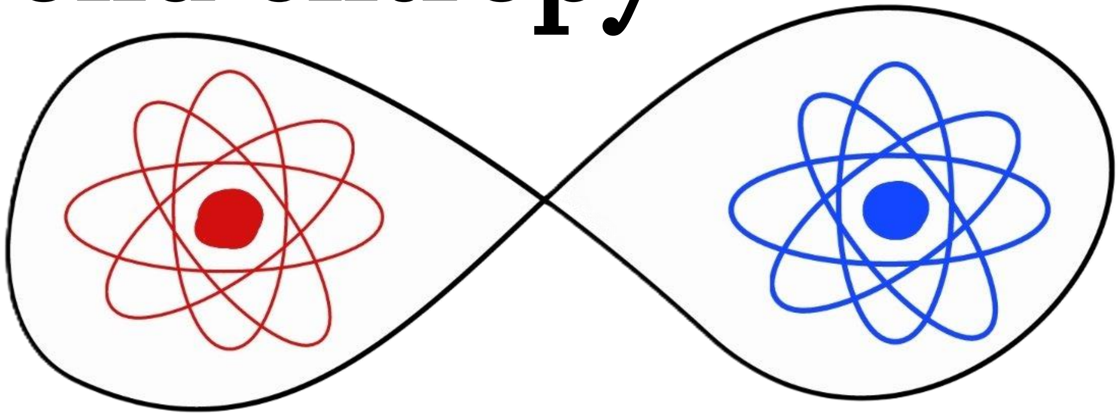
Thermodynamic signatures of Gaussian entanglement beyond entropy

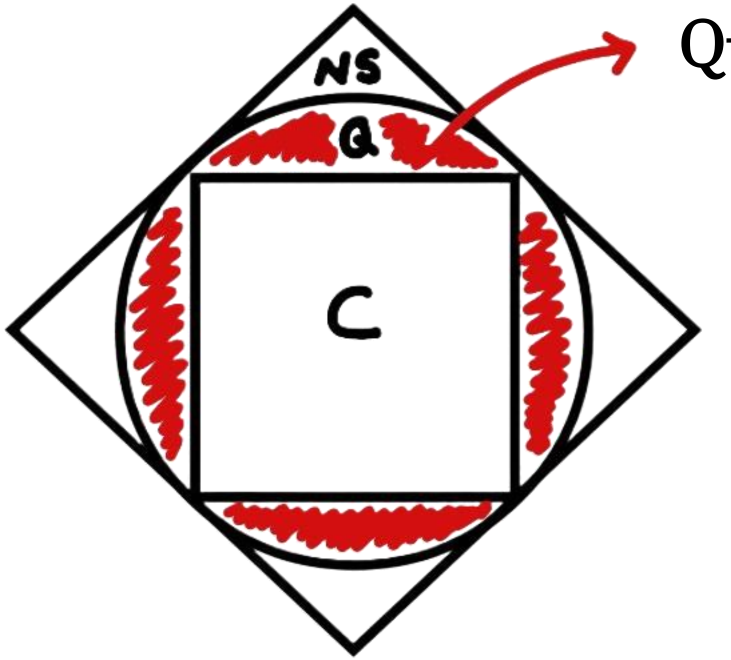


Bea Polo, Federico Centrone,
Alimuddin Mir & Gerardo Adesso

ArXiv preprint:

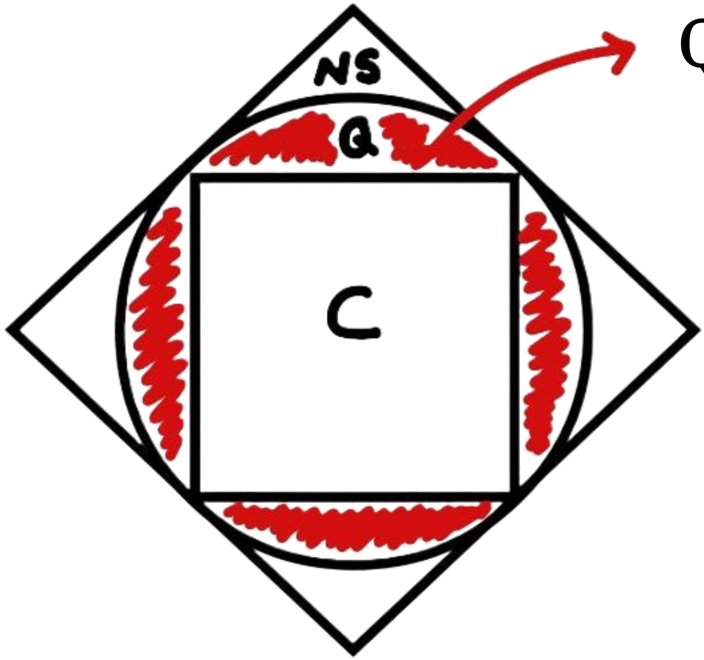
International Conference on Quantum Energy
Padova, 3rd June 2025

1. INTRODUCTION



Quantum correlations

1. INTRODUCTION



Quantum correlations → QUANTUM ADVANTAGE

- Q Communication
- Q Computation
- Q Simulation

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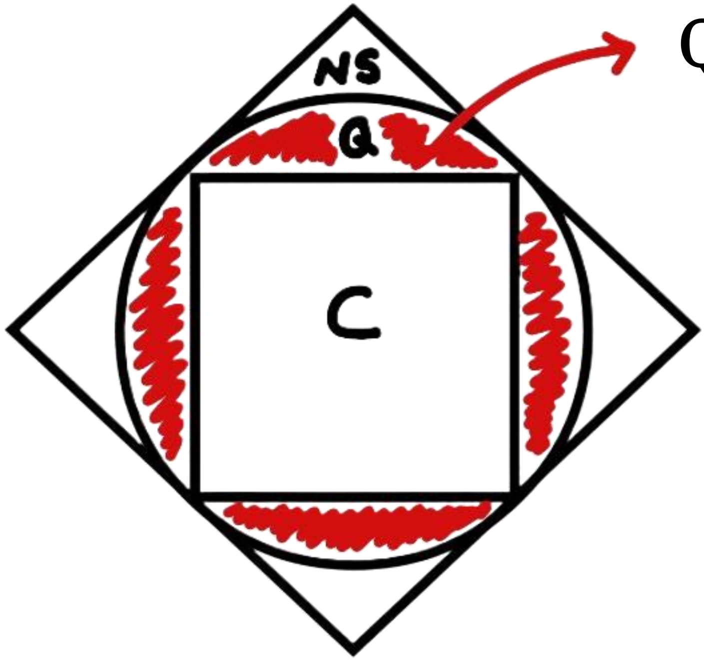

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DETECT and QUANTIFY

Mutual information, conditional entropy, accessible information…

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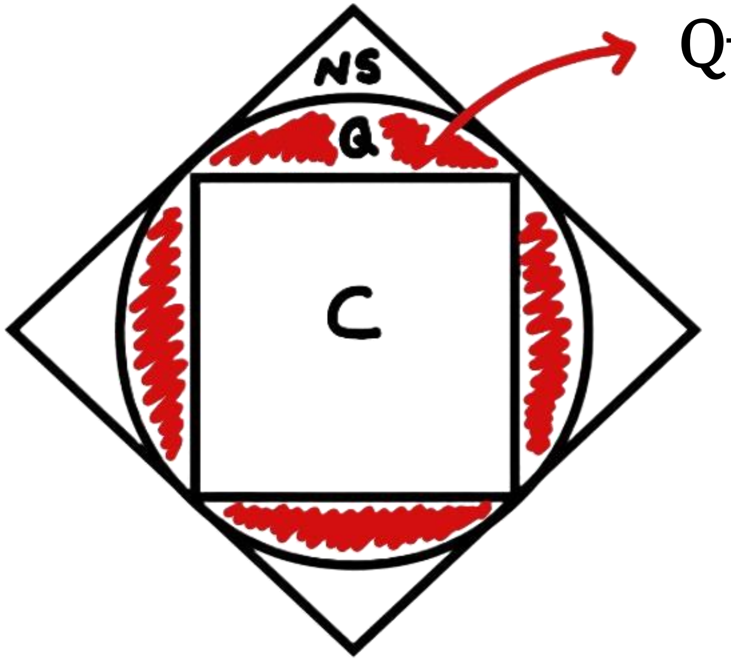


Quantum correlations → QUANTUM ADVANTAGE

Q Communication

Q Computation

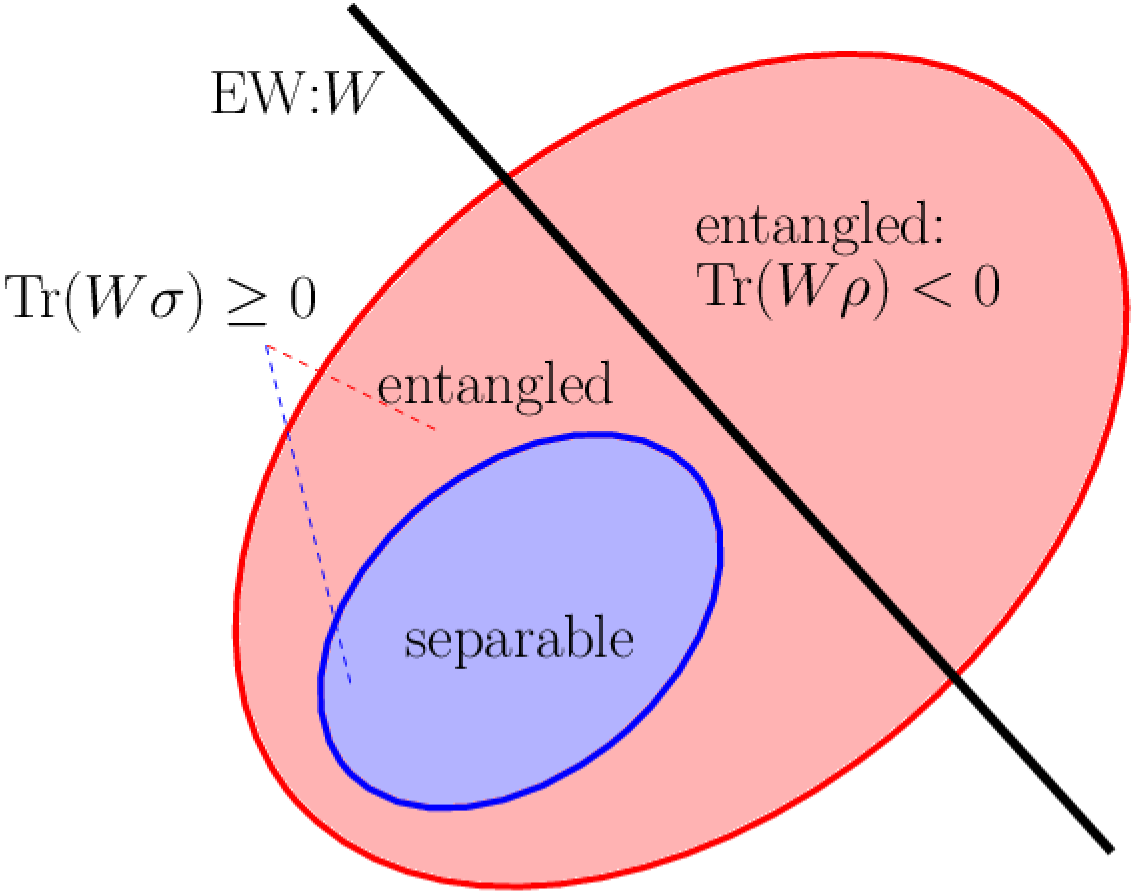
Q Simulation

DETECT and QUANTIFY

Mutual information, conditional entropy, accessible information… require full tomography!! (resource expensive)

1. INTRODUCTION

Entanglement witnesses



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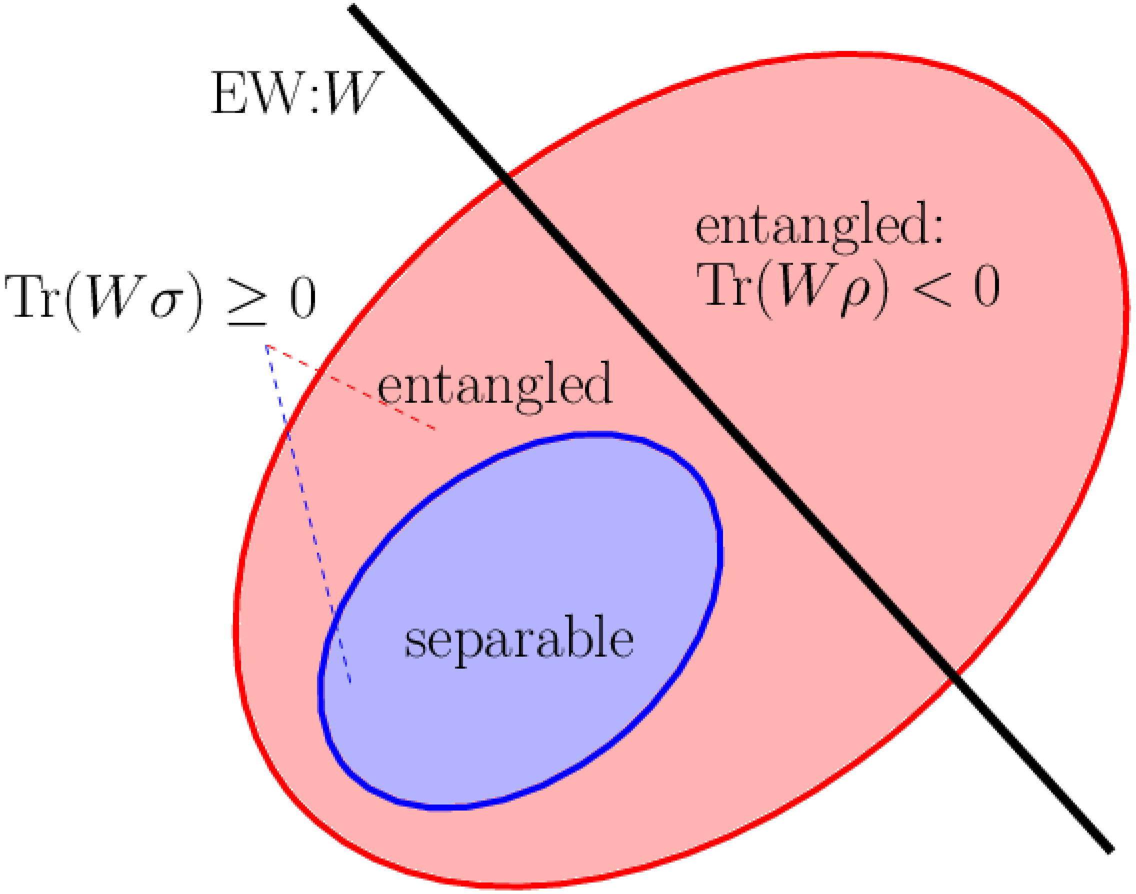

NOT necessary BUT sufficient

certification of the presence of

entanglement in the measured state

W ??

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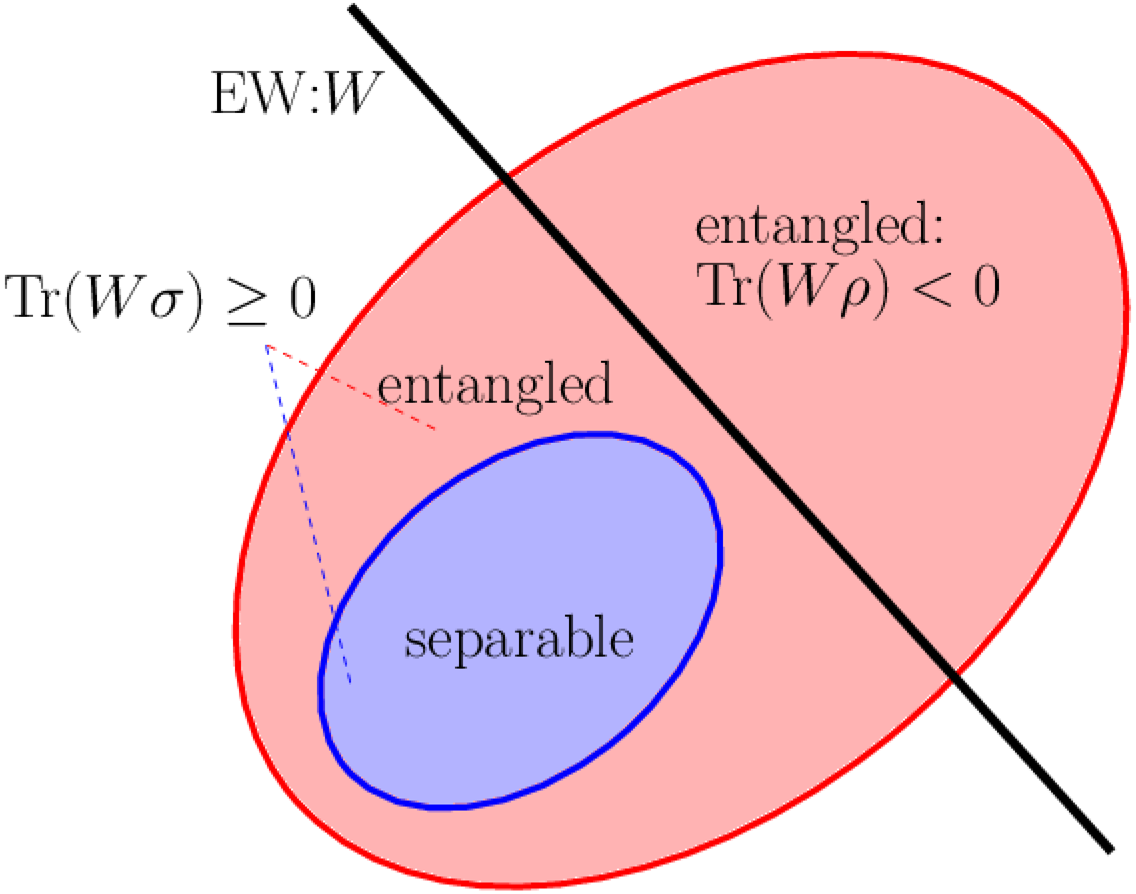

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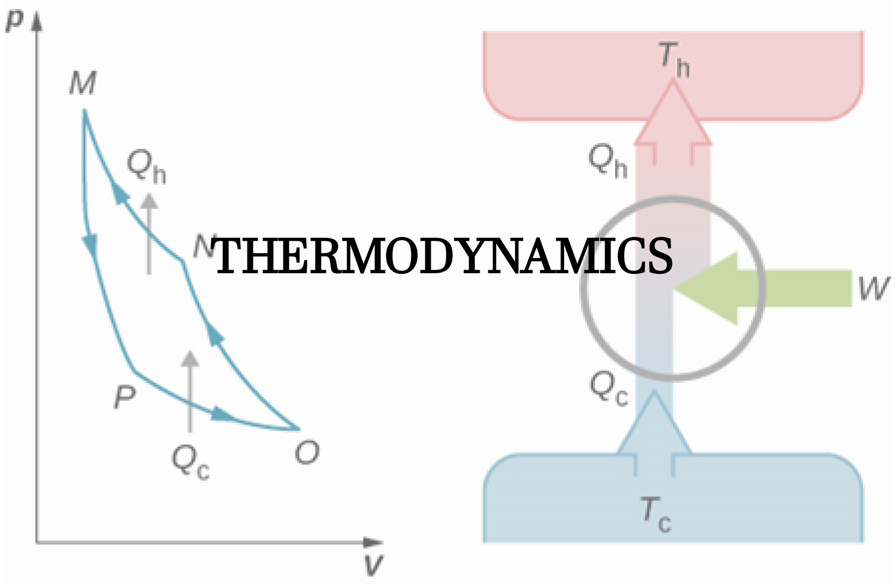

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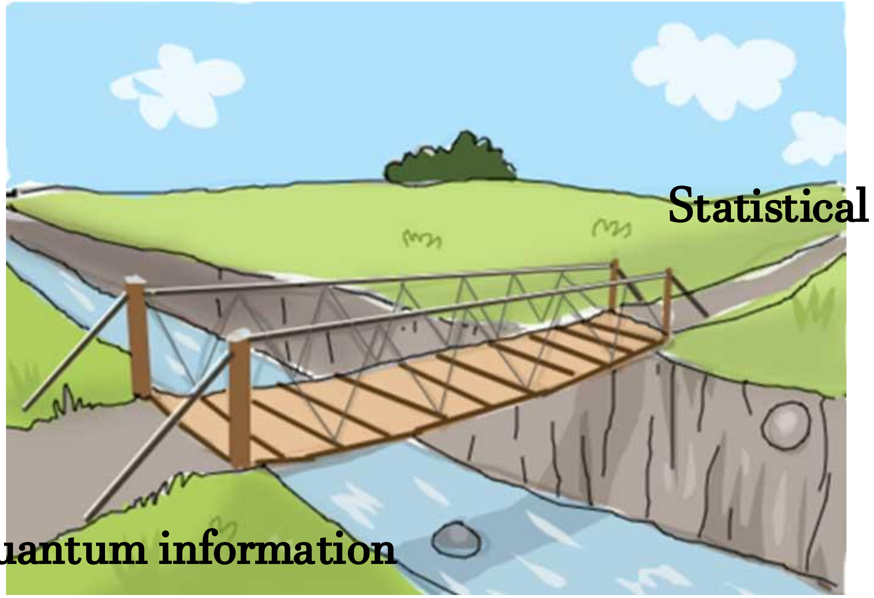

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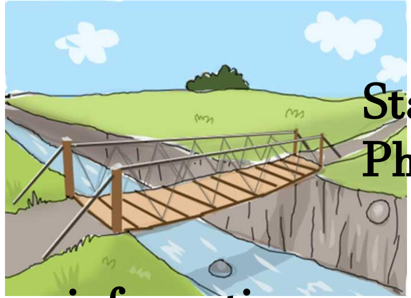

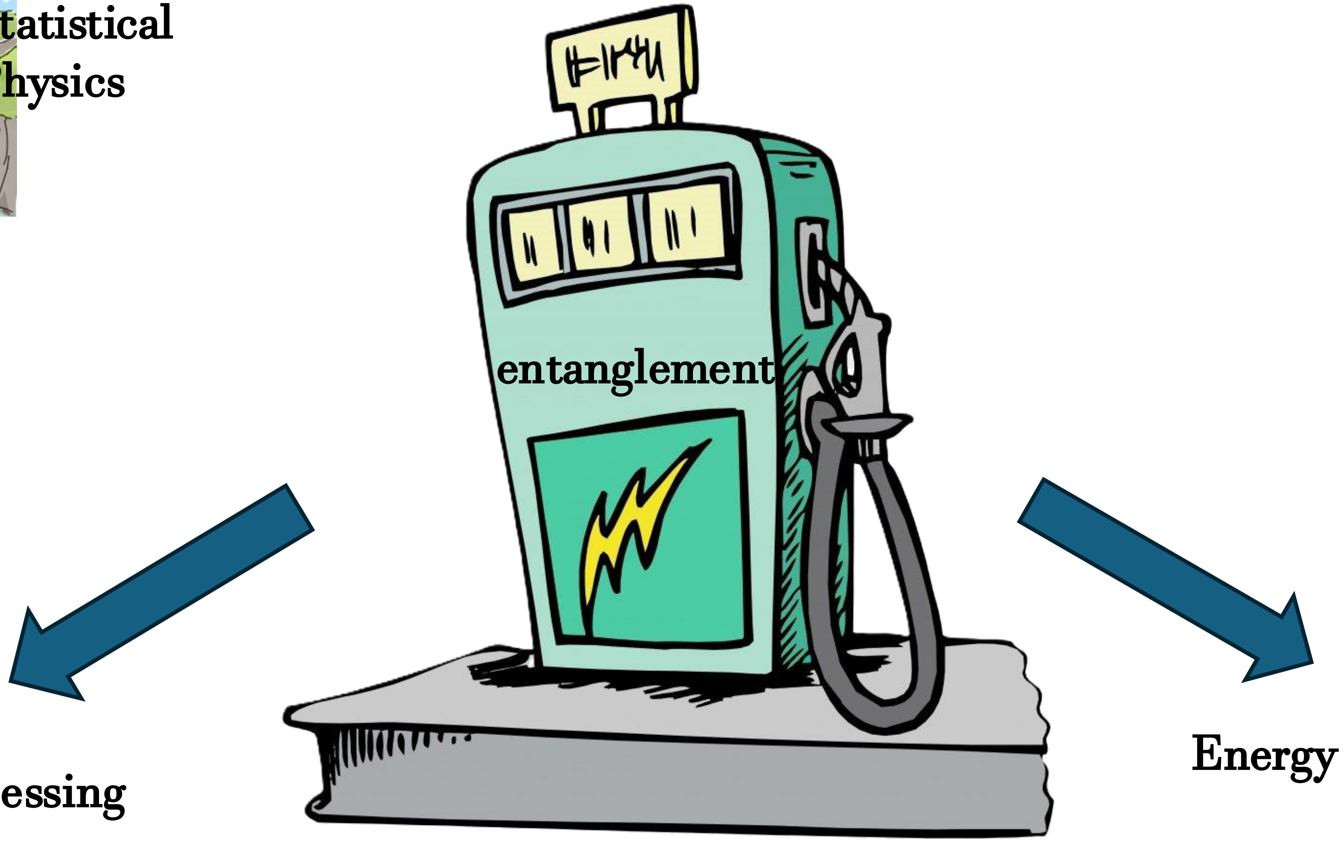

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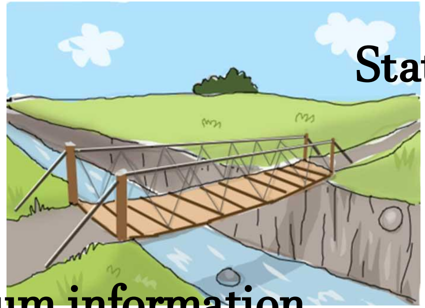
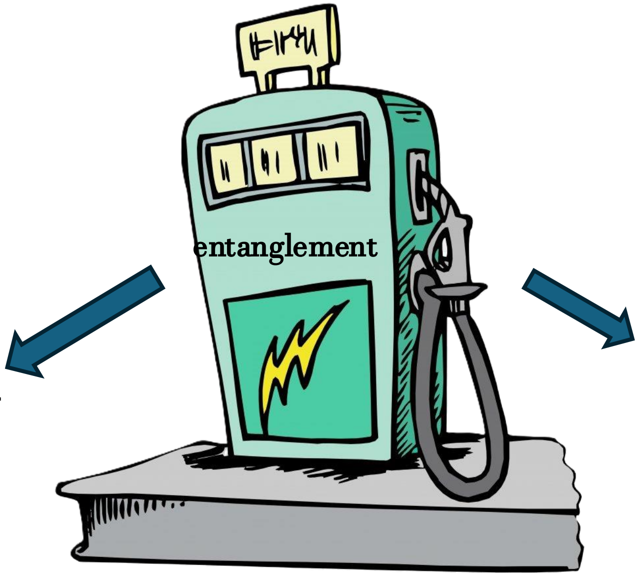


Statistical Physics

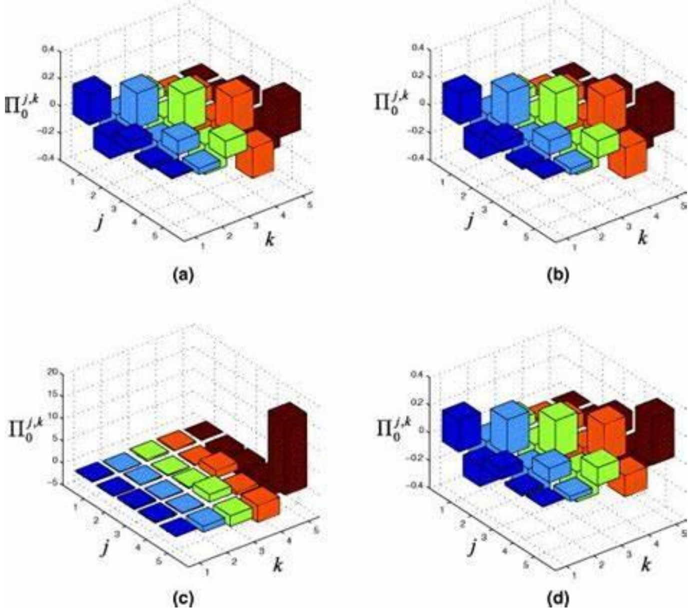
Quantum information



Information processing

Energy processing

Less number of copies as compared to tomography?



1. INTRODUCTION

WORK EXTRACTION

Coupling system to thermal bath

Unitary evolution until no more work can be extracted (in a single copy)

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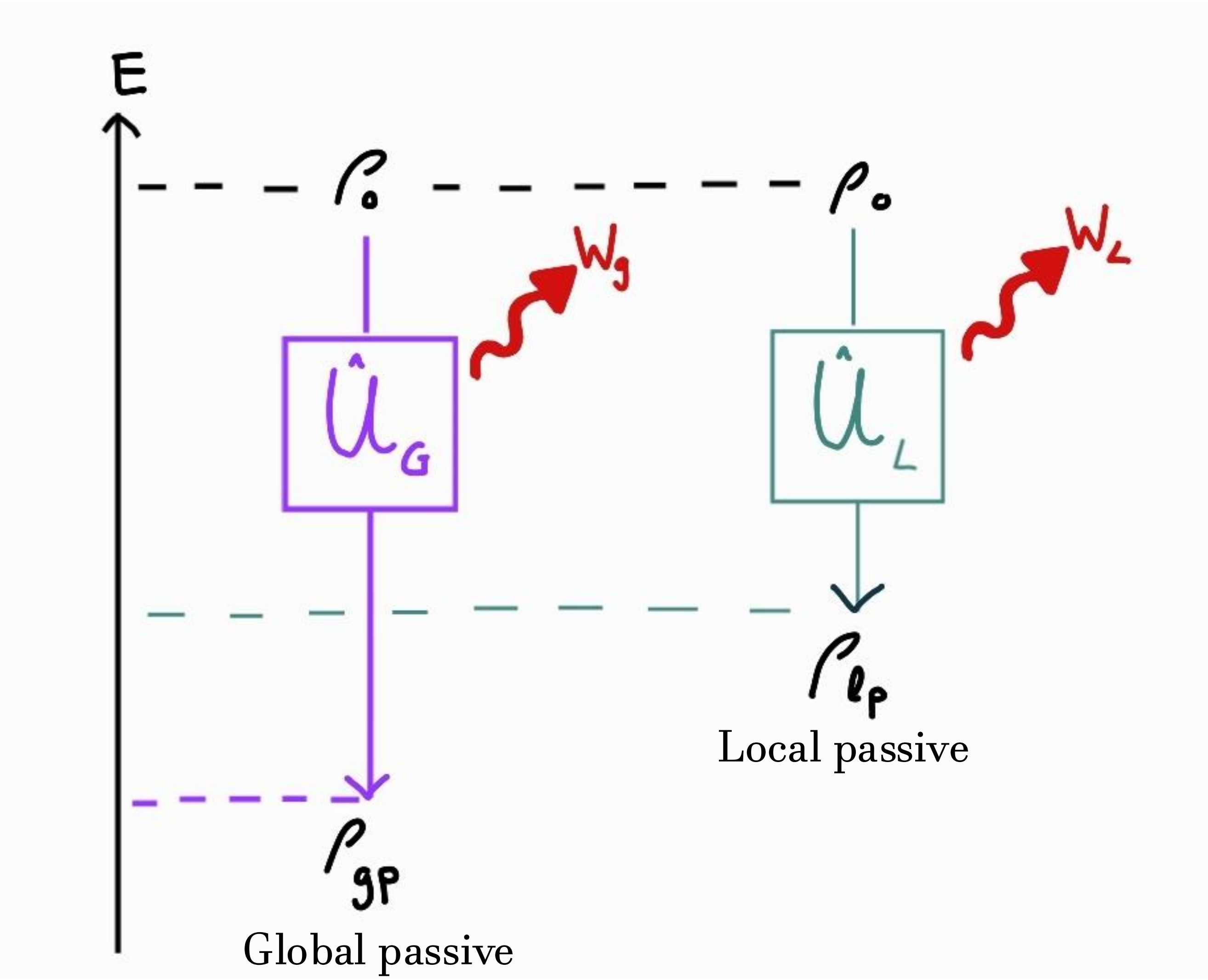
Coupling system to thermal bath

Unitary evolution until no more work can be extracted (in a single copy)

$$\rho_0 \rightarrow \boxed{\hat{U} = e^{-i\hbar \int_0^\tau dt \;\; H+V(t)}} \rightarrow \underbrace{\rho' = \hat{U}^\dagger \rho_0 \hat{U}}_{passive \;\; state}$$

1. INTRODUCTION

2 types of passitivity:



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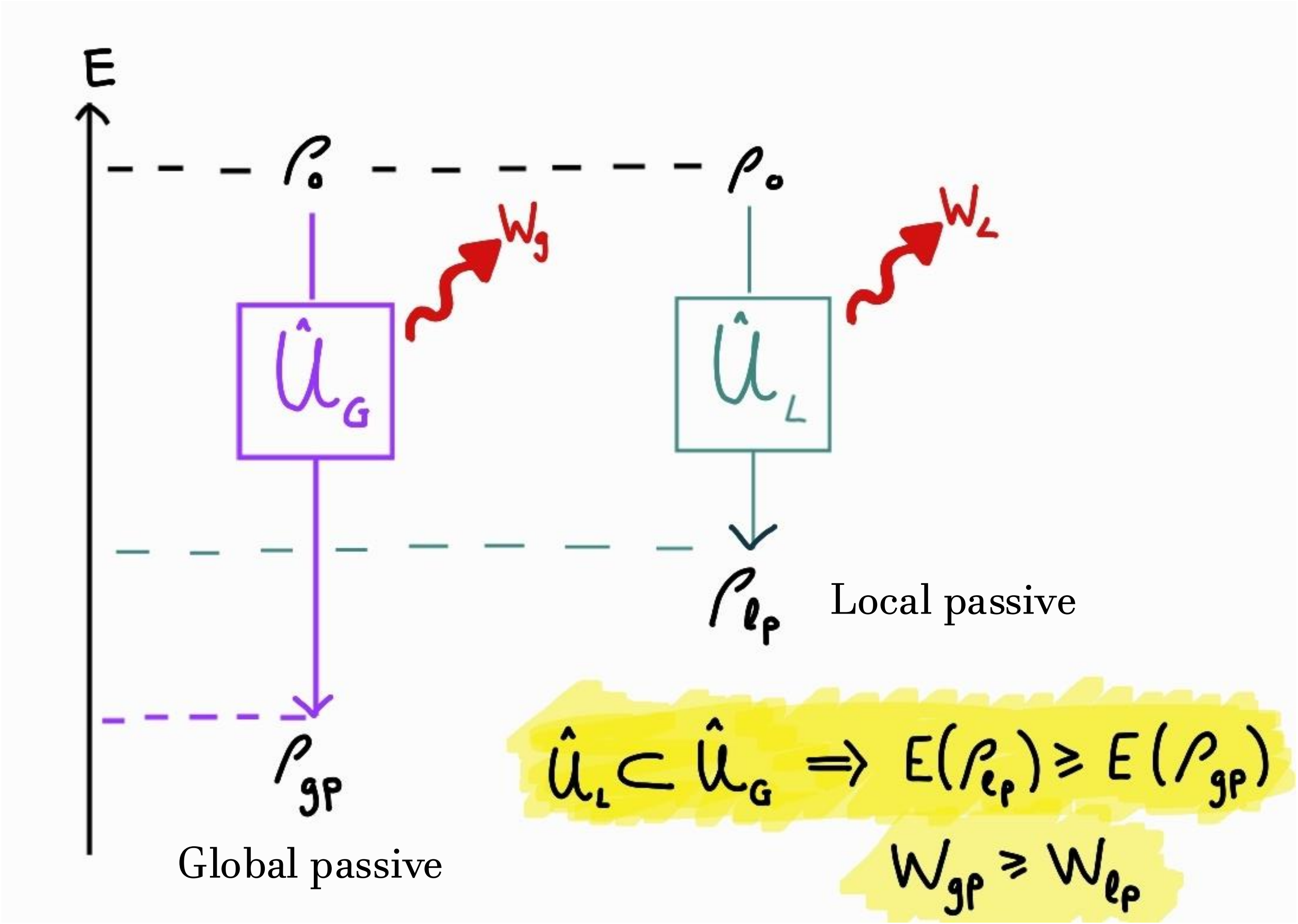

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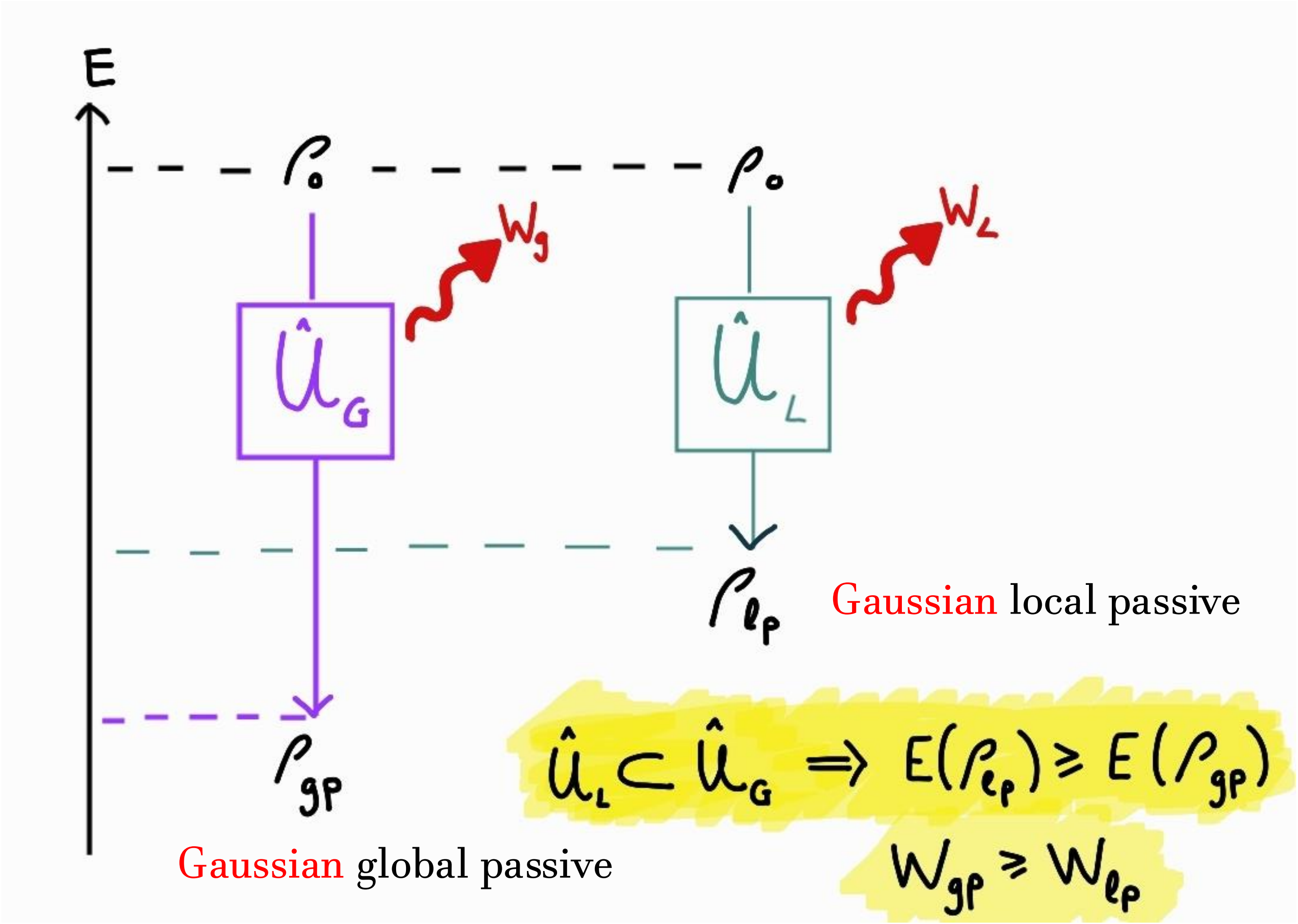

2. ERGOTROPIC GAP as ENTANGLEMENT WITNESS

$$\Delta\mathcal{E} = E_{\text{loc p}} - E_{\text{glob p}} \quad \Longleftrightarrow \quad \text{Entanglement}$$

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⟷ **Entanglement**

Bound on ergotropic gap for bipartite separable states

Mir Alimuddin*, Tamal Guha†, and Preeti Parashar‡

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Phys. Rev. A **99**, 052320 – **Published 14 May, 2019**

DOI: https://doi.org/10.1103/PhysRevA.99.052320

Valid entanglement measure ***for pure bipartite states in DV framework***

- Zero for product states
- Non-zero for all entangled states
- Monotonically decreasing under LOCC

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Not an entanglement monotone

for mixed states

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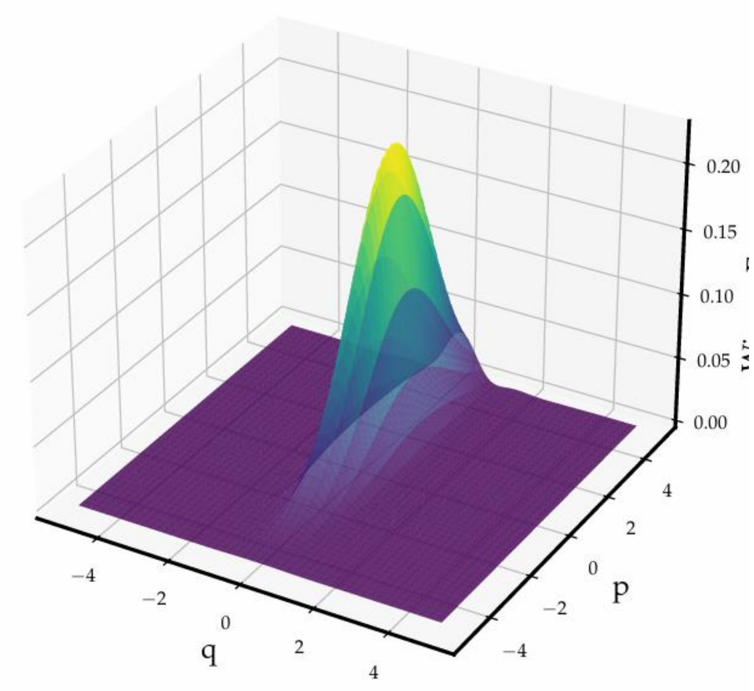
Extension to Continuous Variables???

Gaussian case



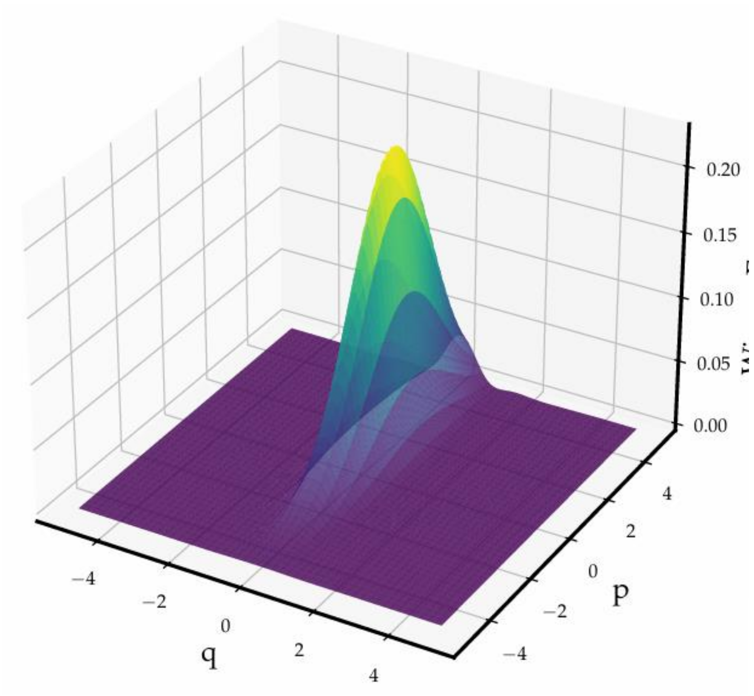
- Easily-tractable subset of continuous-variable states (few parameters needed for their characterization)

- Two-mode Gaussian study ⟶ generalizable to any bipartite Gaussian if certain symmetries are present (locally symmetric).

- Parametrization (Bloch-Messiah):

temperature factors	squeezing	beam-splitter angle	frequencies
$k_1, k_2 \longrightarrow \begin{cases} k = \dfrac{k_1 + k_2}{2} \\ \gamma = \dfrac{k_1 - k_2}{2} \end{cases}$	z_1, z_2	θ	$\omega_1, \omega_2 \longrightarrow \begin{cases} \omega = \omega_1 \\ \alpha = \dfrac{\omega_2}{\omega_1} \end{cases}$

Gaussian case



- Easily-tractable subset of continuous-variable states (few parameters needed for their characterization)

- Two-mode Gaussian study → generalizable to any bipartite Gaussian if certain symmetries are present.

- Bloch-Messiah parametrization

$$\Delta\mathcal{E}_{\rho_G} \to \Delta\mathcal{E}_{\rho_G}(k, \gamma, z_1, z_2, \alpha, \theta)$$

- But results are independent of the parametrization choice

3. RESULTS

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Pure case $k = 1, \gamma = 0$

- Analogous to DV case
- $\Delta\epsilon$ is a valid measure of entanglement
- $\Delta\epsilon$ is functionally dependent on mutual information

3. RESULTS

Mixed case

Unlike DV, $\Delta\mathcal{E}$ is nonzero if and only if there exist correlations between the two modes

3. RESULTS

<u>Mixed case</u>

First observation: **$\Delta\epsilon$ grows with temperature in CV systems !!**

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Counter intuitive: correlations should decrease with more noise destroying coherence

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Consider relative ergotropic gap instead of absolute one:

$$\Delta\mathcal{E}_{rel} = \frac{E_{\text{loc passive}} - E_{\text{glob passive}}}{E_{\text{glob passive}}} \quad \text{for } E_{\text{glob passive}} \neq 0$$

3. RESULTS

Mixed case

$$\Delta\mathcal{E}_{rel} = \frac{E_{\text{loc passive}} - E_{\text{glob passive}}}{E_{\text{glob passive}}} \quad \text{for } E_{\text{glob passive}} \neq 0$$

Functionally independent of mutual information!!!

3. RESULTS

Mixed case

$$\Delta\mathcal{E}_{rel} \leq B$$ for ρ separable ???

3. RESULTS

Mixed case

$$\Delta\epsilon_{rel} \leq B$$

for ρ separable

Impose PPT-based separability condition for Gaussian states

3. RESULTS

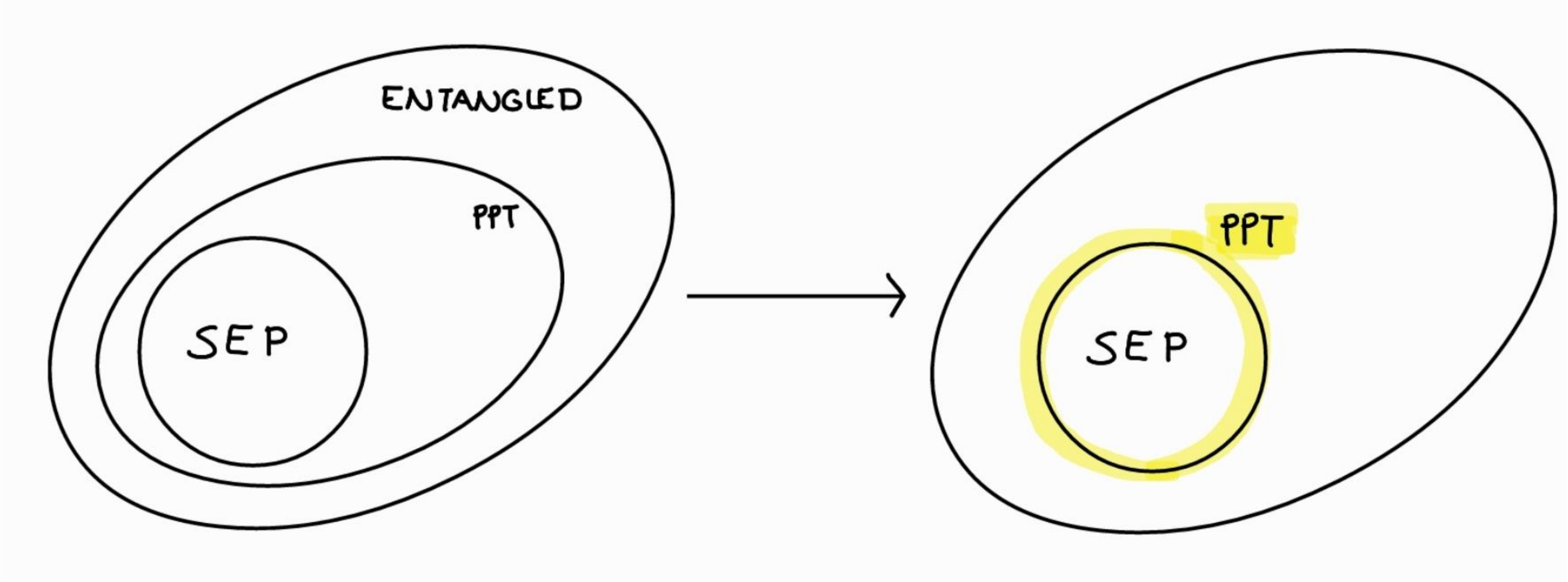
Mixed case

$$\Delta\mathcal{E}_{rel} \leq B^{\text{sep}} \qquad \rho \text{ separable}$$

$$\Downarrow$$

$$B^{\text{sep}} \rightarrow B^{\text{sep}}(k, \gamma, \alpha)$$

Since PPT is necessary AND sufficient for separability of two-mode Gaussian states...



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$$\Delta\mathcal{E}_{rel} \geq B^{\mathrm{ent}} \quad \rho \text{ entangled}$$

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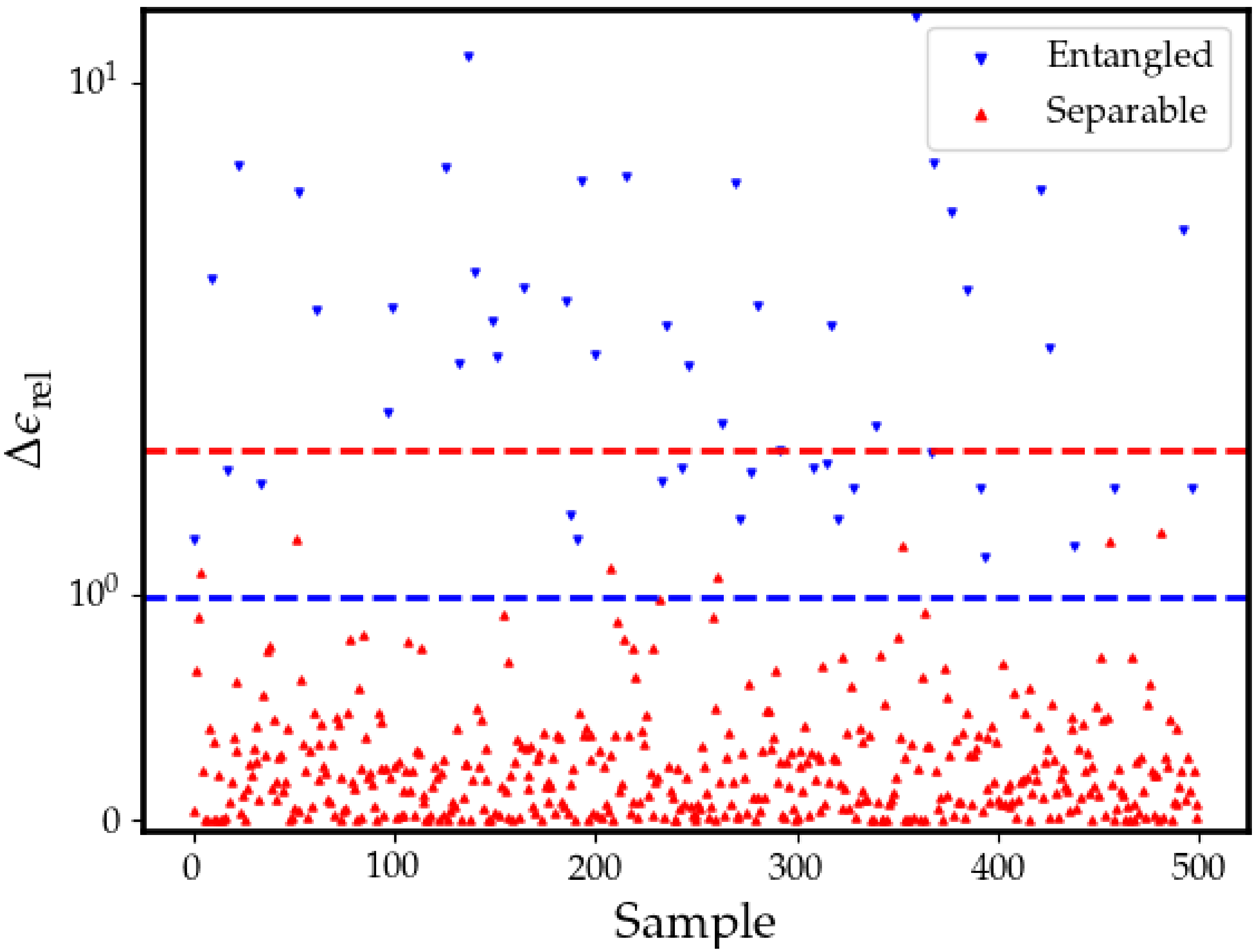
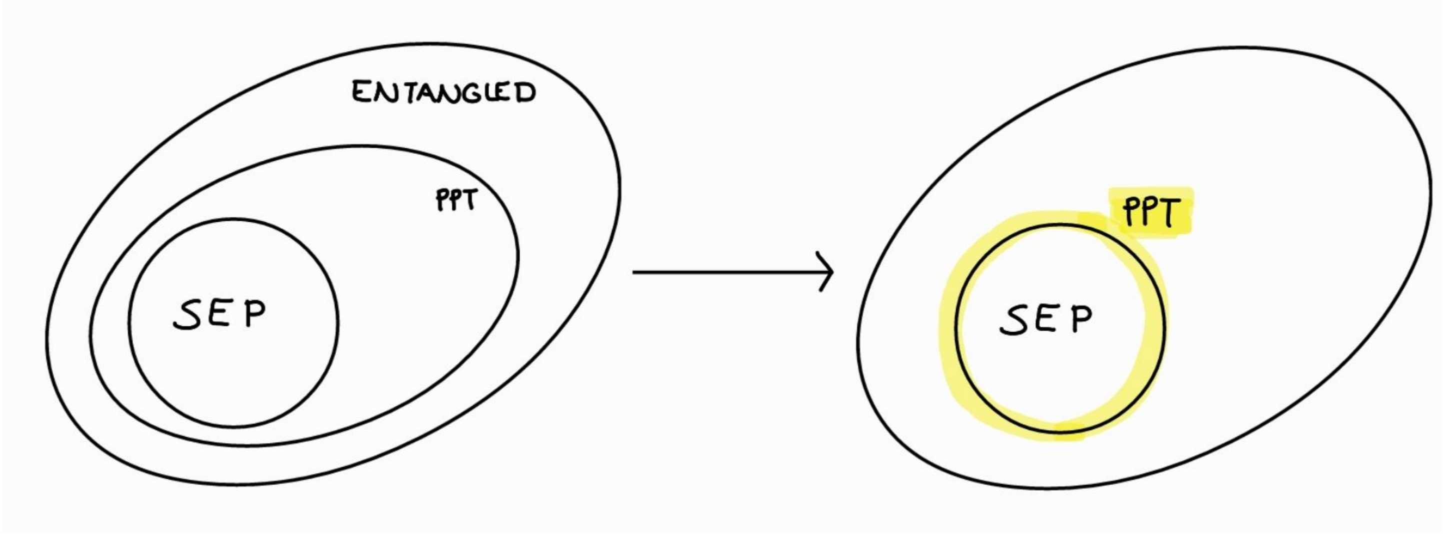
3. RESULTS



Entangled
Separable
10^1
10^0
0
$\Delta\epsilon_{rel}$
0
100
200
300
400
500
Sample

3. RESULTS

For locally symmetric bipartite Gaussians, PPT is necessary and sufficient:



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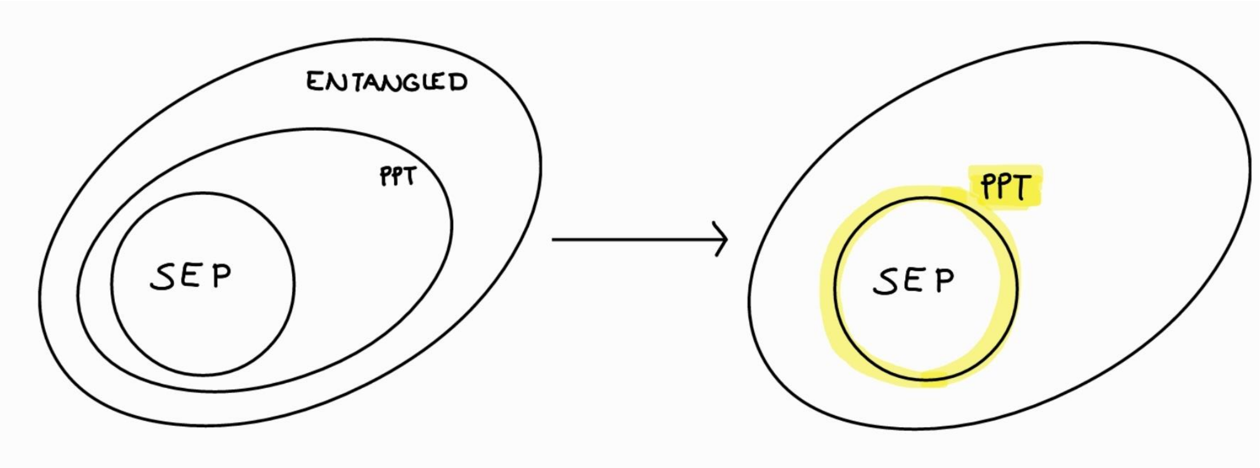
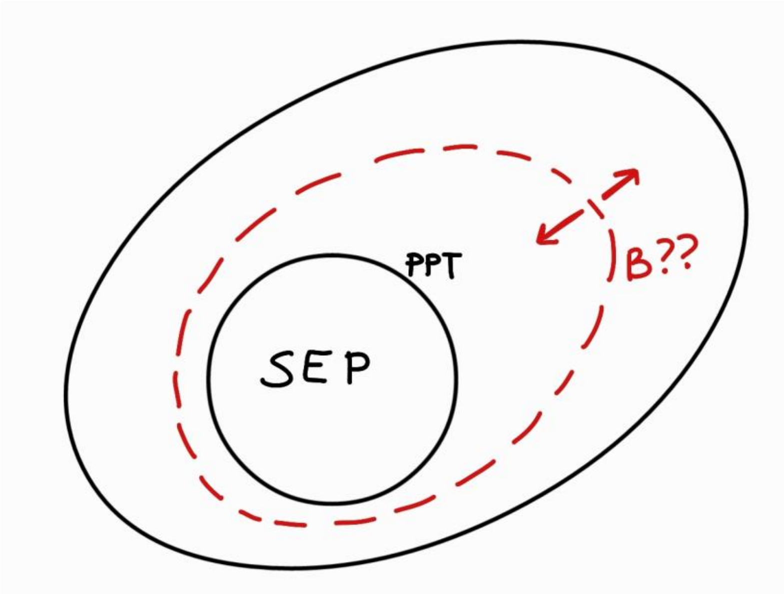


Question: how “good” is our criterion at detecting entangled states?



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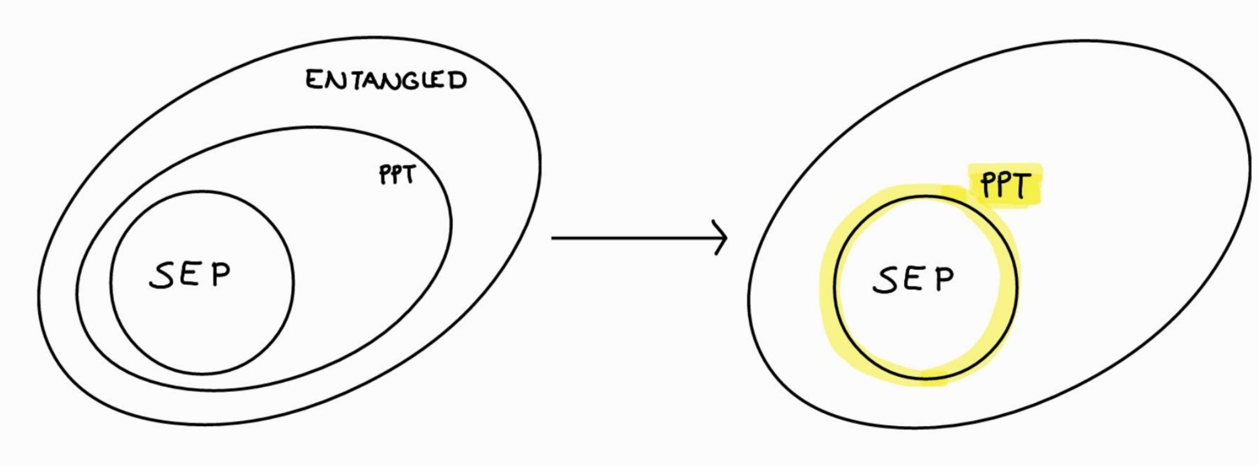

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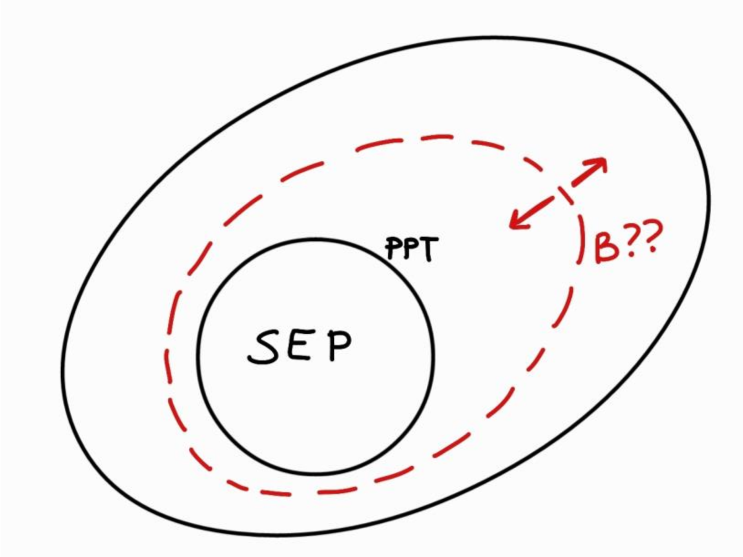

Faithfulness of our witness depends on the distance between B^{ent} and B^{sep}

3. RESULTS

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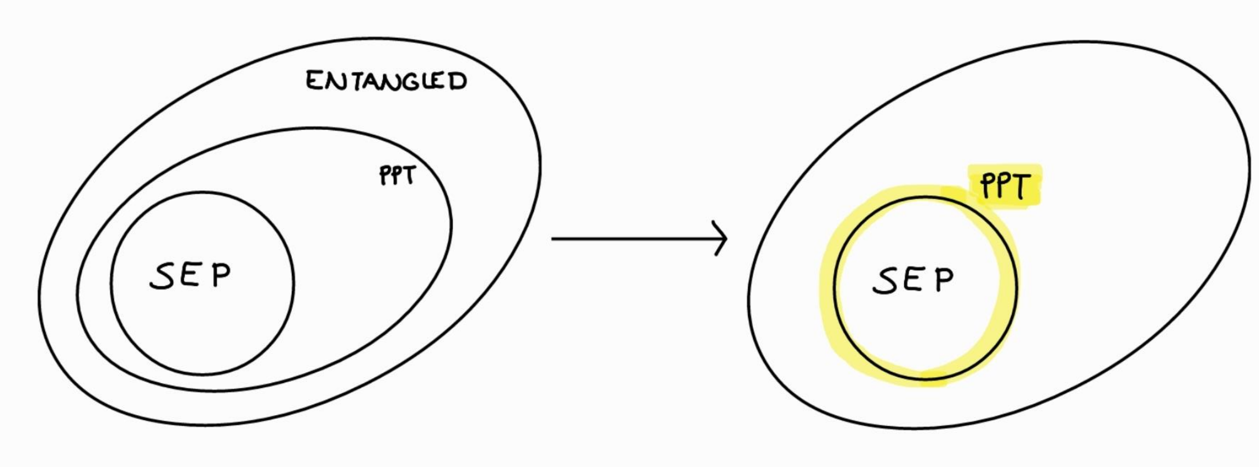

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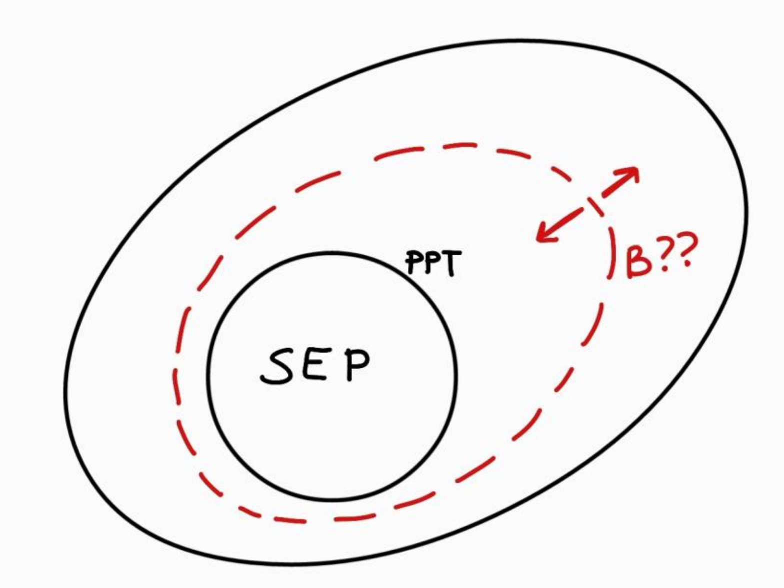
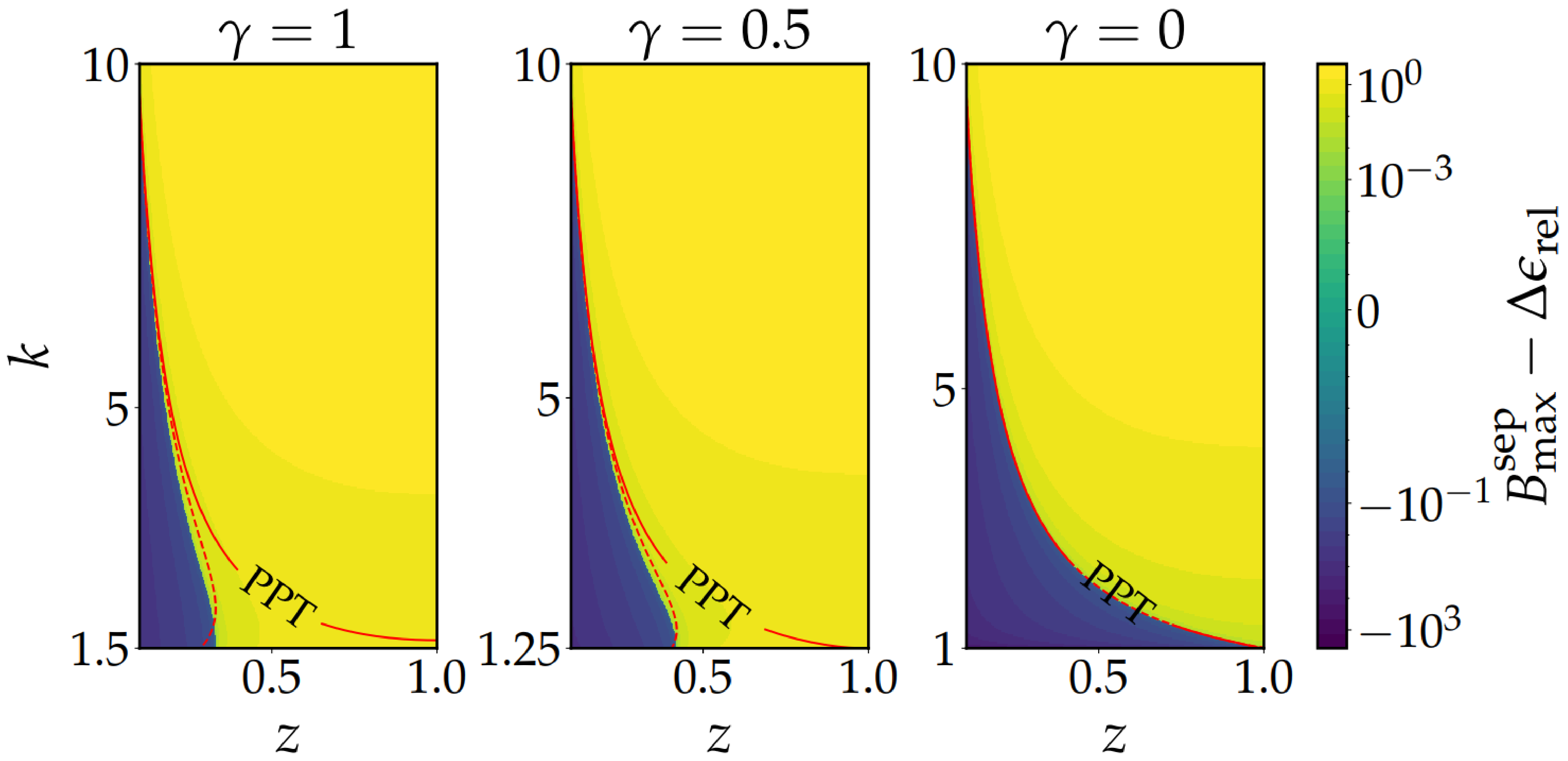


We study the parametric family of two-mode-squeezed states:

$$TMS(z, k, \gamma)$$

3. RESULTS



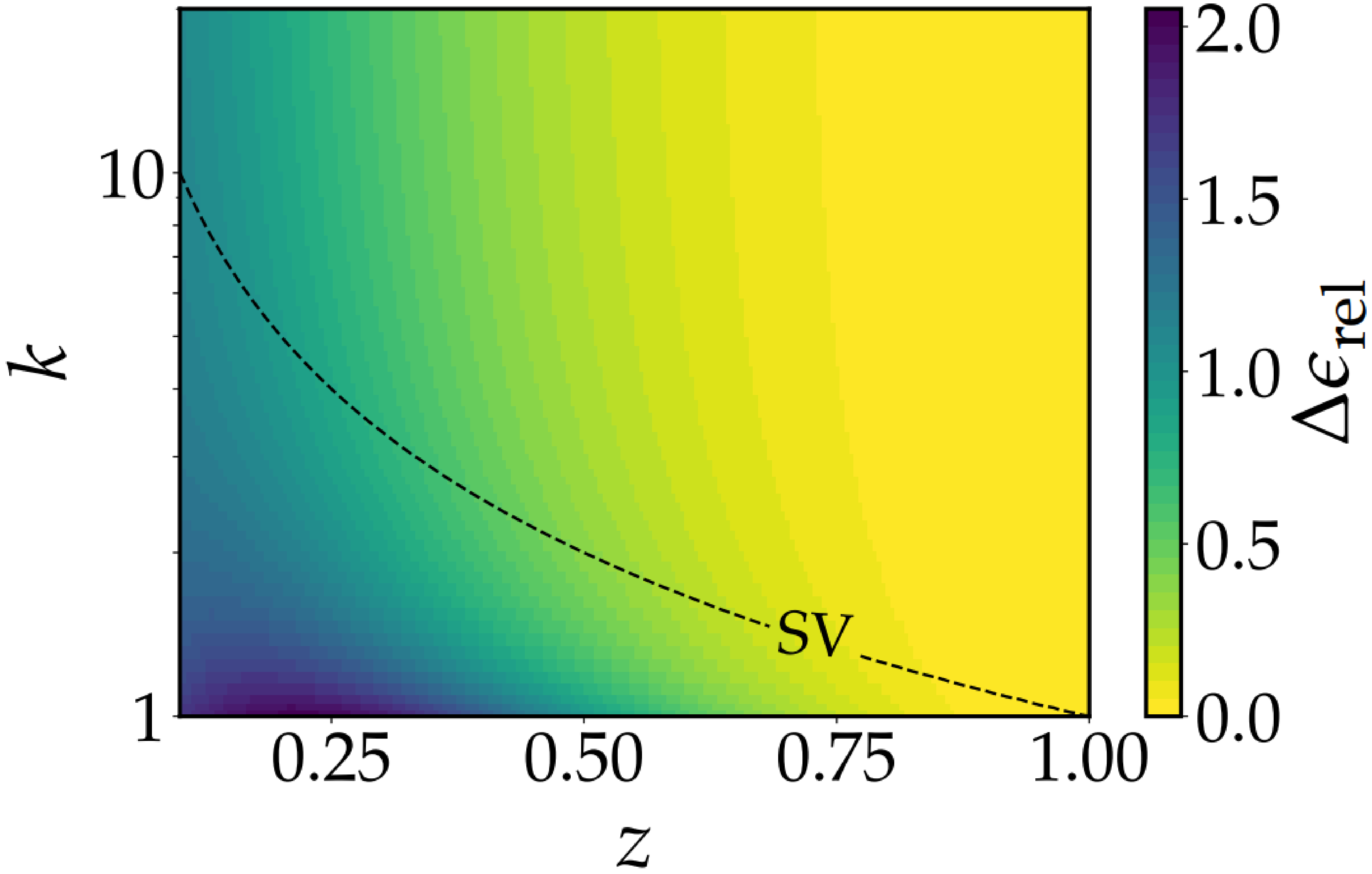
$\gamma = 1$
$\gamma = 0.5$
$\gamma = 0$
10
5
1.5
1.25
1
k
0.5
1.0
z
PPT
10^0
10^{-3}
0
-10^{-1}
-10^3
$B_{\max}^{\text{sep}} - \Delta\epsilon_{\text{rel}}$

Non-Gaussian case

- We study the behaviour of Gaussian relative ergotropic gap on non-Gaussian states
- Focus on TMS states (Gaussian) that undergo a photon subtraction (non-Gaussian) in one of the modes
- Apply SV criterion as information-theoretic witness of entanglement

$$\langle a^\dagger a\rangle_\rho \langle b^\dagger b\rangle_\rho - \langle ab\rangle_\rho \langle a^\dagger b^\dagger\rangle_\rho < 0 \implies \rho \text{ entangled}$$

Non-Gaussian case



4. CONCLUSIONS and OUTLOOK

- Relative ergotropic gap as a witness of entanglement for Gaussian mixed bipartite states (locally symmetric)

- Independent quantity from mutual information

- Bound on the maximum gap for two-mode Gaussian separable states

- Bound on the minimum gap for two-mode Gaussian entangled states

- Necessary and sufficient criterion for states with symmetric thermal fluctuations (equivalent to PPT)

- Possible extension of the applicability of the criterion to more complex non-Gaussian states

4. CONCLUSIONS and OUTLOOK

- Generalization to broader classes of non-Gaussian states
- Multipartite scenario
- Efficiency benchmark $\left(\#\ \text{copies}\right)$ with respect to quantum tomography

BACK-UP SLIDES

Gaussian relative ergotropic gap:

$$\Delta\mathcal{E}_{rel} = \frac{1}{(k-1)(1+\alpha)+\gamma(1-\alpha)} \cdot \Bigg[\sqrt{(k+\gamma)^2 cos^4\theta + (k-\gamma)^2 sin^4\theta + (k^2-\gamma^2) cos^2\theta sin^2\theta \left(\frac{z_1^2+z_2^2}{z_1 z_2}\right)}$$
$$+\alpha\sqrt{(k-\gamma)^2 cos^4\theta + (k+\gamma)^2 sin^4\theta + (k^2-\gamma^2) cos^2\theta sin^2\theta \left(\frac{z_1^2+z_2^2}{z_1 z_2}\right)} - [k(1+\alpha)+\gamma(1-\alpha)]\Bigg]$$

Two-mode Gaussian separability condition:

$$(k^4+\gamma^4-2k^2\gamma^2-2k^2-2\gamma^2+1)z_1z_2 \geq 4sin^2\theta cos^2[(z_1^2+z_2^2)(k^2-\gamma^2)-z_1z_2(2k^2+2\gamma^2)]$$

Bound for separable and entangled states:

$$B^{\text{sep}} = \frac{\frac{1+\alpha}{2}\sqrt{1 + k^4 + \gamma^4 + 2k^2 + 2\gamma^2 - 2k^2\gamma^2 + 8k\gamma} - [k(1+\alpha) + \gamma(1-\alpha)]}{(k-1)(1+\alpha) + \gamma(1-\alpha)}$$

$$B^{\text{ent}} = \frac{\frac{1+\alpha}{2}\sqrt{1 + k^4 + \gamma^4 + 2k^2 + 2\gamma^2 - 2k^2\gamma^2 - 8k\gamma} - [k(1+\alpha) + \gamma(1-\alpha)]}{(k-1)(1+\alpha) + \gamma(1-\alpha)}$$

Distinction between DV and CV:

In continuous-variable systems,

$$\Delta\mathcal{E} = k \cdot (\text{correlations}) \propto k$$