Frustrating quantum batteries

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Frustrating quantum batteries, PRX Quantum 5, 030319

Quantum batteries

- Work reservoirs: quantum mechanical systems used to temporarily store energy that can later be extracted and transferred to a second system in a coherent way[1].
- The simplest example of a thermal machine.
- QBs can achieve more efficient and rapid charging processes compared to classical systems. [2,3,4]

[1] R. Alicki, M. Fannes, PRE 87, 042123 (2013)

[2] G. M. Andolina, M. Keck, A. Mari, V. Giovannetti, and M. Polini, PRB 99, 205437 (2019)

[3] F. C. Binder, S. Vinjanampathy, K. Modi and J. Goold, New J. Phys. 17 075015 (2015)

[4] F. Campaioli, F. A. Pollock, F. C. Binder, L. Céleri, J. Goold, S. Vinjanampathy, and K. Modi, PRL. 118, 150601 (2017)

Quantum batteries

Open questions:

- stabilization of stored energy[5].
- Investigation of optimal energy transfer protocols[6].
- Experimental realization.
- ...

[5] S. Gherardini, F. Campaioli, F. Caruso, and F. C. Binder, Phys. Rev. Res. 2, 013095 (2020)

[6] S. Tirone, R. Salvia, and V. Giovannetti, Phys. Rev. Lett. 127, 210601 (2021)

Our proposal

We design a complete protocol for the realization of an efficient QB based on a quantum many-body system.

- Charging.
- Stabilization of stored energy against decoherence.
- Work transfer to an ancillary system.

Quantum Ising battery

- Our QB is based on a quantum Ising chain in a transverse field.

$$H = J \sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x - h \sum_{j=1}^{N} \sigma_j^z$$

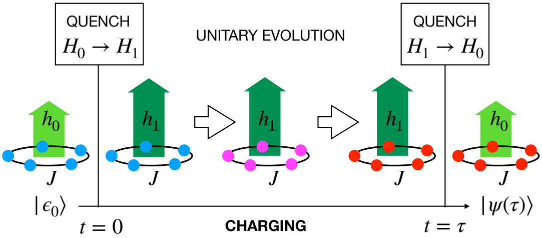
- It can be realized on Rydberg atoms platforms.

1. Charging protocol

- We start from the ground-state of the Ising chain with transverse field h_0

$$\rho_0 = |\epsilon_0\rangle \langle\epsilon_0| \, .$$

- To charge the battery we perform a global quantum quench in the magnetic field



- The energy stored in the battery is $E_{in} = \text{Tr}(\rho_B H) - \text{Tr}(\rho_0 H)$.

How much work can we extract?

- An important characterization of a QB is given in terms of the amount of energy that we can extract from it.
- Extraction is performed applying a transformation that brings the battery to a final state with lower energy.

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- Extraction is performed applying a transformation that brings the battery to a final state with lower energy.
- This can be done through a unitary transformation U.
- The amount of energy extracted during the process can be quantified as

$$E_{ex} = \mathrm{Tr}\Big(U\rho_B U^\dagger H_B\Big) - \mathrm{Tr}(\rho_B H_B).$$

How much work can we extract?

- The goal is to maximize the amount of energy that can be extracted.
- The maximum amount of work extractable with a unitary transformation is called the ergotropy W:

$$W = \max_{U} \left\{ \mathrm{Tr}\Big(U \rho_B U^\dagger H_B \Big) - \mathrm{Tr}(\rho_B H_B) \right\}.$$

How much work can we extract?

- In order to perform this maximization, it's useful to consider the spectral decompositions of ρ_B and H_B:

$$\rho_B = \sum_j r_j \left| r_j \right\rangle \left\langle r_j \right| ,$$

$$H = \sum_j \epsilon_j \left| \epsilon_j \right\rangle \left\langle \epsilon_j \right|$$

with $\epsilon_j \leq \epsilon_{j+1}$ and $r_j \geq r_{j+1}$.

How much work can we extract?

- Since a unitary transformation cannot modify the eigenvalues of the density matrix, the maximum amount of work is extracted by bringing the battery to the final passive, equilibrium state $\sigma = \sum_j r_j \left|\epsilon_j\right\rangle \left\langle\epsilon_j\right|$.

[7] A. E. Allahverdyan, R. Balian and T. M. Nieuwenhuizen, "Maximal work extraction from finite quantum systems", Europhysics letters, **67**, 565 (2004)

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- Therefore[7]

$$W = \sum_{j,k} \epsilon_j r_k \big(\left| \left\langle \epsilon_j | r_k \right\rangle \right|^2 - \delta_{jk} \big).$$

- The ergotropy thus represents a figure of merit that tells us the maximum amount of energy that we can extract from our quantum battery.

[7] A. E. Allahverdyan, R. Balian and T. M. Nieuwenhuizen, "Maximal work extraction from finite quantum systems", Europhysics letters, **67**, 565 (2004)

How much work can we extract?

- For our QB, since the charging protocol is unitary and ρ_B is a pure state, then $W = E_{in}$.
- However, decoherence effects might reduce the purity of the battery and, as a consequence, its ergotropy.

Decoherence effects

- To take these effects into account we consider a situation in which after the quench the battery state converges to the diagonal ensemble[8]:

$$\rho_B = \sum_n p_n \left|\epsilon_n\right\rangle \left\langle\epsilon_n\right| .$$

- For a diagonal battery state $\rho_B = \sum_n p_n \left|\epsilon_n\right\rangle \left\langle\epsilon_n\right|$ the ergotropy is given by

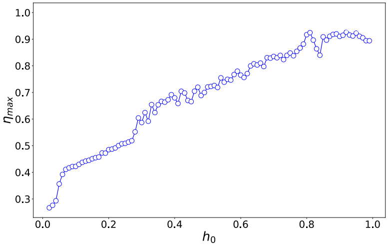
$$W = \mathrm{Tr}\{\rho_B H\} - \sum_n r_n \epsilon_n,$$

where the $\{r_n\}$ is a rearrangement of $\{p_n\}$ such that $r_n \geq r_{n+1}$.

[8] G. J. Milburn, "Intrinsic decoherence in quantum mechanics", Physical Review A, **44**, 5401 (1991)

2. Stabilization of the stored energy

- We measure the fraction of energy available as ergotropy $\eta = \frac{W}{E_{in}}$.



η_{max} vs h_0 for $N = 25$, $\Delta h = 0.01$.

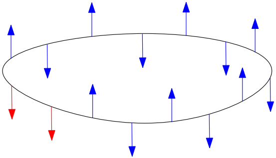
- η increases as we get close to the critical point $h_0 = h^* = |J| = 1$.
- Gap vanishes $\Rightarrow$ closer energy levels $\Rightarrow$ higher ergotropy!
- Idea: use topologically frustrated chain!

Topologically frustrated quantum Ising chain

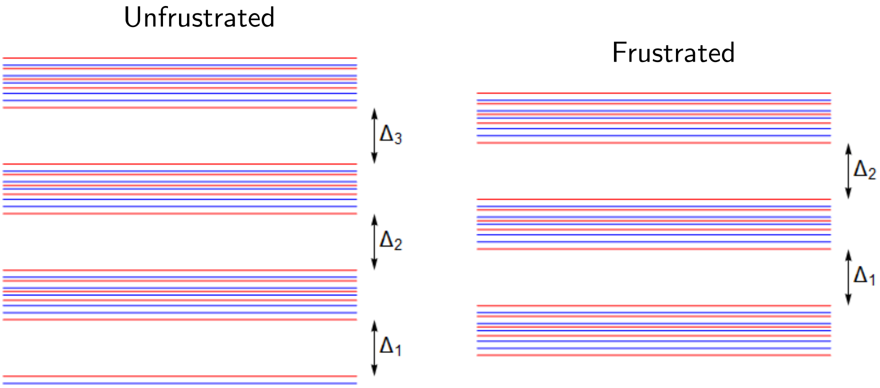
- Same model, but different boundary conditions.

$$H = J \sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x - h \sum_{j=1}^{N} \sigma_j^z$$

- Dominant antiferromagnetic (AFM) interactions ($J > 0$);
- Frustrated boundary conditions (FBCs):
  - odd number of spins N;
  - periodic boundary conditions (PBCs).



Frustration closes the gap

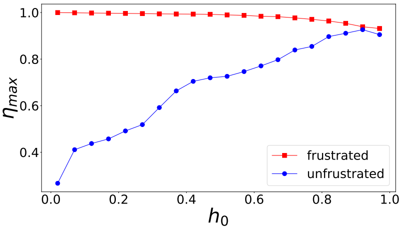


- Gap closing as N^{-2} in the frustrated model[9] [10].
- Potential to increase the ergotropy!

[9] V. Marić, S. M. Giampaolo and F. Franchini, "Quantum phase transition induced by topological frustration", Communications Physics **3**, 220 (2020)

[10] A. G. Catalano, D. Brtan, F. Franchini and S. M. Giampaolo, "Simulating continuous symmetry models with discrete ones", Physical Review B, **106**, 125145 (2022)

Frustrated vs Unfrustrated: Robustness to decoherence



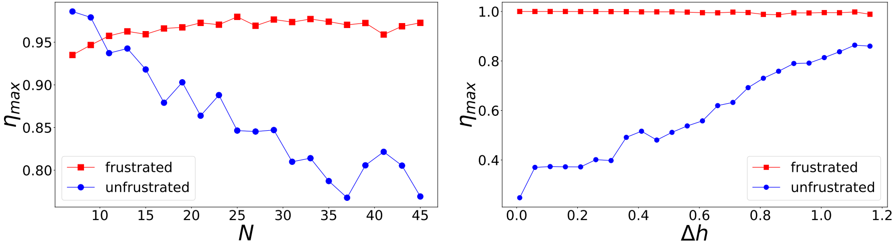
η_{max} vs h_0 for $N = 25$, $\Delta h = 0.01$.

$$\Delta_{AFM} = \frac{2h_0}{1 - h_0}\frac{\pi^2}{N^2} + O(N^{-4}).$$

- Frustrated and unfrustrated system are comparable at $h_0 = h^*$;
- frustrated system much more resistent moving towards the classical point: higher η than the critical system.

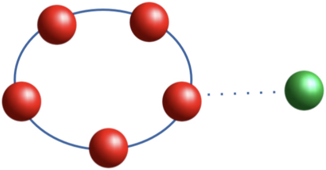
Frustrated vs Unfrustrated: Robustness to decoherence

- This is valid also for different system's sizes and quench amplitudes.



Left: η_{max} vs N for $h0 = 0.751$ and $h1 = 0.7$. Right: η_{max} vs Δh for $h_0 = 0.001$ and $N = 25$.

3. Energy transfer to an ancillary qubit



- We connect the charged battery to an ancillary spin through a single-link interaction:

$$H = J\sum_{j=1}^{N}\sigma_j^x\sigma_{j+1}^x - h\sum_{j=1}^{N}\sigma_j^z + \lambda H_{int} + \omega\sigma_S^z,$$

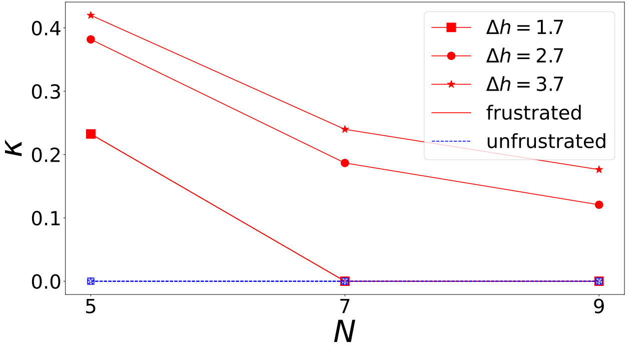
$$H_{int} = \sigma_1^+\sigma_S^- + \sigma_1^-\sigma_S^+.$$

Ergotropy of the ancillary spin

- We measure the fraction of its maximal ergotropy acquired by the ancillary spin:

$$\kappa = \frac{W_S}{2\omega}.$$

- $\kappa = 0$ for the unfrustrated system $\Rightarrow$ only heat transfer;
- $\kappa > 0$ for the frustrated battery $\Rightarrow$ work transfer!



Conclusions

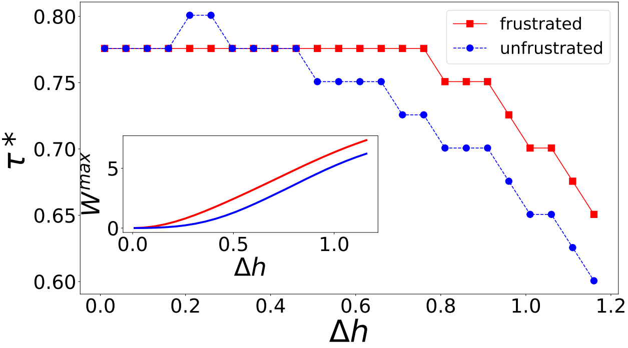
- New charging protocol based on global quantum quench of the external field: easy and efficient.
- Topological frustration enhances the performance of quantum batteries:
  - Robustness to decoherence;
  - higher ergotropy transfer.
- Possibility of realization with Rydberg atoms.

Outlook:

- Other quench protocols even more efficient.
- Beyond quantum battery: other technological applications.

Frustrated vs Unfrustrated: charging time

- We consider as the charging time of the battery the time τ^* needed by the ergotropy to reach its first local maximum.
- Even though τ^* is comparable for frustrated and unfrustrated systems, the former have a higher value of η and W.



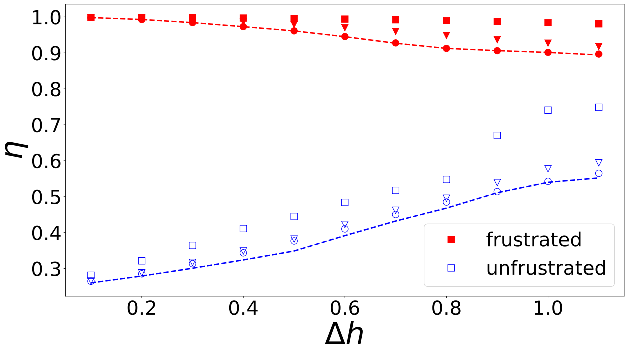
Effects of decoherence on the charging protocol

- If charging is not fast enough, we cannot neglect decoherence effects during the quench protocol.
- We implemented Milburn model[11] for decoherence to the diagonal ensemble in the charging protocol ⇒ modified Schrödinger dynamics:

$$\dot{\rho} = -\imath[H_1, \rho] - \frac{1}{2\nu}[H_1, [H_1, \rho]],$$

with $H_1 = H_B + \Delta h \sum_j \sigma_j^z$.

[11] G. J. Milburn, "Intrinsic decoherence in quantum mechanics", Physical Review A, **44**, 5401 (1991)



η_{max} vs Δh for different values of the decoherence frequency: $\nu = 10, 1, 0.1, 0$ (squares, triangles, circles, dashed line). Data are obtained for a chain of $N = 15$ spins, for $h_0 = 0.001$.

Energy transfer

- We work at $\omega = |J| = 1$, $h = 0.02$ and $\lambda = 0.01$. In this way, even though H_{int} does not commute with the rest of the hamiltonian, energy transfer from the battery to the spin is possible without the interaction absorbing / giving a relevant amount of energy.