Beyond the Higgs with Amplitudes

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Reuven Balkin, Gauthier Durieux, Teppei Kitahara, YS, Yaniv Weiss '21

expanding on methods from:
YS Weiss '18
Durieux Kitahara YS Weiss '19
Durieux Kitahara Machado YS Weiss '20

Planck 2025

Higgs discovery

1890s

beta decay

EFT footprints of W

1982

W

2012

h

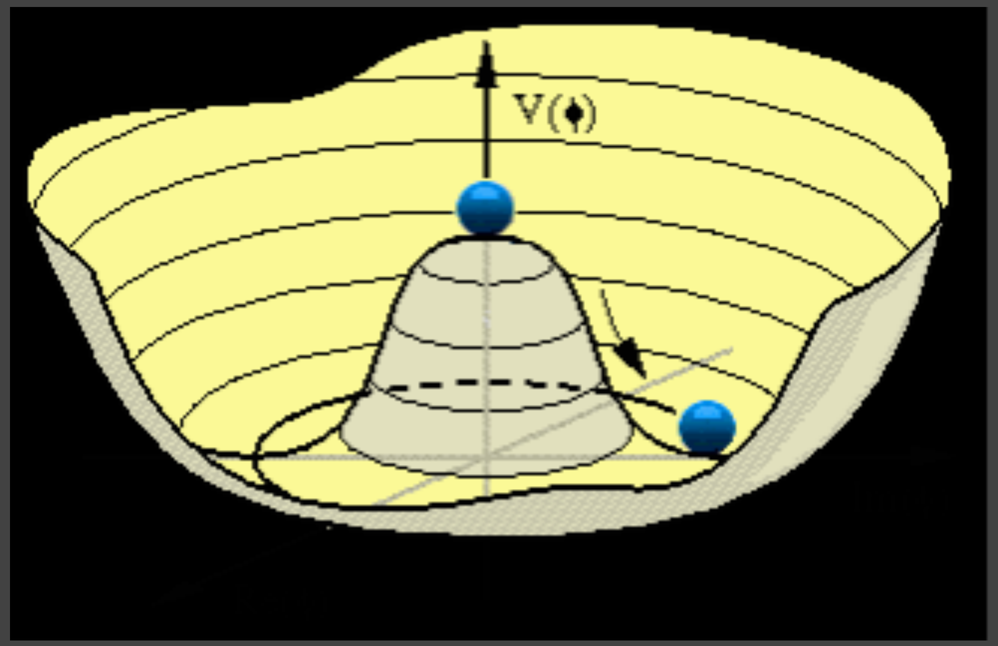
now measuring Higgs

& electroweak sector

interactions

for first time

simple parametrization in terms of Higgs potential:



"by hand:"

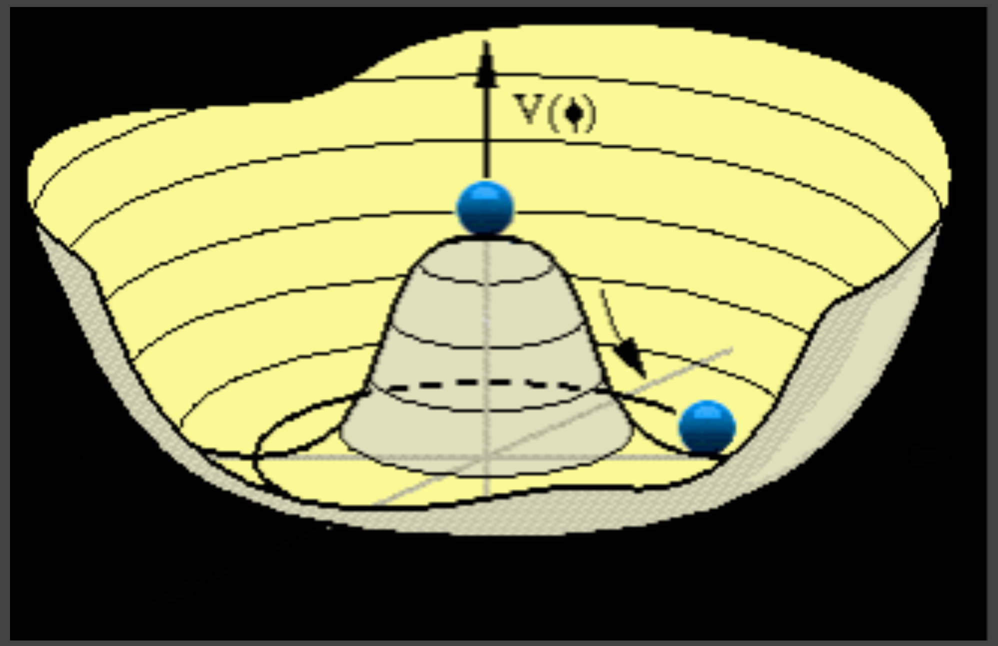
? minimum away from origin

? 246 GeV scale

? stable against radiative corrections

motivate new physics

simple parametrization in terms of Higgs potential:



"by hand:"

? minimum away from origin

? 246 GeV scale

? stable against ra

motivate new

tuned: but not enough to give up on Naturalness

$$10^{-4} \neq 10^{-32}$$

Higgs discovery

1890s

beta decay

EFT footprints of W

1982

W

2012

h

??

EFTs

Higgs discovery

1890s

beta decay

EFT footprints of W

1982

W

2012

h

??

EFTs

- test of SM
- search for new particles above LHC reach

EFT extensions of SM (electroweak): challenges:

- bases of SMEFT operators: ~1000 @ dim-6, more @ dim-8 ..

    counting, explicit constructions [field redefinitions, IBP, EOMs]

- derivation of low-E SMEFT predictions: Lagrangian in broken theory:

    redefine parameters from “input” physical masses, couplings

- comparing to experiment: a multi-dim mapping

$$\mathscr{L} = \sum_i c_i \mathscr{O}_i(\phi_1, \ldots, \phi_n) \;\leftrightarrow\; \text{observables}$$

global analyses

+ different EFT extensions:

SM only or SM + a few light new fields

SMEFT: SU(3)xSU(2)xU(1) at Λ

h in Higgs doublet; large scale separation possible: $\Lambda \gg v$

HEFT: SU(3)xU(1)$_{EM}$ h gauge singlet; no scale separation: $\Lambda \sim v$

counting dimensions ambiguous, UV matching ambiguous

eg Alonso Jenkins Manohar ‘15

…

Dawson Fontes Quezada-Calonge Sanz-Cillero ‘23

“sick” EFT : eg integrated out fields with masses from EWSB

HEFT/SMEFT distinction can be obscured by field redefinitions

Cohen Craig Lu Sutherland ’20

…

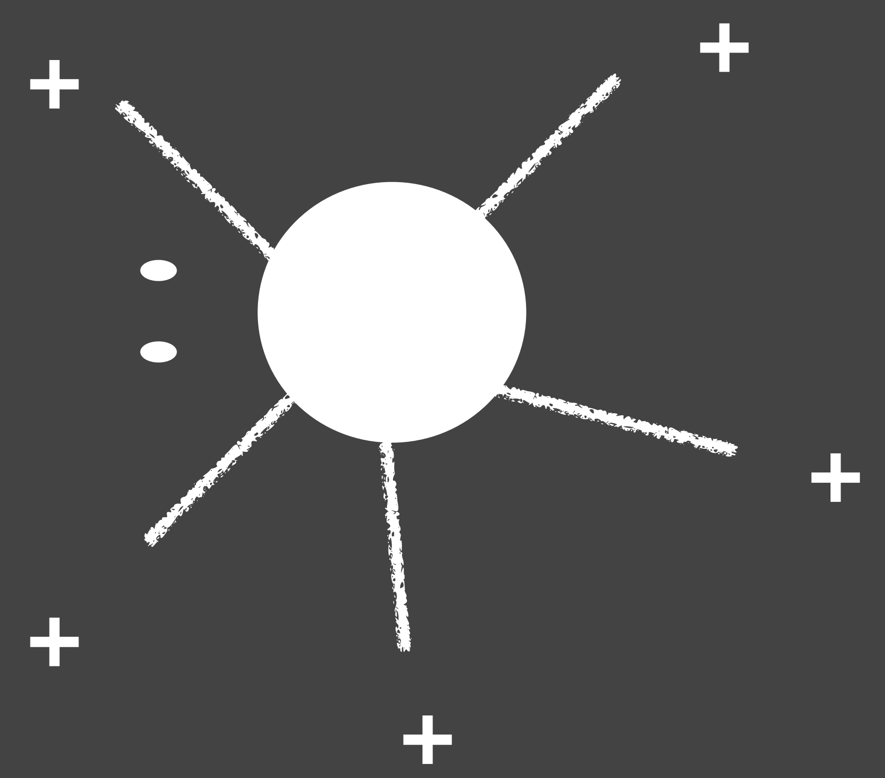
Formulate using **(on-shell) amplitudes:** **construction & counting**

YS Weiss '18

amplitudes: the whole is often SMALLER than the sum of its parts:

gauge boson amplitudes: many Feynman diagrams (~10 million for tree 10-gluon):

Mangano Parke review



$= 0$

Parke & Taylor (1986): n-gluons helicity **amplitudes-squared** of definite helicities:

$$|\mathcal{M}_n(+++++\ldots)|^2 = c_n(g,N)\,[\,0 + O(g^4)\,] \tag{1}$$

$$|\mathcal{M}_n(-++++\ldots)|^2 = c_n(g,N)\,[\,0 + O(g^4)\,] \tag{2}$$

$$|\mathcal{M}_n(--+++\ldots)|^2 = c_n(g,N)\,\Big[\,(1\cdot 2)^4 \sum_P \frac{1}{(1\cdot 2)(2\cdot 3)(3\cdot 4)\ldots(n\cdot 1)} + O(N^{-2}) + O(g^2)\,\Big] \tag{3}$$

where $c_n(g,N) = g^{2n-4}N^{n-2}(N^2-1)/2^{n-4}n$ and $(i\cdot j) = p_i\cdot p_j$. The sum is over all permutations, P, of $1\ldots n$. Eqn(3) has the correct dimensions for a so much more efficient than Feynman diagrams

gauge fields —> spin-1 massless particles 2 dof's only

in fact: amplitude bootstrap: gauge symmetry is an *output* rather than an input

bootstrapping amplitudes:

- Lorentz
- global internal symmetries
- Locality
- Unitarity

$\Bigg\}$ rich beautiful & very diverse program

starting with Bern Dixon Kosower ~90's

…

now widely used for EFT applications too gauge symmetry is an *output* rather than an input

consistent interactions of spin-1 particles

in particular: Lie groups   standard QFT textbook example (Schwartz)   Benincasa Cachazo ’08

from

1. 3-points —> group structure consts
2. factorization of 4-pts on 3-pts —> Jacobi identity

also for non-zero mass:

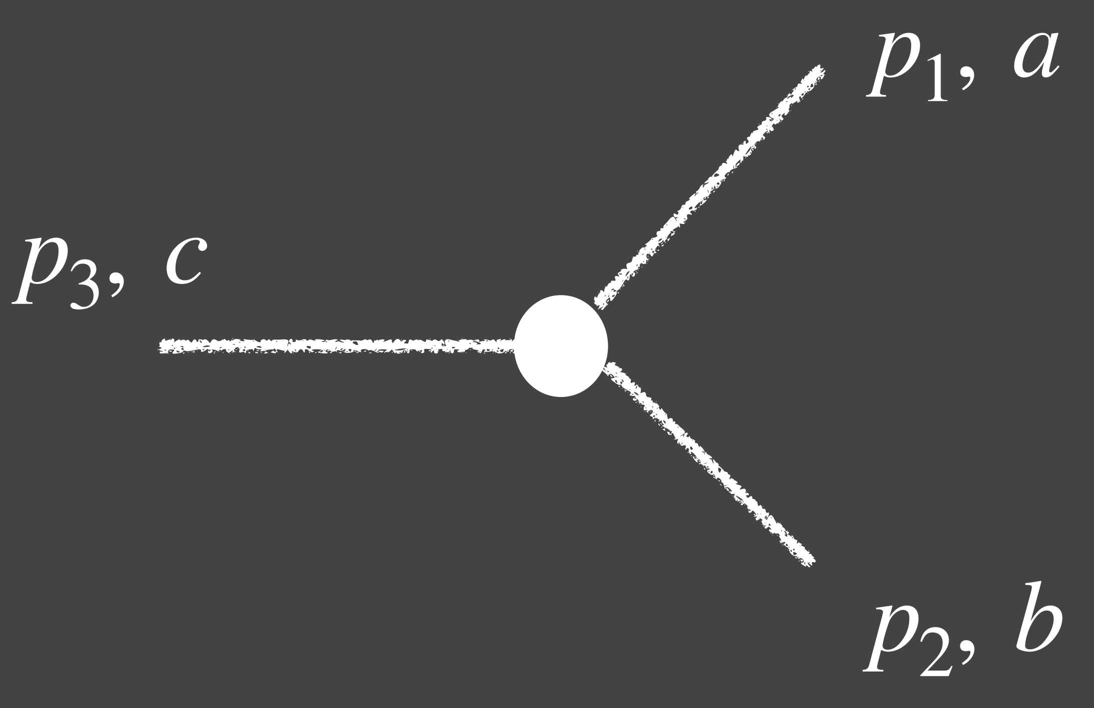
Durieux Kitahara YS Weiss ’19
Liu Yin ‘22

what are the most general interactions of spin-1 particles?

$$C^{abc}\ \big(\langle\mathbf{12}\rangle[\mathbf{23}]\langle\mathbf{31}\rangle+[\mathbf{12}]\langle\mathbf{23}\rangle[\mathbf{31}]\ +\ \text{perm}\big)/M^2\ +\ \mathcal{O}(\text{mass-splittings})$$

Lorentz: completely antisymmetric

$$+C'^{abc}\ \langle\mathbf{12}\rangle\langle\mathbf{23}\rangle\langle\mathbf{31}\rangle/\Lambda^2+\mathbf{C}''^{\mathbf{abc}}\ [\mathbf{12}][\mathbf{23}][\mathbf{31}]/\Lambda^2$$



—> C^{abc} completely antisymmetric
Lie group structure constants

not surprising: gauge symmetry broken by mass terms — restored at high-E

reintroduce via: $W_\mu\ \rightarrow -\dfrac{i}{g}\partial_\mu U\ U^{-1}\ +\ UW_\mu U^{-1}$

(field redefinition; gauge symmetry = redundancy of description) amplitude formulation of EFTs (for standard model/EWSB):

★ blind to field redefinitions — notoriously difficult in EFTs: operator bases; HEFT/SMEFT

★ power of Lorentz symmetry: gauge symmetry is intimately tied to/follows from Lorentz

(unbroken/broken)

On-shell applications to EFTs (massless & massive)

very partial list!

- count (& construct) bases of EFT operators

YS Weiss ’18
Ma Shu Xiao ’19
Remmen Rodd ’19
Aoude Machado ’19
Durieux Kitahara YS Weiss ’19
Li Ren Shu Xiao Yu Zheng ’20
Durieux Machado ’20
De Angelis Accettulli-Huber ’21
De Angelis ’22
…
also used in Henning Melia Murayama ‘15

- selection rules: explain zeros in
    - matrix of anomalous dimensions of EFT operators (loop cuts & generalized cuts)

Cheung Shen ’15
Bern Parra-Martinez Sawyer ’20

    - interference of SM x EFT amplitudes (tree)

Azatov Contino Machado Riva ‘16

- derive anomalous dimensions of EFT operators (loop cuts & generalized cuts)

Barratella Fernandez von Harling Pomarol ’20
Bern Parra-Martinez Sawyer ’20
Jiang Ma Shu ’20
De Angelis Accettulli-Huber ’21
…

…

in some of these:

$$\begin{tikzcd} \text{amplitude} \arrow[d] & \\ \mathscr{L} \arrow[d] & \\ \text{amplitude} \arrow[r] & \text{LHC} \end{tikzcd}$$

$$\begin{tikzcd} \text{amplitude} \arrow[d] \arrow[ddr] & \\ \mathscr{L} \arrow[d] & \\ \text{amplitude} \arrow[r] & \text{LHC} \end{tikzcd}$$

search for new physics:
0th order: new structures
mapping amplitudes <—> observables

Outline

amplitude basics

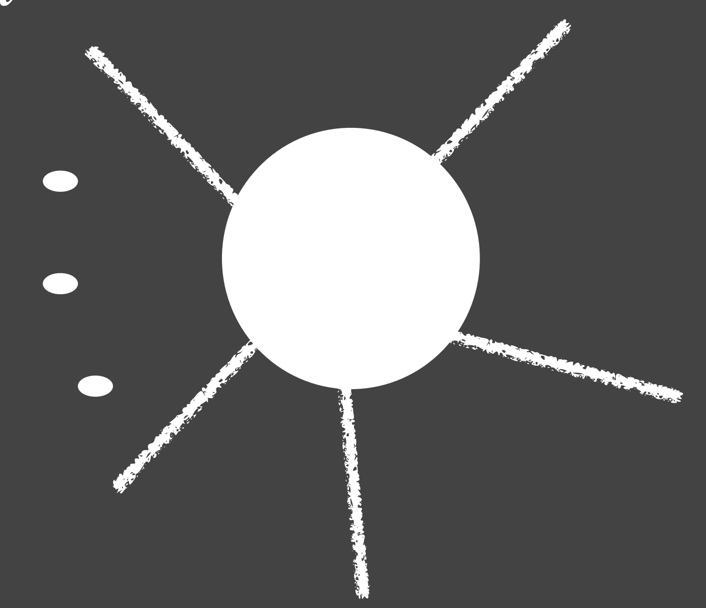
amplitude construction of EFT bases

bottom-up construction: HEFT

on-shell Higgsing: low-E SMEFT   (d=6 + VV production @ d=8)

power of Lorentz: gauge theories, Higgsing, perturbative unitarity & gauge symmetry

writing amplitudes: spinor variables:



amplitude is function of momenta, polarizations ($s = 1/2, \quad s = 1$)

all can be written in terms of massless 2-component spinors:

$$u_+(p) = p] \quad \text{or} \quad u_-(p) = p\rangle$$

$$\bar{u}_+(p) = [p \qquad \bar{u}_-(p) = \langle p$$

—> unified description of amplitudes of different spins

massless-massive relations (Higg mechanism) & relation to Lorentz made transparent

Little group covariant massive spinors of Arkani-Hamed Huang Huang ‘17 massless momenta:

$$p_i = i\rangle[i \quad :$$

Little group: U(1) = Lorentz transformations keeping p_i invariant:

$$i] \rightarrow e^{i\phi}\ i] : \quad charge + 1$$

$$i\rangle \rightarrow e^{-i\phi}\ i\rangle : \quad charge - 1$$

massive momenta: LG covariant spinor formalism

Arkani-Hamed Huang Huang '17

$$p_i = i\rangle^I\,[i_I \qquad I = 1,2$$

Little group SU(2) = Lorentz transformations keeping p_i invariant:

$$i\rangle^I \to W^I_J\ \ i\rangle^J \qquad [i_I \to (W^{-1})^J_I\ \ [\,i_J$$

external leg i :

massless

$i, h = 1/2$ $\quad i]$

$i, h = -1/2$ $\quad i\rangle$

$i, h = +1$ $\quad i]i]$

$i, h = -1$ $\quad i\rangle i\rangle$

massive

$i,\ \ s = 1/2$ $\quad \mathbf{i}]$ or $\mathbf{i}\rangle$

$i,\ \ s = 1$ $\quad \mathbf{i}]\mathbf{i}]$ or $\mathbf{i}\rangle\mathbf{i}\rangle$ or $\mathbf{i}\rangle\mathbf{i}]$

$$\mathbf{i}]\mathbf{i}] \equiv i]^{\{I}\, i]^{J\}}$$

can construct any SU(2) LG rep from symm
combinations of doublets

amplitude = function of spinor products $\langle ij \rangle, \quad [ij]$, or $\langle \mathbf{ij} \rangle, \quad [\mathbf{ij}]$

& Lorentz invariants $s_{ij} = (p_i + p_j)^2$

amplitudes transform under external particle little group

U(1) massless / SU(2) massive

—> selection rules on allowed (spinor) structures

massless-massive amplitude relations

high-energy limit:

$$p = p^{I=1} + p^{I=2} \qquad \equiv k + q$$

HE: $k = \mathcal{O}(E)$ $(\sim p)$ $\qquad q = \mathcal{O}(m^2/E)$

eg, only $\mathbf{p}]^{I=1} \sim p]$ survives; $\qquad \mathbf{p}]^{I=2} = q]$ subleading

—> HE limit: simply unbold spinor structures

massless <—> massive amplitudes : bolding <—> unbolding

Arkani-Hamed Huang Huang ‘17

amplitude construction of EFT bases

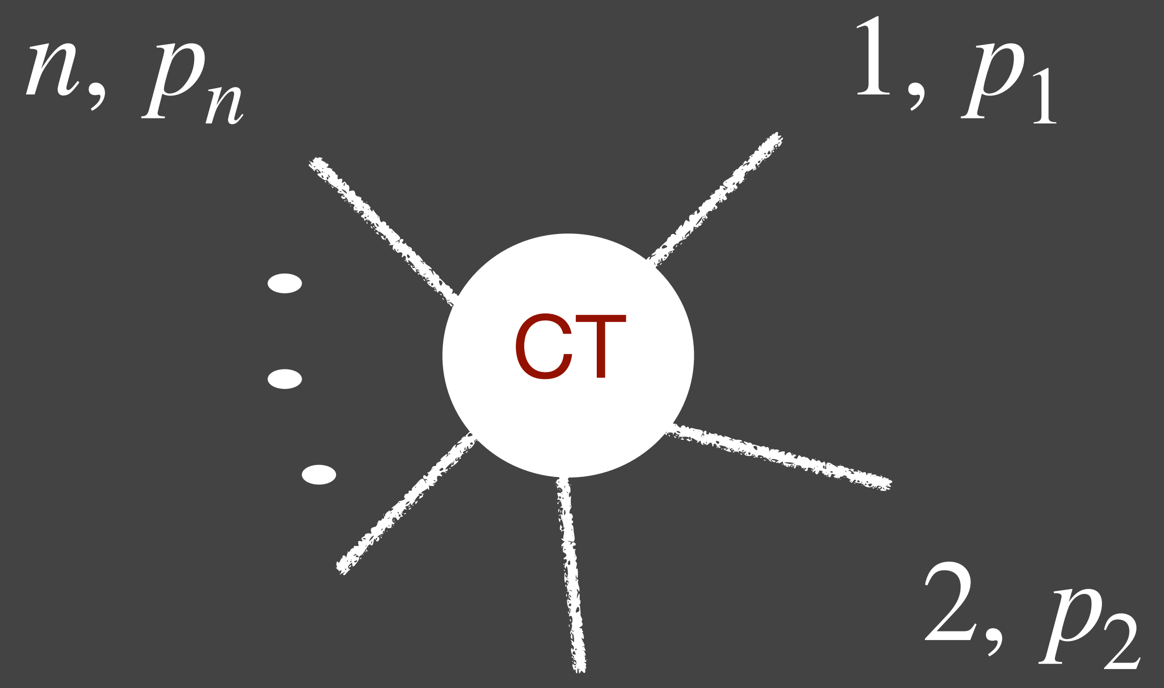
amplitude construction of EFT bases

input: massive (and/or massless) particles, global symmetries

- most general 3-points (renormalizable + higher-dim): consistent with symmetries: Lorentz—LG + internal
- higher-point contact terms: consistent with symmetries: Lorentz—LG + internal, manifestly local in kinematic variables (pole free)

$$\mathscr{L} = \sum_i c_i \mathcal{O}_i(\phi_1, \ldots, \phi_n)$$

1-1 correspondence



$$\propto \sum_i c_i \, (\cdots)_i$$

on top of factorizable part:

contact-terms:

YS Weiss ‘18
Durieux Kitahara YS Weiss; Durieux Machado ’19
Durieux Kitahara Machado YS Weiss ’20
…

$$\frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}}\, P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

local: no poles

$$\frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

carries LG weight; “stripped” off

all Lorentz invariants s_{ij}

“stripped contact term” SCT

$$\frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

carries LG weight; “stripped” of all Lorentz invariants s_{ij}

“stripped contact term”   SCT

polynomial in Lorentz invariants s_{ij}

subject to kinematical constraints,

eg, $s_{12} + s_{13} + s_{23} = \sum m^2$

derivative expansion

$$\frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}}\,P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

carries LG weight; “stripped” of
all Lorentz invariants s_{ij}
“stripped contact term”   SCT

polynomial in Lorentz
invariants s_{ij}
subject to kinematical constraints,
eg, $s_{12} + s_{13} + s_{23} = \sum m^2$

derivative expansion

1. Find SCT basis
2. Multiply SCT by polynomial of invariants
3. Make sure no redundancies re-introduced

for s=0,1/2,1 relevant for SM-EFTs:

m=0 generic SCTs & CTs

full list of 4-pt massive SCTs

Durieux Machado ’19

Durieux Kitahara Machado YS Weiss ’20

use this for a bottom-up construction of

EFT extensions of SM:

input: physical particles (massive & massless)

SU(3)xU(1)

higgs = SU(3)xU(1) singlet

gives **HEFT** amplitudes

4-pt HEFT contact-terms:

Liu Ma YS Waterbury ’23

- CT basis with $E^2, \quad E^3, \quad E^4$ growth

[Dong Ma Shu Zhou ’22: HEFT operators]

- corresponding to $d \leq 8$ HEFT operators

- clear identification of operator dimension from dim-analysis:

factors of $p]p\rangle$ (external massive vector) $\rightarrow p]p\rangle/M$

any extra powers of E compensated by powers of Λ

—> read off dimension of operator (4-pt ampl is dim-less)

—> E/M terms in amplitudes; reflect non-locality

Durieux Kitahara Machado YS Weiss ’20

vs SMEFT where bad HE behavior must cancel (will see: gauge invariance <—> perturbative unitarity) full set of HEFT (+SMEFT) contact terms with E^2 growth: (mostly dim-6 operators)

LOW ENERGY AMPLITUDES

Massive amplitudes	E^2 contact terms
$\mathcal{M}(WWhh)$	$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{WWhh}(\mathbf{12})^2$
$\mathcal{M}(ZZhh)$	$C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$
$\mathcal{M}(gghh)$	$C^{\pm\pm}_{gghh}(12)^2$
$\mathcal{M}(\gamma\gamma hh)$	$C^{\pm\pm}_{\gamma\gamma hh}(12)^2$
$\mathcal{M}(\gamma Zhh)$	$C^{\pm}_{\gamma Zhh}(1\mathbf{2})^2$
$\mathcal{M}(hhhh)$	C_{hhhh}
$\mathcal{M}(f^cfhh)$	$C^{\pm\pm}_{ffhh}(\mathbf{12})$
$\mathcal{M}(f^cfWh)$	$C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^cfZh)$	$C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^cf\gamma h)$	$C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{1}3)(\mathbf{2}3)$
$\mathcal{M}(q^cqgh)$	$C^{\pm\pm\pm}_{qqgh}(\mathbf{1}3)(\mathbf{2}3)$
$\mathcal{M}(f^cff^cf)$	$C^{\pm\pm\pm\pm,1}_{ffff}(\mathbf{12})(\mathbf{34})$, $C^{--++}_{ffff}\langle\mathbf{12}\rangle[\mathbf{34}]$, $C^{-+-+}_{ffff}\langle\mathbf{13}\rangle[\mathbf{24}]$, $C^{-++-}_{ffff}\langle\mathbf{14}\rangle[\mathbf{23}]$ $C^{\pm\pm\pm\pm,2}_{ffff}(\mathbf{13})(\mathbf{24})$, $C^{++--}_{ffff}[\mathbf{12}]\langle\mathbf{34}\rangle$, $C^{+-+-}_{ffff}[\mathbf{13}]\langle\mathbf{24}\rangle$, $C^{+--+}_{ffff}[\mathbf{14}]\langle\mathbf{23}\rangle$

$(12) = [12]$ or $\langle 12\rangle$

bold: massive particle

C's: Wilson coefficients

most suppressed by $\bar{\Lambda}^2$
(amplitude dim-less)

Ma Liu YS Waterbury 2301.11349

$C^{\pm\pm}_{WWhh}(\mathbf{12})^2$

Massive amplitudes	E^2 contact terms
$\mathcal{M}(WWhh)$	$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{WWhh}(\mathbf{12})^2$
$\mathcal{M}(ZZhh)$	$C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$
$\mathcal{M}(gghh)$	$C^{\pm\pm}_{gghh}(12)^2$
$\mathcal{M}(\gamma\gamma hh)$	$C^{\pm\pm}_{\gamma\gamma hh}(12)^2$
$\mathcal{M}(\gamma Zhh)$	$C^{\pm}_{\gamma Zhh}(1\mathbf{2})^2$
$\mathcal{M}(hhhh)$	C_{hhhh}
$\mathcal{M}(f^c fhh)$	$C^{\pm\pm}_{ffhh}(\mathbf{12})$
$\mathcal{M}(f^c fWh)$	$C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c fZh)$	$C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c f\gamma h)$	$C^{\pm\pm\pm}_{ff\gamma h}(1\mathbf{3})(\mathbf{23})$
$\mathcal{M}(q^c qgh)$	$C^{\pm\pm\pm}_{qqgh}(1\mathbf{3})(\mathbf{23})$
$\mathcal{M}(f^c ff^c f)$	$C^{\pm\pm\pm\pm,1}_{ffff}(\mathbf{12})(\mathbf{34})$, $C^{--++}_{ffff}\langle\mathbf{12}\rangle[\mathbf{34}]$, $C^{-+-+}_{ffff}\langle\mathbf{13}\rangle[\mathbf{24}]$, $C^{-++-}_{ffff}\langle\mathbf{14}\rangle[\mathbf{23}]$ $C^{\pm\pm\pm\pm,2}_{ffff}(\mathbf{13})(\mathbf{24})$, $C^{++--}_{ffff}[\mathbf{12}]\langle\mathbf{34}\rangle$, $C^{+-+-}_{ffff}[\mathbf{13}]\langle\mathbf{24}\rangle$, $C^{+--+}_{ffff}[\mathbf{14}]\langle\mathbf{23}\rangle$

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$C^{\pm\pm}_{WWhh}(\mathbf{12})^2$ $\quad [\mathbf{12}]^2$ or $\langle\mathbf{12}\rangle^2$ → HEFT: $1/\bar{\Lambda}^2$ SMEFT: $1/\Lambda^2$

$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$ → HEFT: $1/M^2$ SMEFT: $1/\Lambda^2$

SMEFT: this term required to cancel E^2/M^2 of **low-E factorizable** amplitude:

$$C^{00}_{WWhh} = C^{00,\mathrm{fac}}_{WWhh} + C^{00,\mathrm{CT}}_{WWhh}$$

$$\frac{1}{M^2} \qquad \frac{1}{\Lambda^2}$$

not an independent Wilson coeff; determined by 3-pts (bottom-up)

independent Wilson coeff; from dim-6 SMEFT

$C^{\pm\pm}_{WWhh}(\mathbf{12})^2$ $[\mathbf{12}]^2$ or $\langle\mathbf{12}\rangle^2$ $\to$ HEFT: $1/\bar{\Lambda}^2$ SMEFT: $1/\Lambda^2$

$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$ $\to$ HEFT: $1/M^2$ SMEFT: $1/\Lambda^2$

SMEFT: this term required to cancel E^2/M^2 of **low-E factorizable** amplitude:

HEFT: N/A

$$C^{00}_{WWhh} = C^{00,\mathrm{fac}}_{WWhh} + C^{00,\mathrm{CT}}_{WWhh}$$

$$W^\mu W_\mu h^2$$

in principle : $\frac{1}{M^2}$ $\frac{1}{\bar{\Lambda}^2}$ but no real distinction between $M \sim v \sim \bar{\Lambda}$

HEFT: indep CT with indep Wilson coefficient

Full set of HEFT E^3, E^4 terms

HEFT; **naive** SMEFT dim's

# of indep structures/couplings

4.1.7 W^+W^-ZZ

$$
\begin{array}{rcll}
0000: & [\mathbf{12}][\mathbf{34}]\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle, [\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle + (3\leftrightarrow 4) & (4;8) & \# = 2 \\
++00: & [\mathbf{12}]^2[\mathbf{34}]\langle\mathbf{34}\rangle;\ \mathrm{PF} & (6;8) & \# = 2 \\
+0+0: & \{[\mathbf{12}][\mathbf{34}][\mathbf{13}]\langle\mathbf{24}\rangle, [\mathbf{14}][\mathbf{23}][\mathbf{13}]\langle\mathbf{24}\rangle\} + (3\leftrightarrow 4); (1\leftrightarrow 2);\ \mathrm{PF} & (6;8) & \# = 8 \\
00++: & [\mathbf{34}]^2[\mathbf{12}]\langle\mathbf{12}\rangle;\ \mathrm{PF} & (6;8) & \# = 2 \\
+-00: & [\mathbf{13}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{24}\rangle;\ \mathrm{PF} & (6;8) & \# = 2 \\
+0-0: & \{[\mathbf{12}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{34}\rangle + (3\leftrightarrow 4), (1\leftrightarrow 2)\};\ \mathrm{PF} & (6;8) & \# = 4 \\
00+-: & [\mathbf{13}][\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{24}\rangle + (3\leftrightarrow 4) & (6;8) & \# = 1 \\
++++: & \{[\mathbf{12}]^2[\mathbf{34}]^2, [\mathbf{13}]^2[\mathbf{24}]^2 + (3\leftrightarrow 4)\};\ \mathrm{PF} & (8;8) & \# = 4 \\
++--: & [\mathbf{12}]^2\langle\mathbf{34}\rangle^2;\ \mathrm{PF} & (8;8) & \# = 2 \\
-+-+: & [\mathbf{14}]^2\langle\mathbf{23}\rangle^2 + (3\leftrightarrow 4);\ \mathrm{PF} & (8;8) & \# = 2
\end{array}
\tag{9}
$$

At order E^5 several new $vvvv$ SCTs become independent in the $(+000)$, $(+++0)$, and $(++-0)$ helicity categories.

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PF = parity flip
angle <—> square

useful to classify by
"helicity category"
helicities of unbolded SCT

(leading HE amplitude)

do new SCTs appear at higher dim's and where

SCT bases may or may not be exhausted in different HEFT 4pt's:

$$\frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

eg: VVVV: new-independent SCTs typically appear at E^5 but all of type $(\,..\,)^2(\,..\,)^2$

Zhhh: one at dim-7: $\tilde{s}_{12}[\mathbf{121}\rangle + \tilde{s}_{13}[\mathbf{131}\rangle + \tilde{s}_{14}[\mathbf{141}\rangle \quad (7;8) \quad \# = 1$

remaining one at dim-13: $(\tilde{s}_{12} - \tilde{s}_{13})(\tilde{s}_{12} - \tilde{s}_{14})(\tilde{s}_{13} - \tilde{s}_{14})([\mathbf{1231}] - \langle\mathbf{1231}\rangle)$

ZZhh: SCT basis exhausted at dim-8:

$$\begin{array}{cc} [\mathbf{131}\rangle[\mathbf{232}\rangle + [\mathbf{141}\rangle[\mathbf{242}\rangle,\, \tilde{s}_{12}[\mathbf{12}]\langle\mathbf{12}\rangle & (6;8) \quad \# = 2 \\ \tilde{s}_{12}[\mathbf{12}]^2;\ \mathrm{PF} & (8;8) \quad \# = 2 \\ [\mathbf{1(3-4)2}\rangle^2 + \langle\mathbf{1(3-4)2}]^2 & (8;8) \quad \# = 1 \end{array}$$

Results: mapping of all possible kinematic structures in 2 to 2 amplitudes:

completely model-independent

Experiment:

$$\mathscr{A} = \underbrace{\frac{[\,\cdot\cdot\,]\cdot\cdot\langle\,\cdot\cdot\,\rangle}{\Lambda^{\#}}}_{\text{SCT scattering angle and decay angles}} \quad \underbrace{P\left(\frac{s}{\Lambda^2},\frac{t}{\Lambda^2}\right)}_{\text{scattering angle}}$$

? construct observables to isolate

novel structures not appearing in SM

back to the SMEFT: is counting dimensions that useful?

imagine a UV model that gives

$$\mathscr{L}^{(6)} = \sum_i c_i\, \mathscr{O}_i^{(6)}(\phi_1, \ldots, \phi_n)$$

with all c_i of same order

more a UV nightmare rather than dream.. (baroque & tuned)

Results: mapping of all possible kinematic structures in 2 to 2 amplitudes:

completely model-independent

$$\mathscr{A} = \underbrace{\frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}}}_{\substack{\text{SCT}\\ \text{scattering}\\ \text{angle}\\ \text{and}\\ \text{decay angles}}} \; \underbrace{P\left(\frac{s}{\Lambda^2}, \frac{t}{\Lambda^2}\right)}_{\substack{\text{scattering}\\ \text{angle}}}$$

Theory: different SCTs typically from integrating out different UV fields

(with derivative expansion given by polynomial in invariants)

reasonable UV models are likely sparse in SCT space at given dim

see also

Chang Chen Liu Luty ’22

Chang et al ’23

SMEFT amplitude bases
@ low-energy

1. derive amplitudes of massless SU(3)xSU2)xU(1) theory

as above

2. (on-shell) Higgs to get massive amplitudes

on-shell Higgsing

Balkin Durieux Kitahara YS Weiss ’21

Liu Ma Waterbury YS ’23

+ Northey YS Soreq Ueda in progress

main focus here: contact-term part:

starting with contact-terms of massless SMEFT

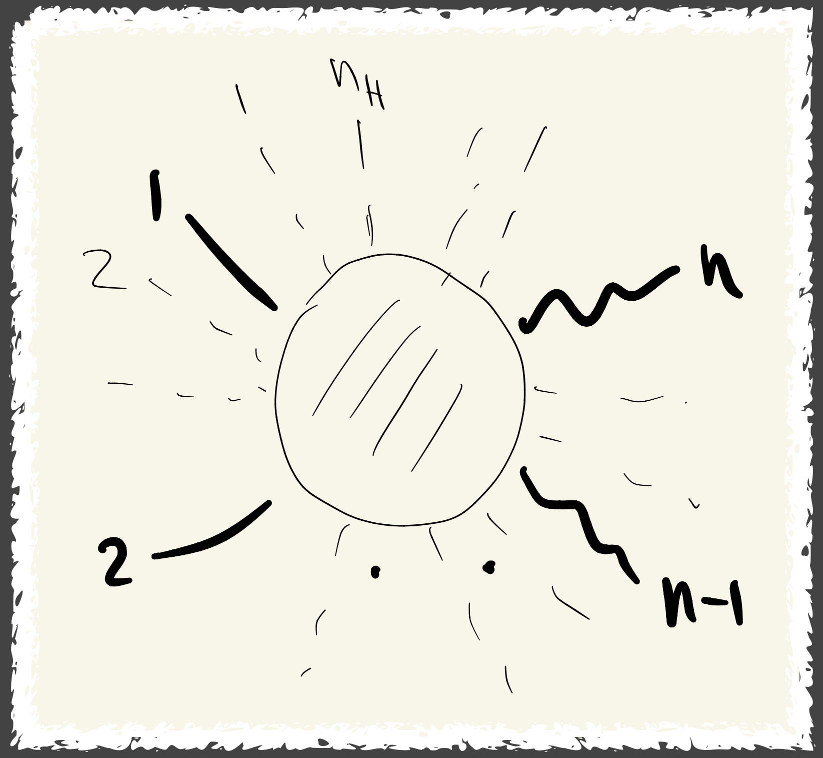
how do we get the massive LE contact terms?

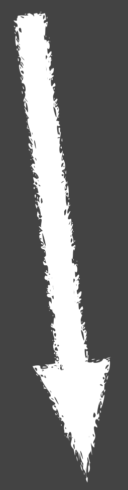
IR unification of UV amplitudes Arkani-Hamed Huang Huang ‘17

N=4 Coulomb branch amplitudes Craig Elvang Kiermaier Slatyer ‘11

high-energy massless amplitudes :

$$A(k_1, .., k_n),\ A(k_1, .., k_n;\ H(q_1), ..),\ \ A(k_1, .., k_n;\ H(q_1), H(q_2), ..),\ \ ...$$





low-energy massive amplitudes: $M(k_1, .., k_n)$

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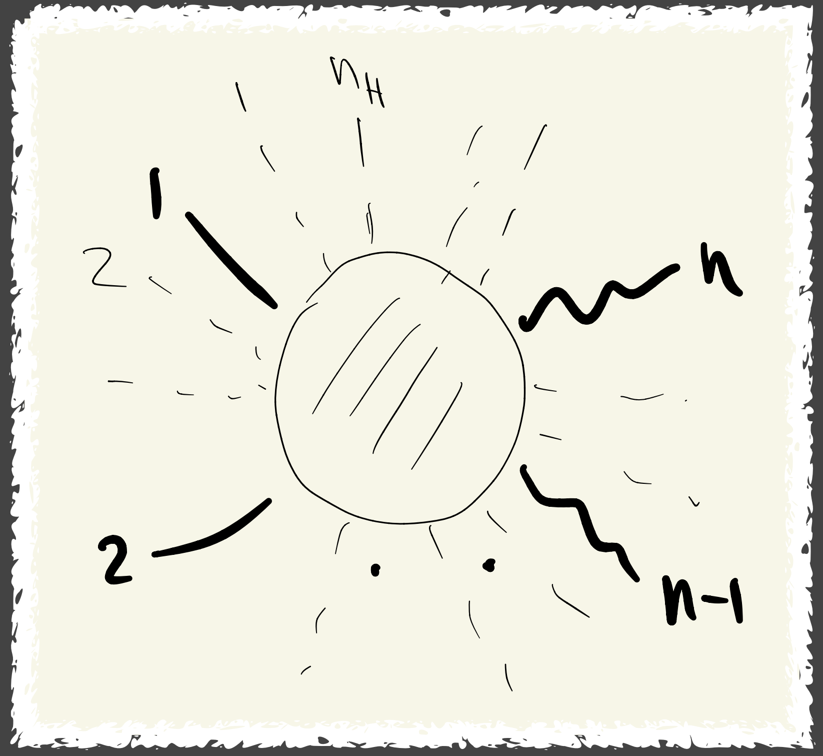

match at $E \gg v$ $\qquad q_i \to\ 0 \qquad \left[\ q_i \sim v^2/E \to 0\ \right]$

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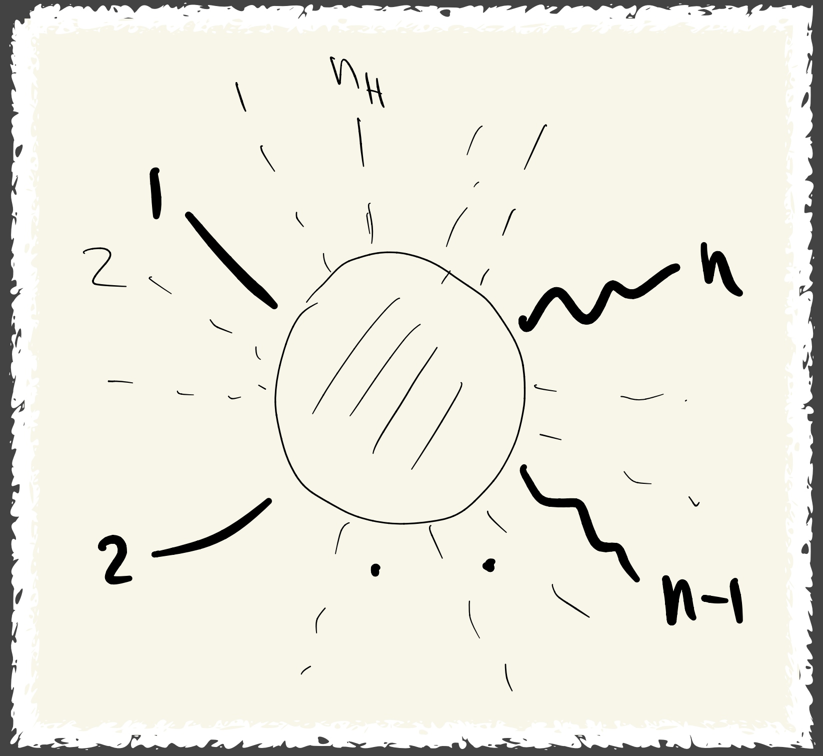

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see also
Cheung Helset Parra-Martinez ‘21

low-energy massive amplitudes: $\quad M(k_1,..,k_n)$

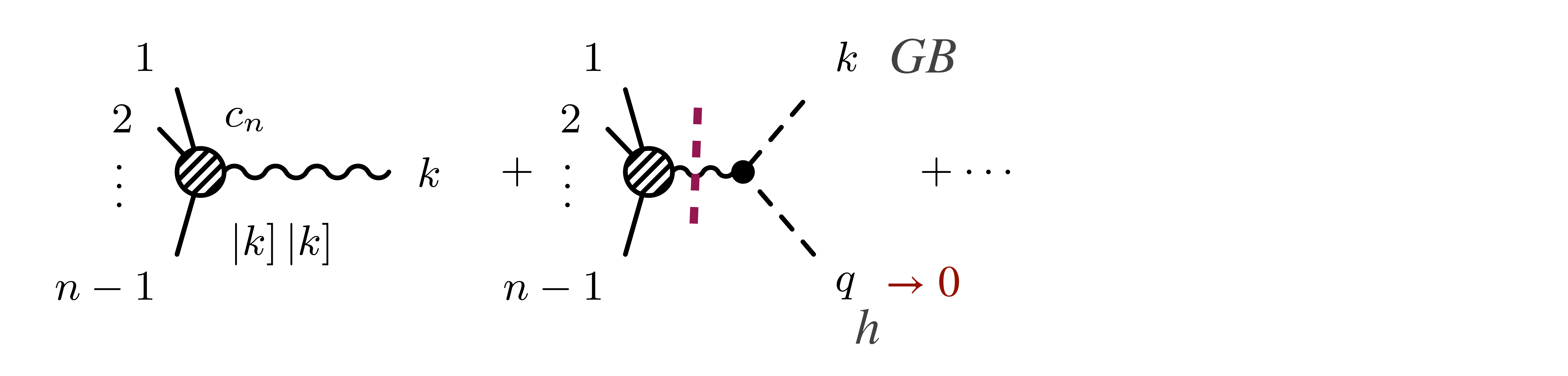
$$M_n(1,\ldots,n)\ =\ A_n(1,\ldots,n) + v \lim_{(qn)\sim v\to 0} A_{n+1}(1,\ldots,n;H(q)) + \cdots$$

1. generating mass: external leg

- light-like k —> p=k+q; $(k+q)^2 = m^2$
- nonzero spin: massless polarization —> massive polarization

n-pt amplitude with external vector n

(n+1)-pt amplitude with external Higgses n, (n+1): $(k+q)^2 = m^2 \to 0$



$$\propto \left(\frac{1}{(k+q)^2} = \frac{1}{m^2} \right) \times \;(A_3 \propto g)$$

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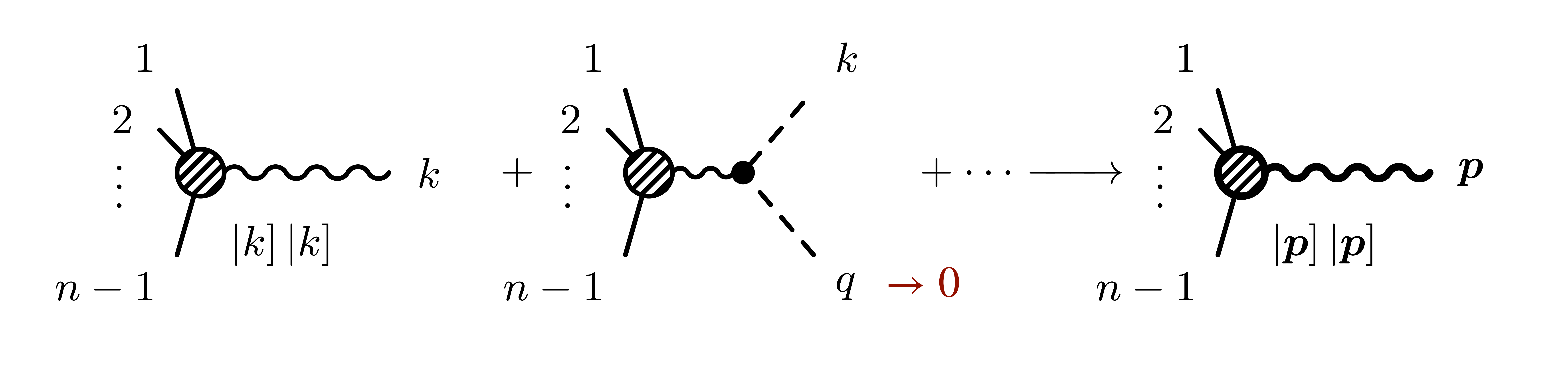

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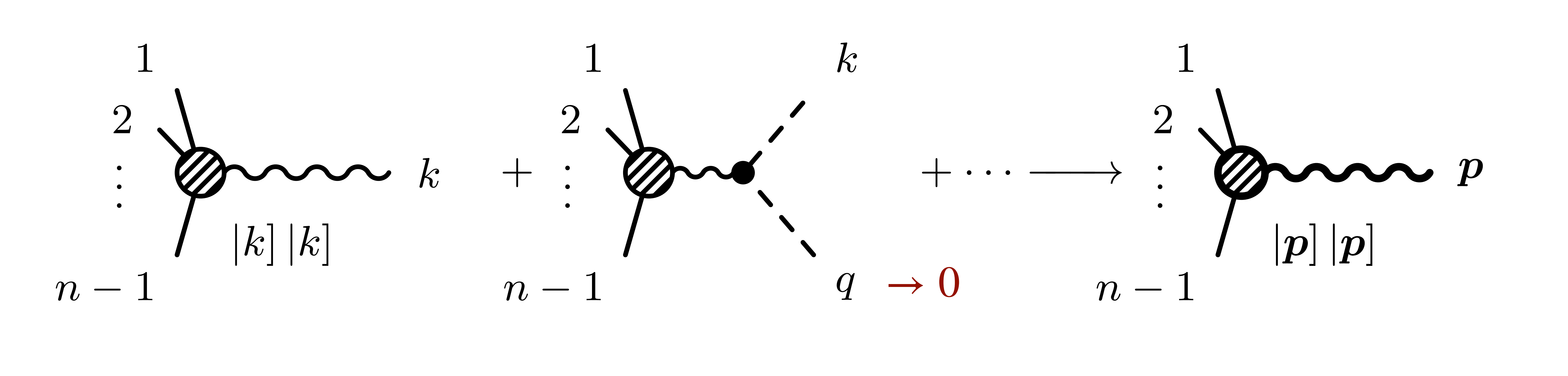
$$\propto \left(\frac{1}{(k+q)^2} = \frac{1}{m^2} \right) \times \ (A_3 \propto g)$$



n-pt amplitude
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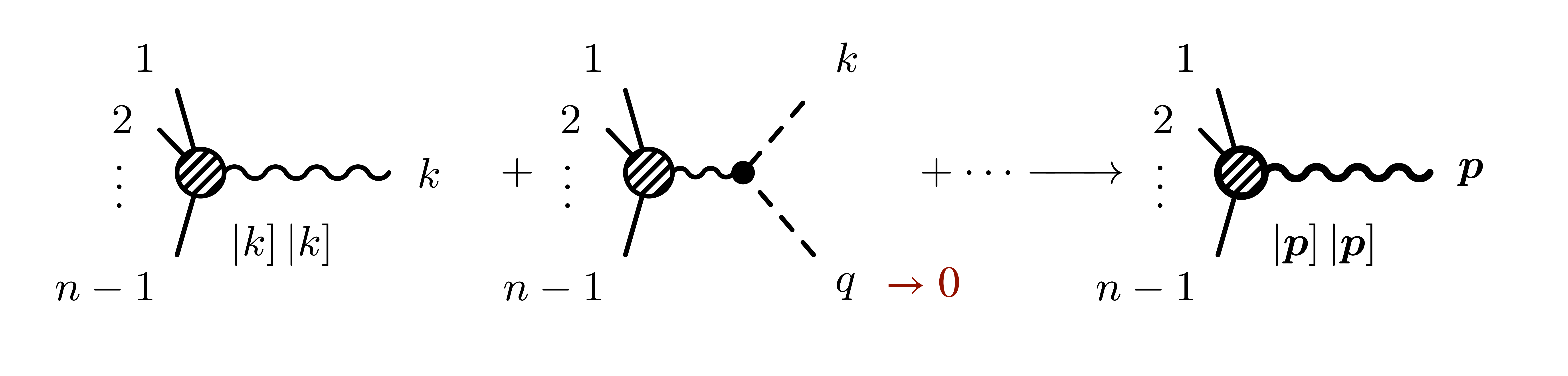
soft Higgs leg supplies
second lightlike
momentum to form
massive momentum

$$\mathbf{p} = k + q$$



n-pt amplitude
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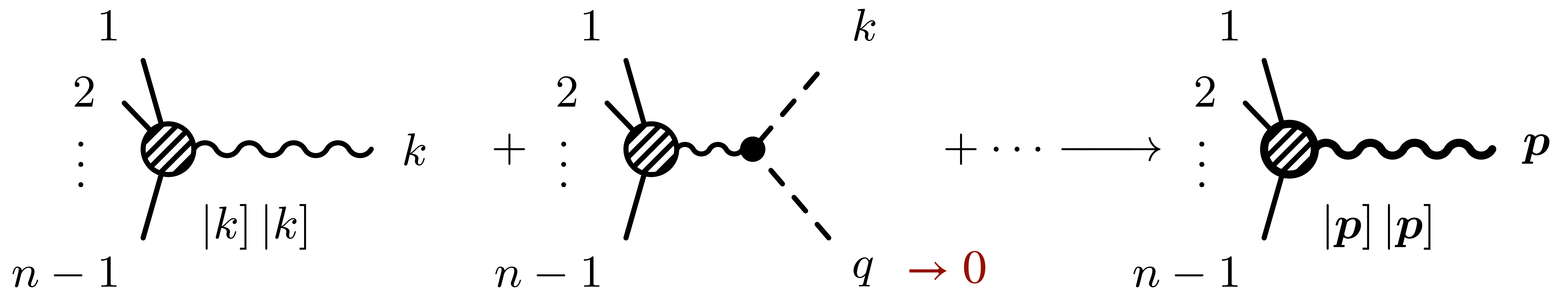
symmetrization over LG indices: exchanging k, q in Higgs legs

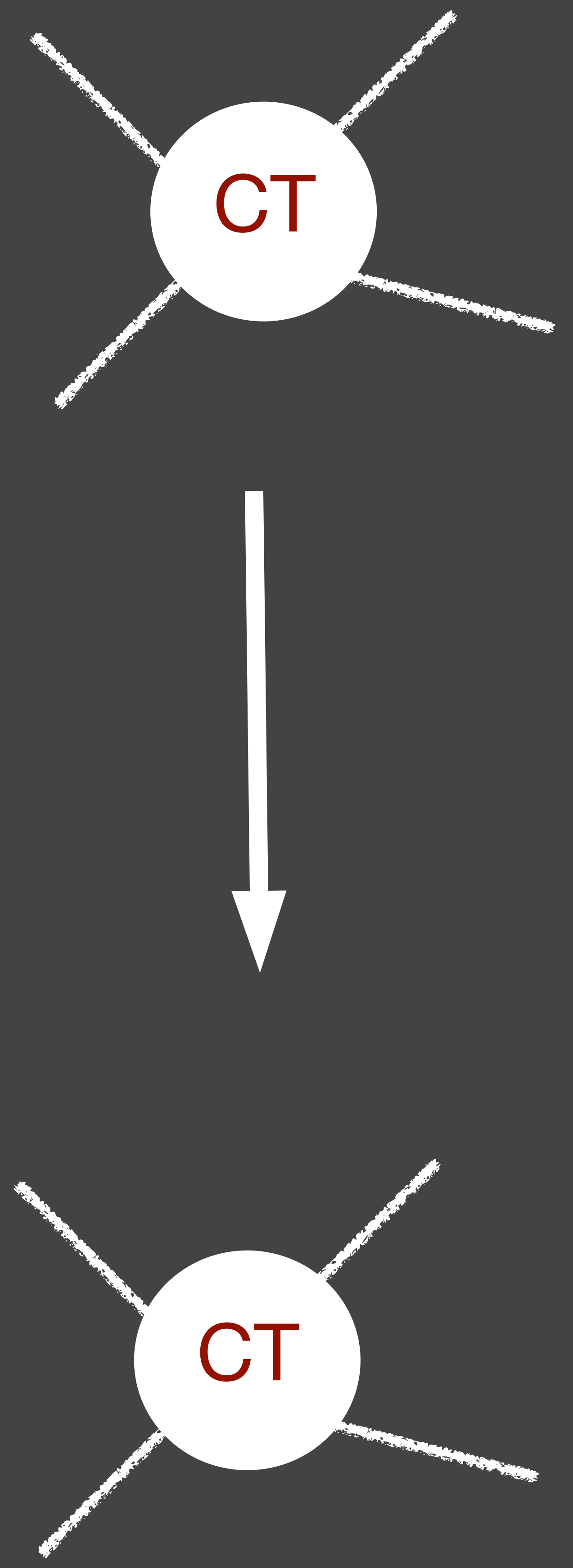


n-pt amplitude with external massive vector n

massless spinor structure gets bolded $k]k] \to \mathbf{p}]\mathbf{p}]$

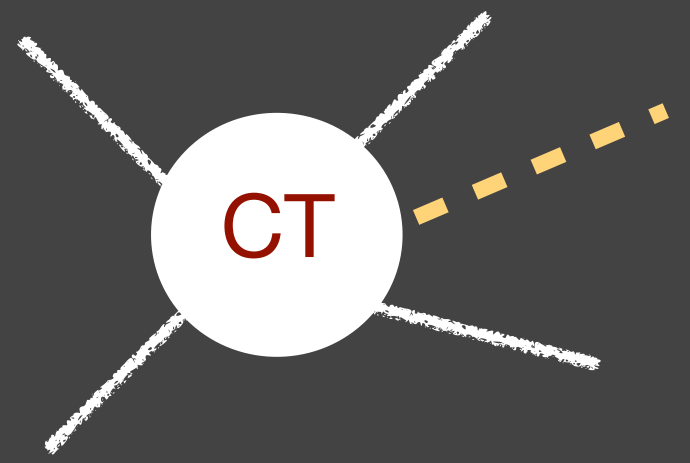
2. Contact-term basis (= EFT input = new couplings)

each high-E contact term: $c_i\,S_i$



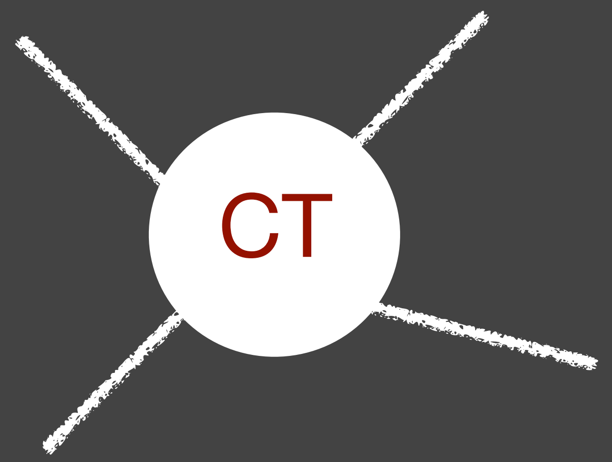
low-E contact term: $c_i\,\mathbf{S}_\mathbf{i}$

may need extra Higgs leg(s):



$c_i \, S_i$

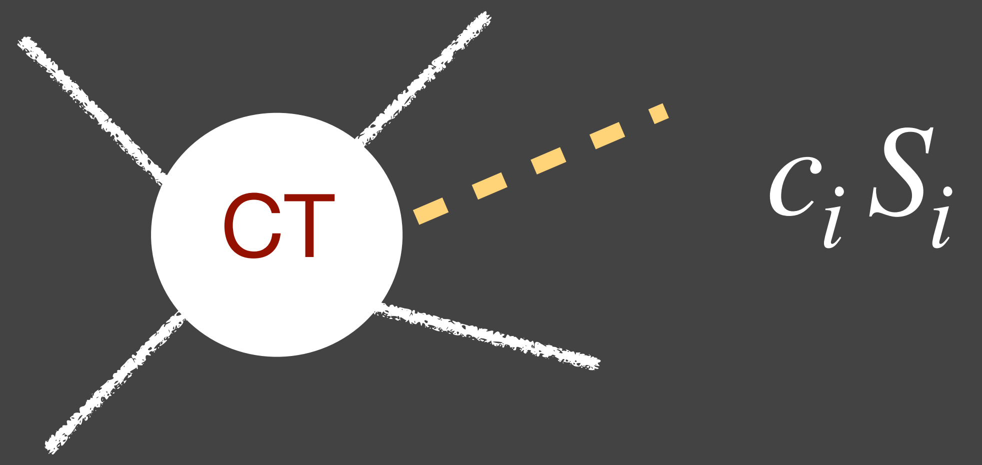
low-E contact term:



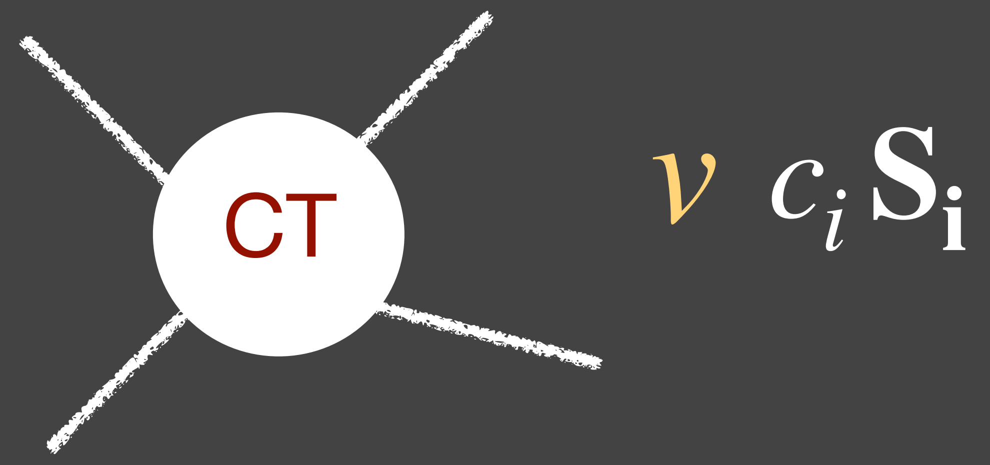
$v \;\; c_i \, \mathbf{S_i}$

$$C_n = c_n + \; v c_{n+1} + \cdots$$

may need extra Higgs leg(s):



low-E contact term:



$$C_n = c_n + v c_{n+1} + \cdots$$

+ higher-orders:

corrections to LE couplings

via factorizable ampls:

wave function renorm's

simple prescription for obtaining LE contact terms:

to get any LE n-pt CT:

consider HE massless CTs with same n legs $+ n_H = 0,1,\dots$ external Higgs legs:

$c_i^{(n_H)}$ x (kinematic structure)

take Higgs momenta = 0

$c_i^{(n_H)} \to v^{n_H}\, c_i^{(n_H)}$ x (kinematic structure)

next treat (kinematic structure) — simply bold as saw above massless fermion: $i] \;\rightarrow\; \mathbf{i}]$

massless vector $i]i] \;\rightarrow\; \mathbf{i}]\mathbf{i}]$

massless scalar with momentum insertion $p_i \;= i]\langle i$

—> 1. massive *scalar* CT with momentum insertion $\mathbf{p_i}$

—> 2. massive *vector CT* $p_i = i]\langle i \;\rightarrow\; \mathbf{i}]\langle\mathbf{i}$

(longitudinal vector from Goldstone boson) from Lorentz symmetry pov:

covariantize massless spinor structure wrt SU(2) LG   bolding

—> Higgsing   mass must be proportional to VEV & coupling (gauge/Yukawa)

VV pair production from dim=8 SMEFT: $V = W, Z, \gamma, g$

Goldberg Liu YS 2407.07945

all low-energy 4-pt CTs generated by dim-8 SMEFT

$VV \to VV$ $\bar{f}f \to VV$ .. (massless fermions)

- dim-8 is leading effect (dim-6 SMEFT merely corrects SM-3pts; easy to see from amplitudes)
- good at $M_V \sim E \ll \Lambda$ (not just high-E where EFT may be unreliable)
- sensitivity to anomalous Higgs self couplings
- up/down quark SU(2) relations broken:

eg: $\bar{u}uW^+W^-$, $\bar{d}dW^+W$ have different Wilson coefficients first happens at dim-8

VV pair production from dim=8 SMEFT: $V = W,\ Z,\ \gamma,\ g$

Goldberg Liu YS 2407.07945

+ differences between HEFT and SMEFT:

- number of independent couplings: eg, for 4W: 21 in HEFT, 13 in SMEFT

- some structures are not generated in the dim-8 SMEFT   due to SU(2)xU(1)

only one VVVV: $W^+W^+W^-W^-$: $[12]\langle 12\rangle[34]^2$

more “missing” entries in ffVV:

$f^c f W^+W^-$ and $f^c f' W^\pm Z$				
Structure/Amplitude	$\overline{U}UW^+W^-$ or $\bar{U}UZZ$	$\overline{D}DW^+W^-$ or $\bar{D}DZZ$	$\overline{U}DW^-Z$	$\overline{D}UW^+Z$
$[\mathbf{23}]\langle \mathbf{14}\rangle[\mathbf{3}(1-2)\mathbf{4}\rangle$	✓	✓	–	–
$[\mathbf{3}4]^2[2(\mathbf{3}-\mathbf{4})1\rangle$	–	–	–	–
$f^c f Z(\gamma/g)$ and $f^c f' W^\pm(\gamma/g)$				
Structure	$\overline{U}UZ(\gamma/g)$	$\overline{D}DZ(\gamma/g)$	$\overline{U}DW^-(\gamma/g)$	$\overline{D}UW^+(\gamma/g)$
$\langle \mathbf{13}\rangle[24][4(1-2)\mathbf{3}\rangle$	✓	✓	–	–
$[\mathbf{3}4]^2[2(\mathbf{3}-4)1\rangle$	–/✓	–/✓	–	–

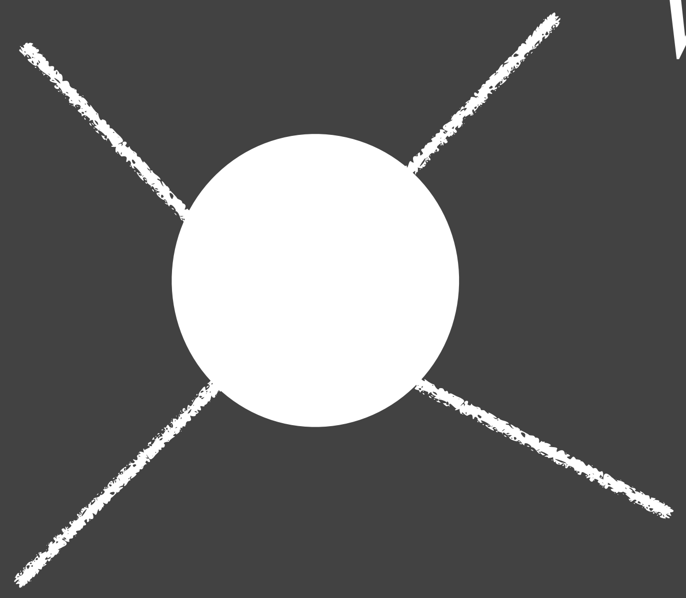
Table 2. Missing kinematic structures in the dimension-8 SMEFT $f^c f W^+W^-$ and $f^c f' W^\pm Z$ amplitudes. A ✓ means a structure is generated at dimension-8 for the given amplitude, while – means it is not.

also transparent in amplitude framework:

gauge invariance <—> perturbative unitarity

low-E SM particles + perturbative unitarity —> SU(3)xSU(2)xU(1) SM

bottom-up HEFT construction: amplitude featuring massive vector:



$W \quad \mathbf{p}]\mathbf{p}], \quad \mathbf{p}\rangle\mathbf{p}\rangle, \; or \quad \frac{1}{M}\mathbf{p}]\mathbf{p}\rangle$

$$\frac{1}{M}\mathbf{p}]\mathbf{p}\rangle \equiv \frac{1}{M} p]^{\{I} p\rangle^{J\}}$$

$(HE: \quad p]^1 \sim \sqrt{E} \quad p\rangle^1 \sim M/\sqrt{E})$

long. polarization: $(I, J) = (1,2)$ get $\frac{p]^{\{I}p\rangle^{J\}}}{M} \sim \frac{\sqrt{E}\cdot\sqrt{E}}{M}$: bad energy growth

transverse polarization: $(I, J) = (1,1)$ get $\sim \frac{p^1\rangle}{M}$

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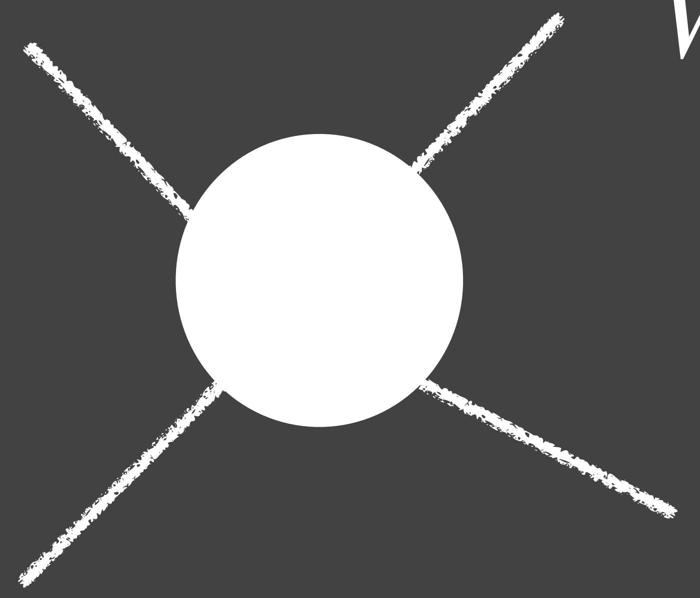

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finite arbitrary spinor

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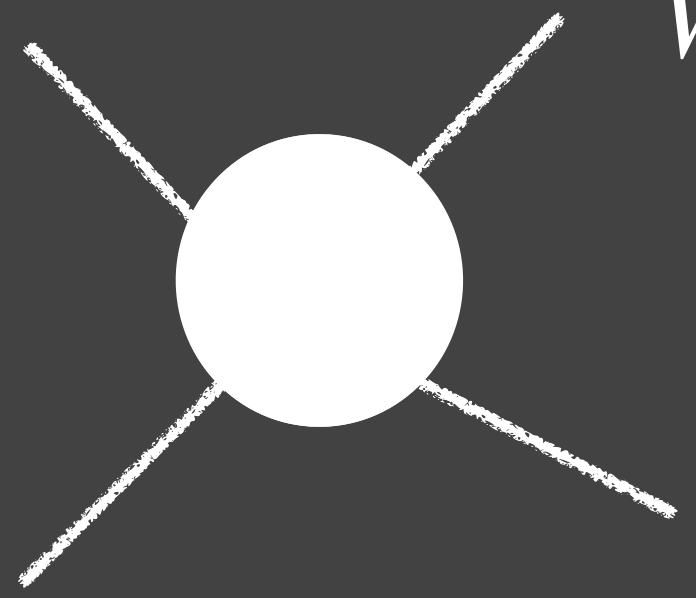

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finite arbitrary spinor

these terms appear in HEFT; cancel in SMEFT cancellation of bad energy growth <—> cancellation of spurious spinor dependence

reference spinor of massless

gauge boson polarization

Liu Ma YS Waterbury ’23

gauge invariance <—> perturbative unitarity

power of Lorentz symmetry: equivalence of gauge invariance & perturbative unitarity:

two components of one massive LG tensor

to conclude:

- LHC: new measurements of Higgs, electroweak interactions — many for first time
- Amplitude formulations of EFTs provide truly model-independent parametrization of possible new effects: search for new physics/test of SM
- Theory: new perspectives on gauge theories: massless & Higgsed

Thank you!

HEFT CT bases inventory

(more results on operators via on-shell)

- *all HEFT 3-points (+matching to SMEFT)* — *Durieux Kitahara YS Weiss '19*
- *[all generic 3-points for spins up to 3]*
- *all generic 4-pt SCTs for spins 0, 1/2, 1]* — *Durieux Kitahara Machado YS Weiss'20*
- *HEFT 4-points: hggg, Zggg, ffVh, WWhh* — *Shadmi et al '18, Durieux et al '19, Balkin et al '21*

+ some full amplitudes (factorizable + contact terms): ffWh, ffZh, WWhh

- *5V (4W+Z etc)* — *De Angelis '21*
- *Higgs, top 4pts in terms of momenta+polarizations* — *Chang et al '22, '23*
- *all HEFT 4pts up to d=8* ← — *Liu Ma YS Waterbury '23*

example: back to WWhh @ dim-6

LE (HEFT) construction: factorizable amplitude features WWh, hhh 3-pt amplitudes

+ WWhh contact terms

can be determined by on-shell Higgsing: eg,

4H CT —>

$$-\frac{c^{+}_{(H^\dagger H)^2} - 3c^{-}_{(H^\dagger H)^2}}{2}\frac{s_{12}}{2\Lambda^2} \longrightarrow \frac{c^{+}_{(H^\dagger H)^2} - 3c^{-}_{(H^\dagger H)^2}}{2}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

factorizable d=4 WWHH amplitude —> Higgses to same term with 1/M^2 (coeff~ gauge coupling)

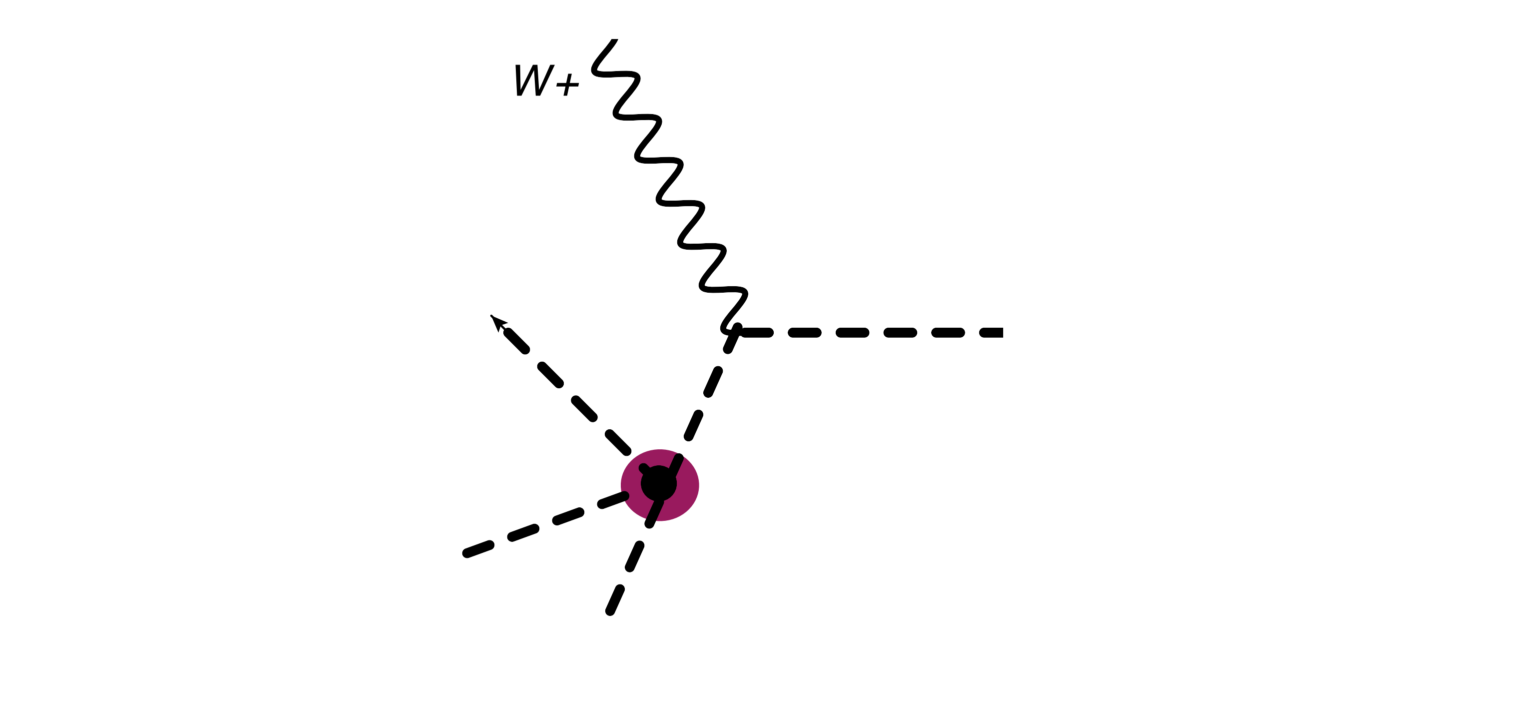
4H CT —> hhh amplitude

example: back to WWhh @ dim-6

d=4 WHH (gauge) —>

$$C^{00}_{WWh}\frac{\langle \mathbf{12}\rangle[\mathbf{12}]}{M_W} \qquad C^{00}_{WWh} = g$$

+ contribution from d=6—> 4H



—>

$$g(1+v^2C)\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{M_W}$$

altogether: all 1/M pieces cancel out in HE limit

could get same result from bottom-up construction plus requiring this cancellation

= perturbative unitarity (all E, E^2 terms suppressed by cutoff Lambda)

or equivalently: requiring cancellation of spurious spinors for transverse Ws

used in Durieux et al to derive relation between Yukawa and fermion mass