Solving the unit commitment problems with quantum annealers.

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Quantum annealing

The technique used to solve optimization problems with a quantum system that evolves based on the time-dependent Schrödinger equation

The quantum system is prepared in the known ground state of an initial Hamiltonian H_{init} and is changed over time into H_{final}

Background

Quantum annealing

Based on the **adiabatic theorem**:
a system stays in an instantaneous ground state if change happens sufficiently slowly.

The formulation is built so that:

$$H(t/t_{max}) = A(t/t_{max})H_{init} + B(t/t_{max})H_{final}$$

Hamiltonian H_{final} encodes optimization problem Functions A and B satisfy $A(0) >> B(0)$ and $A(1) << B(1)$

Unit commitment problem

The unit Commitment (UC) problem involves optimizing the scheduling of power generators to either minimize production costs or maximize revenue.

Vast family of problems: many different aspects of the scheduling can be considered.

Problem Formulation

Constrained optimization

The UC problem is a **continuous variable constrained optimization** problem. Most instances of this problem include three aspects:

- **Time**: each optimization is done in a time window with discrete time steps.
- **Generators**: the targets of the optimization.
- **Forecast**: requirement or conditions for the energy production.

The problem we choose is a **prosumer's earning optimization** problem:

Instead of a network we modeled the activity of a single small prosumer like a residential building. This model takes into consideration added details:

- Green energy production.
- Batteries for energy storage.
- Fixed consumption and production.
- Financial incentives.

Problem formulation

Generators

Each **generator** k is represented by the set

- Energy produced ($E_k(t)$).
- Unitary production cost (cost_k).
- Minimum energy produced (E_k^{min}).
- Maximum energy produced (E_k^{nom}).
- Maximum energy produced at time t ($E_k^{max}(t)$).

Problem formulation

Forecast

The **forecasts** predict energy market prices:

- unitary selling price ($P^{vend}(t)$).
- Unitary buying price ($P^{acqu}(t)$).

Problem formulation

Batteries

Each **battery** a is represented by the set

- Energy stored at time t ($E_a(t)$).
- Energy discharged at time t ($E_a^s(t)$).
- Energy charged at time t ($E_a^c(t)$).
- Maximum energy capacity (E_a^{max}).
- ...

The end goal of optimization is to maximize prosumer earnings based on the forecast. To achieve this we create a cost function:

$$\sum_{t}^{T} \left(E^{vend} P^{vend} + E^{inc} P^{inc} - E^{acqu} P^{acqu} - \sum_{k}^{K} \mathrm{cost}_k E_k(t) \right)$$

And we selected the variables $E_k(t)$, $E_a^s(t)$, and $E_a^c(t)$ to be the **target variable** to optimize.

Problem formulation

Constraints

Finally, to obtain a reasonable solution we need to set some constraints to this problem:

- Range (max and min).
- Charge and discharge.
- Sell and buy.
- Incentives.
- Zero sum.

QUBO formulation

Once we have our constrained model, we need to change that formulation to one that can be embedded in a quantum annealer. To do so we follow these steps:

- Create a **binary representation** of the continuous variables.
- Rewrite the objective function on the basis of the new variables.
- Rewrite each constraint on the basis of the new variables.
- Add the new constraints to the objective function using **hyperparameters**.

Complexity

The number of **time slots** T greatly influences the complexity.

T is a multiplication factor for both the number of constraints (9 constraints for each t) and the number of variables (75 new variables each t).

Real-life scenarios time-slots of 15 minutes over 24 hours (96 time slots).

Complexity

The binarization of big values creates combinations of variables (QUBO matrix entries) with different values.

Adding the constraints to the objective function through hyperparameters changes these values as well.

Optimizing a problem where weights can be between 1 and 10^6 is extremely difficult.

Numerical results

We performed experiments with multiple techniques.

- **Dwave quantum annealer**.
- **Dwave hybrid solver**.
- **Simulated annealing**.

Numerical results

Simulated annealing

Test with 80 periods and 1 battery:

```
Time Used: 4115.8174 seconds
parameters:
(60.0, 12.0, 2.0, 140.0, 2.0, 0.005, 15.0, 900.0, 60.0)
Violated 24 constraints with balance: -130811.22880000001
------------------------------------------------------------------
```

	gen0	c_acc0	s_acc0	E_vend	E_acqu	pen_inc	scelta inc	max_p gen0	E prodotta	E consumata	netto_energia	pvend	pacqu	costo(t)	ricavo(t)	CF(t)	PF(t)
0	0.0	0	0	0	590	0.0	prod	0.0	0.0	590.4	-4.000000e-01	1.0147	2.0147	1188.6730	0.000	590.4	0.0
1	0.0	8	0	0	576	0.0	prod	0.0	0.0	573.6	2.400000e+00	0.9920	1.9920	1147.3920	0.000	565.6	0.0
2	0.0	64	0	0	618	0.0	prod	0.0	0.0	620.4	-2.400000e+00	0.9945	1.9945	1232.6010	0.000	556.4	0.0
3	0.0	16	0	0	576	0.0	prod	0.0	0.0	575.2	8.000000e-01	0.9965	1.9965	1149.9840	0.000	559.2	0.0
4	0.0	0	0	0	575	0.0	prod	0.0	0.0	586.4	-1.140000e+01	0.9914	1.9914	1145.0550	0.000	586.4	0.0
5	0.0	0	0	0	632	0.0	prod	0.0	0.0	630.8	1.200000e+00	1.1153	2.1153	1336.8696	0.000	630.8	0.0
6	7.0	0	0	0	703	9.8	prod	6.0	9.8	720.4	-7.600000e+00	1.3324	2.3324	1639.6772	10.780	720.4	2.8
7	0.0	0	0	0	768	32.0	prod	67.2	32.0	790.8	9.200000e+00	1.5790	2.5790	1980.6720	35.200	790.8	32.0
8	185.0	0	0	0	641	268.2	prod	182.4	268.2	908.8	4.000000e-01	1.4292	2.4292	1557.1172	295.020	908.8	83.2
9	151.0	0	0	0	650	283.8	prod	302.0	283.8	933.2	6.000000e-01	1.5088	2.5088	1630.7200	312.180	933.2	132.8
10	377.0	80	0	0	512	535.8	prod	378.8	535.8	1048.4	-6.000000e-01	1.4086	2.4086	1233.2032	589.380	968.4	158.8
11	400.0	0	0	0	418	578.8	prod	396.8	578.8	997.2	-4.000000e-01	1.1564	2.1564	901.3752	636.680	997.2	178.8
12	223.0	0	0	0	523	408.6	prod	420.8	408.6	931.6	0.000000e+00	1.0052	2.0052	1048.7196	449.460	931.6	185.6
13	119.0	0	0	0	637	294.6	prod	400.4	294.6	932.0	-4.000000e-01	1.0024	2.0024	1275.5288	324.060	932.0	175.6
14	232.0	0	0	0	568	386.4	prod	368.0	386.4	952.8	1.600000e+00	1.1528	2.1528	1222.7904	425.040	952.8	154.4
15	127.0	64	0	0	741	244.2	prod	277.2	244.2	987.6	-2.400000e+00	1.2879	2.2879	1695.3339	268.620	923.6	117.2
16	127.0	64	0	0	779	202.6	prod	173.6	202.6	983.6	-2.000000e+00	1.3466	2.3466	1828.0014	222.860	919.6	75.6
17	68.0	0	0	0	844	97.6	prod	71.6	97.6	941.6	0.000000e+00	1.4490	2.4490	2066.9560	107.360	941.6	29.6
18	7.0	0	0	0	967	14.2	prod	18.0	14.2	981.2	1.136868e-13	1.6550	2.6550	2567.3850	15.620	981.2	7.2
19	0.0	10	0	0	1024	0.4	prod	0.8	0.4	1002.4	2.200000e+01	1.5899	2.5899	2652.0576	0.440	992.4	0.4
20	0.0	32	0	0	642	0.0	prod	0.0	0.0	643.2	-1.200000e+00	1.1000	2.1000	1348.2000	0.000	611.2	0.0
21	0.0	0	0	0	585	0.0	prod	0.0	0.0	585.2	-2.000000e-01	1.0480	2.0480	1198.0800	0.000	585.2	0.0
22	0.0	64	0	0	639	0.0	prod	0.0	0.0	640.8	-1.800000e+00	1.0221	2.0221	1292.1219	0.000	576.8	0.0
23	0.0	49	0	0	640	0.0	prod	0.0	0.0	630.6	9.400000e+00	1.0670	2.0670	1322.8800	0.000	581.6	0.0
24	0.0	0	0	0	607	0.0	prod	0.0	0.0	607.6	-6.000000e-01	1.0678	2.0678	1255.1546	0.000	607.6	0.0
25	0.0	0	17	0	639	17.0	prod	0.0	17.0	668.8	-1.280000e+01	1.1316	2.1316	1362.0924	18.700	668.8	0.0
26	13.0	0	0	0	719	15.8	prod	11.2	15.8	734.8	0.000000e+00	1.4030	2.4030	1727.7570	17.380	734.8	2.8
27	47.0	0	0	0	711	79.0	prod	126.0	79.0	789.6	4.000000e-01	1.4619	2.4619	1750.4109	86.900	789.6	32.0
28	431.0	64	0	0	448	514.2	prod	342.0	514.2	962.4	-2.000000e-01	1.4000	2.4000	1075.2000	565.620	898.4	83.2
29	334.0	0	0	0	448	466.8	prod	566.0	466.8	914.4	4.000000e-01	1.2918	2.2918	1026.7264	513.480	914.4	132.8
30	639.0	63	0	0	216	797.8	prod	710.0	797.8	1015.8	-2.000000e+00	1.1854	2.1854	472.0464	877.580	952.8	158.8
31	767.0	91	0	0	127	945.8	prod	743.6	945.8	1079.0	-6.200000e+00	1.1549	2.1549	273.6723	1040.380	988.0	178.8
32	237.0	0	0	135	640	422.6	prod	789.2	422.6	926.4	1.200000e+00	1.0372	2.0372	1303.8080	604.882	926.4	185.6
33	623.0	125	0	0	256	798.6	prod	750.8	798.6	1057.4	-2.800000e+00	1.0601	2.0601	527.3856	878.460	932.4	175.6
34	159.0	0	0	0	637	313.4	prod	689.6	313.4	950.0	4.000000e-01	1.1803	2.1803	1388.8511	344.740	950.0	154.4

time = 4115.8 s constraint violated = 3.3% (24/720)

Numerical results

Quantum annealer

Test with 2 periods and 0 batteries:

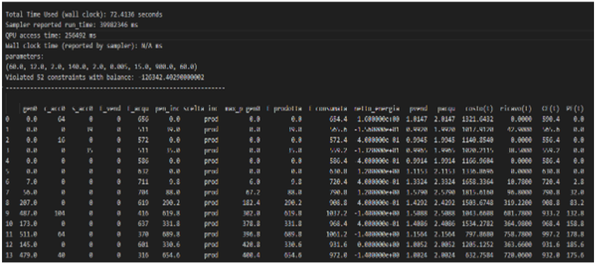
```
Best chain_strength found: 1 and annealing_time = 20
Time Used: 7.9846 seconds
Parameters:
(100.0, 100.0, 100.0, 100.0, 100.0, 0.1, 100.0, 500.0, 100.0)
Violated 3 constraints with balance: -869.4837000000002
-------------------------------------------------------------


    gen0  E_vend  E_acqu  pen_inc scelta inc  max_p gen0  E prodotta  \
0  294.0       0     409    684.4       cons       358.0       294.0
1  292.0       0     445    292.0       prod       417.2       292.0

   E consumata  netto_energia   pvend   pacqu   costo(t)  ricavo(t)  CF(t)  \
0        684.4           18.6  1.0328  2.2328   913.2152     752.84  684.4
1        741.6           -4.6  1.1153  2.3153  1030.3085     321.20  741.6

   PF(t)
0    0.0
1    0.0
```

Broken chain $= 100/100$ (avg chain break: 0.9814)

Numerical results

Hybrid solution

Test with 80 periods and 1 battery:



```
Total Time Used (wall clock): 72.4136 seconds
Sampler reported run_time: 30082346 ms
QPU access time: 256492 ms
Wall clock time (reported by sampler): N/A ms
parameters:
(60.0, 12.0, 2.0, 140.0, 2.0, 0.005, 15.0, 900.0, 60.0)
Violated 52 constraints with balance: -126342.40290000002
-----------------------------------------------------------------
```

	gen0	c_acc0	s_acc0	E_vend	E_acqu	pen_inc	scelta inc	max_p gen0	E prodotta	E consumata	netto_energia	pvend	pacqu	costo(t)	ricavo(t)	CF(t)	PF(t)
0	0.0	64	0	0	656	0.0	prod	0.0	0.0	654.4	1.600000e+00	1.0147	2.0147	1321.6432	0.0000	590.4	0.0
1	0.0	0	[illegible]	0	[illegible]	[illegible]	prod	0.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0
2	0.0	16	0	0	572	0.0	prod	0.0	0.0	572.4	4.000000e-01	0.9945	1.9945	1140.8540	0.0000	556.4	0.0
3	0.0	0	[illegible]	0	[illegible]	[illegible]	prod	0.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0
4	0.0	0	0	0	586	0.0	prod	0.0	0.0	586.4	4.000000e-01	0.9914	1.9914	1166.9684	0.0000	586.4	0.0
5	0.0	0	0	0	632	0.0	prod	0.0	0.0	630.0	1.200000e+00	1.1153	2.1153	1336.0896	0.0000	630.0	0.0
6	7.0	0	0	0	711	9.8	prod	6.0	9.8	720.4	4.000000e-01	1.3324	2.3324	1658.3364	10.7800	720.4	2.8
7	56.0	0	0	0	704	88.0	prod	67.2	88.0	790.0	1.200000e+00	1.5790	2.5790	1815.6100	96.8000	790.0	32.0
8	207.0	0	0	0	619	200.2	prod	182.4	200.2	908.8	4.000000e-01	1.4202	2.4202	1503.6748	319.2200	908.8	83.2
9	487.0	104	0	0	416	619.8	prod	302.0	619.8	1037.2	-1.400000e+00	1.5008	2.5008	1043.6608	681.7800	933.2	132.8
10	173.0	0	0	0	637	331.8	prod	378.8	331.8	968.4	4.000000e-01	1.4006	2.4006	1534.2782	364.9800	968.4	158.8
11	511.0	64	0	0	370	689.8	prod	396.8	689.8	1061.2	-1.400000e+00	1.1504	2.1504	797.8600	758.7800	997.2	178.8
12	145.0	0	0	0	601	330.6	prod	420.8	330.6	931.6	0.000000e+00	1.0052	2.0052	1205.1252	363.6600	931.6	185.6
13	479.0	40	0	0	316	654.6	prod	400.4	654.6	972.0	-1.400000e+00	1.0024	2.0024	632.7584	720.0000	932.0	175.6

time $= 72.4$ s constraint violated $= 7.2\%$ $(52/720)$

Conclusions

Transforming the UC formulation to a QUBO problem revealed some criticalities:

- The range of values a variable can assume is very big.
- More comprehensive the formulation is the more complex the hyperparameter tuning is.
- Current Hardware cannot support a full quantum execution.

Conclusions

At the same time, we also found that:

- A QUBO formulation is feasible without special techniques.
- Hybrid solver performs slightly worse than classical simulated annealing in terms of accuracy but using a fraction of the time.
- We found that hyperparameters tuned on small problems work on bigger ones as well.

End

Thank you.