Potential NRQE(C)D: Matching (NR)QE(C)D and the Schroedinger equation

Antonio Pineda

Universitat Autònoma de Barcelona & IFAE

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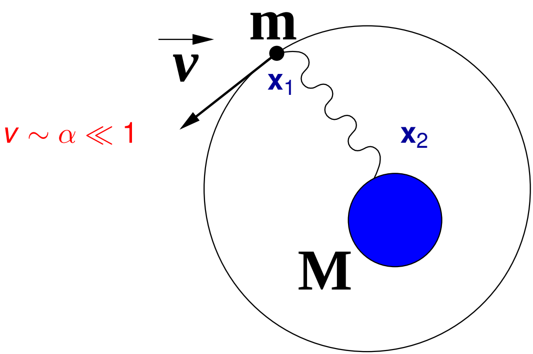

$$\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2 \qquad \mathbf{X} = \frac{m}{m+M}\mathbf{x}_1 + \frac{M}{m+M}\mathbf{x}_2$$

$$H = \frac{\mathbf{p}^2}{2m} + V(r) \qquad V(r) = -\frac{Z_1 Z_2 \alpha}{r}$$

Scales: hard, soft, ultrasoft; $m \gg mv \gg mv^2$...

Motivation

(My) Real Motivation: to understand the connection between non-relativistic (NR) Quantum Mechanics and Quantum Field Theories.
"Physical Systems":
NR (bound state) systems:

- QED: positronium, Hydrogen-like/exotic atoms, atomic physics ...
- QCD: Heavy Quarkonium
- And now gravity!!

Tool: **Effective Field Theories ≡ Factorization**
Why?: There is a hierarchy of different scales (hard, soft and ultrasoft).

$$m \gg mv \gg mv^2 \,, \quad (\Lambda_{QCD})$$

EFTs are especially useful in these situations.
1) Perturbative calculations much easier and systematic.
2) Nonperturbative information is parameterized in a model independent way.
3) Power counting.

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Index

1) Introduction

A) Kinematical regime: *s* plane

B) Warming up: Matching (NR)QFT with a Quantum mechanics description

2) Matching QCD to NRQCD
Relativistic Feynman diagrams $\leftarrow$

3) Matching NRQCD to pNRQCD (getting the potential)
Non-Relativistic (HQET-like) Feynman diagrams $\leftarrow$

4) Observable: Spectrum

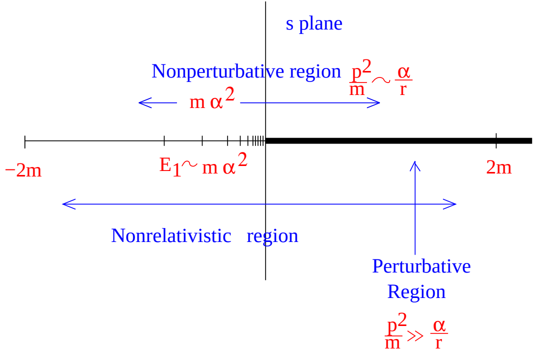
A) Quantum mechanics perturbation theory $\leftarrow$

B) Ultrasoft loops (lamb shift) $\leftarrow$

5) Positronium spectrum

Exercise: Compute the $O(m\alpha^5)$ correction to the spectrum and some $O(m\alpha^6)\times$logs corrections

kinematical situation



1st approximation: ∞ number of NR (bound states) free particles
Unusual situation in EFTs. What we will get is somewhat unusual from the EFT point of view.

$$\mathcal{L} = \sum_n \psi_n^\dagger(\mathbf{X}, t)\left(i\partial_0 + \frac{\boldsymbol{\nabla}_X^2}{2M} - E_n + i\epsilon\right)\psi_n(\mathbf{X}, t)$$

$\psi_n(\mathbf{X})$ represents the quark-antiquark bound state

Our case

$$\mathcal{L} = \sum_n \psi_n^\dagger(\mathbf{X}, t)(i\partial_0 + \frac{\boldsymbol{\nabla}_X^2}{2M} - E_n + i\epsilon)\psi_n(\mathbf{X}, t)$$

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Path integral formulation:

$$Z = \int \Pi D\psi_n^\dagger D\psi_n e^{i\int d^4X(\mathcal{L}+\psi_n^\dagger\eta_n+\eta_n^\dagger\psi_n)}$$

Connection with quantum mechanics (?):

particle-antiparticle wave function: $\Psi(\mathbf{X}, \mathbf{x}) = \Psi_{\mathbf{x}}(\mathbf{X})$

$\mathbf{X}$ = Center of mass coordinate

$\mathbf{x}$ = relative coordinate

Ansatz: Promote $\Psi(\mathbf{X}, \mathbf{x})$ to a field

$$Z = \int D\Psi(X, x)^\dagger D\Psi(X, x) e^{i\int d^4X d^3\mathbf{x}(\mathcal{L}+\Psi^\dagger J(X,x)+J^\dagger(X,x)\Psi)}$$

$$\mathcal{L} = \Psi^\dagger(i\partial_0 - \hat{h} + i\epsilon)\Psi$$

where

$$\hat{h} = \hat{h}_{\mathbf{X}} + \hat{h}_{\mathbf{x}}\,, \quad \hat{h}_{\mathbf{X}} = -\frac{\boldsymbol{\nabla}_X^2}{2M}\,, \quad \hat{h}_{\mathbf{x}} = -\frac{\boldsymbol{\nabla}_x^2}{2\mu_r} + V(\mathbf{x})$$

We have traded E_n for $V(\mathbf{x})$. The point is that we will able to relate $V(\mathbf{x})$ with some Green functions in the underlying theory and in some kinematical regime to compute it perturbatively.
Change of basis: $\Psi(\mathbf{X},\mathbf{x}) = \sum_n \phi_n(\mathbf{x})\psi_n(\mathbf{X})$,
where $\phi_n(\mathbf{x})$ is a function and $\psi_n(\mathbf{X})$ a field and

$$\hat{h}_{\mathbf{x}}\phi_n(\mathbf{x}) = E_n\phi_n(\mathbf{x})$$

$$Z = N\int \Pi D\psi_n^\dagger D\psi_n e^{i\int d^4X(\sum_n \psi_n^\dagger(i\partial_0+\frac{\boldsymbol{\nabla}_X^2}{2M}-E_n+i\epsilon)\psi_n+\int d^3x\sum_n(\phi_n^*\psi_n^\dagger J+J^\dagger\phi_n\psi_n))}$$

$$\int d^3x\phi_{n'}^*\hat{h}_{\mathbf{x}}\phi_n = \delta_{nn'}E_n$$

$$\int d^3x\phi_{n'}^*\phi_n = \delta_{nn'}$$

We have closed the connection between both formulations
Infinite number of states $\leftrightarrow$ Integro-diferential equation (Schrödinger equation)

So...

We want to obtain an effective field theory, Potential Non-Relativistic QCD, which describes the heavy quarkonium dynamics and profits from the hierarchy $m \gg mv \gg mv^2$

$$\left. \begin{array}{l} \left(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_s^{(0)}(r)\right)\Phi(\mathbf{r}) = 0 \\ +\text{corrections to the potential} \\ +\text{interaction with other low} \\ \quad \text{energy degrees of freedom} \end{array} \right\} \text{potential NRQCD} \qquad E \sim mv^2$$

The starting point is $V_s^{(0)} = -C_f \frac{\alpha_s}{r}$.

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How to get there?

We will introduce a hierarchy of EFTs when sequentially integrating out each scale (only one scale in each step, strong simplification).

QCD

↓ Integrating out the hard scale (m)

NRQCD — Caswell, Lepage

↓ Integrating out the soft scale (mv)

pNRQCD — Soto, Pineda

NRQCD: the scale m

- Degrees of freedom
- Symmetries
- Cutoff

NRQCD has an ultraviolet cutoff Λ such that $m \gg \Lambda \gg$ any other dynamical scale in the problem. $\Psi = \psi + \chi$

$$\mathcal{L}_{NRQCD} = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi} \left\{ \frac{\mathbf{D}^2}{2m} + c_F\, g \frac{\mathbf{\Sigma} \cdot \mathbf{B}}{2m} + c_D\, g \frac{\gamma^0 (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D})}{8m^2} \right.$$

$$\left. + i c_S\, g \frac{\gamma^0 \mathbf{\Sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m^2} + \frac{\mathbf{D}^4}{8m^3} \right\} \Psi$$

$$-\frac{1}{4} c_1 F_{\mu\nu} F^{\mu\nu} + \frac{d_2}{m^2} g F_{\mu\nu} D^2 g F^{\mu\nu} + \frac{d_3}{m^2} g^3 f_{ABC} F^A_{\mu\nu} F^B_{\mu\alpha} F^C_{\nu\alpha}$$

$$\delta\mathcal{L}_{NRQCD} = \frac{d_{ss}}{m_1 m_2} \psi_1^\dagger \psi_1 \chi_2^\dagger \chi_2 + \frac{d_{sv}}{m_1 m_2} \psi_1^\dagger \boldsymbol{\sigma} \psi_1 \chi_2^\dagger \boldsymbol{\sigma} \chi_2$$

$$+ \frac{d_{vs}}{m_1 m_2} \psi_1^\dagger \mathrm{T}^a \psi_1 \chi_2^\dagger \mathrm{T}^a \chi_2 + \frac{d_{vv}}{m_1 m_2} \psi_1^\dagger \mathrm{T}^a \boldsymbol{\sigma} \psi_1 \chi_2^\dagger \mathrm{T}^a \boldsymbol{\sigma} \chi_2 \,.$$

Caswell, Lepage, Thacker

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NRQED: the scale m

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Matching QCD to NRQCD: the scale m

$c_i = 1 + O(\alpha_s)$, $c_1 = 1 + O(\alpha_s^2)$ (relevant α_s at low energies), $d's = O(\alpha_s)$.

$$c_i \sim 1 + \alpha_s \left(A \log \frac{m}{\mu} + B \right) \qquad d_i \sim \alpha_s \left(1 + \alpha_s \left(A \log \frac{m}{\mu} + B \right) \right)$$

One major problem: Matching QCD to NRQCD with dimensional regularization.

Solution: Teach dimensional regularization that

$$m >> |\mathbf{p}|, E, \Lambda_{QCD}$$

HOW?

Analytical expansion over the three-momentum and residual energy in the integrand before the integration is made in both the full and the effective theory.

QCD

$$\int d^4q f(q, m, |\mathbf{p}|, E) = \int d^4q f(q, m, 0, 0) + O\left(\frac{E}{m}, \frac{|\mathbf{p}|}{m}\right)$$

NRQCD

$$\int d^4q f(q, |\mathbf{p}|, E) = \int d^4q f(q, 0, 0) = 0\,!!$$

Dimensional regularization. The computation of loops in the effective theory just gives zero.

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Final rules:

- QCD tree level Feynman diagrams $\rightarrow$ non-relativistic reduction. Leading contribution to the NRQCD matching coefficients (equivalent to perform a Foldy-Wouthysen transformation for the bilinear piece of the Lagrangian plus the annihilation terms for the four-fermion operators).
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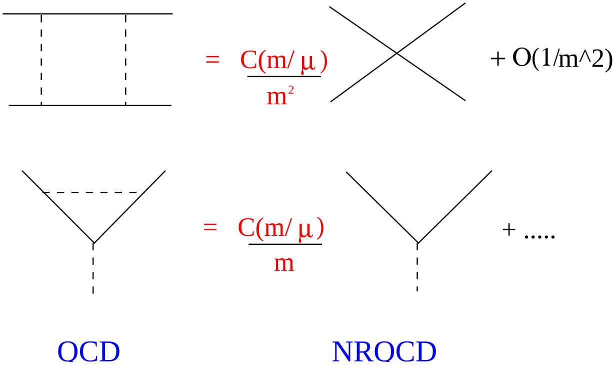
- Matching to some given order in α and $1/m$, i.e. to $O(\alpha^n/m^s)$.
- One matches loops in QCD with only one scale (the mass) to tree level diagrams in NRQCD.



Manohar; Soto, Pineda

Other problem

Power counting of NRQCD in the perturbative situation.

Previous work: Labelle → Multipole expansion (QED)
Manohar, Luke; Grinstein, Rothstein; Savage, Luke
Reformulation of the problem:
Soto, Pineda→How would we like the effective theory for Q–$\bar{Q}$ systems near threshold to be?
1) We do not want to describe all the degrees of freedom included in NRQCD, but rather only those with US energy.
2) We want to get a closer connection with a Schrödinger-like formulation.
We name potential NRQCD this new effective theory.
Beneke, Smirnov→ threshold expansion. Rigorous diagrammatic study of the (perturbative) momentum regions: hard, soft, potential, ultrasoft.
hard: particles with $E \sim |\mathbf{p}| \sim m$ → NRQCD
soft: particles with $E \sim |\mathbf{p}| \sim mv$ → pNRQCD
potential: particles with $E \sim mv^2$, $|\mathbf{p}| \sim mv$
ultrasoft: particles with $E \sim |\mathbf{p}| \sim mv^2$

Brambilla, Pineda, Soto, Vairo; Beneke, Smirnov; Kniehl, Penin; Kniehl, Penin, Smirnov, Steinhauser; ⋯

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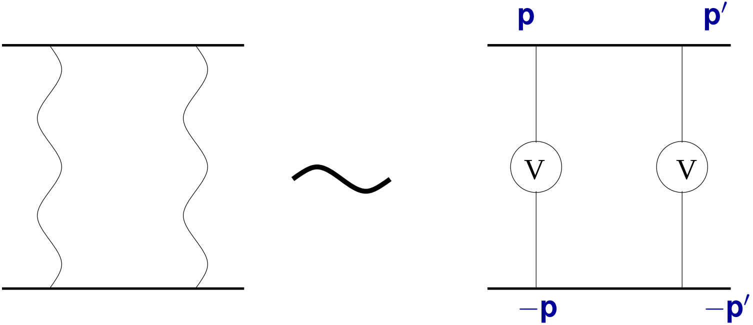
Physical Picture



$$I \sim \int \frac{d^4 q}{(2\pi)^4} V(p,q) \frac{1}{E/2 + q^0 - \mathbf{q}^2/2m + i\epsilon} \frac{1}{E/2 - q^0 - \mathbf{q}^2/2m + i\epsilon} V(q,p')$$

$$V(p,q) \sim \frac{1}{(p-q)^2}$$

Counting (different possibilities):

A) $E \sim mv^2 \quad p^0, p'^0 \sim q^0 \sim |\mathbf{p}| \sim \mathbf{q} \sim \mathbf{p}' \sim mv$

Static propagator (time independent contribution: correction to the potential):

$$\frac{i}{q^0 + i\epsilon} \to I \sim \delta V(\mathbf{p} - \mathbf{q})$$

B) $E \sim p^0, p'^0 \sim q^0 \sim mv^2 \quad |\mathbf{p}| \sim \mathbf{q} \sim \mathbf{p}' \sim mv$

Nonrelativistic propagator (leading contribution):

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This is nothing but the usual NR Quantum Mechanics!!, I can be written as

$$I \sim \langle \mathbf{p} | \hat{V} \frac{1}{E - \mathbf{p}^2/m + i\epsilon} \hat{V} | \mathbf{p}' \rangle$$

This can be done to any order considering ladder loops. Therefore,

$$i\mathcal{A} = -i \langle \mathbf{p} | \left(\hat{V} + \hat{V} \frac{1}{E - \mathbf{p}^2/m + i\epsilon} \hat{V} + \ldots \right) | \mathbf{p}' \rangle \,.$$

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pNRQCD: the scale mv

The integration of the mv scale gives rise to potential terms. The Lagrangian is local in time but not in space.

- Degrees of freedom
- symmetries
- cutoff

pNRQCD has two ultraviolet cut-offs, ν_{us} and ν_p. ν_{us} fulfils the relation $\mathbf{p}^2/m \ll \nu_{us} \ll |\mathbf{p}|$ and is the cut-off of the energy of the quarks, and of the energy and the momentum of the gluons. ν_p fulfils $|\mathbf{p}| \ll \nu_p \ll m$ and is the cut-off of the relative momentum of the quark–antiquark system, $\mathbf{p}$.

Power counting/scales

Scales: m, p, $1/r$, $\Lambda_{mp} = \{\Lambda_{QCD}, mv^2, ...\}$

Dimensionless quantities:

$$\boxed{\frac{p}{m},\ \alpha_s,\ \frac{1}{m\,r},\ \boxed{\Lambda_{mp} r} \ll 1}$$

The multipole expansion can be used in the new EFT.

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L'_{NRQCD}, gluons multipole expanded (only ultrasoft gluons).

$$V_s^{(0)} \equiv -C_F \frac{\alpha_{V_s}}{r}.$$

$$\frac{V_s^{(1)}}{m} \equiv -\frac{C_F C_A D_s^{(1)}}{2mr^2}.$$

$$\frac{V_s^{(2)}}{m^2} = -\frac{C_F D_{1,s}^{(2)}}{2m^2}\left\{\frac{1}{r}, \mathbf{p}^2\right\} + \frac{C_F D_{2,s}^{(2)}}{2m^2}\frac{1}{r^3}\mathbf{L}^2 + \frac{\pi C_F D_{d,s}^{(2)}}{m^2}\delta^{(3)}(\mathbf{r})$$
$$+\frac{4\pi C_F D_{S^2,s}^{(2)}}{3m^2}\mathbf{S}^2\delta^{(3)}(\mathbf{r}) + \frac{3C_F D_{LS,s}^{(2)}}{2m^2}\frac{1}{r^3}\mathbf{L}\cdot\mathbf{S} + \frac{C_F D_{S_{12},s}^{(2)}}{4m^2}\frac{1}{r^3}S_{12}(\hat{\mathbf{r}}),$$

where $S_{12}(\hat{\mathbf{r}}) \equiv 3\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_1\,\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ and $\mathbf{S} = \boldsymbol{\sigma}_1/2 + \boldsymbol{\sigma}_2/2$.

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Matching NRQCD to pNRQCD

Same idea than in NRQCD:

Expansion in the scales that are left in the effective theory. We integrate out the scale k (transfer momentum between the quark and antiquark).

Analytical expansion of **1/m** (and therefore **p**) and **E** before the integration is made in both the full and the effective theory. Effectively HQET-like rules (HQET quark propagator).

NRQCD

$$\int d^4q f(q,k,|\mathbf{p}|,E) = \int d^4q f(q,k,0,0) + O\left(\frac{E}{k},\frac{|\mathbf{p}|}{k}\right) \qquad \text{potentials}$$

pNRQCD

$$\int d^4q f(q,|\mathbf{p}|,E) = \int d^4q f(q,0,0) = 0\,!!$$

Dimensional regularization. The computation in the effective theory just gives zero.

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Final rules:

- Matching to some given order in α and $1/m$, i.e. $O(\alpha^n/m^s)$.
- NRQCD tree level Feynman diagrams $\rightarrow$ non-relativistic reduction. They give the leading contribution to the potentials.
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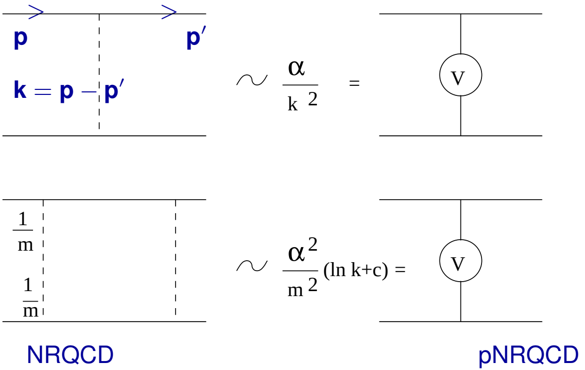
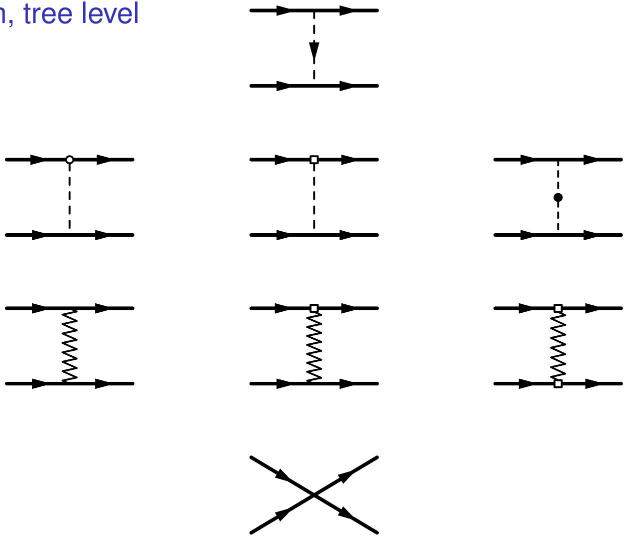


Positronium, tree level



Order $1/m^2$

$$\tilde{V}^{(b)} = \frac{\pi\alpha_{\rm eff}(k)}{2}\left[Z_p\frac{c_D^{(\mu)}}{m_\mu^2}+Z_\mu\frac{c_D^{(p)}}{m_p^2}\right],$$

$$\tilde{V}^{(c)} = -i2\pi\alpha_{\rm eff}(k)\frac{(\mathbf{p}\times\mathbf{k})}{\mathbf{k}^2}\cdot\left\{Z_p\frac{c_S^{(\mu)}\mathbf{s}_1}{m_\mu^2}+Z_\mu\frac{c_S^{(p)}\mathbf{s}_2}{m_p^2}\right\},$$

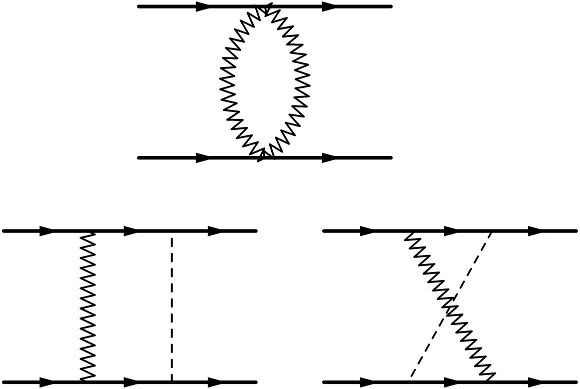
$$\tilde{V}^{(d)} = -Z_\mu Z_p 16\pi\alpha\left(\frac{d_2^{(\mu)}}{m_\mu^2}+\frac{d_2^{(\tau)}}{m_\tau^2}+\frac{d_{2,NR}}{m_p^2}\right),$$

$$\tilde{V}^{(e)} = -Z_\mu Z_p\frac{4\pi\alpha_{\rm eff}(k)}{m_\mu m_p}\left(\frac{\mathbf{p}^2}{\mathbf{k}^2}-\frac{(\mathbf{p}\cdot\mathbf{k})^2}{\mathbf{k}^4}\right),$$

$$\tilde{V}^{(f)} = -\frac{i4\pi\alpha_{\rm eff}(k)}{m_\mu m_p}\frac{(\mathbf{p}\times\mathbf{k})}{\mathbf{k}^2}\cdot\left(Z_p c_F^{(\mu)}\mathbf{s}_1+Z_\mu c_F^{(p)}\mathbf{s}_2\right),$$

$$\tilde{V}^{(g)} = \frac{4\pi\alpha_{\rm eff}(k)c_F^{(1)}c_F^{(2)}}{m_\mu m_p}\left(\mathbf{s}_1\cdot\mathbf{s}_2-\frac{\mathbf{s}_1\cdot\mathbf{k}\mathbf{s}_2\cdot\mathbf{k}}{\mathbf{k}^2}\right),$$

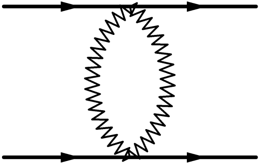
$$\tilde{V}^{(h)} = -\frac{1}{m_p^2}\left\{(c_{3,NR}^{pl_i}+3c_{4,NR}^{pl_i})-2c_{4,NR}^{pl_i}\mathbf{S}^2\right\}.$$

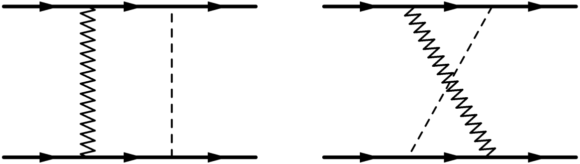


$\overline{\text{MS}}$ scheme:

$$\tilde{V}^{(a)}_{1\,loop} = \frac{Z_\mu^2 Z_p^2 \alpha^2}{m_\mu m_p}\left(\log\frac{\mathbf{k}^2}{\mu^2} - \frac{8}{3}\log 2 + \frac{5}{3}\right),$$

$$\tilde{V}^{(b,c)}_{1\,loop} = \frac{4 Z_\mu^2 Z_p^2 \alpha^2}{3 m_\mu m_p}\left(\log\frac{\mathbf{k}^2}{\mu^2} + 2\log 2 - 1\right).$$





$\overline{\mathrm{MS}}$ scheme:

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QCD

Previous work: Gupta, Radford, Repko; Titard, Ynduráin
Static two loops: Schroeder; Peter
Static three loops: Anzai, Kiyo, Sumino; Smirnov^2, Steinhauser
Logs: Brambilla, Soto, Vairo, Pineda; Kniehl, Penin; Hoang, Manohar, Stewart; Brambilla, Garcia, Soto, Vairo
Finite terms $1/m$ and $1/m^2$ potentials: Kniehl, Penin, Smirnov, Steinhauser
Renormalization group improved expressions: Soto, Pineda; Pineda; Brambilla, Garcia, Soto, Vairo; Hoang, Stahlhofen
...

$$\alpha_{V_s} = \alpha_s(r)\left\{1 + (a_1 + 2\gamma_E\beta_0)\frac{\alpha_s(r)}{4\pi} + \cdots\right\},$$

$$D_s^{(1)} = \alpha_s^2(r)\left\{1 + \frac{2}{3}(4C_F + 2C_A)\frac{\alpha_s}{\pi}\ln\mu r + \cdots\right\},$$

$$D_{d,s}^{(2)} = \alpha_s(r)\left\{1 + \frac{\alpha_s}{\pi}\left(\frac{2C_F}{3} + \frac{17C_A}{3}\right)\ln mr + \frac{16}{3}\frac{\alpha_s}{\pi}\left(\frac{C_A}{2} - C_F\right)\ln\mu r\right\},$$

...

Wave-function description

$\rightarrow$ project to the quark-antiquark sector.

$$\int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \Psi(\mathbf{x}_1,\mathbf{x}_2)\psi(x_1)\chi_c(x_2)|0\rangle$$

$$H\int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \Psi(\mathbf{x}_1,\mathbf{x}_2)\psi^\dagger(\mathbf{x}_1)\chi_c^\dagger(\mathbf{x}_2)|0\rangle = \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 (\hat{h}\Psi(\mathbf{x}_1,\mathbf{x}_2))\psi^\dagger(\mathbf{x}_1)\chi_c^\dagger(\mathbf{x}_2)|0\rangle$$

For QED (multipole expansion)

$$\begin{aligned} L_{pNRQED} &= \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \Psi^\dagger(\mathbf{x}_1,\mathbf{x}_2)\left(iD_0 + \frac{\mathbf{D}^2_{\mathbf{x}_1}}{2m_1} + \frac{\mathbf{D}^2_{\mathbf{x}_2}}{2m_2} - V(\mathbf{x},\mathbf{p})\right)\Psi(\mathbf{x}_1,\mathbf{x}_2) \\ &= \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \Psi^\dagger(\mathbf{x}_1,\mathbf{x}_2)\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2_{\mathbf{x}}}{m} + \frac{\boldsymbol{\nabla}^2_{\mathbf{X}}}{4m} \right. \\ &\quad \left. - e\mathbf{x}\cdot\boldsymbol{\nabla}A_0(\mathbf{X}) - 2ie\frac{\mathbf{A}(\mathbf{X})\cdot\boldsymbol{\nabla}_{\mathbf{x}}}{m} - V(\mathbf{x},\mathbf{p})\right)\Psi(\mathbf{x}_1,\mathbf{x}_2) \end{aligned}$$

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New fields: Singlet S and US photons

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New fields: Singlet S and US gluons (and Octet (O) for QCD)

Gauge transformation:

$S(\mathbf{x},\mathbf{X},t) \rightarrow S(\mathbf{x},\mathbf{X},t) \quad O(\mathbf{x},\mathbf{X},t) \rightarrow g(\mathbf{X},t)O(\mathbf{x},\mathbf{X},t)g^{-1}(\mathbf{X},t)$

pNRQED Lagrangian at $O(r)$

$$
\begin{aligned}
\mathcal{L}_{pNRQED} &= S^\dagger \left(i\partial_0 - V_s^{(0)}(\mathbf{x}) \right) S \\
&+ g V_A(\mathbf{x}) S^\dagger \mathbf{x} \cdot \mathbf{E}\, S \\
&- S^\dagger \left(\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}(\mathbf{x})}{m^n} \right) S,
\end{aligned}
$$

$$V = V(c(\nu_s/m), d(\nu_s/m, \nu_p/m), r, \nu_s, \nu_{us}) \sim \sum_n c_n(\nu; m, r)\alpha_s^n(\nu)$$

Interpolating fields:

$$Q_2^\dagger(\mathbf{x}_2, t)\phi(\mathbf{x}_2, \mathbf{x}_1; t)Q_1(\mathbf{x}_1, t) = Z_s^{1/2}(\mathbf{x})S(\mathbf{X}, \mathbf{x}, t)$$

pNRQCD Lagrangian at $O(r)$

$$\mathcal{L}_{pNRQCD} = \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - V_s^{(0)}(\mathbf{x}) \right) S + O^\dagger \left(iD_0 - V_o^{(0)}(\mathbf{x}) \right) O \right\}$$

$$+gV_A(\mathbf{x})\mathrm{Tr}\left\{ O^\dagger \mathbf{x}\cdot\mathbf{E}\, S + S^\dagger \mathbf{x}\cdot\mathbf{E}\, O \right\} + g\frac{V_B(\mathbf{x})}{2}\mathrm{Tr}\left\{ O^\dagger \mathbf{x}\cdot\mathbf{E}\, O + O^\dagger O \mathbf{x}\cdot\mathbf{E} \right\}$$

$$-\mathrm{Tr}\left\{ S^\dagger \left(\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}(\mathbf{x})}{m^n} \right) S - O^\dagger \left(\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_o^{(n)}(\mathbf{x})}{m^n} \right) O \right\},$$

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$$Q_2^\dagger(x_2)\phi(\mathbf{x}_2, \mathbf{X}; t)T^a\phi(\mathbf{X}, \mathbf{x}_1; t)Q_1(x_1) = Z_o^{1/2}(\mathbf{x})O^a(\mathbf{X}, \mathbf{x}, t)$$

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Potential = Wilson loops = HQET-like Feynman diagrams ←
3) Observable: Spectrum or decays
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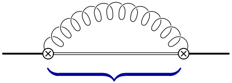

$1/(E - V_o^{(0)} - \mathbf{p}^2/m)$

$$\delta G_s \sim \frac{1}{h_s^{(0)} - E} \int \frac{d^3\mathbf{k}}{(2\pi)^{D-1}} \mathbf{r} \frac{k}{k + h_o^{(0)} - E} \mathbf{r} \frac{1}{h_s^{(0)} - E}$$

$$\sim \frac{1}{h_s^{(0)} - E} \mathbf{r} (h_o^{(0)} - E)^3 \left\{ \frac{1}{\epsilon} + \gamma + \ln \frac{(h_o^{(0)} - E)^2}{\mu_{us}^2} + C \right\} \mathbf{r} \frac{1}{h_s^{(0)} - E}$$

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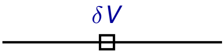
B) Quantum mechanics perturbation theory (both at weak and strong coupling)$\leftarrow$

$$\delta G_s \sim \frac{1}{h_s^{(0)} - E} \delta V_s \frac{1}{h_s^{(0)} - E} + \frac{1}{h_s^{(0)} - E} \delta V_s \frac{1}{h_s^{(0)} - E} \delta V_s \frac{1}{h_s^{(0)} - E} + \cdots$$

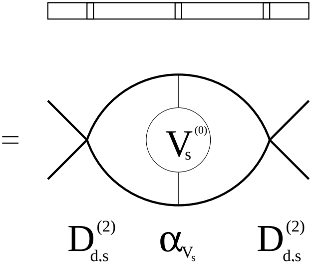


Ultraviolet divergences are governed by the short-distance behavior of the potentials, i.e. by perturbation theory. Therefore, they can be computed and absorbed in the matching coefficients of the currents or in the potentials.

Example:

$$\delta(r)\frac{1}{h_s^{(0)}-E}\delta(r) \to \delta(r)\frac{1}{\mathbf{p}^2/m-E}\frac{C_f\alpha_s}{r}\frac{1}{\mathbf{p}^2/m-E}\delta(r)$$

Since the singular behavior of the potential loops appears for $\mathbf{p}^2/m \gg \alpha_s/r$, a perturbative expansion in α_s is licit. The divergences can be absorbed in the matching coefficients of the local potentials (proportional to the $\delta^{(3)}(\mathbf{r})$) providing with the renormalization group equations.



$$\langle \mathbf{r}=0|\frac{1}{E-\mathbf{p}^2/m}C_f\frac{\alpha_{V_s}}{r}\frac{1}{E-\mathbf{p}^2/m}|\mathbf{r}=0\rangle$$

$$\sim \int\frac{\mathrm{d}^d p'}{(2\pi)^d}\int\frac{\mathrm{d}^d p}{(2\pi)^d}\frac{m}{\mathbf{p}'^2-mE}C_f\frac{4\pi\alpha_{V_s}}{\mathbf{q}^2}\frac{m}{\mathbf{p}^2-mE}\sim -C_f\frac{m^2\alpha_{V_s}}{16\pi}\frac{1}{\epsilon},$$

where $D=4+2\epsilon$ and $\mathbf{q}=\mathbf{p}-\mathbf{p}'$. This divergence is absorbed in $D_{d,s}^{(2)}$.

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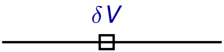
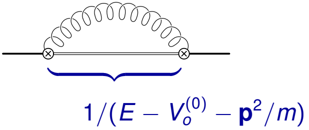
Relativistic Feynman diagrams ←
2) Matching NRQCD to pNRQCD. Integrating out the soft scale, mv
Potential = Wilson loops = HQET-like Feynman diagrams ←
3) Observable: Spectrum or decays
Corrections to the Green Function ($h_s^{(0)} = \mathbf{p}^2/m + V_s^{(0)}$)

$$G_s^{(0)}(E) = \frac{1}{h_s^{(0)} - E} \longrightarrow G_s(E) = P_s \frac{1}{h_s^{(0)} - H_I - E} P_s = G_s^{(0)} + \delta G_s$$

- A) Ultrasoft loops (only at weak coupling: lamb shift-like): $\mathbf{x} \cdot \mathbf{E}$ ←
- B) Quantum mechanics perturbation theory (both at weak and strong coupling)←





Positronium

$$L_{pNRQED} = \int d^3\mathbf{x} d^3\mathbf{X} S^\dagger(\mathbf{x},\mathbf{X},t)$$

$$\left\{ i\partial_0 - \frac{\mathbf{p}^2}{m} + \frac{\alpha}{|\mathbf{x}|} + \frac{\mathbf{p}^4}{4m^3} - \frac{\delta^{(3)}(\mathbf{x})}{m^2}\left(\pi\alpha\left(c_D - 2{c_F}^2\right) + d_s + 3d_v - 16\pi\alpha d_2\right)\right.$$

$$+\frac{\alpha}{2m^2}\frac{1}{|\mathbf{x}|}\left(\mathbf{p}^2 + \frac{1}{\mathbf{x}^2}\mathbf{x}\cdot(\mathbf{x}\cdot\mathbf{p})\mathbf{p}\right) - \frac{\delta^{(3)}(\mathbf{x})}{m^2}\mathbf{S}^2\left(\pi\alpha\frac{4}{3}{c_F}^2 - 2d_v\right)$$

$$-\frac{\alpha}{4m^2}\frac{1}{|\mathbf{x}|^3}\mathbf{L}\cdot\mathbf{S}\,(2c_S + 4c_F) - \frac{\alpha {c_F}^2}{4m^2}\frac{1}{|\mathbf{x}|^3}S_{12}(\mathbf{x}) - \delta V(\mathbf{x})$$

$$\left. +\mathbf{x}\cdot e\mathbf{E}(\mathbf{X},t)\right\} S(\mathbf{x},\mathbf{X},t)\,.$$

$$\delta V = \frac{\delta^{(3)}(\mathbf{x})}{m^2}\left(\frac{\alpha^2}{3} - \frac{7\alpha^2}{3}\log\mu^2\right) - \frac{7\alpha^2}{6\pi m^2}\mathrm{reg}\frac{1}{|\mathbf{x}|^3}$$

The positronium spectrum can be obtained with $O(m\alpha^5)$ accuracy with this Lagrangian. The cancellation of the scale dependence coming from the different scales involved in the problem is nicely seen.

The calculations can be performed using dimensional regularization.

$$
\begin{aligned}
E_{n,l,j} = 2m - m\frac{\alpha^2}{4n^2} + \frac{m\alpha^4}{8}\Bigg\{ & -\frac{4}{n^3(2l+1)} + \frac{11}{8n^4} \\
& -\frac{2\alpha}{3\pi}\frac{\delta_{l0}}{n^3}\Bigg(9\log\alpha + 7\log n + 8\log R(n,l) - 14\log 2 \\
& \qquad -\frac{49}{15} - 7\left(\sum_{k=1}^{n}\frac{1}{k} + \frac{n-1}{2n}\right)\Bigg) \\
& -\frac{16\alpha}{3\pi}\frac{1-\delta_{l0}}{n^3}\left(\log R(n,l) + \frac{7}{16}\frac{1}{l(l+1)(2l+1)}\right) \\
& +\frac{14}{3}\frac{\delta_{l0}\delta_{s1}}{n^3}\left\{1 + \frac{3\alpha}{7\pi}\left(-\frac{32}{9} - 2\log 2\right)\right\} + \frac{(1-\delta_{l0})\delta_{s1}}{l(2l+1)(l+1)n^3}C_{j,l}\Bigg\},
\end{aligned}
$$

where $\log R(n,l) = \log\frac{2\langle E_{n,l}\rangle}{m\alpha^2}$ is called the Bethe logarithm and

$$
C_{j,l} = \begin{cases}
-\frac{l+1}{2\,l-1}\left(2(3l-1) + \frac{\alpha}{\pi}(4\,l-1)\right) & ,\ j = l-1\ , \\
-2 - \frac{\alpha}{\pi} & ,\ j = l\ , \\
\frac{l}{2\,l+3}\left(2(3l+4) + \frac{\alpha}{\pi}(4\,l+5)\right) & ,\ j = l+1\ .
\end{cases}
$$

CONCLUSIONS

The use of Effective Field Theories facilitates the understanding of the dynamics of NR systems:

Model independent and systematic.

Disentangles the physics associated to each scale. One scale at a time. Payoff: Possible to efficiently perform calculations in DIMENSIONAL REGULARIZATION.

Possible to connect Quantum Field Theories and a NR Quantum-mechanical formulation of the NR systems.
Payoff: Efficient implementation of the MULTIPOLE EXPANSION

Systematic POWER COUNTING.

Possible to obtain renormalization group equations and resum logarithms.

Plenty of Observables: **Decay widths ($\Gamma(n^3S_1 \to e^+e^-)$, $\Gamma(n^1S_0 \to \gamma\gamma)$), Bottomonium sum rules. Determination of m_b. t–$\bar{t}$ production near threshold. Determination of m_t. QED and atomic/hadronic physics, ...**

AND NOW GRAVITY!!

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