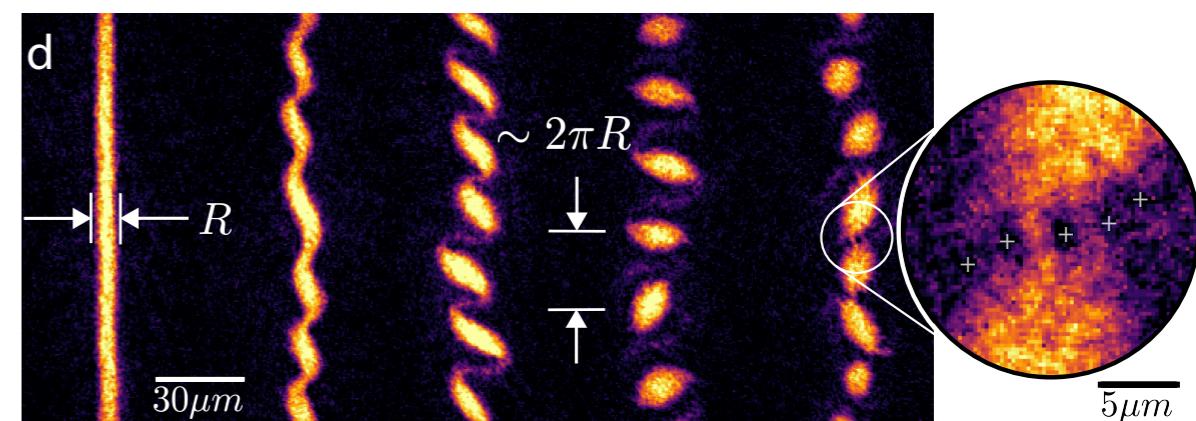
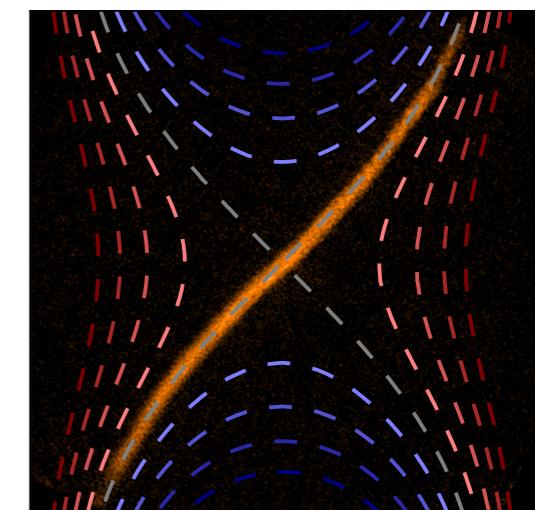
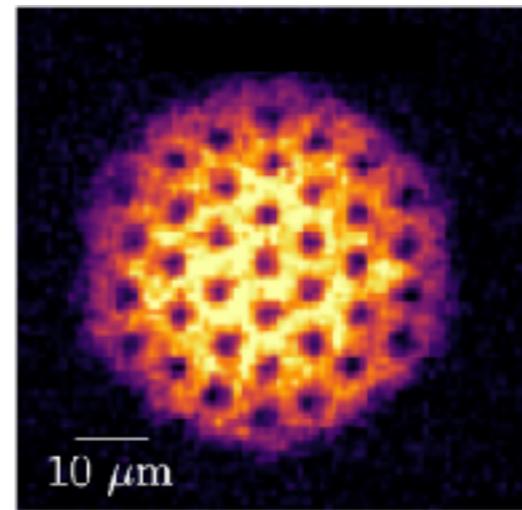
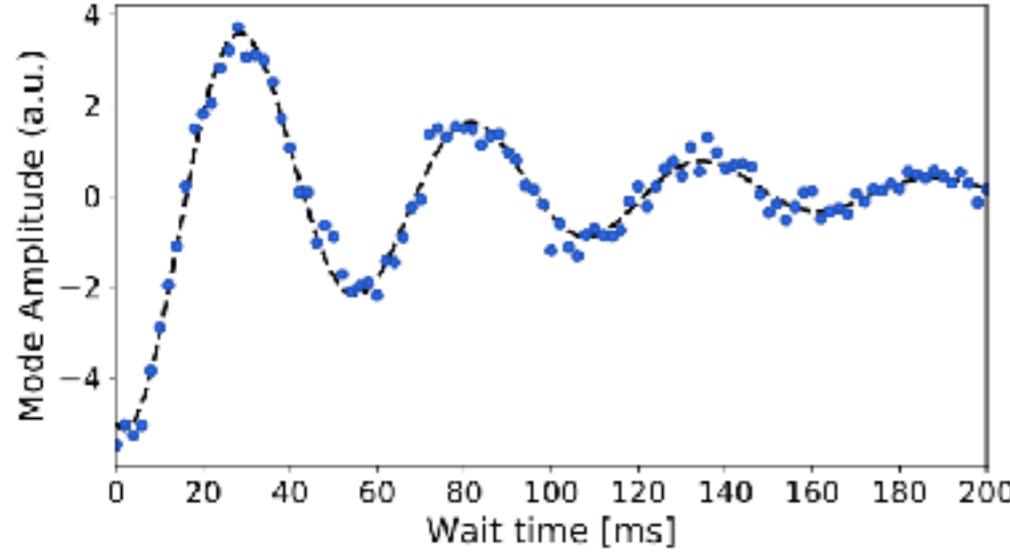
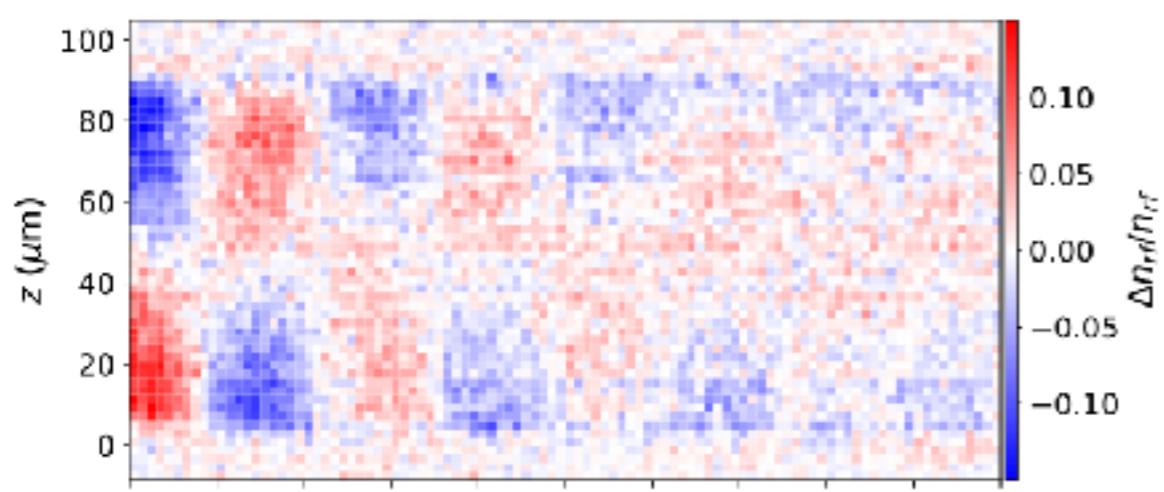


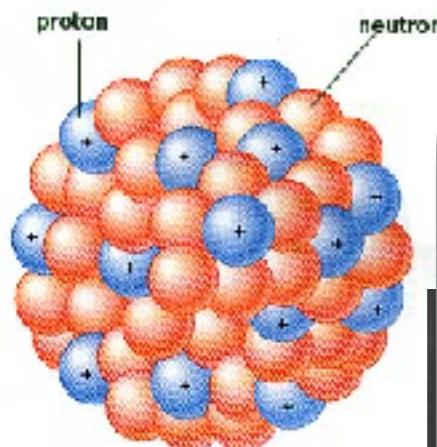
# Dynamics and transport in strongly-interacting quantum fluids

Richard Fletcher



# Two routes to strongly-correlated matter

## Strong interactions



$10^{-15}$  m

- No small parameter
- Sign problem (for fermions)
- Breakdown of quasiparticles/Fermi liquid
- Macroscopically degenerate
- Exponential growth of Hilbert space
- Non-perturbative behaviour
- ...

Equilibrium properties difficult  
-> Dynamic and transport even harder

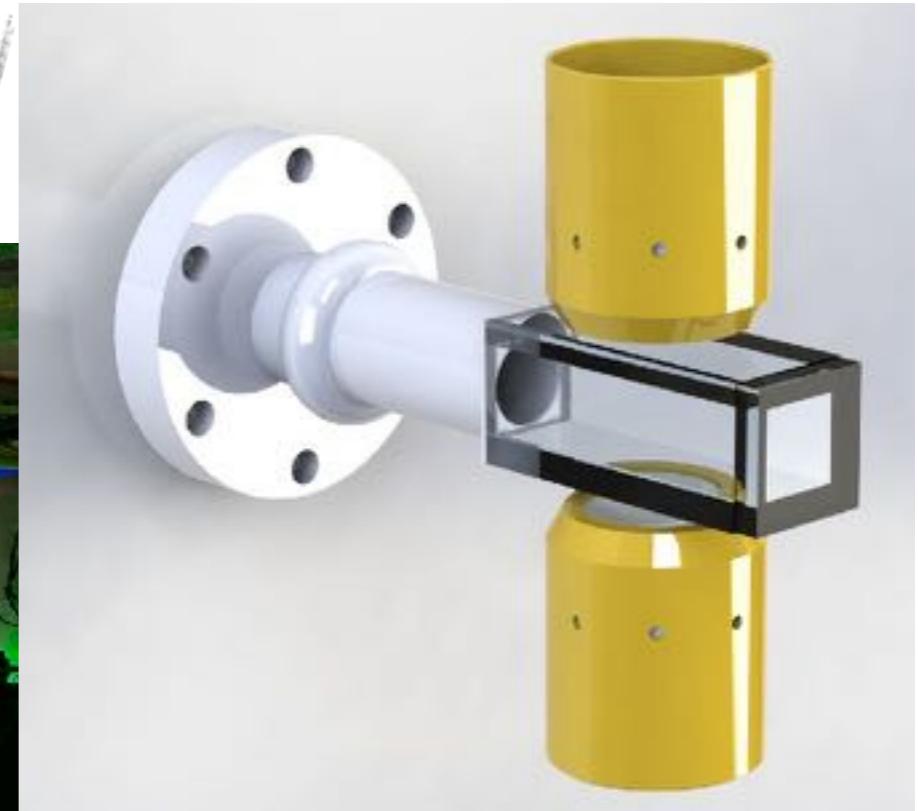
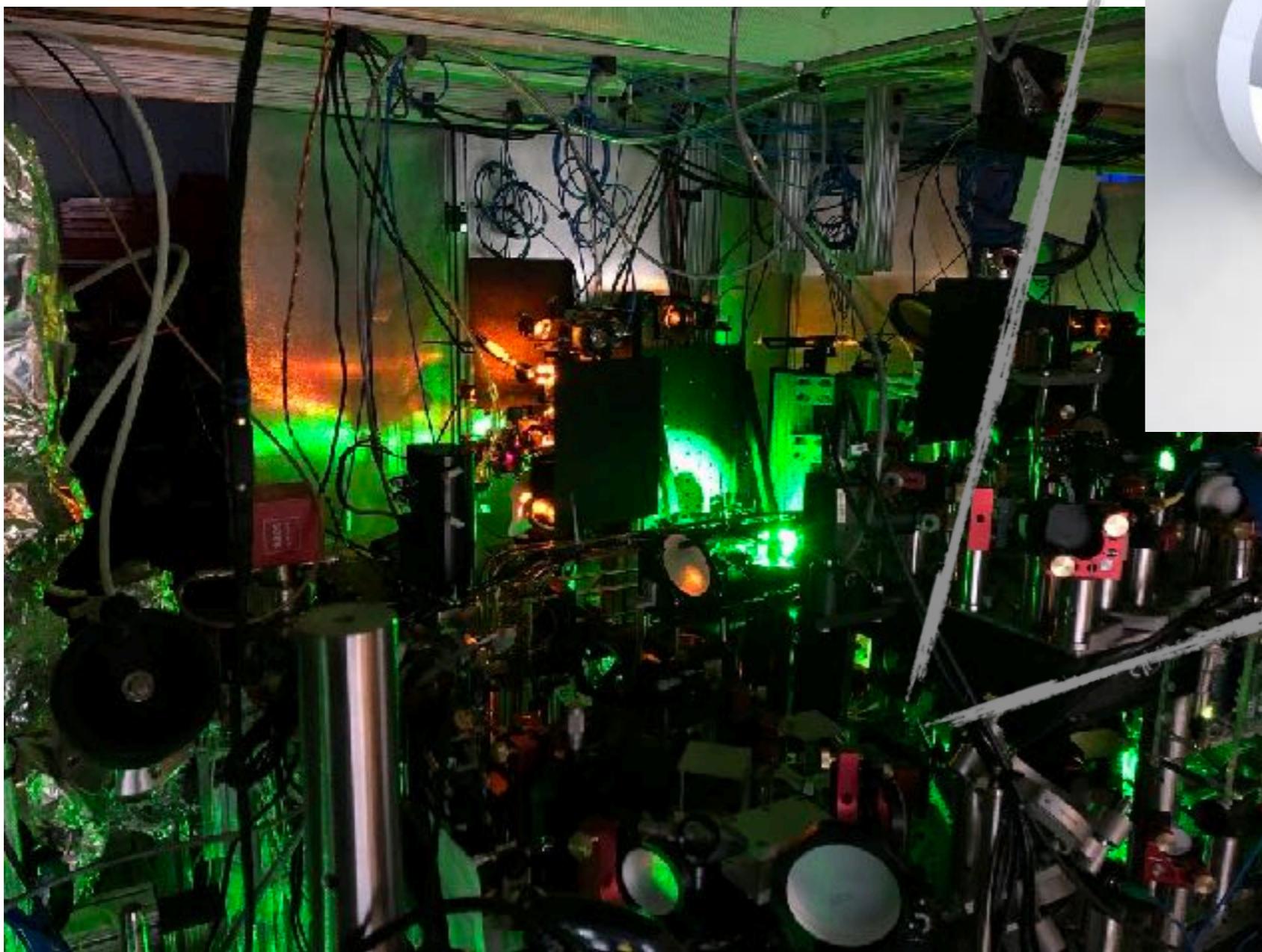
## Quenched kinetic energy

degenerate  
systems reorganise into  
correlated states

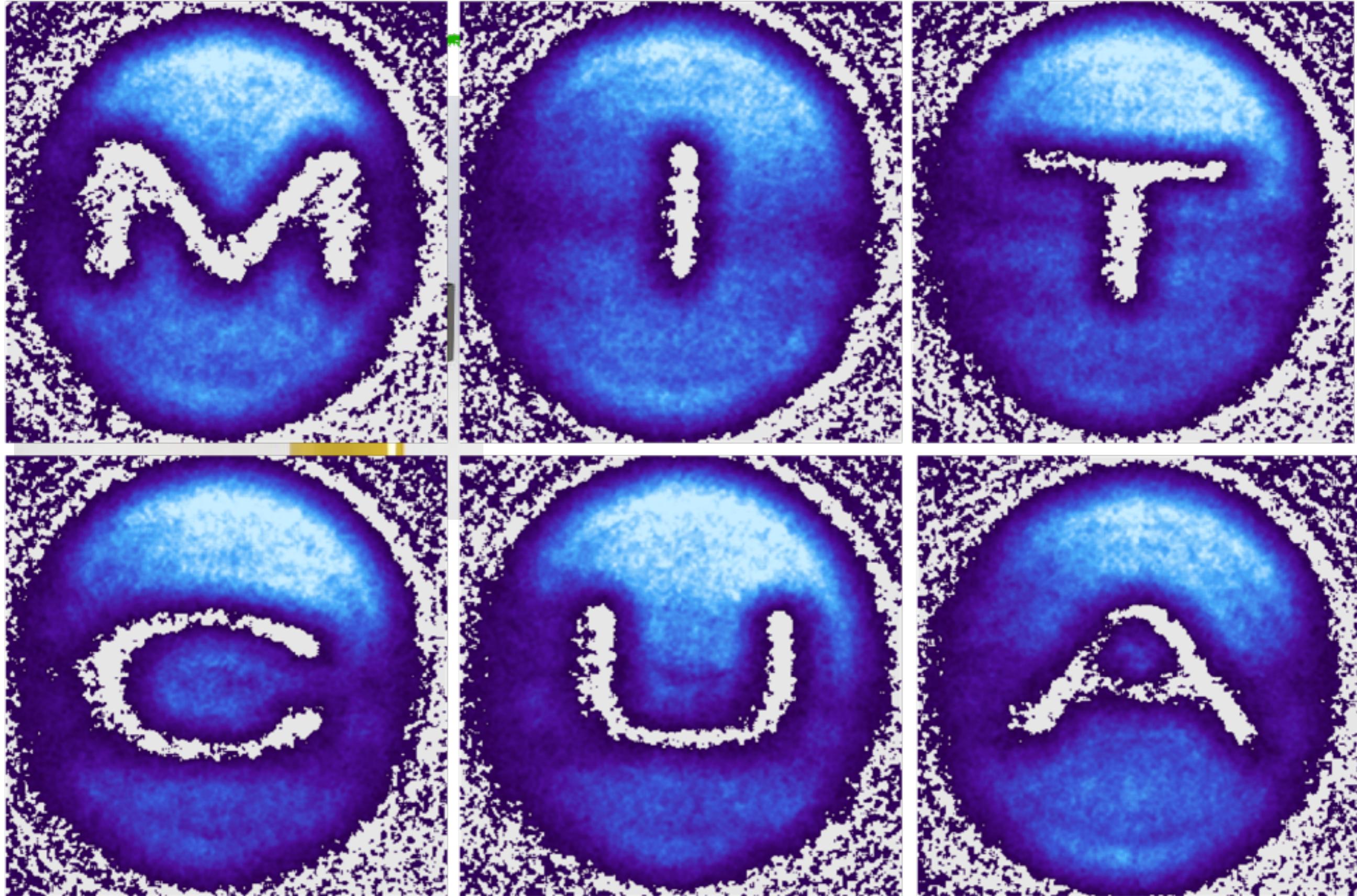
ipationless flow  
fractional quasiparticles  
fractional statistics

..P. Jarillo-Herrero  
556, 43–50 (2018)

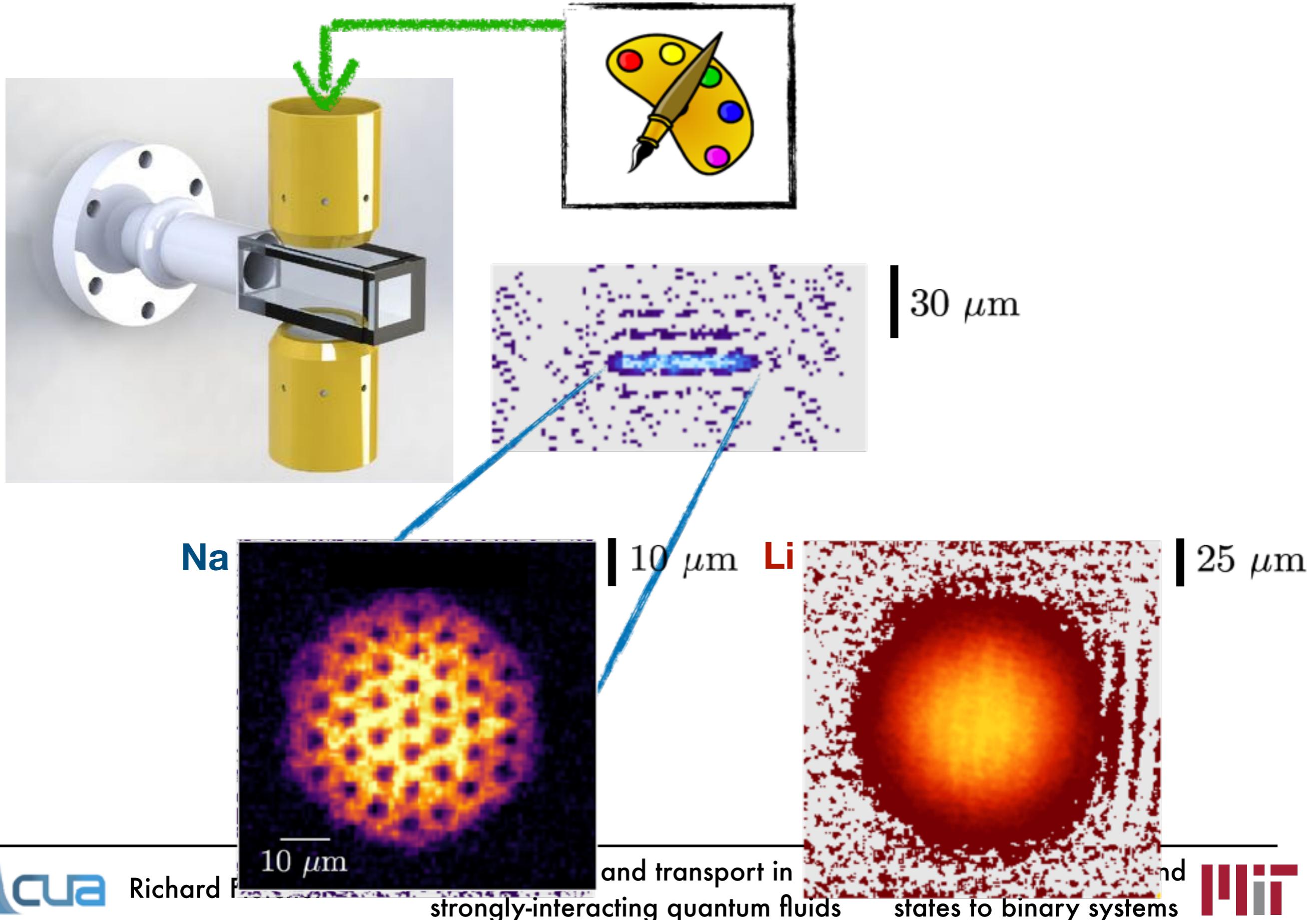
# Ultracold quantum gas platform



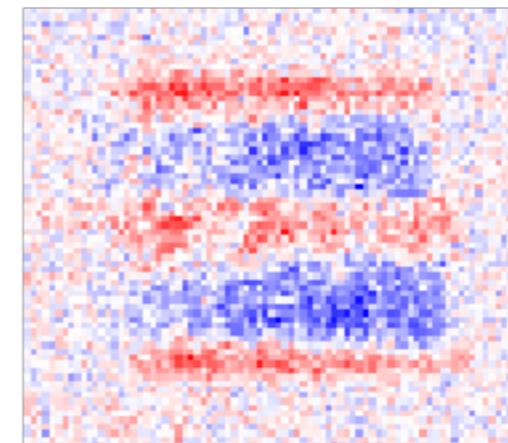
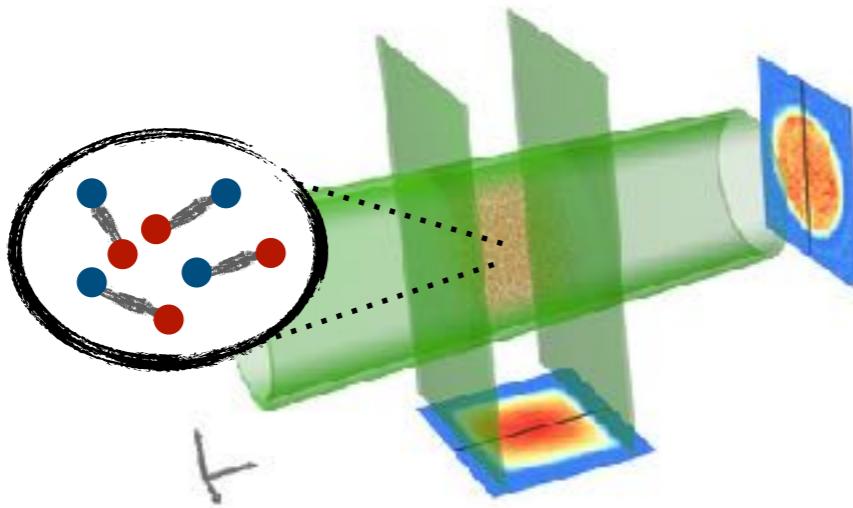
# Ultracold quantum gas platform



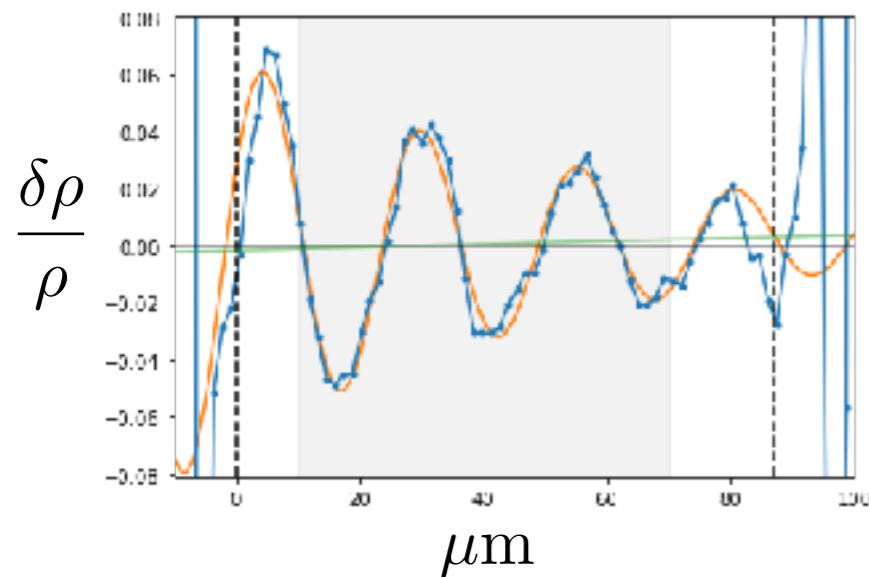
# Ultracold quantum gas platform



# 1. Banging the Fermi drum



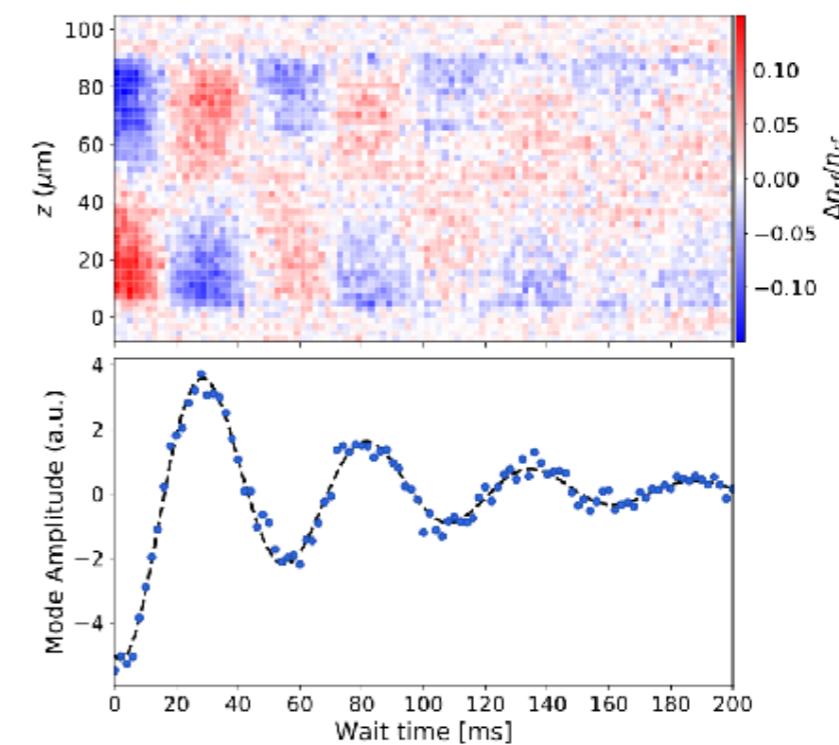
## 'Universal' diffusion of first sound



$$\omega^2 = c^2 k^2 + i\omega D k^2$$

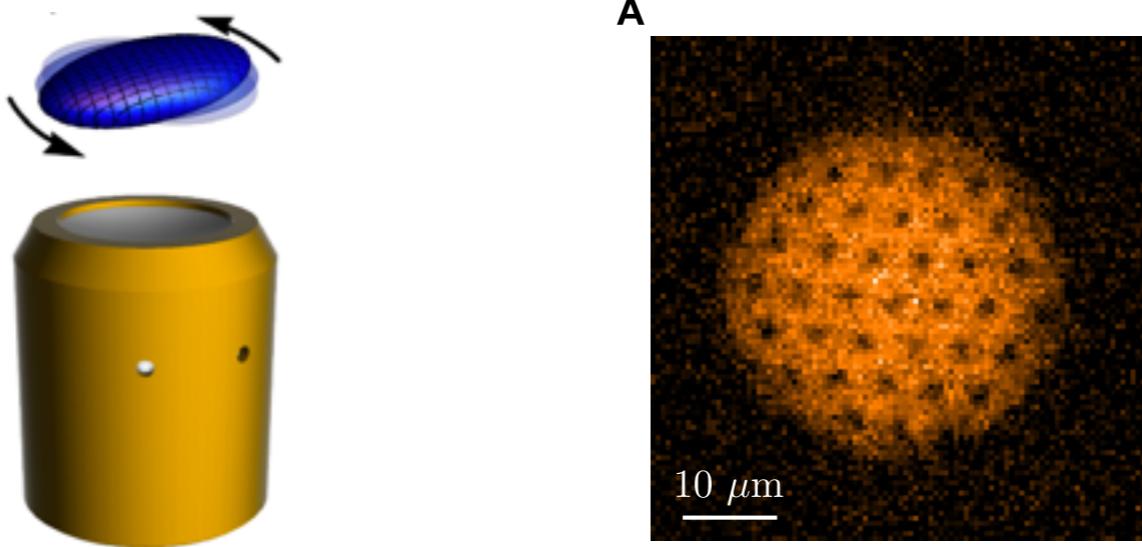
$$\Gamma = D k^2$$

## In situ thermometry of second sound

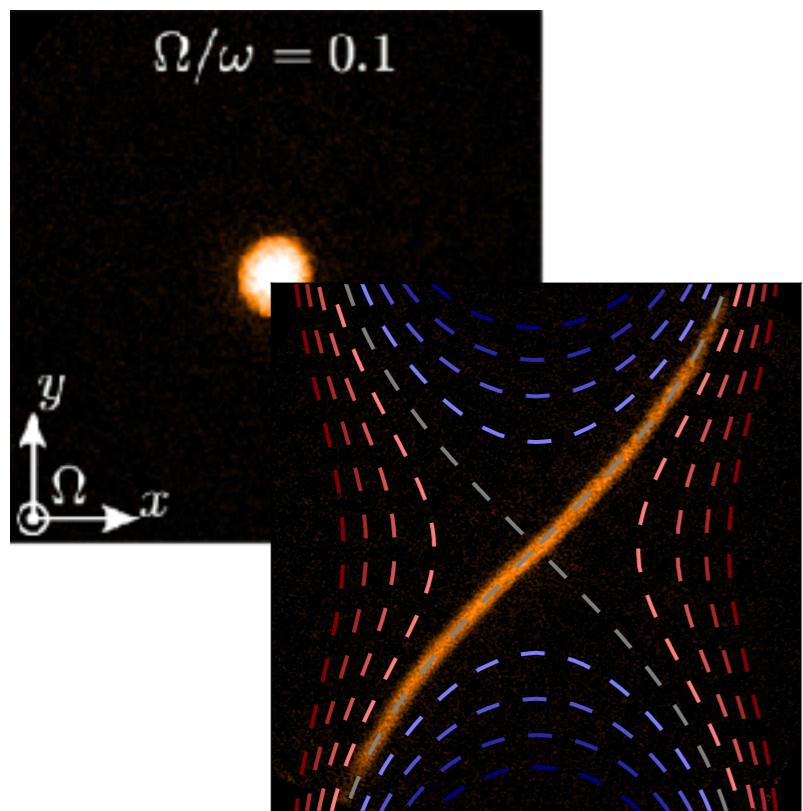


P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, M. Zwierlein,  
arXiv:1909.02555 (2019), Science, accepted

## 2. *In situ* physics of a rotating quantum gas

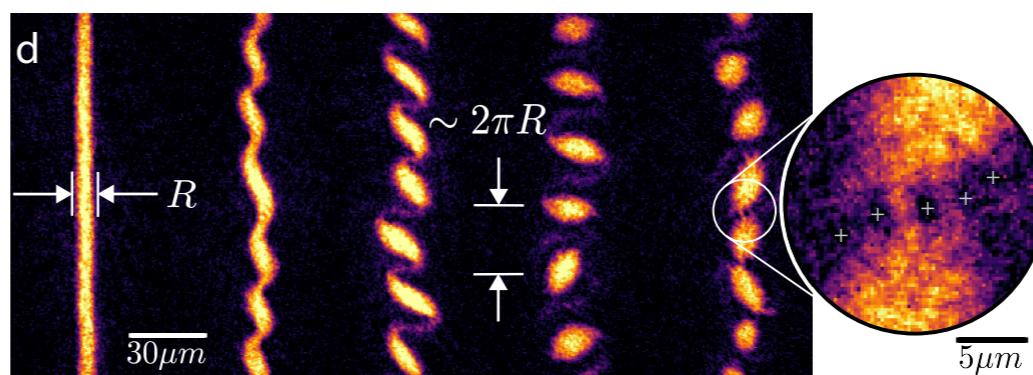


### Geometric squeezing

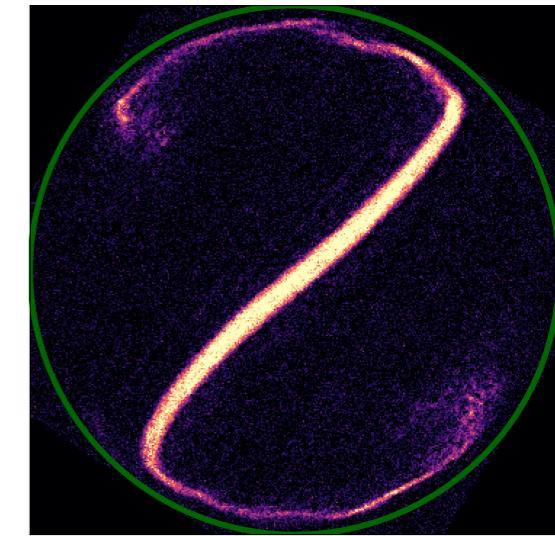


### Dynamics of superfluid under gauge field + interactions

Flat LL dispersion  $\rightarrow$  purely interaction-driven physics

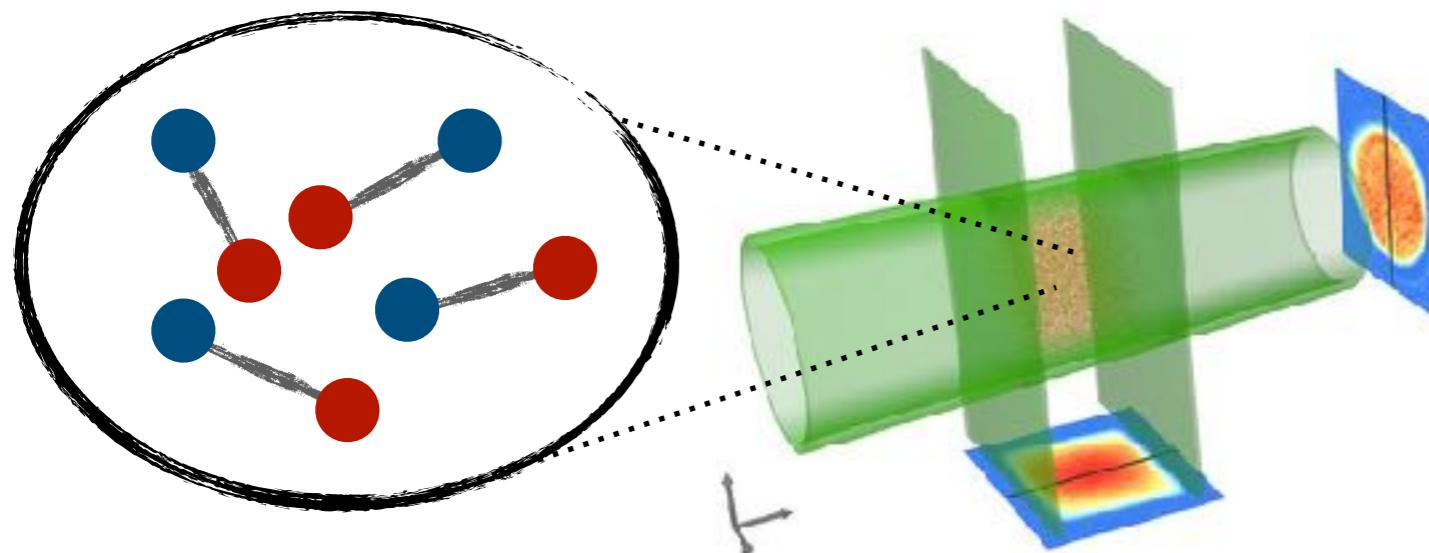


### Chiral edge modes



R. Fletcher, A. Shaffer, C. Wilson, P. Patel, Z. Yan, V. Crepel, B. Mukherjee, M. Zwierlein,  
arXiv:1911.12347 (2019) Science, accepted

# Unitary Fermi gas



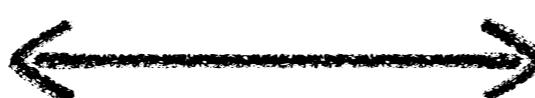
Unitary Fermi gas       $a \rightarrow \infty$

## Box potential

- Critical phenomena
- Correlation functions
- Transport properties
- Spectral response
- ...

Two lengthscales:

$$\lambda_{\text{th}}, n^{-1/3}$$



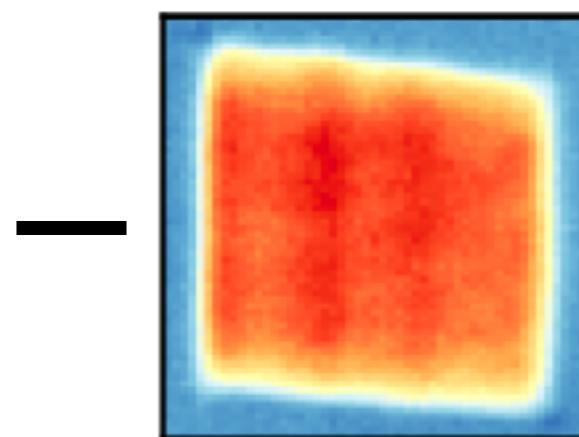
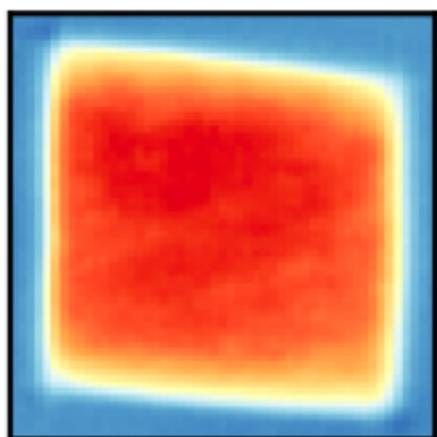
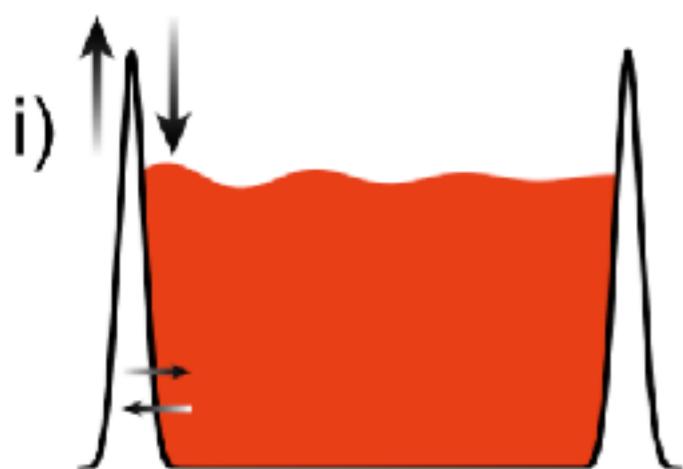
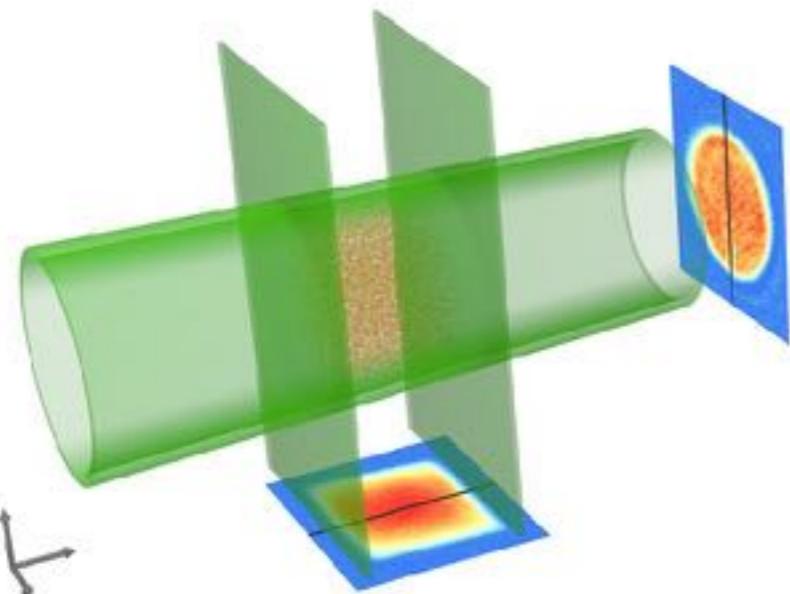
Two energy scales:

$$k_B T, E_F$$

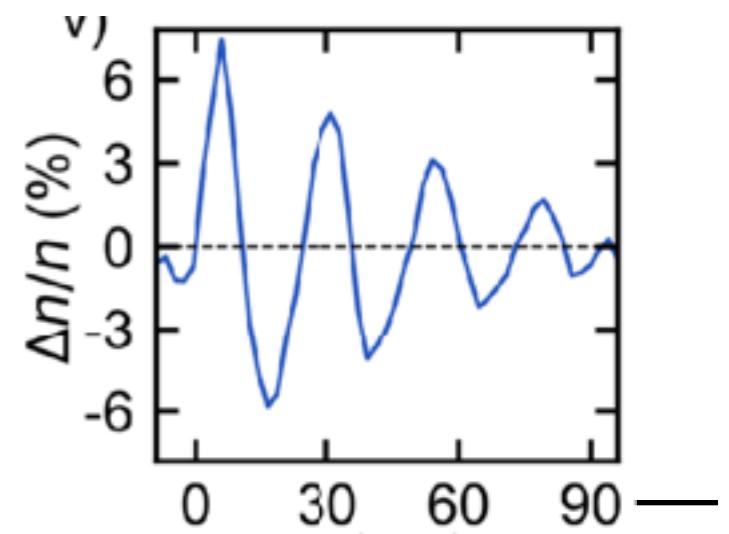
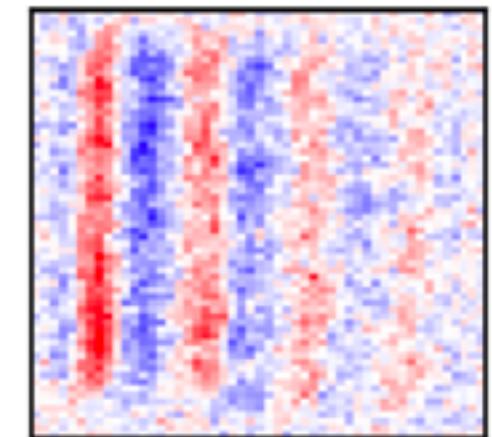
Scale invariant system:

$$E(T) = \frac{3}{5} N E_F \times f\left(\frac{T}{T_F}\right)$$

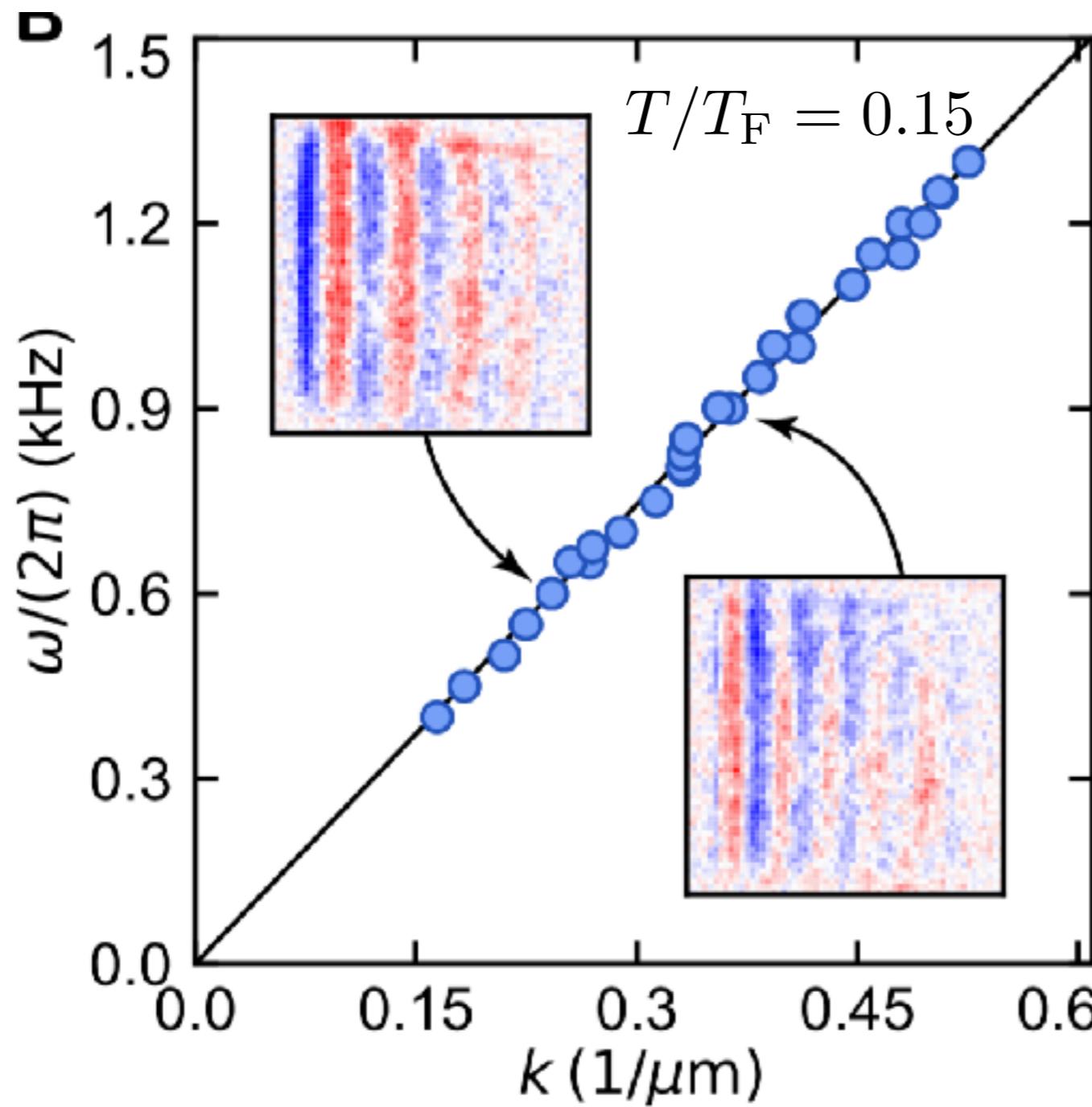
# Injecting sound waves



=



# Injecting sound waves



Extract sound speed

$$c = 15.5(4) \text{ mm/s}$$

Prediction from EoS:

$$c = 15.2(4) \text{ mm/s}$$

$$E=(0.9)mc^2$$

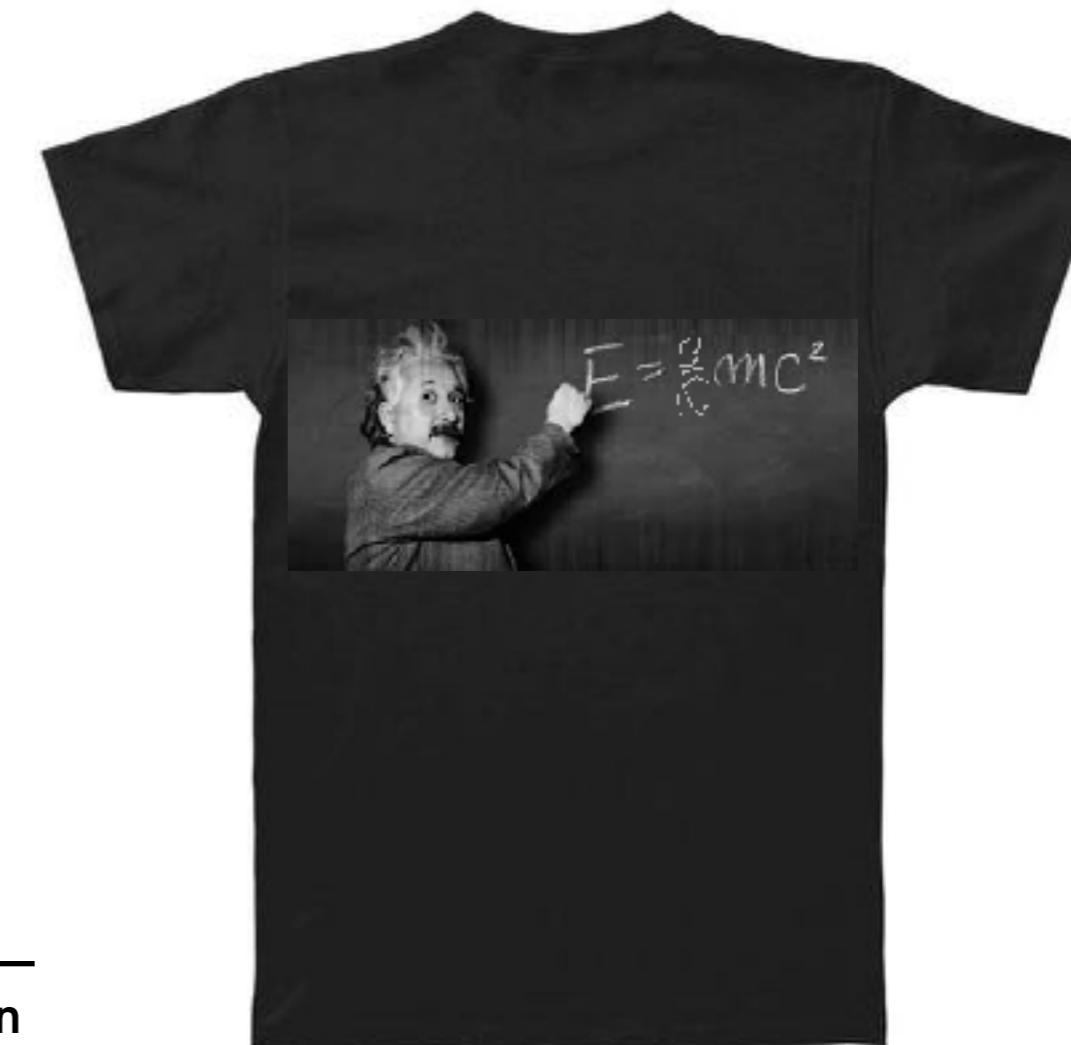
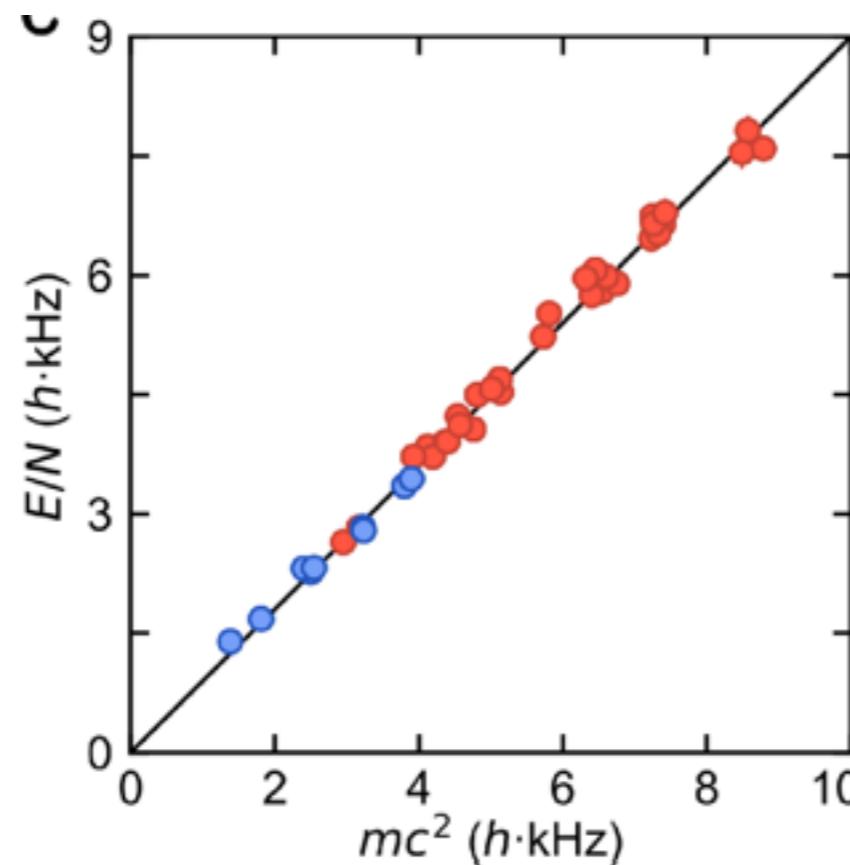
## Consequence of scale invariance

$$c^2 = \frac{\partial P}{\partial \rho} \Big|_s = \frac{V^2}{N} \frac{\partial^2 E}{\partial V^2} \Big|_s$$

$$E \propto V^{-2/3} \quad \text{if} \quad E \rightarrow \frac{E}{\lambda^2}$$

$$\curvearrowright E/N = \frac{9}{10} mc^2$$

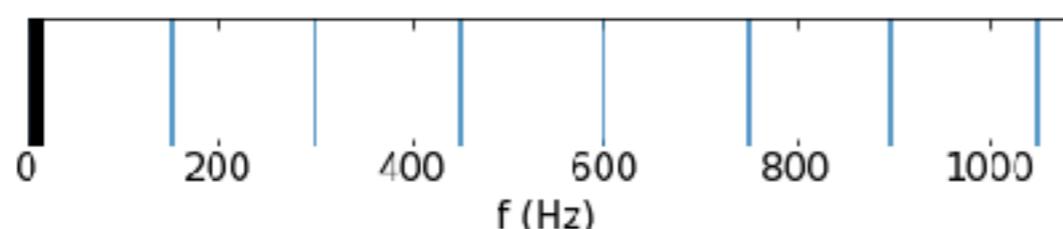
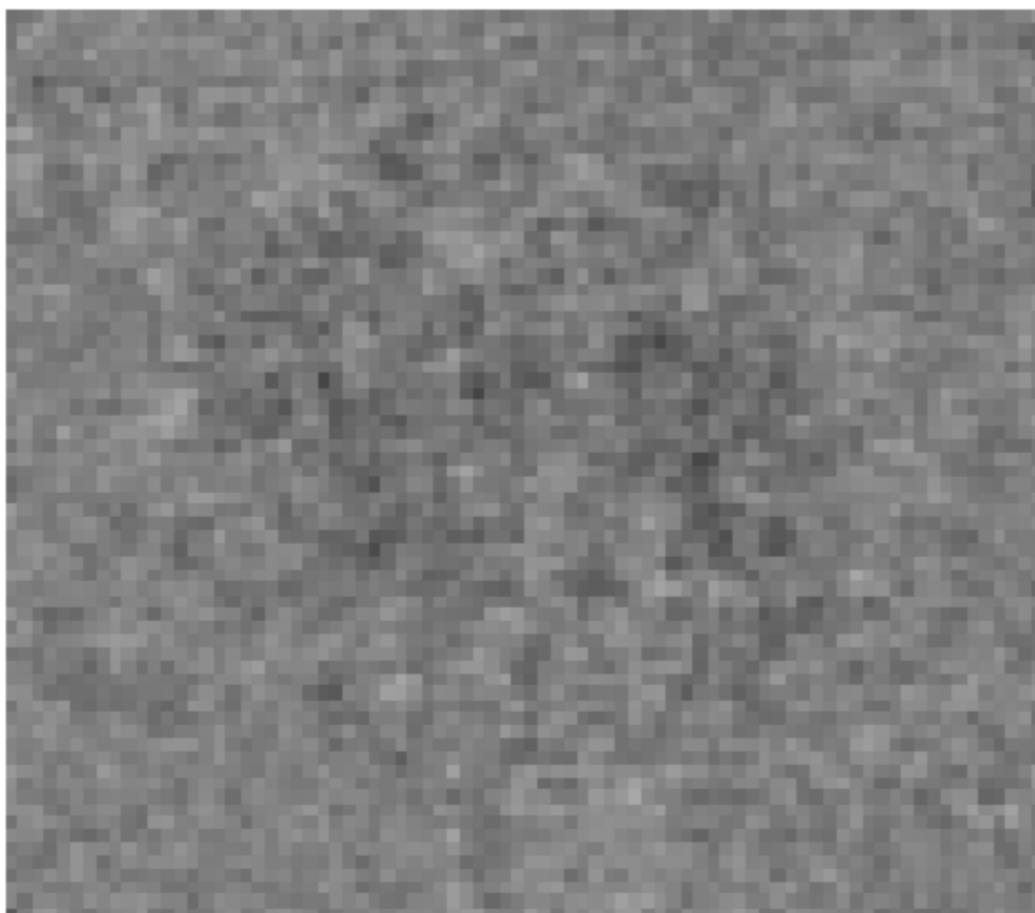
- Normal fluid
- Superfluid



# Attenuation of sound waves

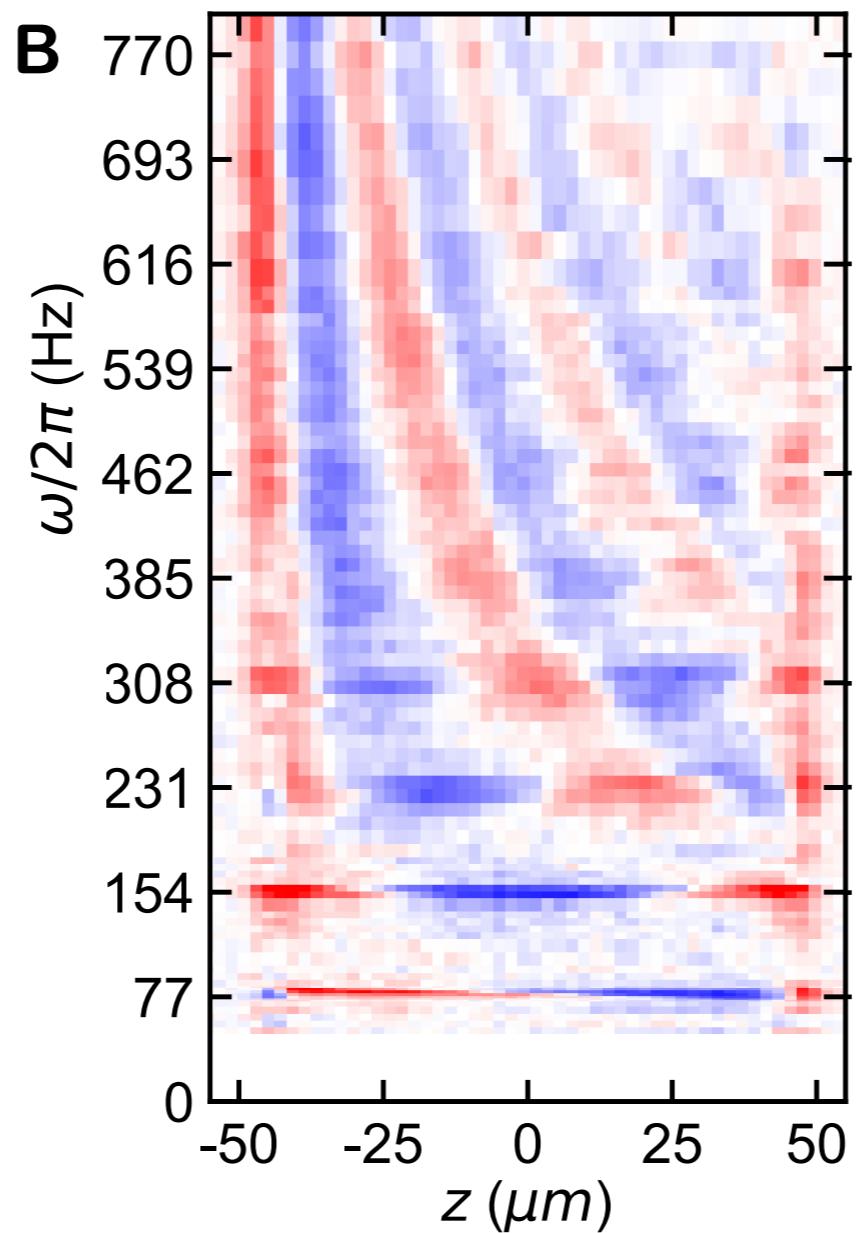
One-sided shaking: restricted to  $t < \frac{L}{c}$

Instead: drive resonant modes of box



Integrate along 'boring' direction  
and plot as function of drive  
frequency...

# Attenuation of sound waves



# Hydrodynamic equations

Mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial z} = 0$$

Continuity equation

Momentum

$$\frac{\partial j}{\partial t} + \frac{\partial p}{\partial z} = \frac{4}{3} \eta \frac{\partial^2 v}{\partial z^2}$$

Navier-Stokes equation - **viscosity** damps velocity gradients

Heat

$$\frac{\partial s}{\partial t} + \frac{\partial (vs)}{\partial z} = \frac{\kappa}{T} \frac{\partial^2 T}{\partial z^2}$$

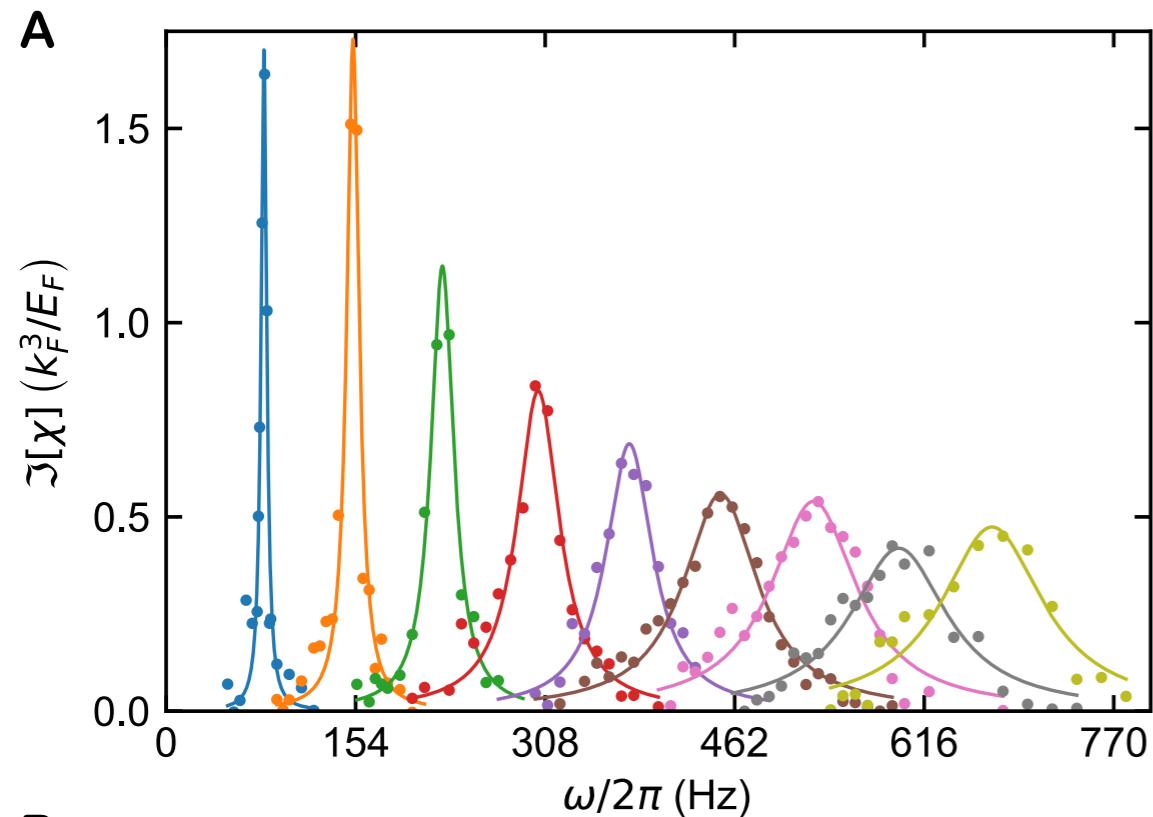
**Thermal conduction** damps temperature variations, increases entropy

$$\omega^2 = c^2 k^2 + i\omega D k^2$$

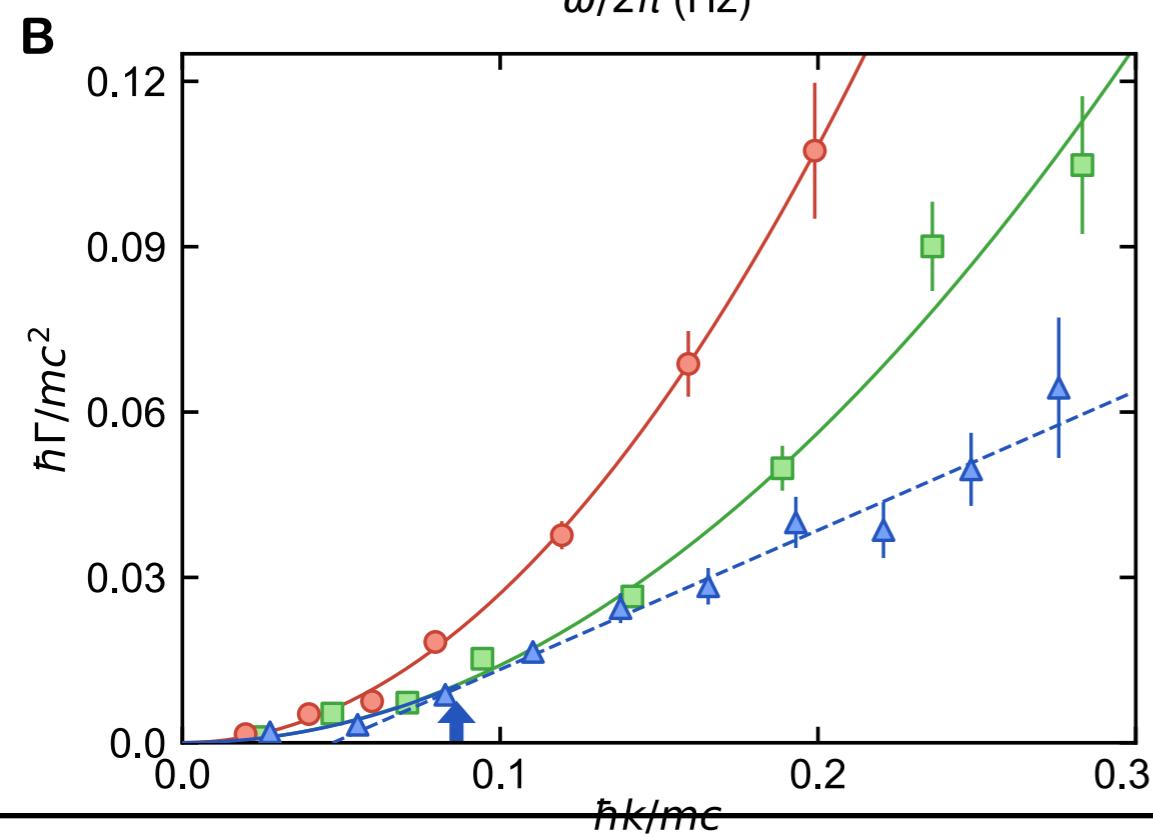
$$\Gamma = D k^2$$

$$D = \frac{4}{3} \frac{\eta}{\rho} + \frac{\kappa}{\rho} \left( \frac{1}{c_v} - \frac{1}{c_p} \right)$$

# Attenuation of sound waves



Spectral response broadens with frequency



- $T/T_F = 0.36$
- $T/T_F = 0.21$
- $T/T_F = 0.13$

Obtain diffusivity as function of temperature

# Diffusivity of first sound

Dimensional grounds:

$$D \sim v l \sim v_F \frac{k_F^2}{n} \sim \frac{\hbar}{m}$$

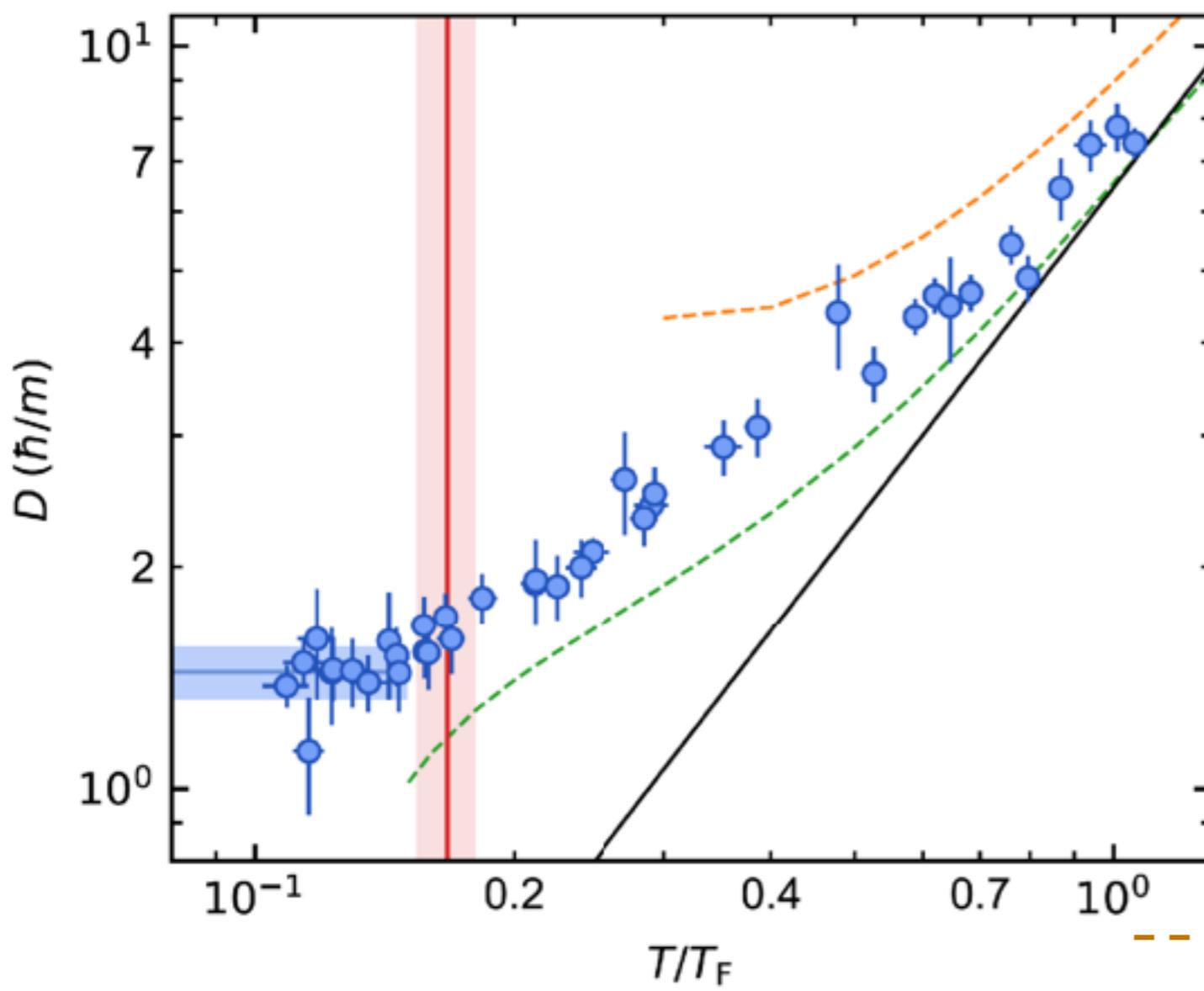
$$D = \frac{\hbar}{m} f_D \left( \frac{T}{T_F} \right)$$

# Diffusivity of first sound

Dimensional grounds:

$$D \sim v l \sim v_F \frac{k_F^2}{n} \sim \frac{\hbar}{m}$$

$$D = \frac{\hbar}{m} f_D \left( \frac{T}{T_F} \right)$$



High temperature limit:

$$D \sim v l \quad v \sim \frac{\hbar}{m\lambda} \quad l \sim \frac{1}{n\lambda^2}$$

$$D = \beta \frac{\hbar}{m} \left( \frac{T}{T_F} \right)^{3/2}$$

Schäfer et al., PRA  
2010

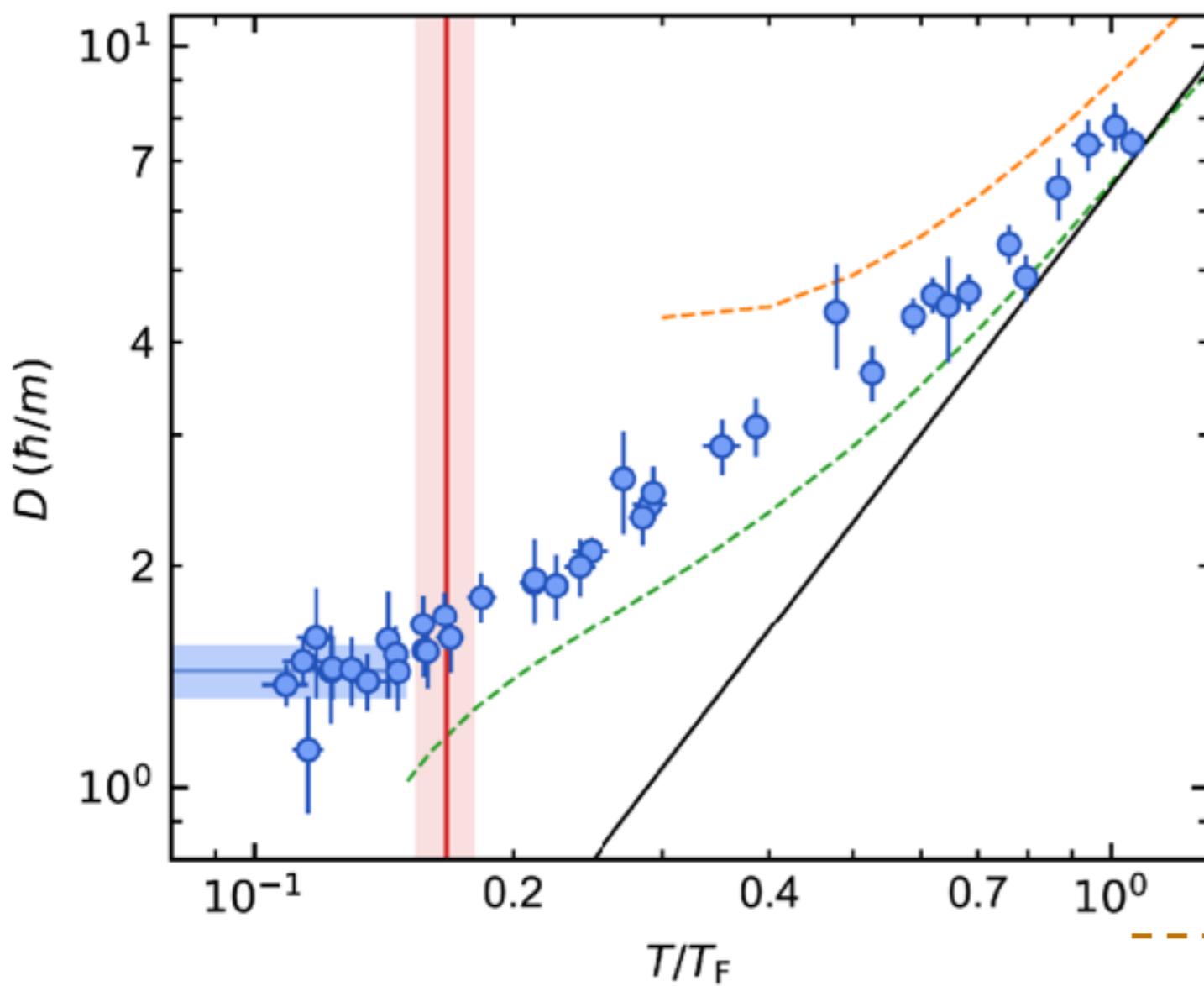
$\frac{7\eta}{3\rho}$   $\eta$  from Enss, Haussmann, Zwerger, Ann.  
Phys. (2011)

# Diffusivity of first sound

Dimensional grounds:

$$D \sim v l \sim v_F \frac{k_F^2}{n} \sim \frac{\hbar}{m}$$

$$D = \frac{\hbar}{m} f_D \left( \frac{T}{T_F} \right)$$



**Low temperature limit:**

$$D \sim v l \frac{n_q}{n} \quad l \sim \frac{1}{n_q \sigma} \\ \sim \frac{v}{\sigma n}$$

**Gapped excitations:**

$$v \sim v_F \quad \sigma \sim \frac{1}{k_F^2} \left( \frac{\Delta}{E_F} \right)^2$$

$$D \sim \frac{\hbar}{m} \left( \frac{E_F}{\Delta} \right)^2$$

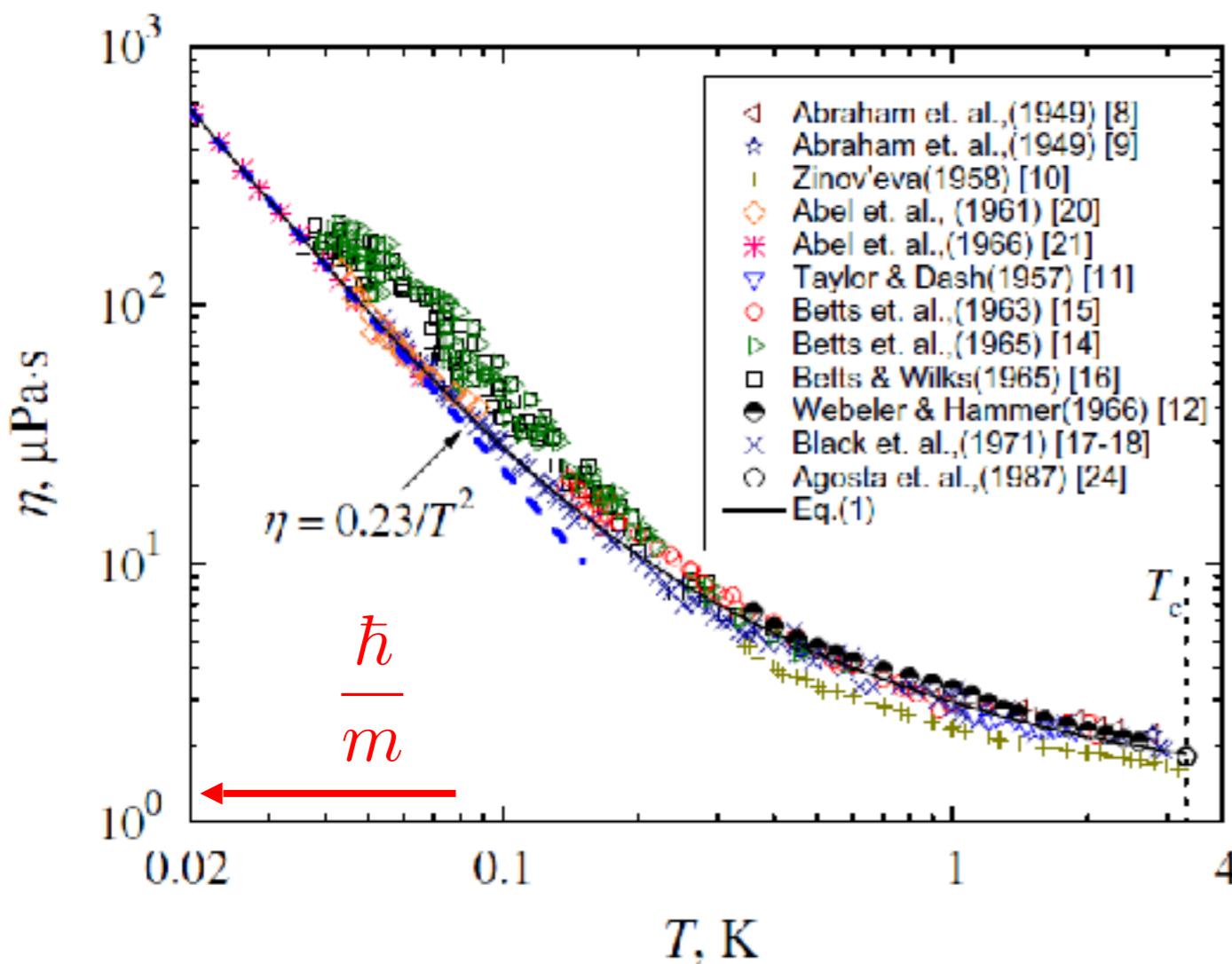
Schäfer et al., PRA  
2010

$\frac{7 \eta}{3 \rho}$   $\eta$  from Enss, Haussmann, Zwerger, Ann. Phys. (2011)

# Comparison to 3He

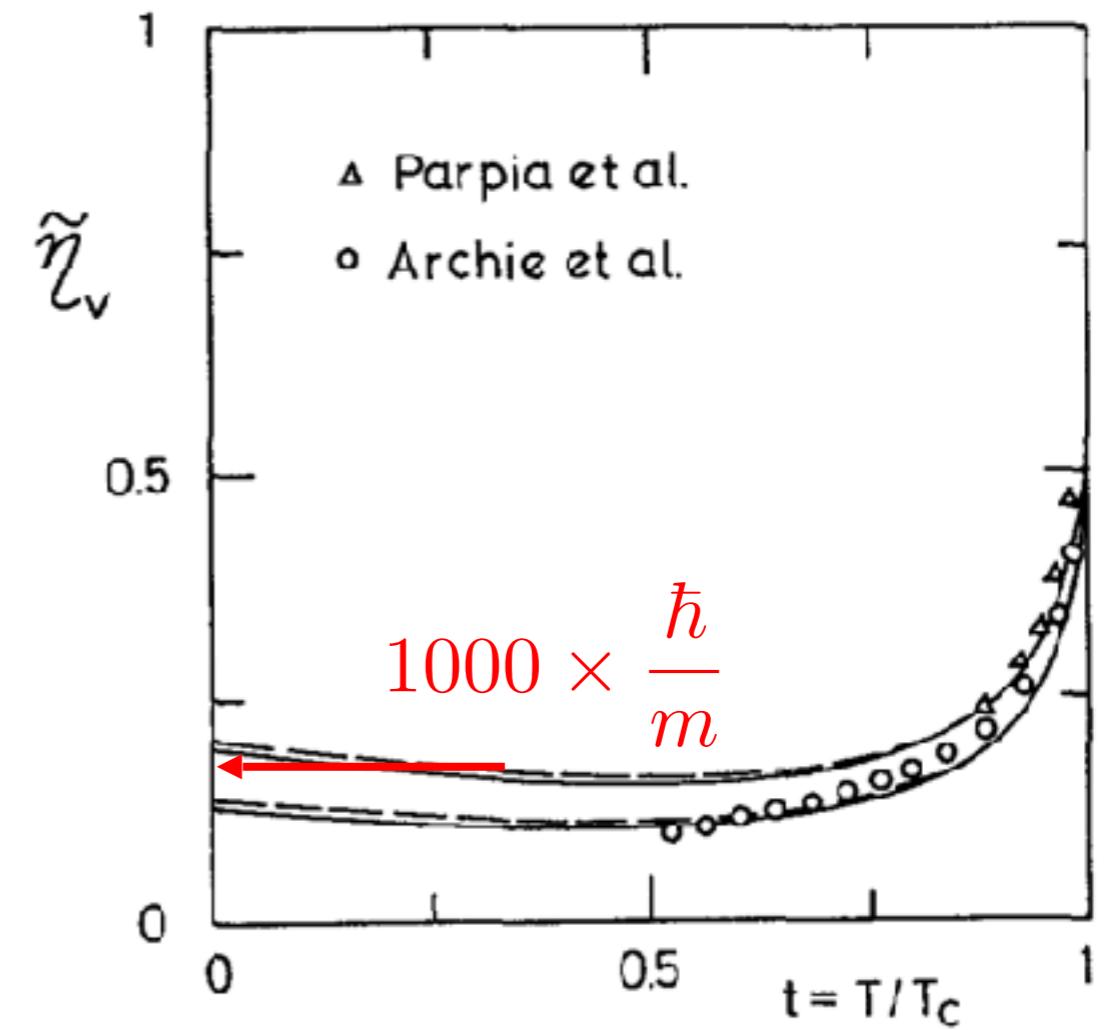
$$D \sim \frac{\hbar}{m} \left( \frac{E_F}{\Delta} \right)^2$$

Above T<sub>c</sub>, Fermi liquid



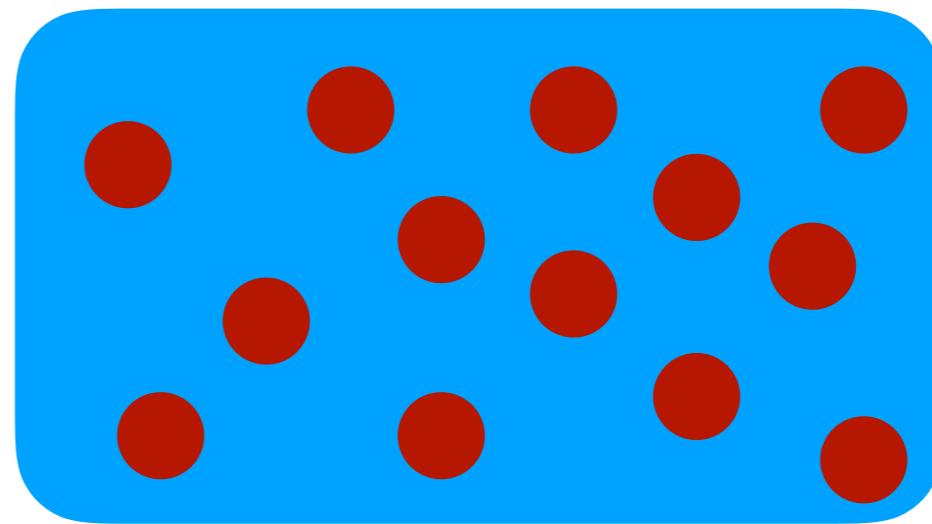
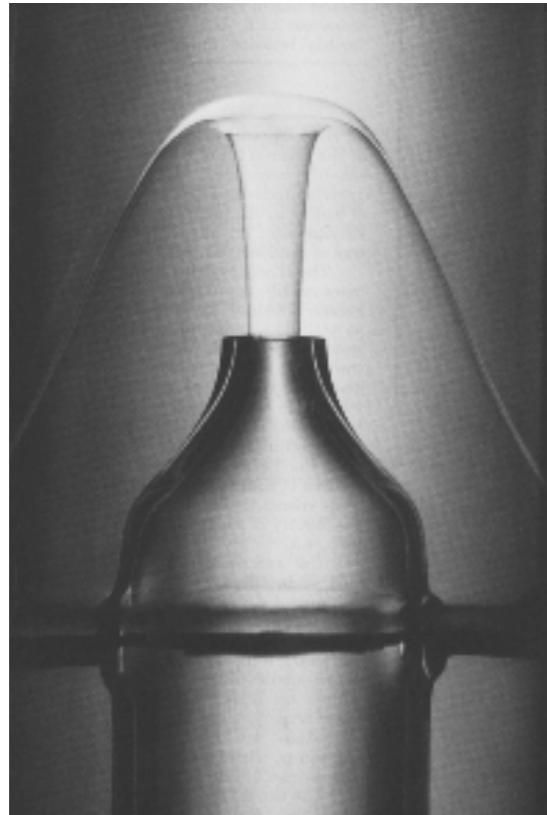
Huang et al., Cryogenics 52, 538 (2012)

Below T<sub>c</sub>, constant D



Ono et al., JLTP 48, 167 (1982)

# Two-fluid hydrodynamics



Superfluid

Normal fluid

# In situ spectroscopy

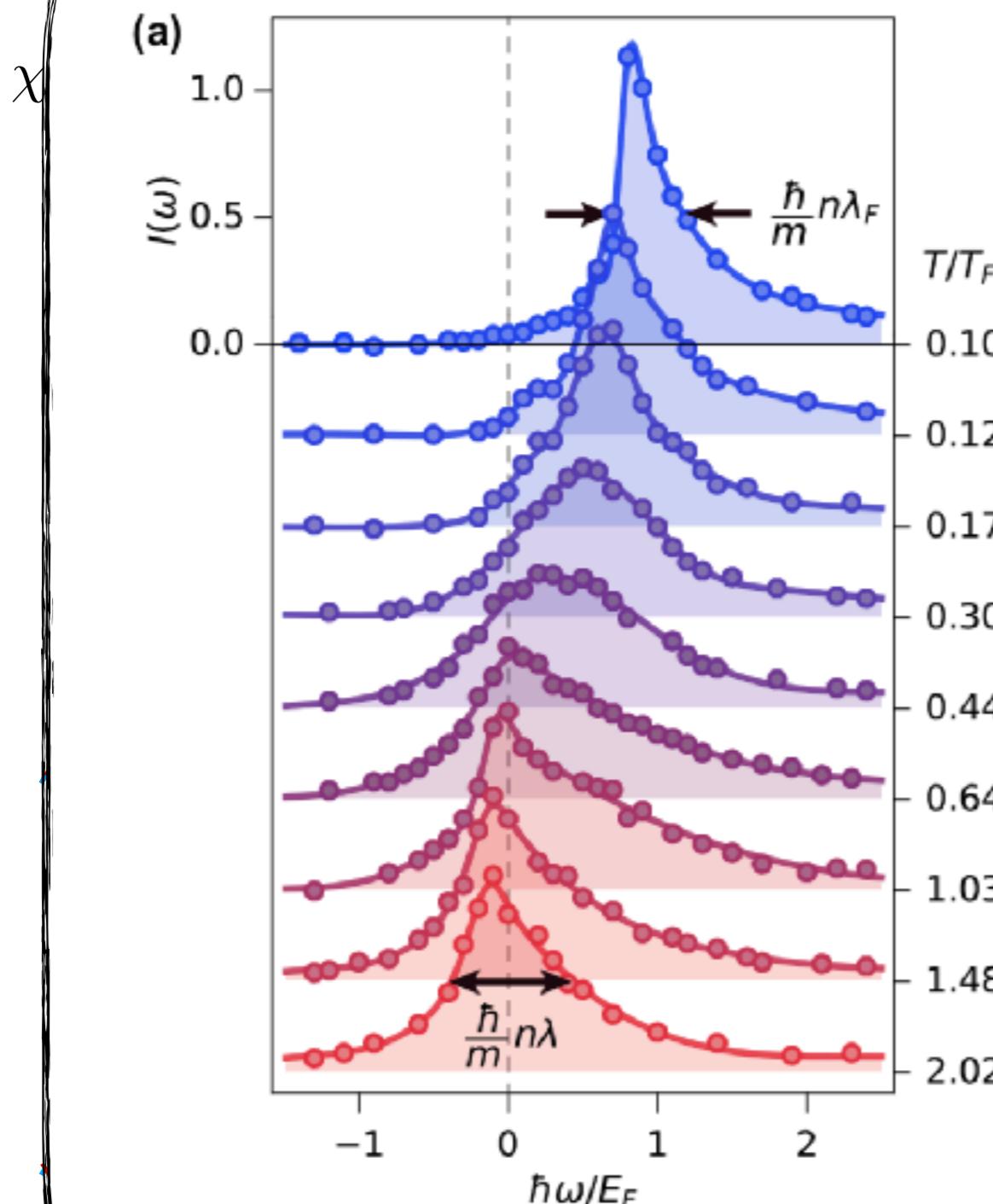
Density resp

## Spectroscopy of unitary Fermi gas

Martin 1965

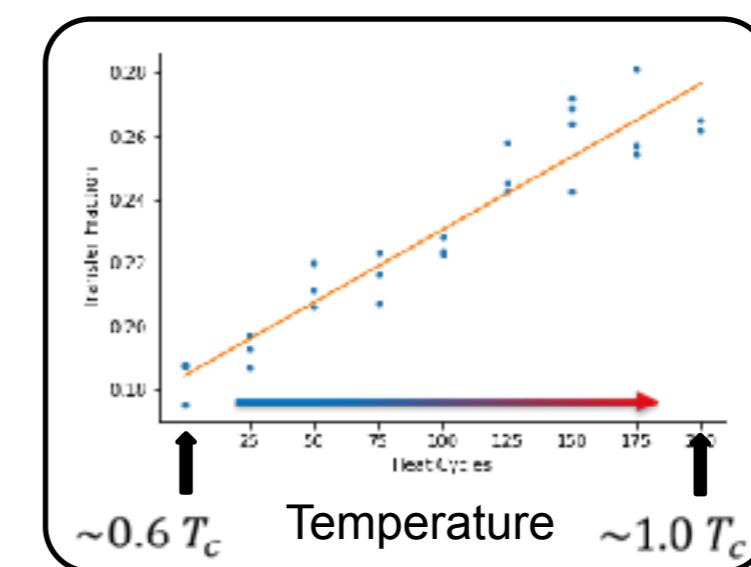
First sound

Second sound

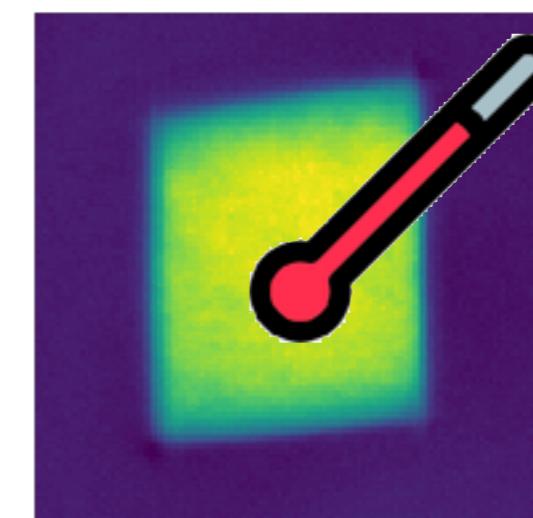
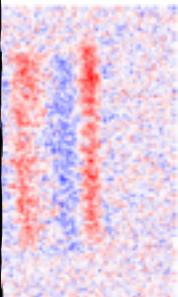


Biswaroop Mukherjee, Parth B. Patel, Zhenjie Yan, Richard J. Fletcher, Julian Struck, and Martin W. Zwierlein  
Phys. Rev. Lett. 122, 203402 (2019)

Number of flipped spins gives probe of local temperature



$s^2$

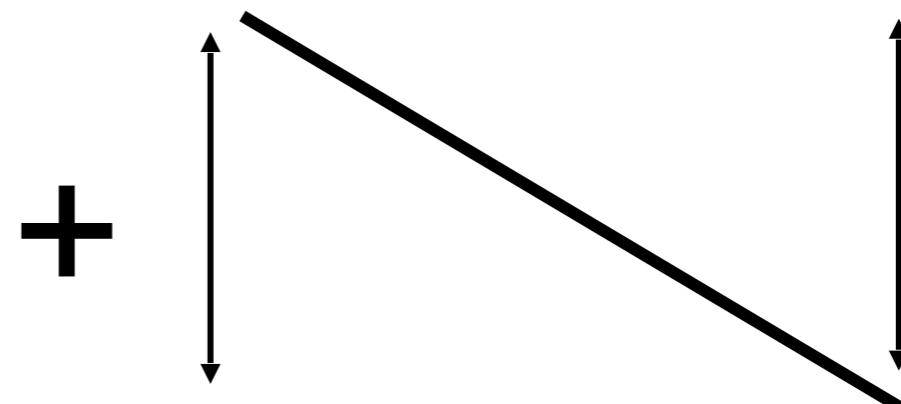
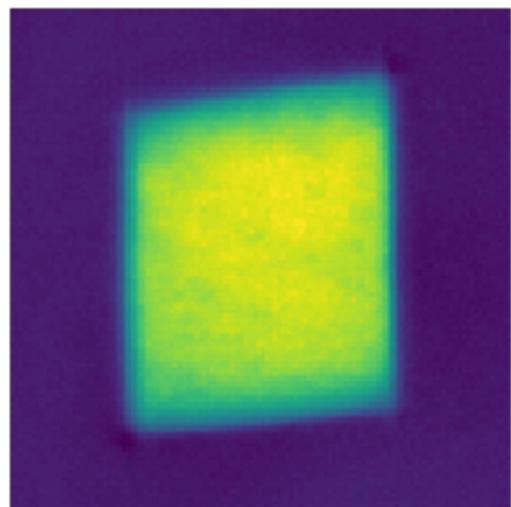


# Injection of second sound

(Isentropic) expansivity of UFG

$$\left. \frac{\partial \rho}{\partial T} \right|_s = -\frac{3}{2} \frac{\rho}{T}$$

Drive temperature wave using potential (coupled to density)

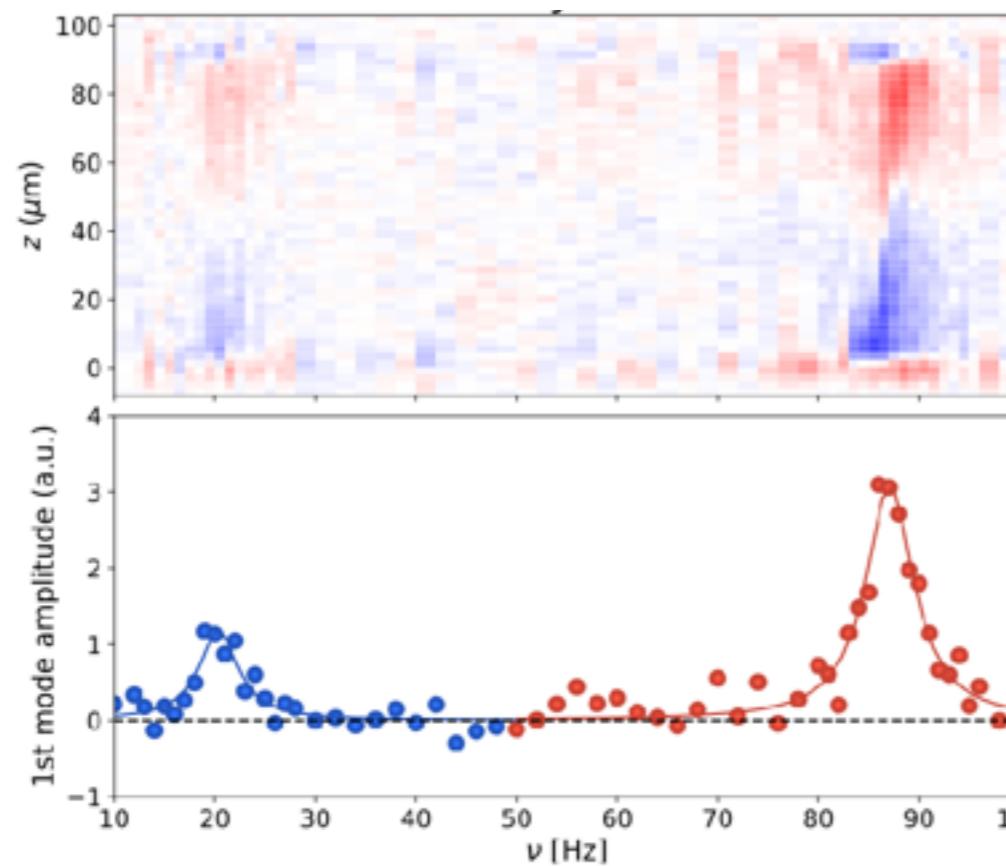


Observation of Second Sound in Quasi-1D geometry:  
Sidorenkov et al., Grimm, Stringari&Piatevskii, Nature 2013

On bosons: weakly interacting situation: JILA (Debbie Jin 1996), MIT  
Hydrodynamic gas: van der Straten

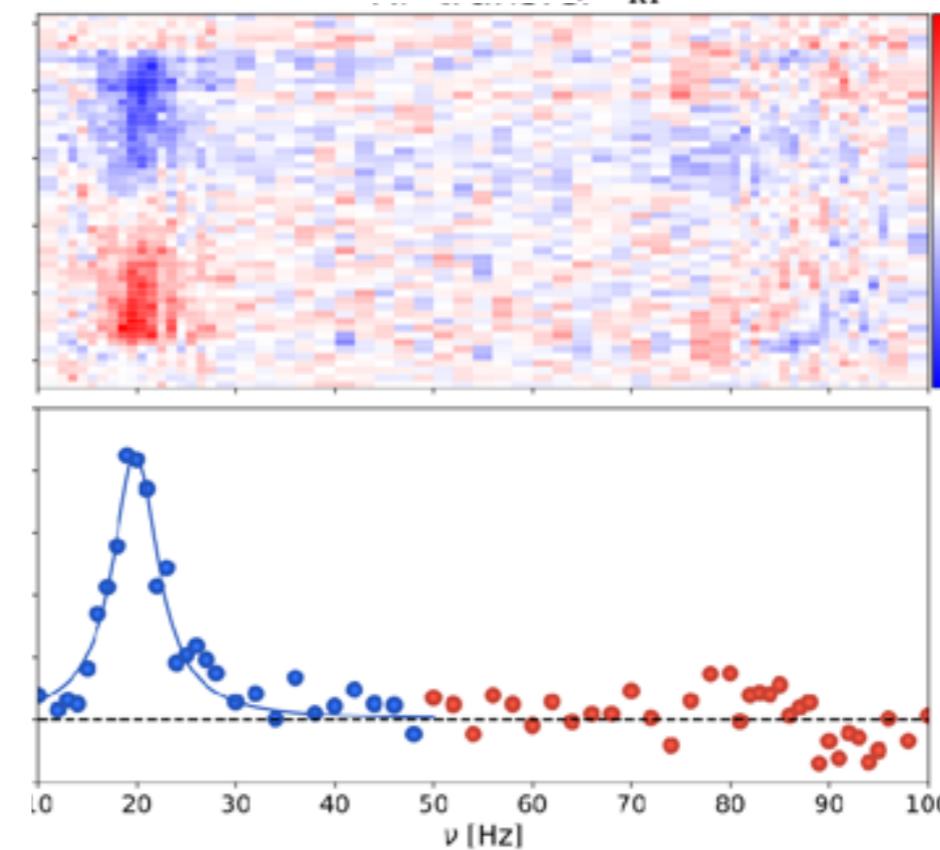
# Observation of first and second sound

Density Probe

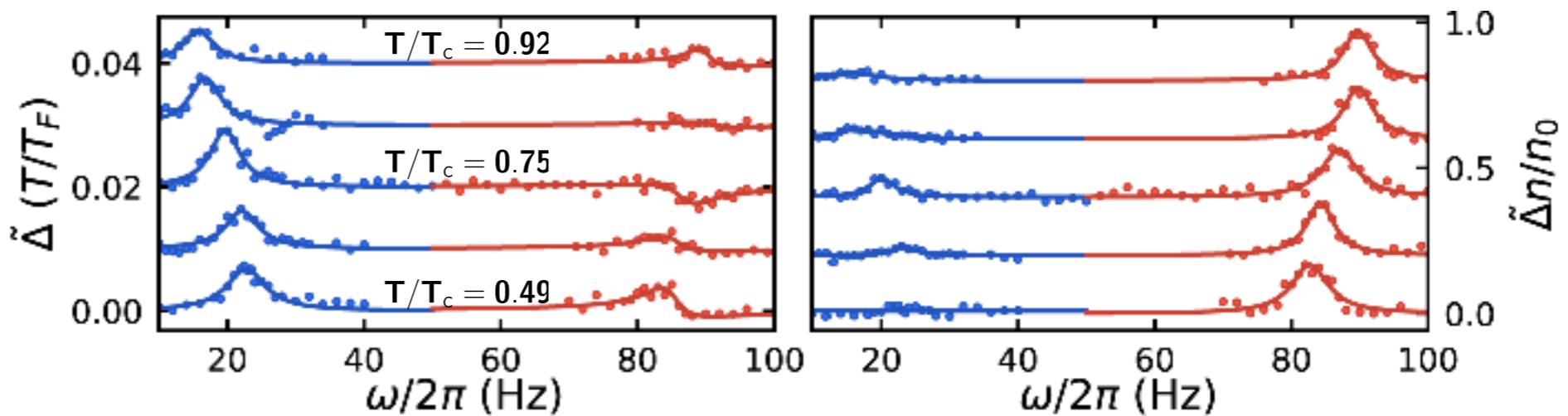


Temperature Probe

after RF transfer,  $n_{RF}$



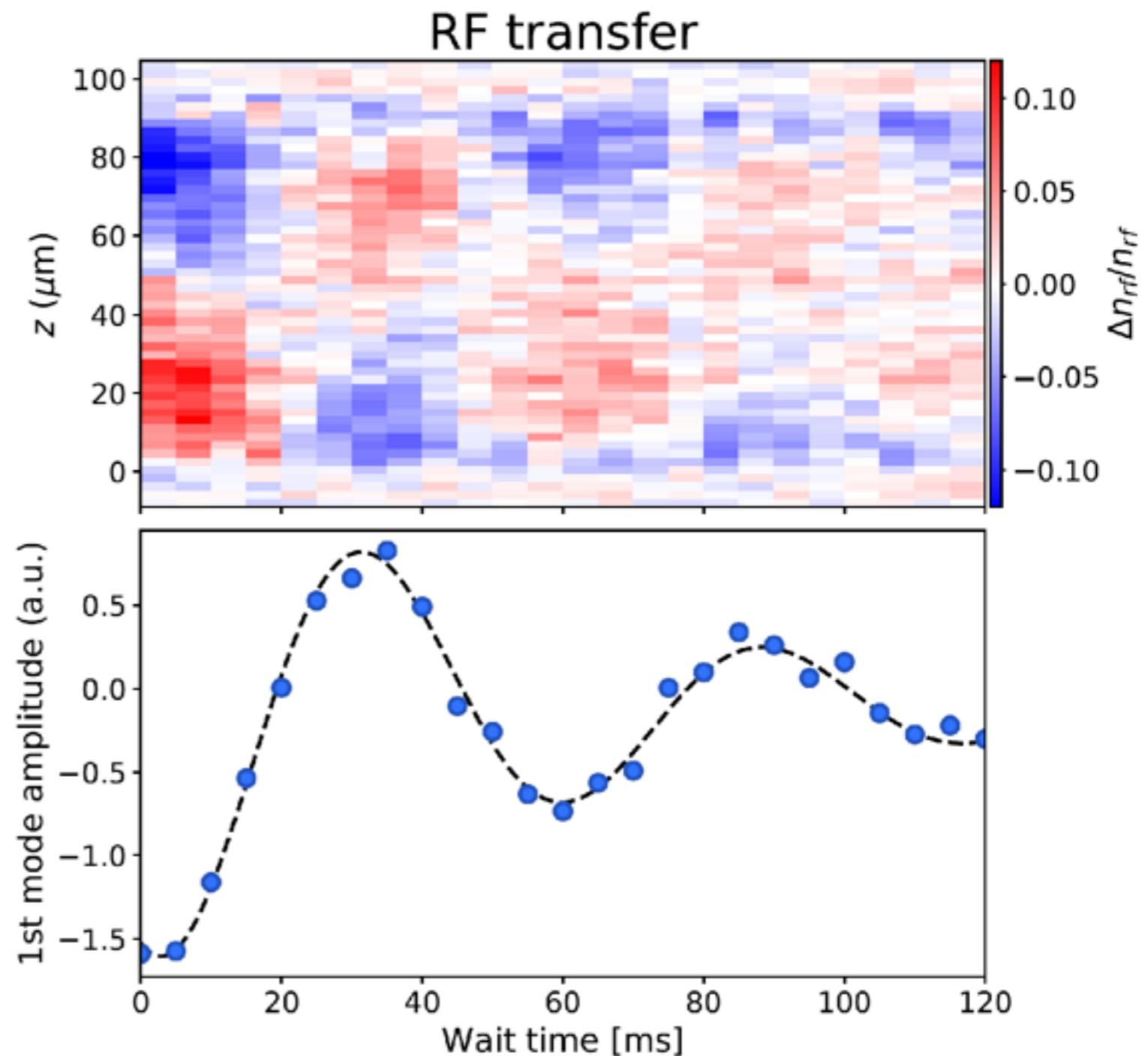
→ Speeds and decay rates of first and second sound



Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, M. Zwierlein, in prep. (2020)

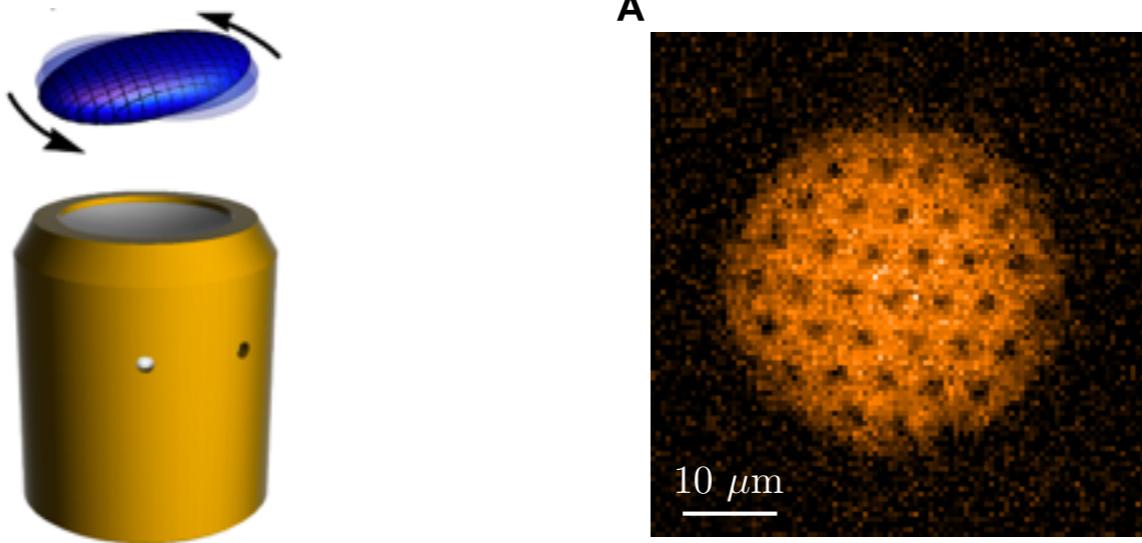
# Damping of second sound

Decay of standing wave of second sound

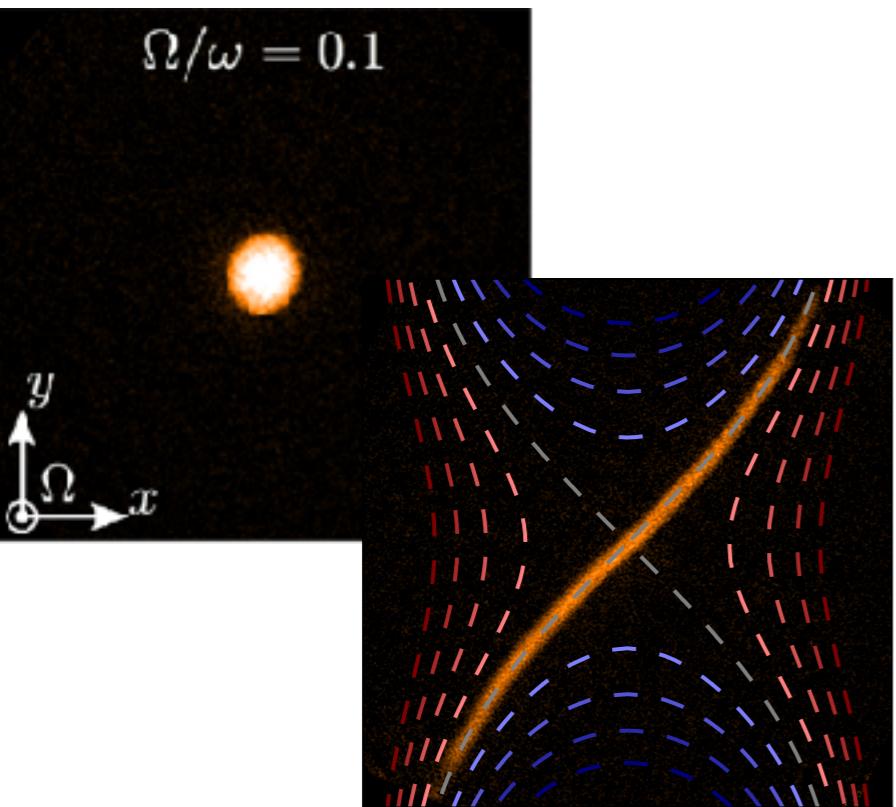


Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, M. Zwierlein, in prep. (2020)

## 2. *In situ* physics of a rotating quantum gas

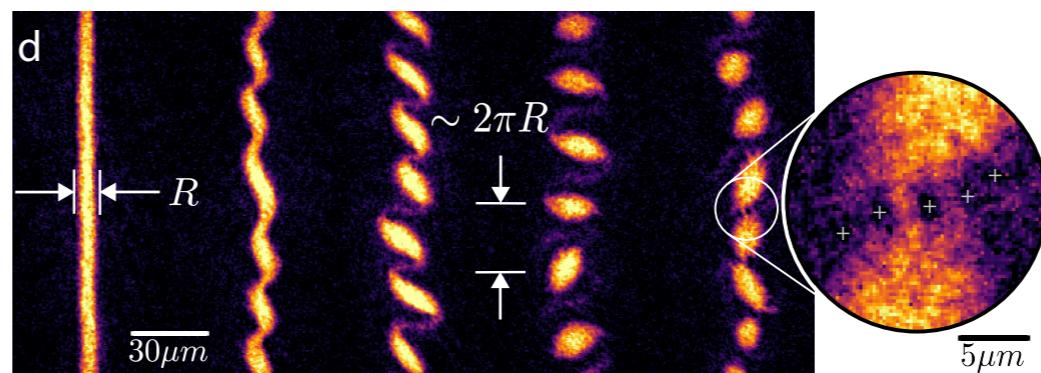


### Geometric squeezing

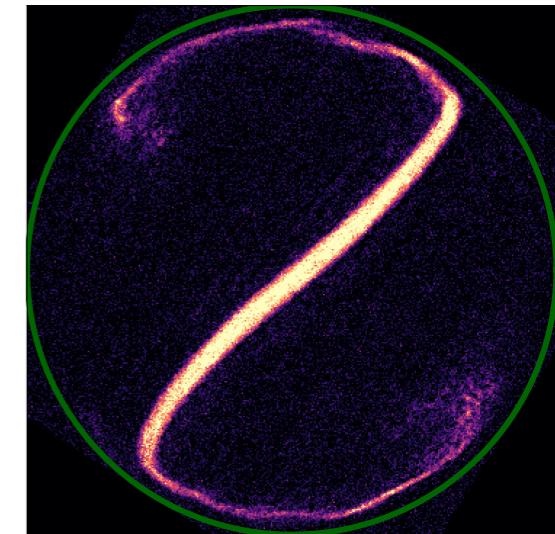


### Dynamics of superfluid under gauge field + interactions

Flat LL dispersion  $\rightarrow$  purely interaction-driven physics

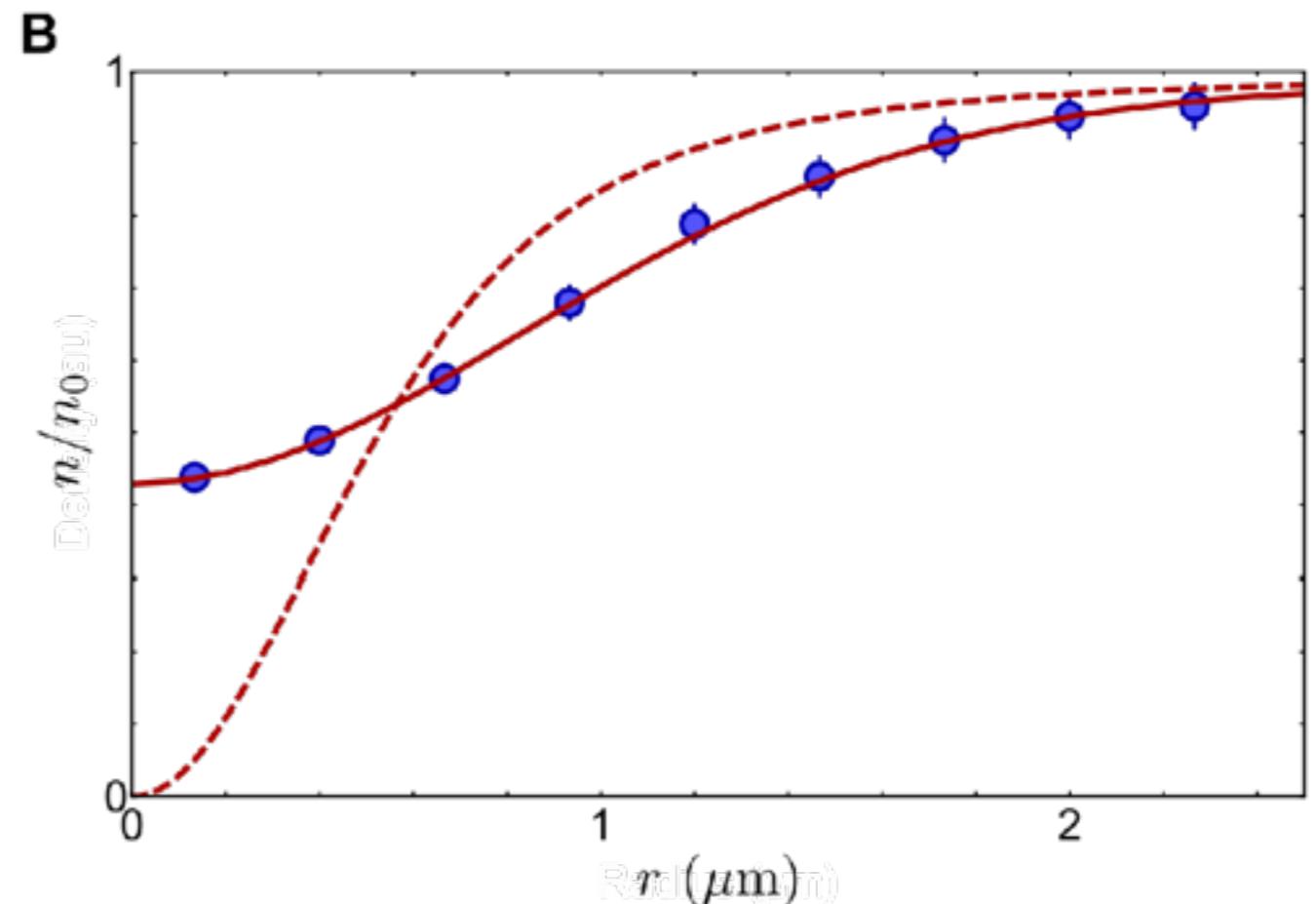
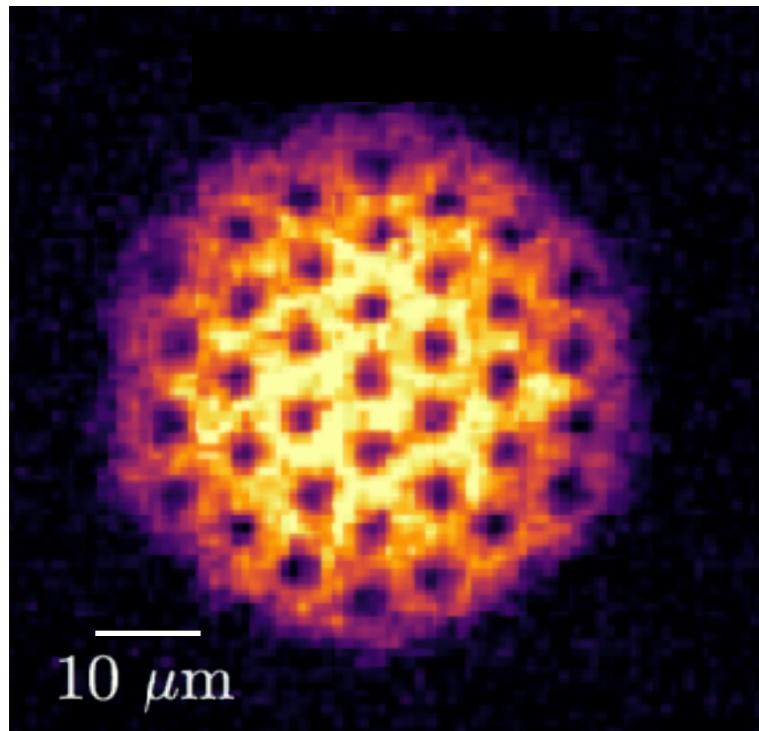


### Chiral edge modes

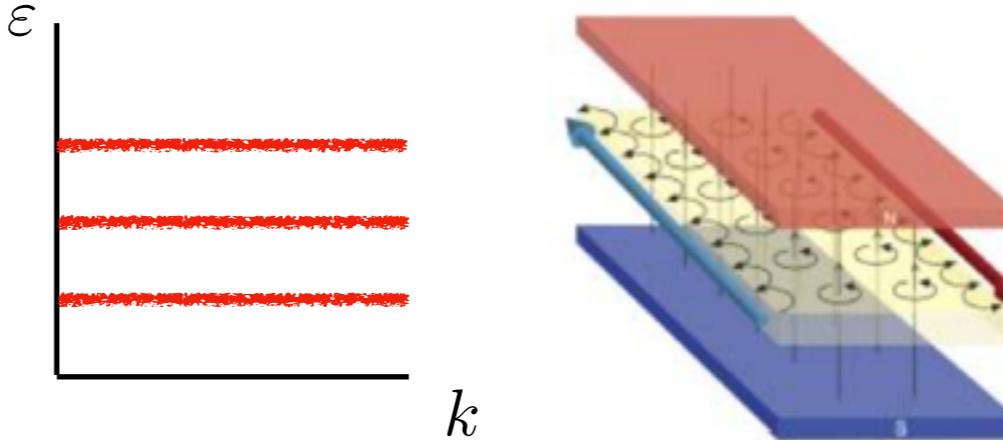


R. Fletcher, A. Shaffer, C. Wilson, P. Patel, Z. Yan, V. Crepel, B. Mukherjee, M. Zwierlein,  
arXiv:1911.12347 (2019) Science, accepted

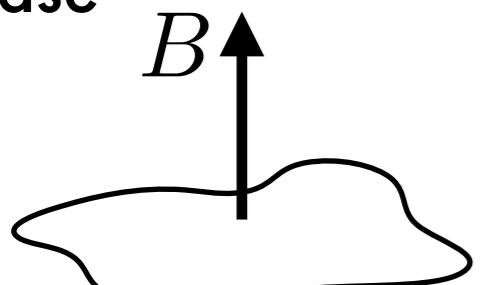
# In situ detection of quantum vortices



# Artificial magnetic fields:

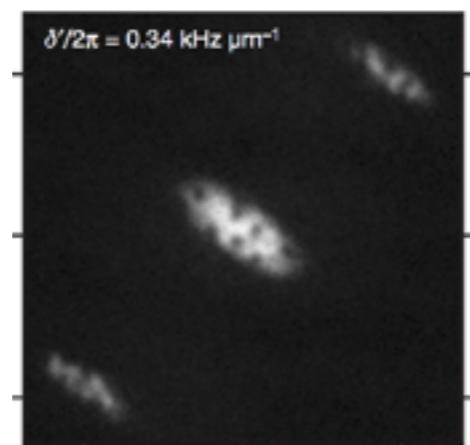
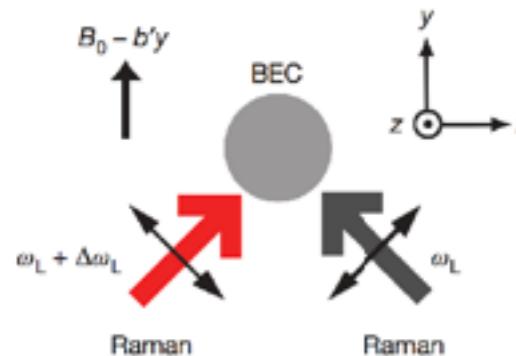


Construct analog of Aharonov-Bohm phase



$$\phi = \frac{q}{m} \int \vec{B} \cdot d\vec{S}$$

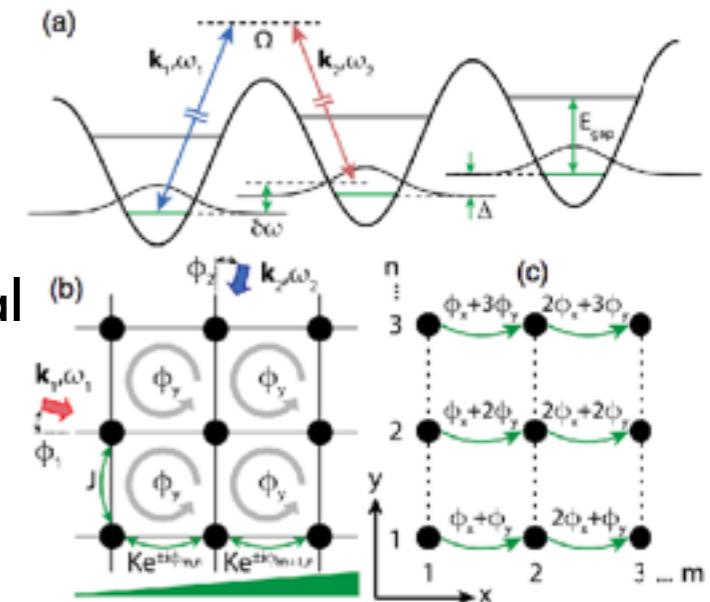
## Spin-orbit coupling



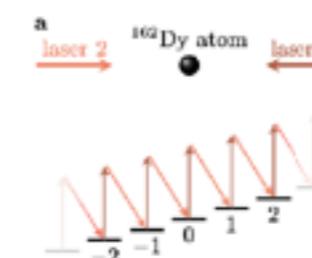
Y.-J. Lin, R. L. Compton, K. Jimenez-Garcia, J. V. Porto,  
I. B. Spielman, Nature 462, 628 (2009)

## Direct phase imprinting

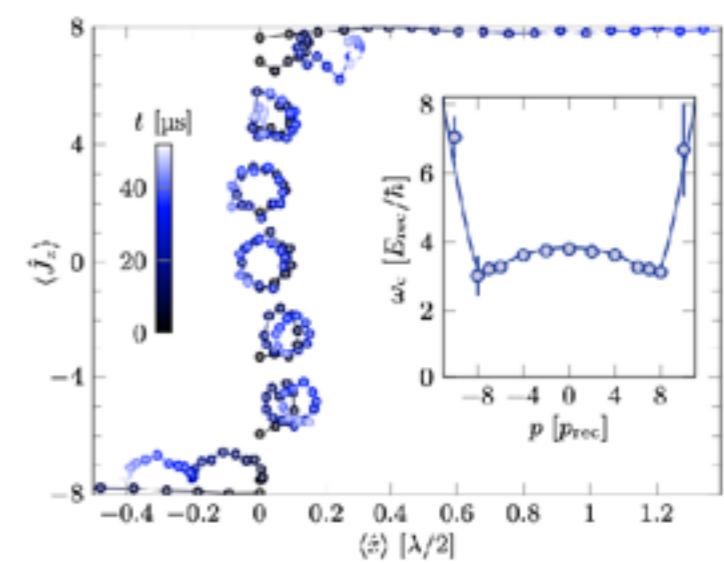
M. Aidelsburger, M. Atala  
M. Lohse, J. T. Barreiro,  
B. Paredes, I. Bloch,  
PRL 111, 18530 (2013)



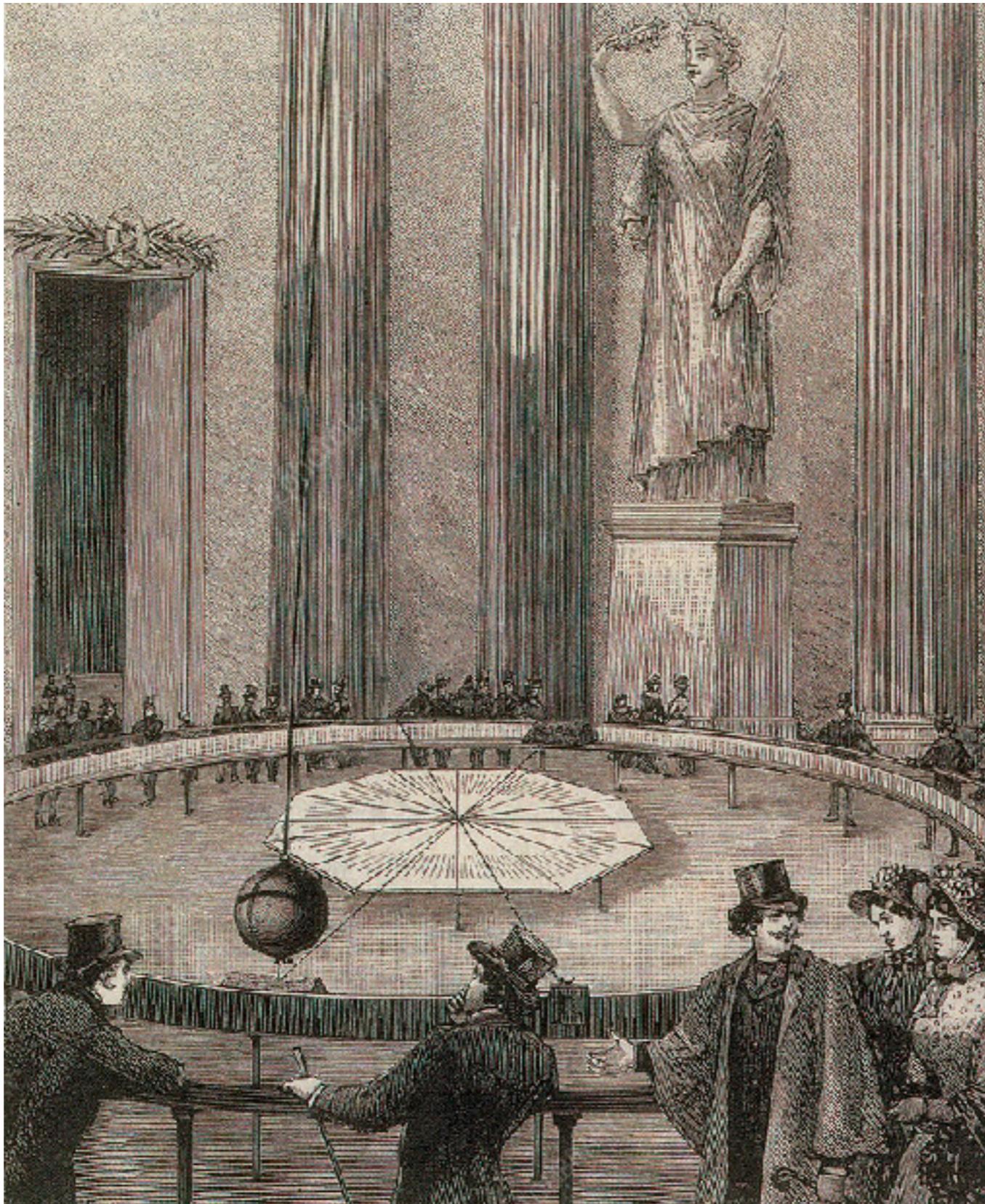
H. Miyake, G. A. Siviloglou, C. J. Kennedy, W. C.  
Burton, W. Ketterle, PRL 111, 185302 (2013)



T. Chalopin et al  
arxiv:2001.01664  
2020

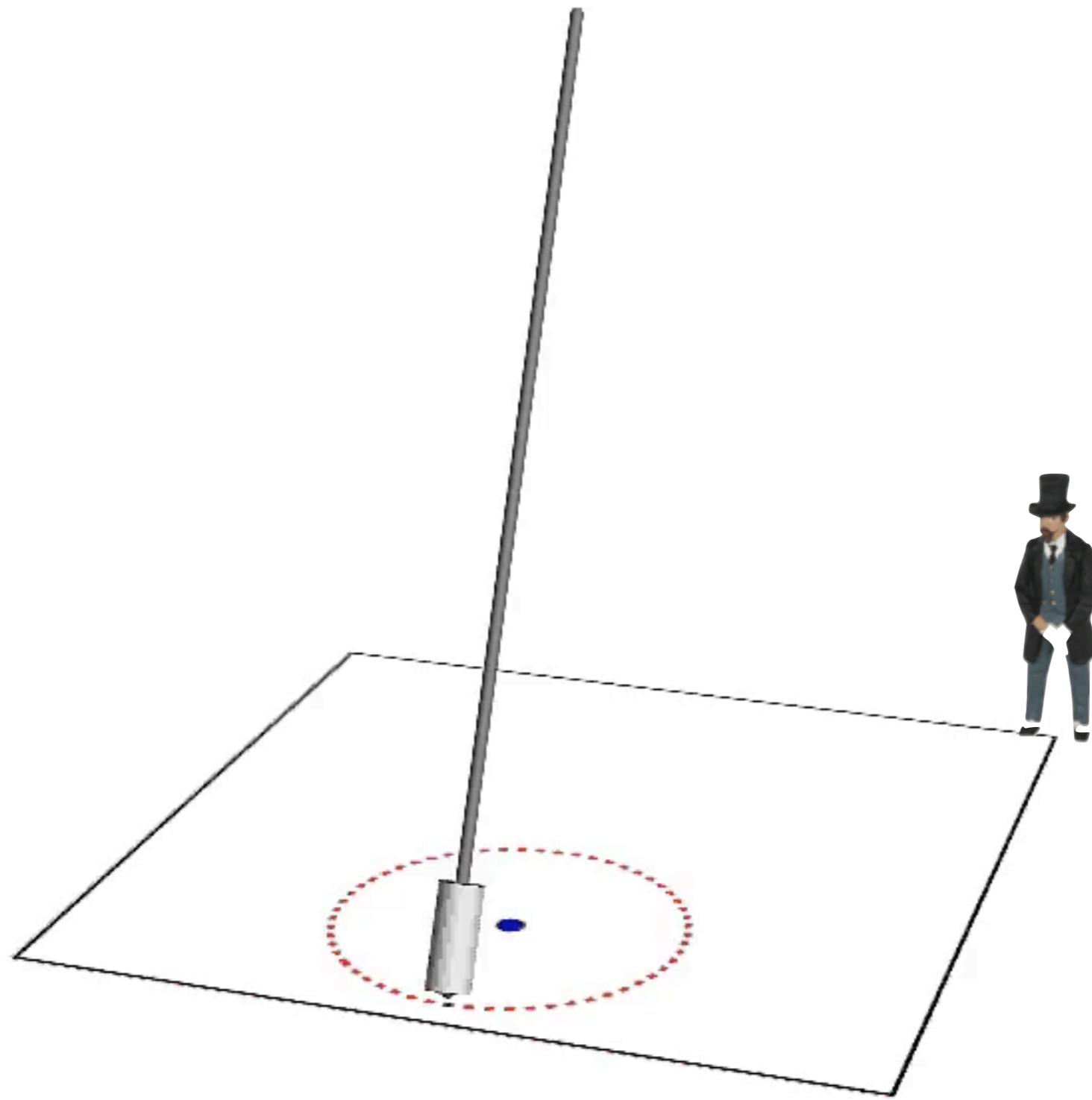


# Basics: Foucault pendulum



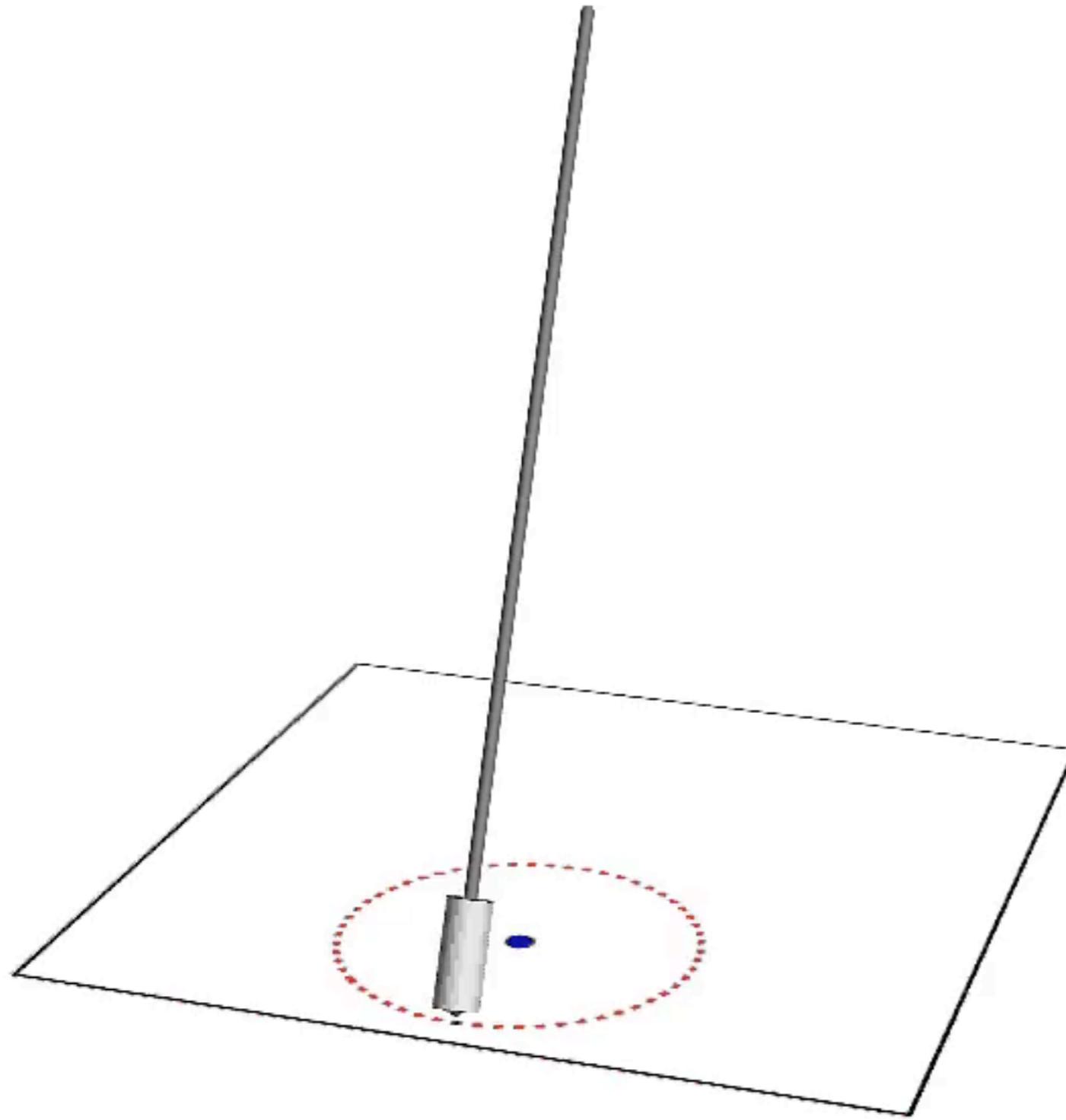
# Basics: Foucault pendulum

$$\Omega = 0.1$$

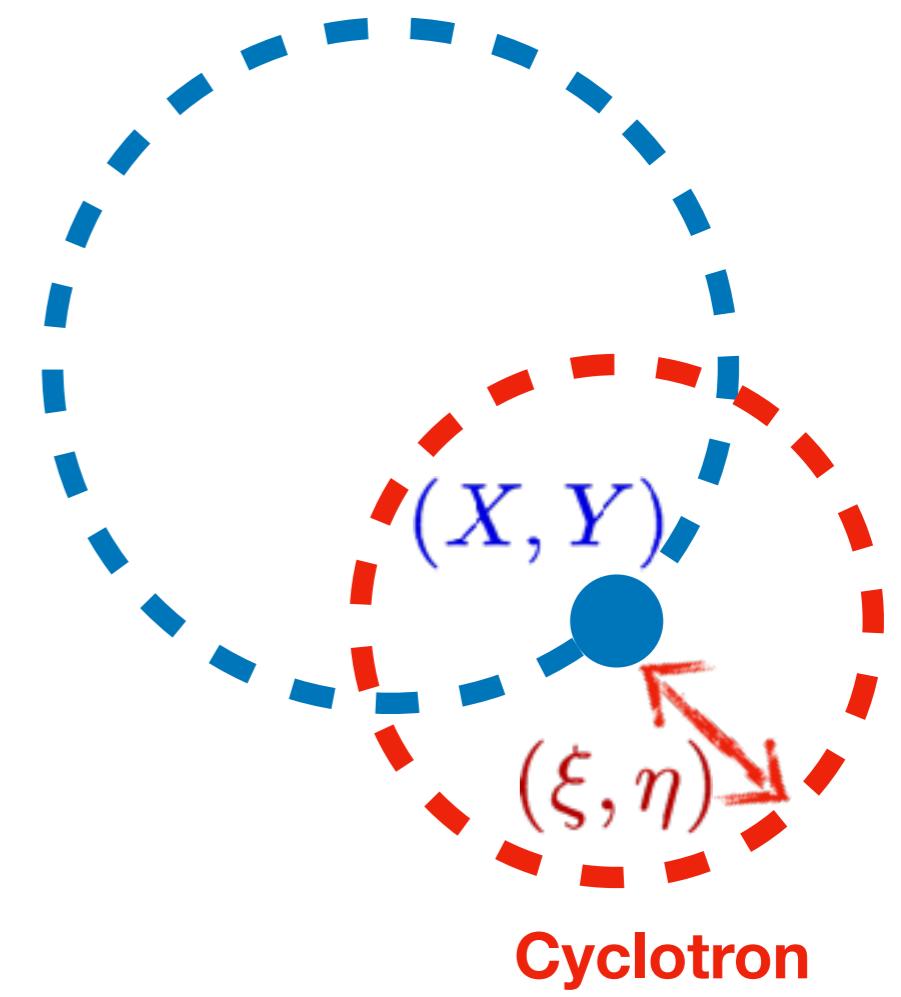


# Basics: Foucault pendulum

$$\Omega = 0.1$$

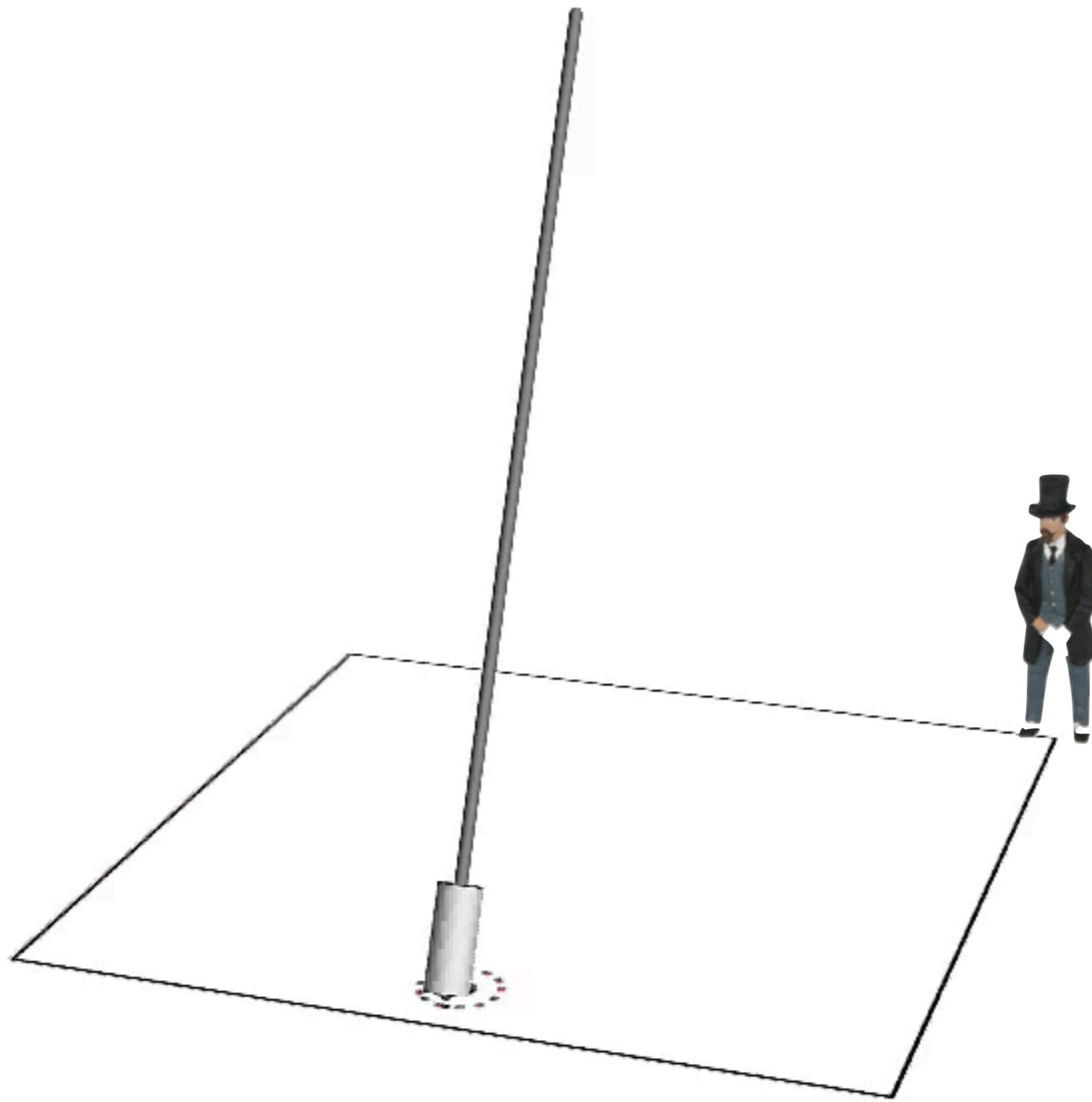


Guiding centre



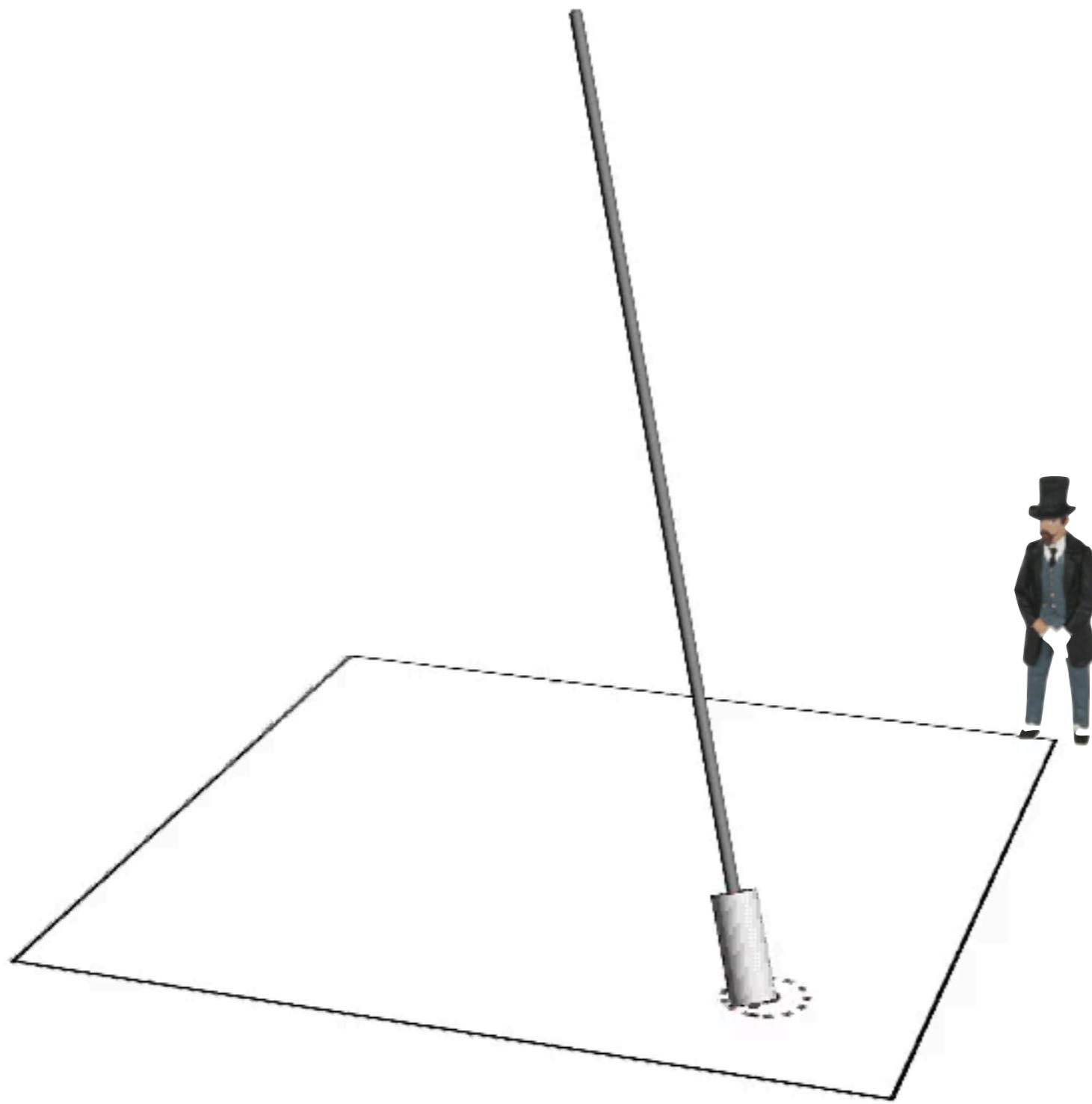
# Basics: Foucault pendulum

$$\Omega = 0.8$$



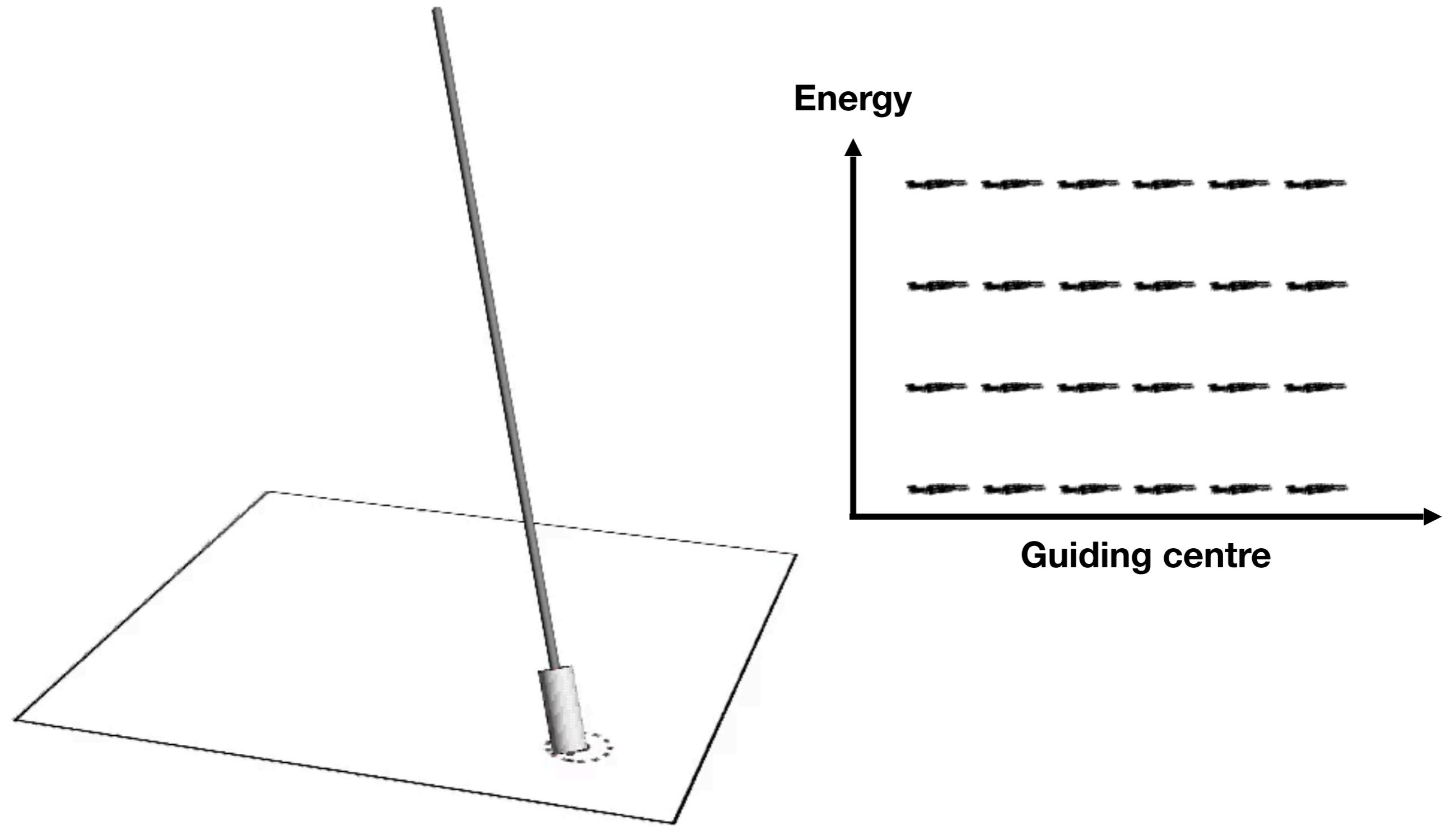
# Basics: Foucault pendulum

$$\Omega = 1$$



# Basics: Foucault pendulum

$$\Omega = 1$$



# Quantum description

$$H = \hbar\omega(\omega + \Omega)(\xi^2 + \eta^2) + \hbar\omega(\omega - \Omega)(X^2 + Y^2)$$

$$= \hbar(\omega + \Omega)\hat{a}^\dagger\hat{a} + \hbar(\omega - \Omega)\hat{b}^\dagger\hat{b} + \hbar\omega$$

**Cyclotron**

**Guiding centre**

$$\hat{a} \sim \hat{\xi} + i\hat{\eta}$$

$$\hat{b} \sim \hat{X} - i\hat{Y}$$

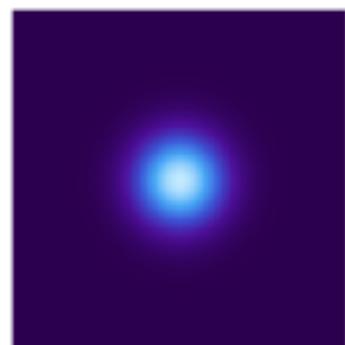
$$[\hat{\xi}, \hat{\eta}] = il_B^2$$

$$[\hat{X}, \hat{Y}] = -il_B^2$$

$$l_B = \sqrt{\frac{\hbar}{2m\omega}}$$

If potential  $V(x, y)$  varies slowly, can't resolve cyclotron motion

$$V(x, y) \rightarrow V(X, Y)$$

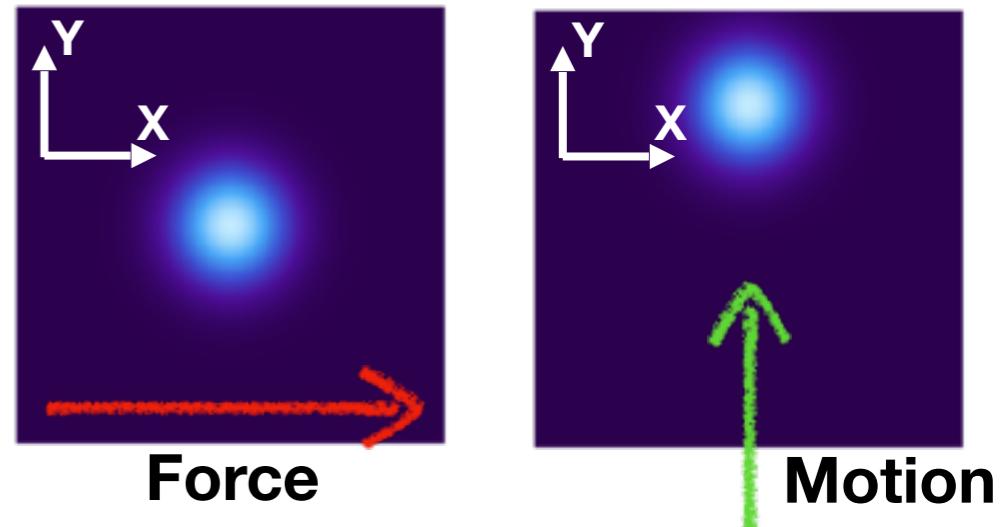


Particle moves within non-commutative space

# Life in a non-commuting space $[\hat{X}, \hat{Y}] = -il_B^2$

Transverse Hall response  $V \sim FX$

$$\hat{U}(t) = \exp\left(-\frac{i}{\hbar}Vt\right) = \hat{T}_Y\left(\frac{Ft}{2m\Omega}\right)$$



Isopotential flow

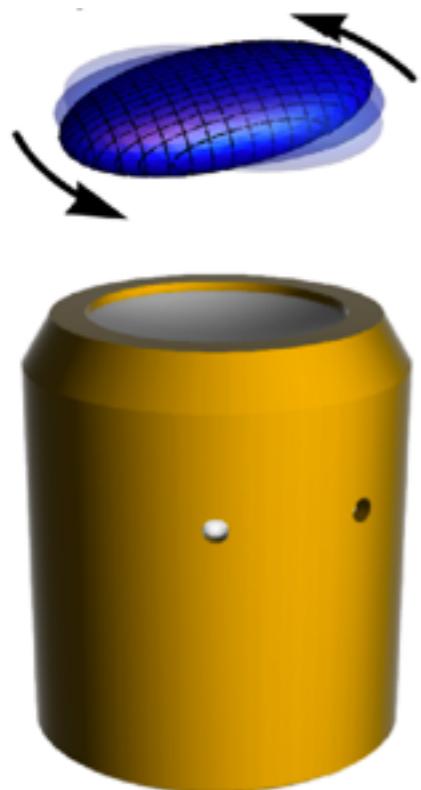
$$\vec{v}_d = \frac{1}{2m\omega} \hat{\Omega} \times \nabla V \quad \nabla \cdot \vec{v}_d = 0$$

Time evolution under generic spatial potential

=

Geometric transformation

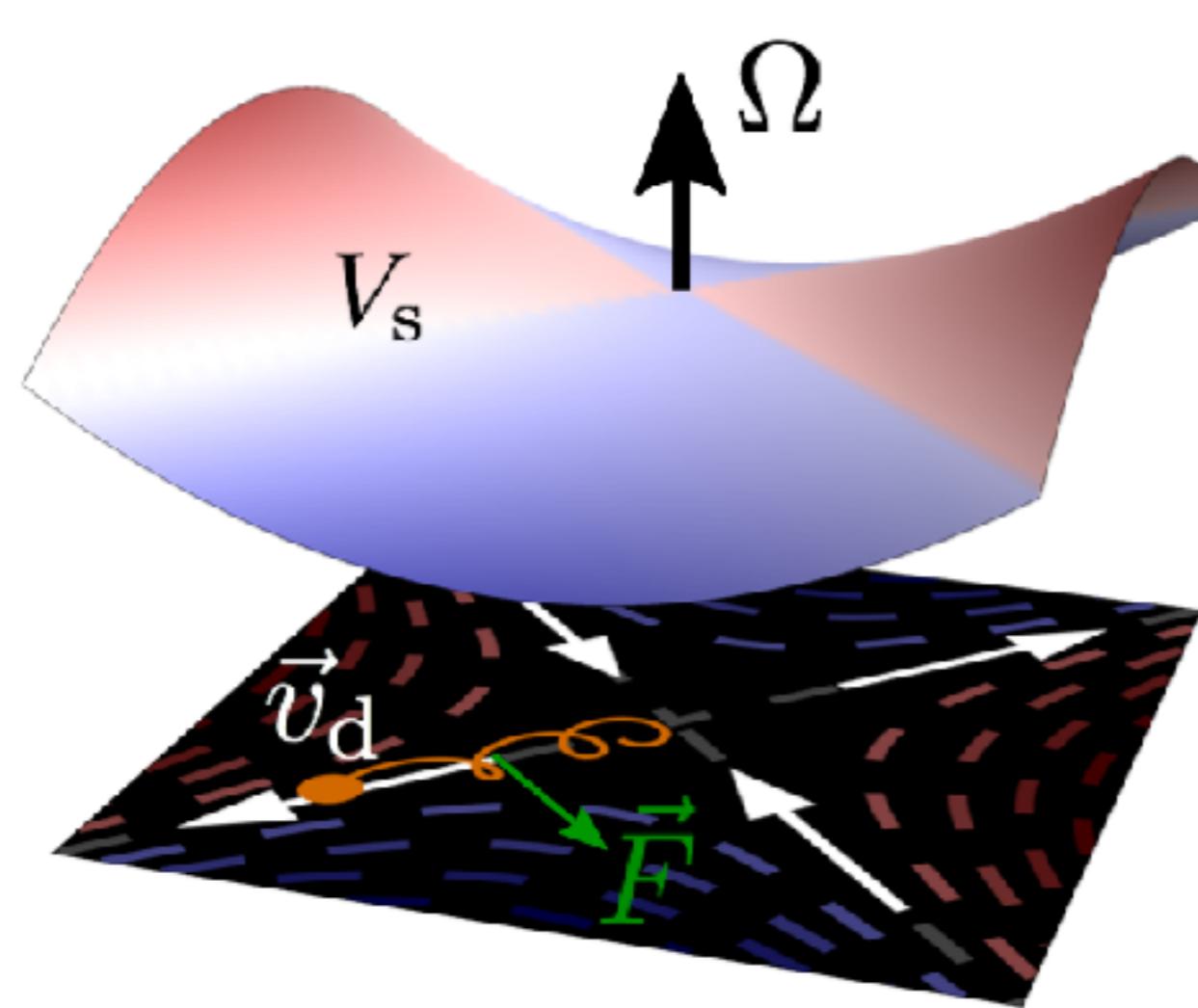
# Our experiment: BEC in a rotating elliptical trap



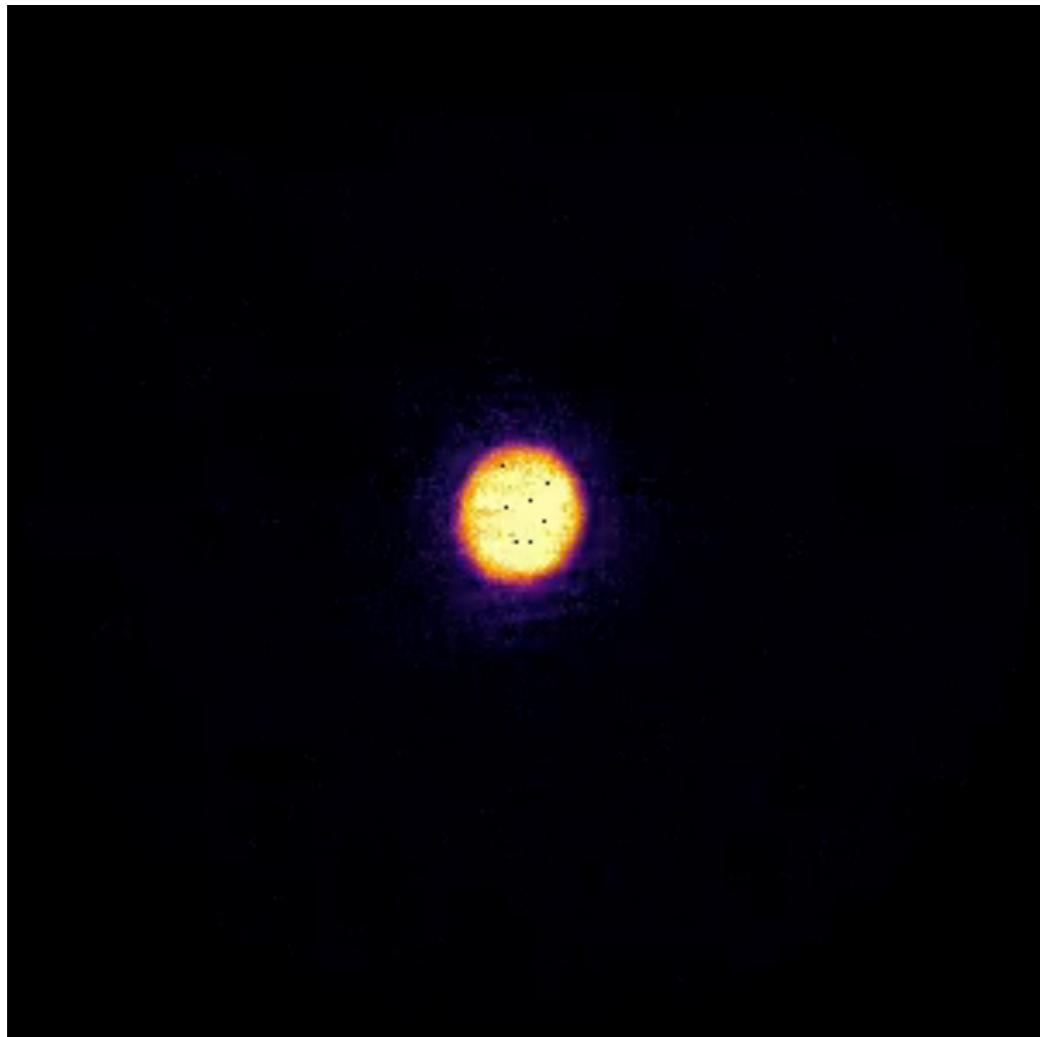
Elliptical harmonic potential

$$V = \frac{1}{2}m\omega^2 [(1 + \varepsilon)x^2 + (1 - \varepsilon)y^2]$$

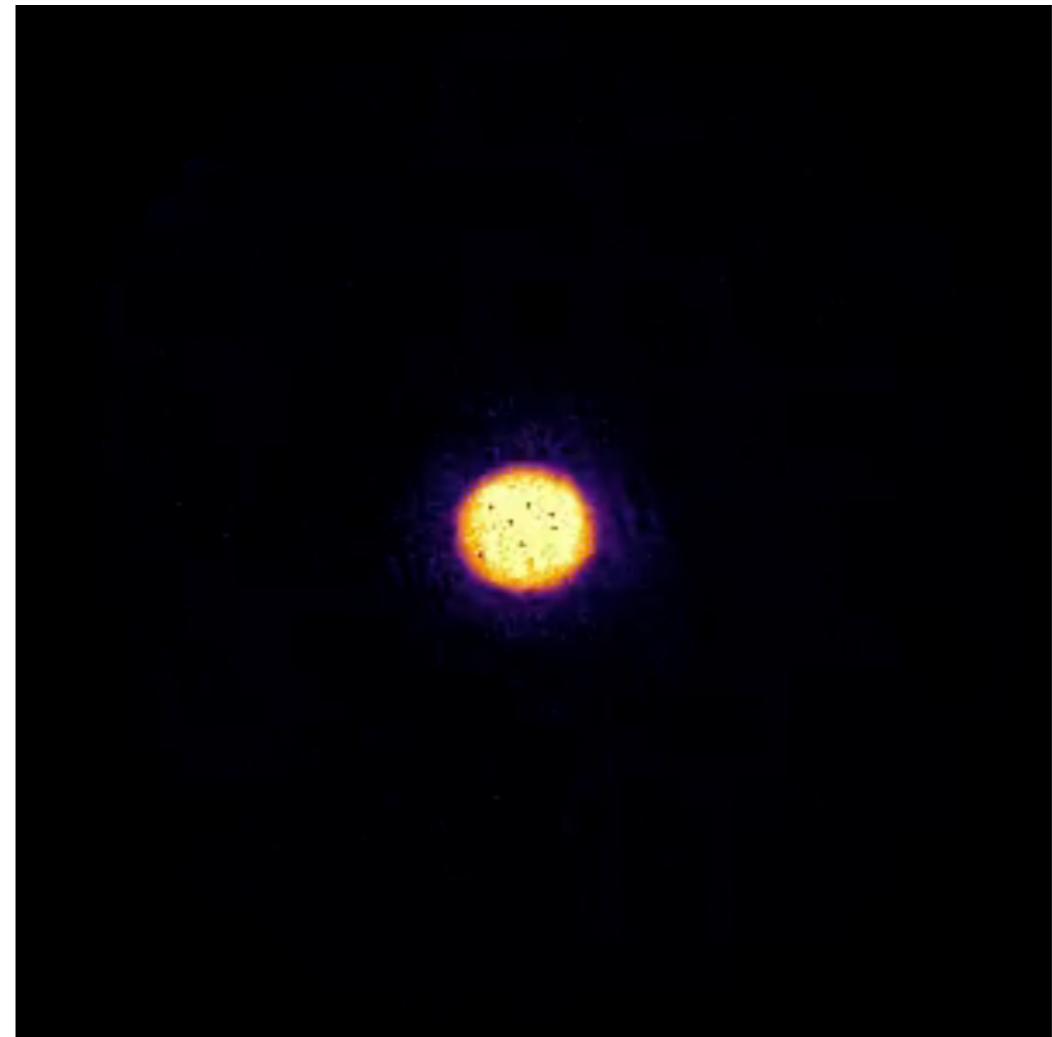
Rotate at frequency  $\Omega$ , ramp up to trap frequency  $\omega$  and hold



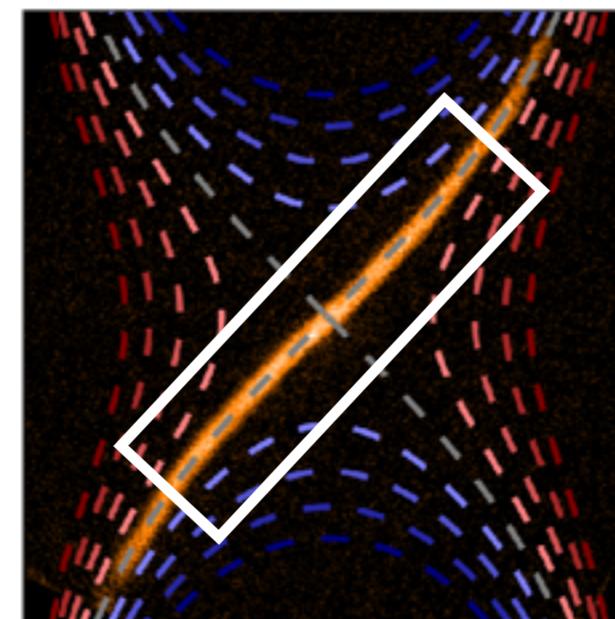
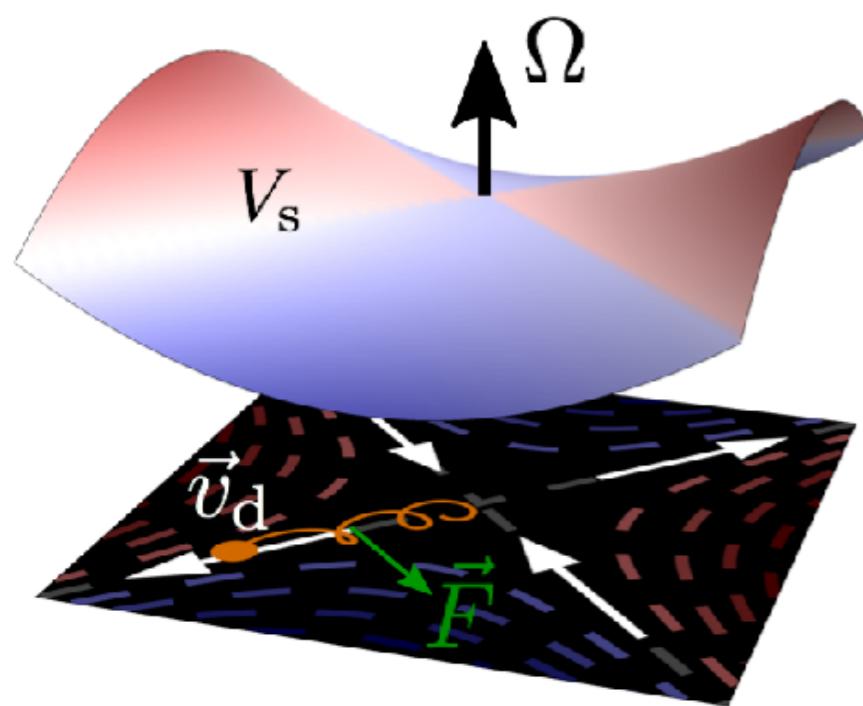
**Lab frame**



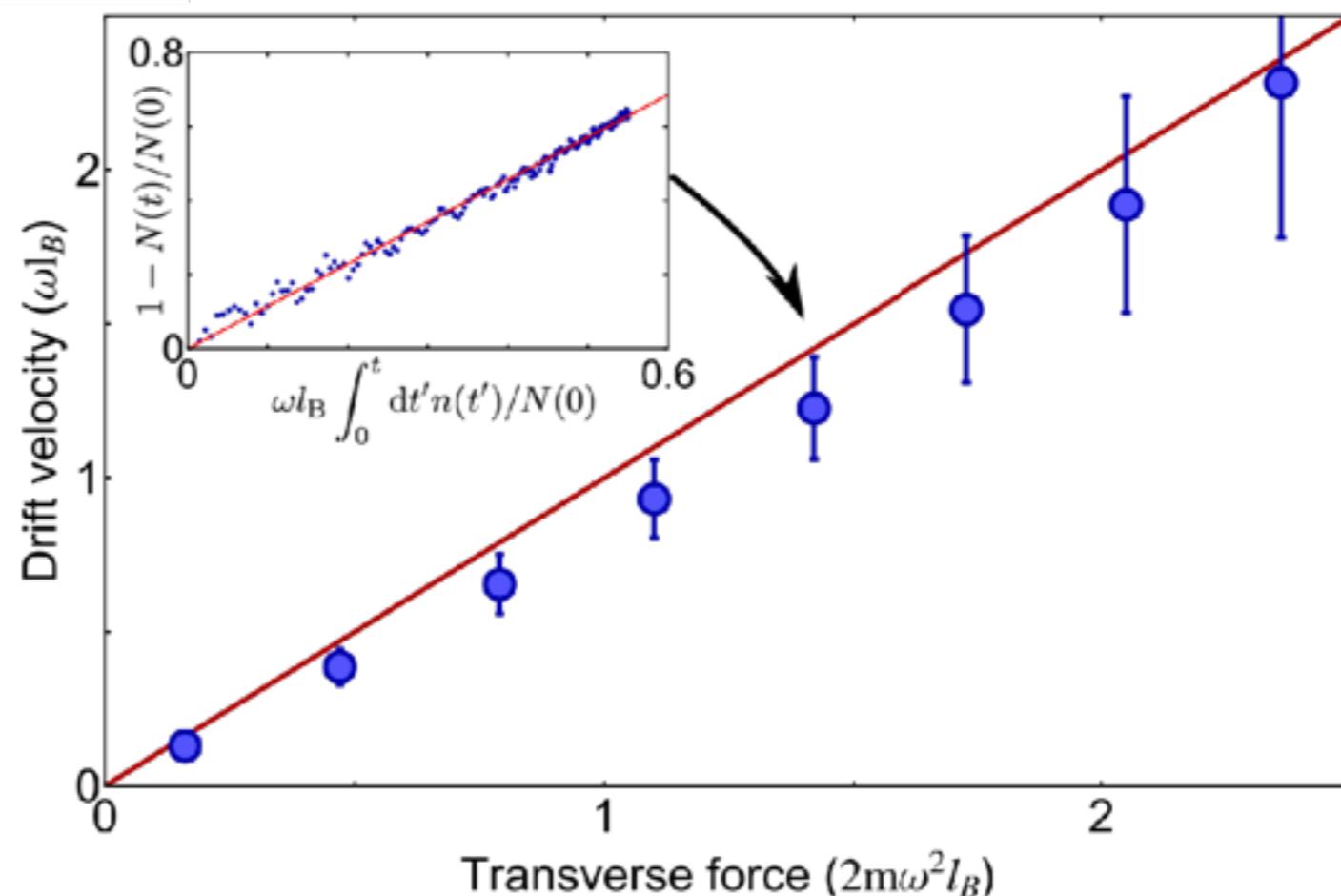
**Rotating frame**



# Drift velocity



$$\frac{dN}{dt} = -2v_d n$$



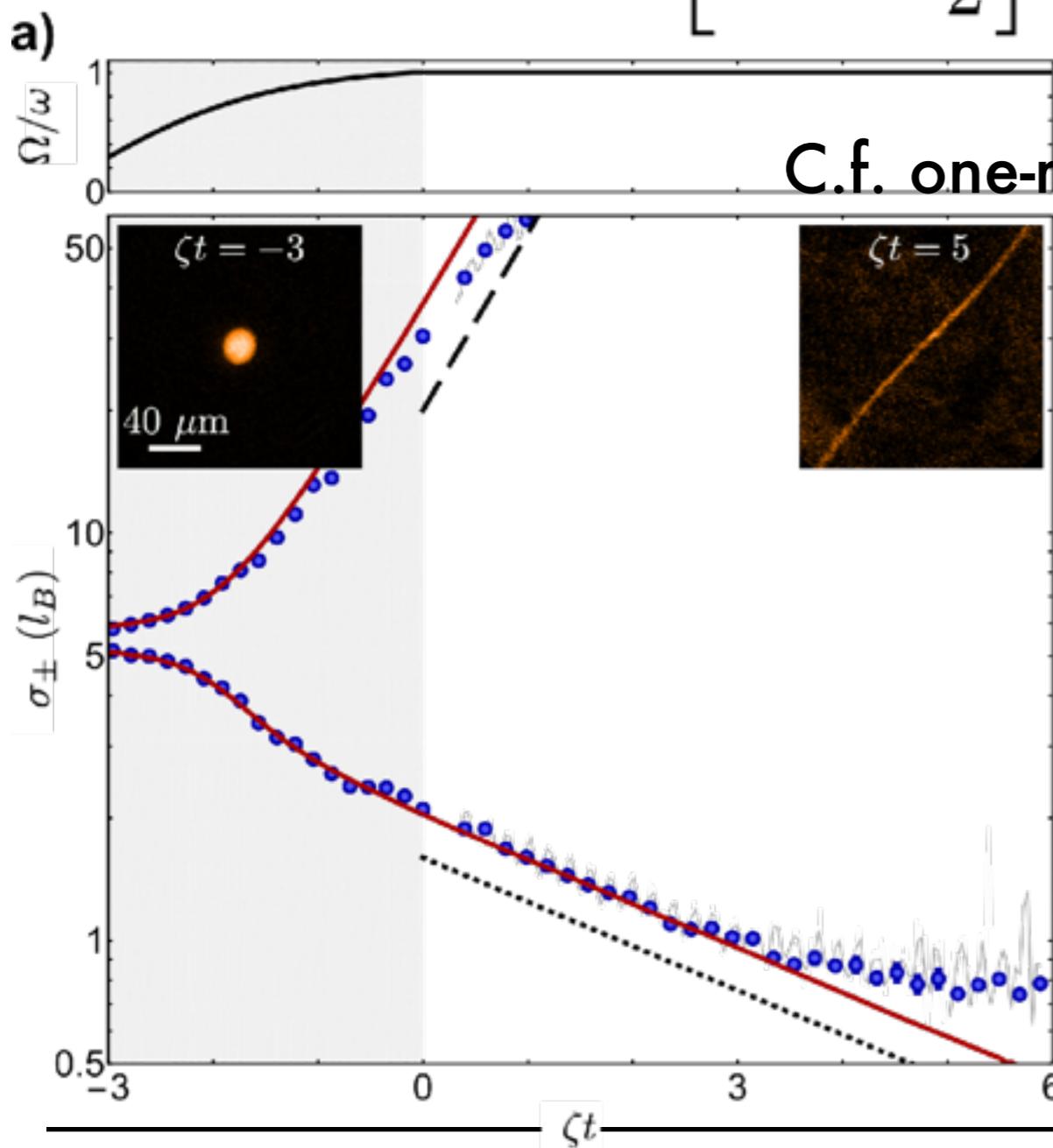
# Geometric squeezing

Hamiltonian under saddle potential

$$\hat{H} = 2\hbar\omega \left[ \hat{a}^\dagger \hat{a} + \frac{1}{2} \right] + \frac{\hbar\zeta}{2} \left[ \hat{b}^\dagger \hat{b} + \hat{b}^\dagger \hat{b}^\dagger \right]$$

Squeezing rate

$$\zeta = \frac{\varepsilon\omega}{2}$$



C.f. one-mode squeezing operator:

$$\hat{S}(\alpha) = \exp \left( \alpha^* \hat{b}^\dagger \hat{b} - \alpha \hat{b}^\dagger \hat{b}^\dagger \right)$$

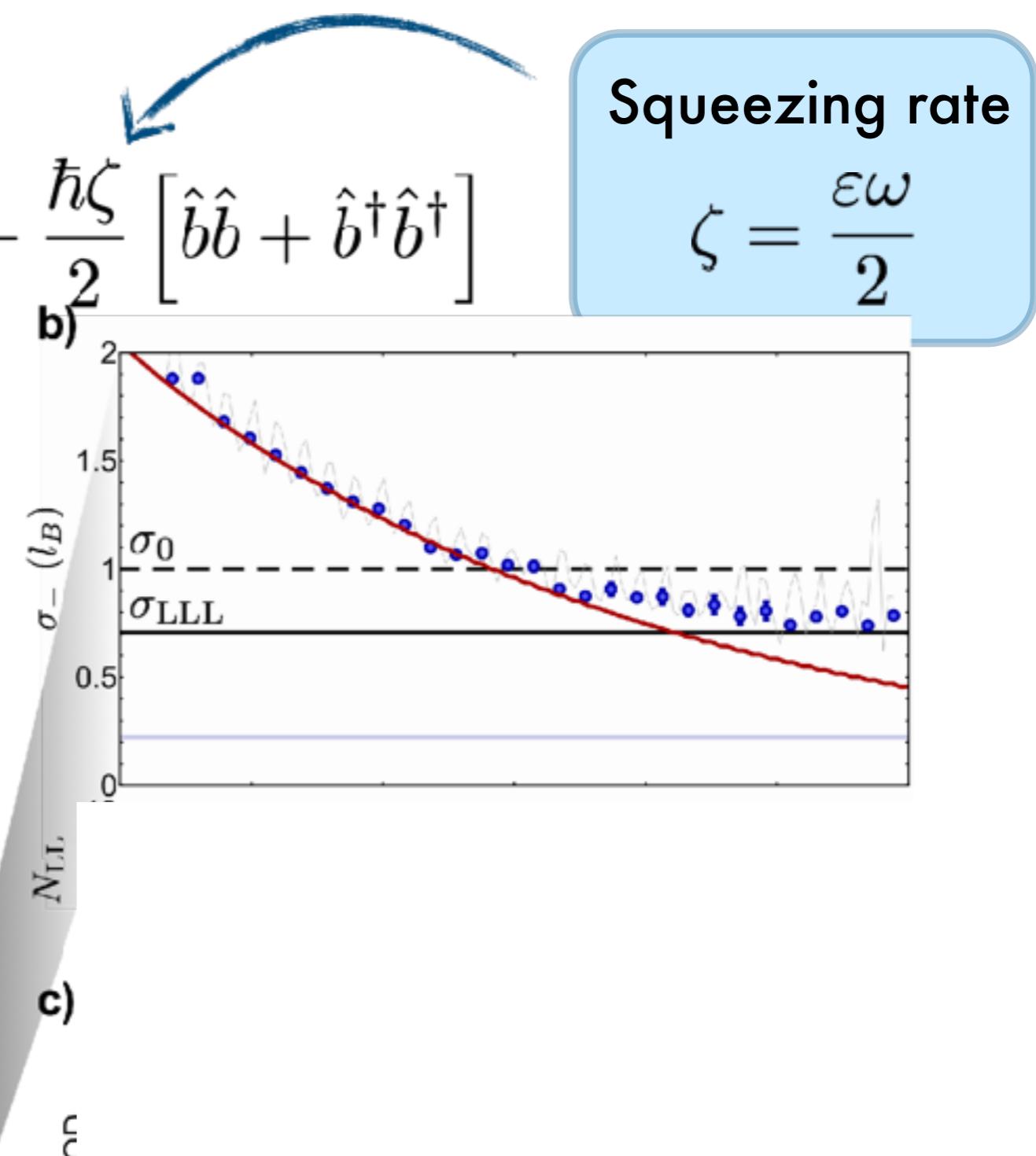
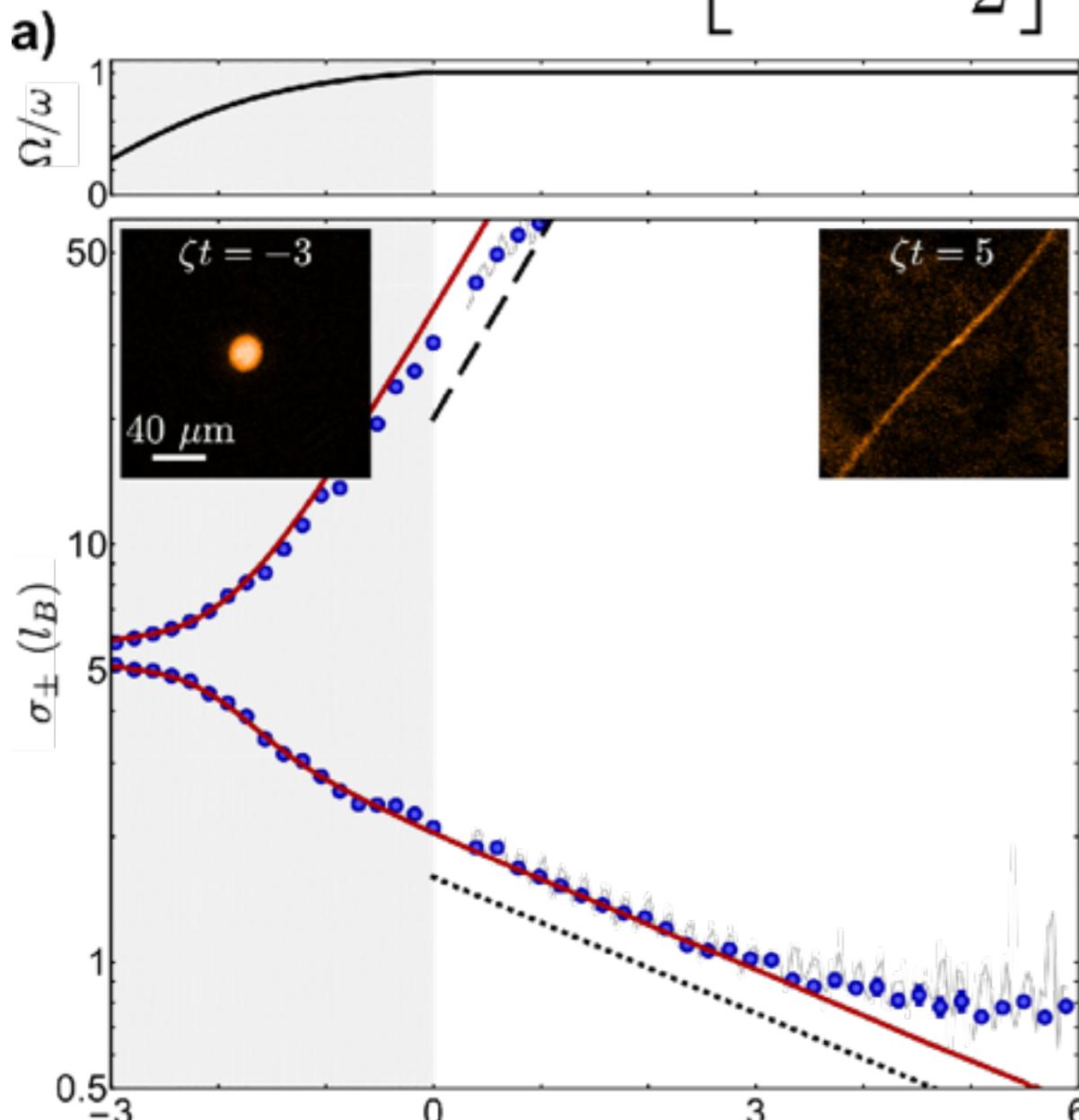
# Geometric squeezing

## Hamiltonian under saddle potential

$$\hat{H} = 2\hbar\omega \left[ \hat{a}^\dagger \hat{a} + \frac{1}{2} \right] + \frac{\hbar\zeta}{2} \left[ \hat{b}^\dagger \hat{b} + \hat{b}^\dagger \hat{b}^\dagger \right]$$

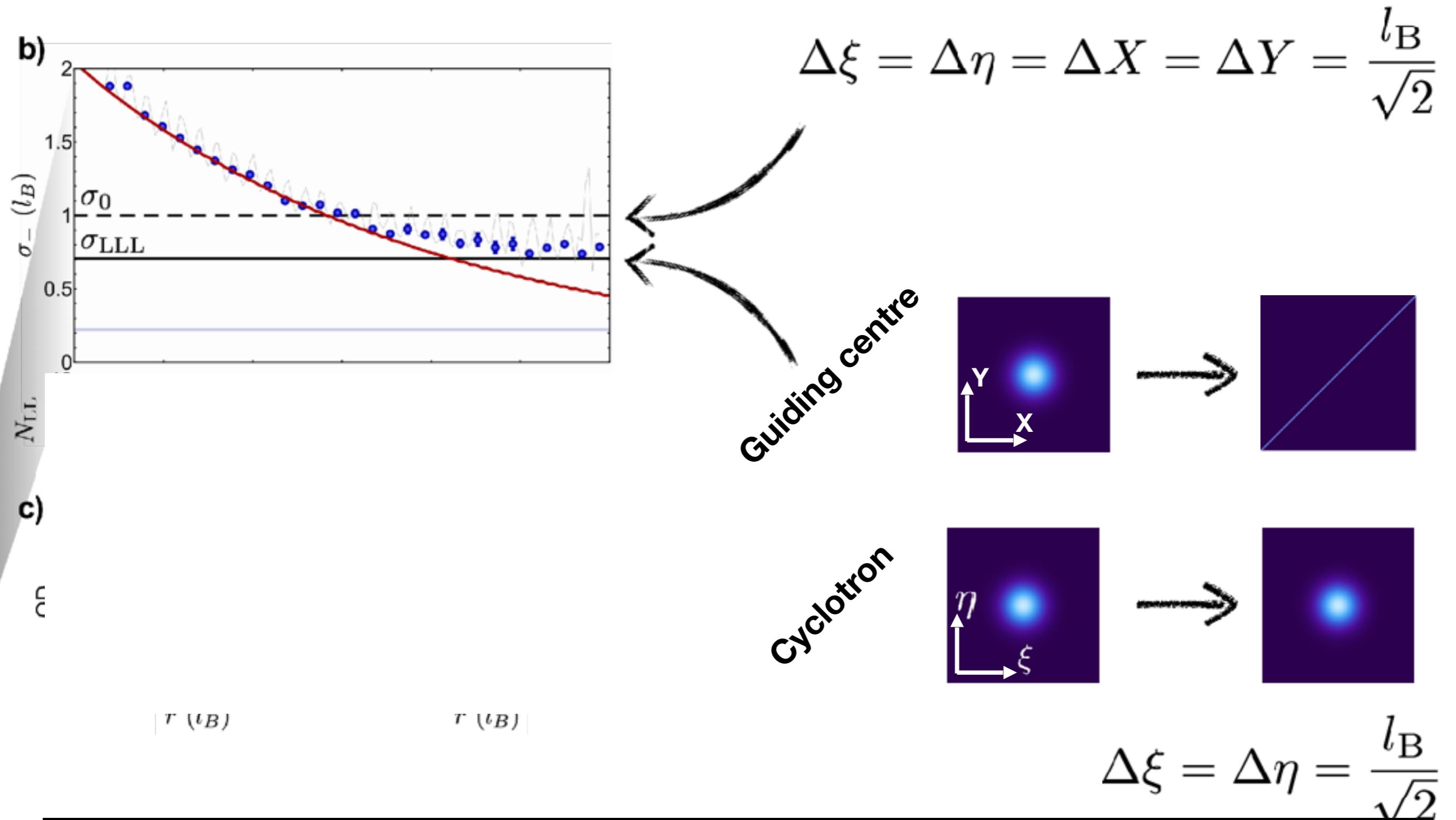
Squeezing rate

$$\zeta = \frac{\varepsilon\omega}{2}$$

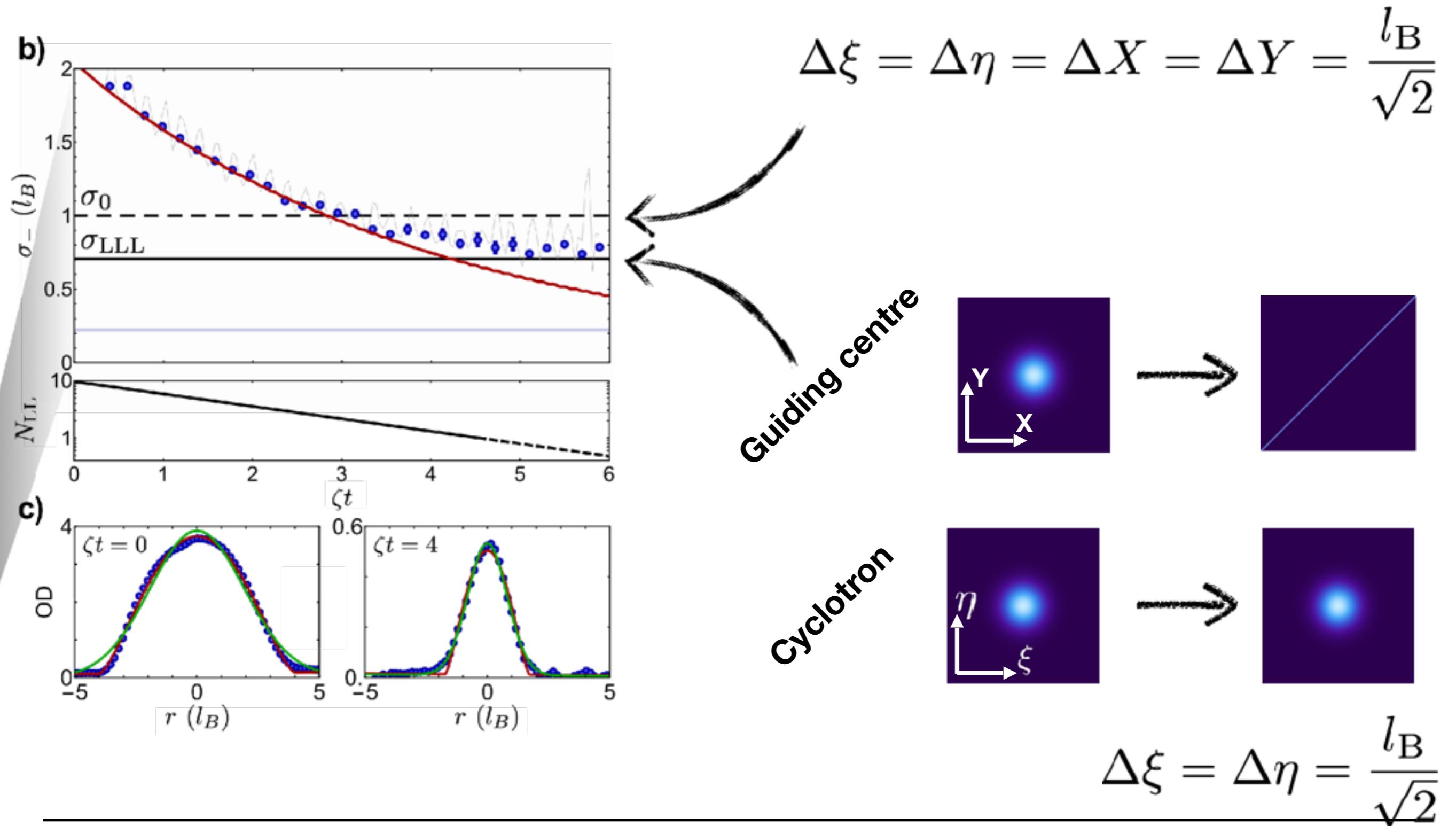


Dynamics and transport in  
strongly-interacting quantum fluids

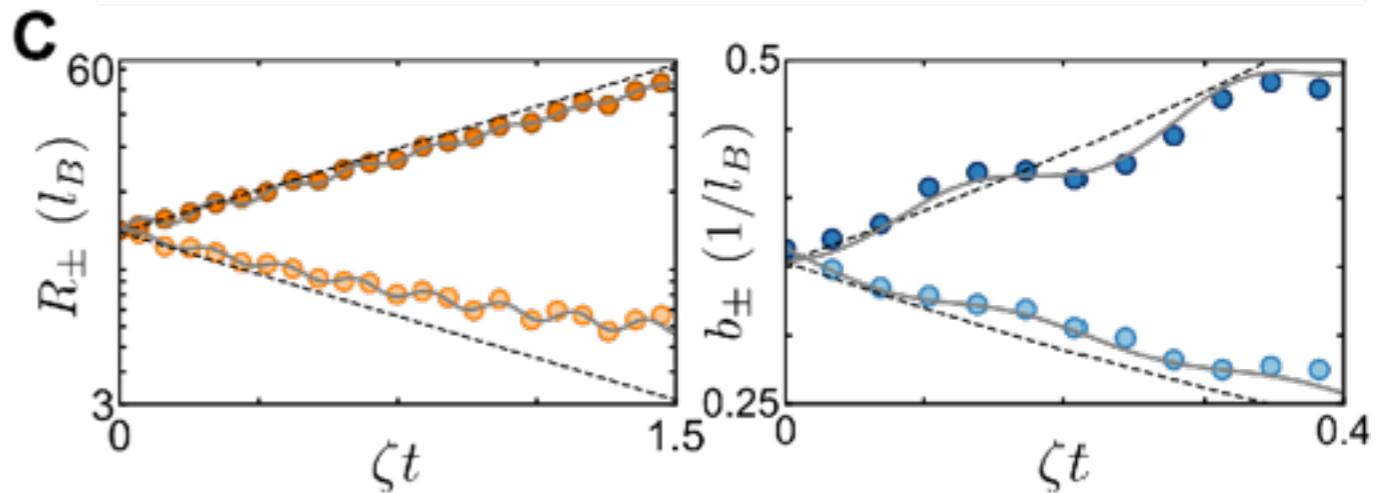
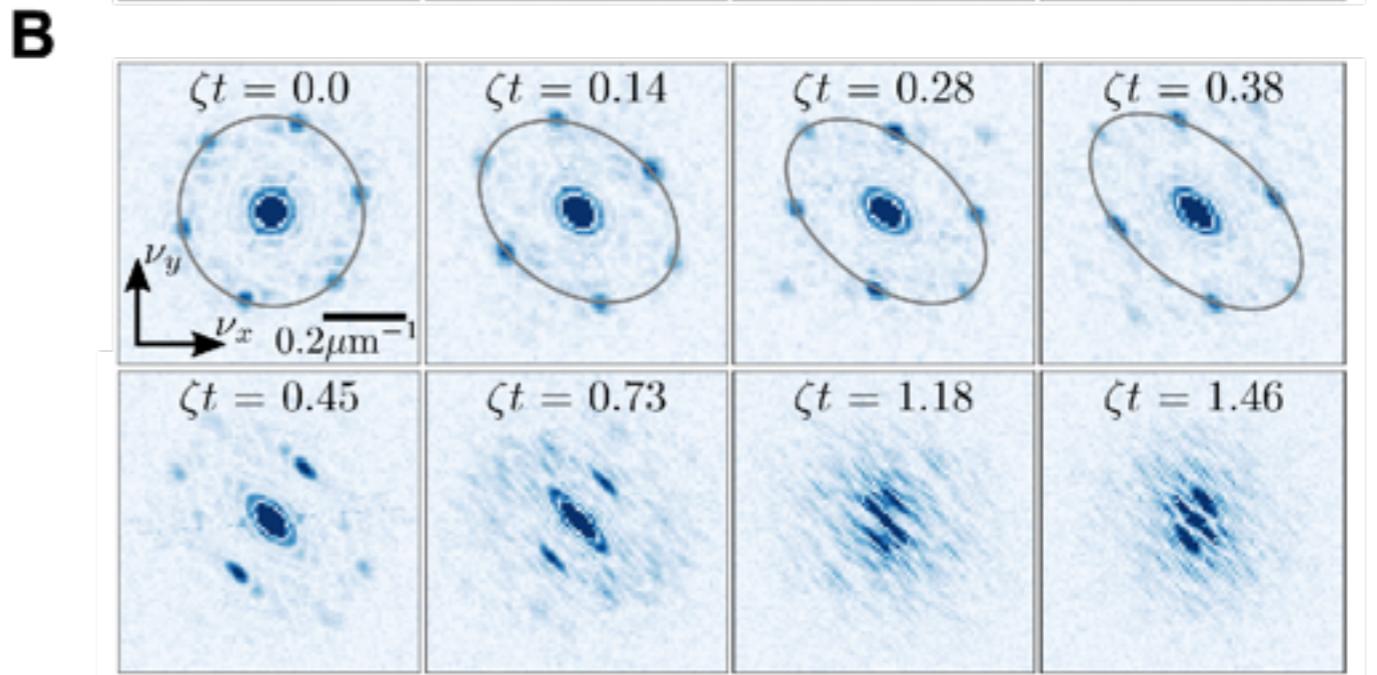
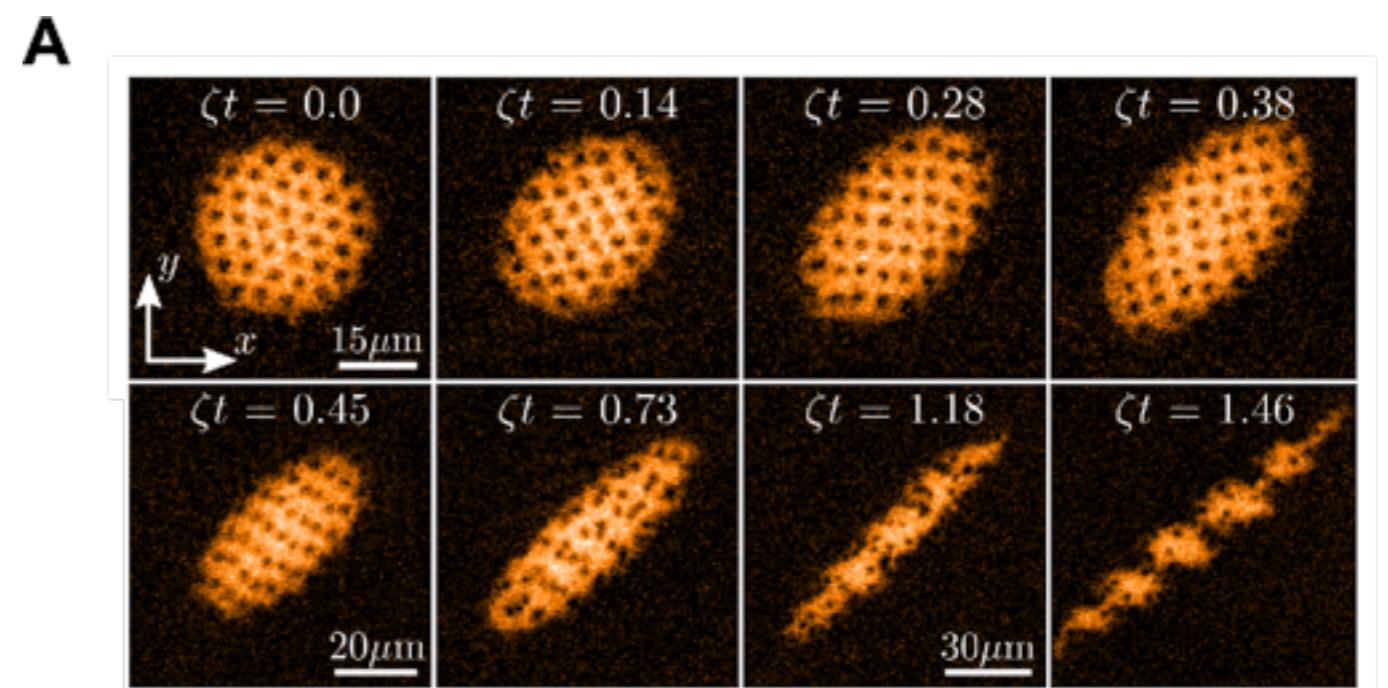
# Geometric squeezing

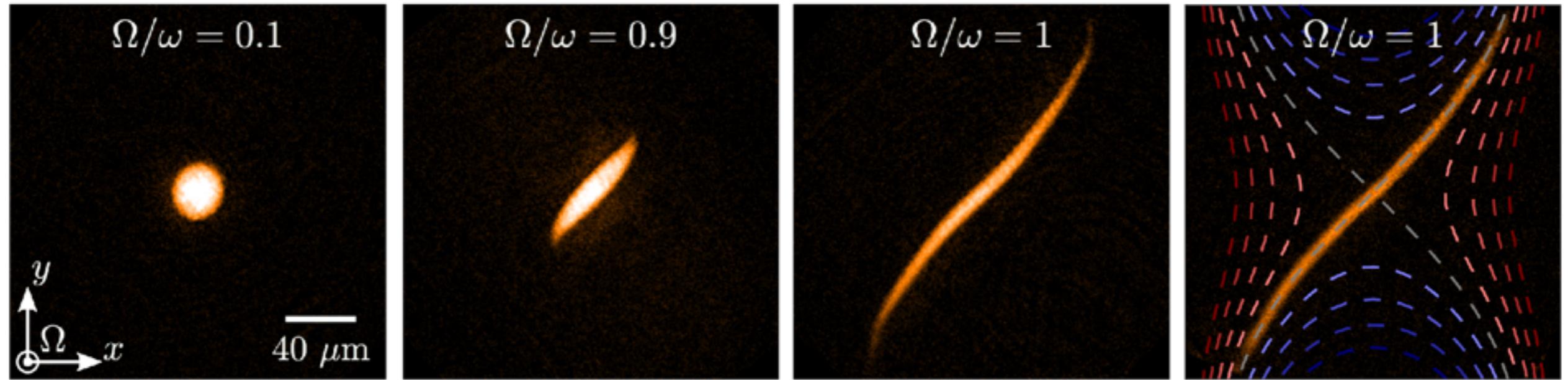


# Geometric squeezing

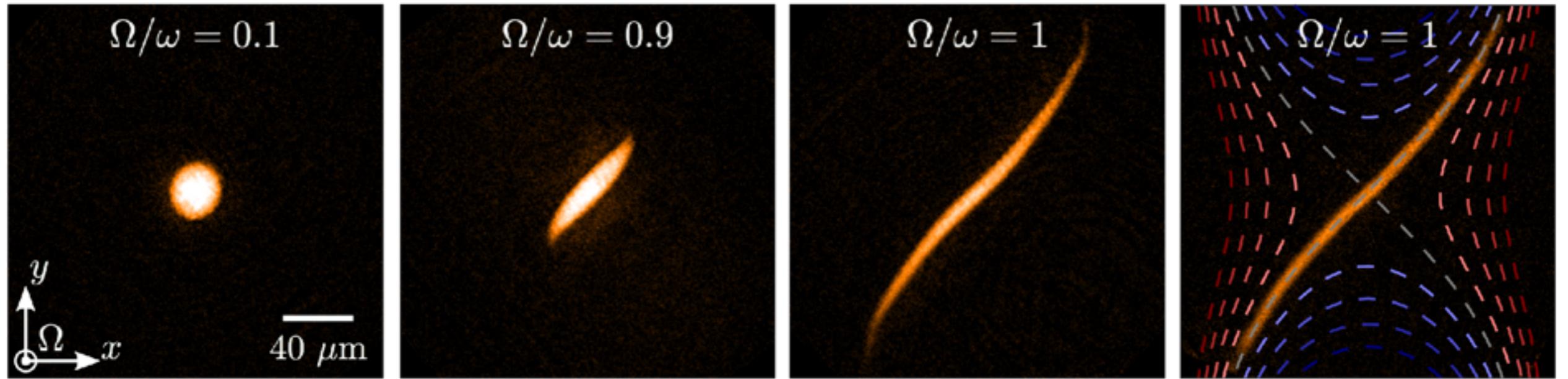


# Geometric squeezing of a vortex lattice





R. Fletcher, A. Shaffer, C. Wilson, P. Patel, Z. Yan, V. Crepel, B. Mukherjee, M. Zwierlein, arXiv:1911.12347 (2019) Science, accepted

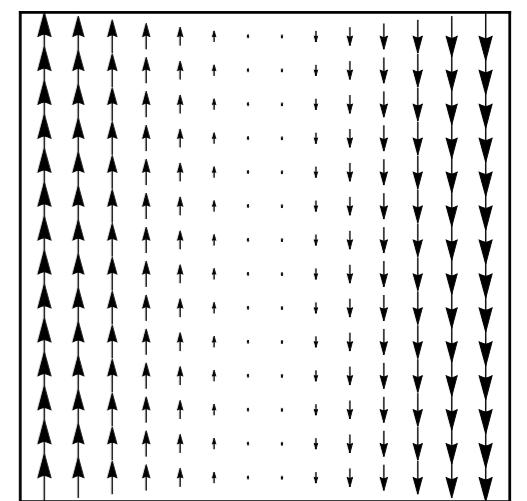
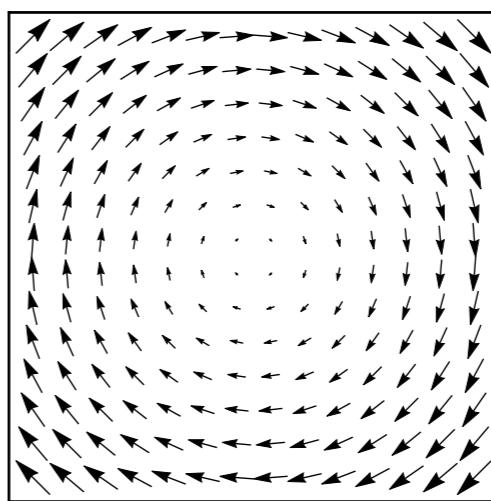


R. Fletcher, A. Shaffer, C. Wilson, P. Patel, Z. Yan, V. Crepel, B. Mukherjee, M. Zwierlein, arXiv:1911.12347 (2019) Science, accepted

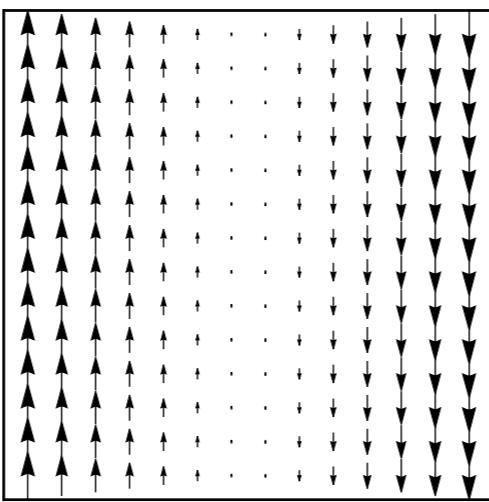
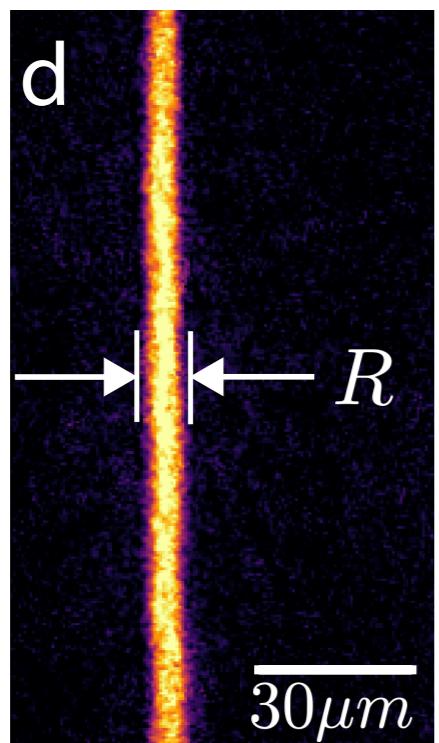
Transformation from a 'symmetric gauge' to 'Landau gauge' condensate

$$\hat{H} = \int d^2r \hat{\Psi}^\dagger \left[ \frac{(\hat{\mathbf{p}} - q\mathbf{A})^2}{2m} + \frac{g}{2} \hat{\Psi}^\dagger \hat{\Psi} \right] \hat{\Psi}$$

$$\mathbf{v} = \frac{1}{m} (\hbar \nabla \theta - q\mathbf{A})$$

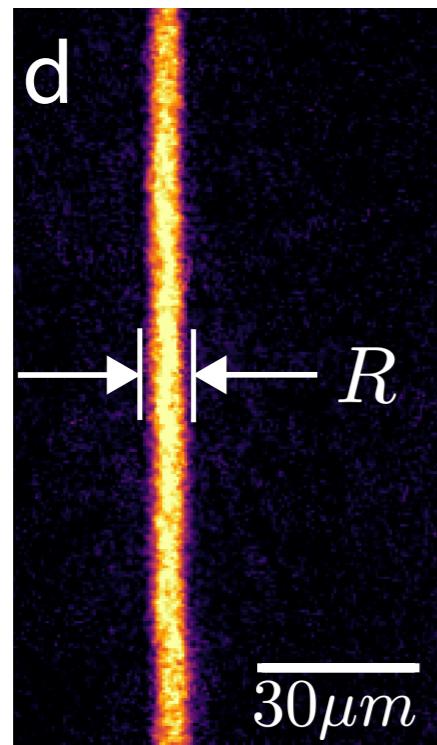


# Landau gauge condensate

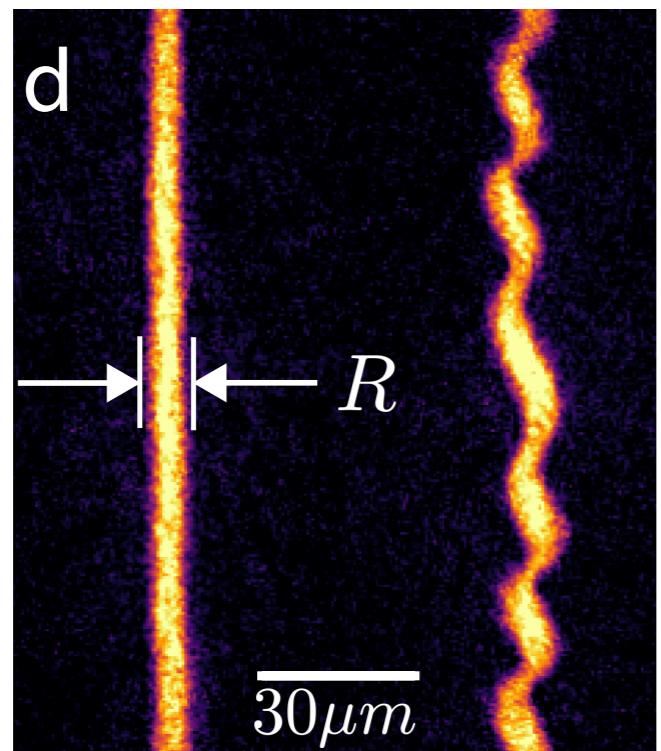


$$\hat{H} = \int d^2r \hat{\Psi}^\dagger \left[ \frac{\hat{p}_x^2}{2m} + 2m\omega^2 \left( \hat{x} - \frac{\hat{p}_y l_B^2}{\hbar} \right)^2 + \frac{g}{2} \hat{\Psi}^\dagger \hat{\Psi} \right] \hat{\Psi}$$

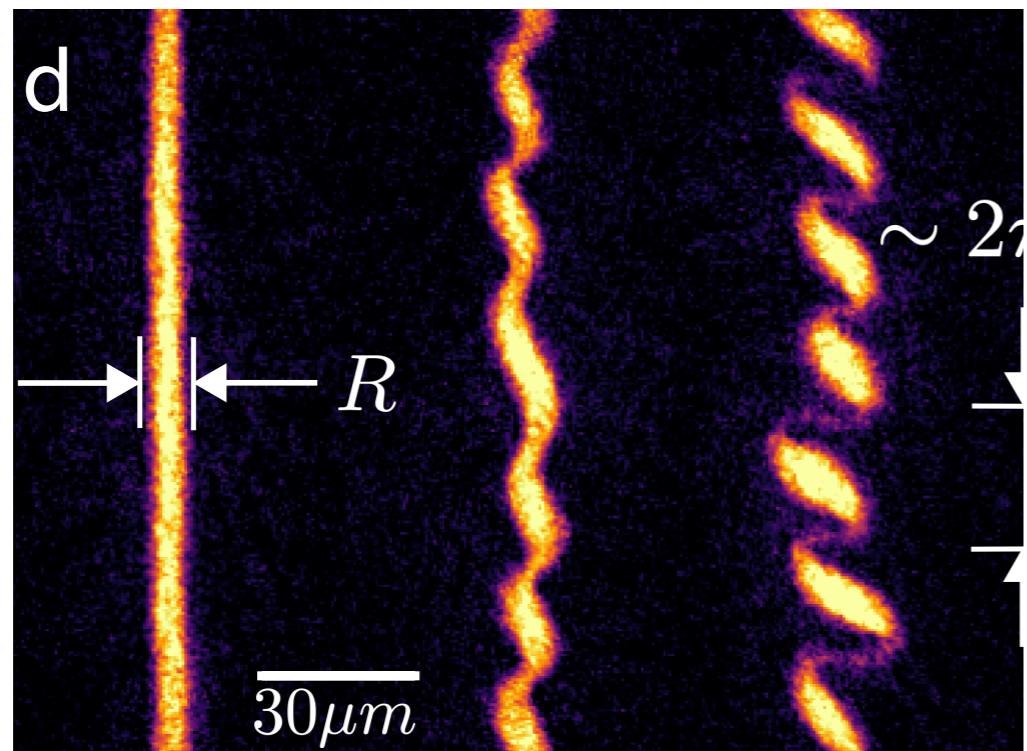
# Interplay of gauge field and interactions



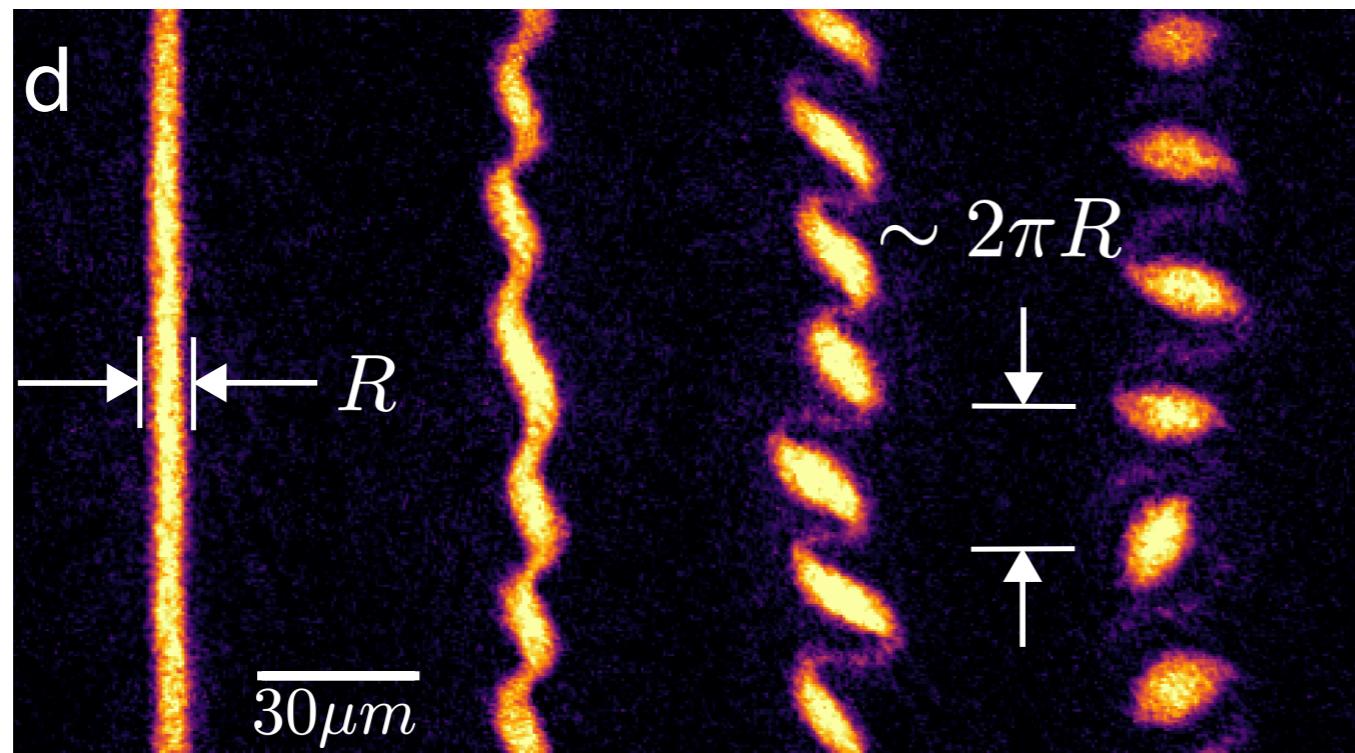
# Interplay of gauge field and interactions



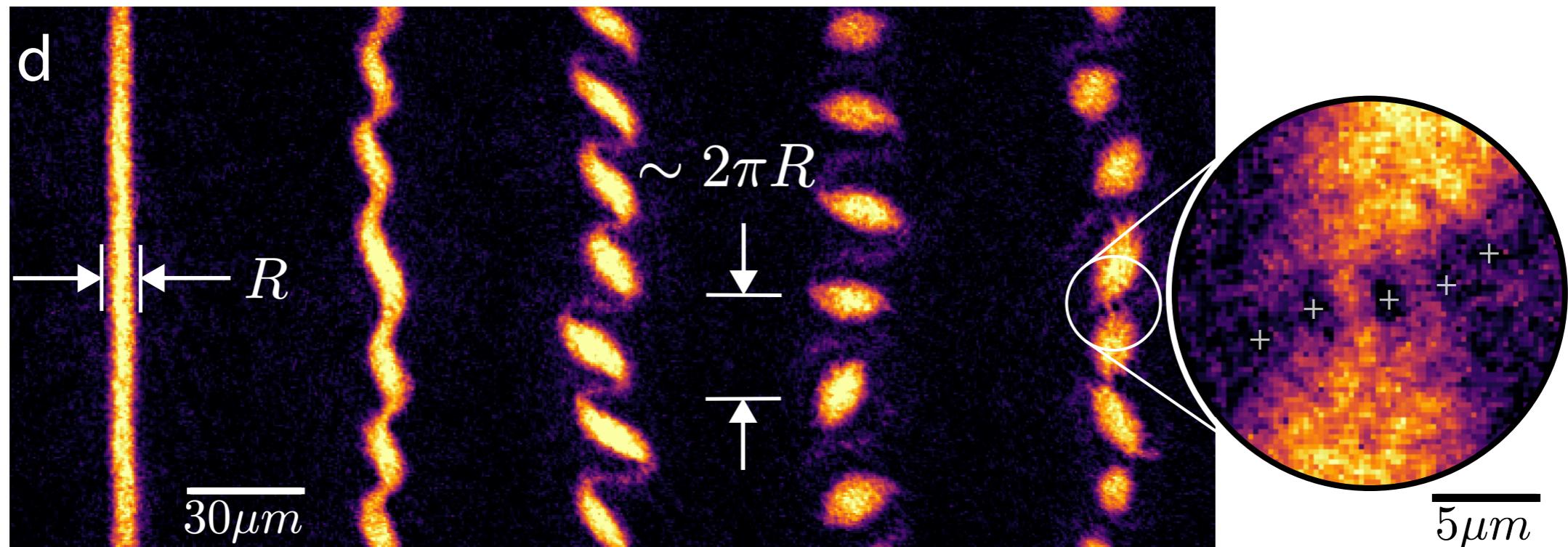
# Interplay of gauge field and interactions



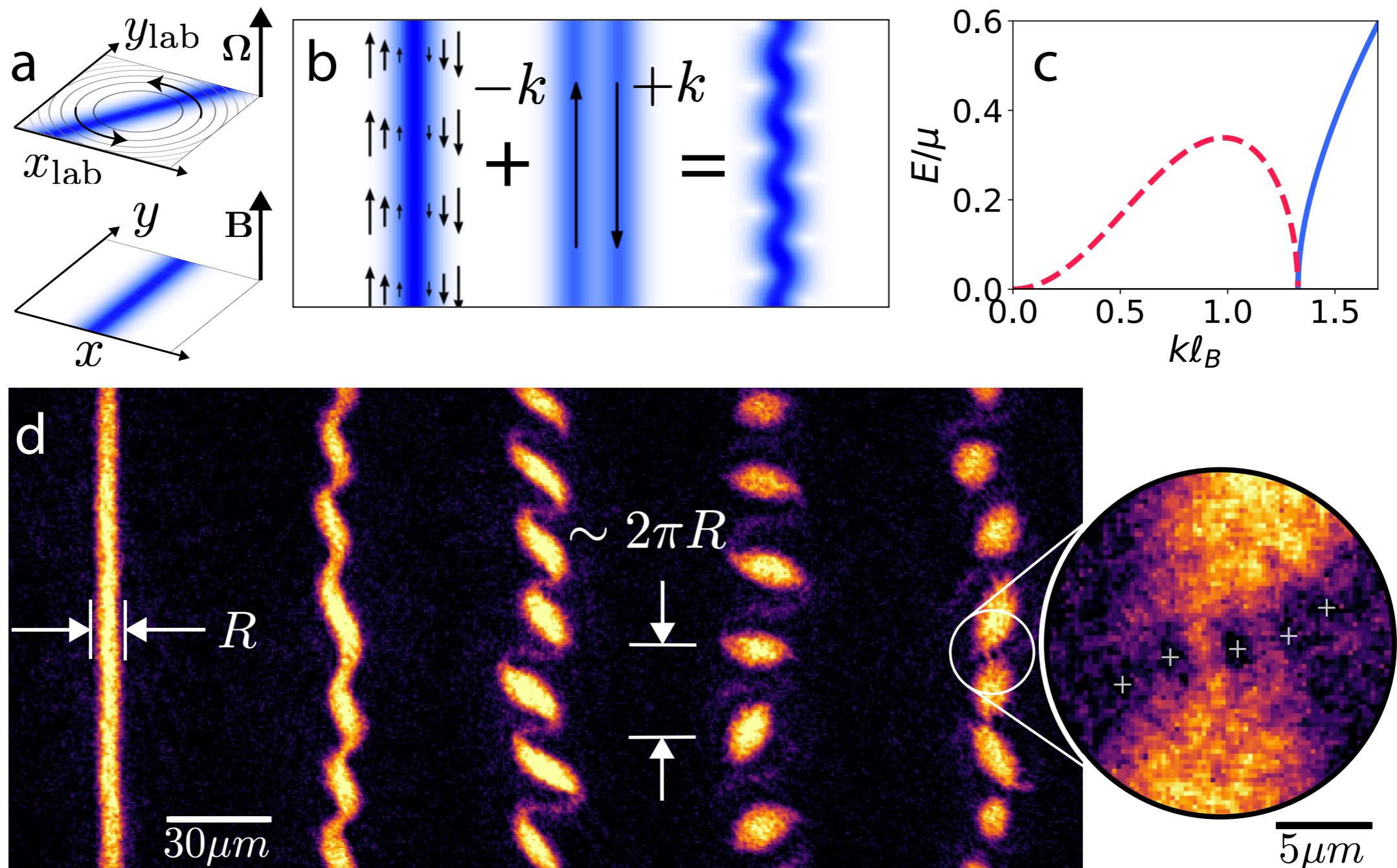
# Interplay of gauge field and interactions



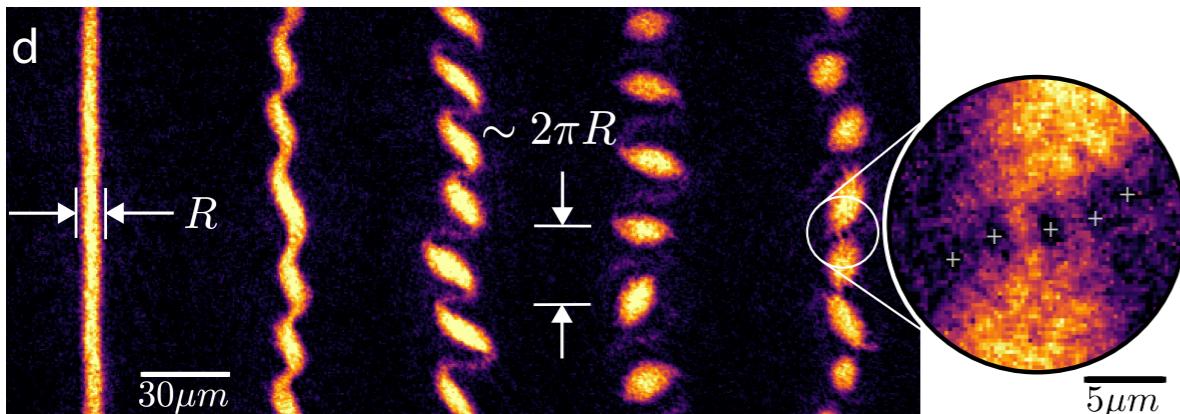
# Interplay of gauge field and interactions



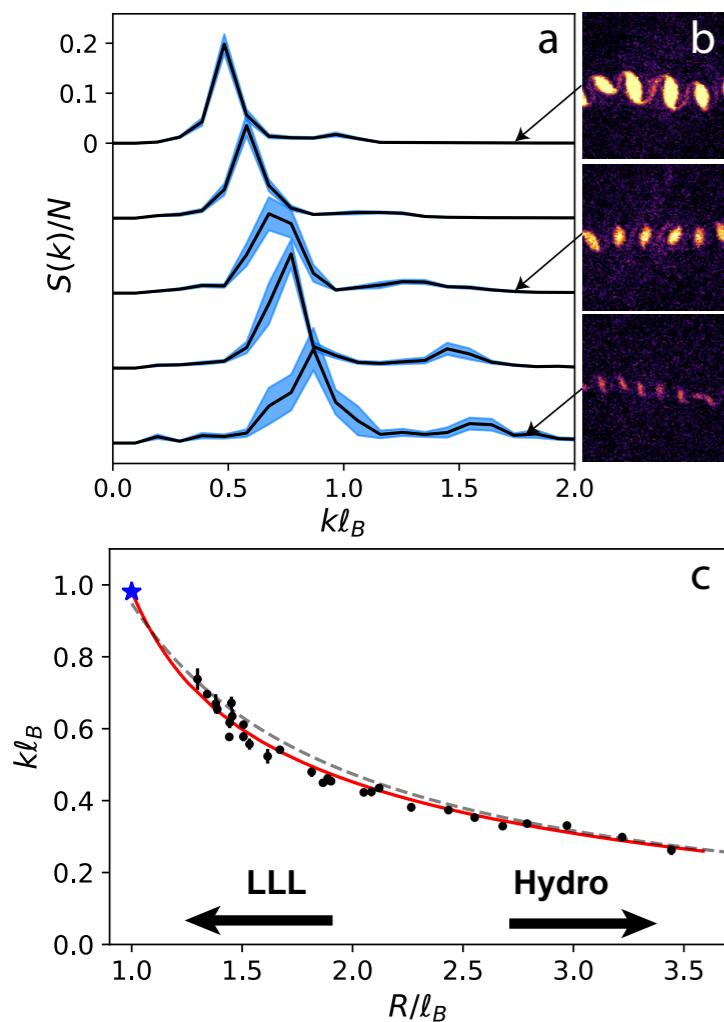
# Interplay of gauge field and interactions



# Instability of the Landau gauge wavefunction

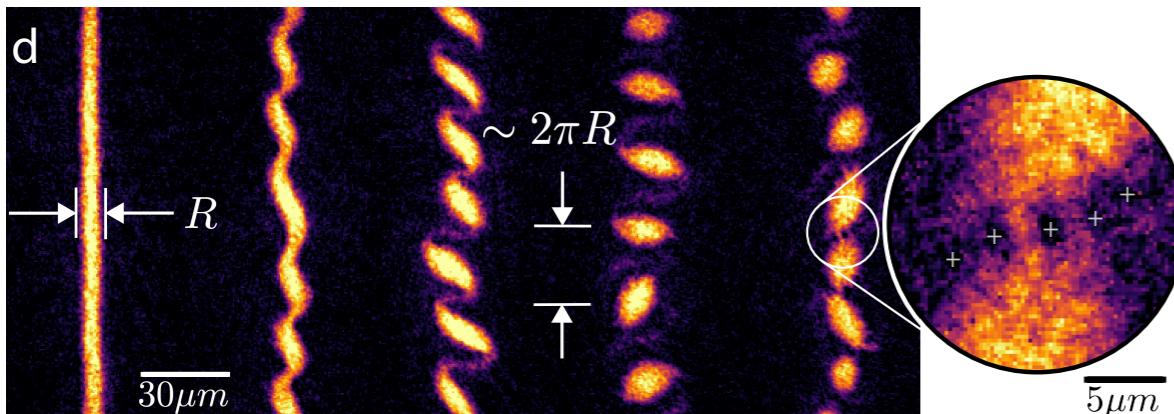


Spontaneous crystallisation of superfluid  
Occurs in the absence of any single-particle dynamics

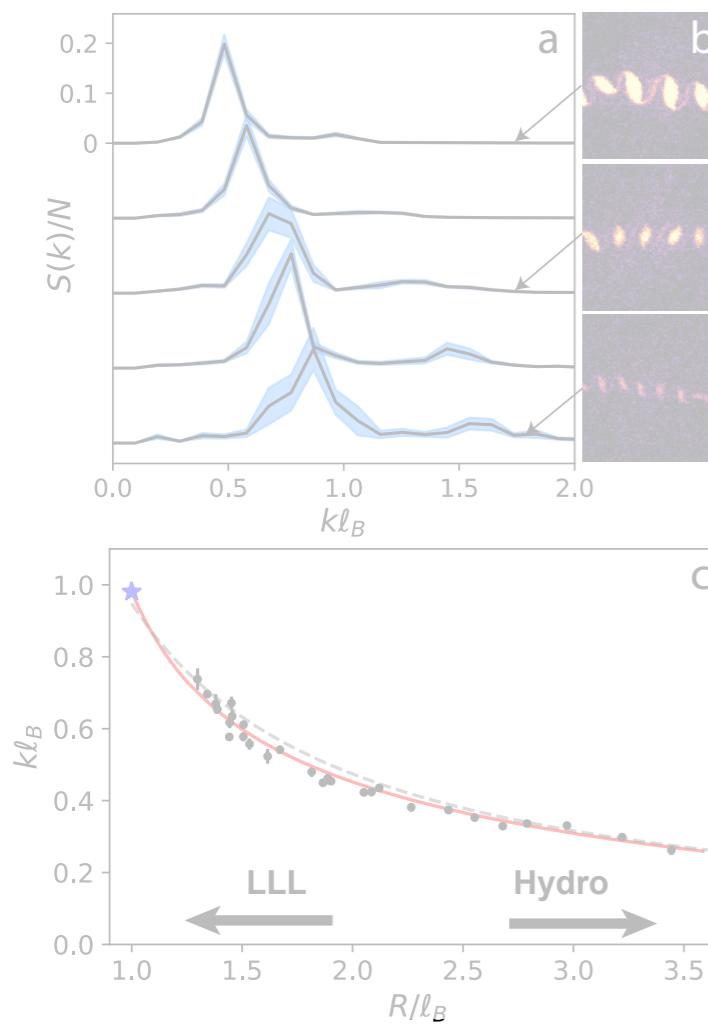


Crystal lengthscale

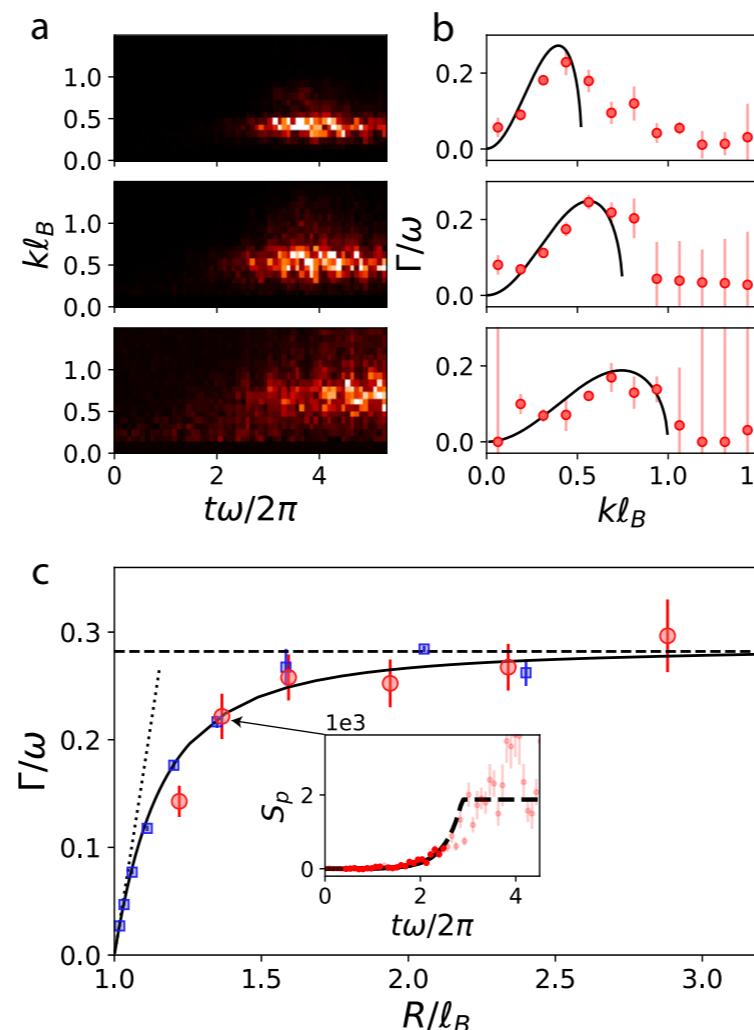
# Instability of the Landau gauge wavefunction



Spontaneous crystallisation of superfluid  
Occurs in the absence of any single-particle dynamics

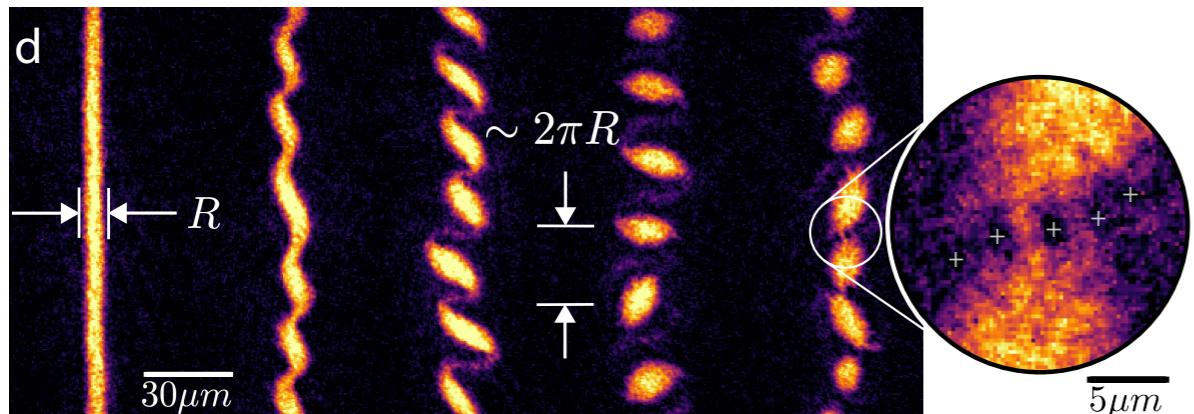


Crystal lengthscale

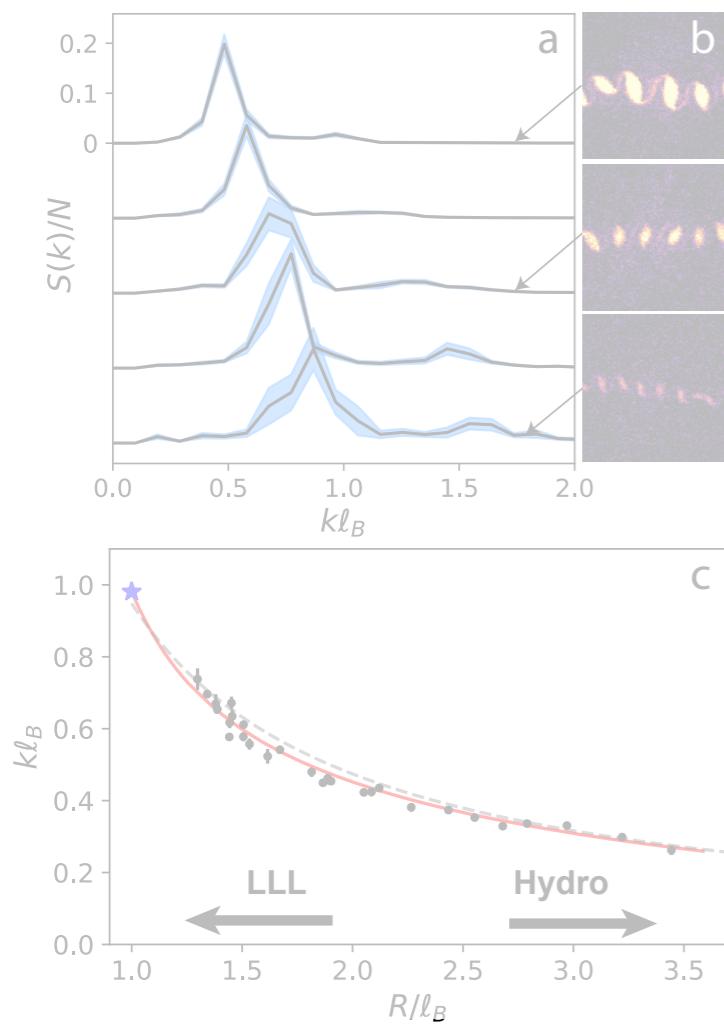


Crossover to LLL dynamics

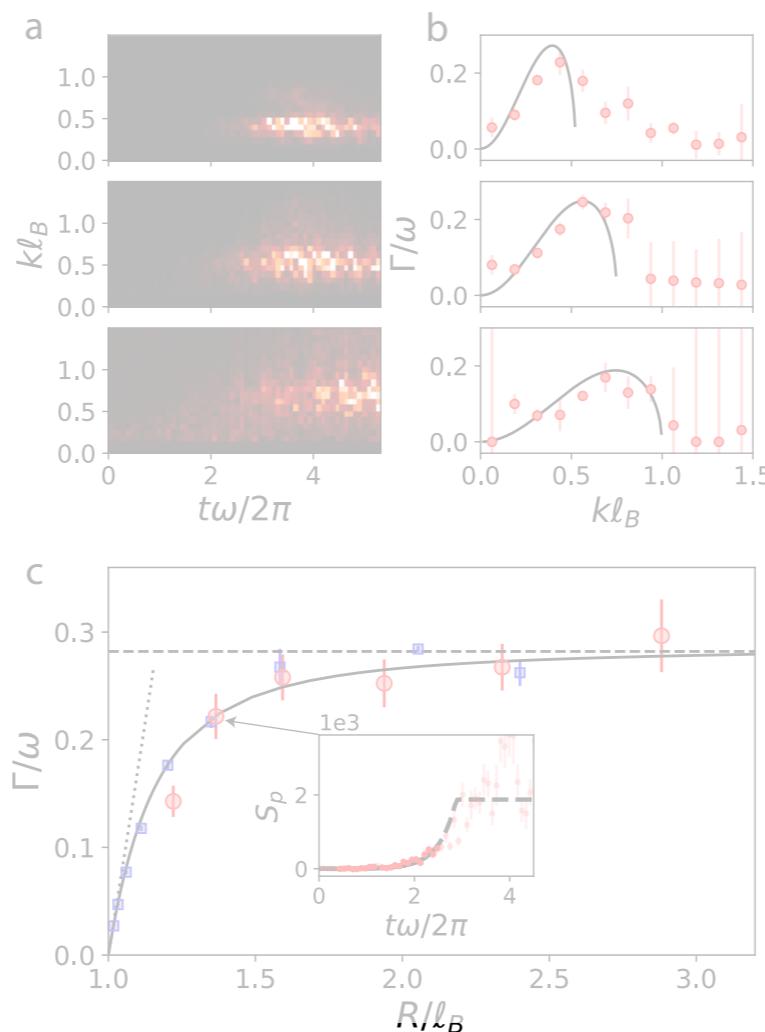
# Instability of the Landau gauge wavefunction



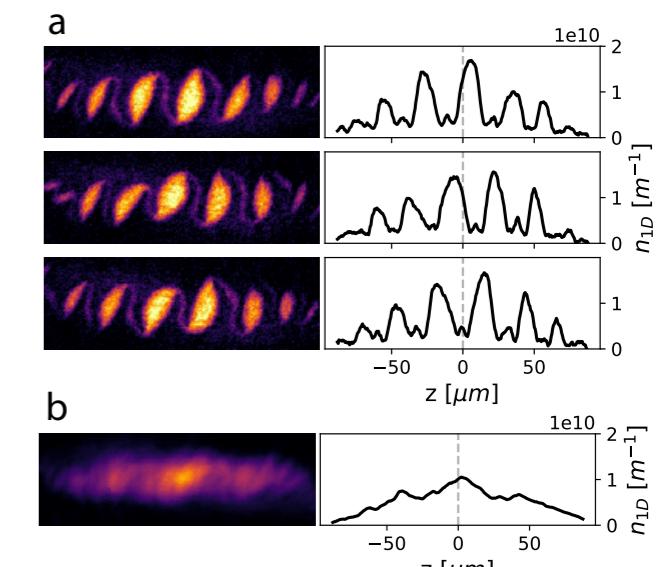
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Crystal lengthscale

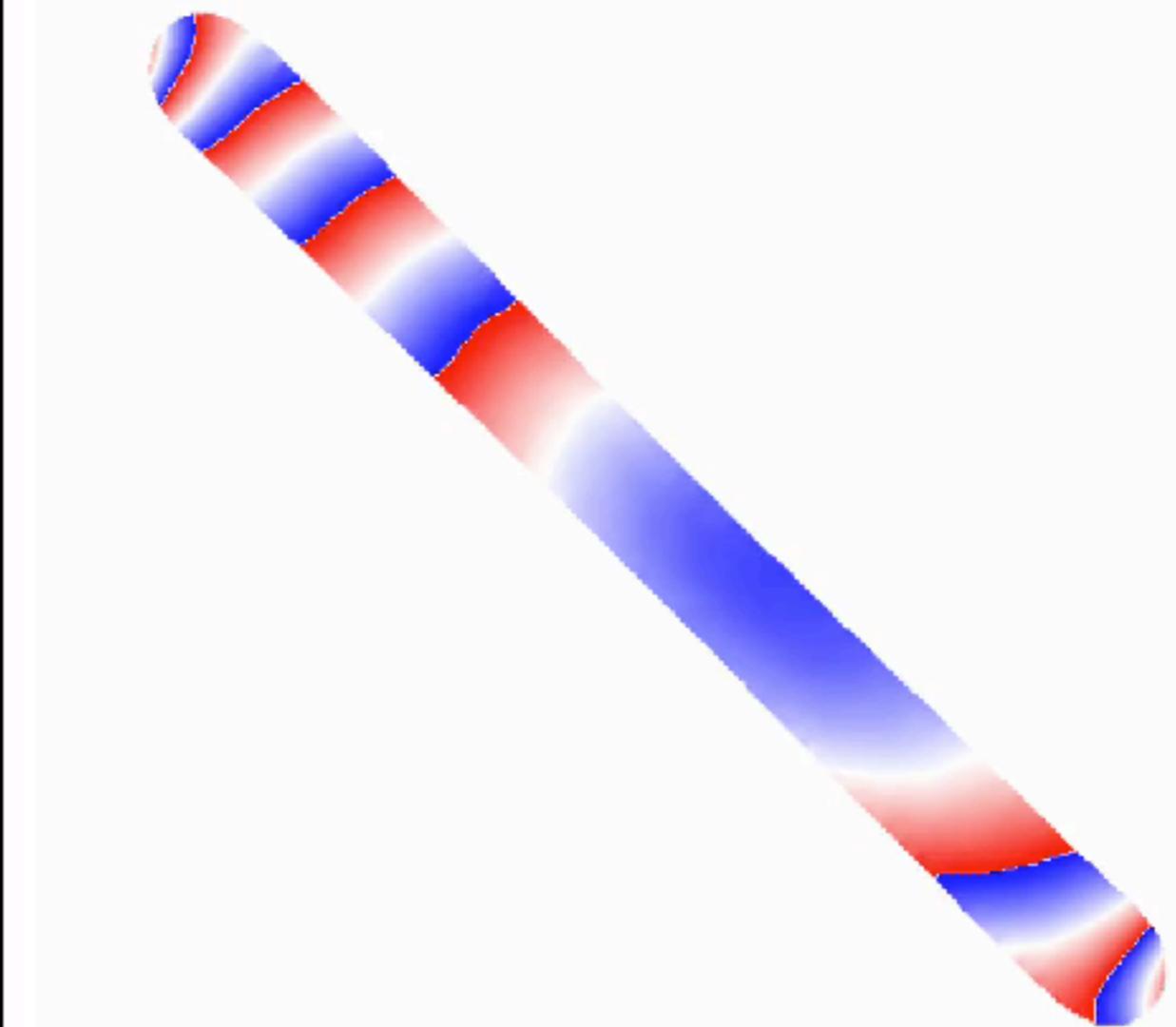
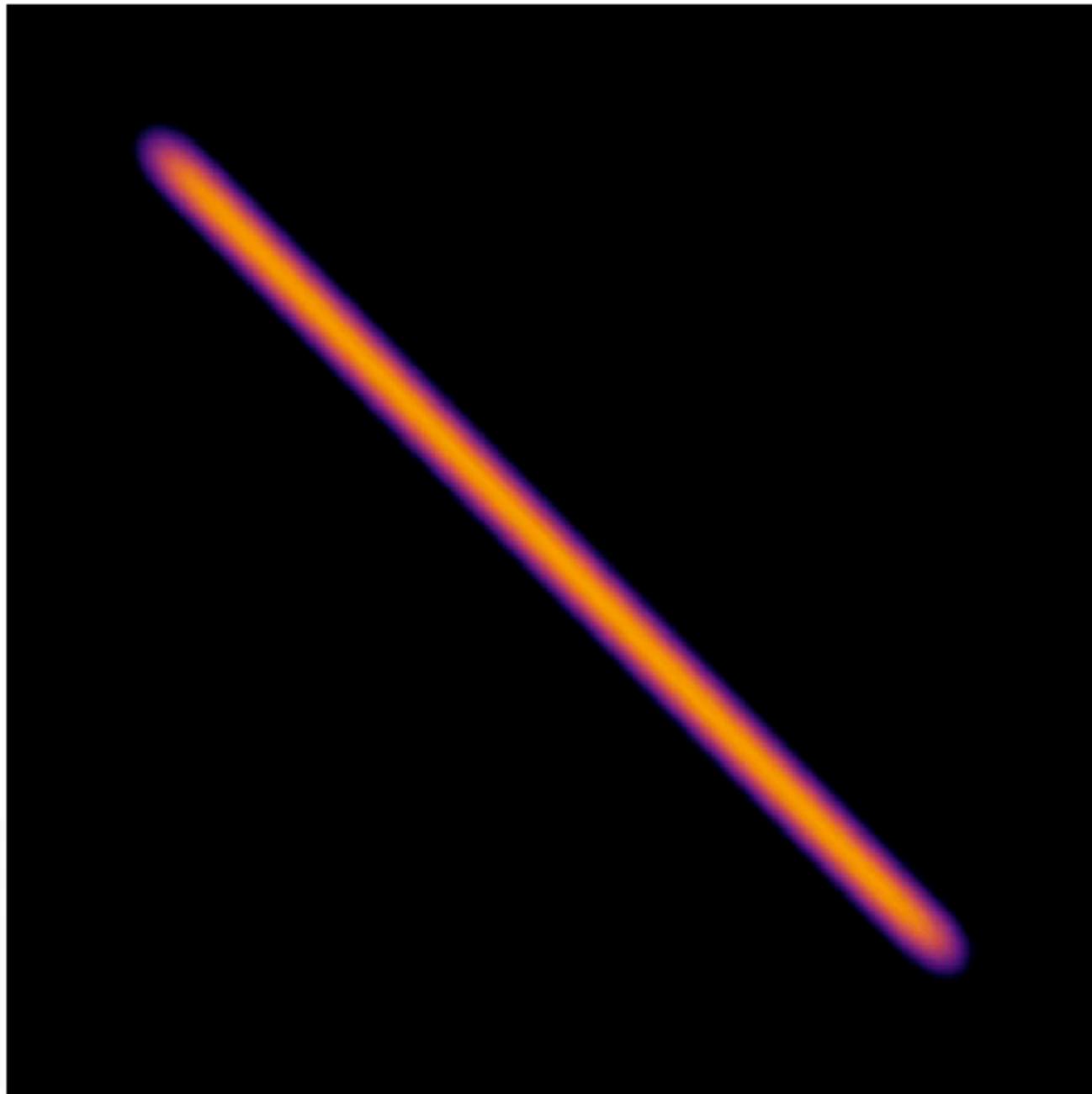


Crossover to LLL dynamics

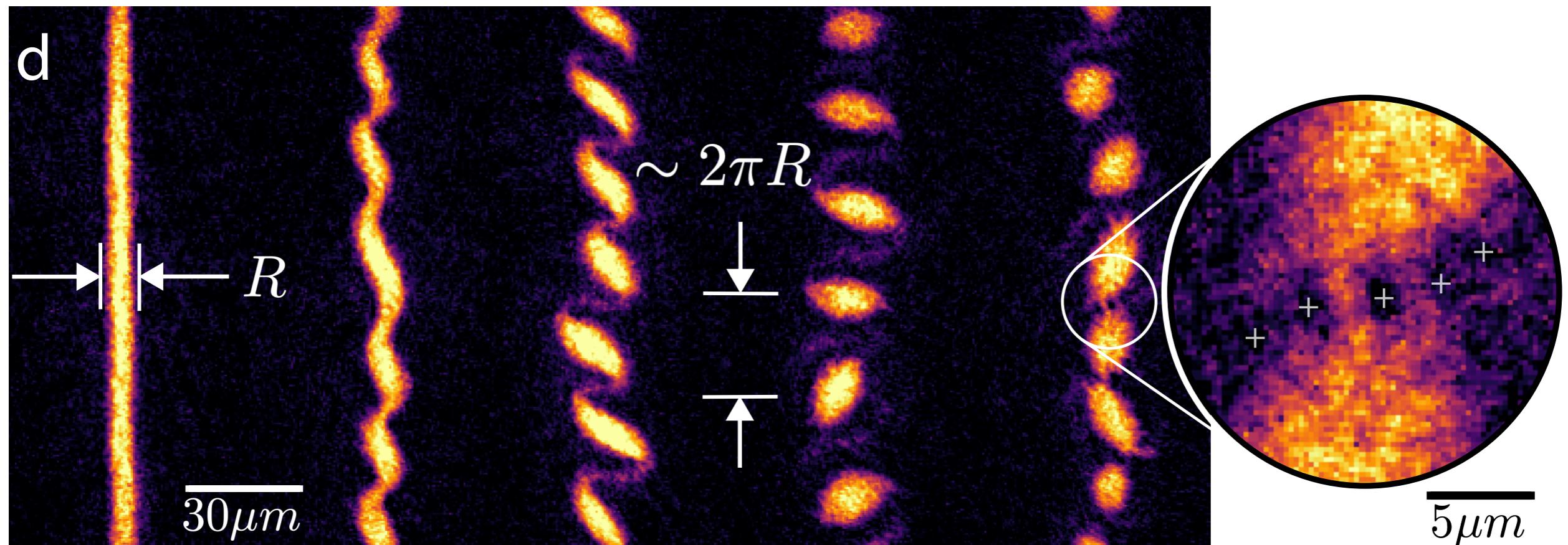


Spontaneous symmetry breaking

# Alternate 'vortex dynamics' view



# Alternate 'vortex dynamics' view



# The team



Parth Patel



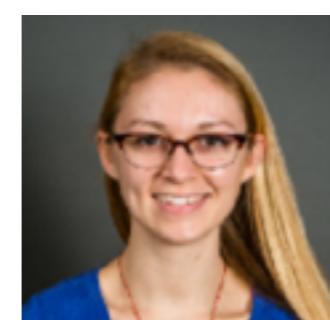
Biswaroop  
Mukherjee



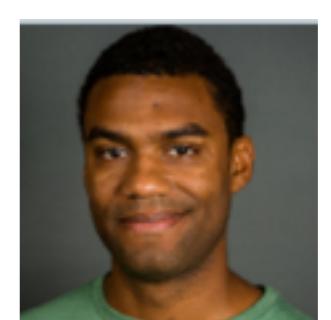
Zhenjie Yan



Dr. Julian  
Struck -> LKB



Airlia Shaffer



Cedric Wilson



Valentin Crepel



+Bola Malek  
Lev Kendrick  
Thatcher Chamberlin



Martin Zwierlein

+ CUA  
community



Richard Fletcher

Dynamics and transport in  
strongly-interacting quantum fluids

EFT methods from bound  
states to binary systems



# The team

From July 2020



Parth Patel



Biswaroop  
Mukherjee



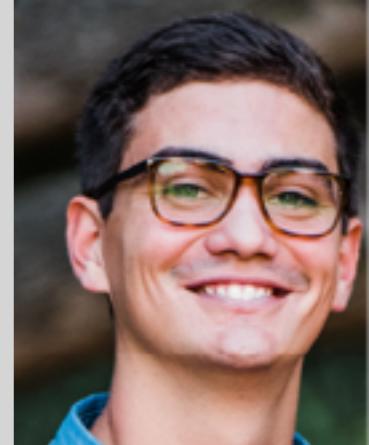
Zhenjie Yan



Dr. J.  
Struck



Jasmine Kalia



Jared Rivera



Martin Zwierlein  
+ CUA  
community



Richard Fletcher

Dynamics and transport in  
strongly-interacting quantum fluids

EFT methods from bound  
states to binary systems



# Thank you!



Richard Fletcher

Dynamics and transport in  
strongly-interacting quantum fluids

EFT methods from bound  
states to binary systems

