The art of being precise

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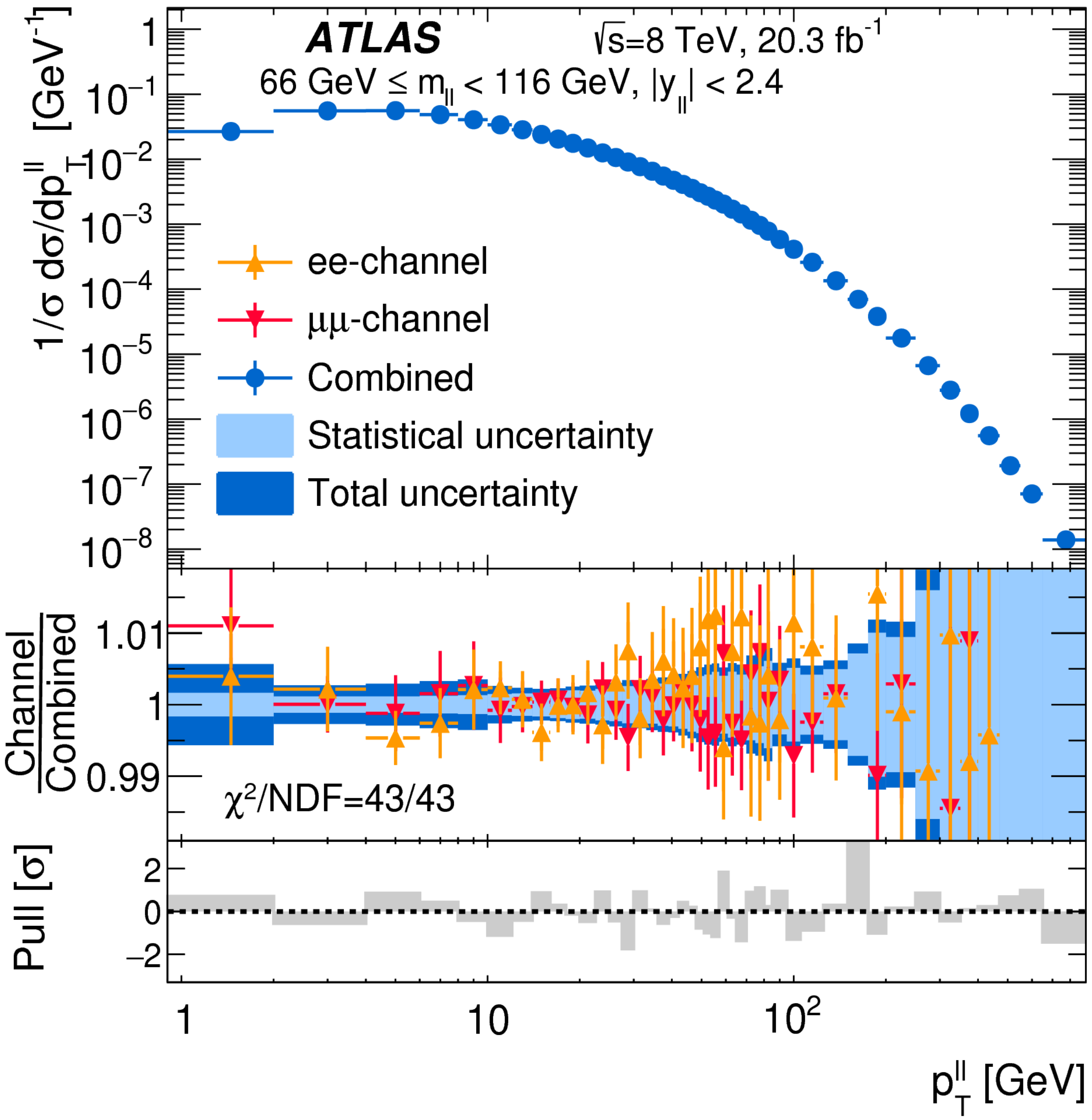
Role of precision

- test the Standard Model (SM) with highest precision and controlled theory uncertainties
- constrain fundamental parameters and parton distribution functions (PDFs)
- probe New Physics via small deviations from SM predictions
- optimise experimental analyses through design of better observables/selection criteria suitable for precision calculations
- guide future collider designs and priorities

Potential of LHC and future machines, in particular e^+e^- colliders, cannot be exploited without precision theory predictions

Precision a reality

Z-boson kinematics to below a percent



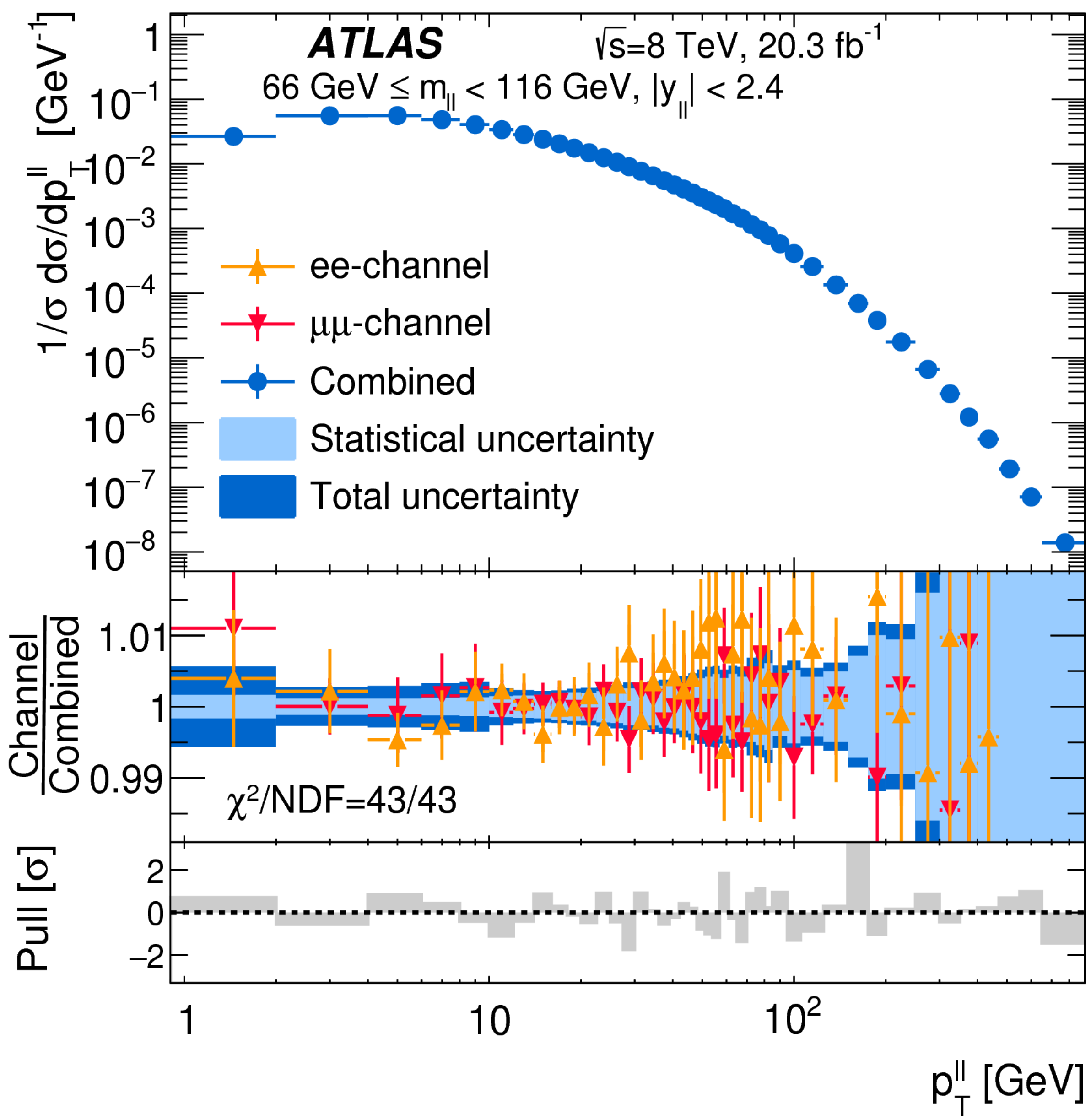
Z-boson transverse momentum used recently to extract strong coupling with high precision

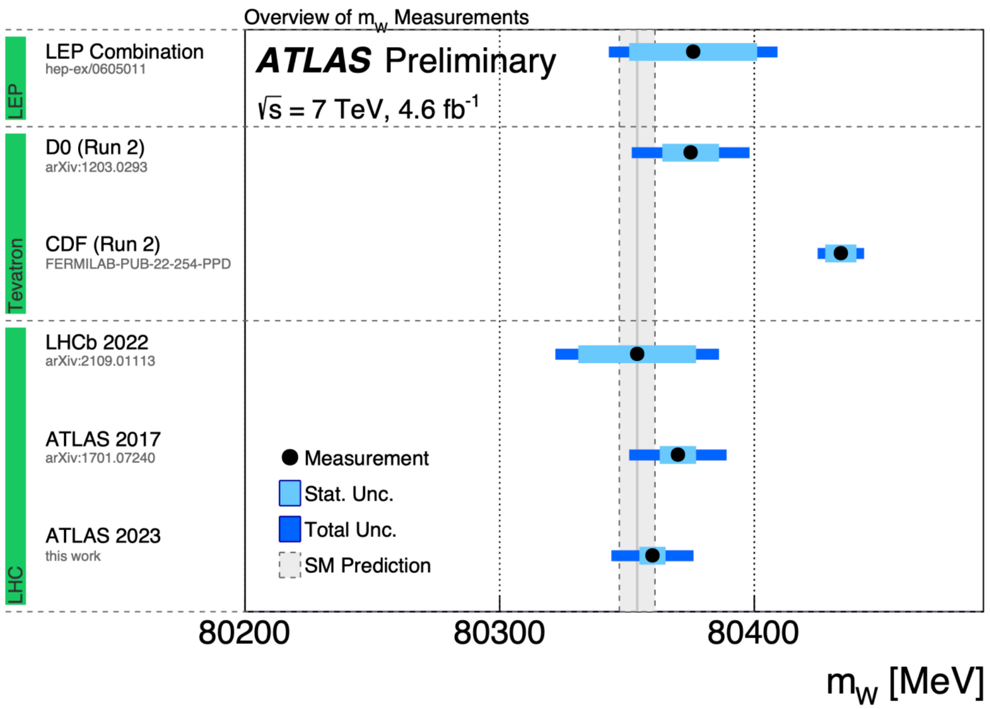
$$\alpha_s(m_Z) = 0.11828^{+0.00084}_{-0.00088}$$

ATLAS, 2309.12986

Precision a reality

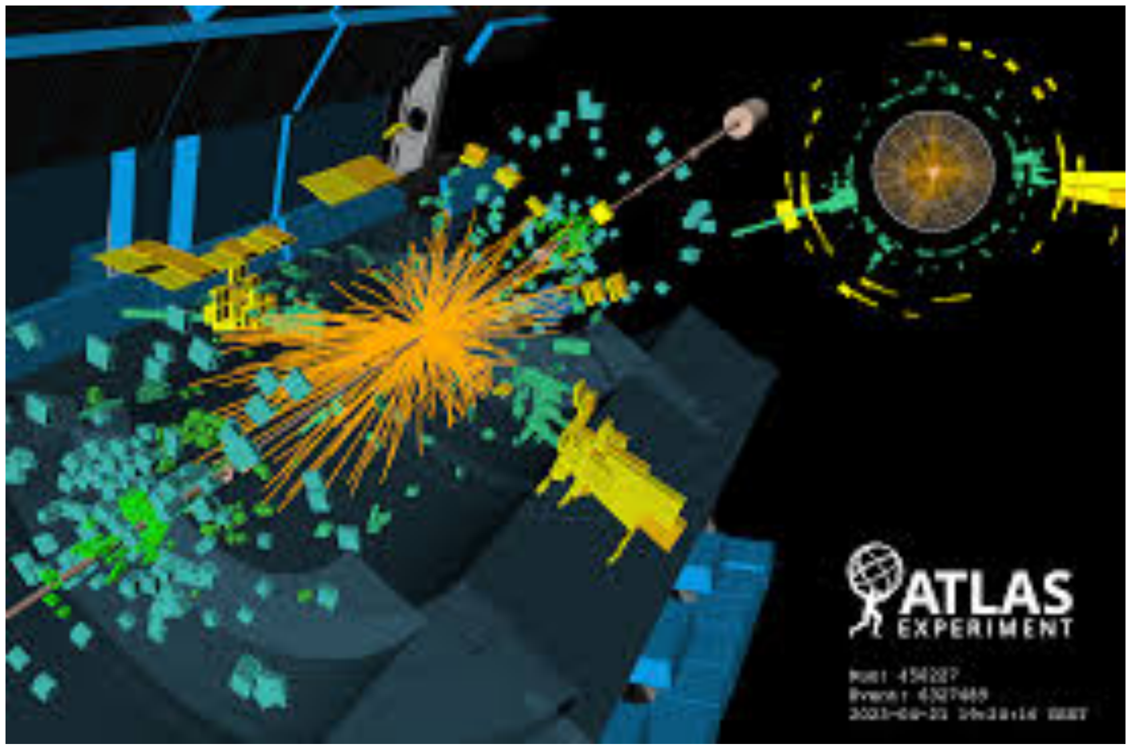
Z-boson kinematics to below a percent

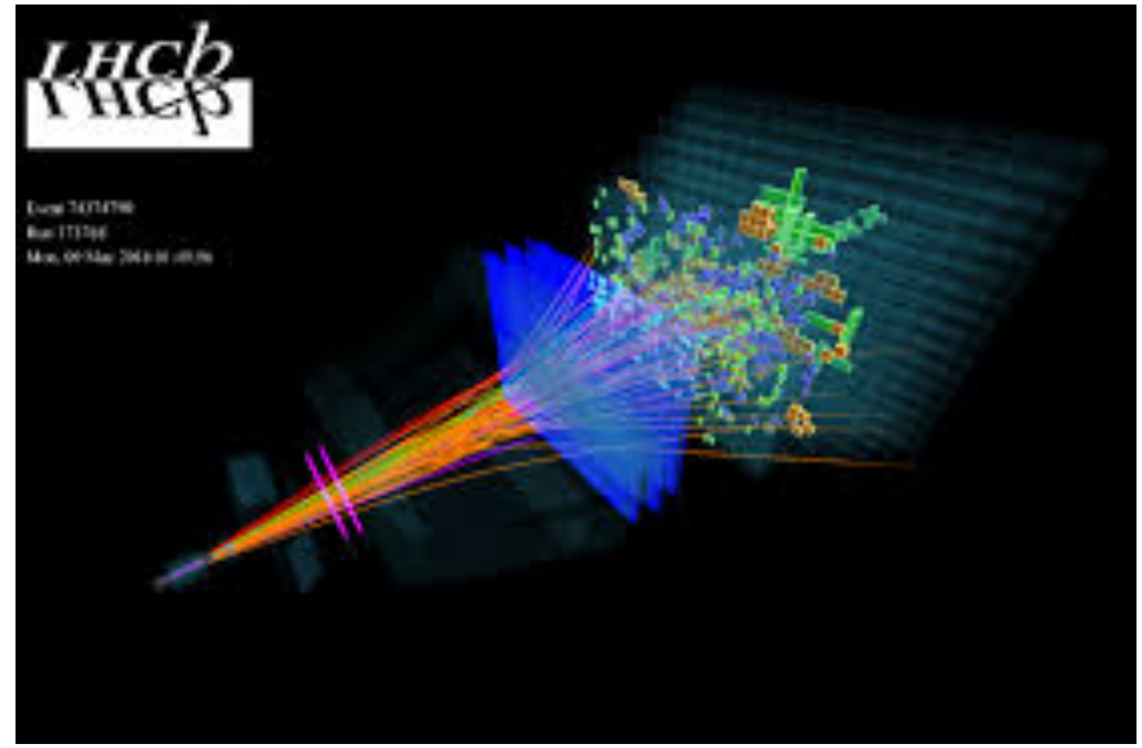


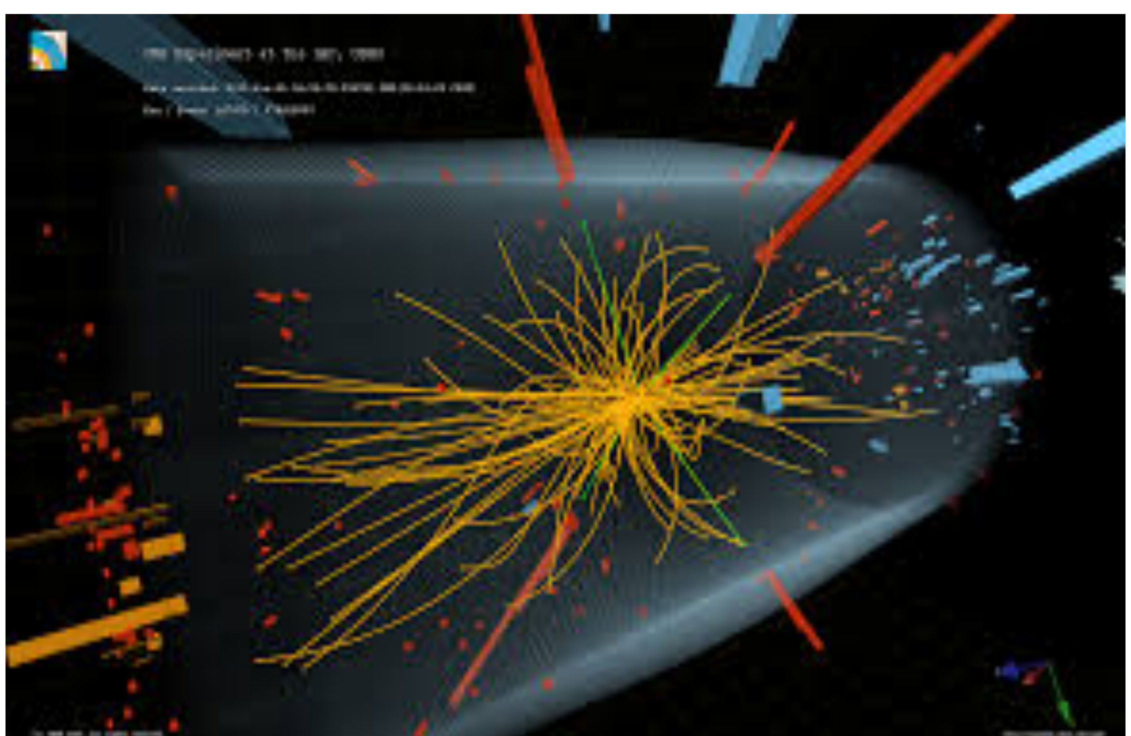


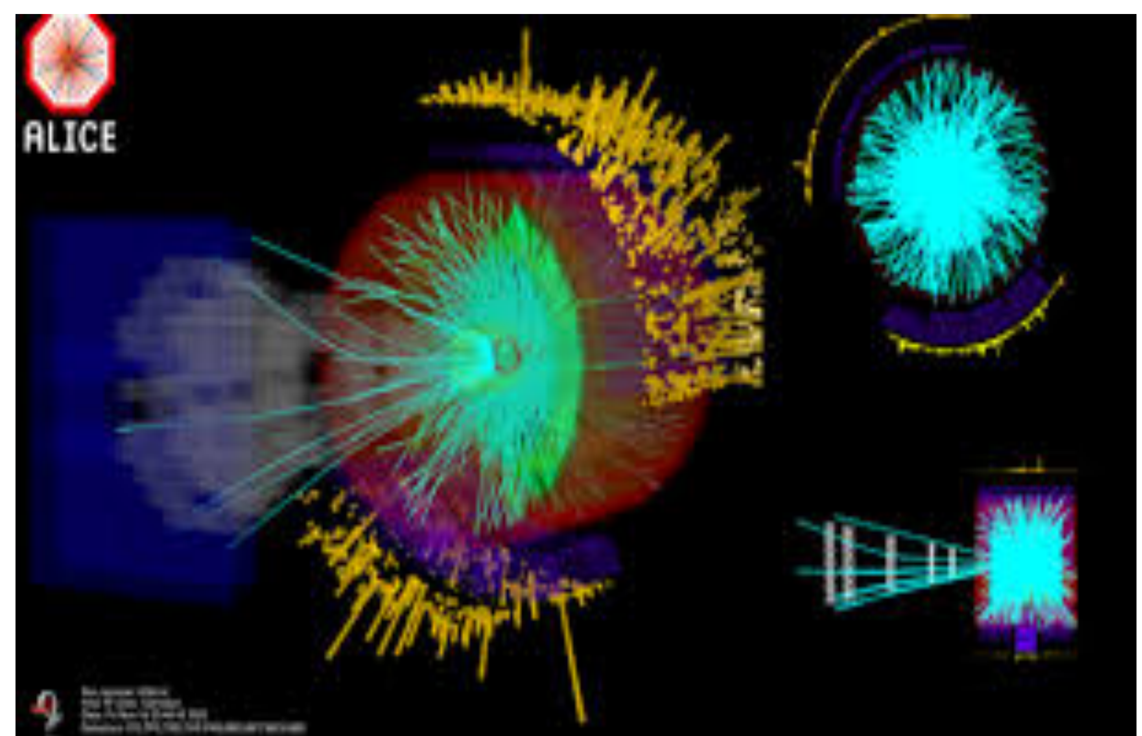
Crucial input for W-boson mass extraction

Collider events: real world

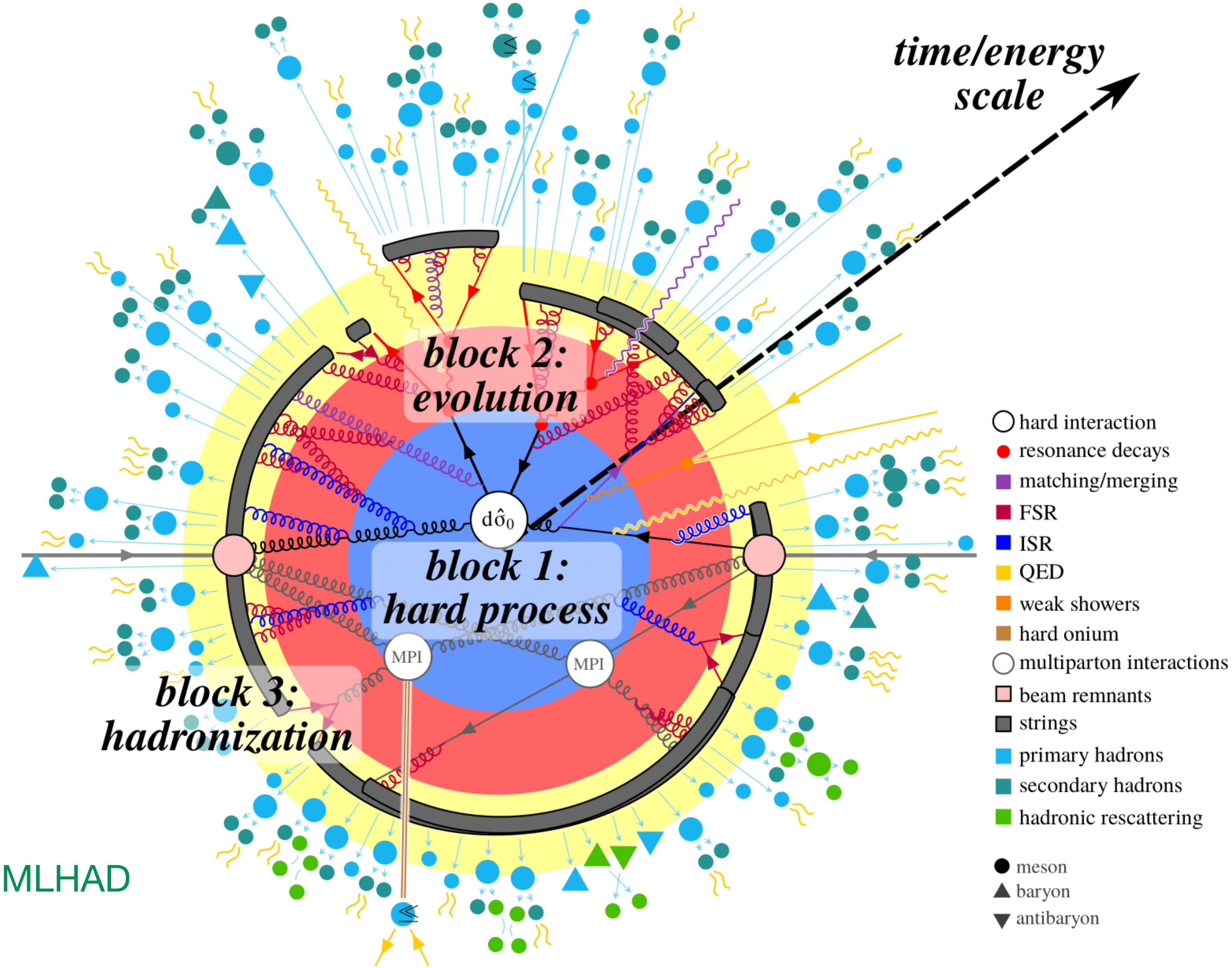








Theorist point of view



from MLHAD

Clearly, a multilateral challenge

NLO: one-loop amplitudes

Main **one-loop amplitude providers**:

- **BlackHat(https://blackhat.hepforge.org/)**
- **Collier (https://collier.hepforge.org)**
- **GoSam (https://gosam.hepforge.org)**
- **Golem95 (https://golem.hepforge.org/)**
- **Helac-NLO/Helac-1Loop (https://helac-phegas.web.cern.ch)**
- **Ninja (https://ninja.hepforge.org/)**
- **Njet (https://www.physik.hu-berlin.de/de/pep/tools/njet)**
- **NLOX (http://www.hep.fsu.edu/~nlox/)**
- **OpenLoops (https://openloops.hepforge.org)**
- **Recola/Recola2 (https://recola.hepforge.org)**

✓ Fast, automated generation and numerical evaluation of one-loop amplitudes

✓ Easy interface with Sherpa, Herwig, POWHEG, and others

✓ QCD only, or full SM (QCD+EW)

NLO: general purpose tools

1. **MadGraph5_aMC@NLO (https://launchpad.net/mg5amcnlo)**
   - Full automation of NLO QCD and EW
   - Process generation via FeynRules/UFO. Supports parton showers via aMC@NLO
2. **Sherpa+OpenLoops (https://sherpa.hepforge.org, https://openloops.hepforge.org)**
   - SHERPA handles phase space, subtraction, matching, and showering
   - OpenLoops provides fast NLO matrix elements. Efficient for multi-leg processes
3. **Herwig+Matchbox (https://herwig.hepforge.org)**
   - Herwig's Matchbox module enables automated NLO QCD corrections and matching
   - Works with external amplitude providers (OpenLoops, MadGraph, etc.)
4. **POWHEG-BOX (http://powhegbox.mib.infn.it)**
   - NLO with matching to parton showers (POWHEG method)
   - Semi-automated; requires user input for new processes
5. **MCFM (https://mcfm.fnal.gov)**
   - Parton-level code for NLO calculations (less automated)
   - Mostly SM processes. Mostly based on analytic calculations, very stable and fast

...

NLO automation

NLO QCD calculations are now fully automated, thanks to:

✓ the ability to reduce one-loop amplitudes to master integrals

✓ analytic knowledge of all relevant master integrals

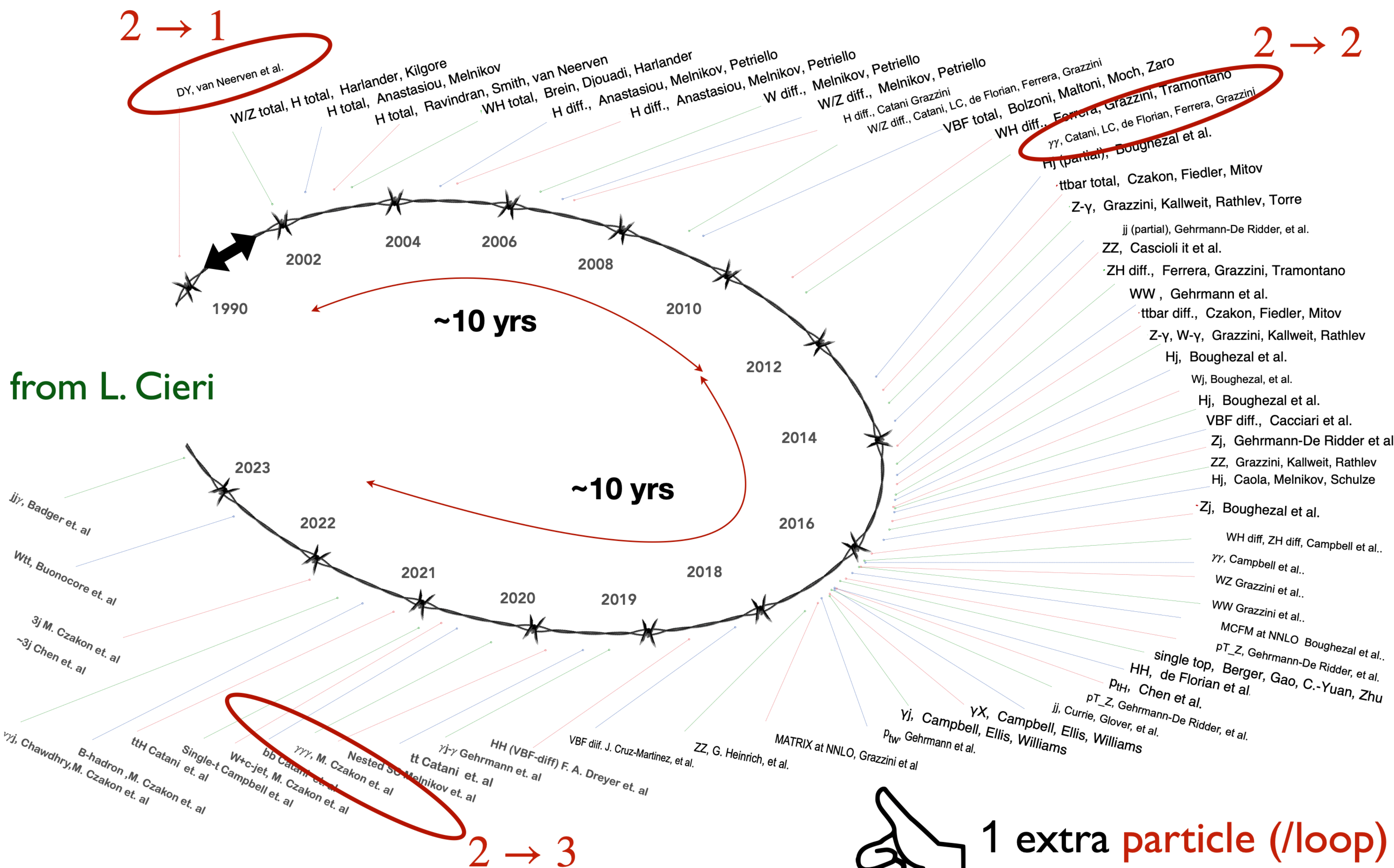
✓ a systematic understanding of how to handle intermediate divergences (e.g. subtraction/slicing methods)

✓ the availability of tools for computing tree-level real radiation

Open challenges at NLO

- numerical stability and efficiency for high multiplicity processes
- NLO corrections to loop-induced processes

NNLO: status

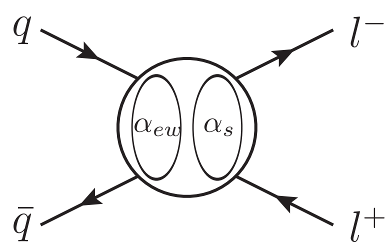


1 extra particle (/loop)
every 10 years

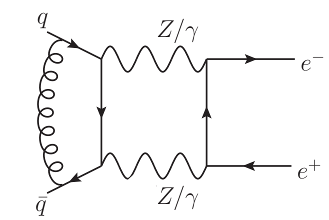
⇒ ***no full automation yet***

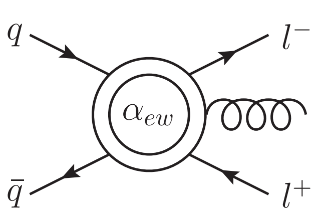
NNLO: ingredients

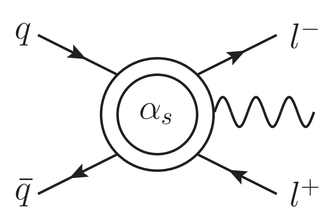
<u>Three main ingredients at NNLO:</u>



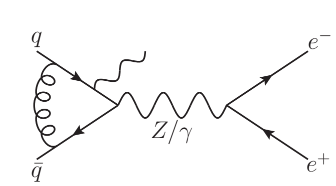
1. two-loop virtual + one-loop squared

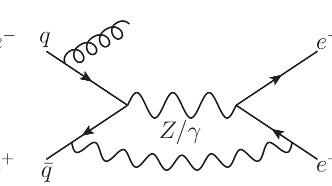




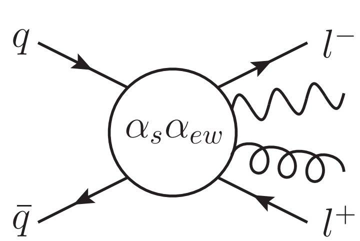


2. one-loop with one extra radiation

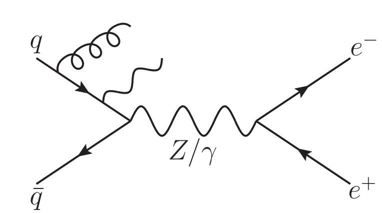


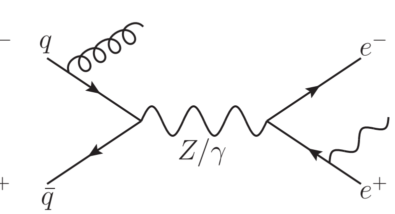


[issue:numerical stability in unresolved regions]



3. Tree-level double real



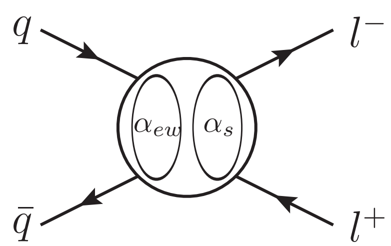


diagrams from C. Signorile-Signorile

Sample diagrams shown for mixed QCD-EW corrections to Drell-Yan production

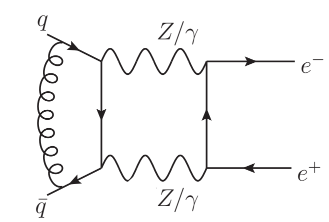
NNLO: ingredients

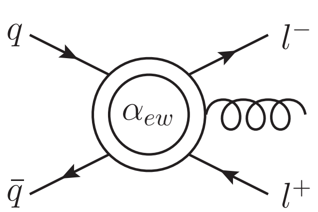
Three main ingredients at NNLO:

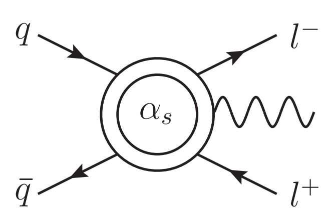


1. two-loop virtual + one-loop squared

The hardest challenge ✗

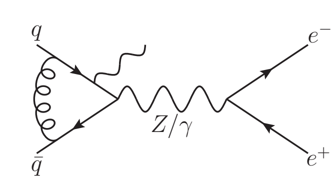


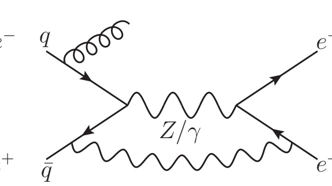


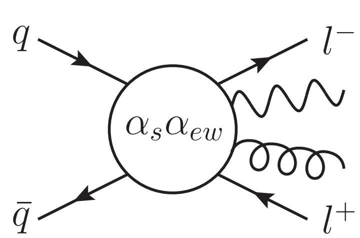


2. one-loop with one extra radiation ✓

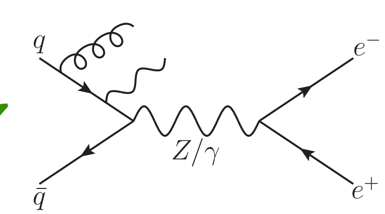
[issue:numerical stability in unresolved regions]

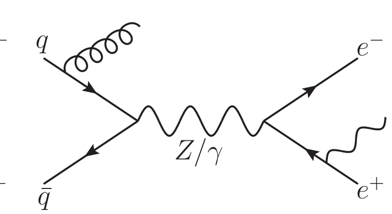






3. Tree-level double real ✓

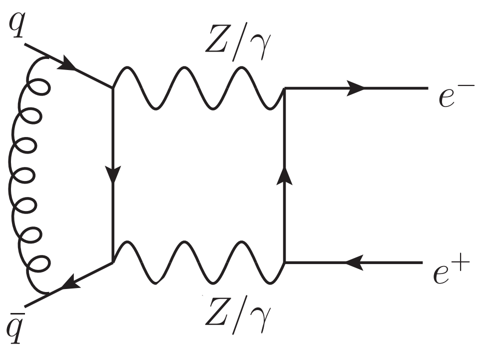




diagrams from C. Signorile-Signorile

Sample diagrams shown for mixed QCD-EW corrections to Drell-Yan production

2-loop calculations



Feynman rules

$$C_{\mu_1\ldots\mu_n\nu_1\ldots\nu_n}(\{p_{\mathrm{ext.}}\})\int \frac{d^D l_1}{(2\pi)^4}\frac{d^D l_2}{(2\pi)^4}\frac{{l_1}^{\mu_1}\ldots {l_1}^{\mu_n}{l_2}^{\nu_1}\ldots {l_2}^{\nu_m}}{D_1\ldots D_N}$$

$$D_i = q_i^2 - m_i^2$$

- The goal is to express a large number of tensor integrals appearing in Feynman diagrams as linear combinations of a small set of master integrals.
- Well established reduction-techniques, e.g.
  - projection on form factors
  - integration-by-parts (IBP) identities
  - Lorentz invariance identities
  - dimensional shift relations
  - …

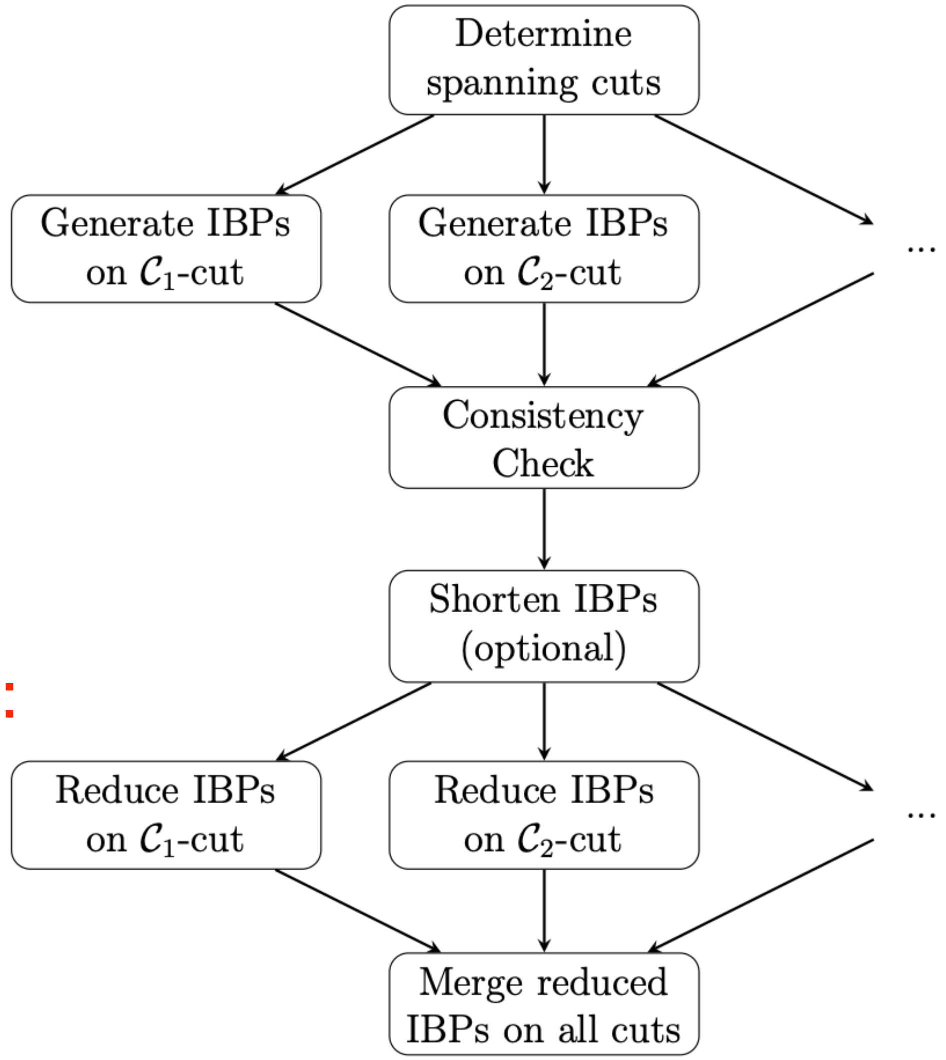
2-loop challenge #1: reduction

While the flowchart is rather straightforward:

- define integral family
- generate IBP identities
- solve Laporta algorithm

The challenges involved are substantial and complex:

- explosion in number of terms and algebraic complexity
- finding minimal, linearly independent topologies not trivial
- finding and exploiting symmetries



from 2502.20778 [NeatIBP 1.1+Kira]

2-loop challenge #1: reduction

Main public tools for IBP reduction of two-loop Integrals:

1. **AIR** (https://people.phys.ethz.ch/~pheno/air/)
   - Automates IBP reduction for scalar and tensor integrals to master integrals.
2. **BLADE** (https://gitee.com/multiloop-pku/blade)
   - Fast IBP reduction in block triangular form
2. **FiniteFlow** (https://github.com/peraro/finiteflow-mathtools/)
   - multivariate functional reconstruction using finite fields and dataflow graphs.
3. **Fire** (https://gitlab.com/feynmanintegrals/fire)
   - Uses the Laporta algorithm for efficient IBP reduction of multi-loop integrals.
4. **Kira** (https://gitlab.com/kira-pyred/kira)
   - High-performance tool for IBP reduction, particularly for multi-loop integrals.
5. **LiteRed** (https://github.com/rnlg/LiteRed2)
   - A C++ program for IBP reduction, optimized for speed and simplicity.
6. **NeatIBP** (https://github.com/yzhphy/NeatIBP)
   - small-size integration-by-parts relations for Feynman integrals.
7. **Reduze** (https://reduze.hepforge.org/)
   - Automates IBP reduction for scalar and tensor integrals, support both one- and two-loop level

Steady progress allows reduction of increasingly complicated processes

2-loop challenge #2: masters

At NLO: the full set of master integral for any process is fully known analytically

At NNLO: the complete set of master integrals is not known. Often non-trivial to verify linear independence

Techniques for evaluation:

- difference and differential equations (DE), including auxiliary mass method
- Mellin–Barnes representations
- sector decomposition
- expansion in limits
- iterated integrals
- …
- ongoing development of **new analytic and numerical methods**

2-loop challenge #2: masters

Most used public codes:

1. **pySecDec (https://github.com/gudrunhe/secdeco)**
   - Numerical evaluation of master integrals using sector decomposition
2. **FIESTA (https://gitlab.com/feynmanintegrals/fiesta)**
   - Numerical evaluation of master integrals using sector decomposition
3. **AMFLOW (https://gitlab.com/multiloop-pku/amflow)**
   - Numerical evolution of master integrals using finite flow method

- powerful checks of analytic calculations
- used directly in numerical/phenomenological implementations
- if the evaluation is very slow, used to construct integration grids to be just interpolated on-the-fly

2-loop challenge #2: masters

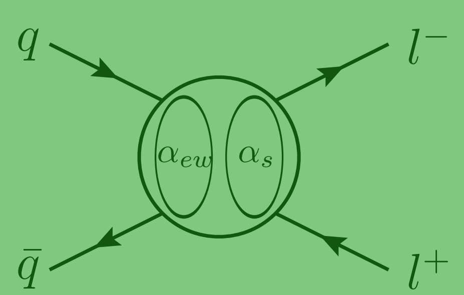
Open challenges:

- Elliptic and beyond:
  Master integrals with elliptic or more general functions (e.g. modular forms) are not expressible in terms of multiple polylogarithms; the theory of these functions is still being developed.
- Numerical evaluation stability:
  Some integrals can only be evaluated numerically, but the precision and speed required for collider applications are often challenging/prohibitive.
- Automation bottlenecks:
  While IBP and DE methods are largely automated, choosing good bases and simplifying expressions still requires expert intervention.

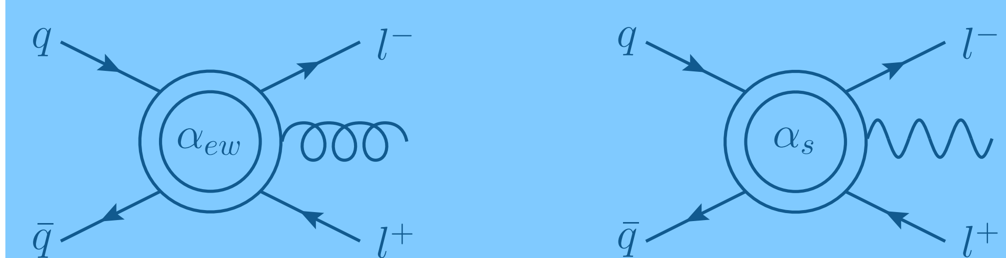
Frontier: 2 to 2 process with full off-shell legs, 2-loop integrals with 3-4 scales, multi-leg QCD-EW mixed contributions …

NNLO challenge #3: singularities

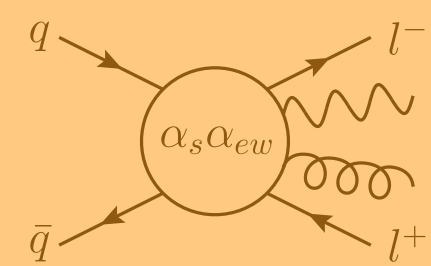
While full results are finite, double-virtual, real-virtual and double-real amplitudes involve intermediate singularities. Since they live in different phase-spaces (Φ_n, Φ_{n+1}, and Φ_{n+2}) the cancellation of singularities must occur *before numerical evaluation*.



Two-loop virtual Φ_n



Real-virtual Φ_{n+1}



Double real-Φ_{n+2}

NNLO challenge #3: singularities

Two core ideas

Slicing

finite ⇒ numerical evaluation

$$\sigma(X) = \int_0^{\tau_{\rm cut}} \frac{d\sigma_{\rm approx}(X)}{d\tau} + \int_{\tau_{\rm cut}} \frac{d\sigma(X)}{d\tau}$$

approximate evaluation close to soft-collinear limits

- Non-local in the phase space
- Dependence on the slicing parameter

Subtraction

finite ⇒ numerical evaluation

$$\int d\Phi_D |\mathcal{M}|^2 F_{\rm J} = \int d\Phi_4 \left(|\mathcal{M}|^2 F_{\rm J} - K\right) + \int d\Phi_D K$$

soft-collinear counterterms ⇒ poles in 1/ε

- Local in the phase space
- Construction and integration of counterterms hard/process specific

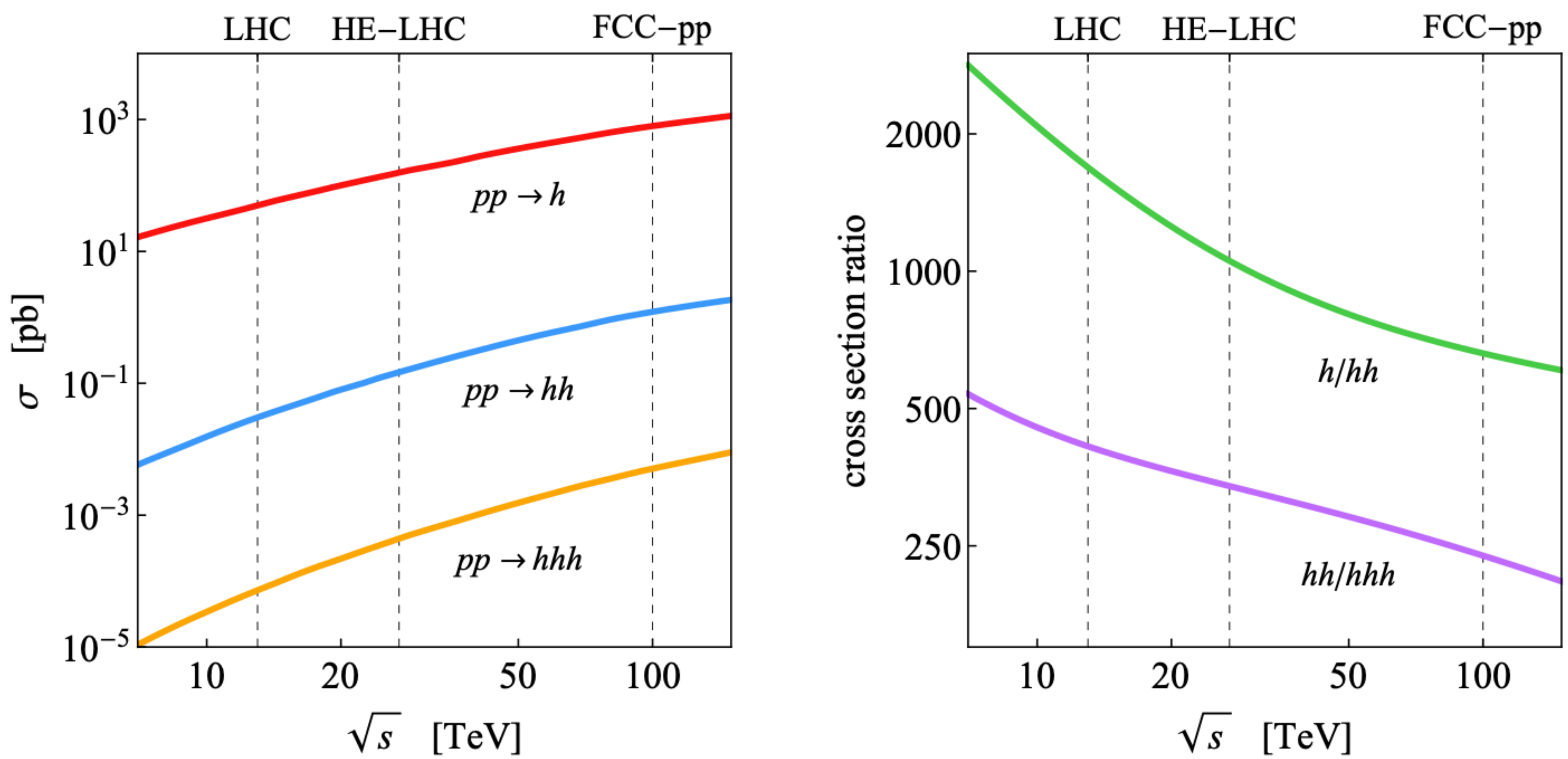
Many practical realisations of these ideas at two-loops and beyond

NNLO challenge #3: singularities

Method	Local?	Analytic?	Examples	Public tool
Antenna subtraction [Gehrmann-De Ridder et al. ’05]	✓	✓	e^+e^- → 3 jets, DIS, Z/H+jet, dijets, diphoton, VBF Higgs …	NNLOJet
q_T subtraction [Catani, Grazzini ’07]	✗	✓	Colour singlet processes (2 to 2) , tt, bb, ttH, ttW	HNNLO, DYNNLO, MCFM
N-jettiness subtraction [Boughezal et al. ’15; Gaunt et al. ’15]	✗	✓	V+jet, H+jet, top decay, single top	Geneva, MCFM
Sector improved residue subtraction [Czakon ’10; Czakon, Heymes ’14]	✓	✗	tt, Higgs+jet, inclusive jet, W+jet, 3γ, 2γ +jet, γ + 2 jet,3 jets, B-hadrons	STRIPPER (private)
Local analytic subtraction [Magnea, Maina, Pelliccioli, Signorile-S., Torielli, Ucciratial. ’18-’20]	✓	✓	e^+e^- → 2 jets	Private code
Nested soft–collinear subtraction [Caola, Melnikov, Rontsch ’17]	✓	✓	VH, VBF QCD-EW Drell Yan	Private code
ColourFull subtraction [Del Duca, Duhr, Somogyi, Trocsanyi ’05]	✓	✓	e^+e^- → 3 jets, H → bb, H	NNLOCAL
Projection to Born [Cacciari, Dreyer, Karlberg, Salam, GZ ’15]	✓	✓	VFB H, DIS, H → bb, HH, t-channel single top [also jet in DIS, H → bb, gg → @ N3LO)	Private codes
Loop Treel duality [Bierenbaum, Catani, Draggiotis, Rodrigo ’10, Capatti, HIrschi, Kermanschah, Pelloni, Ruijl ’19]	✓	✗	γ^*→ 2 jets, γ^*→ tt	Private code
Locally finite two-loops [Anastasiosu, Sterman ’18]	✓	✗	Multi-photon in e+e-, multi-Higgs and EW bosons finite finite top-mass, electroweak production through gluon fusion	Private code

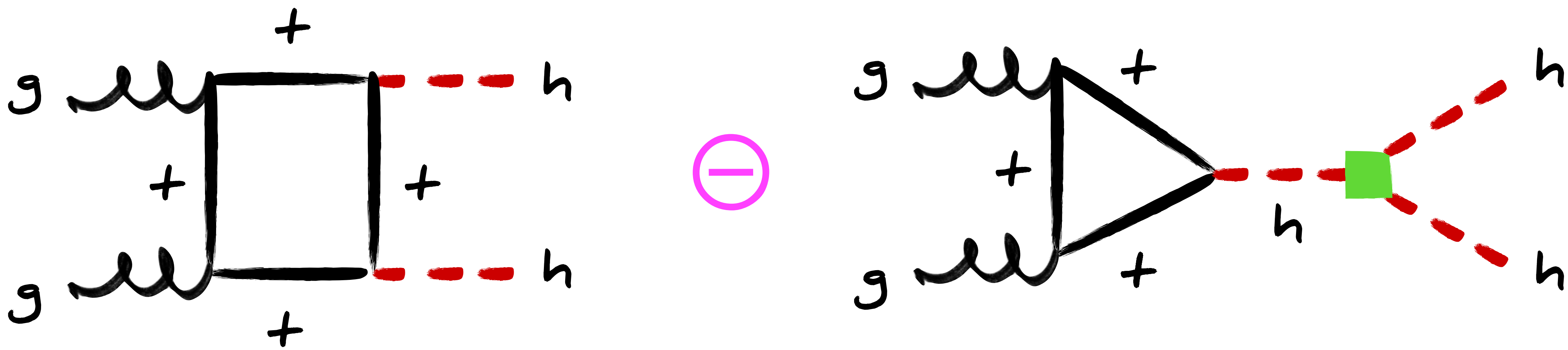
Example: $gg \to HH$

Bizon et al. 1810.04665



- Double-Higgs production golden channel for the direct measurement of the Higgs self-coupling, the footprint of the SM Higgs boson and of Higgs potential
- Small cross section because of accidental cancellations and phase space effect
- Interesting also in the context of possible new resonances (2HDM, MSSM, RS, …)

gg → HH: LO



LO amplitude ✓

Eboli, Marques, Novaes, Natale ’87
Glover, van der Bij ’88

Going beyond LO highly non-trivial

gg → HH: beyond LO

NLO QCD:

- large-m_t [Dawson, Dittmaier, Spira '98] [Grigo, Hoff, Melnikov, Steinhauser '13]
- numeric [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]
- large-m_t + threshold exp. Padé [Gröber, Maier, Rauh '17]
- high-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-p_T expansion [Bonciani, Degrassi, Giardino, Gröber '18]
  + high-energy expansion [Bagnaschi, Degrassi, Gröber '23]

NNLO QCD:

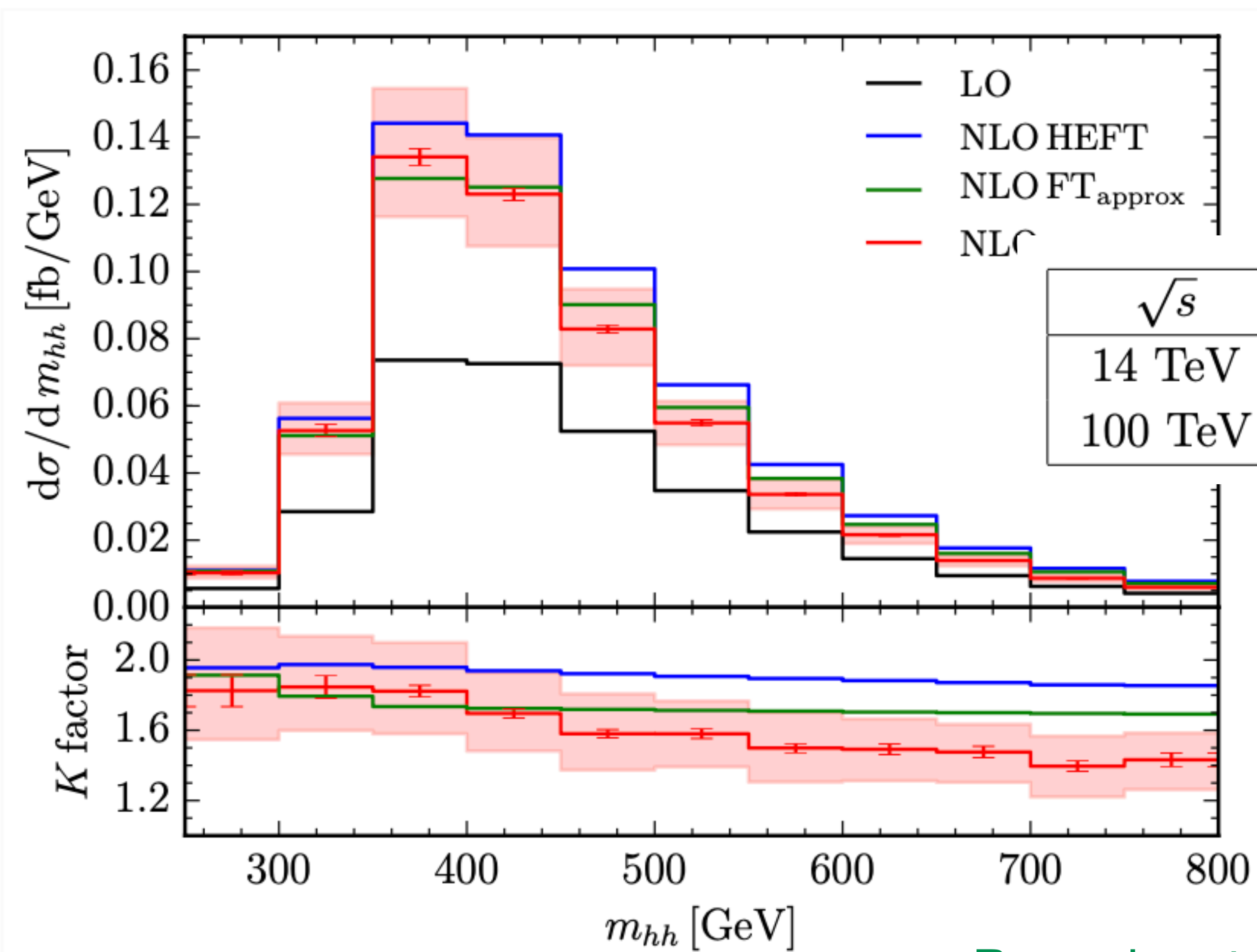
- large-m_t virtuals [de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15, Davies; Steinhauser '19]
- HTL+numeric real ("FTapprox") [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-m_t reals [Davies, Herren, Mishima, Steinhauser '19 '21]
- light fermion corrections at $p_T = 0$ [Davies, Schönwald, Steinhauser '23]

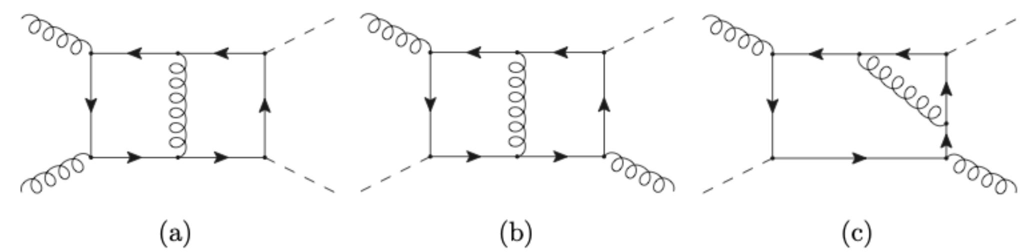
N3LO QCD:

- Wilson coefficient C_{HH} [Spira '16; Gerlach, Herren, Steinhauser '18]
- HTL [Chen, Li, Shao, Wang '19]

from Kay Schönwald

gg → HH: beyond LO





$\sqrt{s}$	LO	B-i. NLO HEFT	NLO FT$_{approx}$	NLO
14 TeV	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$34.26^{+14.7\%}_{-13.2\%}$	$32.91^{+13.6\%}_{-12.6\%}$
100 TeV	$731.3^{+20.9\%}_{-15.9\%}$	$1511^{+16.0\%}_{-13.0\%}$	$1220^{+11.9\%}_{-10.7\%}$	$1149^{+10.8\%}_{-10.0\%}$

Borowka et al. ’16

As for single Higgs production very large NLO corrections and large residual uncertainties (m_t scheme choice?) ⇒ strong motivation to improve precision

gg → HH: approx NNLO

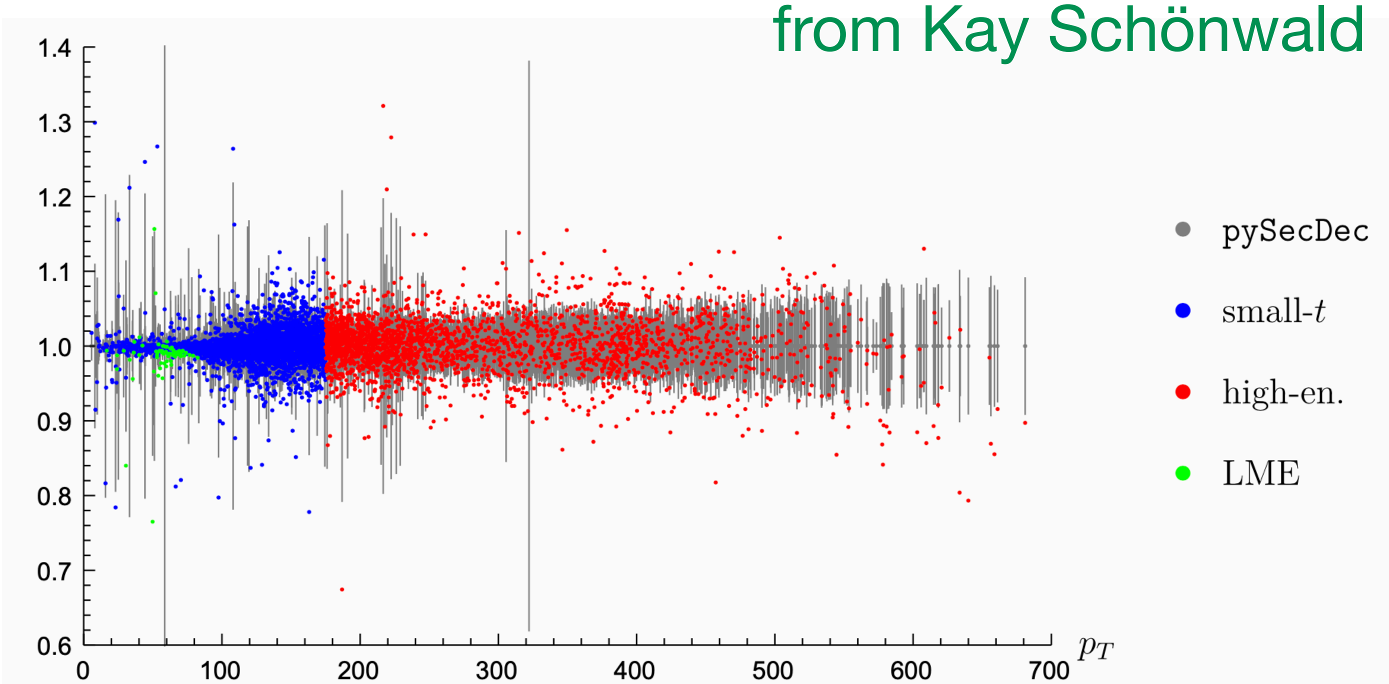
Since a full NNLO (3-loop) is out of reach, clever approximations developed:

- High energy expansion $s, |t|, \gg m_t^2, m_H^2$
- Small-t expansion $s, m_t^2, \gg |t|, m_H^2$

Idea is to fully cover the physically relevant phase space with the two approximations (reminiscent, and exploits, the strategy of regions, according to which full results are recovered by summing over all relevant regions at integrand level)

Robust validation using overlap regions, lower orders and numerical results

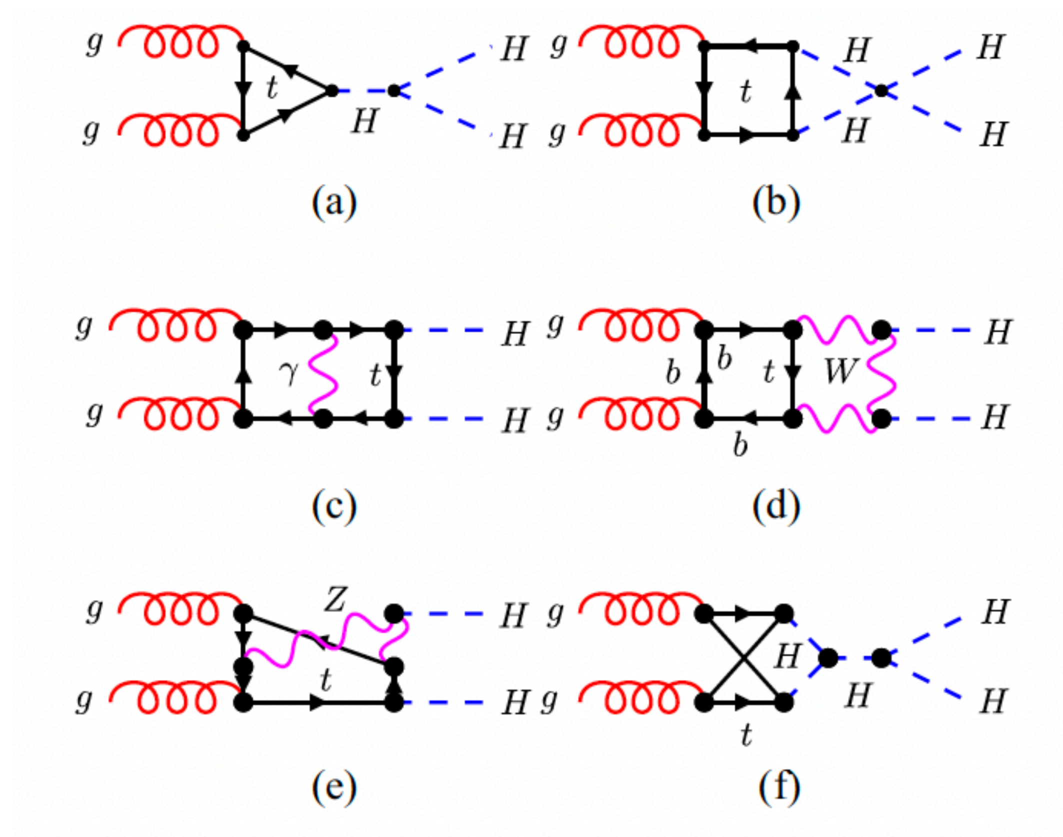
from Kay Schönwald



These and similar approximations (soft Higgs, massification, threshold exp., large N_c, …) are expected to become increasingly relevant as we aim to **halve theoretical uncertainties** — a common target in future projections.

gg → HH: electroweak?

Electroweak effects of order a few percent for inclusive cross-sections, but enhanced in tails of distributions, where New Physics effects might appear, because of large Sudakov logarithms.



Scales involved:

$$s, t, M_H^2, M_W^2, m_t^2, M_Z^2, m_b^2, \ldots$$

Given the # of scales involved, even the calculation of the leading electroweak effects is analytically extremely challenging. As for QCD corrections, expand in regions to obtain approximate results. Alternatively numerical results available.

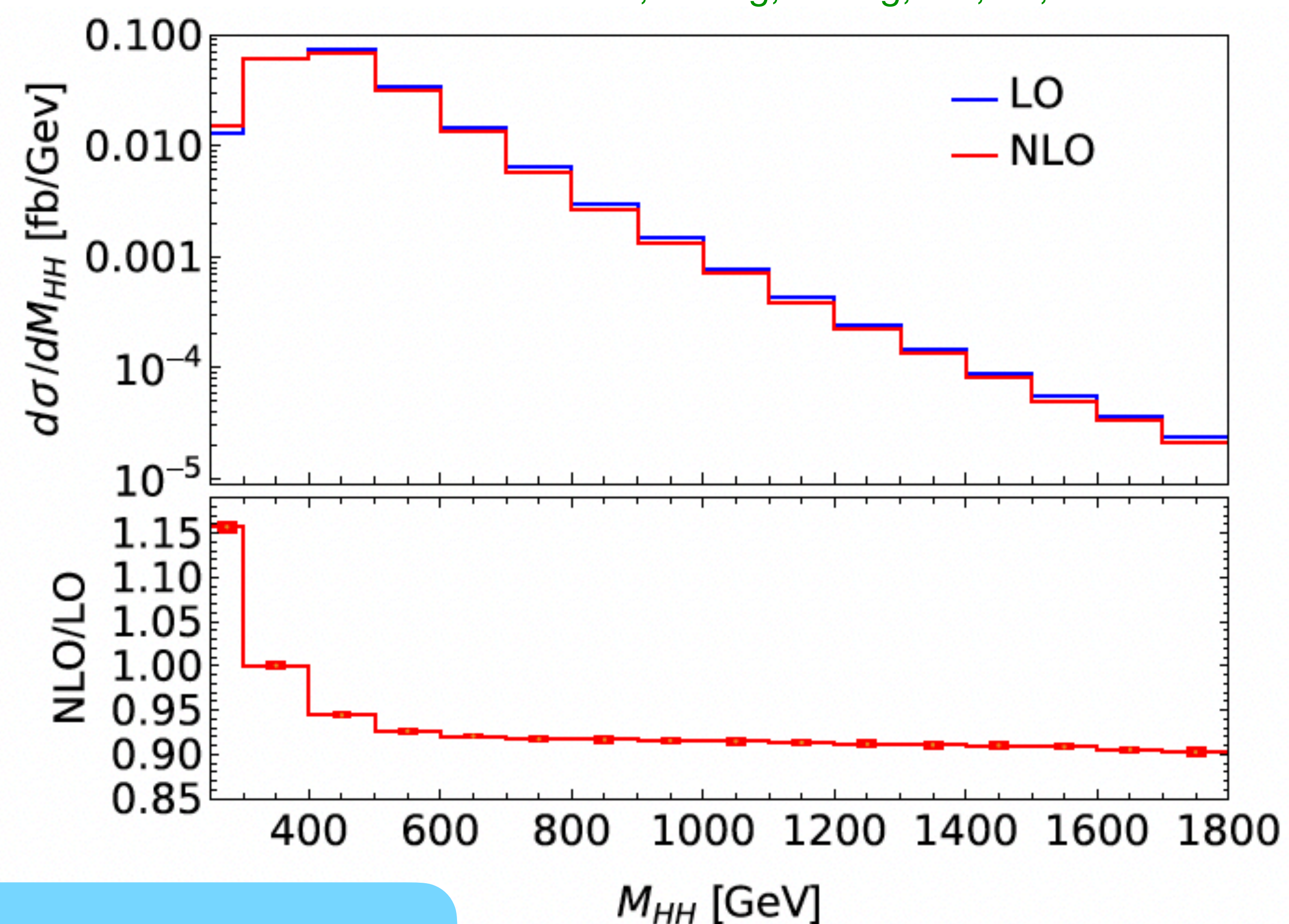
gg → HH: electroweak?

NLO EW effects are known fully numerically:

Bi, Huang, Huang, Ma, Yu, 2311.16963

TABLE I. LO and NLO EW corrected integrated cross sections (in fb) with $\sqrt{s} = 14$ TeV based on 1.8×10^4 reweighted events. The uncertainties arise from statistical errors in phase space integration.

μ	$M_{HH}/2$	$\sqrt{p_T^2 + m_H^2}$	m_H
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$ factor	0.958(1)	0.957(1)	0.954(1)



4-5% on inclusive cross section

10-15% on distributions

When targeting % precision, EW or mixed QCD-EW effects must be fully under control.

Analytic versus numerical?

Rapid progress is happening on both the analytical and numerical fronts. Each has clear strengths and limitations — but their synergy is driving remarkable advances in precision calculations.

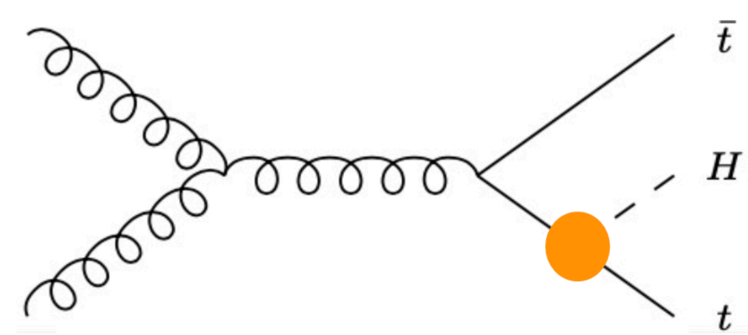
Analytic	Numerical
Fast, efficient	Slow, less efficient
Arbitrary precision	Limited precision (less so recently)
Complexity increases dramatically with the number of scales involved	More suitable for multi-loop and multi-scale problems
Analytic control of function (logs etc), allows insight into the result and control	Hard to extract analytic structure (limits, etc.)

Approximate ttH at NNLO

Catani et al., 2210.04846

Two-loop pp → ttH amplitudes still missing.

Idea: approximate with amplitudes with a soft Higgs emitted off heavy quarks



	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\mathrm{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\mathrm{NLO,H}}\vert_{\mathrm{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\mathrm{NNLO,H}}\vert_{\mathrm{soft}}$	−2.980(3)	2.622(0)	−239.4(4)	65.45(1)

Test the procedure at NLO

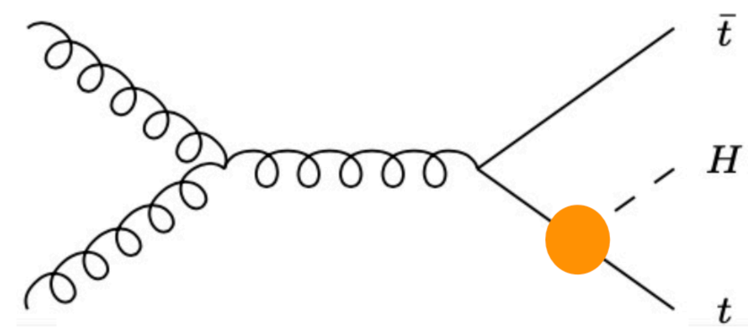
- approximation not that great and works better for qq then gg channel

Approximate ttH at NNLO

Catani et al., 2210.04846

Two-loop pp → ttH amplitudes still missing.

Idea: approximate with amplitudes with a soft Higgs emitted off heavy quarks



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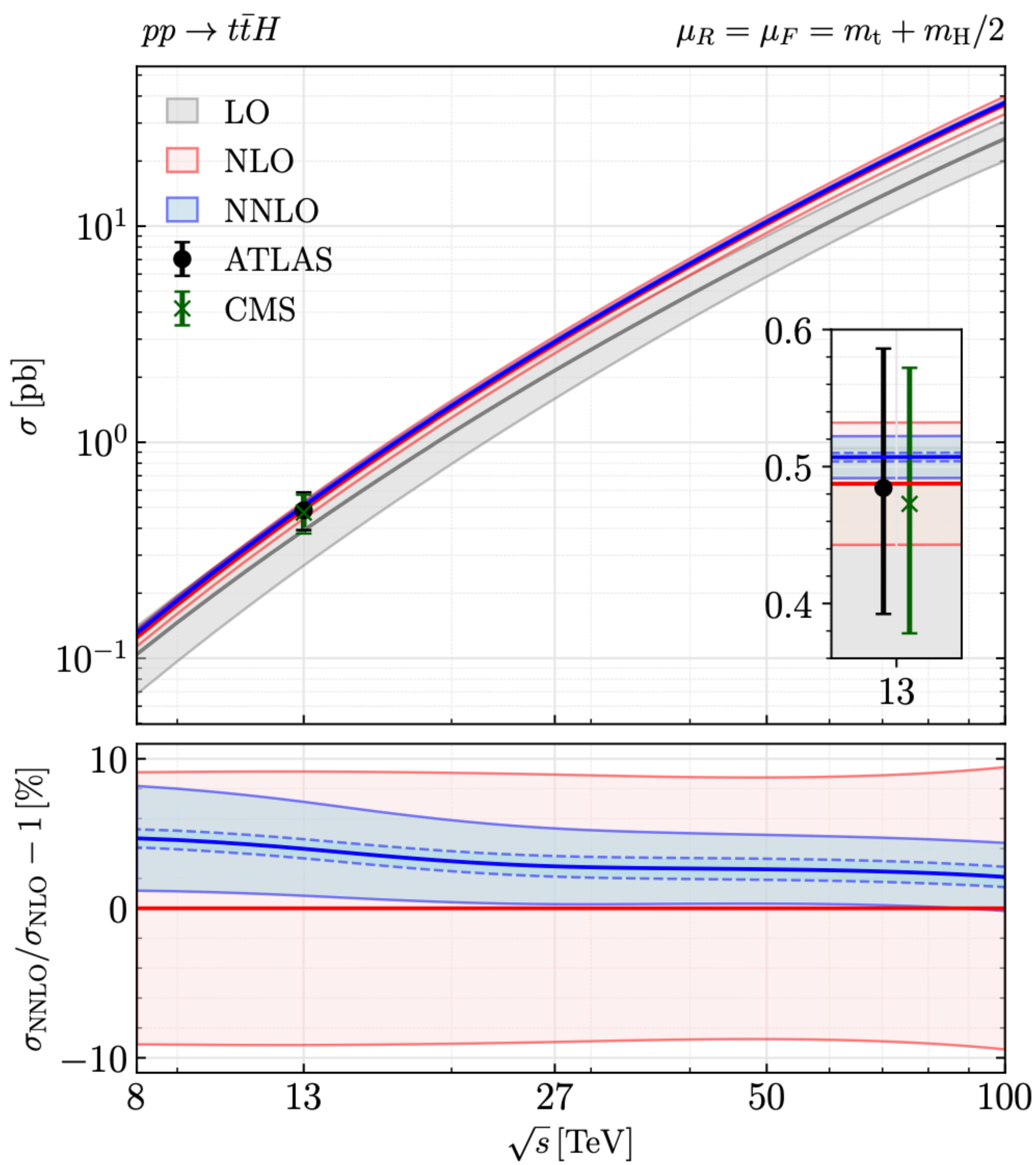
Size of approx. NNLO

- but two-loop corrections are very small (below a %)

Approximate ttH at NNLO

Catani et al., 2210.04846



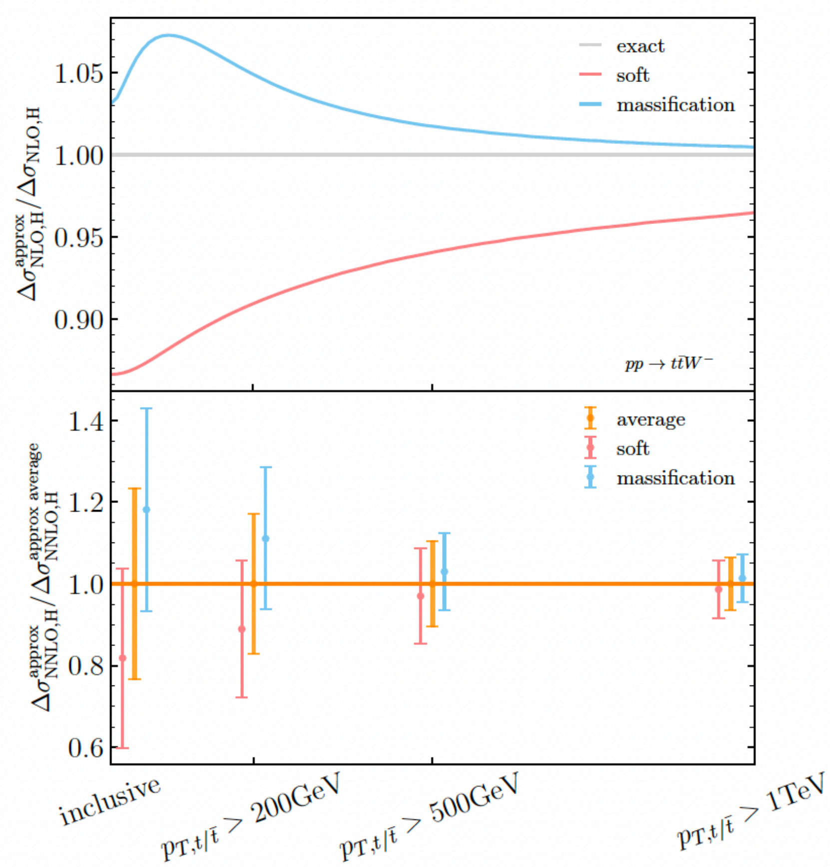
⇒ estimated uncertainty on the total cross section at the few percent level

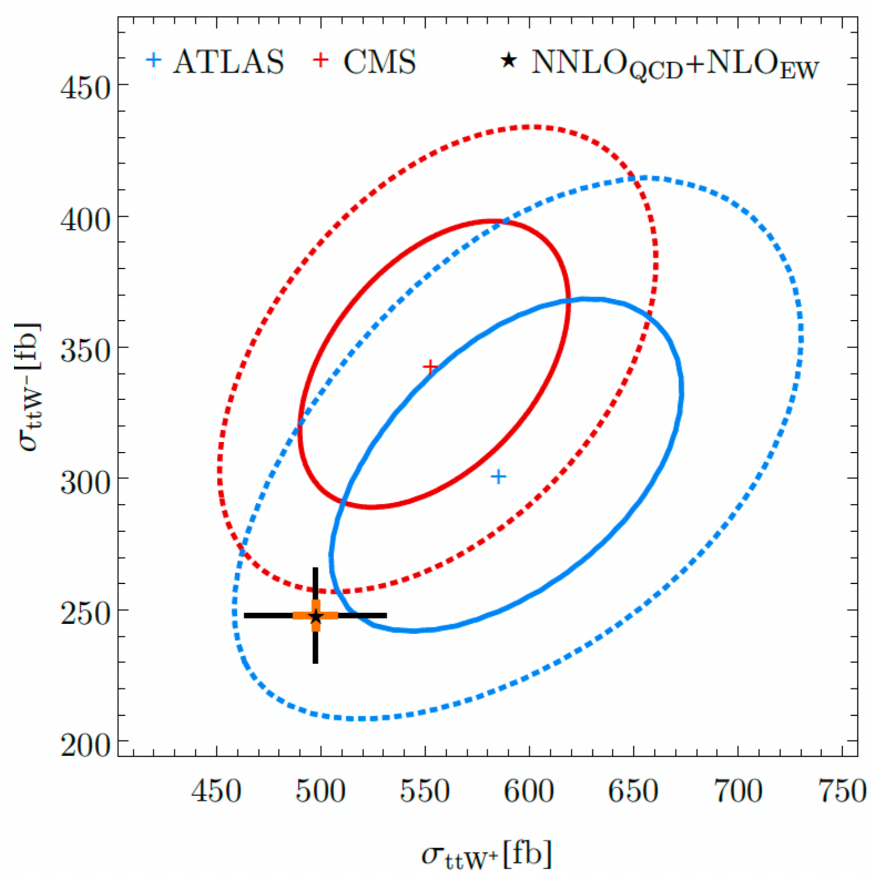
Interesting to validate this once full NNLO is available

Approximate ttW at NNLO

Buonocore et al., 2306.16311

Two approximations (“soft W” and “massification”) estimate missing two-loop ttW corrections, yielding 10% accuracy at NLO and similar results at NNLO, even outside their ideal validity.



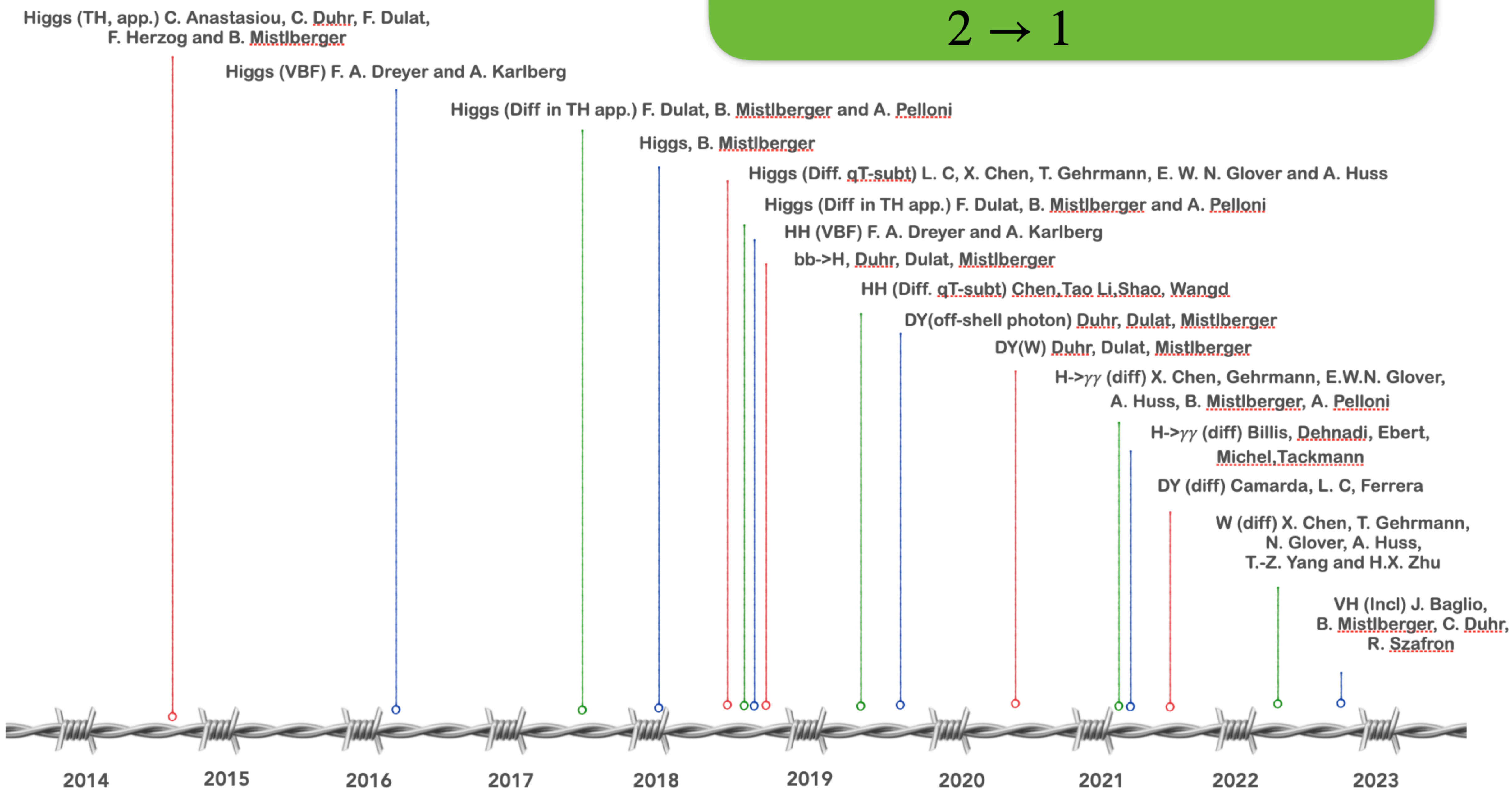


Clever approximations enable precision for key processes (ttH, ttW, ttZ, …) crucial to constrain SM parameters (y_t, g_{ttZ}, …).

N³LO status

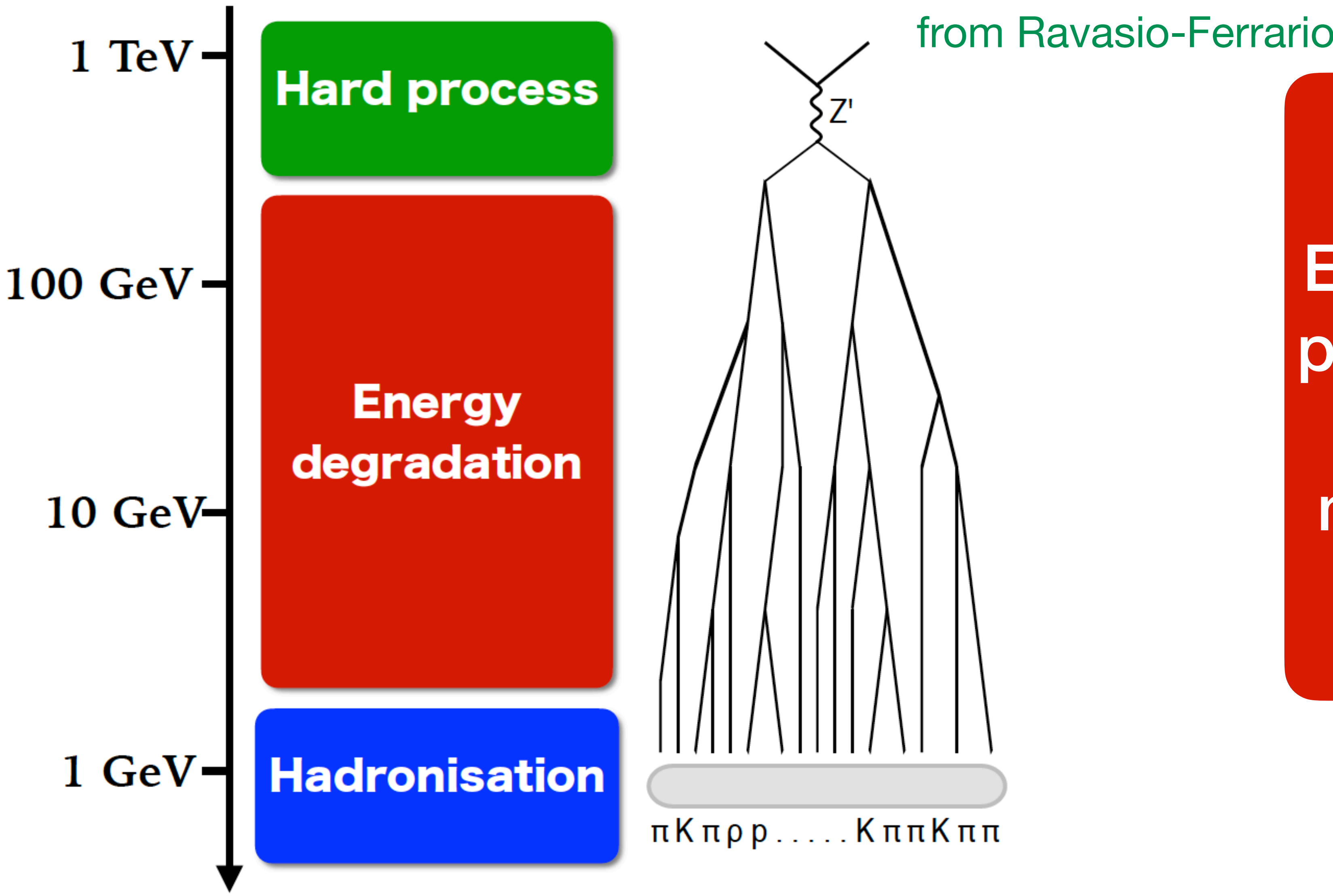
DY-like (no color in final state)

$2 \to 1$



from L. Cieri

Parton shower



Parton shower:
Energy degradation of particles from the hard collision, producing more particles during evolution

Parton showers are ubiquitous at the LHC

Modelling QCD processes, event simulation, background estimates, unfolding, detector simulation, …

Modern parton showers

The development of parton shower has seen a dramatic change in recent years. Key new elements of modern parton showers include

- Improvements in the accuracy of the parton shower
- Numerical procedure to validate the accuracy
- Understanding that some parton showers have lower accuracy
  $\Rightarrow$ can be disregarded when assessing theory uncertainties

ALARIC, DEDUCTOR, PANSCALES, HERWIG7, …

A revolution in parton shower developments is ongoing. Will be crucial for Run 3, HL-LHC and FCC.

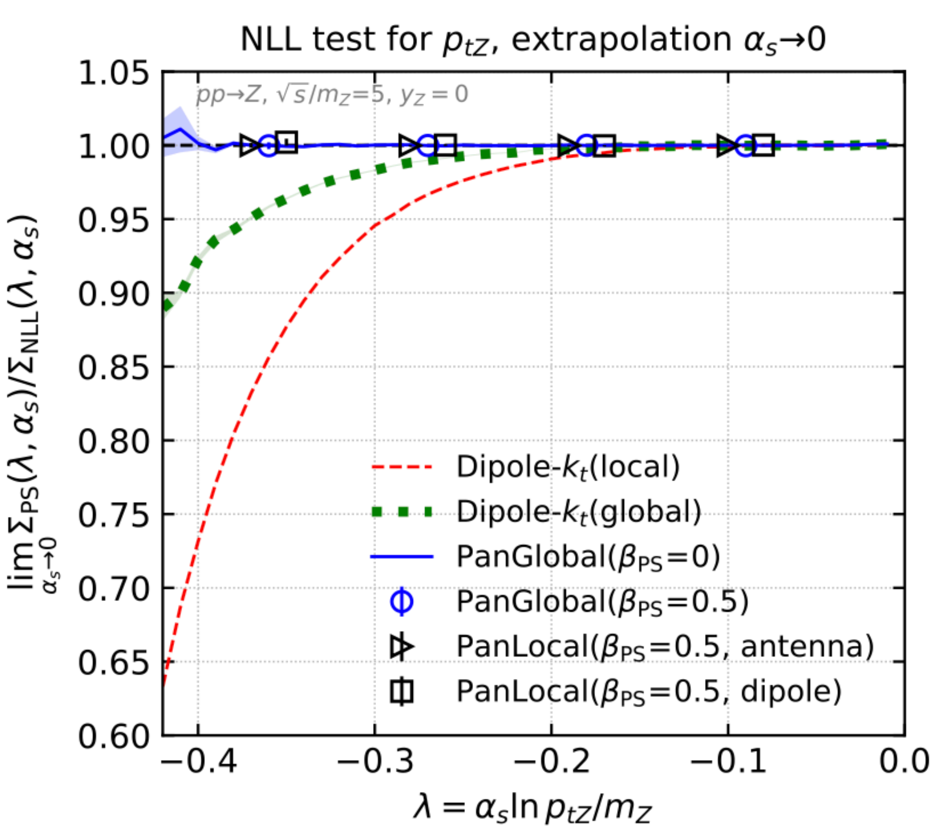
Parton showers

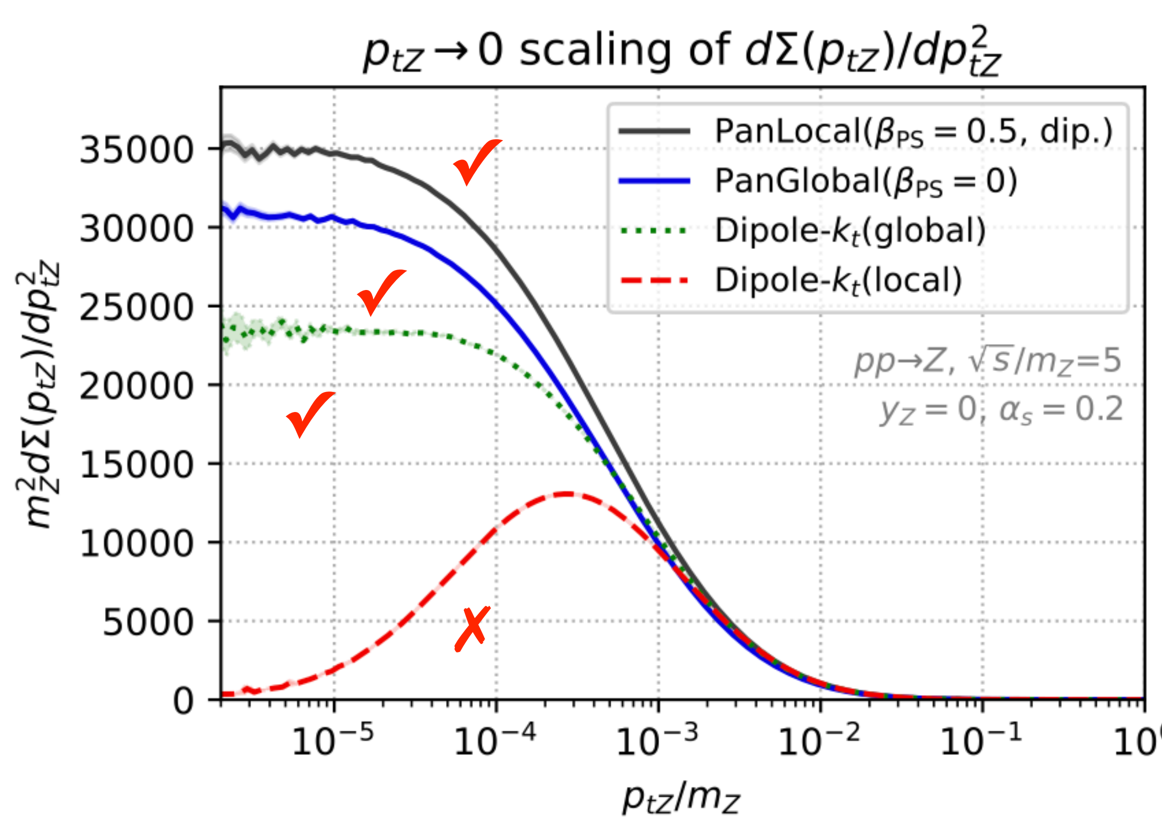
Andersson et al ’92; Nagy & Soper 2009; Dasgupta et al 2018, …

Recoil and other shower design should respect **absence of cross-talk between disparate scales**, i.e. QCD factorisation

Recent work in improving logarithmic accuracy of the shower and quantifying the associated uncertainty

Validation of accuracy using analytic resummations. e.g.

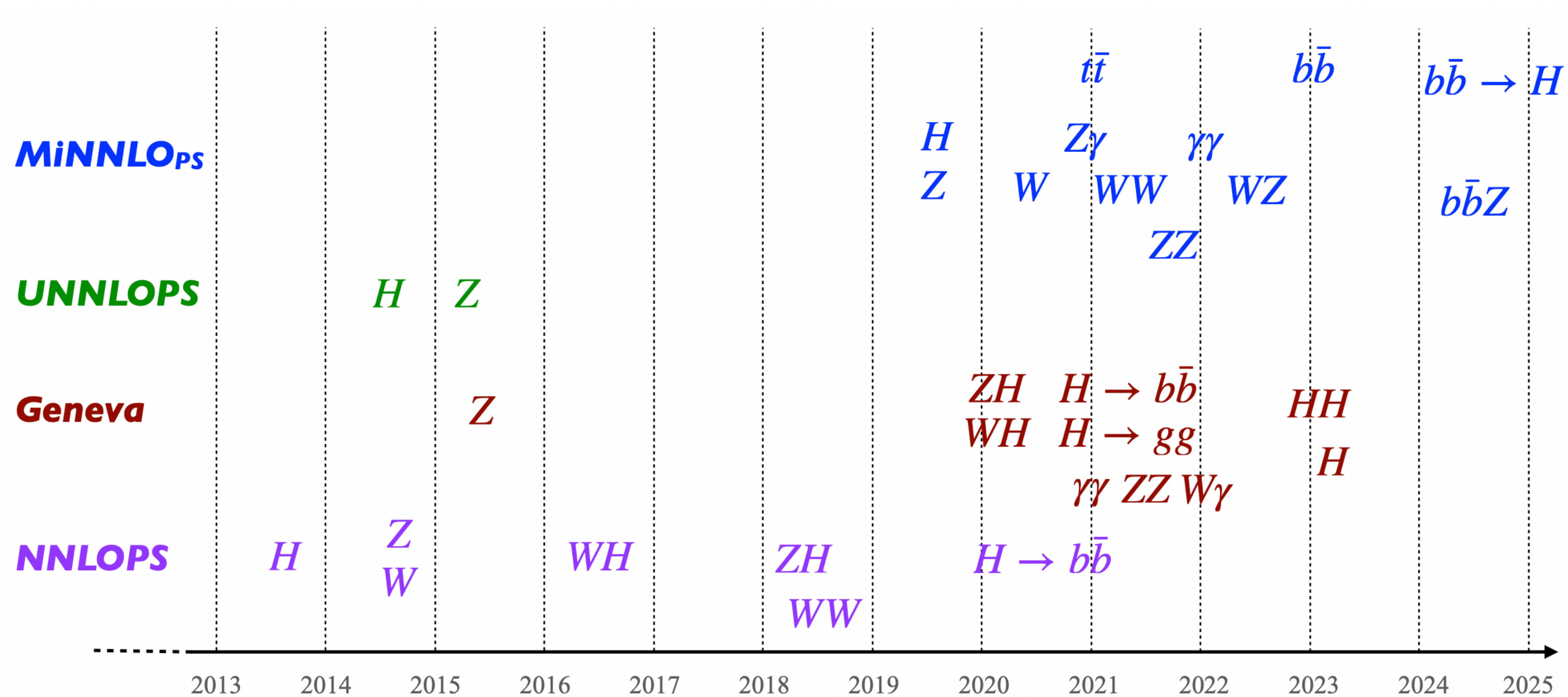




Van Beekveld et al. 2207.09467

Parton shower matching

Different methods developed. NNLOPS with leading logarithmic accuracy in the shower well understood



Not yet clear how to preserve accuracy of more accurate showers in the matching

Art versus automation

Automation is key to making precision scalable and accessible, but it can foster the misconception that precision is merely about brute force. Brute force is insufficient for LHC precision. Deep understanding & breakthroughs prerequisite for practical automation.

PYTHIA

The POWHEG BOX

QCDLoop

Caravel

SHERPA

H7

MADGRAPH5

aMC@NLO

The art of being precise

Art of precision calculations lies in

- identifying what truly matters — and focusing only on that
- knowing what to expect: recognizing key structures, anticipating results, and understanding their numerical impact
- developing original and innovative methods of calculation
- designing new observables that reveal subtle structures, reduce theoretical or experimental uncertainties, or unlock novel phenomena
- using abstraction to find general principles that enable automation, rather than replace insight

Precision is not the absence of creativity—it's a canvas for it.

"…Feynman brought computations to the masses… " J. Schwinger

Conclusion

Progress beyond expectations $\Rightarrow$ remarkable success of theorists

- Progress not due to cranking old machinery but driven by new ideas and developments of new calculational methods
- Strong complementary between numerical and analytical methods

Many calculations eagerly awaited and in sight in the next five years

[in the meantime, clever and robust approximations reduce theory uncertainties]

Perturbative QCD well on track to keep up with experimental precision