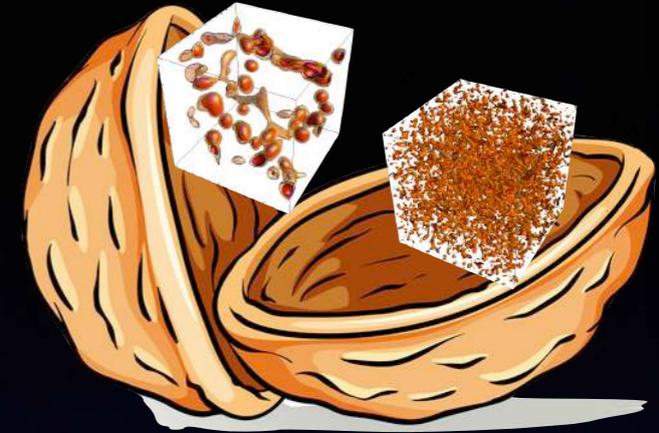


ϕ

Reheating in a nutshell

Reheating : /,ri:'hi:tɪŋ/ *noun*

Process of transfer of energy from a de Sitter space to radiation through the oscillations of a *classical homogeneous* field (the inflaton)

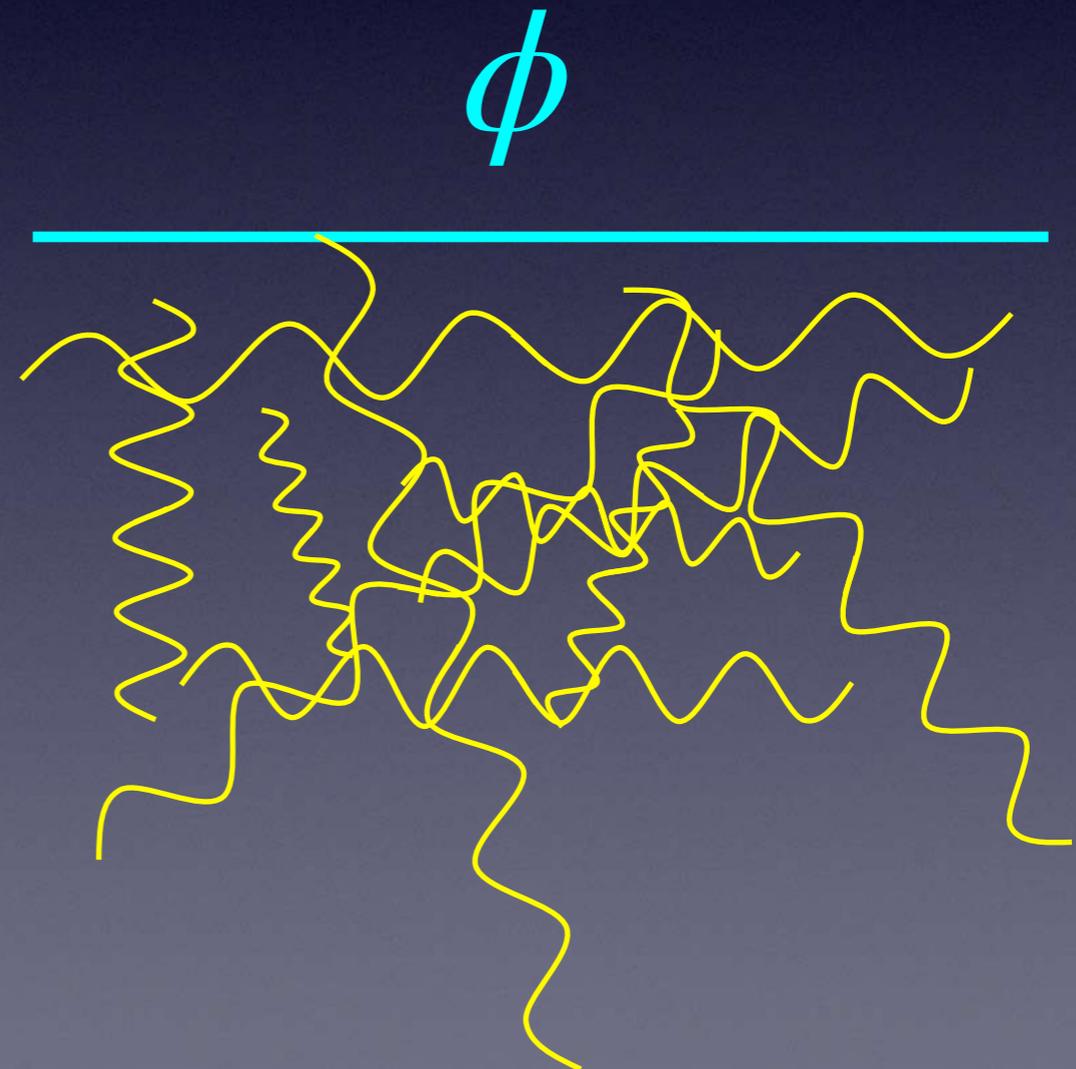
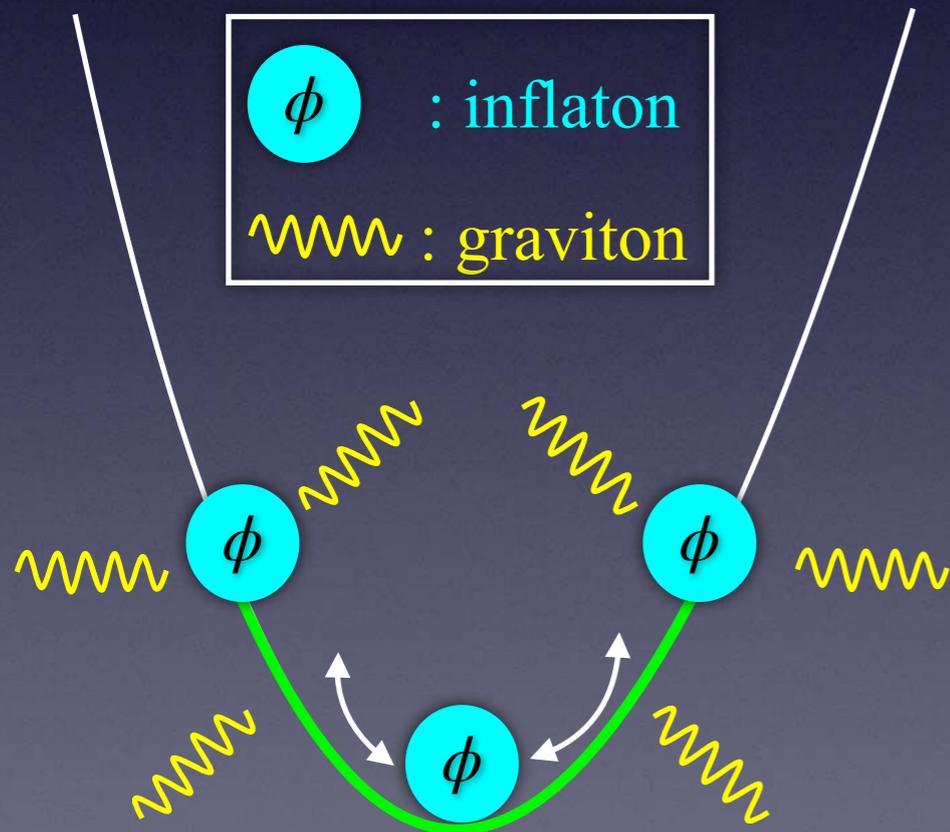
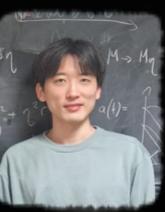
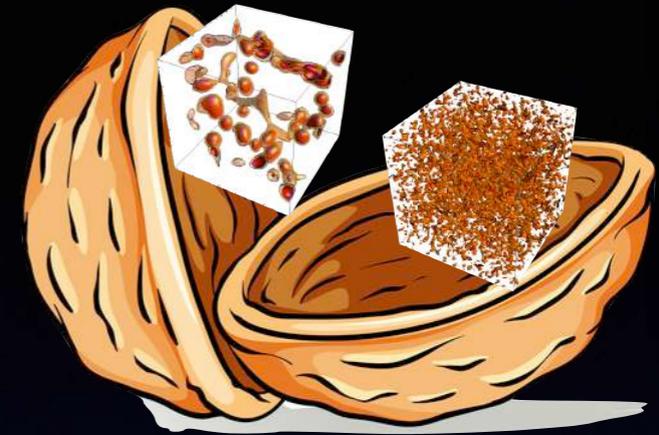


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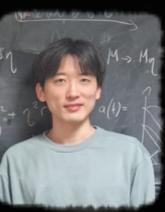
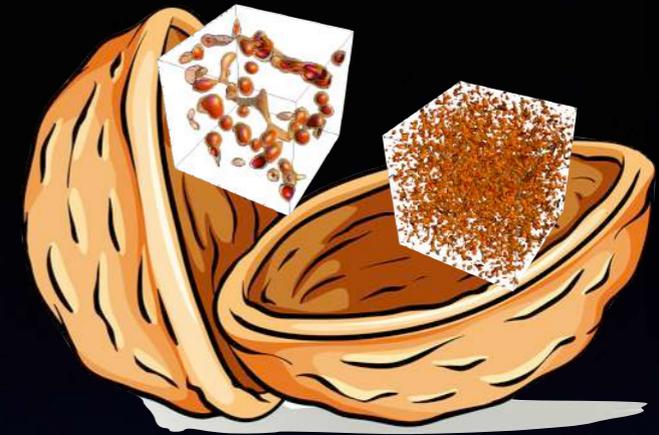
Reheating in a nutshell

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$$

$$\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\frac{d\rho_{\phi}}{dt} + 3H\rho_{\phi} = -\Gamma_{\phi}\rho_{\phi}$$

$$\frac{d\rho_{GW}}{dt} + 4H\rho_{GW} = +\Gamma_{\phi}\rho_{\phi}$$



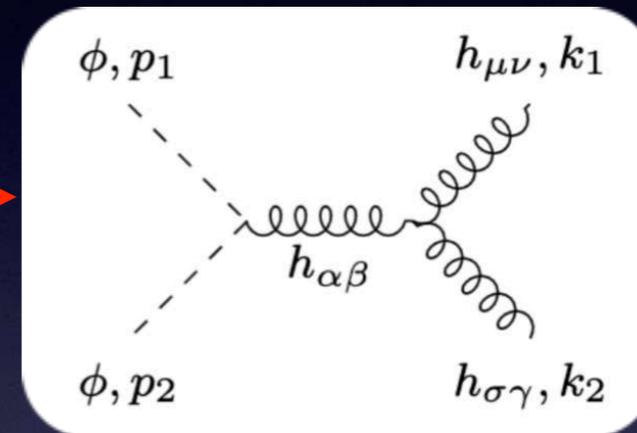
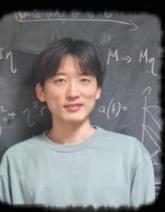
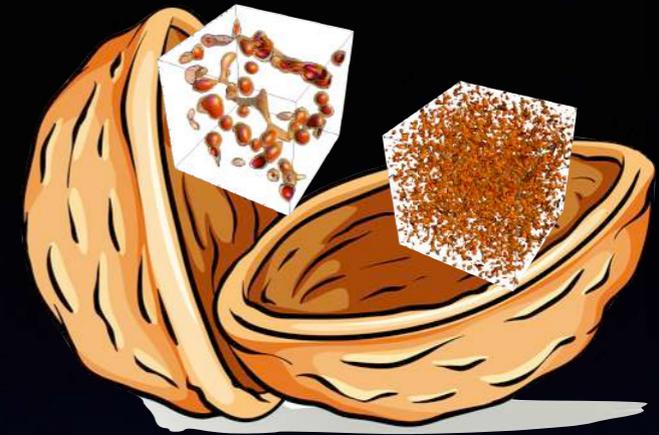
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Feynman
approach

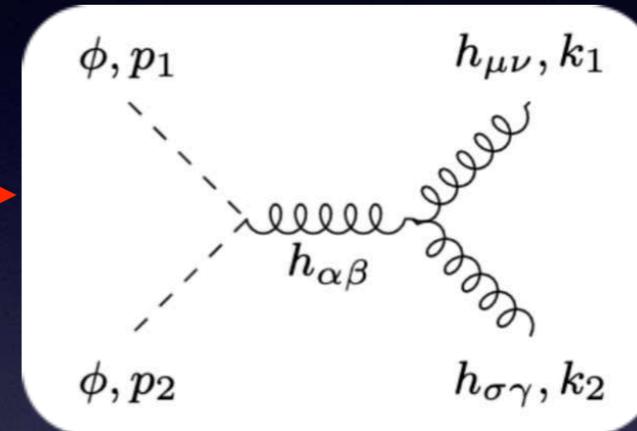
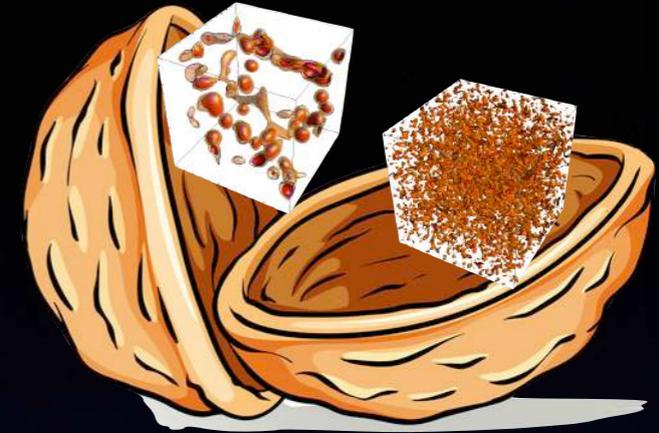
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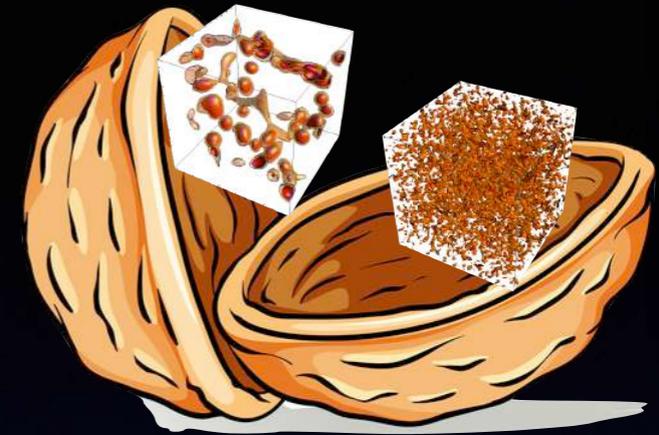
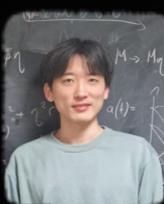
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Feynman
approach

Bogoliubov approach : this corresponds to the mode which *never exited the horizon* during inflation

Reheating in a nutshell

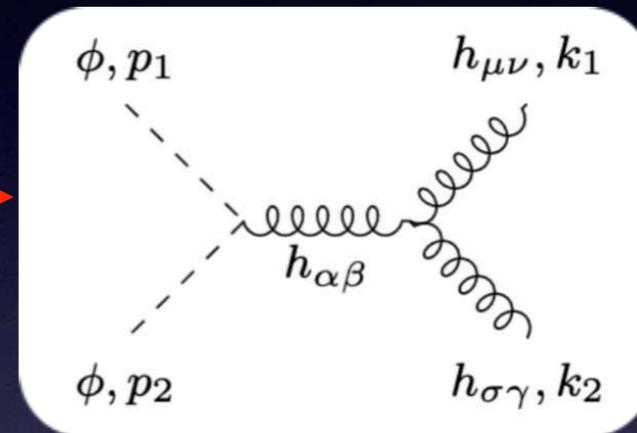


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Feynman approach

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IR PGW (bogo) :

UV PGW (\Leftrightarrow Feynman)

