PN approximation: new results from NRGR

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S. Foffa, RS, W. J. Torres Bobadilla, arXiv:2010.13730
G. L. Almeida, S. Foffa, RS, arXiv:2008.06195, JHEP in press
L. Blanchet, S. Foffa, F. Larrouturou, arXiv:1912.12359, PRD (2020)
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S. Foffa, R. Porto, I. Rothstein, RS, arXiv:1903.05118, PRD (2019)
S. Foffa, P. Mastrolia, RS, C. Sturm, W. J. Torres Bobadilla, arXiv:1902.10571, PRL '19
S. Foffa, P. Mastrolia, RS, C. Sturm, arXiv:1612.00482, PRD '17

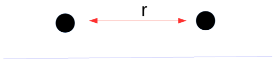
Oct 30th, 2020, EFT methods from Bound States to Binary Systems

Outline

Fundamental aspects of the EFT formalism

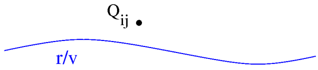
NRGR & PN approximation to GR: Small expansion parameter v, related to metric perturbation $v^2 \sim \frac{GM}{r}$

Near zone, $D \sim r$



Describe conservative dynamics

Far zone, $D \gtrsim \lambda = r/v$



conservative + dissipative

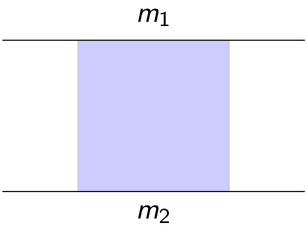
EFT framework pioneered by W. Goldberger and I. Rothstein, PRD '06

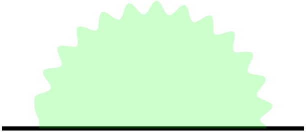
for spin R. Porto PRD '06

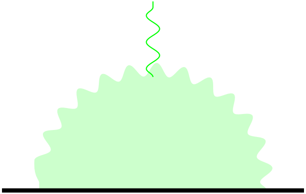
PN approximation for compact binary systems

	Near	Far
World-line	$-m_a \int dt + \int dx^\mu S^{ij}\omega_{\mu ij}$ $\int dt \left(Q_{ij}(S) R_{0i0j} + R^2_{\mu\nu\rho\sigma} + \dots\right)$	$\int d^4x \Big(E h_{00} + \frac{1}{2} S^{ij} h_{0i,j}$ $+Q_{ij} \underbrace{E^{ij} + O_{ijk} E^{ij,k}}_{v} + J_{ij} B_{ij} \dots \Big)$
Bulk	$\frac{1}{16\pi G} \int d^4x \left[R - \frac{1}{2}\left(g^{\alpha\beta}\Gamma^\mu_{\alpha\beta}\right)^2\right]$	
5PN	Foffa+ '19, '20 (partial) Blümlein+ '20	Foffa+ '19 (Blümlein '20)
6&7 PN	×	Blanchet+ '20 (logs)

Near vs. Far zone graphs







And 1 pt diagrams $\rightarrow$ radiation
In this talk in-out formalism
no radiation to ∞

- Interactions at high loop $\rightarrow$ lengthy expressions, double copy?
  See e.g. Goldberger+ '18, Chen '18, Almeida+ '20

Method of regions

Method of regions: Internal graviton momentum can be expanded following the scaling:

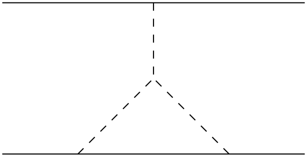
hard	(m, m)	quantum	×
soft	$(\|\vec{q}\|, \|\vec{q}\|)$	quantum	×
potential	$(v/r, 1/r)$	classical	✓
radiation	$(v/r, v/r)$	classical	✓

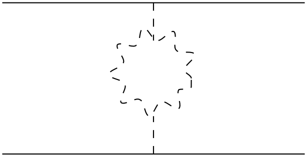
and then integrated over the full phase space

Only potential and radiation gravitons exchanged in classical processes: theory in terms of world lines selects diagrams that do not send source off-shell

Potential graviton $\rightarrow$ small change in energy wrt momentum, dominate classically

Ex. of classical/quantum connected diagrams





Compensating divergences Near - Far

- Near IR/Far UV

$$V \supset GM\dddot{Q}_{ij}^2\left(\frac{1}{\epsilon_{UV}}+\log(\mu\omega)\right)+GMm_1m_2r^2\dot{a}_1^i\dot{a}_{2i}\left(\frac{1}{\epsilon_{IR}}+\log(\mu r)\right)$$

Theory at short and large distances have compensating spurious divergences, finite terms derived straightforwardly (Manohar+ '07, Jantzen '12)

- Near zone UV divergences canceled by local counterterms:

$$G^2m_a^3\int d\tau\left(a^\mu\dot{v}_\mu+R_{\mu\nu}v^\mu v^\nu\right)$$

Foffa+ '19

$a^\mu=0=R_{\mu\nu}$ on the equations of motion

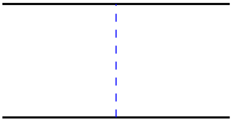
- No far zone IR divergences
- From far zone alone → leading UV logs in the Energy function at all orders via Renormalization group flow (see later in this talk)

Goldberger+ '13, Blanchet + '20

Outline

Near zone conservative dynamics

The potential V (via Feynman Green function):



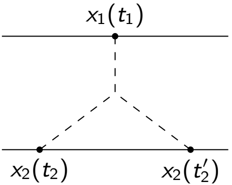
$$
\begin{aligned}
V & \propto \int dk_0 d^3k \frac{e^{-ik_0t_{12}+i\vec{k}\cdot(\vec{x}_1(t_1)-\vec{x}_2(t_2))}}{k^2-k_0^2-i\epsilon} = \int dk_0 d^3k \frac{e^{-ik_0t_{12}+i\vec{k}\cdot\vec{x}_{12}}}{k^2}\left(1+\frac{k_0^2}{k^2}+\ldots\right)\\
& = \delta(t_1-t_2)\int d^3k \frac{e^{i\vec{k}\cdot\vec{x}_{12}}}{k^2}\left(1+\frac{\partial_{t_1}\partial_{t_2}}{k^2}+\ldots\right)\\
& = \int d^3k \frac{e^{i\vec{k}\cdot\vec{x}_{12}}}{k^2}\left(1-\frac{\vec{k}\cdot\vec{v}_1\vec{k}\cdot\vec{v}_2}{k^2}+\ldots+\frac{\vec{k}\cdot\frac{d^{n-1}\vec{v}_1}{dt^{n-1}}\vec{k}\cdot\frac{d^{n-1}\vec{v}_2}{dt^{n-1}}}{k^{2n}}\right)
\end{aligned}
$$

Near zone amplitude integrands clearly bad behaved for $k\to 0$ at high PN-order
Straightforward fix: add the contribution of far-zone, for demonstration see e.g.

Manohar+ '07, Jentzen '12, Blumlein+ '20

(IR) Divergences in the Near zone

IR divergences due to the splitting into Near and Far



In the full theory:

$$
\begin{aligned}
V \quad &\supset \quad \int dt_{1,2,2'} d^4p\, e^{ip_\mu(x_1^\mu(t_1)-x_2^\mu(t_2))} \frac{p^\alpha p^\beta}{p^2} \int d^4k \frac{e^{ik^\mu(x_2(t_2)-x_2(t_2'))}}{(p-k)^2k^2} \\
&= \quad \int dt_{1,2,2'} d^4p e^{ip_\mu(x_1^\mu(t_1)-x_2^\mu(t_2))} \frac{p^\alpha p^\beta}{p^2} \Delta(p^\mu(x_2(t_2)-x_2(t_2')))
\end{aligned}
$$

after near/far breaking:

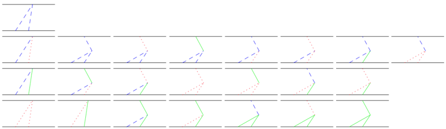
$$
\int dt\, d^3p\, e^{i\vec{p}(\vec{x}_1-\vec{x}_2)} \frac{p^i p^j}{|\mathbf{p}^2|} \int d^3k \frac{1}{|\mathbf{k}|^2|\mathbf{p}-\mathbf{k}|^2} \left(1+\ldots+\frac{\omega^4}{|\mathbf{k}|^4}+\ldots\right)
$$

$$
= \int dt\, d^3p e^{i\vec{p}\cdot\vec{x}_{12}} \frac{p^i p^j}{|\mathbf{p}|^3} \left\{ 1+\ldots+\frac{1}{|\mathbf{p}|^4}\left[(\vec{p}\cdot\vec{v}_1)^3(\vec{p}\cdot\vec{v}_2)^3+\ldots+\underbrace{\vec{p}\cdot\dot{\vec{a}}_1\vec{p}\cdot\dot{\vec{a}}_2}_{IR\ \ divergence}\right]\right\}
$$

Diagrams proliferations

For PN expansion it is useful to distinguish gravity polarizations, e.g. G_N^2

2 topologies



23 Feynman diagrams at 4PN $(G^2 v^3)$

- - - - - $\sim \phi \to v^0$; ··········· $\sim A_i \to v^i$; ——— $\sim \sigma_{ij} \to v^i v^j$;

However only 1 of the two topologies is intrinsically G_N^2

Factorizable vs. prime diagrams

Number grows exponentially

3PN	prime top	prime dgrs	fac	4PN	prime dgrs	fac
G_N	1	3	0		3	0
G_N^2	1	16	3		18	23
G_N^3	5	31	22		158	54
G_N^4	12	0	8		171	146
G_N^5				25	25	25

S. Foffa, RS PRD'19

At 5PN the $G_N^5 v^2$ sector has 40 prime topologies ($\sim$ 700 prime diagrams) and 1232 fac. diags

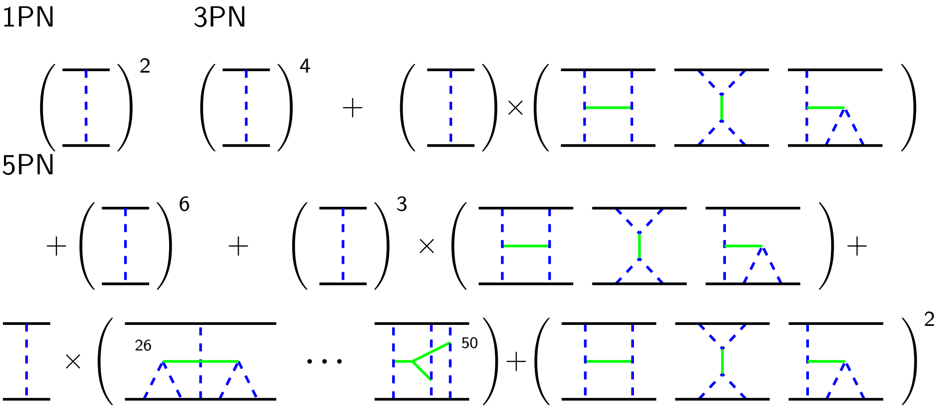
Effective action does not efficiently store perturbative G_N information

NRGR can help tackling the high n $G_N^n v^{0,2}$ side (orthogonal to PM)

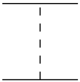
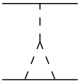
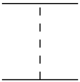
Factorizable diagrams at G^6 5PN

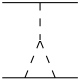
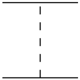
5PN G^6 static sector has 0 prime toplgs and 154 fac dgrs
Static sectors at $2n+1$-PN order have no prime sector:
impossible to build prime digrs with $2n+1$ m(ass) insertions and m-ϕ and $\phi^2\sigma$ vertices

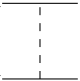
Foffa, Mastrolia, RS, Sturm, W. Torres Bobadilla, PRL '19



Factorizable diagrams at G^5 5PN

At G^5v^2: $\Big($  $\Big)^5$ +  $\times$ $\Big($  $\Big)^3$

$+ \left(31 G_N^3 \text{prime dgrs}\right) \times \Bigg($  $+ \Big($  $\Big)^2\Bigg)$

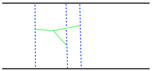
$+ \left(171 G_N^4 v^2 \text{prime dgrs}\right) \times \Big($  $\Big)^2$

They amount to 1232 out of 1907 G^5v^2 diagrams

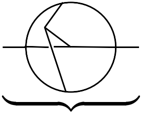
Foffa, RS. W. J. Torres Bobadilla, 2010.14550

EFT and amplitude: tale of a happy marriage

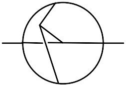
The other obstruction to scalability of the NRGR PN calculation program is the computation of master integrals
E.g. in the static 4PN sector (i.e. G_N^5) one meets



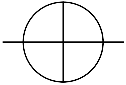
$$= -\mathrm{i}\,(8\pi G_N)^5 \left(\frac{(d-2)}{(d-1)}\; m_1 m_2\right)^3$$



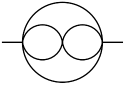
$$\int_{k_{1,2,3,4}} \frac{N_{50}}{k_1^2\; k_2^2\; k_3^2\; k_4^2\; k_{12}^2\; k_{34}^2\; \hat{k}_{24}^2\; p_{13}^2\; \hat{p}_{14}^2}$$



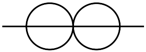
$= c_1$



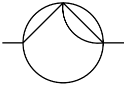
$+c_2$



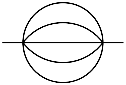
$+c_3$



$+c_4$



$+c_5$

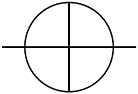


in terms of 4-loop self-energy diagrams in gauge theory

Reduction in terms of master integrals

No new master integrals at 5PN, 4PN ones did it all

Foffa, Mastrolia, RS, Sturm '17



$$= \frac{e^{2\varepsilon\gamma_E}}{s^{2-2\varepsilon}(4\pi)^{4+2\varepsilon}}\left\{\frac{1}{2\varepsilon^2}-\frac{1}{2\varepsilon}-4+\frac{\pi^2}{24} -\varepsilon\left[9-\pi^2\left(\frac{13}{8}-\log 2\right)-\frac{77}{6}\zeta_3\right]+\mathcal{O}\left(\varepsilon^2\right)\right\}$$

Numerical result obtained via Summertime by Lee& Mingulov
analytic result via PSLQ algorithm, fitting trascendentals to numerical result

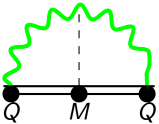
Confirmed up to $O(\varepsilon^0)$ by Damour, Jaranowski '18

Outline

All leading logs in $E_{circ}(x)$

$$E_{circ} = -\frac{M\nu}{2}x\left(1+\frac{16\nu x^2}{15\beta_I}\left[\left(1+24\beta_I x^3\log x\right)x^{4\beta_I x^3}-1\right]\right) \qquad \beta_I = -\frac{214}{105}$$

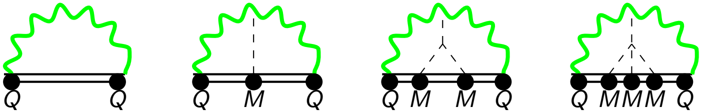
- In PN approximation Log terms arise from tail processes at 4PN order, non-local (but causal) effective term in conservative dynamics ($x \sim v^2 \sim Gm/r \equiv \gamma$):



$$\mathcal{L} = \frac{M\nu}{2}v^2 + \ldots + \frac{2G^2M}{5}\dddot{Q}_{ij}(t)\int d\tau \log(\tau)\ddddot{Q}_{ij}(t-\tau)\ldots$$
$$\to E_{circ} = -\frac{M\nu x}{2}\left(1+\ldots+\frac{448}{15}\nu x^5\log x+\ldots\right)$$

which turns local on circular orbits

- Expansion in $GM\omega = \frac{GM}{r}\times r\omega \sim v^3$:



LO tail: Blanchet, Damour PRD ('88)

Leading Logs at all orders

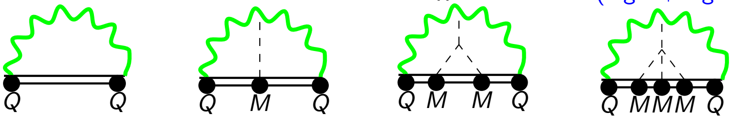
Real part E:

Self-E $\sim (\ddot{Q})(\dddot{Q}) \sim 0$

tail^1 $\sim GM(\dddot{Q})^2 \log t$
$x^4 \log x$

tail^2 $\sim (GM)^2(\dddot{Q})(\ddddot{Q}) \log \sim (GM)^2(\dddot{Q})(\ddddot{Q})$
$x^{11/2}$

tail^3 $\sim (GM)^3 (\log + \log^2) \times (\dddot{Q})(\dddot{Q})$
$x^7(\log x + \log^2 x)$



Imaginary $\frac{dE}{dt}$:

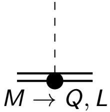
LO flux

tail^1 $\sim \pi x^{3/2}$

tail^2 $\sim x^3 log(x)$

tail^3 $\sim ? \times x^{9/2}$

Other insertions possible:



but do not contribute to leading E-logs: $\nu^2 x^{3n+1} \log^n x$ from tail^{2n-1}

Renormalization group enables to compute all leading logs: E-logs formula extends logs in $E(x)$ at $O(\nu)$ from self-force expanded up to 22 PN in Kavanagh-Ottewill-Wardell PRD 92 (2015)

Far UV divergences

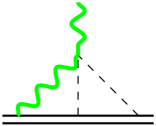
Suppose one had the Far zone theory only: the UV divergence is not compensated by the NZ but it can be renormalized:

- drop the divergence (absorb it with a local counterterm)
- impose $\mu-$independence Goldberger, Ross, Rothstein PRD '14

$$\frac{d\mathcal{L}_{tail}}{d\log\mu}=0 \implies \frac{dM}{d\log\mu}=-\frac{2G^2M}{5}\left(2Q_{ij}^{(1)}Q^{(5)}-2Q_{ij}^{(2)}Q_{ij}^{(4)}+\left(Q_{ij}^{(3)}\right)^2\right)$$

which can be solved by short-circuiting with analog equation for

$$\frac{dQ_{ij}}{d\log\mu}=\frac{214}{105}(GM)^2\,\ddot{Q}_{ij}(t,\mu)$$



Goldberger, Ross PRD '09
(see also Anderson+ '82!)
Adding analogous formula for J (Bernard, Blanchet, Faye, Marchand, Phys. Rev. D97 (2018)) and taking orbital average:

$$M(\mu)=M(\mu_0)-MG^2\sum_{n\geq 1}\frac{(2\log(\mu/\mu_0))^n}{n!}\left(\beta_I G^2M^2\right)^{n-1}\langle Q_{ij}^{(n+2)}Q_{ij}^{(n+2)}\rangle$$

$$L(\mu)=L(\mu_0)-\frac{12MG^2}{5}\sum_{n\geq 1}\frac{(2\log(\mu/\mu_0))^n}{n!}\left(\beta_I G^2M^2\right)^{n-1}\langle Q_{ij}^{(n+1)}Q_{ij}^{(n+2)}\rangle$$

Not quite there for E_{circ}: need for $dE = \omega dL$

Using $Q_{ij}(M,\mu)$ one has (leading log part of) $M(M_0,v,\gamma)$, $L(L_0,v,\gamma)$, adding $dE=\omega dL$ one can compute $r(v)$ on circular orbits: $Energy(r,v) \to E_{circ}(x)$ $(x \equiv (GM\omega)^{2/3})$

$$\gamma \equiv \frac{GM}{r} = x\left[1+\frac{32\nu}{15}\sum_{n\geq 1}\frac{3n-7}{n!}\left(4\beta_I\right)^{n-1}x^{3n+1}\left(\log x\right)^n\right]$$

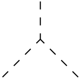
$$\begin{aligned} E &= -\frac{m\nu x}{2}\left[1+\frac{64\nu}{15}\sum_{n\geq 1}\frac{6n+1}{n!}\left(4\beta_I\right)^{n-1}x^{3n+1}\left(\log x\right)^n\right] \\ J &= \frac{m^2\nu}{\sqrt{x}}\left[1-\frac{64\nu}{15}\sum_{n\geq 1}\frac{3n+2}{n!}\left(4\beta_I\right)^{n-1}x^{3n+1}\left(\log x\right)^n\right] \end{aligned}$$

Remarkably $E(x)$ agrees 22PN order $x^{3n+1}(\log x)^n$ (up to $n=7$), expanded self-force result by Kavanagh, Ottewill, Wardell, PRD (2015)

Double copy for EFT

Master integrals have to do with denominators, however numerators can be simplified too by writing $A_{GR} = A_{YM}^2$

Bern, Carrasco, Johansson PRD '08

On-shell three vertices can be mapped: 

$$\begin{array}{ccc} \text{gauge theory} & \rightarrow & \text{gravity theory} \\ gf^{abc}\left(\eta_{\mu\nu}\left(k_1-k_2\right)_\rho+\text{cyclic}\right) & \rightarrow & \sqrt{G_N}\left(\eta_{\mu\nu}\left(k_1-k_2\right)_\rho+\text{cyclic}\right) \\ & & \times\left(\eta_{\mu\nu}\left(k_1-k_2\right)_\rho+\text{cyclic}\right) \\ \text{color factors} & \rightarrow & \text{kinematic numerator} \end{array}$$

Double Copy of Far amplitudes

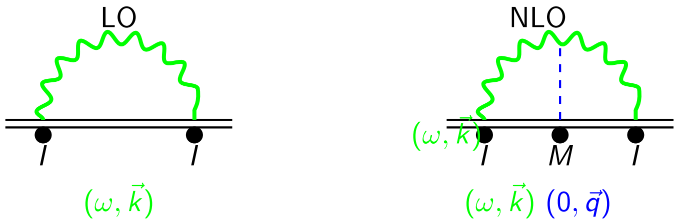
also doubling of world-line vertex  $d_i F_{0i} \to I_{ij} E_{ij}$, $\mu_i F_{ij} \to J_{ij} B_{ij}$

Verified for both electric and magnetic multipoles at NLO in G_N

Goldberger, Ridgway '18, Shen '18, Almeida+ '20

Yet to be verified derivation how YM→ GR mapping propagates from "microscopic physics" to multipoles

Double Copy Far continued



- Electric Self-Energy
  1. LO: $I^{iji_1\dots i_r}(\omega) I^{iji_1\dots i_r}(-\omega) \frac{k_{i_1}\dots k_{i_r} k_{k_1}\dots k_{k_r}}{k^2-\omega^2} \left(\omega^2\delta_{ik} - k_i k_k\right) \left(\omega^2 \delta jl - k_j k_l\right)$
  2. NLO: $I^{iji_1\dots i_r}(\omega) I^{iji_1\dots i_r}(-\omega) \frac{k_{i_1}\dots k_{i_r} k_{k_1}\dots k_{k_r}}{(k^2-\omega^2)((k+q)^2-\omega^2)q^2} k_0^2$
     $\times \left(\omega^2\delta_{ik} - (k+q)_i k_k + q_i q_k\right) \left(\omega^2\delta_{jl} - (k+q)_j q_l + q_j q_l\right)$
- analogously for the magnetic self-Energy at LO and NLO

Gravity+dilaton+anti-symmetric tensor amplitude matches gauge^2

Outline

Conclusion

- NRGR is an efficient method for computations from first-principle
- Divergences well understood (indeed highly constrained hence helpful for sanity checks)
- Conservative+Averaged flux from in-out formalism
- Higher order $\rightarrow$ new master integrals, same problem for any perturbative method (PN, PM…)
- Diagram proliferation: necessary of smart ideas and/or efficient codes (see Peter's talk)

Postponed to 2021!