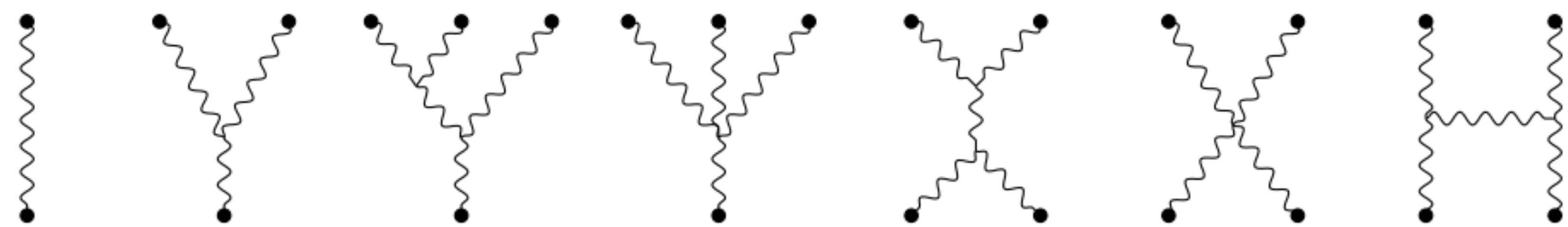
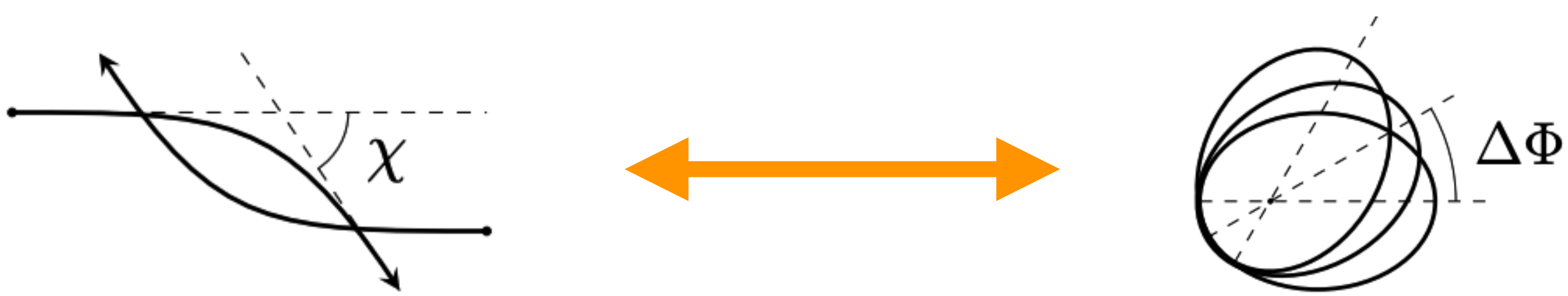
Post-Minkowskian Effective Field Theory Approach to *(Classical)* Gravitational Dynamics





Rafael A. Porto

based on work
with G. Kälin and Z. Liu

1910.03008 1911.09130
2006.01184 2007.04977 2008.06047

DESY.

“EFT methods from bound states to binary systems”

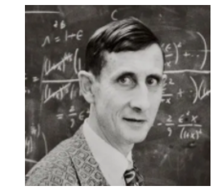
Padova 2020

erc

LHC2LISA

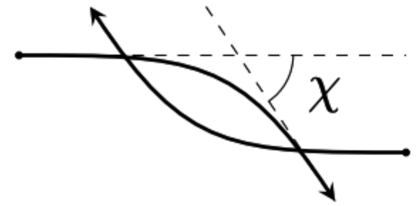
"New directions in science are launched by new tools much more often than by new concepts. The effect of a concept-driven revolution is to explain old things in a new way. The effect of a tool-driven revolution is to discover new things that have to be explained"

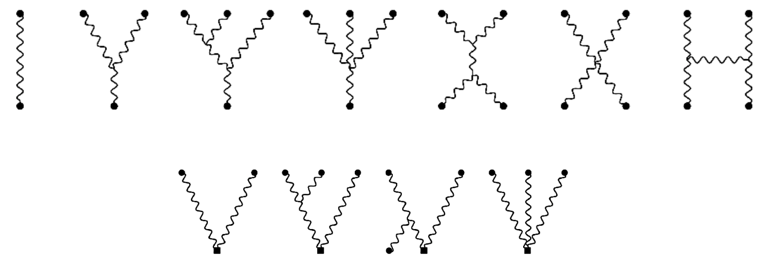
Freeman Dyson, "Imagined Worlds"



- Part I: EFT

  2006.01184
  2007.04977
  2008.06047



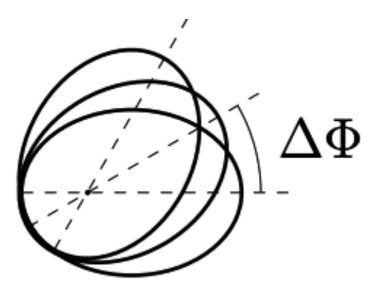


$$\Delta p_a^\mu = -\eta^{\mu\nu} \int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\rm eff}}{\partial x_a^\nu}(x_a(\tau_a))$$

- Part II: B2B

  1910.03008

  1911.09130

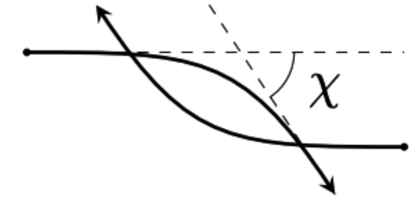


$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$

$$i_r \equiv \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\chi_j^{(2n)}}{(1-2n)j^{2n}}\right)$$

Kalin RAP
2006.01184

EFT for scattering



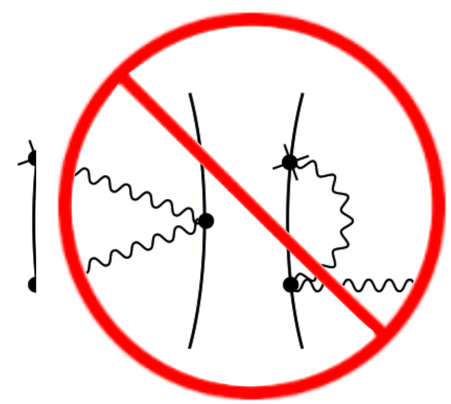
Effective action saddle (classical) approx to any order in G:

(See Ira's, Peter's, Riccardo's talks)

$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

(See Thibault's talk)

We can use the proper time for the WL action (source)



Caveat: No spin
Nor finite-size

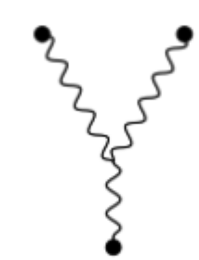
$$S_{\rm pp} = -\sum_a \frac{m_a}{2}\int {\rm d}\tau_a\, g_{\mu\nu}(x_a(\tau_a))v_a^\mu(\tau_a)v_a^\nu(\tau_a)\,.$$

Standard De-Donder propagator (more on ieps later)

$$\langle h_{\mu\nu}(x)h_{\alpha\beta}(y)\rangle = \frac{i}{k^2}P_{\mu\nu\alpha\beta}e^{ik\cdot(x-y)}$$

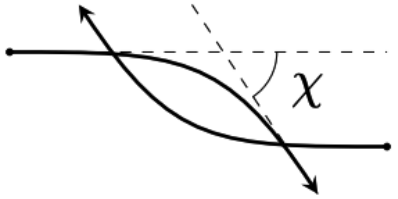
Simplified Feynman rules through GF and total derivatives (but no field redef.)

$$M_{\rm Pl}\mathcal{L}_{hhh} = -\frac{1}{2}h^{\mu\nu}\partial_\mu h^{\rho\sigma}\partial_\nu h_{\rho\sigma} + \frac{1}{2}h^{\mu\nu}\partial_\rho h\partial^\rho h_{\mu\nu} - \frac{1}{8}h\partial_\rho h\partial^\rho h$$
$$+\, h^{\mu\nu}\partial_\nu h_{\rho\sigma}\partial^\sigma h_\mu{}^\rho - h^{\mu\nu}\partial_\sigma h_{\nu\rho}\partial^\sigma h_\mu{}^\rho + \frac{1}{4}h\partial_\sigma h_{\nu\rho}\partial^\sigma h^{\nu\rho}\,.$$



Kalin RAP
2006.01184

EFT for scattering



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(See Thibault's talk)

Post-Minkowskian solution to the equation of motion (Euler eqs.)

$$v_a^\mu(\tau_1) = u_a^\mu + \sum_n \delta^{(n)} v_a^\mu(\tau_a)\,,$$

$$x_a^\mu(\tau_1) = b_a^\mu + u_a^\mu \tau_a + \sum_n \delta^{(n)} x_a^\mu(\tau_a)\,,$$

The true classical motion

Compute total impulse from the action…

$$\Delta p_a^\mu = -\eta^{\mu\nu} \int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\text{eff}}}{\partial x_a^\nu}(x_a(\tau_a))$$

… and deflection angle in the centre-of-mass

$$2\sin\left(\frac{\chi}{2}\right) = \chi - \frac{1}{24}\chi^3 + \mathcal{O}(\chi^5) = \frac{|\Delta \boldsymbol{p}_{1\text{cm}}|}{p_\infty} = \frac{\sqrt{-\Delta p_1^2}}{p_\infty}\,,$$

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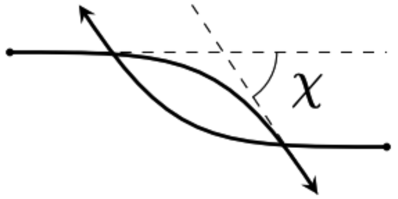
2006.01184

EFT for scattering



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Compute total impulse from the action…

$$\Delta^{(n)} p_a^\mu = \sum_{k\leq n} \Delta^{(n)}_{\mathcal{L}_k} p_a^\mu,$$

Lower order PM terms
Contribute through 'iterations'

$$\Delta^{(n)}_{\mathcal{L}_k} p_a^\mu \equiv -\eta^{\mu\nu} \int_{-\infty}^{+\infty} d\tau_a \left(\frac{\partial}{\partial x_a^\nu} \mathcal{L}_k\Big[b_a + u_a\tau_a + \sum_{r=0}^{n-k} \delta^{(r)} x_a\Big] \right)_{\mathcal{O}(G^n)}.$$

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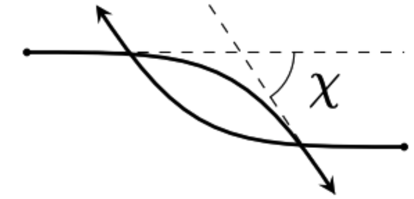
EFT for scattering



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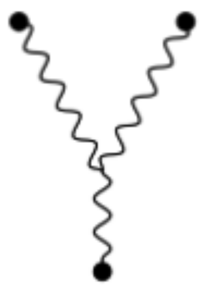


$(v \ll 1)$

Potential Modes $(k_0 \ll |\boldsymbol{k}|)$

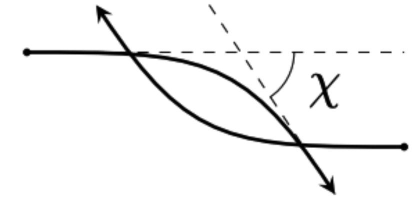
Radiation Modes $(k_0 \sim |\boldsymbol{k}|)$

Dissipative/Flux (Imaginary part with Feynman ieps)

Conservative (Real-part time-reversal invariant)

(relativistic) ‘Soft’ Region

EFT for scattering



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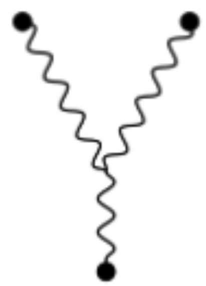


Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes
$(k_0 \sim |\boldsymbol{k}|)$

Conservative
(Real-part
time-reversal invariant)

(relativistic) ‘Soft’ Region

IR finite!

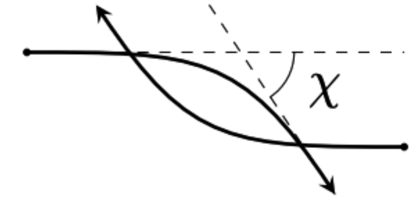
Differential Equations +
b.c. from **entire region**

$$\partial_x \vec{h}(x,\epsilon) = \epsilon\, \mathbb{M}(x)\, \vec{h}(x,\epsilon)$$

Single scale!

(see Julio’s and Carlo’s talks)

EFT for scattering



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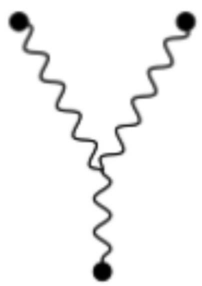

Potential Modes

$(k_0 \ll |\boldsymbol{k}|)$

$$\frac{1}{p_0^2-\boldsymbol{p}^2} \simeq -\frac{1}{\boldsymbol{p}^2}\left(1+\frac{p_0^2}{\boldsymbol{p}^2}+\cdots\right).$$

Off-shell modes: ieps-prescription is irrelevant

<u>But we want to keep all the (special-) relativistic corrections!</u>

Cheung et al. 1808.02489

Bern et al. 1908.01493

Energy integrals + Post-Newtonian resummation

$$\int \frac{{\rm d}k^0}{2\pi}(\cdot) = \frac{i}{2}\left[\sum_{k^0_\star\in H^+}\underset{k^0=k^0_\star}{\rm Res}(\cdot) - \sum_{k^0_\star\in H^-}\underset{k^0=k^0_\star}{\rm Res}(\cdot)\right],$$

Differential Equations + b.c. from **<u>potentials</u>**

$$\partial_x \vec{h}(x,\epsilon) = \epsilon\,\mathbb{M}(x)\,\vec{h}(x,\epsilon)$$

Kalin Liu RAP 2007.04977

Parra-Martinez Ruff and Zeng 2005.04236

Kalin RAP
2006.01184

EFT for scattering: NLO

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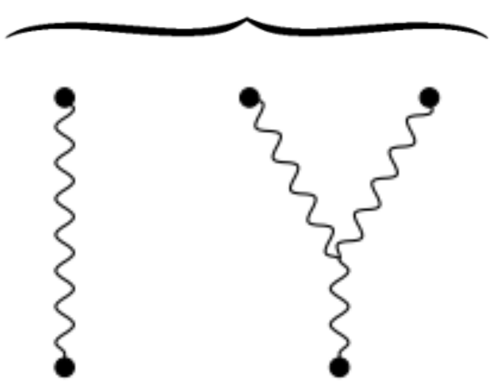
Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

$$\Delta p_a^\mu = -\eta^{\mu\nu}\int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\text{eff}}}{\partial x_a^\nu}(x_a(\tau_a))$$

2PM



$$\Delta p_1^\mu = -\frac{G m_1 m_2 b^\mu}{|b^2|}\left(\frac{2\left(2\gamma^2-1\right)}{\sqrt{\gamma^2-1}} + \frac{3\pi}{4}\frac{\left(5\gamma^2-1\right)}{\sqrt{\gamma^2-1}}\frac{GM}{|b^2|^{1/2}}\right)$$
$$+\,2\frac{m_1 m_2\left(2\gamma^2-1\right)^2}{(\gamma^2-1)^2}\frac{G^2}{|b^2|}\left((\gamma m_2+m_1)u_2^\mu - (\gamma m_1+m_2)u_1^\mu\right)\,,$$

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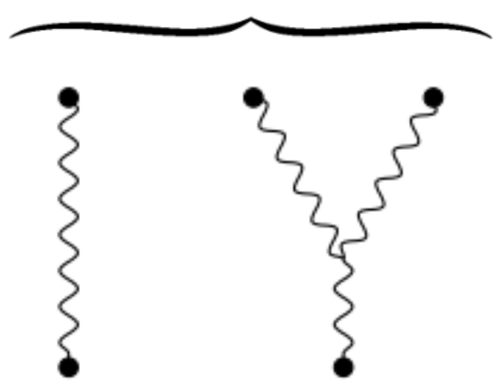
Potential
Modes
$(k_0 \ll |\boldsymbol{k}|)$

Probe limit =
two-body to 2PM
(explicit mass scaling
from diagram+mirror)

Kalin RAP
1910.03008

$$2p_\infty^{\text{test}}(\mathcal{E}_0 \to \gamma)/(2p_\infty(\gamma)) = \Gamma$$

2PM



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On-shell

$$(p_1+\Delta p_1)^2 = p_1^2\,,\quad 2p_1\cdot\Delta p_1 = -\Delta p_1^2\,.$$

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Potential
Modes
$(k_0 \ll |\boldsymbol{k}|)$

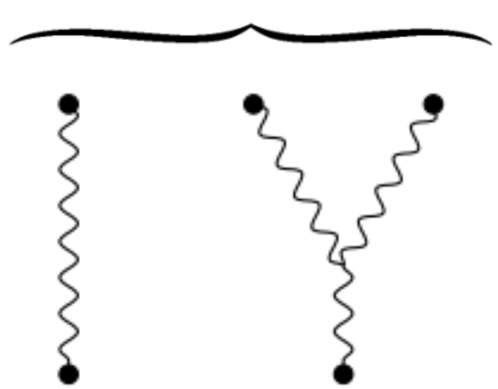
Explicit computation:

$$\Delta^{(2)} p_1^\mu = \Delta^{(2)}_\triangle p_1^\mu + \Delta^{(2)}_u p_1^\mu\,,$$

$$\Delta^{(2)}_\triangle p_1^\mu = -\frac{3m_1m_2^2(5\gamma^2-1)}{256M_{\rm Pl}^4}\frac{\partial}{\partial b_\mu}\int_{k,\ell}\frac{\hat\delta(k\cdot u_2)\hat\delta(\ell\cdot u_2)\hat\delta(\ell\cdot u_1)}{k^2(\ell-k)^2}e^{i\ell\cdot b}\,,$$

$$\Delta^{(2)}_u p_1^\mu = i\frac{m_1m_2^2}{128M_{\rm Pl}^4}\int_{k,\ell}(2\gamma^2-1)^2\,\frac{(\ell^\mu-k^\mu)\ell^2\hat\delta(k\cdot u_2)\hat\delta(\ell\cdot u_2)\hat\delta(\ell\cdot u_1)}{k^2(\ell-k)^2(k\cdot u_1-i\epsilon)^2}e^{i\ell\cdot b}\,.$$

2PM



Contains the iteration:

$$\delta^{(1)}x_1^\mu(\tau_1) = -\frac{m_2}{8M_{\rm Pl}^2}\left((2\gamma^2-1)\eta^{\mu\nu}-2(2\gamma u_2^\mu-u_1^\mu)u_1^\nu\right)\times\int_k\frac{ik_\nu\,\hat\delta(k\cdot u_2)\,e^{ik\cdot b}}{k^2(k\cdot u_1-i0^+)^2}e^{i(k\cdot u_1-i0^+)\tau_1}\,.$$

$$\int\frac{{\rm d}k^0}{2\pi}(\cdot) = \frac{i}{2}\left[\sum_{k^0_\star\in H^+}\mathop{\rm Res}_{k^0=k^0_\star}(\cdot) - \sum_{k^0_\star\in H^-}\mathop{\rm Res}_{k^0=k^0_\star}(\cdot)\right],$$

causality

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2006.01184

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Potential Modes
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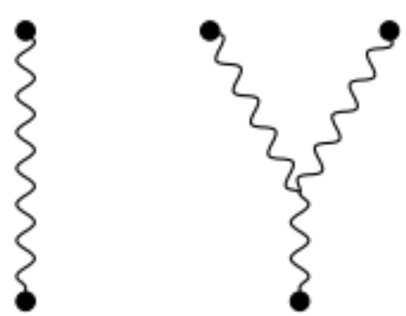
2PM



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$$u^\mu\int_q\frac{\hat\delta(q\cdot u_2)}{(\ell-q)^2q^2(q\cdot u_1-i\epsilon)}\,.\qquad b^\mu\int_q\frac{\hat\delta(q\cdot u_2)}{(\ell-q)^2q^2(q\cdot u_1-i\epsilon)^2}\,.$$

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There is no “box”.
Related to the “crossed-box”

Kalin RAP
2006.01184

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Explicit computation:

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Does not matter at 2PM in D=4 $\quad b\cdot u = 0$

$$\Delta^{(2)}_\triangle p_1^\mu = -\frac{3m_1 m_2^2(5\gamma^2-1)}{256 M_{\rm Pl}^4}\frac{\partial}{\partial b_\mu}\int_{k,\ell}\frac{\hat\delta(k\cdot u_2)\hat\delta(\ell\cdot u_2)\hat\delta(\ell\cdot u_1)}{k^2(\ell-k)^2}e^{i\ell\cdot b}\,,$$

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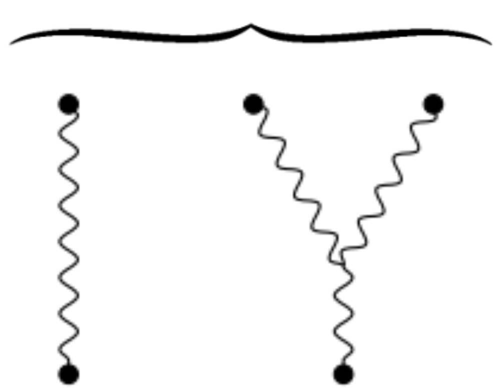

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Related to the "crossed-box"

Vanishes in D=4 (**only!**).

Kalin RAP
2006.01184

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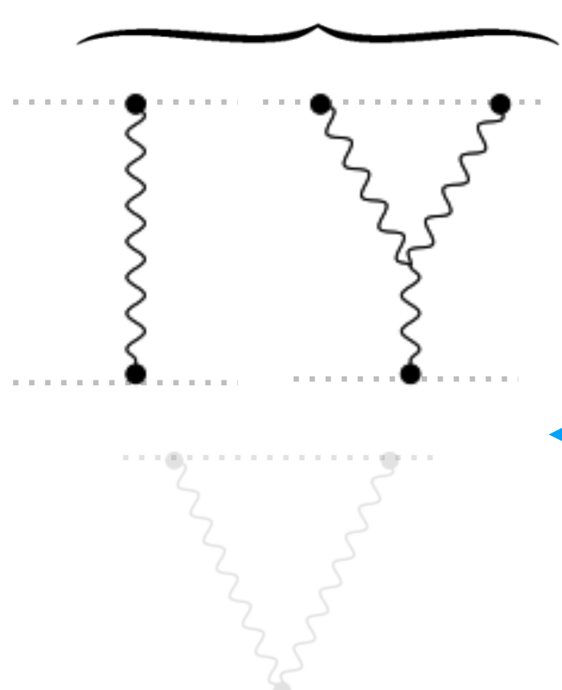
2PM



Cheung et al. 1808.02489 Bern et al. 1908.01493

The left-over part takes the form of an eikonal phase:

$$\theta^{(2)}_{\rm eik} \equiv \frac{\mu M^2}{256 M_{\rm Pl}^4} \frac{3(5\gamma^2-1)}{\sqrt{\gamma^2-1}} \int_{\boldsymbol{k},\boldsymbol{\ell}_\perp} \frac{1}{\boldsymbol{k}^2(\boldsymbol{\ell}_\perp-\boldsymbol{k})^2} e^{-i\boldsymbol{\ell}_\perp\cdot\boldsymbol{b}_\perp} = \frac{3\pi(5\gamma^2-1)}{4\sqrt{\gamma^2-1}} \frac{\mu(GM)^2}{|\boldsymbol{b}_\perp|}\,.$$

$$\theta^{(2)}_{\rm eik}(\boldsymbol{b}_\perp) = \frac{1}{4\mu M\sqrt{\gamma^2-1}} \int_{\boldsymbol{q}_\perp} \mathcal{M}^{(2)}_{\rm cl}(\boldsymbol{q}_\perp) e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}\,,$$

$$\mathcal{M}^{(2)}_{\rm cl}(\boldsymbol{q}) = (5\gamma^2-1)\frac{6\pi^2 G^2\mu^2 M^3}{|\boldsymbol{q}|}\,,$$

Kalin RAP
2006.01184

EFT for scattering: NNLO

Effective action saddle (classical) approx to any order in G:

$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

$$\Delta p_a^{\mu} = -\eta^{\mu\nu}\int_{-\infty}^{+\infty} {\rm d}\tau_a \frac{\partial \mathcal{L}_{\rm eff}}{\partial x_a^{\nu}}(x_a(\tau_a))$$

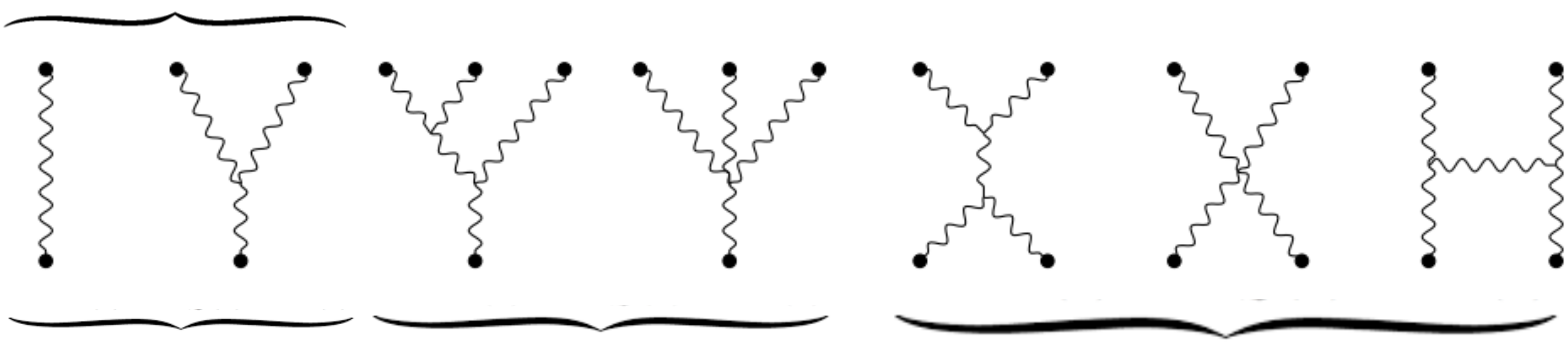
Potential Modes

$(k_0 \ll |\boldsymbol{k}|)$

along the b-direction

$$\int_q \hat{\delta}(q\cdot u_1)\hat{\delta}(q\cdot u_2)\, iq^{\mu}\, t^{s}\, M^{(a,\tilde{a})}_{n_1 n_2; i_1\cdots i_5}(q,\gamma) e^{iq\cdot b}$$

2PM



3PM
(from iterations)

3PM
(from 1pt)

3PM
(from GSF1)

Kalin Liu RAP
2007.04977

EFT for scattering: NNLO

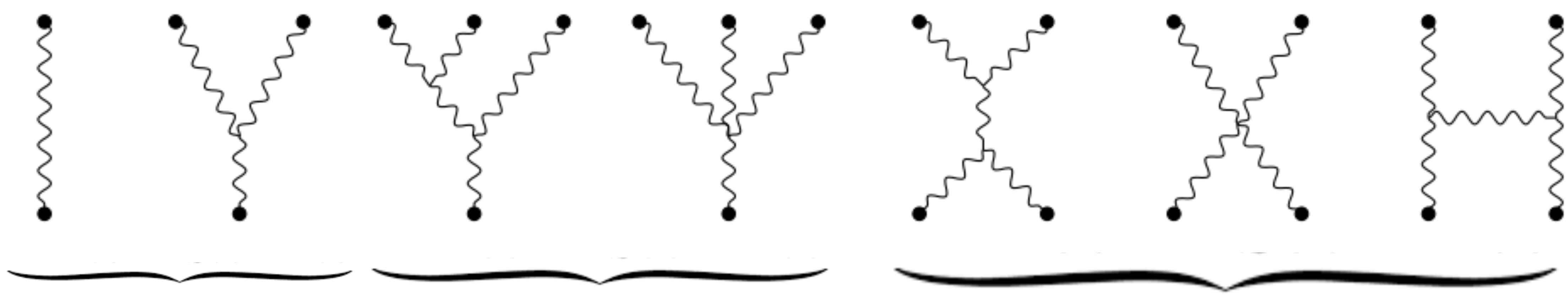
Integrals (one family):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1 \cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1 \cdot u_a)\hat{\delta}(k_2 \cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\tilde{\not{a}}} D_1^{i_1} \cdots D_5^{i_5}},$$

$$A_{1,\not{a}} = k_1 \cdot u_{\not{a}},\; A_{2,\tilde{\not{a}}} = k_2 \cdot u_{\tilde{\not{a}}},\; D_1 = k_1^2,\; D_2 = k_2^2,$$
$$D_3 = (k_1+k_2-q)^2,\; D_4 = (k_1-q)^2,\; D_5 = (k_2-q)^2.$$

***POTENTIAL REGION*: DFQ with b.c. from the static limit of NRGR!**

(See Julio's and Carlo's talk)



3PM
(from iterations)

3PM
(from 1pt)

3PM
(from GSF1)

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2007.04977

EFT for scattering: NNLO

(See Zvi & David and Emil's talk)

Integrals (one family):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1 \cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1 \cdot u_a)\hat{\delta}(k_2 \cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D^{i_1}_1 \cdots D^{i_5}_5},$$

$$A_{1,\not{a}} = k_1 \cdot u_{\not{a}},\ A_{2,\not{\tilde{a}}} = k_2 \cdot u_{\not{\tilde{a}}},\ D_1 = k_1^2,\ D_2 = k_2^2,$$
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***POTENTIAL REGION*: DFQ with b.c. from the static limit of NRGR!**

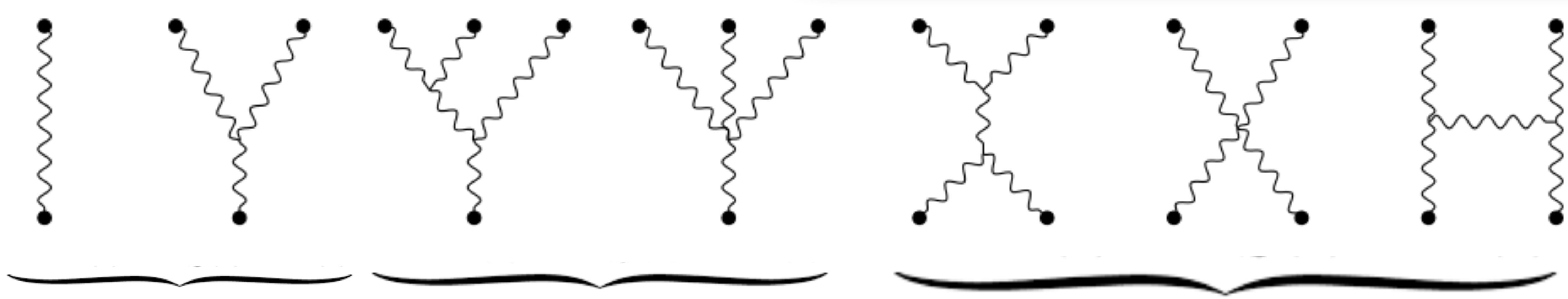
(See Julio's and Carlo's talk)

Advantages:

- ***We land in the soft-expanded cut-version of the integrand***
- No super-classical divergences
- ***On-shell philosophy***: No potential, EFT-matching nor Born iterations

Main Drawback:

- Feynman diagrams (though significantly fewer than NRGR)



3PM
(from iterations)

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3PM
(from GSF1)

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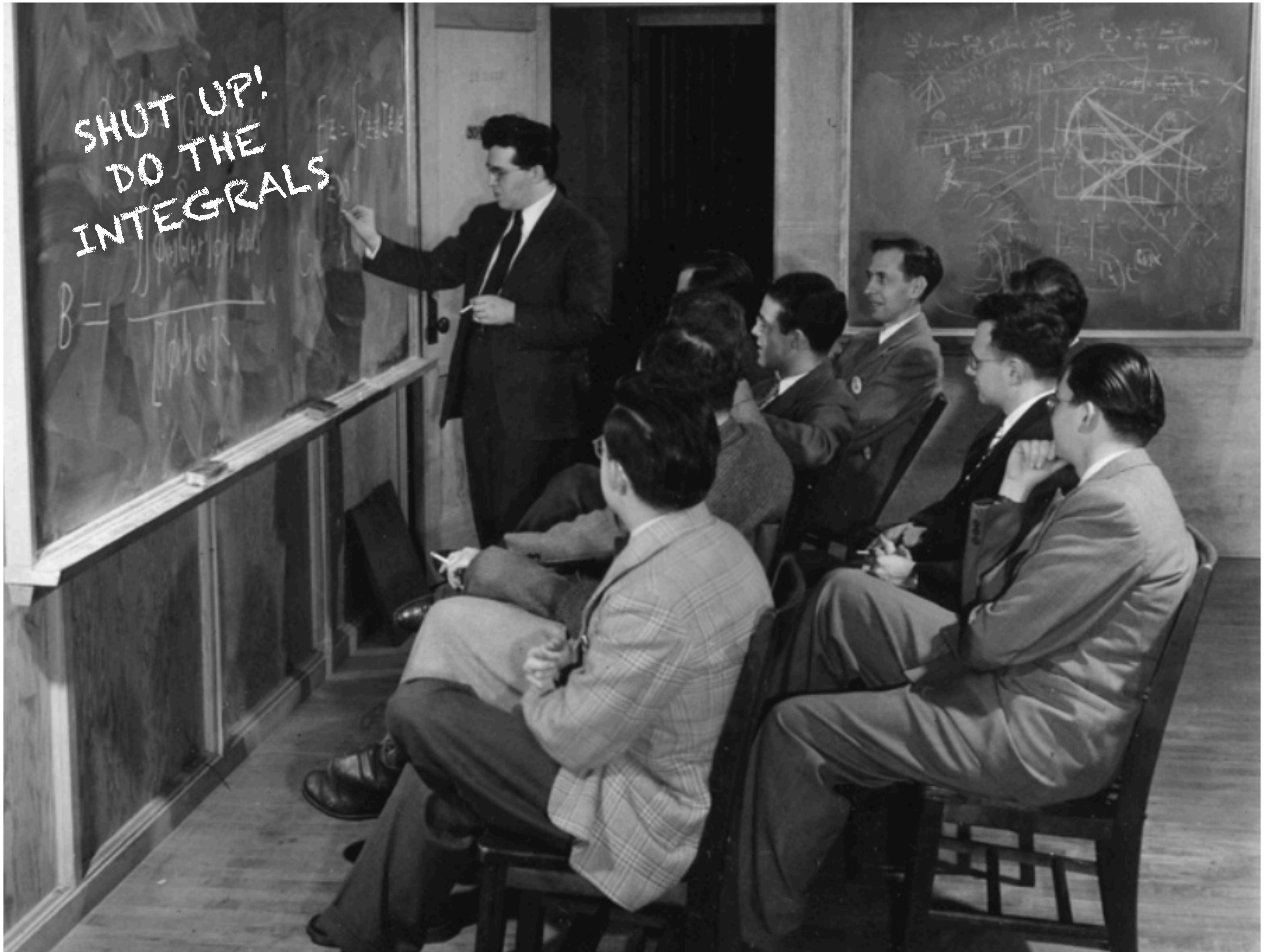
2007.04977

EFT for scattering: NNLO



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EFT for scattering: NNLO

Integrals (one family):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1 \cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1\cdot u_a)\hat{\delta}(k_2\cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D^{i_1}_1 \cdots D^{i_5}_5},$$

$$A_{1,\not{a}} = k_1\cdot u_{\not{a}},\ A_{2,\not{\tilde{a}}} = k_2\cdot u_{\not{\tilde{a}}},\ D_1 = k_1^2,\ D_2 = k_2^2,$$
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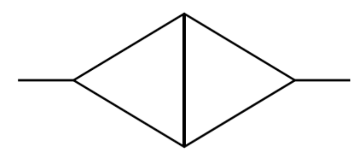
***POTENTIAL REGION*: DFQ with b.c. from the static limit of NRGR!**

"One-point functions":

$$M^{(1,1)}_{n_1,n_2;\cdots} \quad (n_1,n_2) \leq 0,$$

easily computed in the rest-frame

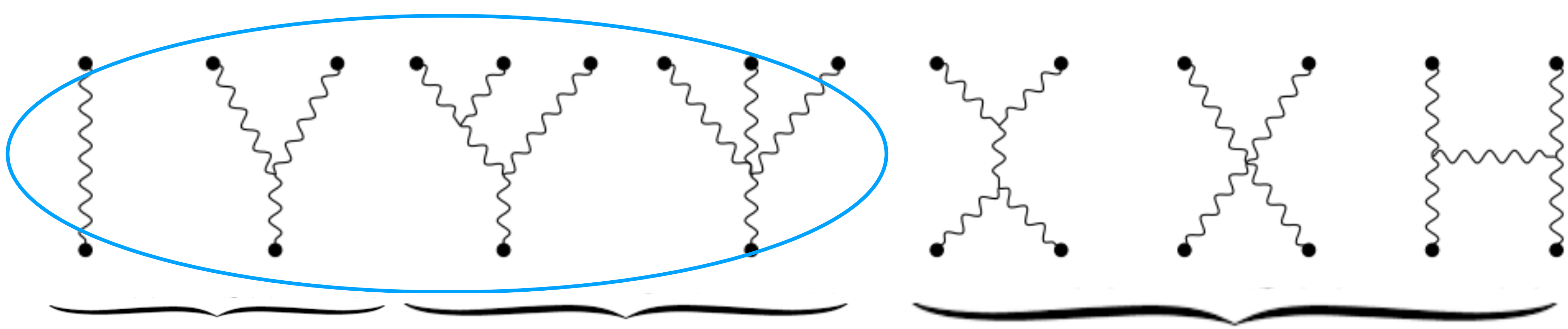
$$u_1 = (1,0,0,0),\ u_2 = (\gamma,\gamma\beta,0,0),$$



$$\beta\gamma = \sqrt{\gamma^2-1}$$

$$\mathbf{K}_{11100} = (4\pi)^{-3} t^{-2\epsilon} e^{2\gamma_E \epsilon} \frac{\Gamma^3(1/2-\epsilon)\,\Gamma(2\epsilon)}{\Gamma(3/2-3\epsilon)}, \ *$$

* poles leads to a contact-term ("soft expansion")



3PM (from iterations)

3PM (from 1pt)

3PM (from GSF1)

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2007.04977

EFT for scattering: NNLO

Integrals (one family):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1\cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1\cdot u_a)\hat{\delta}(k_2\cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D_1^{i_1}\cdots D_5^{i_5}},$$

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$$D_3 = (k_1+k_2-q)^2,\ D_4 = (k_1-q)^2,\ D_5 = (k_2-q)^2.$$

***<u>POTENTIAL REGION</u>*: DFQ with b.c. from the static limit of NRGR!**

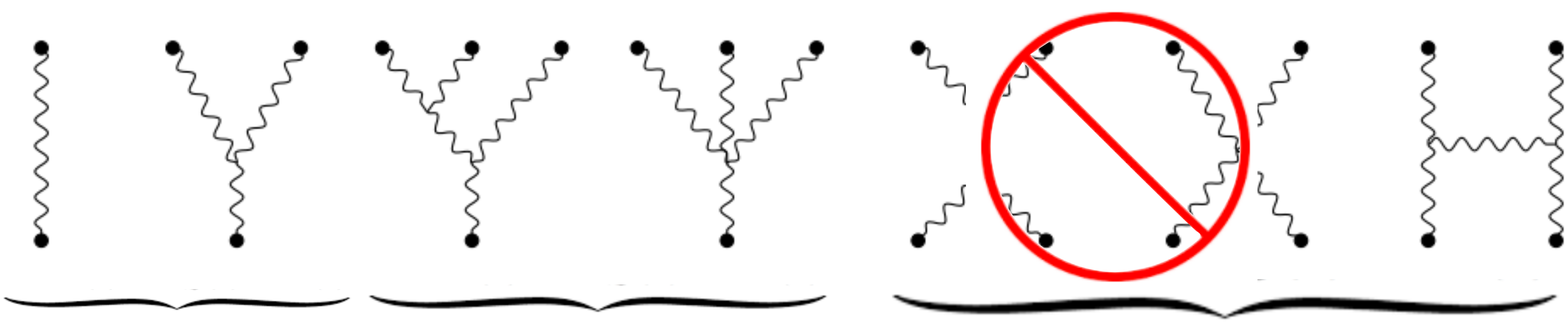
X and YY*

the integrals factorize:

$$\int_{k_1} \frac{\delta(k_1\cdot u_1)}{k_1^2(k_1-q)^2} \times \int_{k_2} \frac{\delta(k_1\cdot u_2)}{k_2^2(k_2-q)^2}$$

Does not lead to a long-range force!

vanish at 3PM (also 2PN)



3PM (from iterations)

3PM (from 1pt)

3PM (from GSF1)

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2007.04977

EFT for scattering: NNLO

Integrals (one family):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1 \cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1\cdot u_a)\hat{\delta}(k_2\cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D_1^{i_1}\cdots D_5^{i_5}},$$

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$$D_3 = (k_1+k_2-q)^2,\ D_4 = (k_1-q)^2,\ D_5 = (k_2-q)^2.$$

***POTENTIAL REGION*: DFQ with b.c. from the static limit of NRGR!**

O(nu) Iterations and H-diagram

One scale DFQ & w/out linear props.

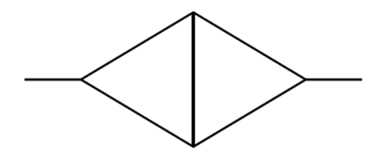
$$\{I_{11111}, I_{11211}, I_{01101}, I_{11011}, I_{00211}, I_{00112}, I_{00111}\},$$

$$I_{i_1\cdots i_5} \equiv M^{(1,2)}_{00;i_1\cdots i_5}.$$

$$\mathrm{d}\vec{f} = \epsilon\left(\mathbb{H}_0\,\mathrm{d}\log(x) + \mathbb{H}_+\,\mathrm{d}\log(1+x) + \mathbb{H}_-\,\mathrm{d}\log(1-x)\right)\vec{f}$$

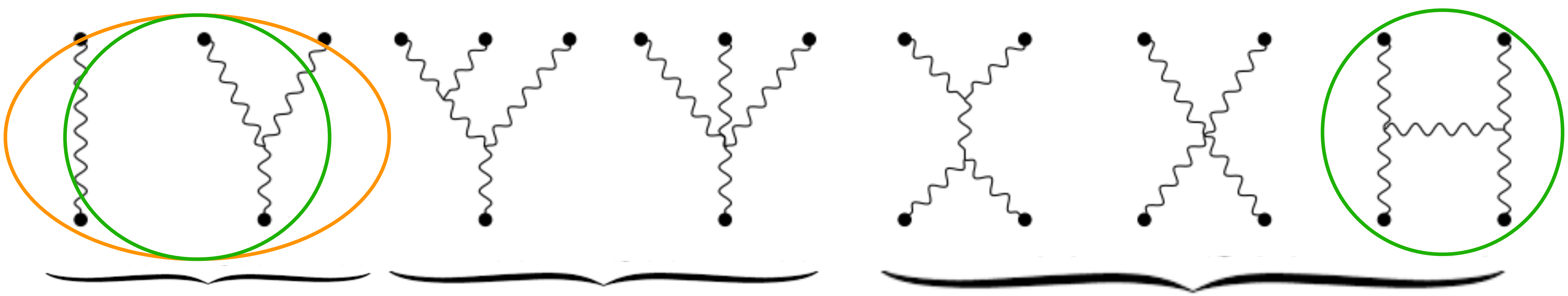
$$\gamma = \frac{1+x^2}{2x},$$

b.c:



$(\gamma \to 1)$

equivalent to cH basis in 2005.04236



3PM
(from iterations)

3PM
(from 1pt)

3PM
(from GSF1)

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2007.04977

EFT for scattering: NNLO

Integrals (one family):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1\cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1\cdot u_a)\hat{\delta}(k_2\cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D_1^{i_1}\cdots D_5^{i_5}},$$

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POTENTIAL REGION: **DFQ with b.c. from the static limit of NRGR!**

O(nu) Iterations and H-diagram

Some of the iteration: *

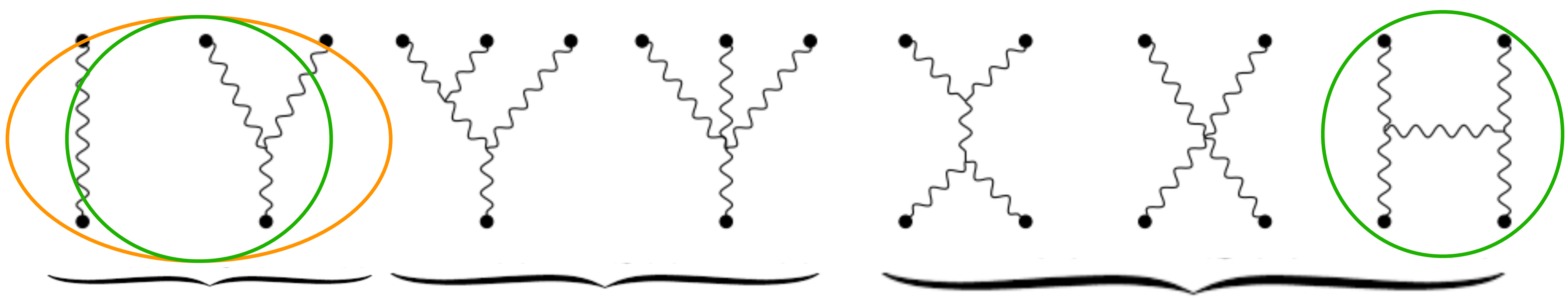
$$\int \hat{d}^D k_2 \hat{d}^D k_3 \frac{\hat{\delta}(k_2\cdot u_1)\,\hat{\delta}(k_3\cdot u_2)}{k_2^2\,k_3^2\,(k_2+k_3-k_1)^2\,(k_2\cdot u_2\pm i0)\,(k_3\cdot u_1\pm i0)}$$

$$f_8 = \epsilon^4\left(\frac{\log^2(x)}{\pi^2}\right) + \text{b.c.} + \mathcal{O}(\epsilon^5)$$

$$\mathcal{I}^{+-} = -\frac{4\pi^2}{6}\int d^{D-2}\ell_1 d^{D-2}\ell_2 \frac{1}{(\ell_1^\perp)^2(\ell_2^\perp)^2(\ell_1^\perp+\ell_2^\perp-q^\perp)^2}$$

$$\mathcal{I} = -\frac{2\pi^4}{3}\frac{\Gamma^3(-\epsilon)\,\Gamma(2\epsilon+1)}{(q^2)^{1+2\epsilon}\,\Gamma(-3\epsilon)} = -\frac{2\pi^2}{\epsilon^2\,q^2}$$

poles cancel!



3PM (from iterations)

3PM (from 1pt)

3PM (from GSF1)

* No crossing in the potential region!

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2007.04977

EFT for scattering: NNLO

Integrals (one family):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1 \cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1 \cdot u_a)\hat{\delta}(k_2 \cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D_1^{i_1} \cdots D_5^{i_5}},$$

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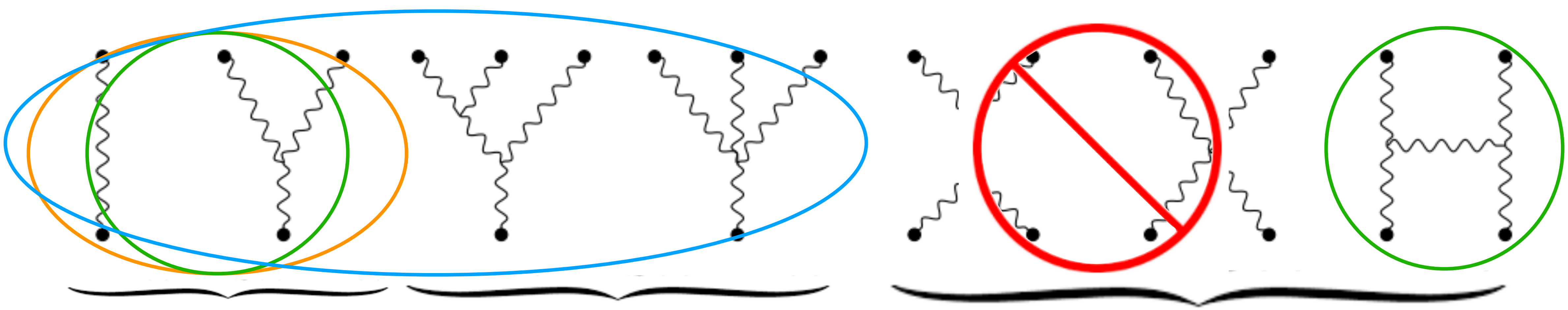
POTENTIAL REGION: **DFQ with b.c. from the static limit of NRGR!**

$$\Delta^{(3)} p_1^\mu = \frac{G^3 b^\mu}{|b^2|^2}\left(\frac{16 m_1^2 m_2^2 (4\gamma^4 - 12\gamma^2 - 3)\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{(\gamma^2-1)} - \frac{4 m_1^2 m_2^2 \gamma (20\gamma^6 - 90\gamma^4 + 120\gamma^2 - 53)}{3(\gamma^2-1)^{5/2}} - \frac{2 m_1 m_2 (m_1^2+m_2^2)(16\gamma^6 - 32\gamma^4 + 16\gamma^2 - 1)}{(\gamma^2-1)^{5/2}}\right)$$
$$+ \frac{3\pi}{2}\frac{(2\gamma^2-1)(5\gamma^2-1)}{(\gamma^2-1)^2}\frac{G^3 M^2 \mu}{|b^2|^{3/2}} \times \left((\gamma m_2 + m_1)u_2^\mu - (\gamma m_1 + m_2)u_1^\mu\right).$$

related to Schw. via b.c. of DFQ

fixed by on-shell condition

Schwarzschild + mirror image



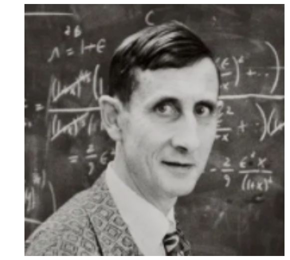
3PM (from iterations)

3PM (from 1pt)

3PM (from GSF1)

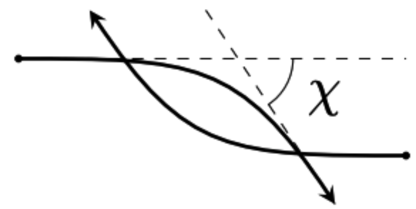
"New directions in science are launched by new tools much more often than by new concepts. The effect of a concept-driven revolution is to explain old things in a new way. The effect of a tool-driven revolution is to discover new things that have to be explained"

Freeman Dyson, "Imagined Worlds"



- Part I: EFT

  2006.01184
  2007.04977
  2008.06047



$$\frac{\chi_b^{(3)}}{\Gamma} = \frac{1}{(\gamma^2-1)^{3/2}}\left[-\frac{4\nu}{3}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25) + \frac{(64\gamma^6-120\gamma^4+60\gamma^2-5)(1+2\nu(\gamma-1))}{3(\gamma^2-1)^{3/2}} - 8\nu(4\gamma^4-12\gamma^2-3)\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}\right],$$

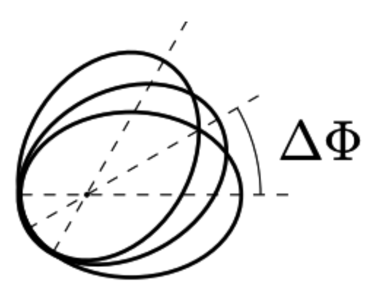
3PM angle <u>agrees</u> with amplitude derivation in

Bern et al. 1908.01493

- Part II: B2B

  1910.03008

  1911.09130

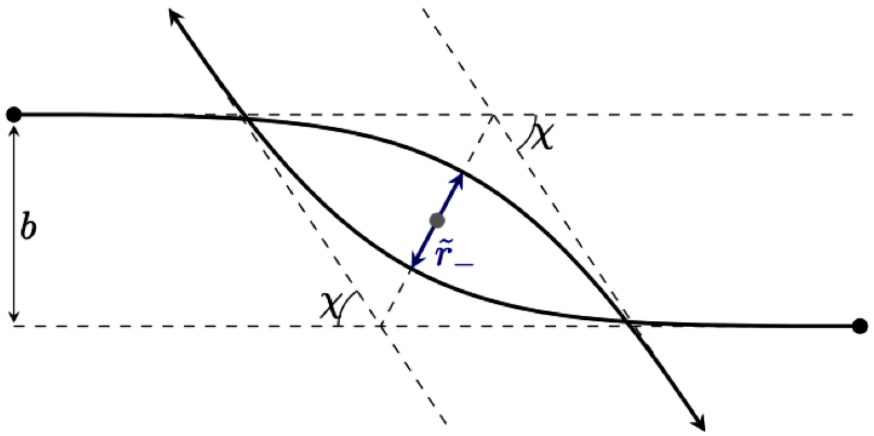


$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$

$$i_r \equiv \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\chi_j^{(2n)}}{(1-2n)j^{2n}}\right)$$

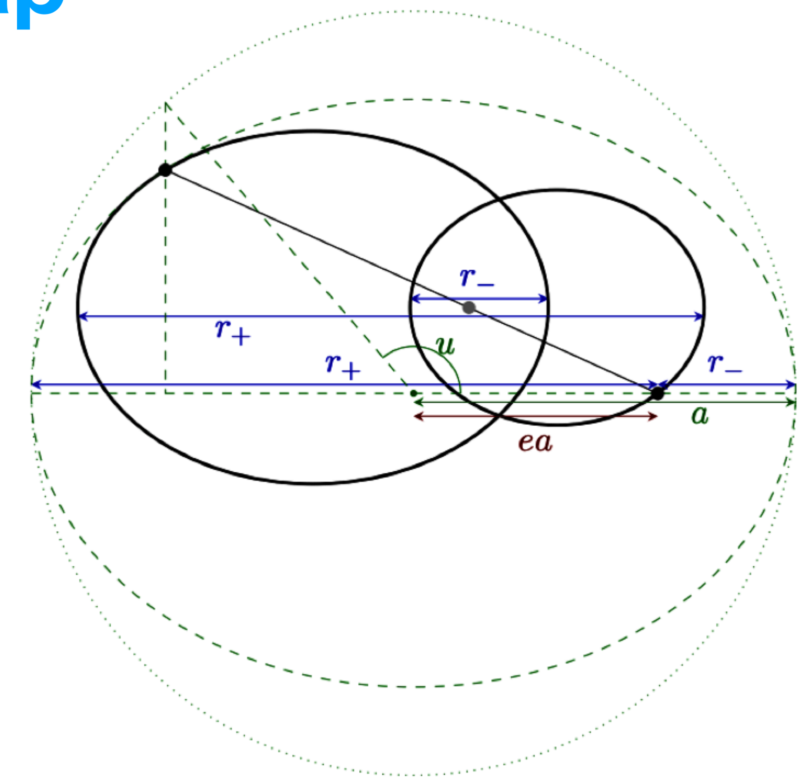
Kalin RAP
1910.03008

Boundary to Bound Map



Can we map gauge -invariant objects?

Do we need the Hamiltonian?



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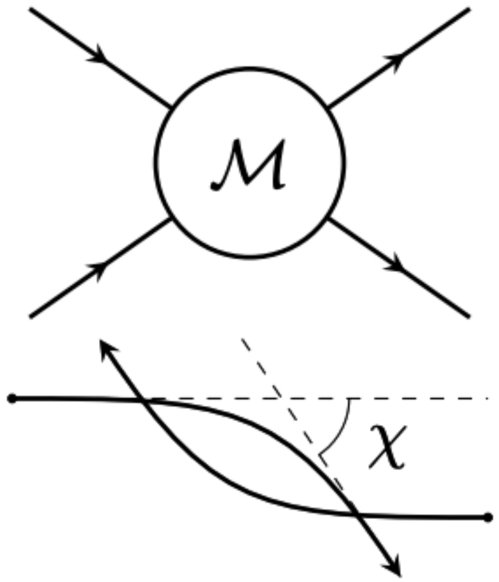
1910.03008

Boundary to Bound Map



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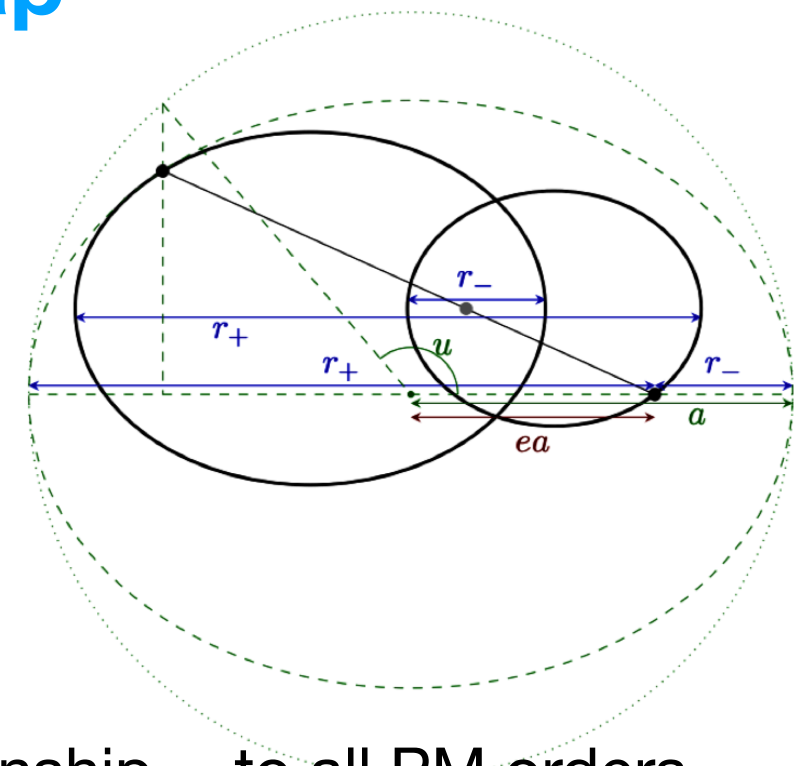

Firsov's formula

$$\bar{p}^2(r,E) = \exp\left[\frac{2}{\pi}\int_{r|\bar{p}(r,E)|}^{\infty} \frac{\chi_b(\tilde{b},E)\mathrm{d}\tilde{b}}{\sqrt{\tilde{b}^2 - r^2\bar{p}^2(r,E)}}\right]$$

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \sum_i P_i(E)\left(\frac{G}{r}\right)^i$$

$$= p_\infty^2(E)\left(1 + \sum_i f_i(E)\left(\frac{GM}{r}\right)^i\right)$$

$$\widetilde{\mathcal{M}}(r,E) = \sum_{n=1}^{\infty} \widetilde{\mathcal{M}}_n(E)\left(\frac{G}{r}\right)^n$$

Algebraic relationship —to all PM orders— scattering angle to coefs. in **Impetus**

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$

'Impetus formula' (local)

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$

Scattering amplitude

$$\widetilde{\mathcal{M}}(r,E) \equiv \frac{1}{2E}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\mathcal{M}(\mathbf{q},\mathbf{p}^2 = p_\infty^2(E))e^{-i\mathbf{q}\cdot\mathbf{r}}$$

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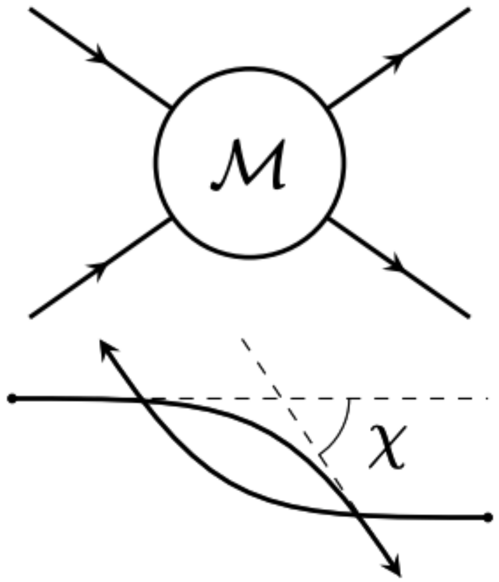
1910.03008

Boundary to Bound Map

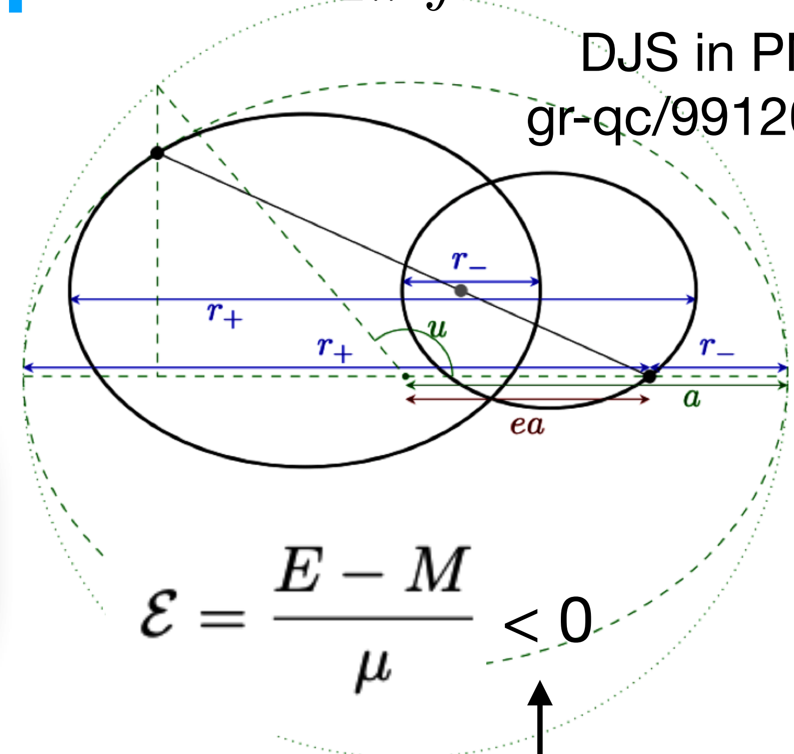
$$\mathcal{S}_r \equiv \frac{1}{2\pi}\oint p_r dr$$

DJS in PN
gr-qc/9912092



B2B radial action through 'impetus'

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$



$$\mathcal{E} = \frac{E-M}{\mu} < 0$$

$$\mathcal{S}_r(J,\mathcal{E}) = \frac{1}{\pi}\int_{r_-}^{r_+} \sqrt{p_\infty^2(\mathcal{E}) + \widetilde{\mathcal{M}}(r,\mathcal{E}) - J^2/r^2}\,\mathrm{d}r$$

$$\delta\mathcal{S}_r(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

Analytic continuation

Scattering amplitude

$$\widetilde{\mathcal{M}}(r,E) \equiv \frac{1}{2E}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\mathcal{M}(\mathbf{q},\mathbf{p}^2 = p_\infty^2(E))e^{-i\mathbf{q}\cdot\mathbf{r}}$$

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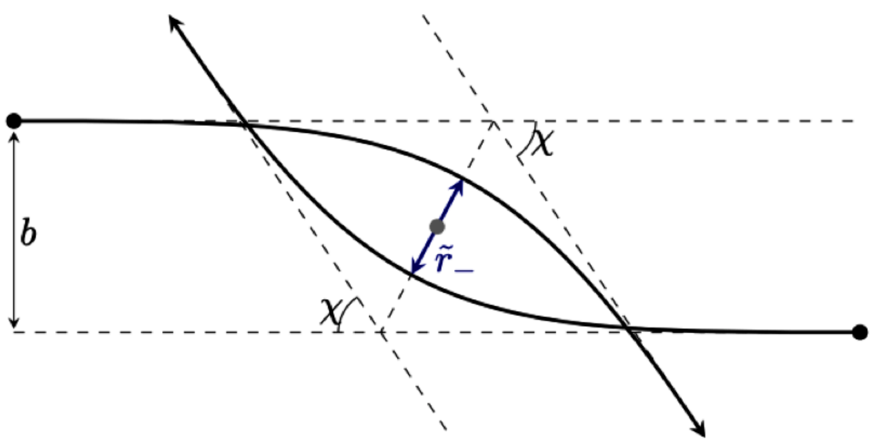
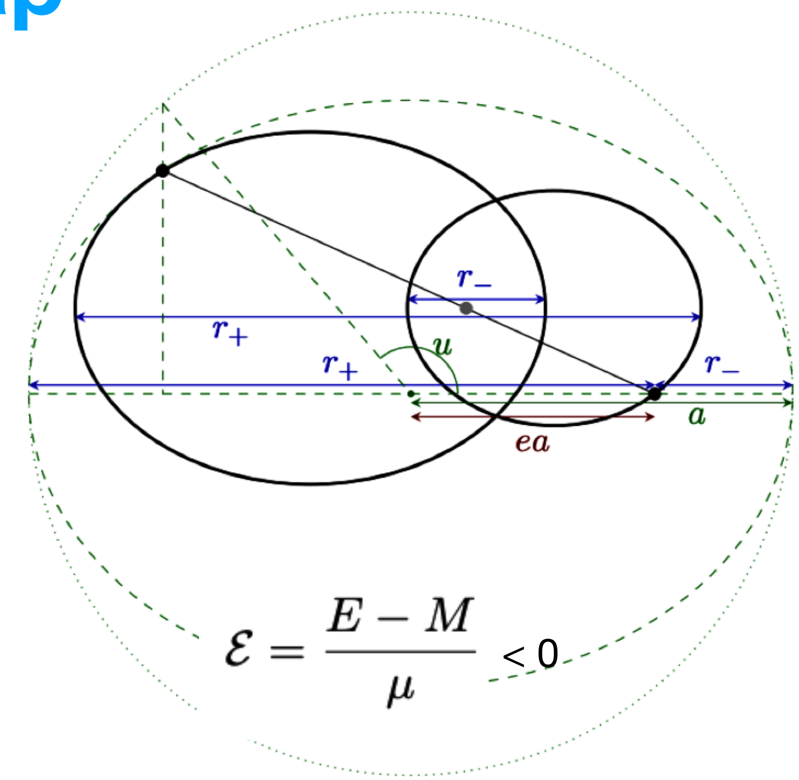
1910.03008

Boundary to Bound Map



Can we relate orbital elements?



$$\mathcal{E} = \frac{E - M}{\mu} < 0$$

$$r = \tilde{a}(\tilde{e}\cosh u - 1) \qquad \text{(Hyperbola)}$$

$$r = a(1 - e\cos u), \qquad \text{(Ellipse)}$$

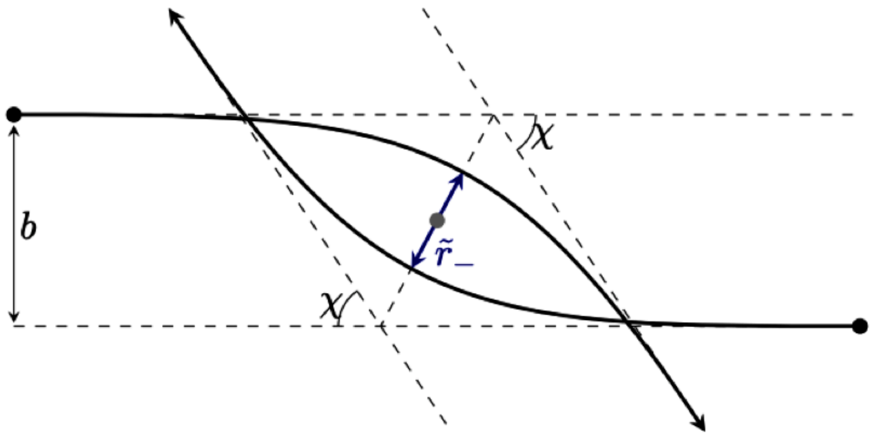
$$-\,\tilde{a} = \frac{\tilde{r}_+ + \tilde{r}_-}{2}, \quad \tilde{e} = \frac{\tilde{r}_+ - \tilde{r}_-}{\tilde{r}_+ + \tilde{r}_-},$$

$$a = \frac{r_+ + r_-}{2}, \quad e = \frac{r_+ - r_-}{r_+ + r_-}.$$

zeros of: $$r^2\left(1 + \sum_i f_i(\mathcal{E})\left(\frac{GM}{r}\right)^i\right) = b^2.$$

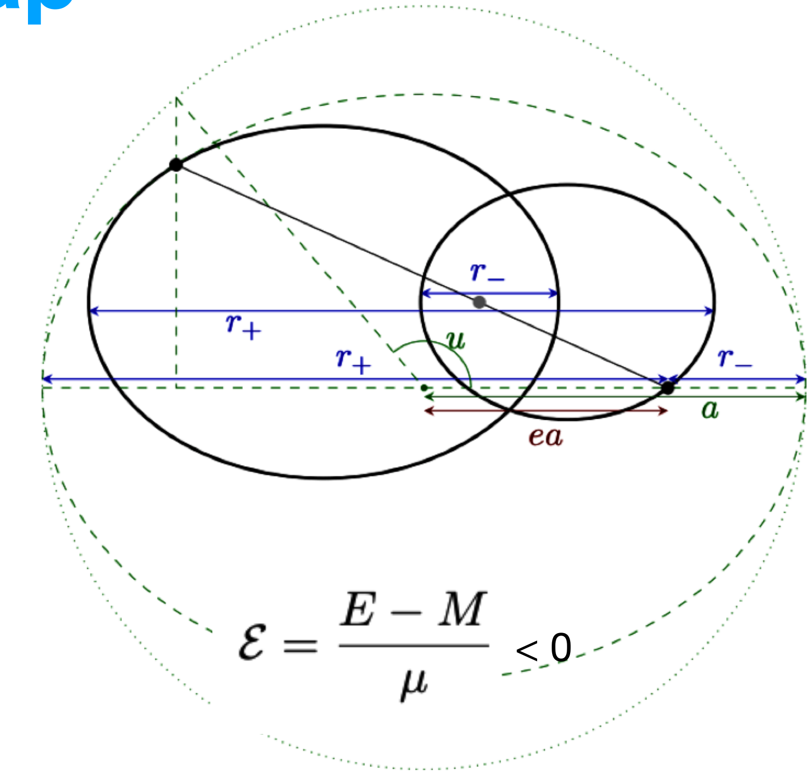
Kalin RAP
1910.03008

Boundary to Bound Map



Analytic Continuation!

impact parameter
and binding energy



Firsov's formula

$$\bar{p}^2(r,E) = \exp\left[\frac{2}{\pi}\int_{r|\bar{p}(r,E)|}^{\infty} \frac{\chi_b(\tilde{b},E)\mathrm{d}\tilde{b}}{\sqrt{\tilde{b}^2 - r^2\bar{p}^2(r,E)}}\right]$$

$$\tilde{r}_- = b\exp\left[-\frac{1}{\pi}\int_b^{\infty} \frac{\chi(\tilde{b},E)\mathrm{d}\tilde{b}}{\sqrt{\tilde{b}^2 - b^2}}\right].$$

$$\tilde{r}_- = b\prod_{n=1}^{\infty} e^{-\frac{(GM)^n \chi_b^{(n)}(\beta)\Gamma\left(\frac{n}{2}\right)}{b^n\sqrt{\pi}\Gamma\left(\frac{n+1}{2}\right)}}.$$

$\beta \equiv \operatorname{arcosh}\gamma$

$$r_-(b,\mathcal{E}) = \tilde{r}_-(ib,\mathcal{E}) \qquad b>0,\ \mathcal{E}<0\,,$$
$$r_+(b,\mathcal{E}) = r_-(-b,\mathcal{E})\,, \qquad b>0\,,$$

$$r_-(J,E) = r_{\min}(ib,i\beta)\,.$$

$$r_-(b,\beta) = ib\prod_{n=1}^{\infty} e^{-\frac{(GM)^n \chi_b^{(n)}(i\beta)\Gamma\left(\frac{n}{2}\right)}{(ib)^n\sqrt{\pi}\Gamma\left(\frac{n+1}{2}\right)}},$$

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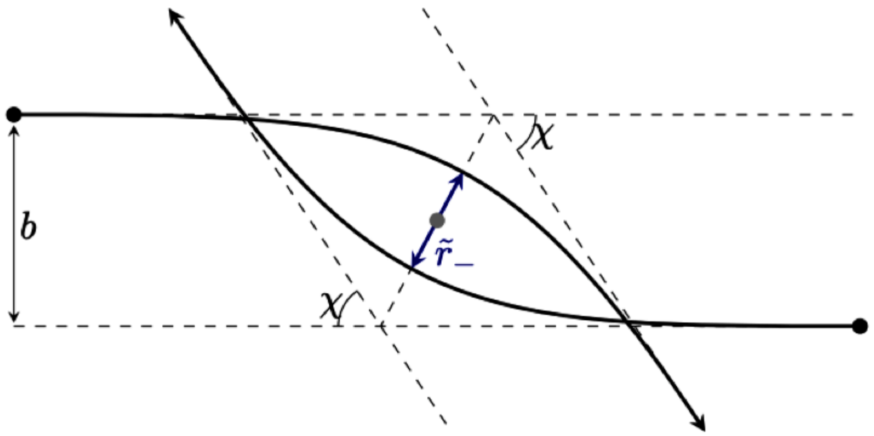
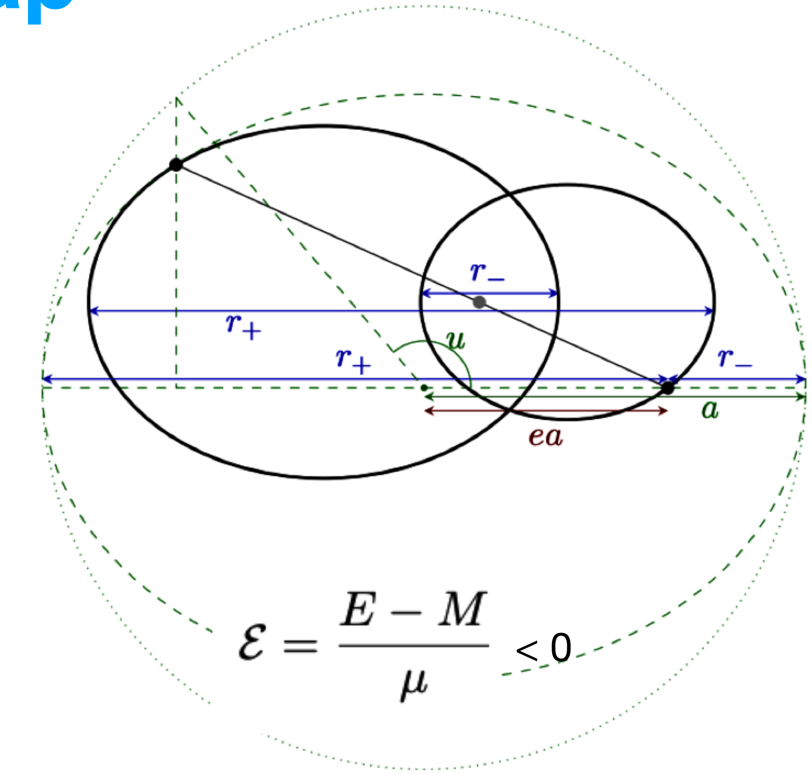
1910.03008

Boundary to Bound Map



Analytic Continuation!

angular momentum
and binding energy



Firsov's formula

$$\bar{p}^2(r,E) = \exp\left[\frac{2}{\pi}\int_{r|\bar{p}(r,E)|}^{\infty} \frac{\chi_b(\tilde{b},E)\mathrm{d}\tilde{b}}{\sqrt{\tilde{b}^2 - r^2\bar{p}^2(r,E)}}\right]$$

$$\tilde{r}_- = b\exp\left[-\frac{1}{\pi}\int_b^{\infty}\frac{\chi(\tilde{b},E)\mathrm{d}\tilde{b}}{\sqrt{\tilde{b}^2-b^2}}\right].$$

$$\tilde{r}_- = b\prod_{n=1}^{\infty} e^{-\frac{(GM)^n \chi_b^{(n)}(\beta)\Gamma\left(\frac{n}{2}\right)}{b^n\sqrt{\pi}\Gamma\left(\frac{n+1}{2}\right)}}.$$

$\beta \equiv \operatorname{arcosh}\gamma$

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$

$$b = J/|p_\infty| > 0\,.$$

$$J = p_\infty b = (-ip_\infty)(ib) > 0,$$

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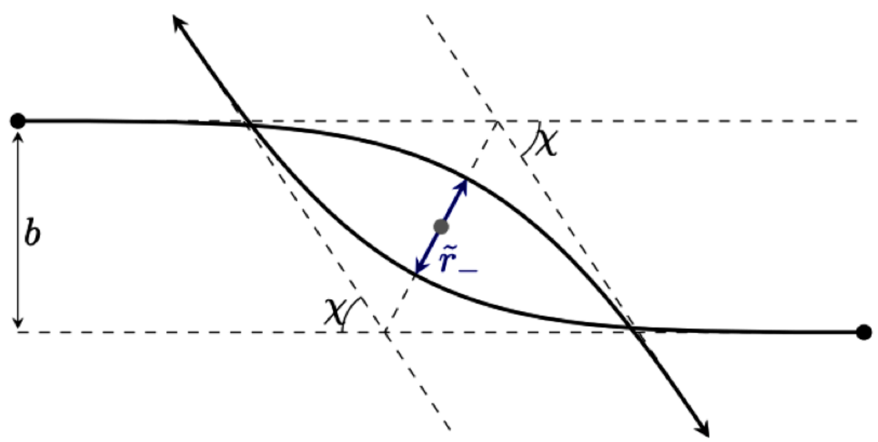
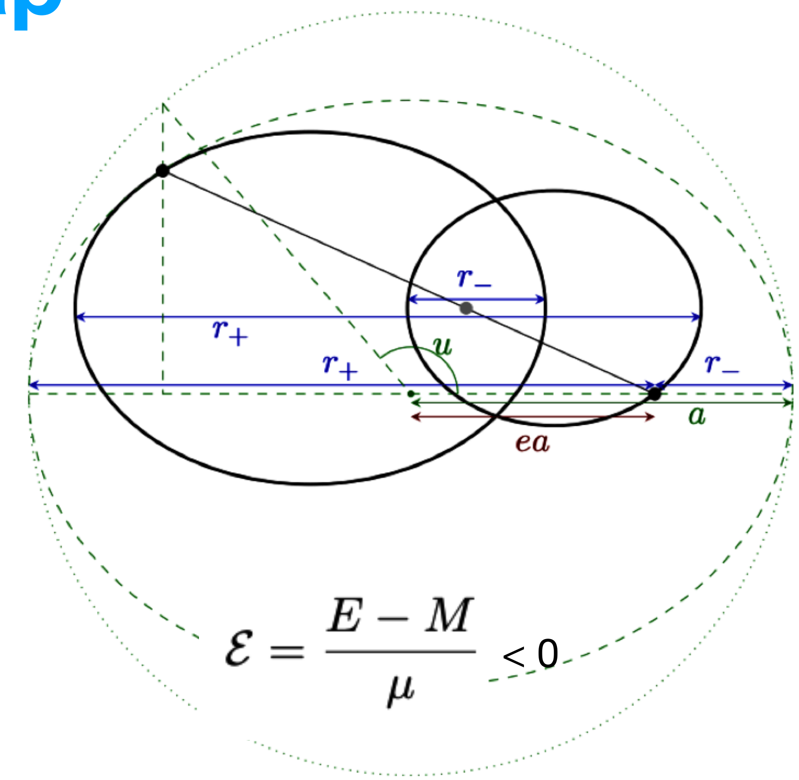
1911.09130

Boundary to Bound Map



What about Observables?

Without **Hamiltonian!**



$$\mathcal{E} = \frac{E-M}{\mu} < 0$$

Scattering angle

$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r$$

Periastron Advanced

$$\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r$$

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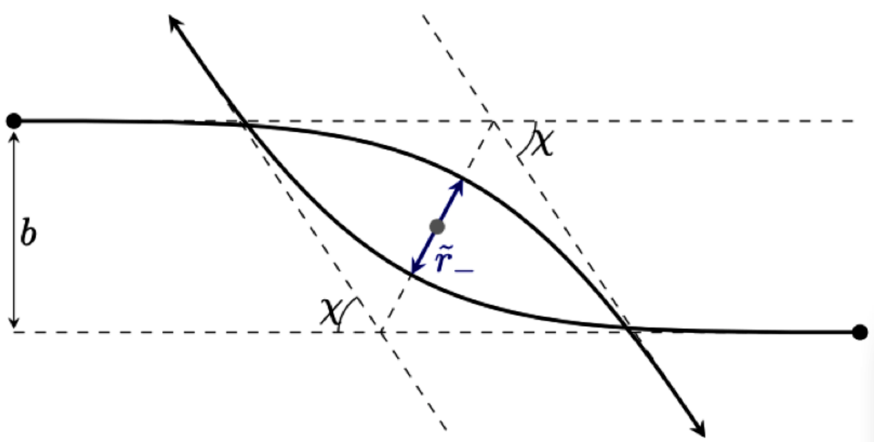
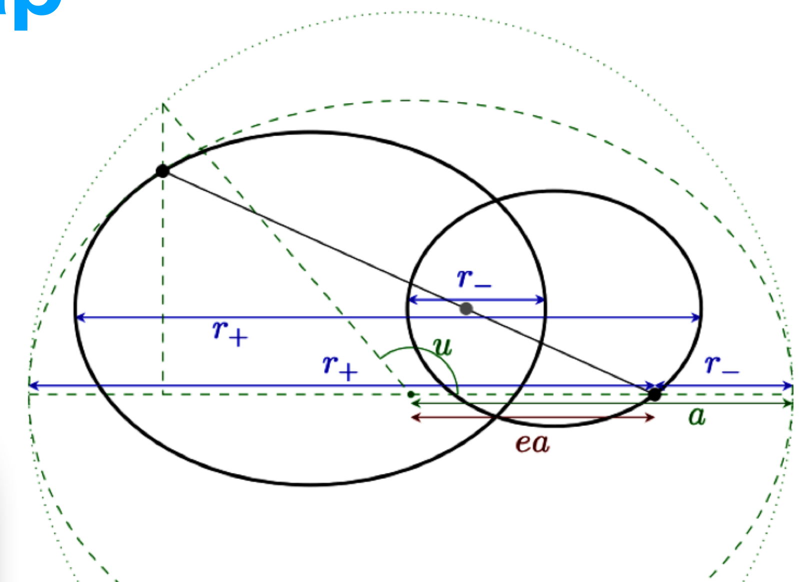
Boundary to Bound Map



Scattering Angle to Periastron adv.

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$



$$\mathcal{E} = \frac{E-M}{\mu} < 0$$

Scattering angle

$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r \,;$$

Periastron Advanced

$$\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r$$

$$4\chi_j^{(4)} = \frac{3\pi\hat{p}_\infty^4}{4}\left(f_2^2 + 2f_1f_3 + 2f_4\right) = \frac{3\pi}{4M^4\mu^4}\left(\widetilde{\mathcal{M}}_2^2 + 2\widetilde{\mathcal{M}}_1\widetilde{\mathcal{M}}_3 + 2p_\infty^2\widetilde{\mathcal{M}}_4\right)$$

$$\frac{\Delta\Phi}{2\pi} = \frac{\widetilde{\mathcal{M}}_2 G^2}{2J^2} + \frac{3(\widetilde{\mathcal{M}}_2^2 + 2\widetilde{\mathcal{M}}_1\widetilde{\mathcal{M}}_3 + 2p_\infty^2\widetilde{\mathcal{M}}_4)G^4}{8J^4} + \mathcal{O}(G^6)\,,$$

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
"EUREKA!"
but, "that's funny..."

—Isaac Asimov

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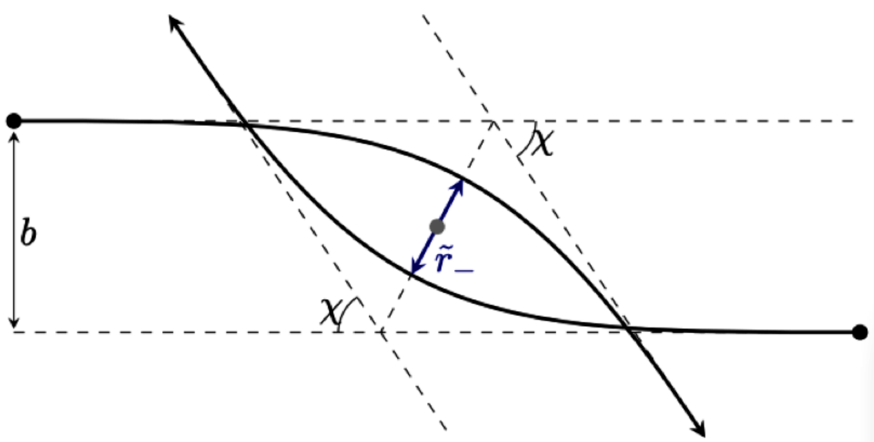
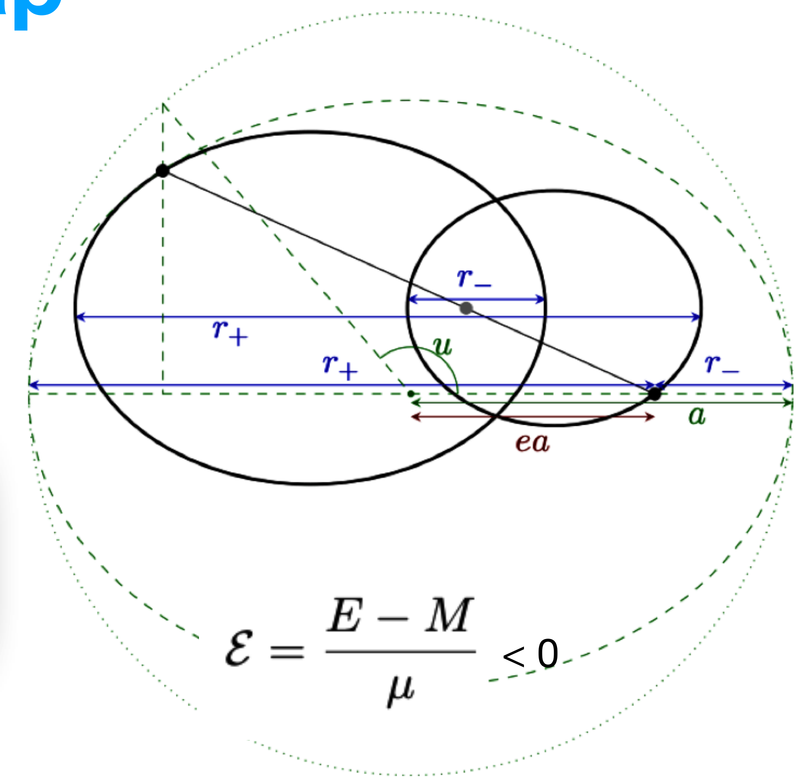
1911.09130

Boundary to Bound Map



Scattering Angle to Periastron adv.

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$



Scattering angle

$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r\,;$$

Periastron Advanced

$$\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r$$

Remarkably!

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})\,, \qquad \mathcal{E}<0\,,$$

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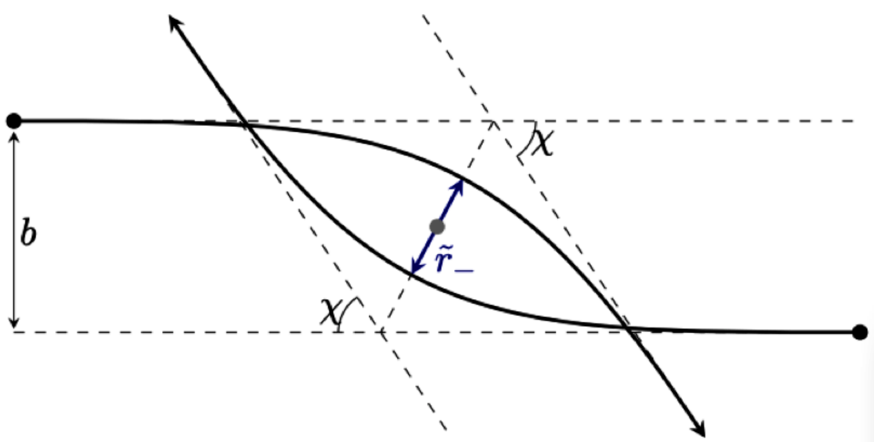
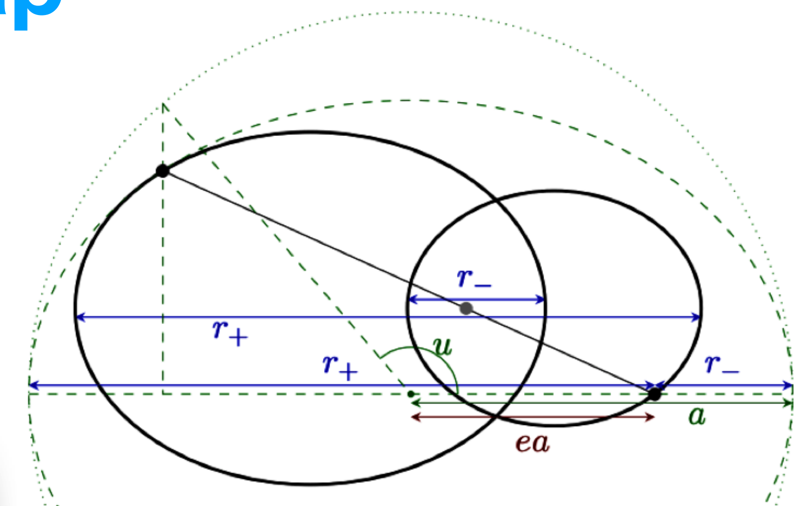
Boundary to Bound Map



B2B radial action through angle

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



$$1 + \frac{\Delta\Phi}{2\pi} = -\frac{\partial i_r}{\partial j}$$

Integrate in dj

$$\frac{\mathcal{S}_r}{GM\mu} = -\left(j + \frac{2}{\pi}\sum_n \chi_j^{(2n)}(\mathcal{E}) \int \frac{dj}{j^{2n}} \right) + \alpha(\mathcal{E}) \, ,$$

fixed by the large-j limit

The (reduced) radial action for the bound problem:

$$i_r(j,\mathcal{E}) \equiv \frac{\mathcal{S}_r}{GM\mu} = \mathrm{sg}(\hat{p}_\infty)\chi_j^{(1)}(\mathcal{E}) - j\left(1 + \frac{2}{\pi}\sum_{n=1} \frac{\chi_j^{(2n)}(\mathcal{E})}{(1-2n)j^{2n}}\right)$$

$$1/j = GM\mu/J$$

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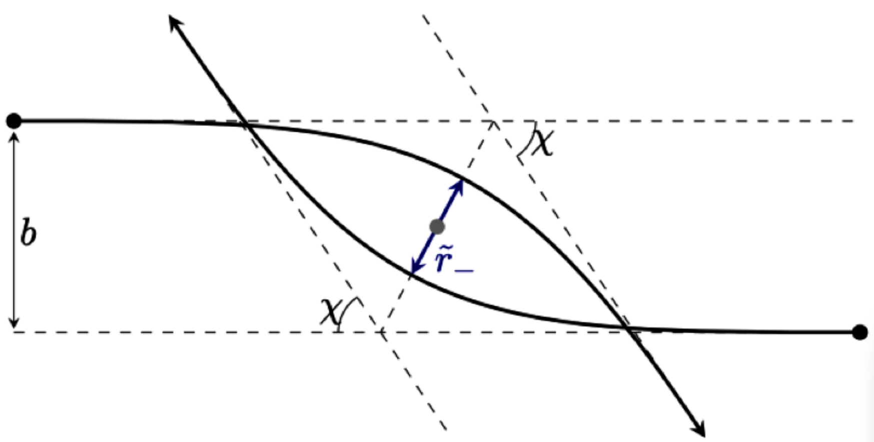
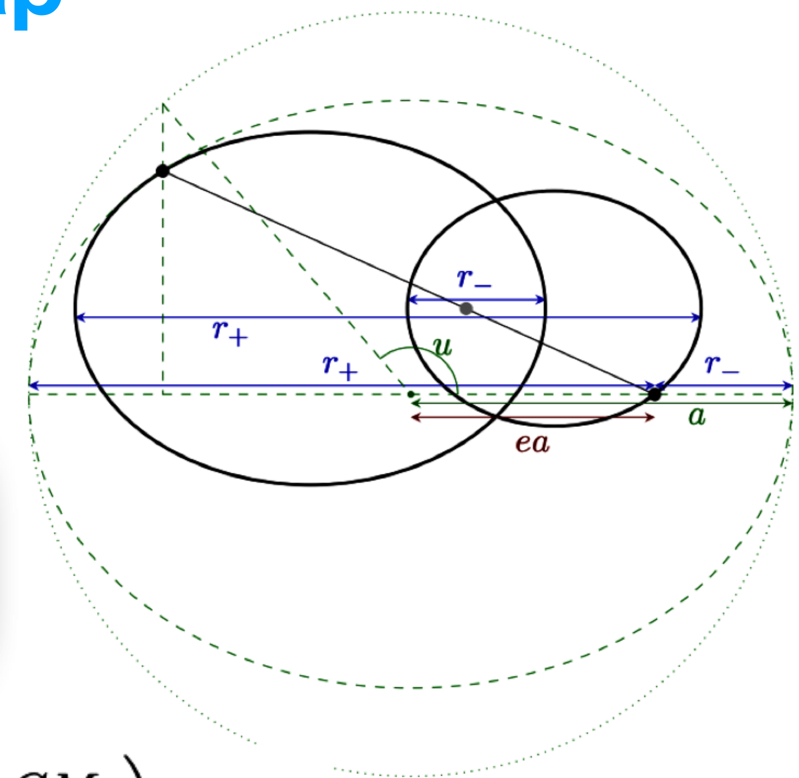
2006.01184

Boundary to Bound Map

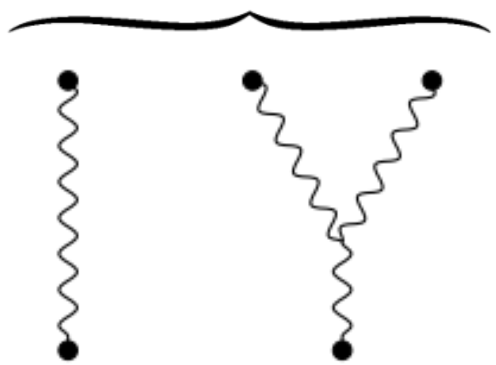


B2B radial action through angle

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



2PM



$$\Delta p_1^\mu = -\frac{G m_1 m_2 b^\mu}{|b^2|}\left(\frac{2\left(2\gamma^2-1\right)}{\sqrt{\gamma^2-1}} + \frac{3\pi}{4}\frac{\left(5\gamma^2-1\right)}{\sqrt{\gamma^2-1}}\frac{GM}{|b^2|^{1/2}}\right)$$
$$+\, 2\frac{m_1 m_2\left(2\gamma^2-1\right)^2}{(\gamma^2-1)^2}\frac{G^2}{|b^2|}\left((\gamma m_2 + m_1)u_2^\mu - (\gamma m_1 + m_2)u_1^\mu\right),$$

Recover the — **remarkably simple!** — radial action to 2PM

$$i_r^{\rm 2PM}(j,\mathcal{E}) = \frac{\hat{p}_\infty}{\sqrt{-\hat{p}_\infty^2}}\chi_j^{(1)}(\mathcal{E}) - j\left(1 - \frac{2}{\pi}\frac{\chi_j^{(2)}(\mathcal{E})}{j^2}\right) = -j + \frac{(2\gamma^2-1)}{\sqrt{1-\gamma^2}} + \frac{3}{4j}\frac{(5\gamma^2-1)}{\Gamma}\,.$$

$$GM\Omega_\phi^{\rm 2PM} = \left(\frac{T_p^{\rm 2PM}}{2\pi GM}\right)^{-1}\left(1 + \frac{\Delta\Phi_{\rm 2PM}}{2\pi}\right) = -\frac{(1-\gamma^2)^{3/2}\Gamma(4j^2\Gamma + 15\gamma^2 - 3)}{j\left(4j\gamma(2\gamma^2-3)\Gamma^3 - 3(1-\gamma^2)^{3/2}(\nu + 5\gamma(2 + (3\gamma-4)\nu))\right)}\,.$$

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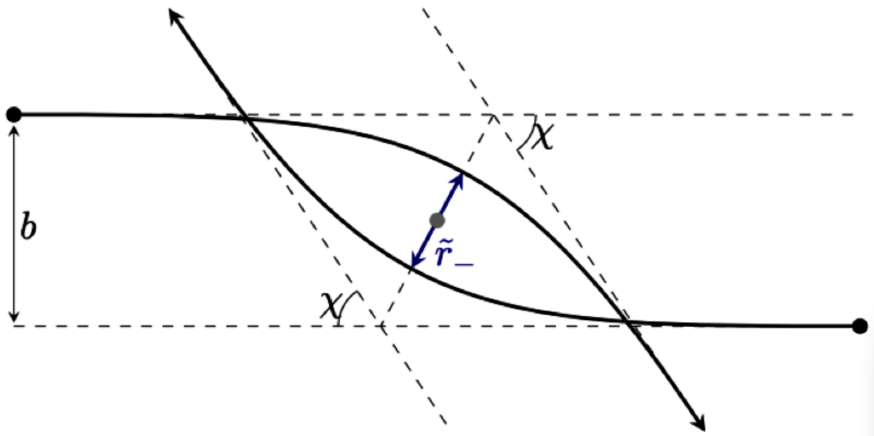
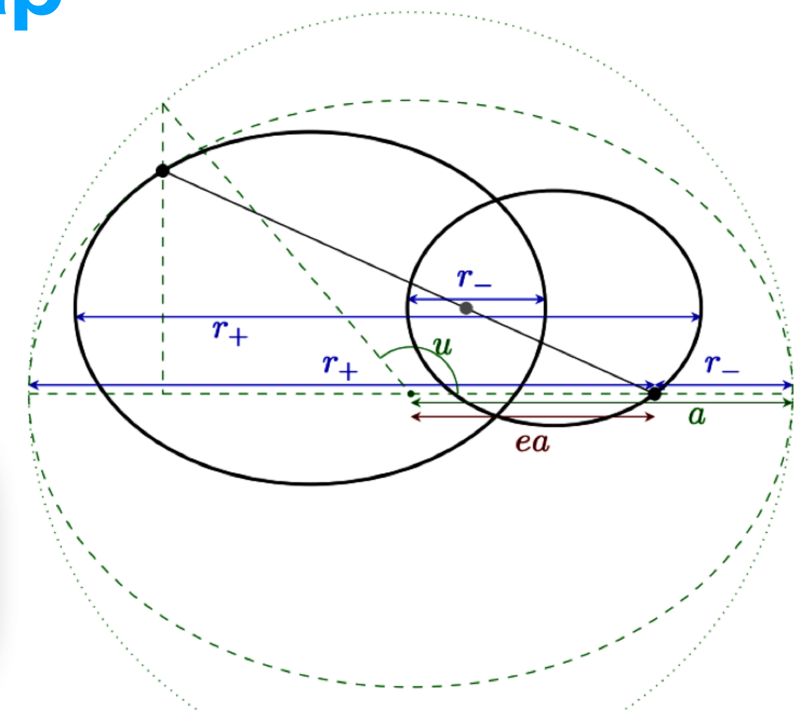
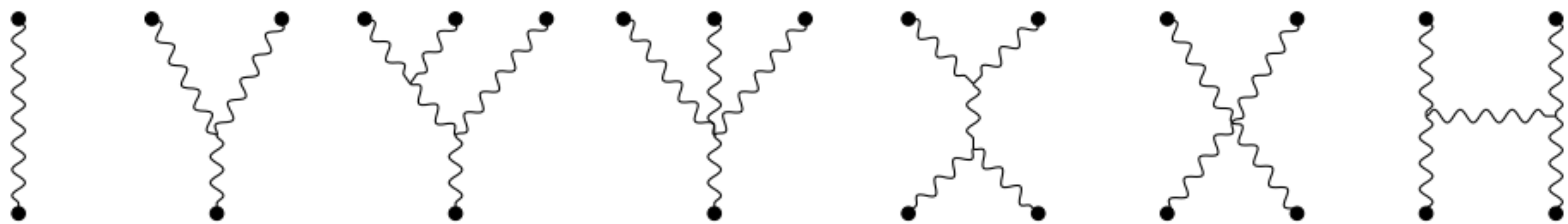
Boundary to Bound Map



B2B radial action through angle

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$





We have the 3PM impulse

$$i_r = \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 - \frac{2}{\pi}\left(\frac{\chi_j^{(2)}}{j^2} + \frac{\chi_j^{(4)}}{3j^4}\right) + \cdots\right)$$

BUT WE DO NOT HAVE THE 4PM ANGLE!

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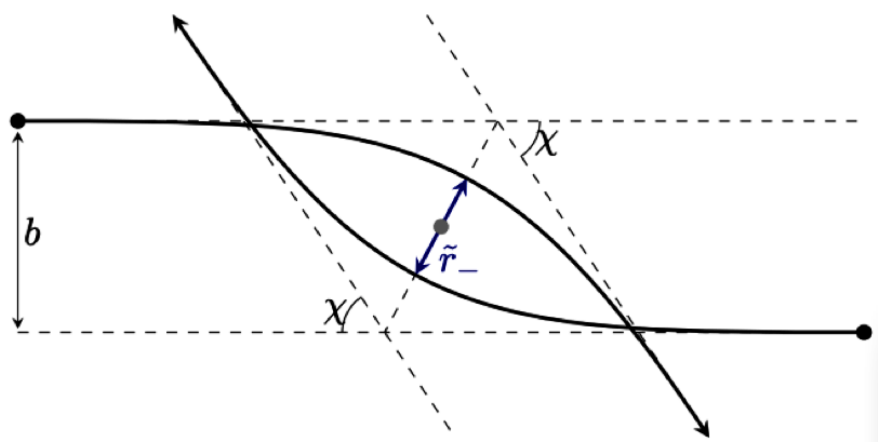
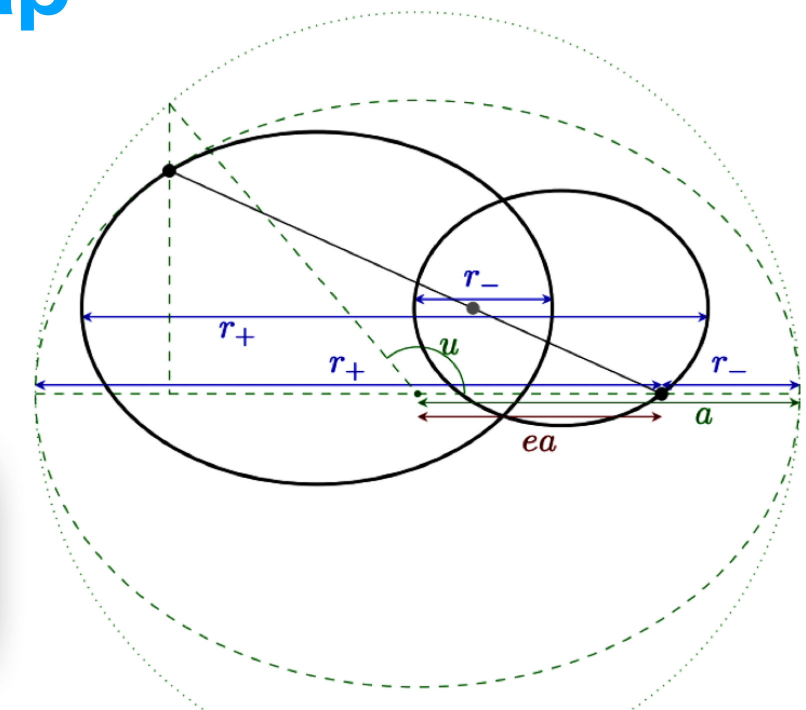
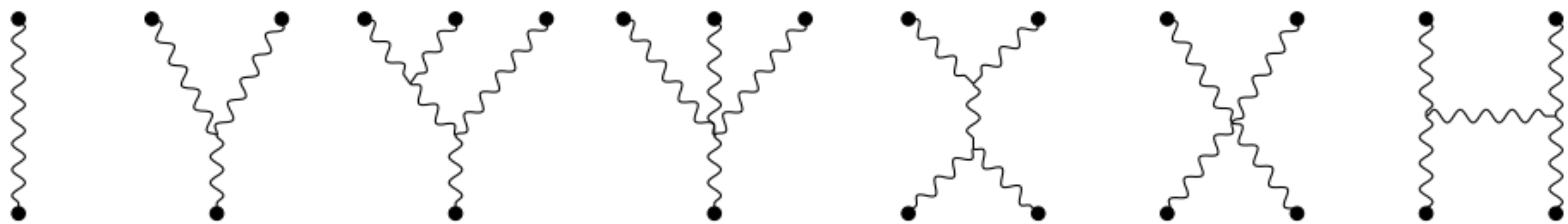
Boundary to Bound Map



B2B radial action through angle

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$





From Firsov:

$$\chi_j^{(3)} = \frac{1}{M^3\mu^3 p_\infty^3}\left(-\frac{P_1^3}{24} + p_\infty^2\frac{P_1P_2}{2} + p_\infty^4 P_3\right)$$

IMPETUS:* Everything you need to know about 3PM

$$\frac{P_3}{M^3\mu^2} = \left(\frac{18\gamma^2-1}{2\Gamma} + \frac{8\nu}{\Gamma}(3+12\gamma^2-4\gamma^4)\frac{\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{\sqrt{\gamma^2-1}} + \frac{\nu}{6\Gamma}\left(6-206\gamma-108\gamma^2-4\gamma^3 + \frac{18\Gamma(1-2\gamma^2)(1-5\gamma^2)}{(1+\Gamma)(1+\gamma)}\right)\right).$$

*We also reconstructed PM Hamiltonian

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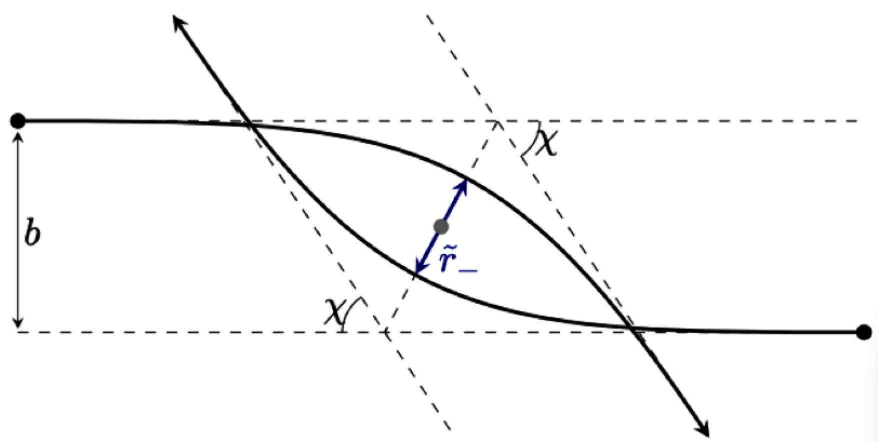
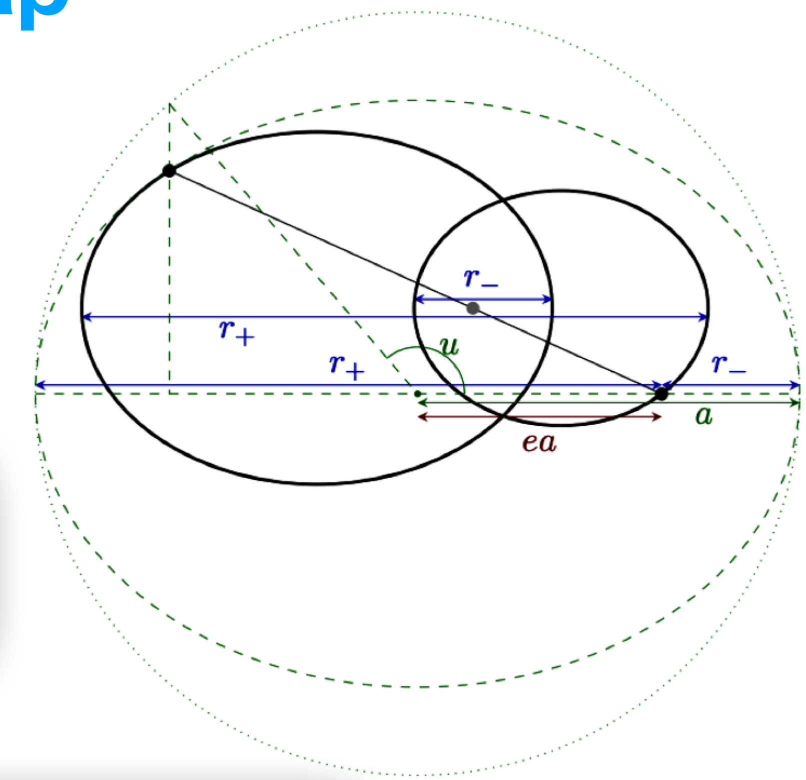
Boundary to Bound Map



B2B radial action through angle

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^{\ell}\right)}\prod_{\ell}\frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$



$$i_r = \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1-\frac{2}{\pi}\left(\frac{\chi_j^{(2)}}{j^2}+\frac{\chi_j^{(4)}}{3j^4}\right)+\cdots\right)$$

From Firsov:

$$\chi_j^{(4)} = \frac{3\pi}{8M^4\mu^4}\left(P_1P_3+\frac{1}{2}P_2^2+p_\infty^2P_4\right),$$

lower-order P_n's enter in the 4PM angle

$$\frac{P_3}{M^3\mu^2} = \left(\frac{18\gamma^2-1}{2\Gamma}+\frac{8\nu}{\Gamma}(3+12\gamma^2-4\gamma^4)\frac{\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{\sqrt{\gamma^2-1}}+\right.$$

$$\left.\frac{\nu}{6\Gamma}\left(6-206\gamma-108\gamma^2-4\gamma^3+\frac{18\Gamma(1-2\gamma^2)(1-5\gamma^2)}{(1+\Gamma)(1+\gamma)}\right)\right).$$

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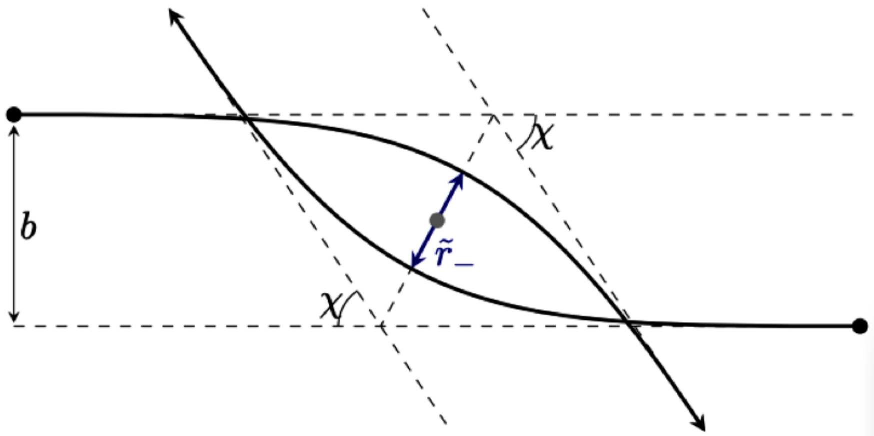
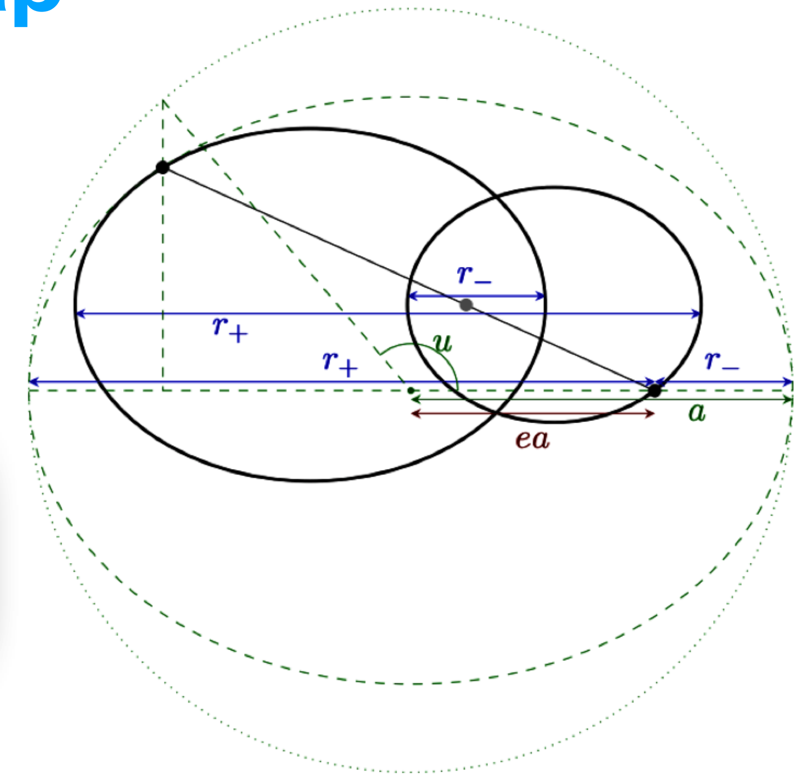
Boundary to Bound Map



B2B radial action through angle

Bound Orbits: $p_\infty^2 \sim \mathcal{E}$ $1/J \sim |p_\infty|$



$$i_r = \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 - \frac{2}{\pi}\left(\frac{\chi_j^{(2)}}{j^2} + \frac{\chi_j^{(4)}}{3j^4}\right) + \cdots\right)$$

$\mathcal{O}(G/J)^6$

$$\chi_j^{(4)} = \frac{3\pi}{8M^4\mu^4}\left(\underbrace{P_1P_3 + \frac{1}{2}P_2^2}\overset{*}{+} p_\infty^2 P_4\right),$$

Missing! **BUT** PN-suppressed (after analytic continuation)

$$\frac{P_3}{M^3\mu^2} = \left(\frac{18\gamma^2-1}{2\Gamma} + \frac{8\nu}{\Gamma}(3+12\gamma^2-4\gamma^4)\frac{\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{\sqrt{\gamma^2-1}} + \frac{\nu}{6\Gamma}\left(6-206\gamma-108\gamma^2-4\gamma^3 + \frac{18\Gamma(1-2\gamma^2)(1-5\gamma^2)}{(1+\Gamma)(1+\gamma)}\right)\right).$$

This pattern is generic! and allows us to perform a **consistent PN-truncation**

***P_n has well-defined static limit**

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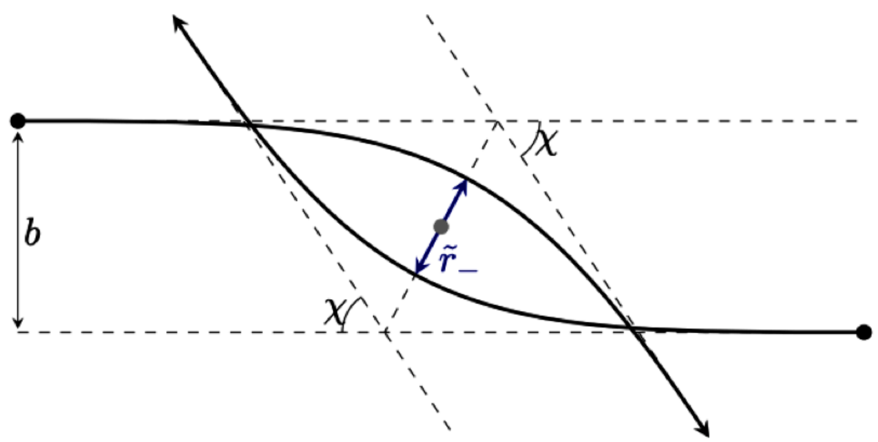
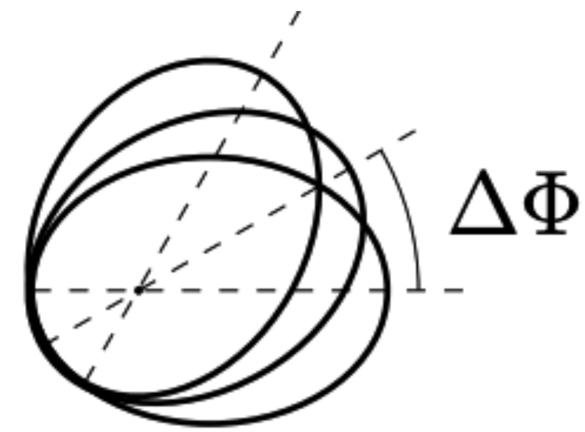
Boundary to Bound Map



B2B radial action through angle

⟷

Full result agrees with literature to 2PN



✓ 1PN ✓ 2PN ✓ 2PN

$$\left(\frac{\Delta\Phi}{2\pi}\right)_{\text{2-loop}} = \frac{3}{j^2} + \frac{3(35-10\nu)}{4j^4} + \frac{3}{4j^2}\left(10-4\nu + \frac{194-184\nu+23\nu^2}{j^2}\right)\mathcal{E}$$

✓ 3PN

$$+\frac{3}{4j^2}\left(5-5\nu+4\nu^2+\frac{3535-6911\nu+3060\nu^2-375\nu^3}{10j^2}\right)\mathcal{E}^2$$

✓ 4PN

$$+\frac{3}{4j^2}\left((5-4\nu)\nu^2+\frac{35910-126347\nu+125559\nu^2-59920\nu^3+7385\nu^4}{140j^2}\right)\mathcal{E}^3$$

$$+\frac{3}{4j^2}\left((5-20\nu+16\nu^2)\,\frac{\nu^2}{4}\right)\mathcal{E}^4+\cdots,$$

The 5PN prediction (confirmed by tutti frutti also at 6PN)

ONE-LOOP EXACT! $\frac{\Delta\Phi}{2\pi} = \frac{\widetilde{M}_2 G^2}{2J^2}$

BUT: 1/j^2 is predicted *to all orders in v*
4PM will complete 1/j^4

$$\chi_j^{(2)} \propto \widetilde{M}_2 = \frac{3M^2\mu^2}{2}\left(\frac{5\gamma^2-1}{\Gamma}\right)$$

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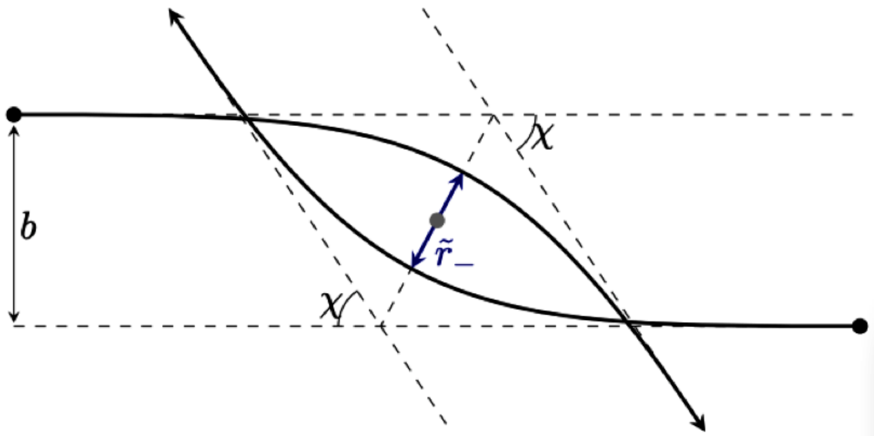
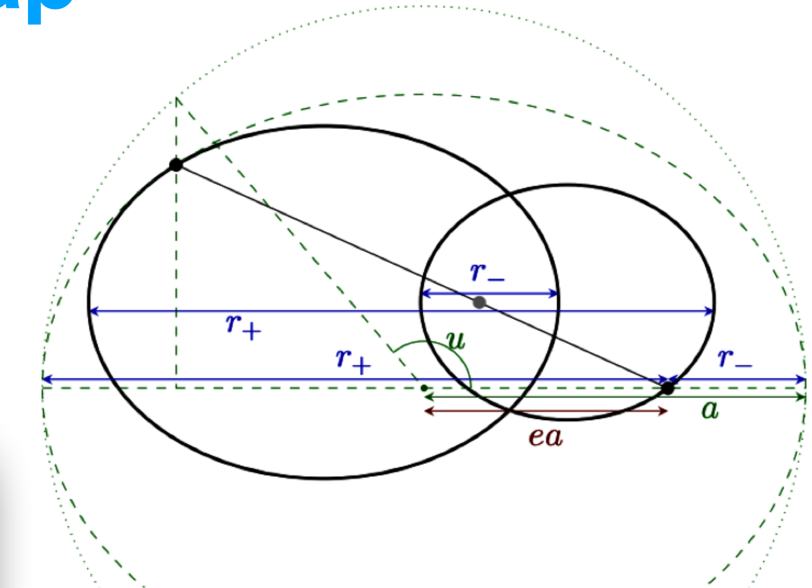
Boundary to Bound Map



Map for *ALL* Dynamical Invariants!

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



$$\delta \mathcal{S}_r(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

$$\Omega_r(j,\mathcal{E}) \equiv \frac{2\pi}{T_p}, \quad \Omega_p(j,\mathcal{E}) \equiv \frac{\Delta\Phi}{T_p},$$

$$\Omega_\phi \equiv \Omega_r + \Omega_p = \frac{2\pi}{T_p}\left(1+\frac{\Delta\Phi}{2\pi}\right).$$

$$\frac{GM\Omega_r^{(L=2)}}{\epsilon^{\frac{3}{2}}} = 1 - \frac{(15-\nu)}{8}\epsilon + \frac{555+30\nu+11\nu^2}{128}\epsilon^2 + \left(\frac{3(2\nu-5)}{2j} - \frac{194-184\nu+23\nu^2}{4j^3}\right)\epsilon^{\frac{3}{2}} + \left(\frac{15(17-9\nu+2\nu^2)}{8j} + \frac{21620-28592\nu+8765\nu^2-865\nu^3}{80j^3}\right)\epsilon^{\frac{5}{2}} + \cdots$$

$$\frac{GM\Omega_\phi^{(L=2)}}{\epsilon^{\frac{3}{2}}} = 1 + \frac{3}{j^2} - \frac{15(2\nu-7)}{4j^4} + \left(\frac{1}{8}(\nu-15) + \frac{15(\nu-5)}{8j^2} - \frac{3(1301-921\nu+102\nu^2)}{32j^4}\right)\epsilon$$
$$+ \left(\frac{3(2\nu-5)}{2j} + \frac{-284+220\nu-23\nu^2}{4j^3} + \frac{3(913-728\nu+106\nu^2)}{j^5}\right)\epsilon^{\frac{3}{2}}$$
$$+ \left(\frac{1}{128}(555+30\nu+11\nu^2) + \frac{3(895-150\nu+51\nu^2)}{128j^2} - \frac{3(-270085+251236\nu-70545\nu^2+7470\nu^3)}{2560j^4}\right)\epsilon^2$$
$$+ \left(\frac{15(17-9\nu+2\nu^2)}{8j} + \frac{31520-34442\nu+10025\nu^2-865\nu^3}{80j^3}\right)\epsilon^{\frac{5}{2}}.$$

$$\epsilon = -2\mathcal{E}$$

3PN match

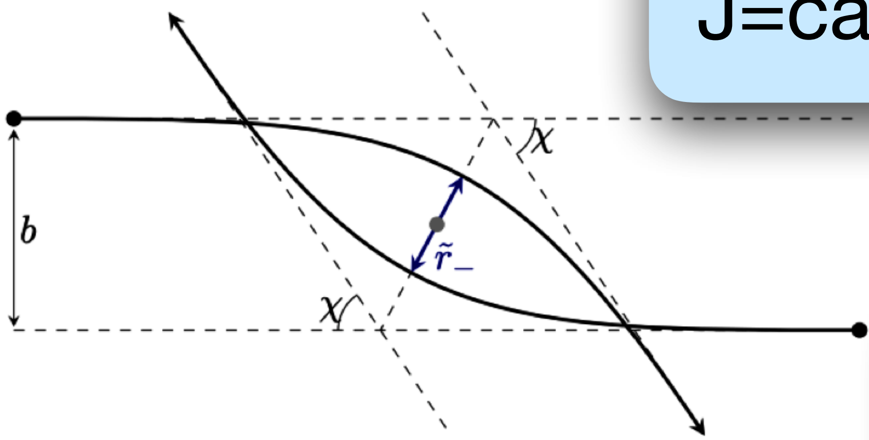
3PN missmatch

Higher orders velocity

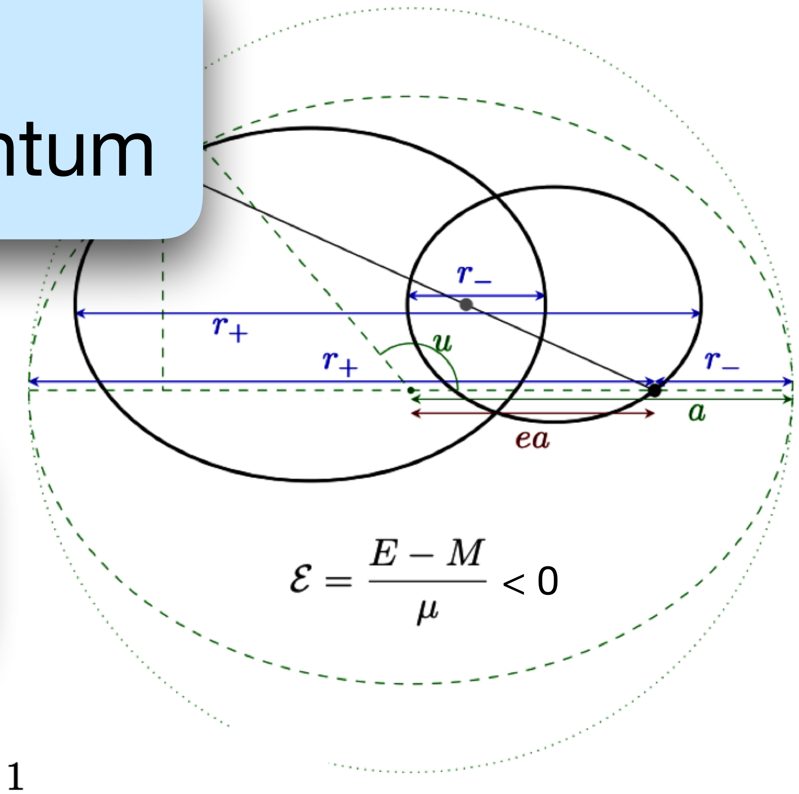
One loop exact (Missing three loop!)

Kalin RAP
1911.09130

Valid for (aligned) spin!*
J=canonical **total** ang. momentum



$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



Angle from
Vines
Steinhoff
Buonanno
1812.00956.

$$\begin{aligned}
\frac{\chi(\ell,a,\epsilon)}{2\pi} = &\left[\frac{1}{\pi}(-\epsilon)^{-\frac{1}{2}} - \frac{(\nu-15)}{8\pi}(-\epsilon)^{\frac{1}{2}} + \frac{35+30\nu+3\nu^2}{128\pi}(-\epsilon)^{\frac{3}{2}}\right]\frac{1}{\ell} \\
&+ \left[3 + \frac{3(2\nu-5)}{4}\epsilon + \frac{3(5-5\nu+4\nu^2)}{16}\epsilon^2 - \frac{7\tilde{a}_+ + \Delta\tilde{a}_-}{2\pi}\epsilon^{-\frac{1}{2}} \right. \\
&\qquad \left. + \frac{5\Delta(\nu-3)\tilde{a}_- + (23\nu-25)\tilde{a}_+}{16\pi}(-\epsilon)^{\frac{3}{2}}\right]\frac{1}{2\ell^2} \\
&+ \left[-\frac{7\tilde{a}_+ + \Delta\tilde{a}_-}{2} - \frac{(\nu-6)\Delta\tilde{a}_- + (7\nu-18)\tilde{a}_+}{2}\epsilon \right. \\
&\qquad - \frac{3\left((15-14\nu+2\nu^2)\Delta\tilde{a}_- + (25-38\nu+14\nu^2)\tilde{a}_+\right)}{16}\epsilon^2 \\
&\qquad \left. - \frac{2}{3\pi}(-\epsilon)^{-\frac{3}{2}} + \frac{33+\nu}{4\pi}(-\epsilon)^{-\frac{1}{2}} + \frac{3003-1090\nu-5\nu^2+128\tilde{a}_+^2}{64\pi}(-\epsilon)^{\frac{1}{2}}\right]\frac{1}{2\ell^3} \\
&+ \left[\frac{3(35+2\tilde{a}_+^2-10\nu)}{4} - \frac{10080-13952\nu+123\pi^2\nu+1440\nu^2}{128}\epsilon \right. \\
&\qquad \left. - \frac{624\Delta\tilde{a}_-\tilde{a}_+ + 24(1-8\nu)\tilde{a}_-^2 - 24(12\nu-61)\tilde{a}_+^2}{128}\epsilon + \cdots\right]\frac{1}{2\ell^4} + \cdots .
\end{aligned}$$

Periastron from
Tessmer
Hartung
Schaefer
1207.6961

*** Likewise we can reconstruct spin-dependent B2B radial action by integration and read off binding energy (spin at 'one-loop' 2PM to appear)**

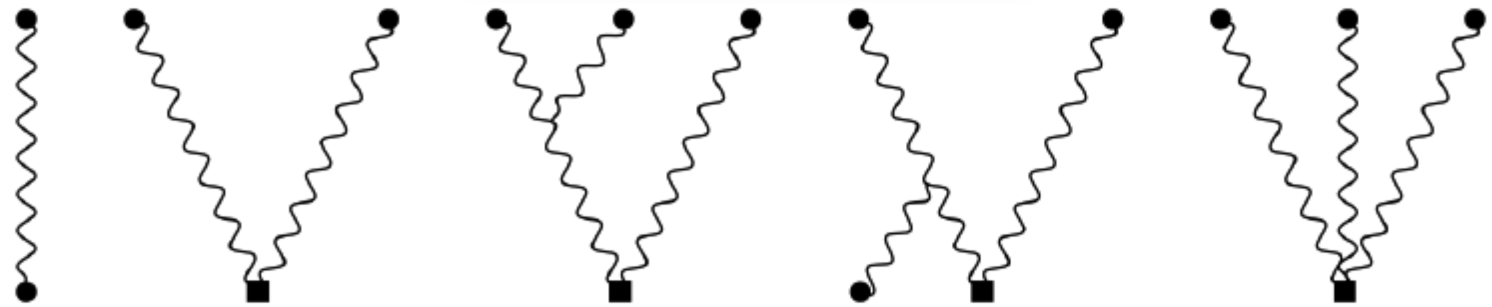
Kalin Liu RAP
2008.06047

EFT approach: Tidal Effects

Bound orbits
$$x = (GM\omega)^{2/3} \sim v^2$$

Quadrupole to G^2v^4 & G^3v^2 and LO octupole agree with Henry et al.

1912.01920



Quadrupole to NLO agrees with Cheng-Solon

2006.06665

Quadrupole/Octupole TLN in binding energy to O(G^3)

$$\Delta\mathcal{E}_{\rm T} = x\Big[18\,\lambda_{E^2}x^5 + 11\big(3(1-\nu)\lambda_{E^2} + 6\,\lambda_{B^2} + 5\nu\,\kappa_{E^2}\big)x^6 + \Big(390\lambda_{\tilde{E}^2} - \frac{13}{28}(161\nu^2 - 161\nu - 132)\lambda_{E^2} - \frac{1326\nu}{7}\kappa_{B^2}$$
$$+\frac{13}{28}(616\nu + 699)\lambda_{B^2} + \frac{13\nu}{84}(490\nu - 729)\kappa_{E^2} + \frac{13}{6}\Delta\bar{P}^{(E,B)}_{8,{\rm stc}}\Big)x^7 + 75\big(45\nu\kappa_{\tilde{E}^2} - (13\nu+3)\lambda_{\tilde{E}^2} + 16\lambda_{\tilde{B}^2}\big)x^8$$
$$-\Big(\frac{85}{36}\left(1083\nu^2 + 1539\nu + 163\right)\lambda_{\tilde{E}^2} + \frac{27200\nu}{3}\kappa_{\tilde{B}^2} - \frac{85}{4}(270\nu + 383)\nu\kappa_{\tilde{E}^2} - \frac{680}{9}(90\nu + 173)\lambda_{\tilde{B}^2} - \frac{17}{6}\Delta\bar{P}^{(\tilde{E},\tilde{B})}_{10,{\rm stc}}\Big)x^9\Big]$$

NNLO terms from PM-static and probe limit

$$\Delta\bar{P}^{(E,B)}_{8,{\rm stc}} = \frac{1326}{7}\nu\kappa_{B^2} + (243 - 90\nu)\nu\kappa_{E^2} + \left(45\nu^2 - \frac{885\nu}{7} + \frac{675}{14}\right)\lambda_{E^2} - \left(234\nu + \frac{837}{14}\right)\lambda_{B^2}\,.$$

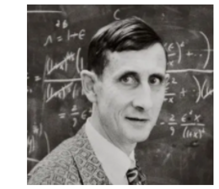
$$\Delta\bar{P}^{(\tilde{E},\tilde{B})}_{10,{\rm stc}} = \frac{1}{3}\left(2050\lambda_{\tilde{E}^2} - 13120\lambda_{\tilde{B}^2}\right) + \mathcal{O}(\nu)\,.$$

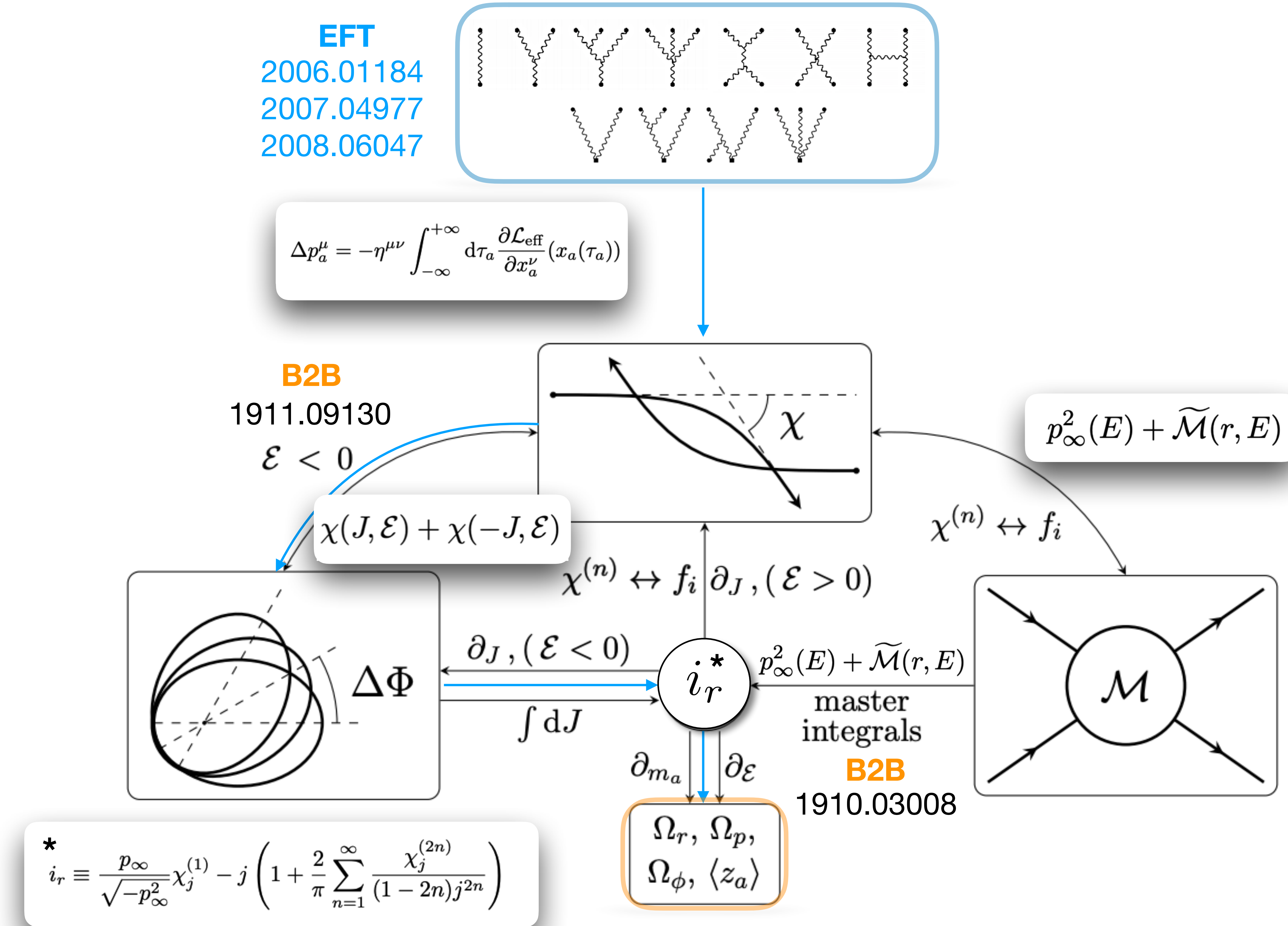
***We also reconstructed the full PM Hamiltonian to NLO**

Boundary to Bound

"New directions in science are launched by new tools much more often than by new concepts. The effect of a concept-driven revolution is to explain old things in a new way. The effect of a tool-driven revolution is to discover new things that have to be explained"

Freeman Dyson, "Imagined Worlds"





Radiation-Reaction

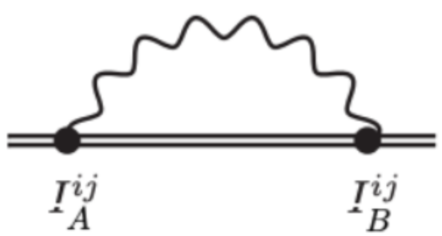
in-in b.c.
cons. vs dissip. from
symmetry in w->-w

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

The radiation-reaction force at LO (multipole expansion)

$iW[\boldsymbol{x}_a^{\pm}] =$ 

$\longrightarrow$ $$W[\boldsymbol{x}_a^{\pm}] = -\frac{G_N}{5}\int dt\, I_-^{ij}(t) I_{+ij}^{(5)}(t)$$

$$(\boldsymbol{a}_a^i)_{\rm rr}(t) = -\frac{2G_N}{5} I^{(5)ij}(t)\boldsymbol{x}_a^j(t).$$ $\longrightarrow$

$$\dot{M} = -\frac{G_N}{5} I^{(1)ij}(t) I^{(5)ij}(t)\,,$$

$$\dot{\mathcal{E}}_N = \frac{8}{15}\frac{G^3 m^2\mu^2}{c^5 r^4}\left\{12v^2 - 11\dot{r}^2\right\}$$

leading cross-term

BUT! we get the angle from the impulse **(integrated in time):**

$$\sqrt{\Delta p^2} \to G\int dt\, I^{(5)ij} x^i x^j \sim G\int dt I^{(5)ij} I^{ij}$$

$$\dot{\mathcal{J}}_N = \frac{8}{5}\frac{G^2 m\,\mu^2}{c^5 r^3}\tilde{\mathbf{L}}_{\mathbf{N}}\left\{2v^2 - 3\dot{r}^2 + 2\frac{Gm}{r}\right\}$$

$$\sqrt{\Delta p^2} \sim G\int dt I^{(3)ij} I^{(2)ij} \sim G\int dt\frac{dL}{dt} \sim G\Delta L \sim G^3$$

$$L^{ij} \equiv -\int d^3\mathbf{x}\,(T^{0i}x^j - T^{0j}x^i)$$

Radiation-Reaction

in-in b.c.
cons. vs dissip. from
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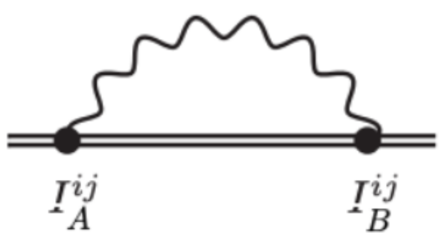
PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

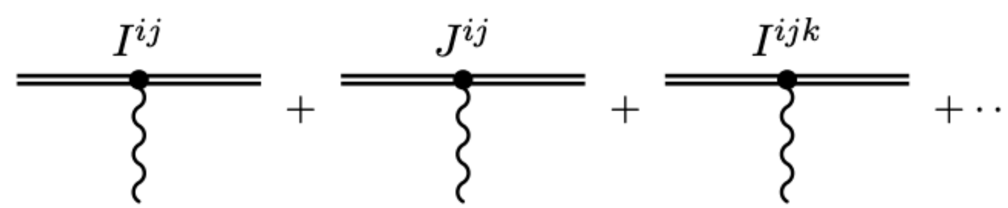
Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

The radiation-reaction force at all orders in the multipole expansion

$iW[\boldsymbol{x}_a^{\pm}] =$



$i\mathcal{A}_h(\omega, \boldsymbol{k}) =$

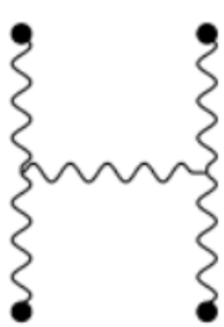


$$= \frac{i}{4M_{\rm Pl}} \epsilon^*_{ij}(\boldsymbol{k}, h) \left[\omega^2 I^{ij}(\omega) + \frac{4}{3} \omega\, \boldsymbol{k}^l \epsilon^{ikl} J^{jk}(\omega) - \frac{i}{3} \omega^2 \boldsymbol{k}^l I^{ijl}(\omega) + \cdots \right] ,$$

The energy would also follow directly by squaring:

$$d\Gamma_h(\boldsymbol{k}) = \frac{1}{T} \frac{d^3\boldsymbol{k}}{(2\pi)^3 2|\boldsymbol{k}|} |\mathcal{A}_h(|\boldsymbol{k}|, \boldsymbol{k})|^2 \;\rightarrow\; P\,\Big|_{h=\pm 2} = \int_{\boldsymbol{k}} |\boldsymbol{k}| d\Gamma_h(\boldsymbol{k})$$

In PMEFT we should re-compute the **soft part** of the H-diagram in the in-in formalism:



$$(I_{\rm H} + I_{\overline{\rm H}})\,\big|_{\ln(-t)} = \frac{1}{64\pi^3} \frac{1}{m^2 t^2} \frac{1}{\sqrt{\sigma^2 - 1}} \operatorname{arcsinh} \sqrt{\frac{\sigma - 1}{2}} \left[\pi + 2i \operatorname{arcsinh} \sqrt{\frac{\sigma - 1}{2}} \right] . \quad \text{(D.3)}$$

Bern et al.
1908.01493

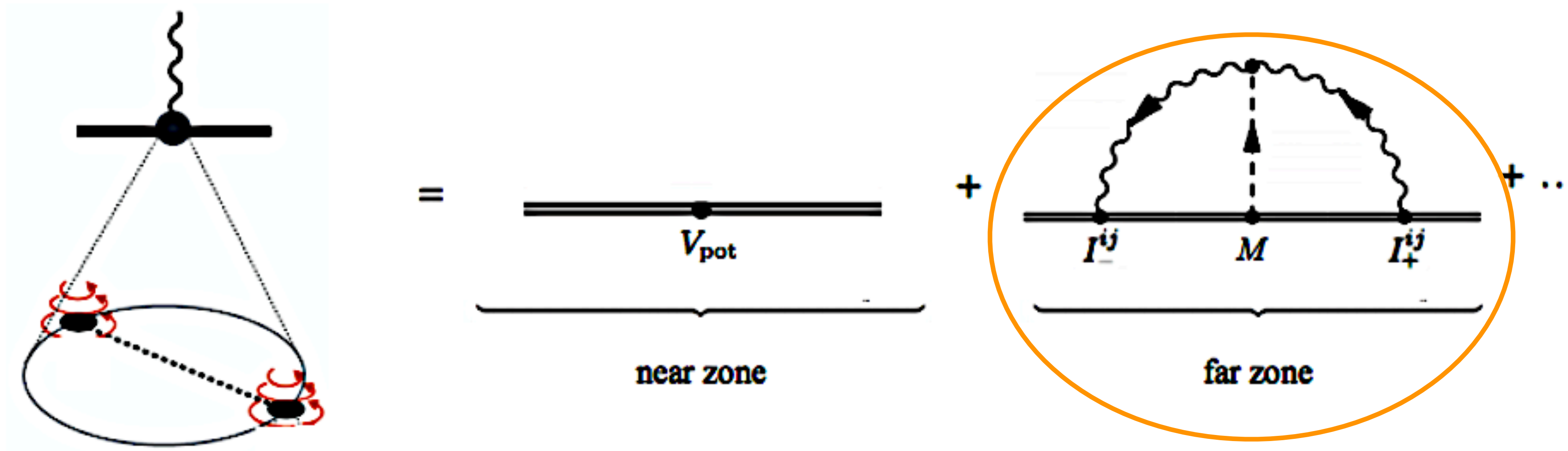
Conservative Radiation-Reaction

in-in b.c.
cons. vs dissip. from symmetry in w->-w

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]



Similarly to NRQCD:

$$\frac{2G_N^2 M}{5} I^{(3)ij} I^{(3)ij} \left(-\frac{1}{\epsilon_{\rm IR}} + 2\log(\mu r) + \cdots\right) + \left(\frac{1}{\epsilon_{\rm UV}} + 2\log(\Omega/\mu) + \cdots\right)$$

$$W_{\rm tail}[\boldsymbol{x}_a^\pm] = \frac{2G_N^2 M}{5} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega^6\, I_-^{ij}(-\omega) I_+^{ij}(\omega) \left[-\frac{1}{(d-4)_{\rm UV}} - \gamma_E + \log\pi - \log\frac{\omega^2}{\mu^2} + \frac{41}{30} + i\pi\,{\rm sign}(\omega)\right].$$

$$\mu \frac{d}{d\mu} V_{\rm ren}(\mu) = \frac{2G_N^2 M}{5} I^{ij(3)}(t) I^{ij(3)}(t)$$

dissipative part

Radiation-Reaction

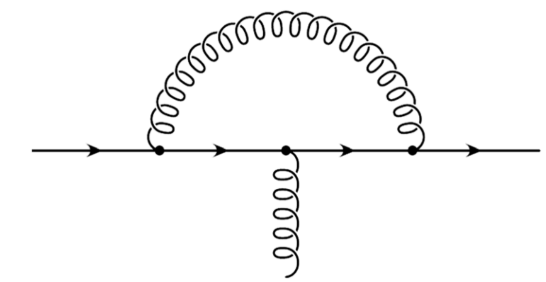
PHYSICAL REVIEW D **96,** 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

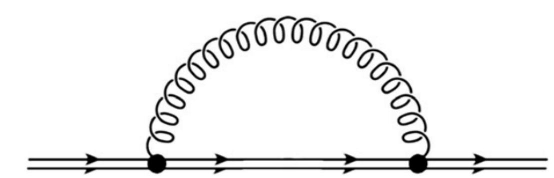
Rafael A. Porto

(See Pineda, Vairo and Aneesh's talks)

Computation in NRQED

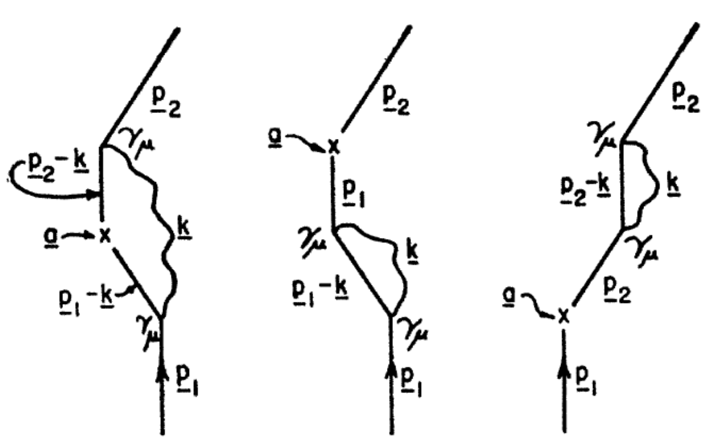


$$\delta E_{n,\ell} = (\delta E_{n,\ell})_{US} + (\delta E_{n,\ell})_{cV} + \cdots$$
$$= \frac{2\alpha_e}{3\pi}\left[\frac{5}{6}e^2\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2} - \sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2 (E_m - E_n)\log\frac{2|E_n - E_m|}{m_e}\right] +$$
$$+\frac{4\alpha_e^2}{3m_e^2}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2 .$$



$$(p^0_{US}, \boldsymbol{p}_{US}) \sim (m_e v^2, m_e v^2)$$

only cancel explicitly in dim. reg.!
(zero-bin subtraction)



a. Eq.12 b. Eq.13 c. Eq.14

Space-Time Approach to Quantum Electrodynamics

R. P. Feynman
Department of Physics, Cornell University, Ithaca, New York
(Received May 9, 1949)

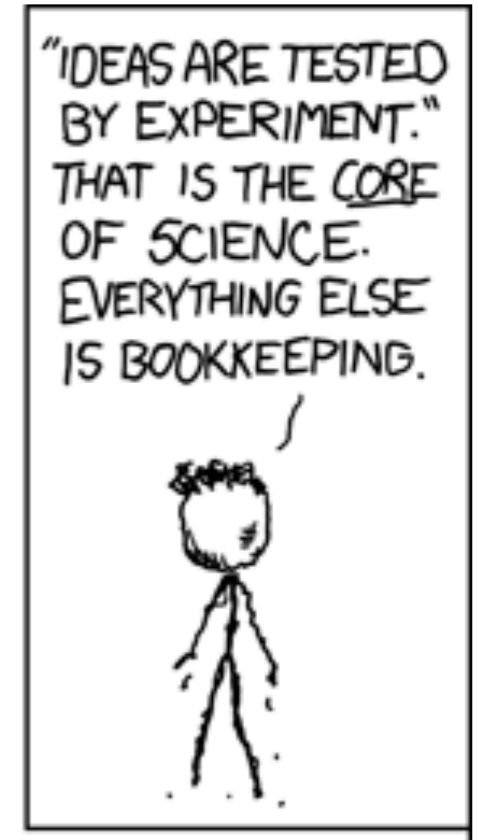
Lamb shift as interpreted in more detail in B.[13]

[13] That the result given in B in Eq. (19) was in error was repeatedly pointed out to the author, in private communication, by V. F. Weisskopf and J. B. French, as their calculation, completed simultaneously with the author's early in 1948, gave a different result. French has finally shown that although the expression for the radiationless scattering B, Eq. (18) or (24) above is correct, it was incorrectly joined onto Bethe's non-relativistic result. He shows that the relation $\ln 2k_{\rm max}-1=\ln\lambda_{\rm min}$ used by the author should have been $\ln 2k_{\rm max}-5/6=\ln\lambda_{\rm min}$. This results in adding a term $-(1/6)$ to the logarithm in B, Eq. (19) so that the result now agrees with that of J. B. French and V. F. Weisskopf,

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
"EUREKA!"
but, "that's funny..."

—Isaac Asimov

Extra Slides



"A method is more important than a discovery, since the right method will lead to new and even more important discoveries."

Lev Landau

Kalin RAP
1910.03008

Reconstruct the Hamiltonian

$$\sqrt{\boldsymbol{p}^2 - \sum_{i=1}^{\infty} P_i(E) \left(\frac{G}{r}\right)^i + m_1^2} + \sqrt{\boldsymbol{p}^2 - \sum_{i=1}^{\infty} P_i(E) \left(\frac{G}{r}\right)^i + m_2^2} = \sum_{i=0}^{\infty} \frac{c_i(\boldsymbol{p}^2)}{i!} \left(\frac{G}{r}\right)^i$$

Solve iteratively — For instance at 3PM $\xi = E_1 E_2 / E^2$

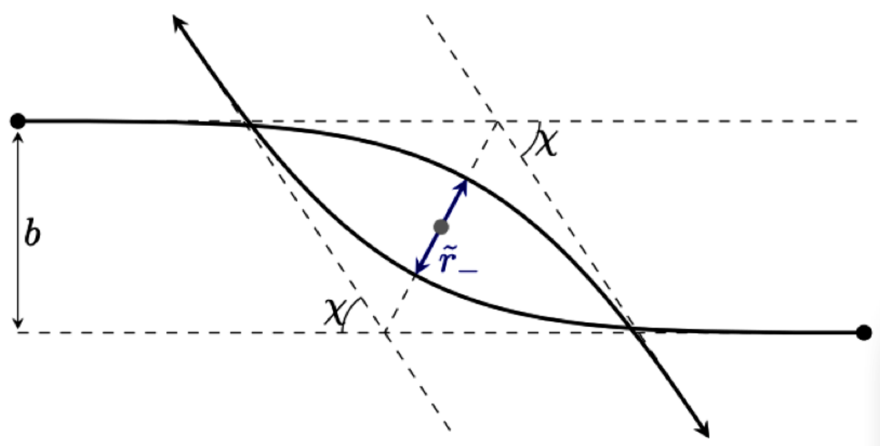
$$\begin{aligned}\frac{c_3(\boldsymbol{p})}{3!} = &- \frac{P_3(E)}{2E\xi} + \frac{(3\xi-1)P_2(E)P_1(E)}{4E^3\xi^3} \\ &+ \frac{(P_2(E)P_1'(E) + P_2'(E)P_1(E))}{4E^2\xi^2} \\ &- \frac{(5\xi^2-5\xi+1)P_1^3(E)}{16E^5\xi^5} - \frac{(9\xi-3)P_1^2(E)P_1'(E)}{16E^4\xi^4} \\ &- \frac{P_1^2(E)P_1''(E)}{16E^3\xi^3} - \frac{P_1(E)(P_1'(E))^2}{8E^3\xi^3},\end{aligned}$$

Use map from momentum to re-write using deflection angle

$$f_n = \sum_{\sigma \in \mathcal{P}(n)} g_\sigma^{(n)} \prod_\ell \left(\widehat{\chi}_b^{(\sigma_\ell)}\right)^{\sigma^\ell} .$$

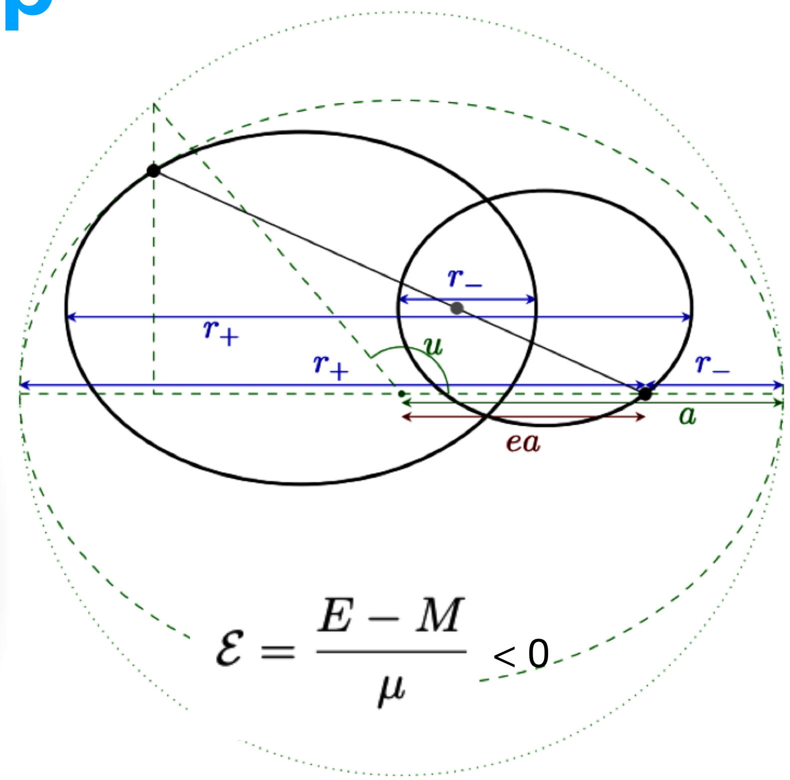
Kalin RAP
1910.03008

Boundary to Bound Map



Analytic Continuation!

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$



$$\mathcal{E} = \frac{E-M}{\mu} < 0$$

Circular orbit

$$r_+(J,\mathcal{E}) = r_-(J,\mathcal{E})$$

First-law binary dynamics:

$$\Omega_{\text{circ}} = \left(\frac{dj(\mathcal{E})}{d\mathcal{E}}\right)^{-1} = \frac{1}{\Gamma}\left(\frac{dj(\mathcal{E})}{d\gamma}\right)^{-1}.$$

Binding energy to 3PM (accurate to 2PN)

$$\epsilon = x\left[1 - \frac{x}{12}(9+\nu) - \frac{x^2}{8}\left(27 - 19\nu + \frac{\nu^2}{3}\right) + \frac{x^3}{32}\left(\frac{535}{6} - \frac{5585\nu}{6} + 135\nu^2 - \frac{35\nu^3}{162}\right) + \frac{x^4}{384}\left(-10171 + \frac{559993}{15}\nu - \frac{34027\nu^2}{3} + \frac{11354\nu^3}{9} + \frac{77\nu^4}{81}\right) + \mathcal{O}(x^5)\right].$$

Controlled to all PN orders by 1PM (strikingly simple!)

$$x_{\text{1PM}} = \frac{(1-\gamma^2)}{((3\gamma - 2\gamma^3)\Gamma)^{2/3}}$$

$$\epsilon = -2\mathcal{E}$$

$$x = (GM\omega)^{2/3} \sim v^2$$

$$\gamma \equiv \frac{1}{2}\frac{E^2 - m_1^2 - m_2^2}{m_1 m_2} = 1 + \mathcal{E} + \frac{1}{2}\nu\mathcal{E}^2\,,$$
$$\Gamma \equiv E/M = \sqrt{1 + 2\nu(\gamma - 1)} = 1 + \nu\mathcal{E}\,.$$

Kalin Liu RAP
2008.06047

EFT approach: Tidal Effects

Bound orbits
$x = (GM\omega)^{2/3} \sim v^2$

Quadrupole to G^2v^4 & G^3v^2 and LO octupole agree with Henry et al.
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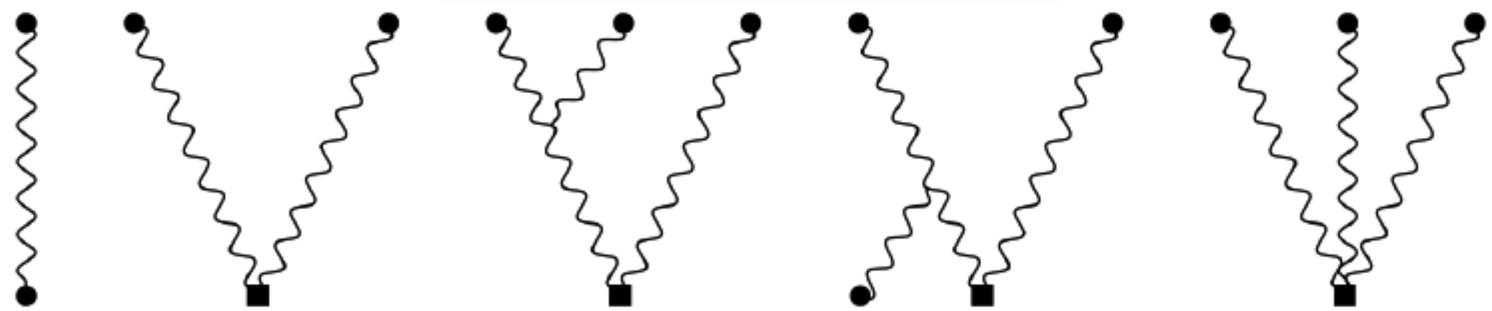

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2006.06665

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$$\Delta\mathcal{E}_{\mathrm{T}} = x\Big[18\,\lambda_{E^2}x^5 + 11\Big(3(1-\nu)\lambda_{E^2} + 6\,\lambda_{B^2} + 5\nu\,\kappa_{E^2}\Big)x^6 + \Big(390\lambda_{\tilde{E}^2} - \frac{13}{28}(161\nu^2 - 161\nu - 132)\lambda_{E^2} - \frac{1326\nu}{7}\kappa_{B^2}$$
$$+\frac{13}{28}(616\nu + 699)\lambda_{B^2} + \frac{13\nu}{84}(490\nu - 729)\kappa_{E^2} + \frac{13}{6}\Delta\bar{P}^{(E,B)}_{8,\mathrm{stc}}\Big)x^7 + 75\big(45\nu\kappa_{\tilde{E}^2} - (13\nu + 3)\lambda_{\tilde{E}^2} + 16\lambda_{\tilde{B}^2}\big)x^8$$
$$-\Big(\frac{85}{36}\left(1083\nu^2 + 1539\nu + 163\right)\lambda_{\tilde{E}^2} + \frac{27200\nu}{3}\kappa_{\tilde{B}^2} - \frac{85}{4}(270\nu + 383)\nu\kappa_{\tilde{E}^2} - \frac{680}{9}(90\nu + 173)\lambda_{\tilde{B}^2} - \frac{17}{6}\Delta\bar{P}^{(\tilde{E},\tilde{B})}_{10,\mathrm{stc}}\Big)x^9\Big]$$

NNLO terms from PM-static and probe limit

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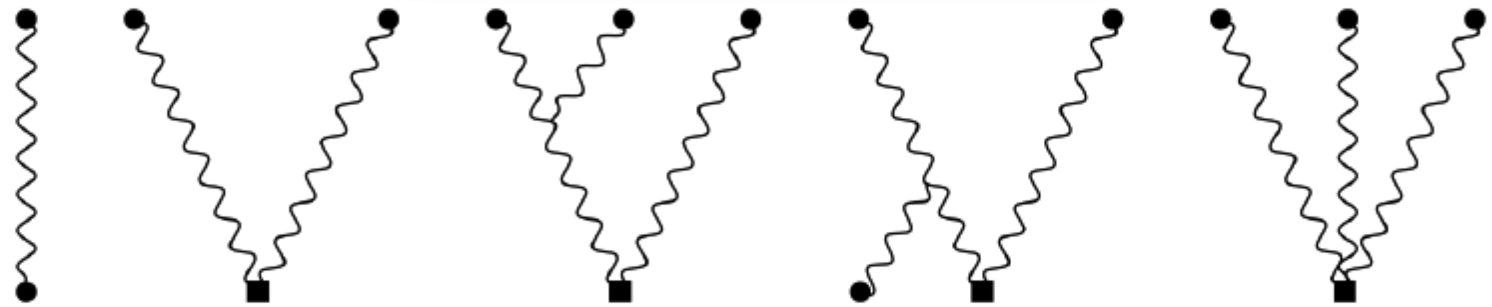
$$\Delta\bar{P}^{(\tilde{E},\tilde{B})}_{10,\mathrm{stc}} = \frac{1}{3}\big(2050\lambda_{\tilde{E}^2} - 13120\lambda_{\tilde{B}^2}\big) + \mathcal{O}(\nu)\,.$$

***We also reconstructed the full PM Hamiltonian to NLO**

Kalin Liu RAP
2008.06047

EFT approach: Tidal Effects

$$\Delta p_a^\mu = -\eta^{\mu\nu} \int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\text{eff}}}{\partial x_a^\nu}(x_a(\tau_a))$$



$$S_{\text{pp}} = \sum_{a=1,2} \int \mathrm{d}\tau_a \Big(-\frac{m_a}{2} g_{\mu\nu} v_a^\mu v_a^\nu + c_{E^2}^{(a)} E_{\mu\nu} E^{\mu\nu} + c_{B^2}^{(a)} B_{\mu\nu} B^{\mu\nu} - c_{\tilde{E}^2}^{(a)} E_{\mu\nu\alpha} E^{\mu\nu\alpha} - c_{\tilde{B}^2}^{(a)} B_{\mu\nu\alpha} B^{\mu\nu\alpha}$$

$$\lambda_{E^2} \equiv \frac{1}{G^4 M^5} \left(m_2 \frac{c_{E^2}^{(1)}}{m_1} + m_1 \frac{c_{E^2}^{(2)}}{m_2} \right),$$

$$\kappa_{E^2} \equiv \lambda_{E^2} + \frac{c_{E^2}^{(1)} + c_{E^2}^{(2)}}{G^4 M^5} = \frac{1}{G^4 M^4} \left(\frac{c_{E^2}^{(1)}}{m_1} + \frac{c_{E^2}^{(2)}}{m_2} \right)$$

$$\begin{aligned}
\frac{\Delta\chi_{(E,B)}}{\Gamma} = {} & \frac{45\pi}{64} \frac{(\gamma^2-1)^2}{(\Gamma j)^6} \Big[\big(35\gamma^4 - 30\gamma^2 - 5\big) \lambda_{B^2} + \big(35\gamma^4 - 30\gamma^2 + 11\big) \lambda_{E^2} \Big] \\
& + \frac{192}{35} \frac{(\gamma^2-1)^{3/2}}{(\Gamma j)^7} \Big[\big(160\gamma^6 - 192\gamma^4 + 30\gamma^2 + 2\big) \lambda_{B^2} + \big(160\gamma^6 - 192\gamma^4 + 72\gamma^2 - 5\big) \lambda_{E^2} \Big] \\
& + \frac{96\nu}{35} \frac{\sqrt{\gamma^2-1}}{(\Gamma j)^7} \kappa_{B^2} \Big[224\gamma^9 - 320\gamma^8 - 728\gamma^7 + 704\gamma^6 + 5488\gamma^5 - 444\gamma^4 + 66262\gamma^3 + 56\gamma^2 + 28084\gamma + 4 \Big] \\
& + \frac{96\nu}{35} \frac{\sqrt{\gamma^2-1}}{(\Gamma j)^7} \kappa_{E^2} \Big[224\gamma^9 - 320\gamma^8 - 728\gamma^7 + 704\gamma^6 + 5628\gamma^5 - 528\gamma^4 + 65982\gamma^3 + 154\gamma^2 + 28329\gamma - 10 \Big] \\
& - \frac{576\nu\sqrt{\gamma^2-1}}{(\Gamma j)^7} \Big[\big(440\gamma^4 + 474\gamma^2 + 32\big)\, \kappa_{B^2} + \big(440\gamma^4 + 474\gamma^2 + 33\big)\, \kappa_{E^2} \Big] a\text{sh}(\gamma)\,,
\end{aligned}$$

Agreement with Cheng-Solon 2006.06665

$$a\text{sh}(\gamma) \equiv (\gamma^2-1)^{-1/2} \sinh^{-1} \sqrt{\tfrac{\gamma-1}{2}}$$

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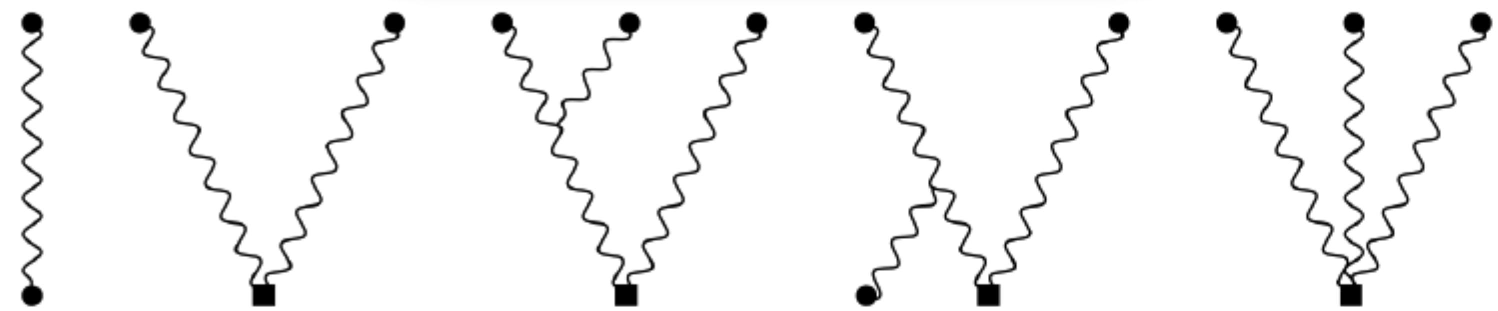
2008.06047

EFT approach: Tidal Effects

$$\Delta p_a^\mu = -\eta^{\mu\nu}\int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\rm eff}}{\partial x_a^\nu}(x_a(\tau_a))$$



$$S_{\rm pp} = \sum_{a=1,2}\int \mathrm{d}\tau_a\Big(-\frac{m_a}{2}g_{\mu\nu}v_a^\mu v_a^\nu + c_{E^2}^{(a)}E_{\mu\nu}E^{\mu\nu} + c_{B^2}^{(a)}B_{\mu\nu}B^{\mu\nu} - c_{\tilde{E}^2}^{(a)}E_{\mu\nu\alpha}E^{\mu\nu\alpha} - c_{\tilde{B}^2}^{(a)}B_{\mu\nu\alpha}B^{\mu\nu\alpha}\Big).$$

$$\lambda_{E^2} \equiv \frac{1}{G^4M^5}\left(m_2\frac{c_{E^2}^{(1)}}{m_1} + m_1\frac{c_{E^2}^{(2)}}{m_2}\right),$$

$$\kappa_{E^2} \equiv \lambda_{E^2} + \frac{c_{E^2}^{(1)} + c_{E^2}^{(2)}}{G^4M^5} = \frac{1}{G^4M^4}\left(\frac{c_{E^2}^{(1)}}{m_1} + \frac{c_{E^2}^{(2)}}{m_2}\right)$$

$$\begin{aligned}\frac{\Delta\chi_{(E,B)}}{\Gamma} = {} & \frac{45\pi}{64}\frac{(\gamma^2-1)^2}{(\Gamma j)^6}\left[(35\gamma^4-30\gamma^2-5)\,\lambda_{B^2} + (35\gamma^4-30\gamma^2+11)\,\lambda_{E^2}\right]\\ & + \frac{192}{35}\frac{(\gamma^2-1)^{3/2}}{(\Gamma j)^7}\left[(160\gamma^6-192\gamma^4+30\gamma^2+2)\,\lambda_{B^2} + (160\gamma^6-192\gamma^4+72\gamma^2-5)\,\lambda_{E^2}\right]\\ & + \frac{96\nu}{35}\frac{\sqrt{\gamma^2-1}}{(\Gamma j)^7}\kappa_{B^2}\left[224\gamma^9-320\gamma^8-728\gamma^7+704\gamma^6+5488\gamma^5-444\gamma^4+66262\gamma^3+56\gamma^2+28084\gamma+4\right]\\ & + \frac{96\nu}{35}\frac{\sqrt{\gamma^2-1}}{(\Gamma j)^7}\kappa_{E^2}\left[224\gamma^9-320\gamma^8-728\gamma^7+704\gamma^6+5628\gamma^5-528\gamma^4+65982\gamma^3+154\gamma^2+28329\gamma-10\right]\\ & - \frac{576\nu\sqrt{\gamma^2-1}}{(\Gamma j)^7}\left[(440\gamma^4+474\gamma^2+32)\ \kappa_{B^2} + (440\gamma^4+474\gamma^2+33)\,\kappa_{E^2}\right]a\mathrm{sh}(\gamma)\,,\end{aligned}$$

Agreement with Cheng-Solon
2006.06665

Notice high-energy limit!

$$a\mathrm{sh}(\gamma) \equiv (\gamma^2-1)^{-1/2}\sinh^{-1}\sqrt{\tfrac{\gamma-1}{2}}$$

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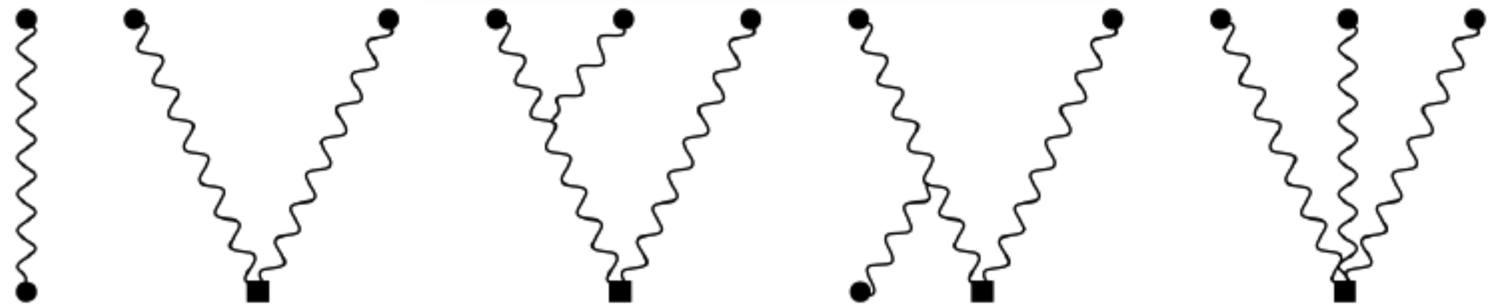
2008.06047

EFT approach: Tidal Effects

$$\Delta p_a^\mu = -\eta^{\mu\nu} \int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\rm eff}}{\partial x_a^\nu}(x_a(\tau_a))$$



$$S_{\rm pp} = \sum_{a=1,2} \int \mathrm{d}\tau_a \Big(-\frac{m_a}{2} g_{\mu\nu} v_a^\mu v_a^\nu + c_{E^2}^{(a)} E_{\mu\nu} E^{\mu\nu} + c_{B^2}^{(a)} B_{\mu\nu} B^{\mu\nu} - c_{\tilde{E}^2}^{(a)} E_{\mu\nu\alpha} E^{\mu\nu\alpha} - c_{\tilde{B}^2}^{(a)} B_{\mu\nu\alpha} B^{\mu\nu\alpha}\,.$$

$$\lambda_{E^2} \equiv \frac{1}{G^4 M^5}\left(m_2 \frac{c_{E^2}^{(1)}}{m_1} + m_1 \frac{c_{E^2}^{(2)}}{m_2} \right),$$

$$\kappa_{E^2} \equiv \lambda_{E^2} + \frac{c_{E^2}^{(1)} + c_{E^2}^{(2)}}{G^4 M^5} = \frac{1}{G^4 M^4}\left(\frac{c_{E^2}^{(1)}}{m_1} + \frac{c_{E^2}^{(2)}}{m_2} \right)$$

$$\begin{aligned}
\frac{\Delta\chi_{(\tilde{E},\tilde{B})}}{\Gamma} = {} & \frac{525\pi}{512(\Gamma j)^8}(\gamma^2-1)^3\Big[(21\gamma^6+385\gamma^4-305\gamma^2+91)\lambda_{\tilde{E}^2}+(21\gamma^6+385\gamma^4-385\gamma^2-21)\lambda_{\tilde{B}^2}\Big] \\
& + \frac{512(\gamma^2-1)^{5/2}}{3003(\Gamma j)^9}\Big[\left(4800\gamma^8+77520\gamma^6-74888\gamma^4+17707\gamma^2+1888\right)\lambda_{\tilde{E}^2} \\
& \qquad + \left(4800\gamma^8+77520\gamma^6-87472\gamma^4+5552\gamma^2-400\right)\lambda_{\tilde{B}^2}\Big] \\
& + \frac{128\nu\sqrt{\gamma^2-1}\,\kappa_{\tilde{B}^2}}{3003(\Gamma j)^9}\Big[27456\gamma^{13}-19200\gamma^{12}+205920\gamma^{11}-271680\gamma^{10}-1589016\gamma^9+950848\gamma^8+22048884\gamma^7 \\
& \qquad -1032064\gamma^6+579540390\gamma^5+395904\gamma^4+826613931\gamma^3-25408\gamma^2+148331040\gamma+1600\Big] \\
& + \frac{128\nu\sqrt{\gamma^2-1}\,\kappa_{\tilde{E}^2}}{3003(\Gamma j)^9}\Big[27456\gamma^{13}-19200\gamma^{12}+205920\gamma^{11}-271680\gamma^{10}-1468896\gamma^9+900512\gamma^8+21724560\gamma^7 \\
& \qquad -980012\gamma^6+580453302\gamma^5+433656\gamma^4+837773079\gamma^3-55724\gamma^2+155291994\gamma-7552\Big] \\
& - \frac{3840\nu\sqrt{\gamma^2-1}}{(\Gamma j)^9}\Big[(7292\gamma^6+19484\gamma^4+7905\gamma^2+288)\kappa_{\tilde{B}^2}+(7292\gamma^6+19644\gamma^4+8141\gamma^2+310)\kappa_{\tilde{E}^2}\Big]\mathrm{ash}(\gamma)\,.
\end{aligned}$$

Notice
high-energy limit!

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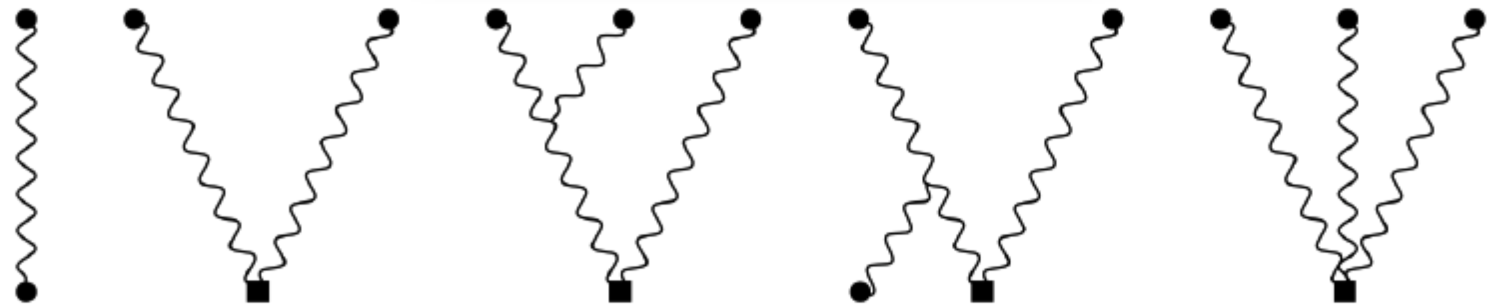
EFT approach: Tidal Effects

$$\Delta p_a^\mu = -\eta^{\mu\nu} \int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\mathrm{eff}}}{\partial x_a^\nu}(x_a(\tau_a))$$



How to reconstruct the radial action?

$$i_r \equiv \frac{p_\infty}{\sqrt{-p_\infty^2}} \chi_j^{(1)} - j \left(1 + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\chi_j^{(2n)}}{(1-2n) j^{2n}}\right)$$

We have n=3

We are missing n=4,5…

$$\frac{\Delta\chi_{(E,B)}}{\Gamma} = \frac{45\pi}{64} \frac{(\gamma^2-1)^2}{(\Gamma j)^6} \left[\left(35\gamma^4 - 30\gamma^2 - 5\right) \lambda_{B^2} + \left(35\gamma^4 - 30\gamma^2 + 11\right) \lambda_{E^2} \right]$$

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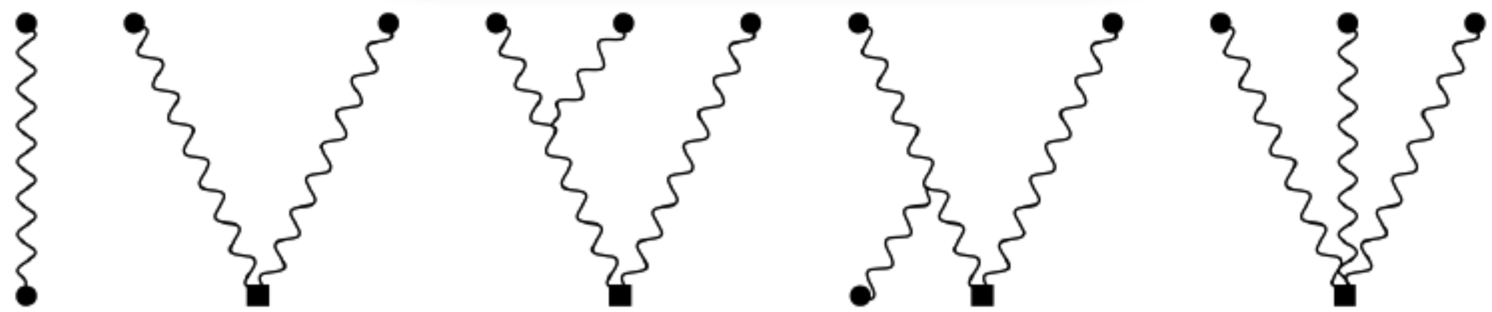
EFT approach: Tidal Effects

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$



How to reconstruct the radial action?

$$i_r \equiv \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\chi_j^{(2n)}}{(1-2n)j^{2n}}\right)$$

We have n=3

We are missing n=4,5…

(But we have P6 & P7 for Quadrupole)

(and P8 & P9 for Octupole)

$$\bar{P}_n \equiv P_n/(\mu^2 M^n) \text{ and } \hat{p}_\infty = p_\infty/\mu,$$

$$\chi_j^{(6)} = \frac{5\pi}{32}\left(\bar{P}_2^3 + 6\bar{P}_1\bar{P}_2\bar{P}_3 + 3\hat{p}_\infty^2\bar{P}_3^2 + 3\hat{p}_\infty^4\bar{P}_6 + \cdots\right)$$

$$\chi_j^{(8)} = \frac{105\pi}{64}\left(\frac{\bar{P}_2^4}{12} + \frac{\bar{P}_1^2\bar{P}_3^2}{2} + \bar{P}_1\bar{P}_2^2\bar{P}_3 + \hat{p}_\infty^2(\bar{P}_1^2\bar{P}_6 + P_2P_3^2) + \hat{p}_\infty^4\left(\bar{P}_1\bar{P}_7 + \bar{P}_2\bar{P}_6\right) + \frac{\hat{p}_\infty^6}{3}\bar{P}_8 + \cdots\right)$$

$$\chi_j^{(10)} = \frac{315\pi}{512}\left(\frac{\bar{P}_2^5}{5} + \bar{P}_1^4\bar{P}_6 + \cdots + 4\hat{p}_\infty^2\left(3\bar{P}_1^2\bar{P}_2\bar{P}_6 + \bar{P}_1^3\bar{P}_7 + \cdots\right) + 6\hat{p}_\infty^4\left(\bar{P}_1^2\bar{P}_8 + 2\bar{P}_1\bar{P}_2\bar{P}_7 + \cdots\right) + 4\hat{p}_\infty^6(\bar{P}_2\bar{P}_8 + \bar{P}_1\bar{P}_9 + \bar{P}_3\bar{P}_7) + \hat{p}_\infty^8\bar{P}_{10} + \cdots\right)$$

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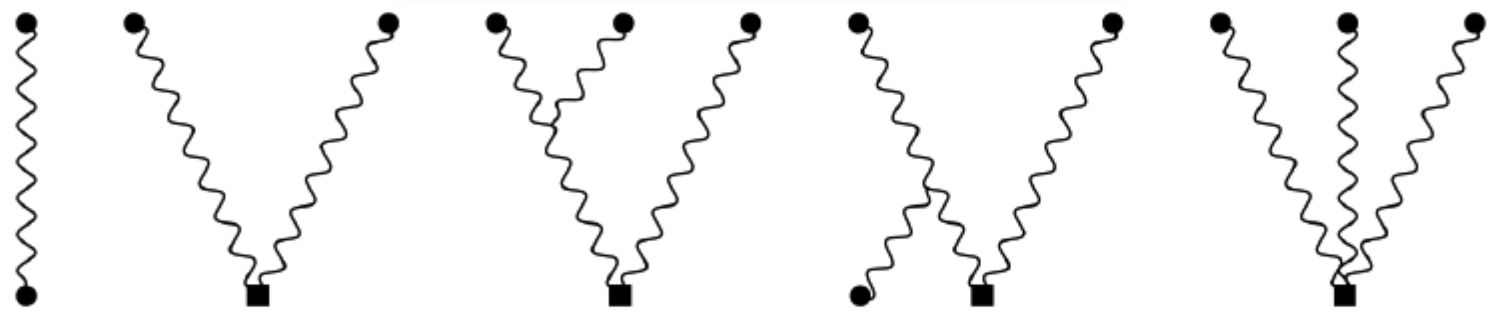
EFT approach: Tidal Effects

Bound orbits

$$p_\infty^2 \sim \mathcal{E} \quad 1/J \sim |p_\infty|$$



How to reconstruct the radial action?

$$i_r \equiv \frac{p_\infty}{\sqrt{-p_\infty^2}} \chi_j^{(1)} - j \left(1 + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\chi_j^{(2n)}}{(1-2n) j^{2n}} \right)$$

We have n=3

We are missing n=4,5…

(But we have P6 & P7 for Quadrupole)

(and P8 & P9 for Octupole)

$$\chi_j^{(6)} = \frac{5\pi}{32} \left(\bar{P}_2^3 + 6 \bar{P}_1 \bar{P}_2 \bar{P}_3 + 3 \hat{p}_\infty^2 \bar{P}_3^2 + 3 \hat{p}_\infty^4 \bar{P}_6 + \cdots \right)$$

$$\chi_j^{(8)} = \frac{105\pi}{64} \left(\frac{\bar{P}_2^4}{12} + \frac{\bar{P}_1^2 \bar{P}_3^2}{2} + \bar{P}_1 \bar{P}_2^2 \bar{P}_3 + \hat{p}_\infty^2 (\bar{P}_1^2 \bar{P}_6 + P_2 P_3^2) + \hat{p}_\infty^4 (\bar{P}_1 \bar{P}_7 + \bar{P}_2 \bar{P}_6) + \frac{\hat{p}_\infty^6}{3} \bar{P}_8 + \cdots \right)$$

$$\chi_j^{(10)} = \frac{315\pi}{512} \left(\frac{\bar{P}_2^5}{5} + \bar{P}_1^4 \bar{P}_6 + \cdots + 4 \hat{p}_\infty^2 (3 \bar{P}_1^2 \bar{P}_2 \bar{P}_6 + \bar{P}_1^3 \bar{P}_7 + \cdots) + 6 \hat{p}_\infty^4 (\bar{P}_1^2 \bar{P}_8 + 2 \bar{P}_1 \bar{P}_2 \bar{P}_7 + \cdots) + 4 \hat{p}_\infty^6 (\bar{P}_2 \bar{P}_8 + \bar{P}_1 \bar{P}_9 + \bar{P}_3 \bar{P}_7) + \hat{p}_\infty^8 \bar{P}_{10} + \cdots \right)$$

Notice the PN-suppression

Counts the number of Pn's which is tied to each PM order

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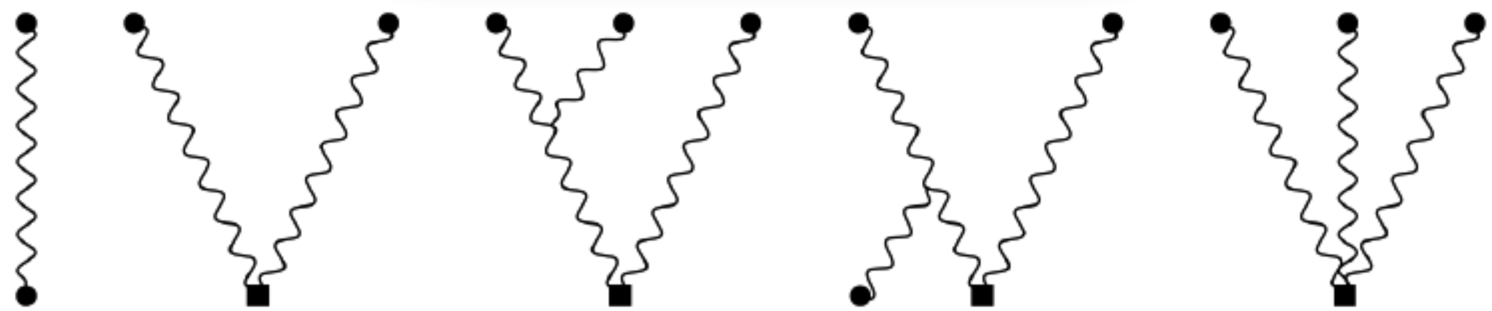
EFT approach: Tidal Effects

Bound orbits

$$p_\infty^2 \sim \mathcal{E} \quad 1/J \sim |p_\infty|$$



The same PN order appears at different PM orders in i_r

$$i_r \equiv \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\chi_j^{(2n)}}{(1-2n)j^{2n}}\right)$$

e.g. recover the full Newtonian Periastron Advance

$$\Delta\Phi_{E^2}^{\rm Newton}(p_\infty, j) = \frac{15\pi}{8}C\frac{p_\infty^4}{j^6}\left[1+14\epsilon^2+21\epsilon^4\right],$$

$$\epsilon = Gm_1m_2/(p_\infty P_\phi) = 1/(p_\infty j)$$

Bini Damour Geralico

2001.00352

$$\chi_j^{(6)} = \frac{5\pi}{32}\left(\bar{P}_2^3 + 6\bar{P}_1\bar{P}_2\bar{P}_3 + 3\hat{p}_\infty^2\bar{P}_3^2 + 3\hat{p}_\infty^4\bar{P}_6 + \cdots\right)$$

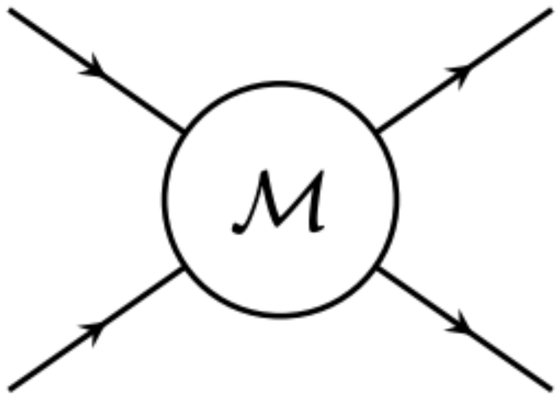
$$\chi_j^{(8)} = \frac{105\pi}{64}\left(\frac{\bar{P}_2^4}{12} + \frac{\bar{P}_1^2\bar{P}_3^2}{2} + \bar{P}_1\bar{P}_2^2\bar{P}_3 + \hat{p}_\infty^2(\bar{P}_1^2\bar{P}_6\right.$$

$$\left. + P_2P_3^2) + \hat{p}_\infty^4\left(\bar{P}_1\bar{P}_7 + \bar{P}_2\bar{P}_6\right) + \frac{\hat{p}_\infty^6}{3}\bar{P}_8 + \cdots\right)$$

$$\chi_j^{(10)} = \frac{315\pi}{512}\left(\frac{\bar{P}_2^5}{5} + \bar{P}_1^4\bar{P}_6 + \cdots + 4\hat{p}_\infty^2(3\bar{P}_1^2\bar{P}_2\bar{P}_6\right.$$

$$+ \bar{P}_1^3\bar{P}_7 + \cdots) + 6\hat{p}_\infty^4(\bar{P}_1^2\bar{P}_8 + 2\bar{P}_1\bar{P}_2\bar{P}_7 + \cdots)$$

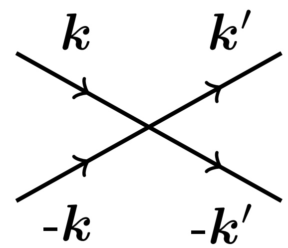
$$\left. + 4\hat{p}_\infty^6(\bar{P}_2\bar{P}_8 + \bar{P}_1\bar{P}_9 + \bar{P}_3\bar{P}_7) + \hat{p}_\infty^8\bar{P}_{10} + \cdots\right)$$



Cheung et al. 1808.02489

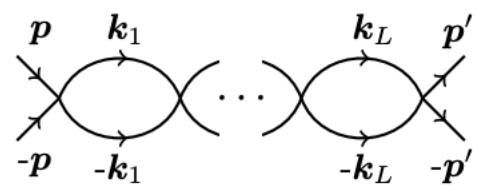
Bern et al. 1908.01493

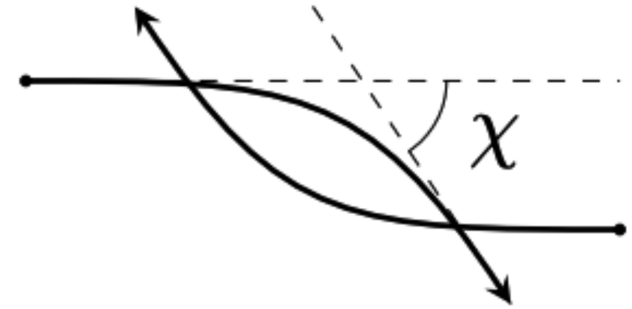
See Zvi's, Radu's and Mikhail's talks

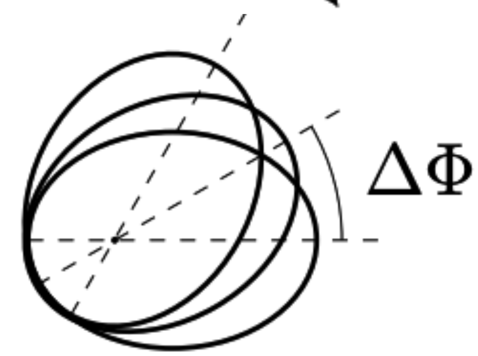


$$= -iV(\boldsymbol{k}, \boldsymbol{k}')$$

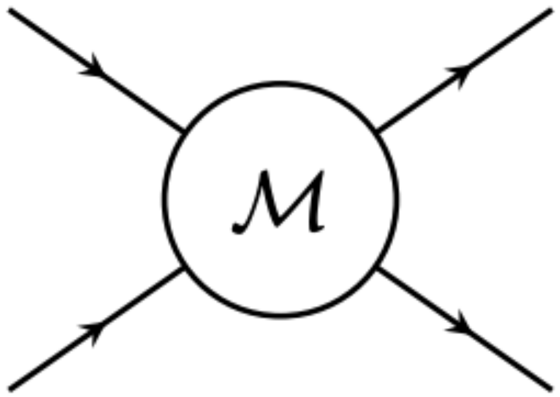
$$M_{\text{EFT}}^{L\text{-loop}} =$$





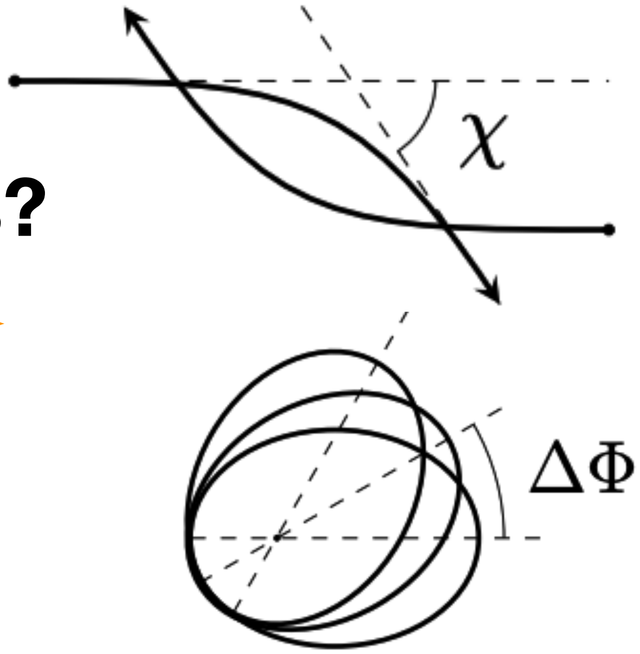


Kalin RAP
1910.03008



Can we map gauge-invariant objects?

Do we need the Hamiltonian?



Kalin RAP
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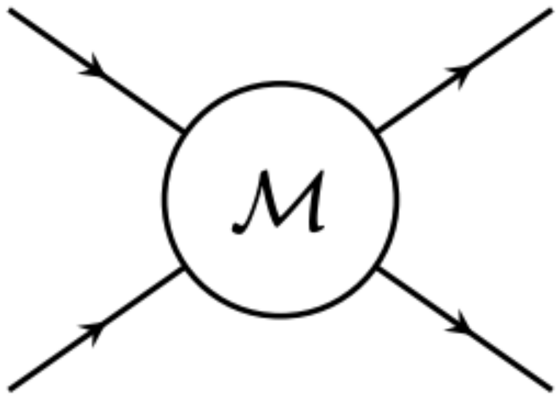

Bern et al.
1908.01493

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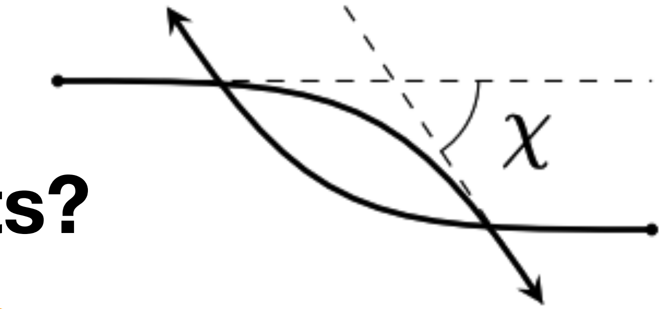
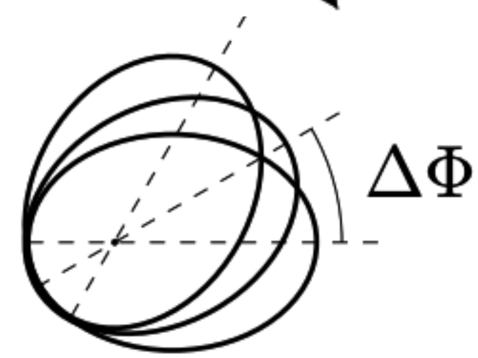




Momentum in the Center-of-Mass

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \sum_i P_i(E)\left(\frac{G}{r}\right)^i = p_\infty^2(E)\left(1+\sum_i f_i(E)\left(\frac{GM}{r}\right)^i\right)$$

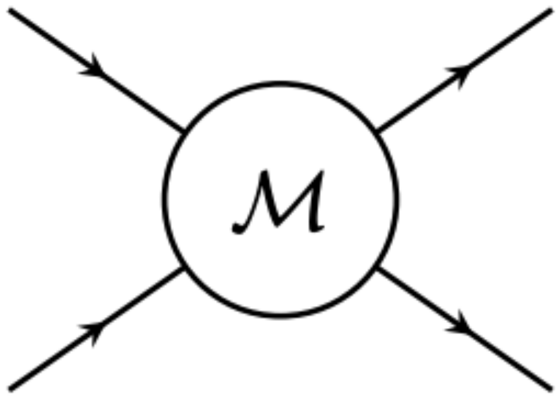
Related to the Fourier transform of the amplitude (relativistic normalization)

$$\widetilde{\mathcal{M}}(r,E) = \sum_{n=1}^{\infty}\widetilde{\mathcal{M}}_n(E)\left(\frac{G}{r}\right)^n \qquad \widetilde{\mathcal{M}}(r,E) \equiv \frac{1}{2E}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\mathcal{M}(\mathbf{q},\mathbf{p}^2=p_\infty^2(E))e^{-i\mathbf{q}\cdot\mathbf{r}}$$

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
"EUREKA!"
but, "that's funny..."

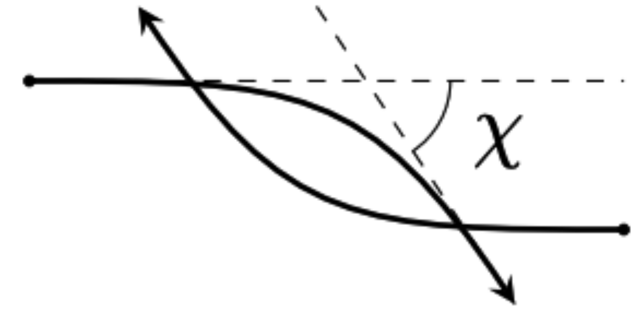
—Isaac Asimov

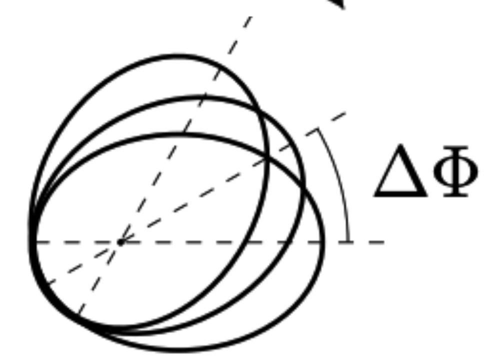
Kalin RAP
1910.03008



Impetus Formula

Do w d the
Har an?





Momentum in the Center-of-Mass

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \sum_i P_i(E)\left(\frac{G}{r}\right)^i = p_\infty^2(E)\left(1+\sum_i f_i(E)\left(\frac{GM}{r}\right)^i\right)$$

Related to the Fourier transform of the amplitude (relativistic normalization)

$$\widetilde{\mathcal{M}}(r,E) = \sum_{n=1}^{\infty} \widetilde{\mathcal{M}}_n(E)\left(\frac{G}{r}\right)^n \qquad \widetilde{\mathcal{M}}(r,E) \equiv \frac{1}{2E}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\mathcal{M}(\mathbf{q},\mathbf{p}^2 = p_\infty^2(E))e^{-i\mathbf{q}\cdot\mathbf{r}}$$

Remarkably! * *

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$

The most exciting phrase
to hear in science,
the one that heralds
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—Isaac Asimov

*IR-finite
*Potential Modes

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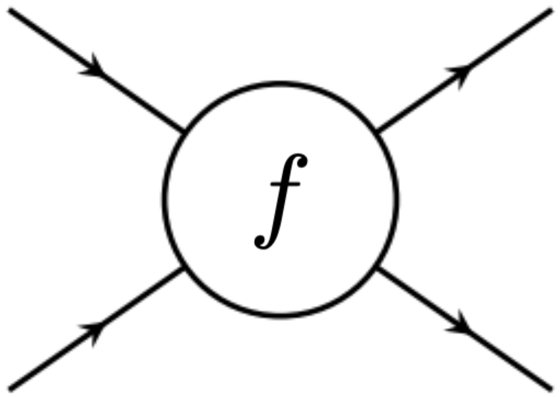
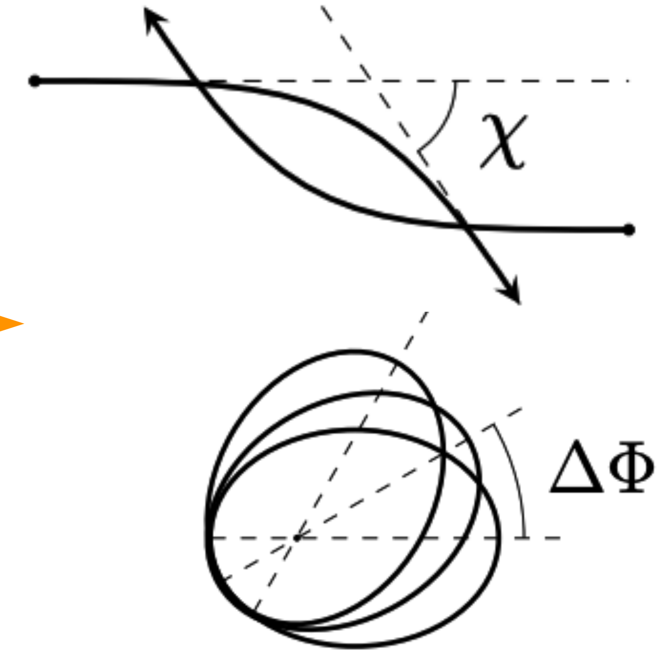


$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$

Sketch of the Proof
(via NR Matching):



Effective Non-Relativistic Schrödinger-type equation

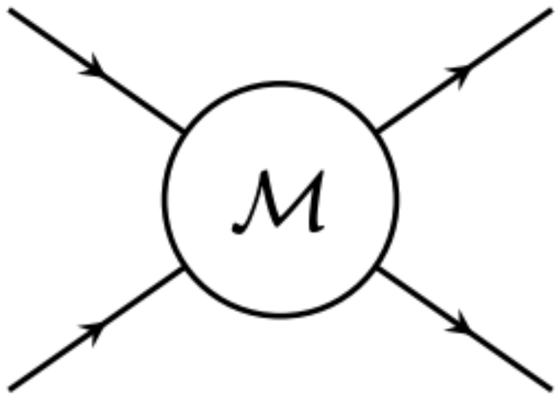
$$H_{\text{eff}}|\psi_{\boldsymbol{p}}(p_\infty)\rangle = \left(\boldsymbol{p}^2 + V_{\text{eff}}\right)|\psi_{\boldsymbol{p}}(p_\infty)\rangle = p_\infty^2(E)|\psi_{\boldsymbol{p}}(p_\infty)\rangle$$

See also Damour's talk

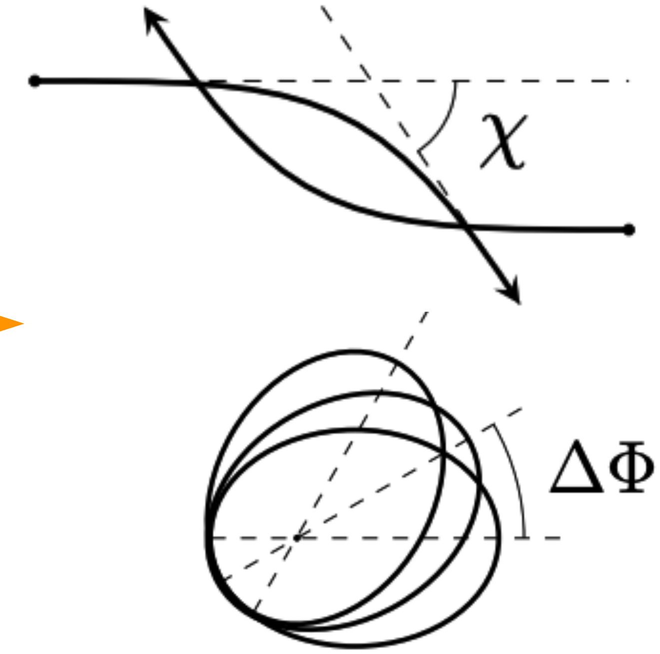
$$4\pi f(\boldsymbol{p},\boldsymbol{q}) = -\underbrace{\langle \boldsymbol{p}+\boldsymbol{q}|V_{\text{eff}}|\boldsymbol{p}\rangle}_{\text{Born}} + \overbrace{\cdots}^{\text{Iterations}}$$

See also Poul's talk

Kalin RAP
1910.03008



$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$



Sketch of the Proof
(via NR Matching):

Effective Non-Relativistic Schrödinger-type equation

$$H_{\text{eff}}|\psi_{\boldsymbol{p}}(p_\infty)\rangle = \left(\boldsymbol{p}^2 + V_{\text{eff}}\right)|\psi_{\boldsymbol{p}}(p_\infty)\rangle = p_\infty^2(E)|\psi_{\boldsymbol{p}}(p_\infty)\rangle$$

Impetus — Iterations

Match to Relativistic

$$4\pi f(\boldsymbol{p},\boldsymbol{q}) = -\langle \boldsymbol{p}+\boldsymbol{q}|V_{\text{eff}}|\boldsymbol{p}\rangle + \cdots$$

CANCEL OUT ON BOTH SIDES!

$$\frac{d\sigma}{d\Omega} = |f(p_\infty^2,\boldsymbol{q})|^2 = \frac{1}{(4\pi)^2(2E)^2}|\mathcal{M}(p_\infty^2,\boldsymbol{q})|^2$$

$$\int \mathrm{d}^d\boldsymbol{l}\, \frac{f^{(\alpha\beta\gamma)}(\boldsymbol{l},\boldsymbol{p},\boldsymbol{q})}{|\boldsymbol{l}|^\alpha|\boldsymbol{l}+\boldsymbol{q}|^\beta(2\boldsymbol{l}\cdot\boldsymbol{p}+\boldsymbol{l}^2)^\gamma}\,, \quad \xrightarrow[\gamma=1]{}$$

Super-classical
IR divergent

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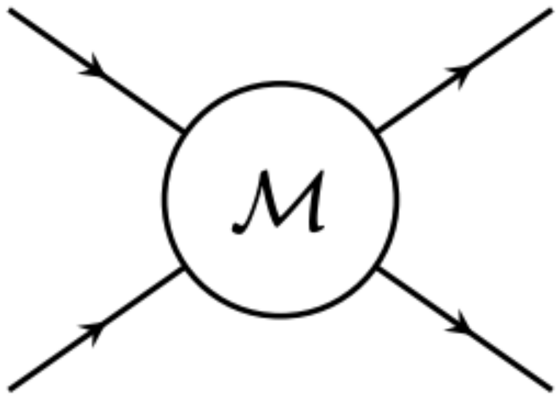

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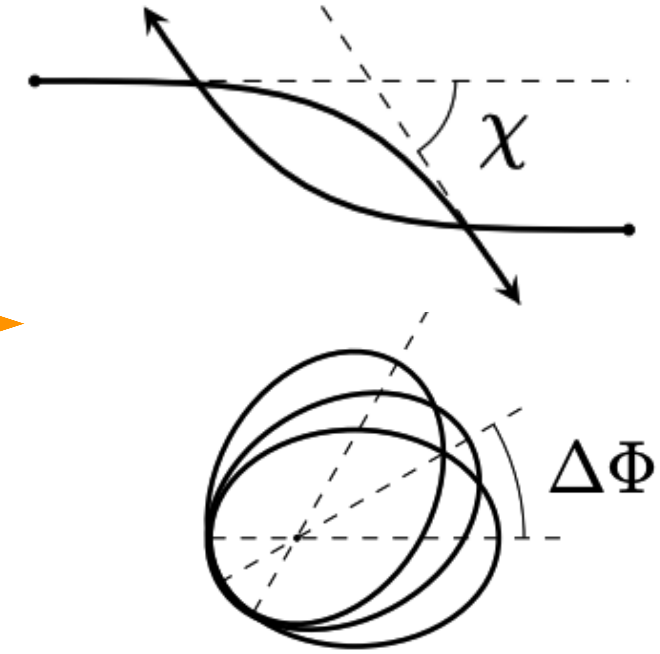

Sketch of the Proof

(inspired by 1811.10950
Kosower Maybee O'Connell)

Introduce a 'scattered momentum' ***at a point*** (in standard QM)

$$\boldsymbol{p}_{\rm sc}^2(r,p_\infty^2) = \psi_{\boldsymbol{p}}^\dagger(\boldsymbol{r},p_\infty)\,(-\nabla^2 - p_\infty^2)\psi_{\boldsymbol{p}}(\boldsymbol{r},p_\infty)$$

$$\Psi_{\mathbf{p}} \sim e^{i\mathbf{p}\cdot\mathbf{r}} + \frac{f(\theta)e^{ipr}}{r}$$

Two contributions (linear and quadratic in the amplitude)

$$\boldsymbol{p}_{\rm sc}^2(r,E) = I_{(1)}(r,E) + I_{(2)}(r,E)$$

Radiation-Reaction

IR-safe by construction!

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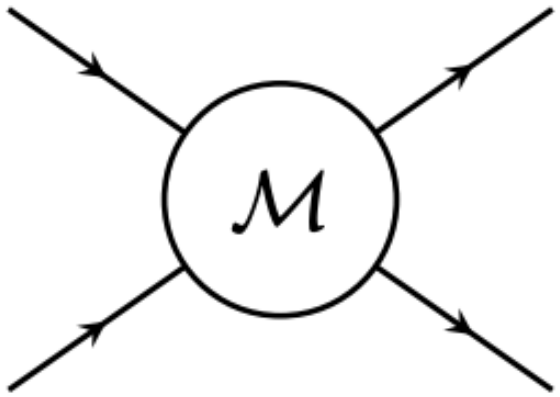
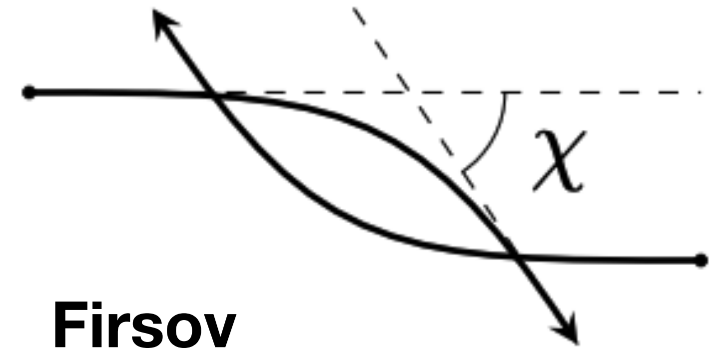
$$\frac{1}{2}\chi(b,E)=\sum_n \chi_b^{(n)}(E)\left(\frac{GM}{b}\right)^n=\sum_n \chi_j^{(n)}(E)\frac{1}{j^n}$$



$$\boldsymbol{p}^2(r,E)=p_\infty^2(E)+\widetilde{\mathcal{M}}(r,E)$$



$$\bar{p}^2(r,E)=\exp\left[\frac{2}{\pi}\int_{r|\bar{p}(r,E)|}^{\infty}\frac{\chi_b(\tilde{b},E)\mathrm{d}\tilde{b}}{\sqrt{\tilde{b}^2-r^2\bar{p}^2(r,E)}}\right]$$

Firsov

$(\bar{p}\equiv|\boldsymbol{p}|/p_\infty)$

angle from momentum (and vice-verse):

$$\chi_b^{(n)}=\frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell\frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$

Directly from the amplitude to all orders!

$$4\,\chi_j^{(4)}=2\hat{p}_\infty^4\sqrt{\pi}\,\Gamma\left(\frac{5}{2}\right)\left(\frac{1}{\Gamma(2)}\frac{f_4^1}{1!}+\frac{1}{\Gamma(1)}\frac{f_2^2}{2!}+\frac{1}{\Gamma(1)}\frac{f_1^1f_3^1}{1!1!}+\frac{1}{\Gamma(-1)}\frac{f_1^4}{4!}\right)$$

$$4\chi_j^{(4)}=\frac{3\pi\hat{p}_\infty^4}{4}\left(f_2^2+2f_1f_3+2f_4\right)=\frac{3\pi}{4M^4\mu^4}\left(\widetilde{\mathcal{M}}_2^2+2\widetilde{\mathcal{M}}_1\widetilde{\mathcal{M}}_3+2p_\infty^2\widetilde{\mathcal{M}}_4\right)$$

(Keep an eye on this expression)

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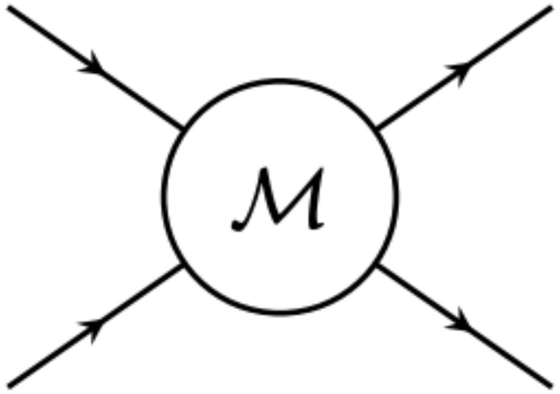

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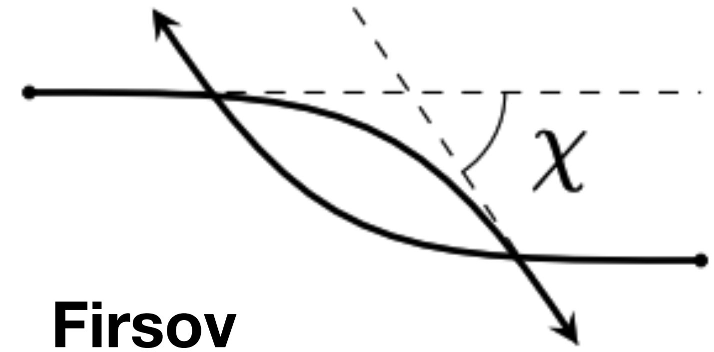

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Firsov

$(\bar{p}\equiv|\boldsymbol{p}|/p_\infty)$

In general f_i's enter to all PM orders:

$$\chi_b^{(2n)}[f_{1,2}]=\frac{\sqrt{\pi}f_2^n\Gamma\left(n+\frac{1}{2}\right)}{2\Gamma(n+1)},\quad n=1,2,\ldots$$

$$\chi_b^{(2n+1)}[f_{1,2}]=\frac{1}{2}f_1 f_2^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{f_1^2}{4f_2^2}\right),\quad n=0,1,\ldots,$$

$$\frac{\chi[f_{1,2}]+\pi}{2}=\frac{1}{\sqrt{1-\mathcal{F}_2y^2}}\left(\frac{\pi}{2}+\arctan\left(\frac{y}{2\sqrt{1-\mathcal{F}_2y^2}}\right)\right)\qquad \mathcal{F}_2\equiv f_2/f_1^2$$