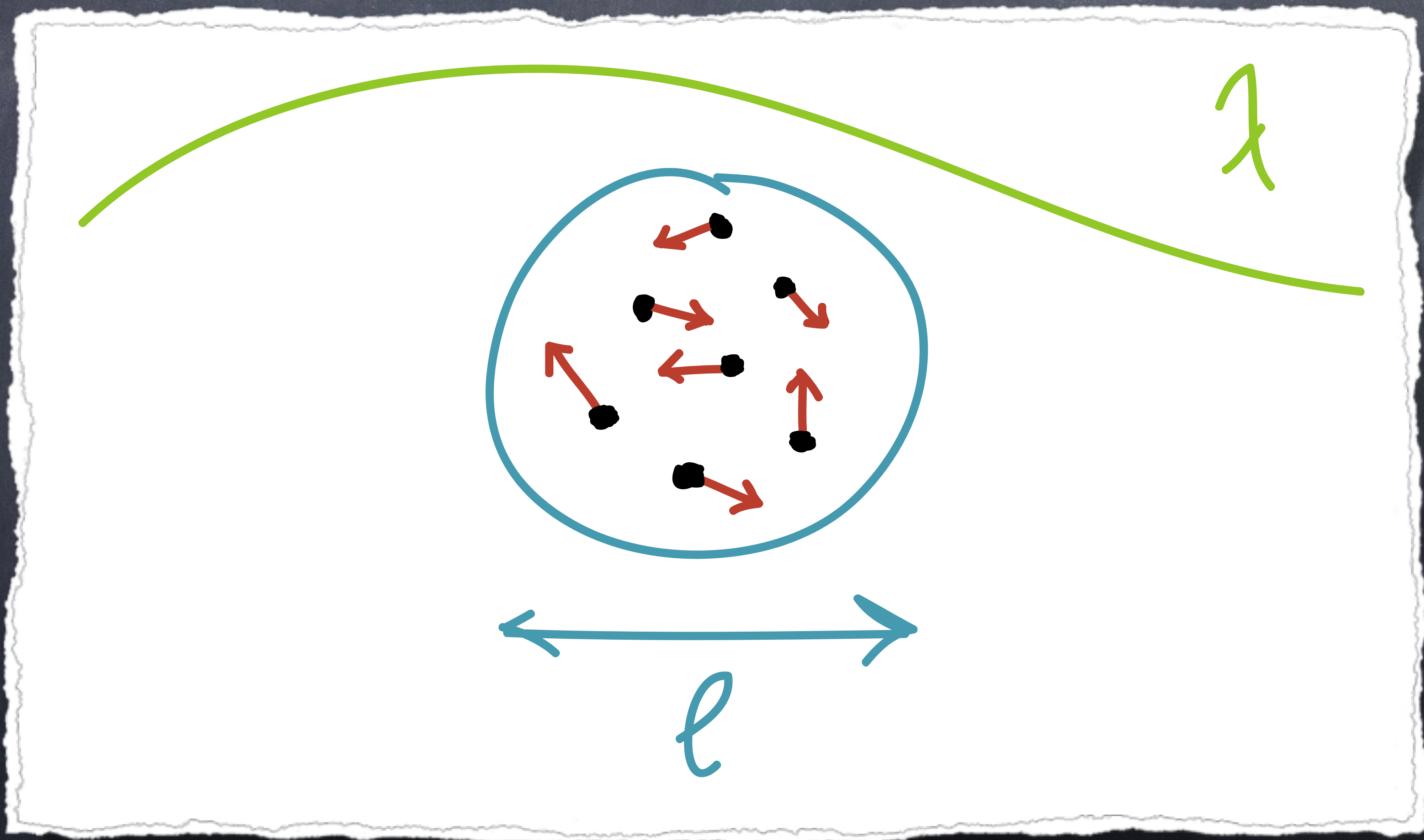
Alberto Nicolis
Columbia University

EFT methods for vortex-sound interactions

w/ Endlich, Esposito, Garcia-Saenz, Horn, Krichevsky, Mitsou, Penco

Hydrodynamics

$\lambda \gg \ell$ $\qquad\qquad$ $\omega \ll 1/\tau$



zero T super-fluid vs. ordinary fluid

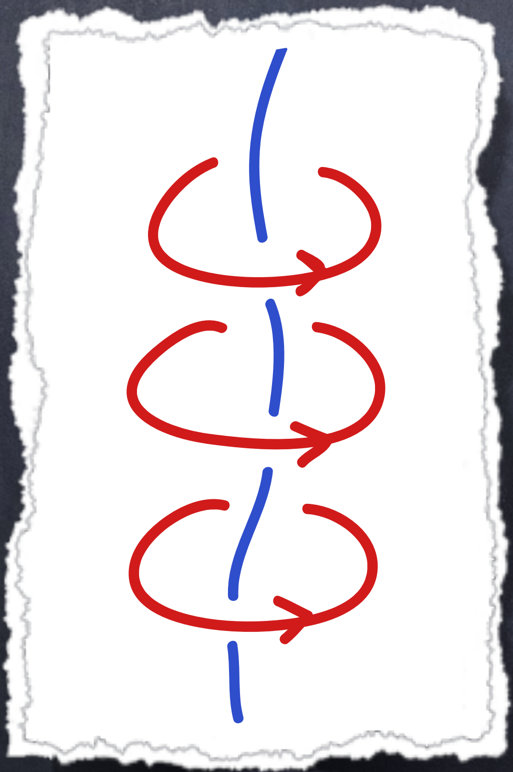
compressional (sound) sector

- Hydrodynamics
- Hydrodynamics

transverse (vortex) sector

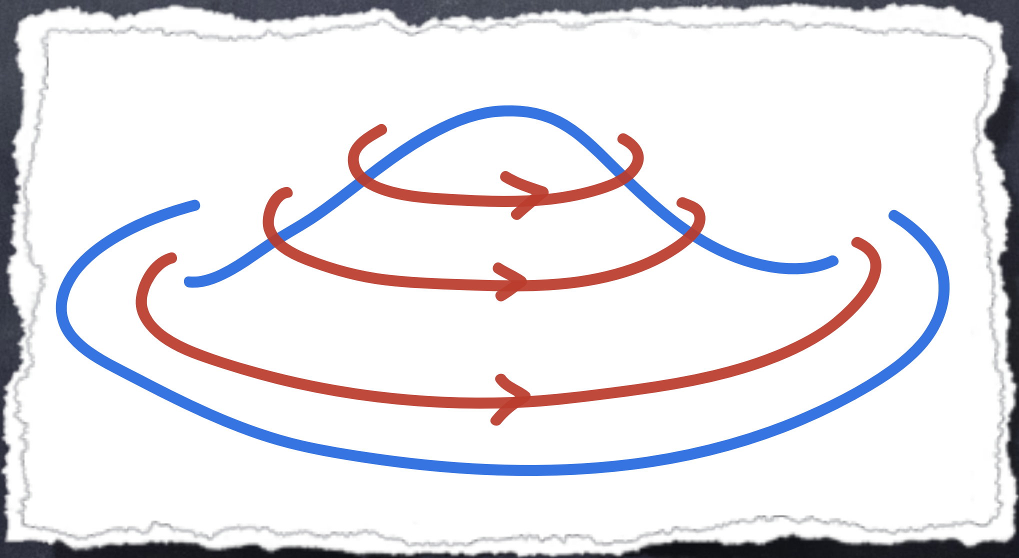
- Hard (gapped)

$$\vec{\nabla} \times \vec{v} = 0$$



- Soft (gapless)

$$\vec{\nabla} \times \vec{v} \neq 0$$

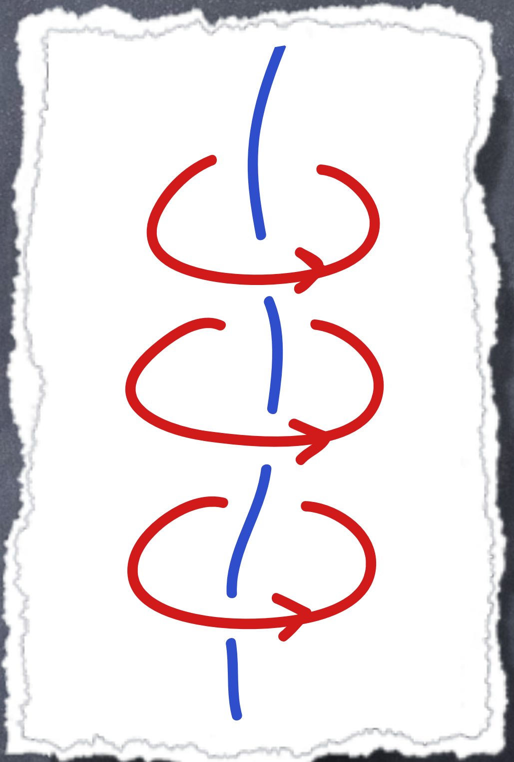


Phenomenology

Vortex dynamics (incompressible limit)

$$\vec{\nabla} \cdot \vec{v} = 0 \qquad \vec{\omega} = \vec{\nabla} \times \vec{v}$$

For vortex lines



$$\Gamma = \oint \vec{v} \cdot d\vec{\ell} \quad \leftrightarrow \quad I$$

$$\vec{v} \quad \leftrightarrow \quad \vec{B}$$

Biot-Savart:

$$\vec{v}(\vec{x}) = -\frac{\Gamma}{4\pi} \int \frac{(\vec{x} - \vec{x}\,')}{|\vec{x} - \vec{x}\,'|^3} \times d\vec{x}\,'$$

1st order EOM!

Unlike $m\vec{a} = \vec{F}_{\text{ext}}$

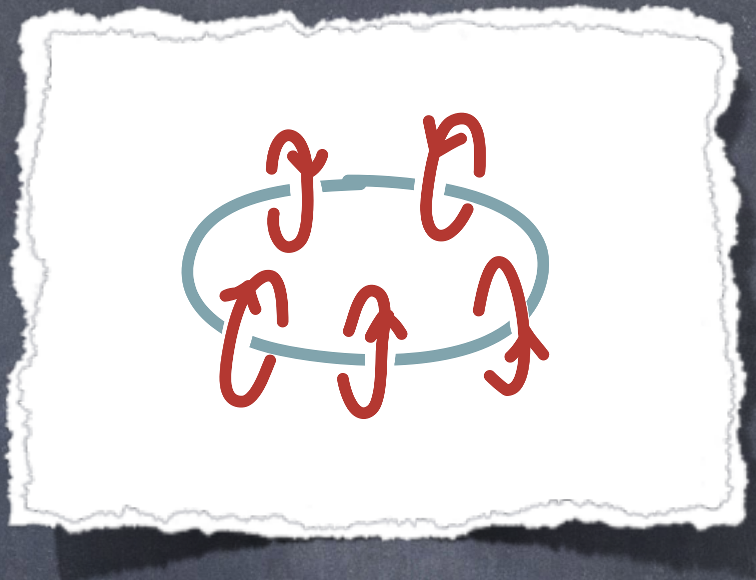
$\Longrightarrow$ No room for "forces"

$\Longrightarrow$ No free initial condition for v

$\Longrightarrow$ Instantaneous v determined by geometry

For vortex rings

$$\vec{v} = \frac{\Gamma}{4\pi R} \log(R/a)\, \hat{n}$$



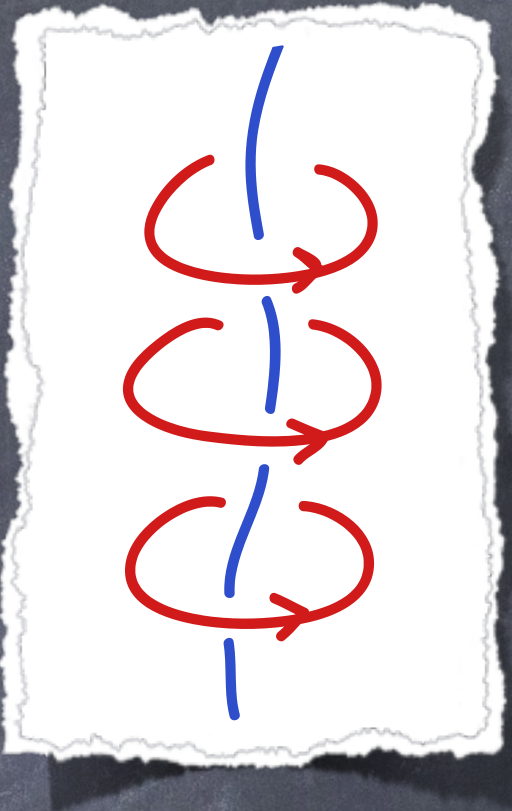
Far away:

$$\vec{v}(\vec{x}) = \vec{B}_{\text{dipole}} \qquad \text{with} \qquad \vec{\mu} = (\pi R^2)\Gamma\, \hat{n}$$

Excitations: Kelvin waves

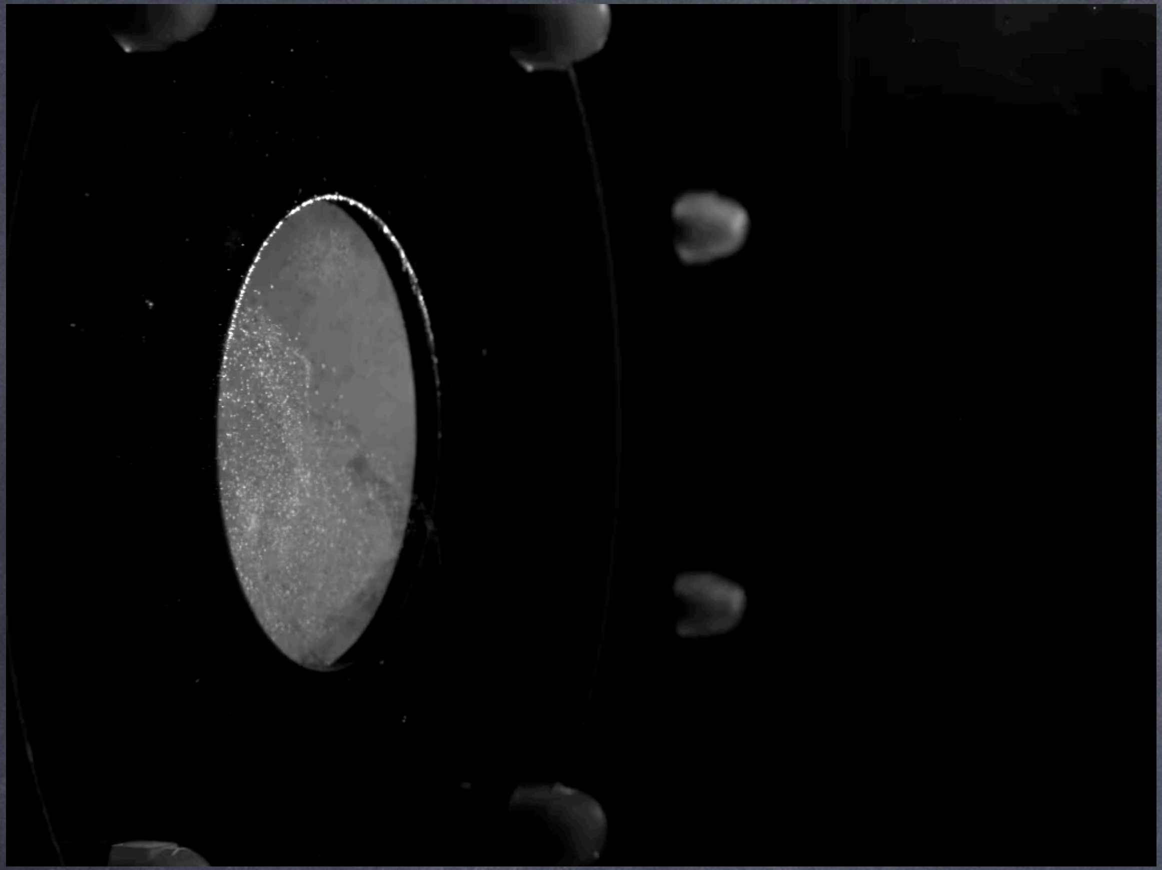
Two modes overall ($\neq 2+2$)

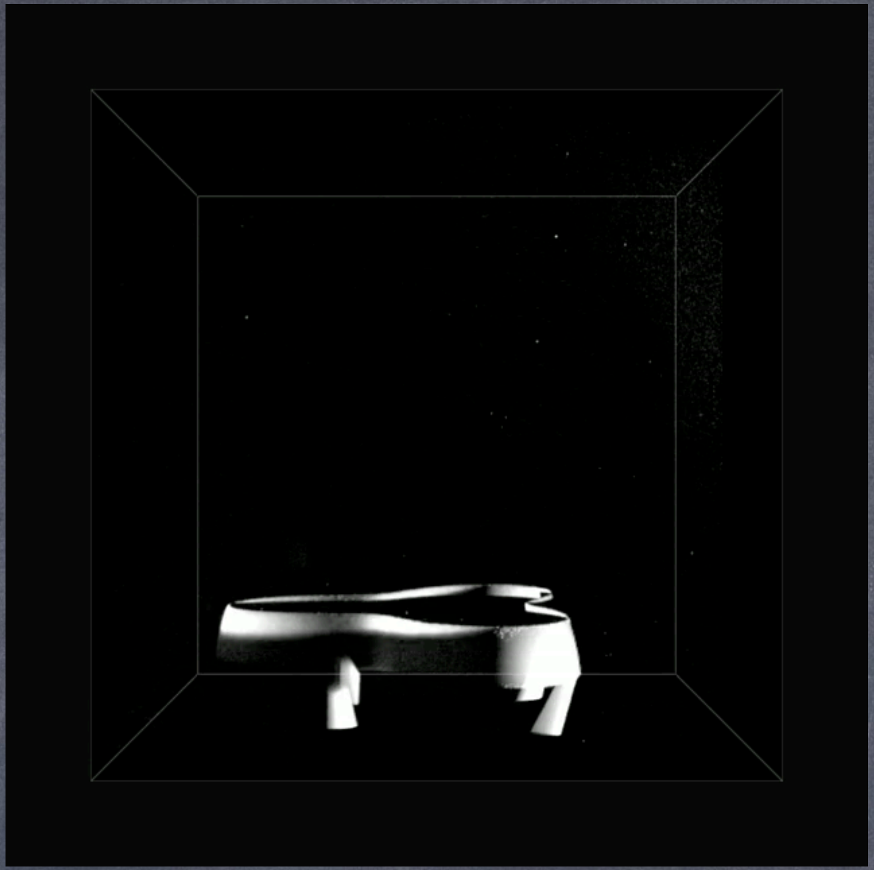
$$\omega_\pm = \frac{\Gamma}{2\pi} k^2 \log(1/ka)$$



- fewer modes than 2-derivative eom
- ``non-local" dispersion relation

(Thomson 1880)





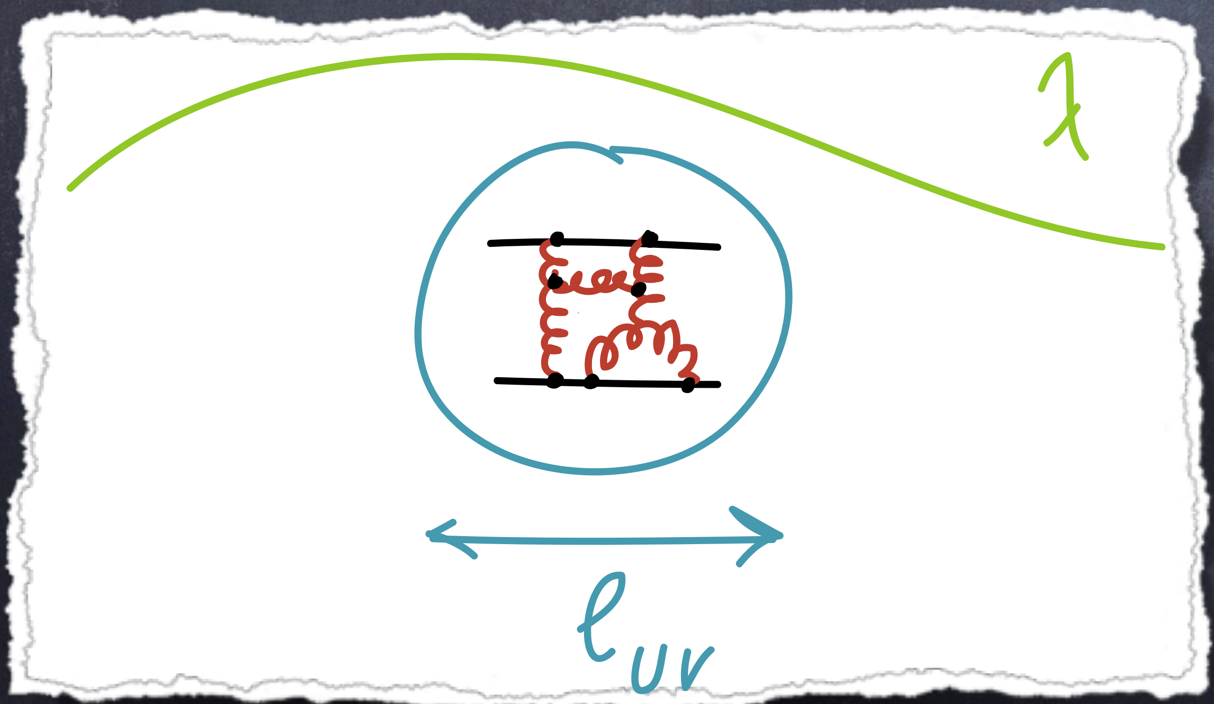
William Irvine

U. of Chicago

How to make sense of their dynamics?

Our approach: Effective field theory

- "Light" degrees of freedom Φ
- Symmetries $\Phi \to G[\Phi]$
- Derivative expansion $\mathcal{L} = f(\Phi, \partial)$



$$(\ell_{UV} \cdot \vec{\nabla})^n \,, \qquad (1/\omega_{UV} \cdot \partial_t)^n$$

$$\omega \ll \omega_{UV}$$
$$\lambda \gg \ell_{UV}$$

Lagrangian vs. eom

- More economical yet more complete
- Automatically conserves energy
- Straightforward to couple to other systems (gravity, EM, etc.)
- Symmetries = conservation laws
- Allows canonical quantization
- ...

For superfluids

(same classical eom as fluids)

- Spontaneously broken Q $\Longrightarrow$ light Goldstone $\phi(x)$
- Finite density for Q $\Longrightarrow$ $\phi(x) = \mu t + \pi(x)$

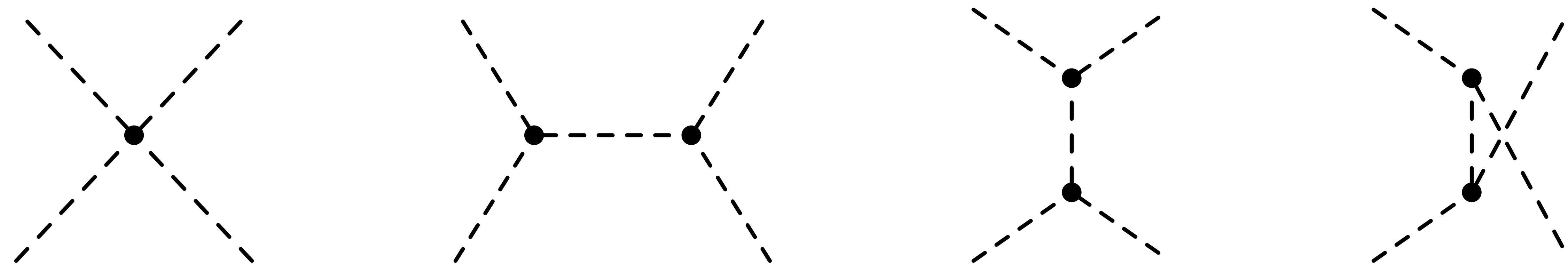
  phonon

- Symmetries: Poincaré + shift $\phi(x) \to \phi(x) + a$
- Bulk action: $S = \int d^4x\, P(X) + \ldots$ $\qquad X \equiv (\partial_\mu \phi)^2$

  eq. of state

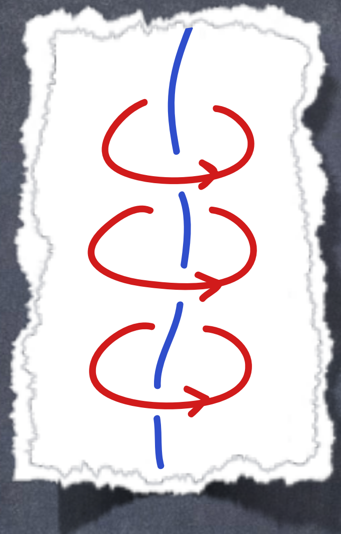
(Son 2002)

Phonon-phonon scattering



$$\frac{d\sigma}{d\Omega} = \frac{p^6}{c_s^2 \rho^2} f(\cos\theta)$$

Coupling to strings

- Problem:  $\phi \to \phi + 2\pi$ ("defect")

- Solution: dual 2-form $\partial_{[\mu}\mathcal{A}_{\nu\rho]} \propto \epsilon_{\mu\nu\rho\alpha}\partial^{\alpha}\phi$

- Analog of dual EM field for magnetic monopoles

- Equivalent bulk dynamics: $S = \int d^4x\, \rho(Y) + \ldots$

eq. of state

$$Y \equiv (\partial_{[\mu}\mathcal{A}_{\nu\rho]})^2$$

Effective string theory

Degrees of freedom & symmetries

Bulk: $\mathcal{A}_{[\mu\nu]}$ $\qquad$ $\mathcal{A}_{\mu\nu} \to \mathcal{A}_{\mu\nu} + \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$

hydro-photon (analog of Coulomb V)

phonon

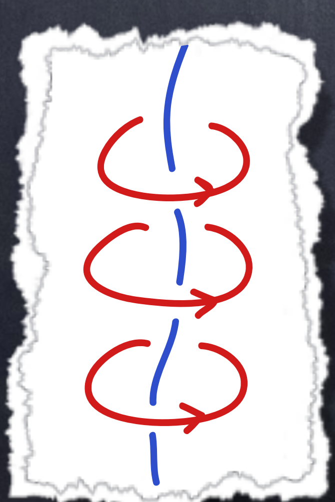
$$\mathcal{A}_{00} = 0$$

$$\mathcal{A}_{0i} = n\, A_i(x) \qquad \vec{\nabla} \cdot \vec{A} = 0$$

$$\mathcal{A}_{ij} = n\, \epsilon_{ijk}(x^k + B^k(x)) \qquad \vec{\nabla} \times \vec{B} = 0$$

String: $\qquad X^\mu(\tau, \sigma)$ $\qquad$ Poincaré 4-vector

arbitrary parameters



$\Rightarrow$ reparametrization invariance

Action

$$S = S_{\rm bulk} + S_{\rm KR} + \dots$$

Bulk: $\int d^4x\, \rho(Y) + \dots$ $\qquad Y \equiv (\partial_{[\mu}\mathcal{A}_{\nu\rho]})^2$

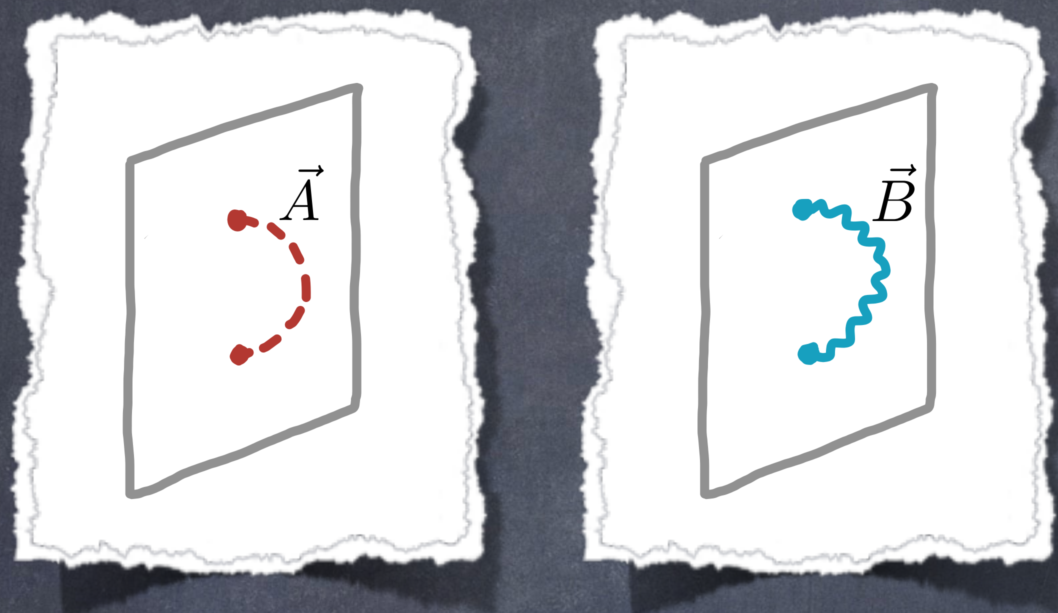
Kalb-Ramond: $\lambda \int d\sigma d\tau\, \mathcal{A}_{\mu\nu}\, \partial_\sigma X^\mu \partial_\tau X^\nu$

(analog of $q \int A_\mu dx^\mu$)

Perturbation theory: $\mathcal{A}_{\mu\nu} \to \text{background} + \vec{A}(x), \vec{B}(x)$

$$X^\mu \to \text{background} + \vec{\pi}(\tau, \sigma)$$

Energy of straight string



$$\frac{dE}{dz} = \frac{n^2\lambda^2}{w^2}\log(L\cdot\Lambda)$$

container size

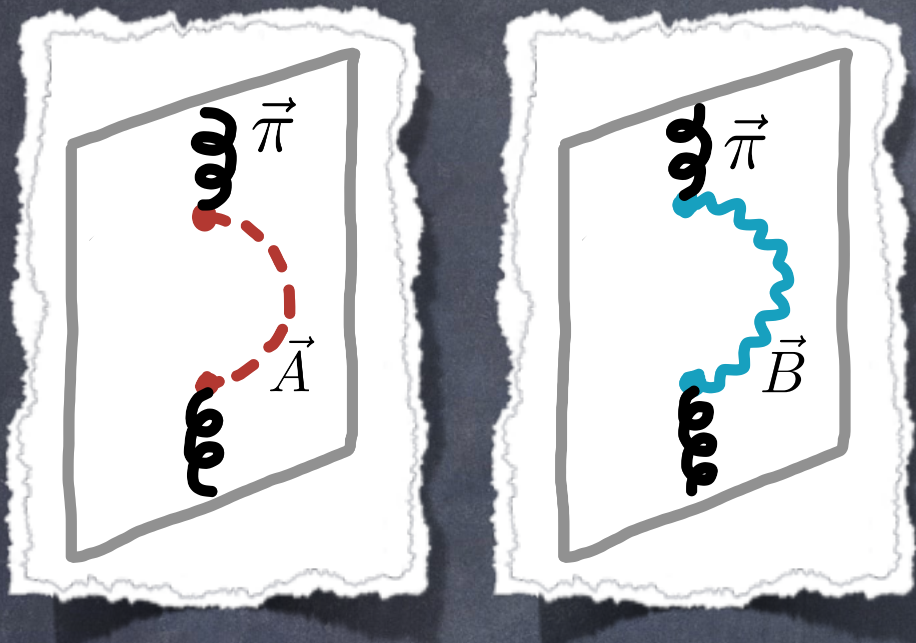
UV cutoff

Add ``tension'': $$S \to S - T\int d\sigma d\tau\,\sqrt{\det\partial_\alpha X^\mu\partial_\beta X_\mu}$$

RG running: $$\frac{d}{d\log\mu}T(\mu) = -\frac{1}{4\pi}\frac{n^2\lambda^2}{w}$$

momentum scale

(nonlinear) Kelvin waves



one time derivative

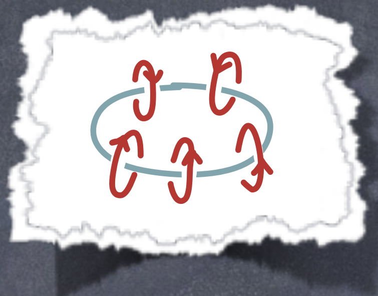
typical momentum

$$S_{\text{eff}}[\vec{\pi}] = \int dt dz \left[n\lambda\, \epsilon_{ab} \pi^a \partial_t \pi^b - T(k)\sqrt{1 + (\partial_z \vec{\pi})^2} \right]$$

$\Rightarrow$ $$\omega_{\pm} = \frac{\Gamma}{4\pi} \log(\mu_0/k)\, k^2$$ (linear)

$\Rightarrow$ "self-pipe" w/ $\vec{v} = \hat{z}\dfrac{\Gamma k}{2\pi}$ (nonlinear)

Small vortex rings



$$\vec{\mu} = \Gamma\, \pi R^2 \hat{n}$$

$$\mathcal{L} = \sum_n \left[\vec{\mu}_n \cdot \dot{\vec{x}}_n + \vec{\mu}_n \cdot (\vec{\nabla} \times \vec{A})\right] - \int d^3x \left(\partial_i A_j\right)^2$$

$$\to \sum_n \left(\vec{\mu}_n \cdot \dot{\vec{x}}_n - \mu_n^{3/2} \log \mu_n\right) - \sum_{n \neq m} \frac{\vec{\mu}_n \cdot \vec{\mu}_m - 3(\vec{\mu}_m \cdot \hat{r})(\vec{\mu}_n \cdot \hat{r})}{r^3}$$

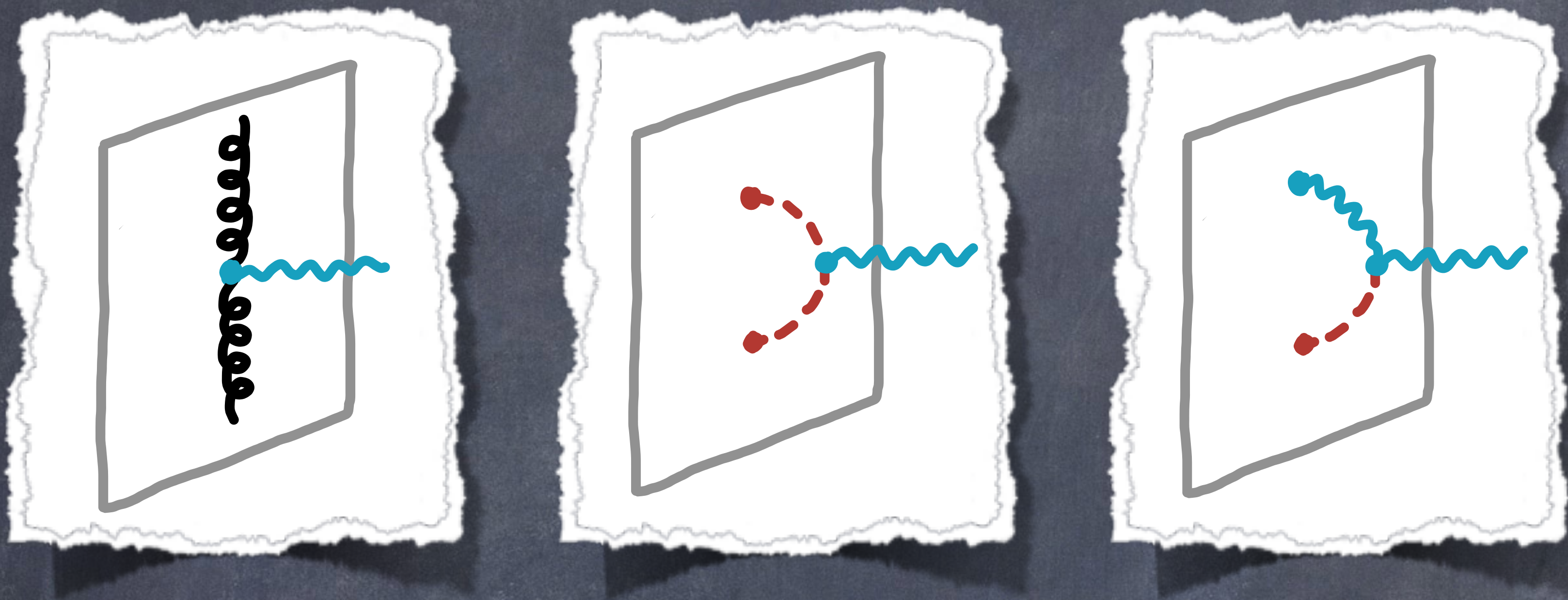
Peculiar conservation laws:

$$\vec{P} = \sum_n \vec{\mu}_n$$

$$\vec{L} = \sum_n \vec{x}_n \times \vec{\mu}_n$$

$$E = \sum_n \mu_n^{3/2} \log \mu_n + \sum_{n \neq m} \frac{\vec{\mu}_n \cdot \vec{\mu}_m - 3(\vec{\mu}_m \cdot \hat{r})(\vec{\mu}_n \cdot \hat{r})}{r^3}$$

Interactions w/ sound



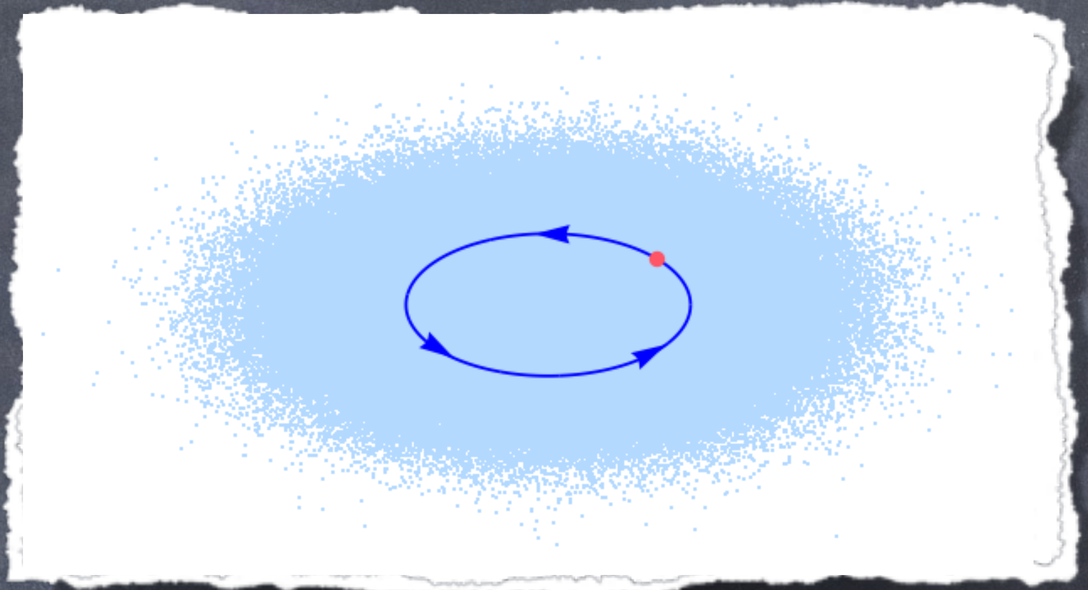
. . .

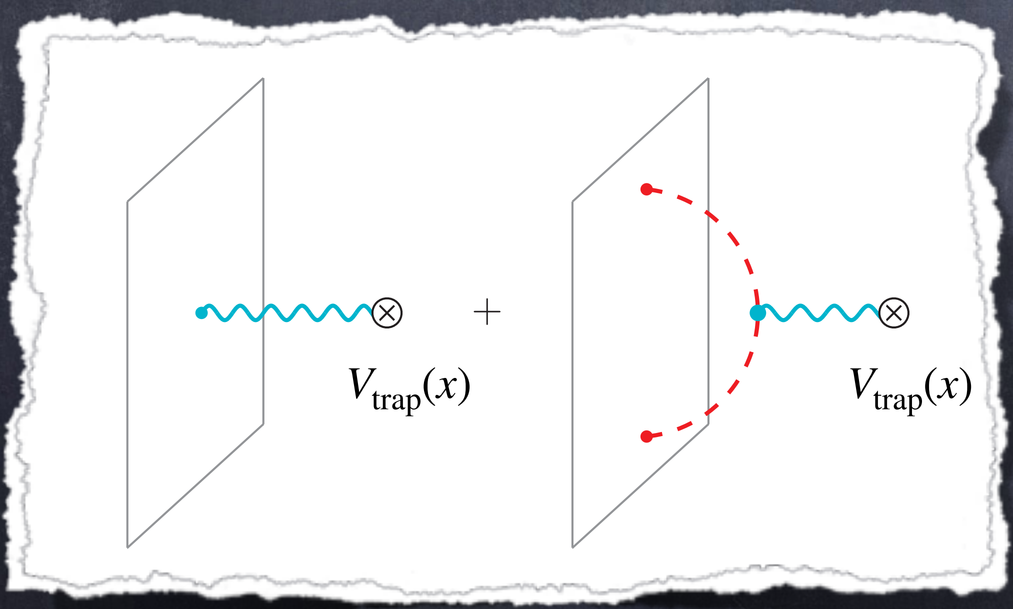
Phonon —> kelvons conversion:

$$\sigma = \frac{1}{16\, w c_s^3}\, \omega^2 q \, \sin^4 \theta \sim \sqrt{\omega^5 / \log \omega}$$

Vortex precession

Fact:

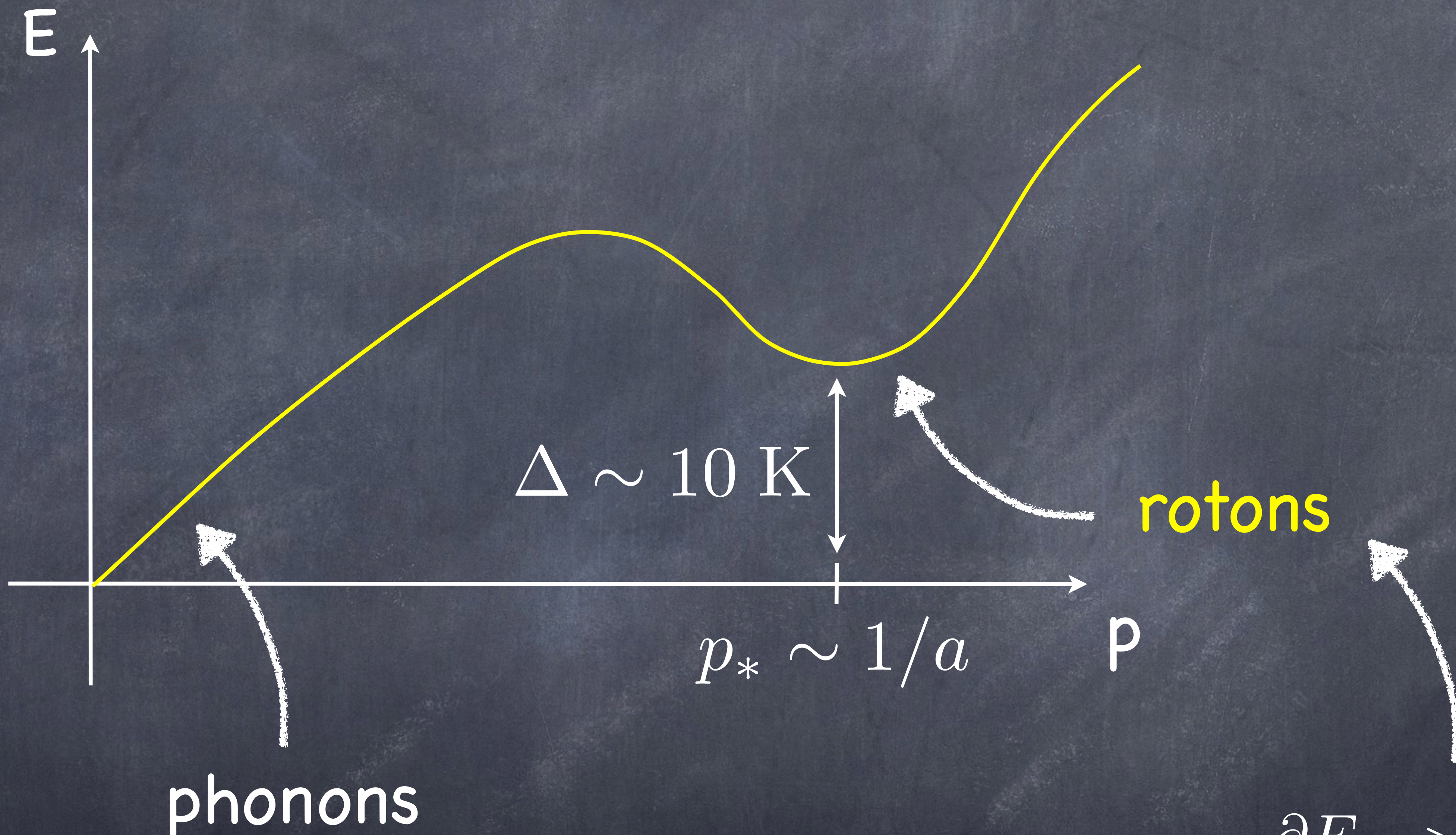




$$\Rightarrow \quad \omega_p = \frac{3\Gamma}{8\pi c_s^2}\,\omega_x\omega_y\,\log(R_\perp/\Lambda)$$

Rotons

Elementary excitations in Helium 4



$$v = c_s$$

$$v = \frac{\partial E}{\partial p} \gtreqless 0$$

(Donnelly 1997)

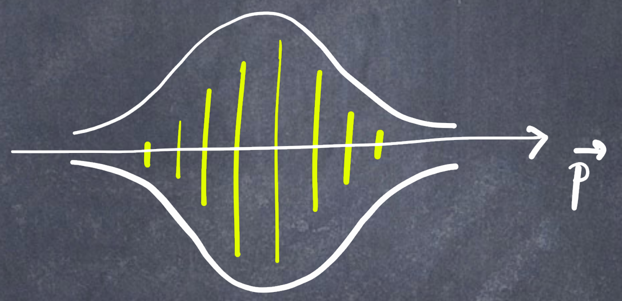
Effective point particle theory

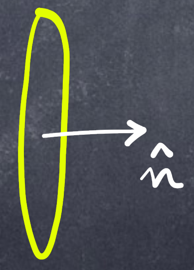
translations $\rightarrow \emptyset$

rotations $\rightarrow$ rotations about velocity

(boosts broken by medium)

Good for

- phonon = 

- vortex ring = 

- roton = ?

Action

data

$$S = \int dt\, f\left(|\dot{\vec{X}} - \vec{u}|, \rho\right)$$

local
fluid velocity

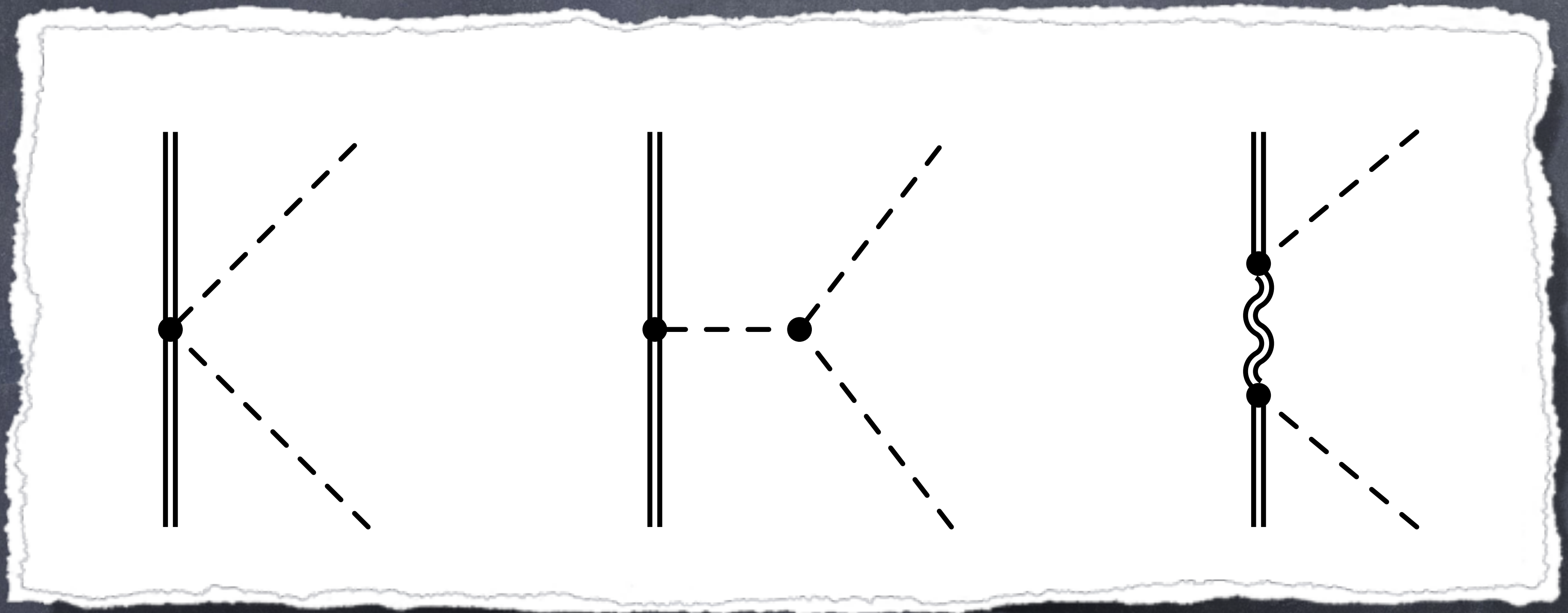
local
density

background
phonons (or flow)

$$f = \text{Legendre transform of } E(p)$$

Describes interactions with soft phonons (or fluid flows)

E.g.:



hard phonon-soft phonon ⟹ matches previous result

roton-soft phonon ⟹ corrections to Landau-Khalatnikov, 1949

On floating and sinking

Archimedes

No obvious Archimedean principle for sound waves, vortex lines, rotons...

EFT again: gravity = gauge field for spacetime symmetries

transformation properties under spacetime symmetries $\Rightarrow$ coupling to gravity

In our case: $\mu \to \mu - \Phi$

Phonons

Phonons "float":

$$m_g = -\frac{d\log c_s}{d\log\rho}\cdot\frac{E}{c_s^2}$$

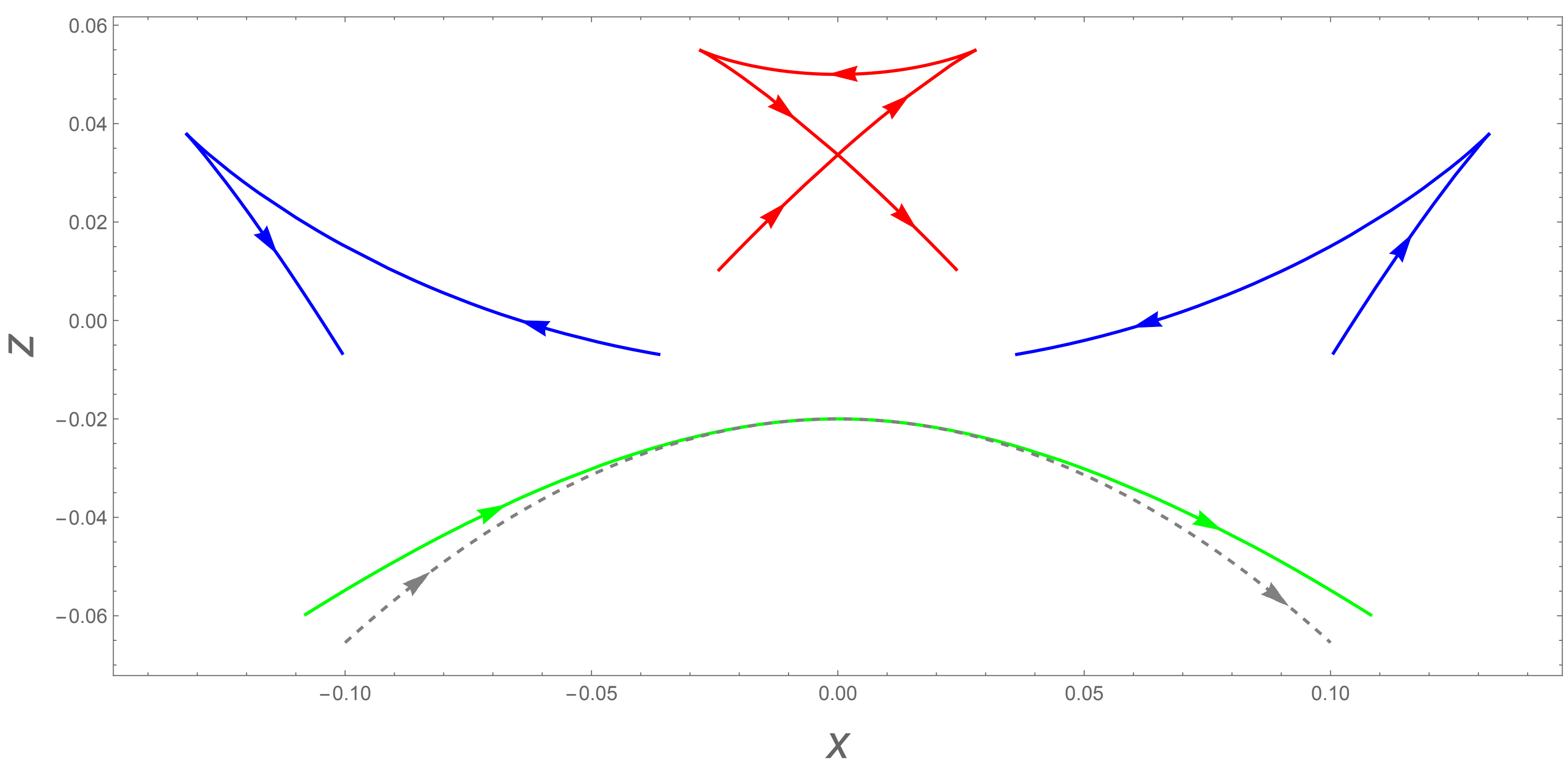
Equivalent to standard refraction:

$$\frac{\cos\theta}{c_s(z)} = \text{const}$$

(Snell's law)

Rotons

Rotons "sink": $$m_g = -\rho \frac{d\Delta}{dP} \sim + \frac{\Delta}{c_s^2}$$



Separation at finite temperature? Measurable?

The mass of sound

$$m_g = -\frac{d \log c_s}{d \log \rho} \cdot \frac{E}{c_s^2}$$

This is also the mass carried by a classical wave:

$$m_g = \int d^3x \, \delta\rho \qquad \text{(at 2nd order)}$$

Can me measure it? Seismic waves? Cold atoms?

Summary

- Vortex lines, vortex rings, rotons: very unconventional mechanical systems
- Important degrees in freedom in superfluids
- EFT: efficient tool to understand them
- Only systematic tool to couple them to bulk fields (sound, gravity, ...)
- Experiments?