Gravitational EFT of Compact Objects

EFT Methods from Bound States to Binary Systems 2020

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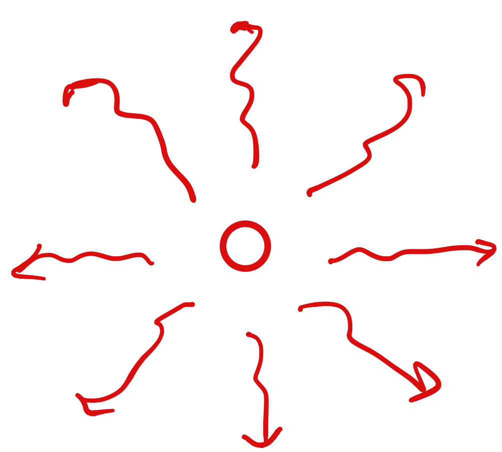
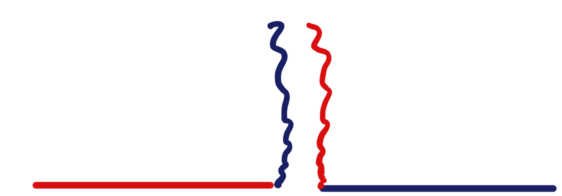
Work done in collaboration with Walter Goldberger

GR and QCD are described by related mathematical structures

- SU(3) Internal Local Gauge Invariance
- Local Space-Time Gauge Invariance

However, even classically they behave remarkably differently





GR

Stable Exact Solution for static sources

(Effectively Abelian)

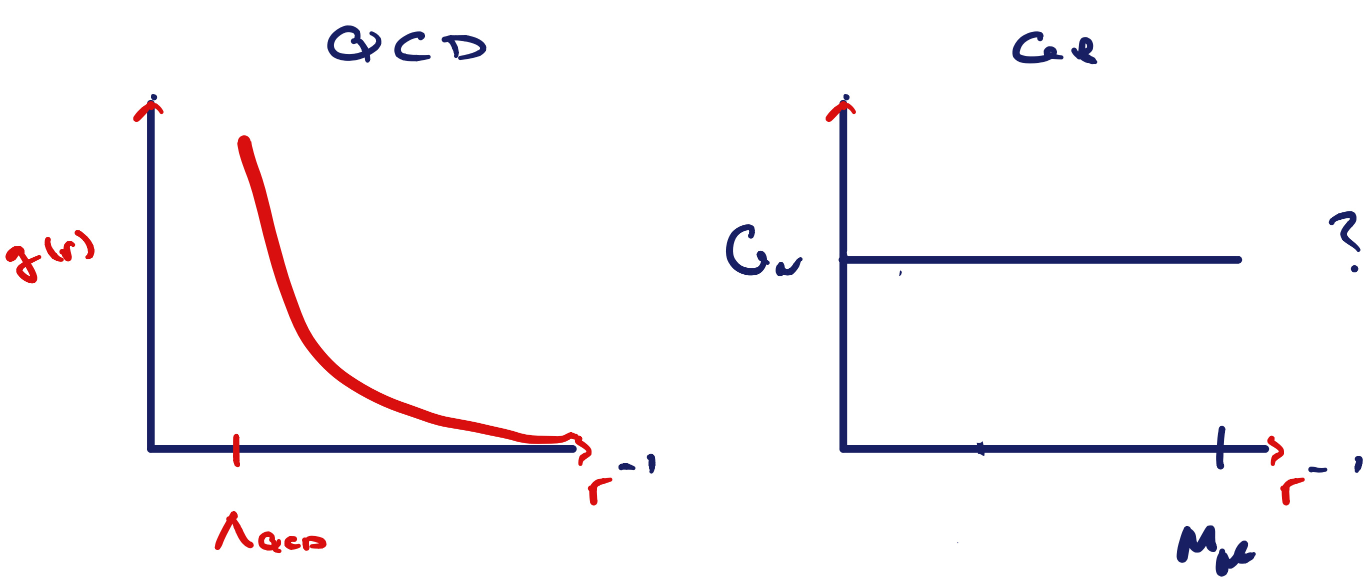
QCD

No Stable Solution for static sources (Mandula 77)

Further Distinction arise as a consequence of the conformal symmetry

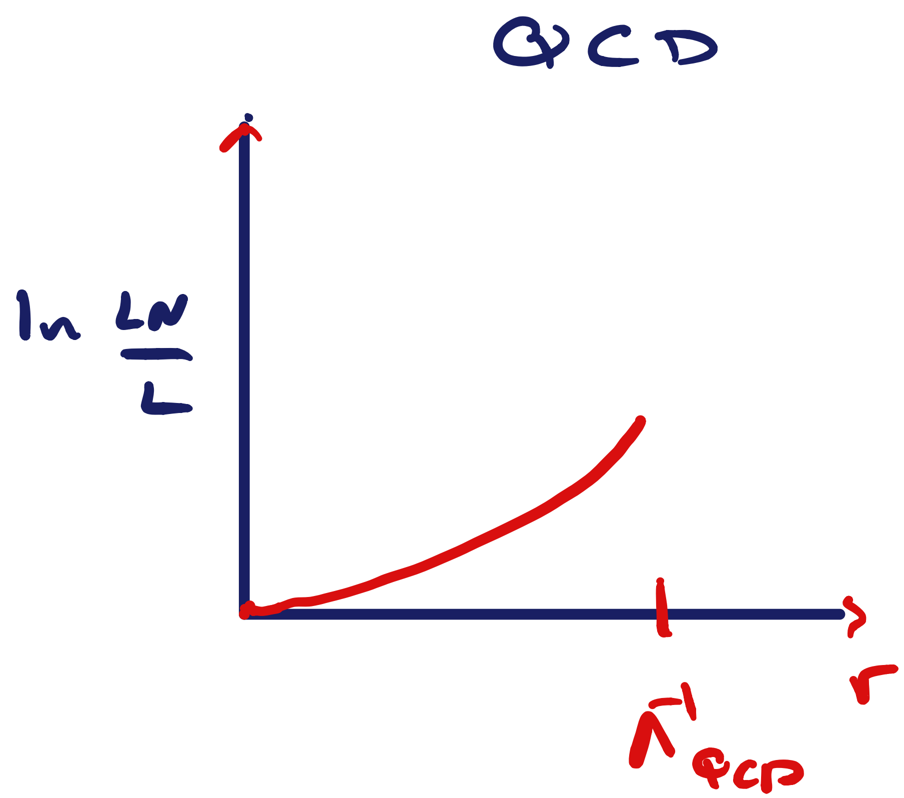
- YM is classically Conformally Invariant
- GR has a Dimensionful Coupling

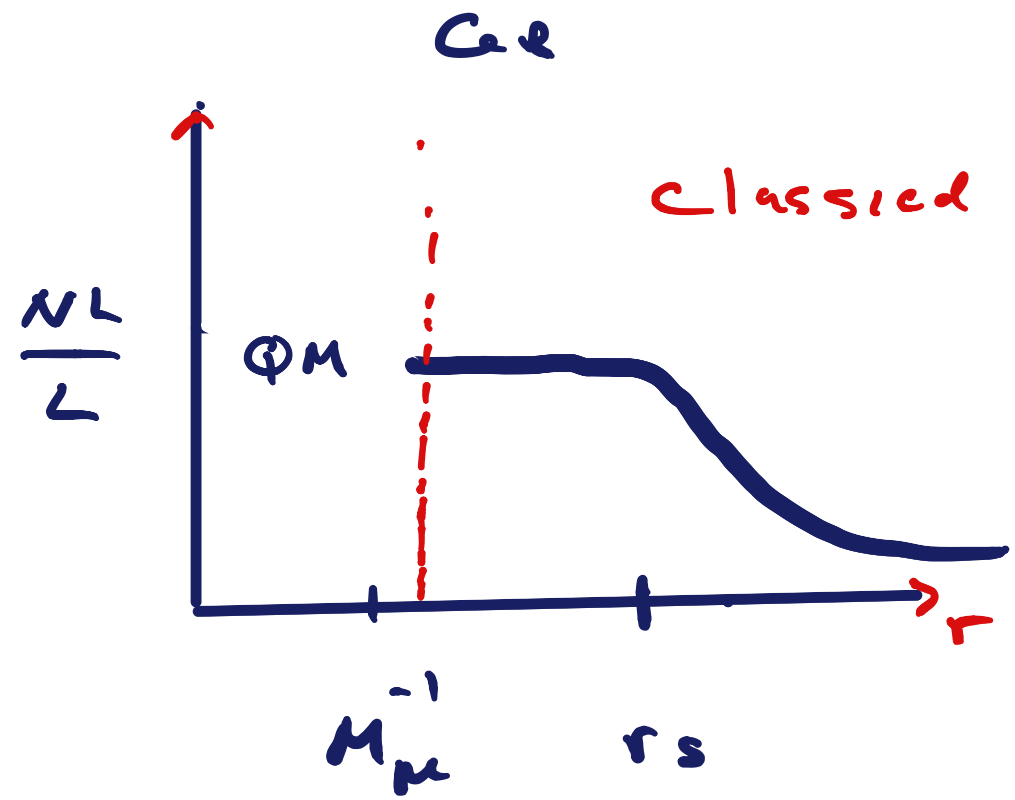
And despite further quantitative relations in the perturbative series for scattering amplitudes (BCJ), this distinction leads to completely different behavior as a function of scale



logs are power suppressed

As a consequence of this the classical/quantum transitions are distinct





NO HIERARCHY Between Classical/Q.M (NO LARGE CASIMIR)

"Large" Charge

$M/M_{pl} >> 1$

In QCD we can calculate a classical potential (ignoring confinement) by hand by ignoring certain regions of loop momenta (see later).

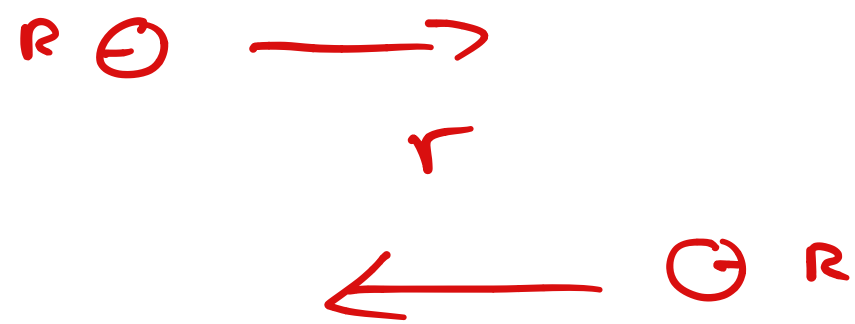
Sharp Analogy with QCD

	Onia	Binary
Short distances	Weak Coupling Coulomb Phase	Strong coupling
Long Distances	Confinement	Minkowski Space
Non-Linearities	Controlled by $\alpha_s \sim v$	Controlled by v^2
Quantum Effects	Controlled by $\alpha_s \sim v$	Controlled by $(M_{pl} r)^2 \sim \hbar / L$

Allows for Strong Classical gravity

We will be interested in processes with hierarchies between scales.

$$R \ll r \sim 1/\sqrt{t}$$



To calculate systematically we have a double expansion:

$$R/r \ll 1$$

	Bound States		**Eikonal**
QCD	$g(1/r) \sim v \ll 1$	**(Re-sum)**	$g^2(t)(s/t) \ll 1$
			free parameter
Gravity	$\frac{GM}{r} \sim v^2 \ll 1$	**(Re-sum)**	$Gs^2/t \ll 1$

In both limits we may treat the incoming particles as classical sources.

$$\frac{1}{(p^2 - m^2 + i\epsilon} \to \frac{1}{q \cdot p + i\epsilon} \qquad q^\mu = (n^\nu, v^\mu)$$

Trivial step one of EFT process (PP approximation).

Two Paths

$$A \sim H S_i S_j$$

theory of Wilson lines

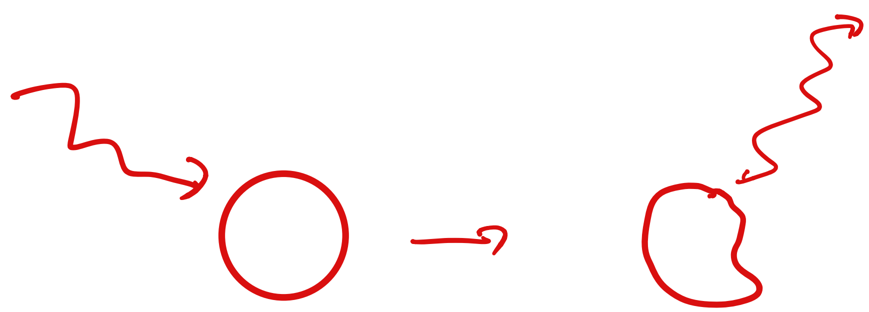
$$S = -m \int ds + S_{EH}$$

$$S = -m \int ds + \int ds\ c^a(s) D_{ab} c^b(s) \qquad D_{ab} c_b = \dot{c}_a + i g v^\mu A_\mu c_b$$

In QCD we are NOT interested in classical sources and this will lead to distinct mode expansions in the EFT.

We will focus on interactions between particles with internal dynamics (e.g. vortices), i.e. not fundamental. For QCD this the length scale is set by confinement (~Fermi) but for gravity the scale is a free parameter (e.g. Schwarzchild radius).

The existence of internal degrees of freedom complicates matters as they are in general gapless (e.g. absorptive).



$$\Delta S = \int ds Q_E(s) \cdot E + Q_B(s) \cdot B + ...$$

Dynamical world line fields whose correlation functions encode response of the system to external perturbations.

First Stage of EFT (PPE)

$$S = -\sum_i m_i \int ds_i + \sum_i \int ds_i (E(s_i) \cdot Q_E^i + B(s_i) \cdot Q_B^i)$$

$$+ \int d^4x \; L_{EH}(g(x))$$

Valid at all scales r>R

Up to this point we have not made any NR approximations.
But we can make further systematic progress if we do.

We have succesfully removed the smallest scale (radius of obejcts) but we still have another hierarchy in the NR limit:

$$r << r/v$$

If we wish to calculate systematically (homogeneous power counting at the level of the action), then we need to separate these scale.

Modal EFT

Decompose modes of fields into kinematic regions

$$\psi(k) = \sum_i \psi_i(k)$$

Each field has support only in a fixed region of k space

Which Modes do we Include?

If we are calculating scattering amplitudes:

-Include all modes which necessary to reproduce all of the non-analytic structure of amplitudes:

- **Coleman Norton Theorem: Cuts arise from physically realizable space-time processes.**

(Beneke+Smirnov)

In NRQCD: $k_s^\mu(mv, m\vec{v})$ $k_{US}^\mu(mv^2, mv^2)$

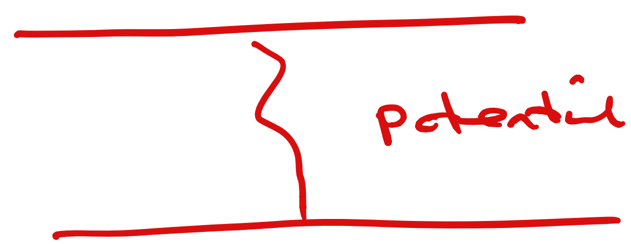
(purely QM)

Radiation and time non-local effects

In NRGR: $k_{US}^\mu(v/r, v/r)$

Also include non-propagating modes which are integrated out to yield effective interactions:

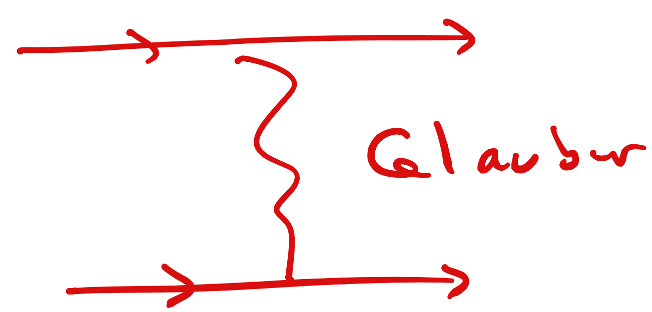
NR



$$\frac{1}{k^2}\,\psi^\dagger\psi\,\psi^\dagger\psi$$

$$k \sim (v, 1/r)$$

EIKONAL



$$\frac{\psi^\dagger\psi\,\psi^\dagger\psi}{\vec{k}_\perp^{\,2}}$$

$$k^+, k^- \approx 0$$

$$\vec{k}_\perp^{\,2} \sim \frac{1}{b^2}$$

Must have a method to ensure that we dont double count when doing integrals (zero bin)

(Manohar+Stewart)

Integrals with overlapping regions

$$I = (I_P - I^P_{US}) + I_{US}$$

$$I^P_{US}(p) = \lim_{\vec{k}\to 0} I_P(p, \vec{k})$$

Crucial to work in dim. reg. to ensure no new regions are introduced and scaleless integrals vanish.

$$S = \int dt \sum_i V_i(r(t)F_i(h_{US}) + \sum_i L(x_i, h_{US})$$

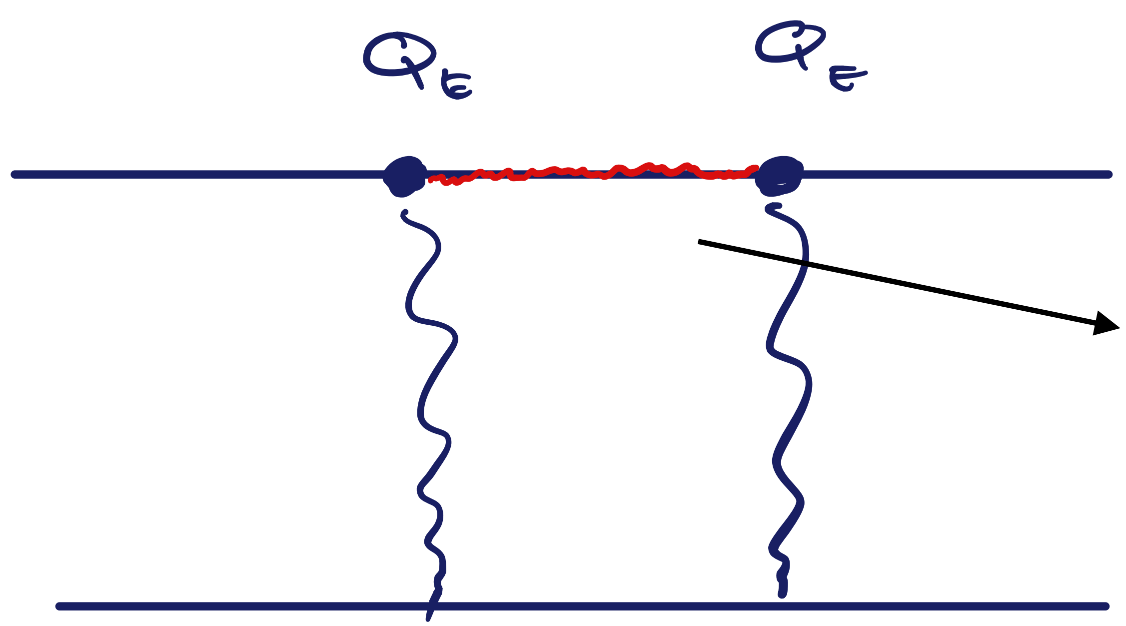
$$S = \sum \int dt_i KE^i - \int \sum_a V_a(r(t)) + \int dt Q_{\mu\nu}(x_i(t))E^{\mu\nu} + ...$$

The potentials can be matched either by the use of scattering amplitudes or calculating directly with the world-lines. Radiation follows by calculating one point amplitudes.

Finite Size Effects

$$\Delta S = \int ds Q_E(s) \cdot E + Q_B(s) \cdot B + ...$$

Effects on equations of motion (CTP formalism)



$$i\theta(t)\langle \Omega \mid [Q_E(t), Q_E(0)] \mid \Omega \rangle$$

$$q_{ij} = \partial_i \partial_j \frac{1}{r}$$

$$F_l^A = -m_A^2 G^2 \int dt' (\partial_l q_{ij}(t)) G_{ijkl}^{B(ret)}(t-t') q_{kl}(t')$$

What can we say about correlation functions?

$$\langle \Omega \mid Q(t_1).....Q(t_n) \mid \Omega \rangle$$

Concentrate on the two point functions for the moment, in particular let us try to construct the Wightman functions as building blocks

$$A_+ = \int dt \langle \Omega \mid Q(t)Q(0)) \mid \Omega \rangle e^{-i\omega t}$$

$$A_- = \int dt \langle \Omega \mid Q(0)Q(t) \mid \Omega \rangle e^{-i\omega t}$$

Let us for the moment confine ourselves to classical processes where

$$G_{ret} = \int \frac{d\omega'}{2\pi} \frac{A_+(\omega') - A_-(\omega')}{\omega - \omega' + i\epsilon} + C$$

What can we say about the analytic structures of correlators?

If no long time tails (i.e. damping exists) then they should be analytic around zero frequency

Dynamical Love number

$$G_R(\omega) = A_1 + i\omega B_1 + A_2\omega^2 + \dots$$

Love Number

Absorptive piece

Implies that the interactions are instantaneous (local in time) and can be contracted to a point.

We can extract these coefficients from matching to other (simpler) observables.

The Love number can be extracted by putting the system and calculating the response

Binnington/Poisson, Damour/Lecian, Kol/Smolkin

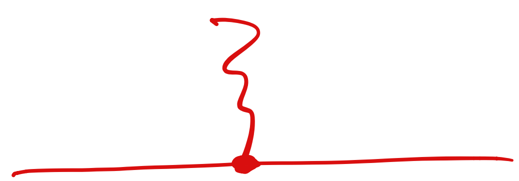
$$\boxed{A_1 = 0}$$

Schwarzschild

The leading order finite size effects for a BH are purely absorptive

$$\sigma_{abs} = \frac{4\pi r_s^6 \omega^4}{45}$$

Starobinsky

$\sigma_{abs}(\omega) \sim \Big|$  $\Big|^2$

$$\sim A_+(\omega)$$

$$F_1^{BHi} = -F_2^{BHi} = (m_a^2 m_b^6 + m_b^2 m_a^6)\frac{8G^7}{5}\left(\frac{1}{\vec{x}^8}(\vec{v} + 2\frac{\vec{x}\cdot\vec{v}}{\vec{x}^2}\vec{x})\right)$$

Exploring the BH EFT

We could just choose to ignore particle emission $(A_-(\omega))$ **in our classical higher order analysis, but this would seem to contradict power counting.**

$\lambda = \omega r_c$ **Classical**

$\delta = \omega / M_{pl}$ **Quantum**

Naively one might think at $A_-(\omega)/A_+(\omega) \sim \delta$

However, this would fly in the face of detailed balance:

Einstein Coefficients : A ~ B

$$A_-(\omega)/A_+(\omega) \sim f(\omega)$$

$$E/T_H \sim (\hbar\omega)/(\hbar/r_s)$$

This is consistent with the fact that Hawking give

$$\langle n(\omega) \rangle = \frac{\Gamma(\omega)}{e^{\beta E} - 1}$$

Recall $$G_{ret} = \int \frac{d\omega'}{2\pi} \frac{A_+(\omega') - A_-(\omega')}{\omega - \omega' + i\epsilon} + C$$

This might lead us to believe that Hawking radiation can lead to a classical (unsuppressed by Planck scale) effects. However,

$$G_{ret} = Tr(\rho\theta(t)[\phi(t), \phi(0)])$$

C number if phi satisfies linear wave equation

Therefore there must be a Cancellation between + and - Wightman functions in any FREE field theory

e.g free field theory in thermal state $F(\omega) = 1/(e^{\beta|\omega|} - 1)$

$$A_+^T = i(2\pi)\delta(k^2 - m^2)(F(\omega) + \theta(\omega)) \quad A_-^T = i(2\pi)\delta(k^2 - m^2)(F(\omega) + \theta(-\omega))$$

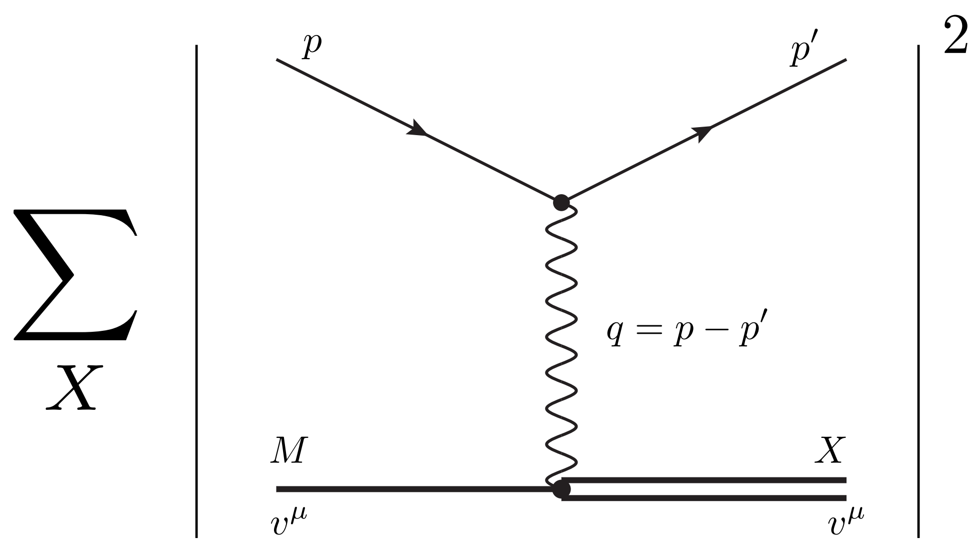
$$G_{ret} = \int \frac{d\omega'}{2\pi} \frac{A_+(\omega') - A_-(\omega')}{\omega - \omega' + i\epsilon} + C \qquad = \frac{1}{k^2 - m^2 + isgn(\omega)\epsilon}$$

Hawking radiation can not show up in Retarded Greens function unless one goes beyond free field theory.

Nonetheless this EFT can yield new results in quantum gravity

What are the effects of hawking radiation on the scattering off of black holes?

Consider inelastic inclusive scattering:



$$|\mathcal{A}_E|^2 = \frac{1}{4m_{Pl}^4}\frac{T}{q^4}A_+^E(\omega)\langle a, b|c, d\rangle \mathcal{A}^E_{ab}\mathcal{A}^E_{cd}$$

$$M \gg E_\phi \gg T_H \gg q \sim b^{-1} \gg \tau_{BH}^{-1}.$$

Excludes the possibility that the final state is Hawking radiation

We can extract $(A_+(\omega))$ from Bekensteins calculation for spontaneous emission from a BH

$$p(m\,|\,n) = \frac{(e^x - 1)e^{xn}\,\Gamma^{m+n}}{(e^x - 1 + \Gamma)^{n+m+1}} \sum_{k=0}^{\min(n,m)} \frac{(-1)^k (m+n-k)!}{k!(n-k)!(m-k)!} \times \left[1 - 2\frac{1-\Gamma}{\Gamma^2}(\cosh x - 1)\right]^k, \quad (18)$$

$$\Gamma^0 \sim \omega^2 \sigma^{abs}_{l=0} \sim \omega^2 r_s^2$$

$$\begin{aligned}
P^{EFT}(0|1) &= \frac{\omega}{2\pi} A_+(\omega) \\
P^{EFT}(1|0) &= \frac{\omega}{2\pi} A_+(-\omega) \\
P^{full}(1|0) &= \frac{\Gamma^{(0)}(\omega)}{x} - \frac{1}{2}\Gamma^{(0)} - \frac{2(\Gamma^{(0)})^2}{x^2} + \frac{\Gamma^{(1)}}{x} \ldots \\
P^{full}(0|1) &= \frac{\Gamma^{(0)}(\omega)}{x} + \frac{1}{2}\Gamma^{(0)} - \frac{2(\Gamma^{(0)})^2}{x^2} + \frac{\Gamma^{(1)}}{x} \ldots
\end{aligned}$$

Matching with wave packets

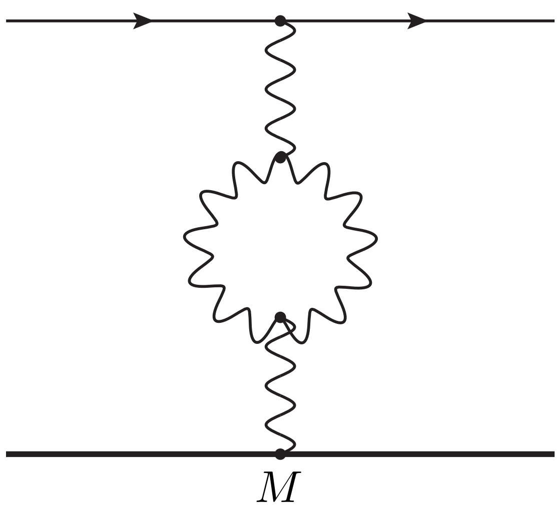
$$A_+^{LO}(\omega) = A_-^{LO}(\omega) = (2\pi)\frac{\Gamma^{(0)}(\omega)}{r_s \omega^2} \sim \lambda/\omega$$

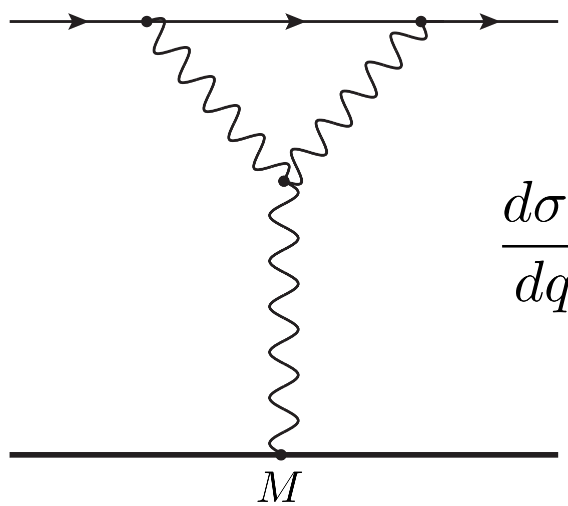
$$\frac{d^3\sigma}{dq^2 d(q\cdot v)} \approx \frac{7G_N r_s^5}{270\pi[(v\cdot p)^2 - m^2]}\left[(v\cdot p)^4 - m^2(v\cdot p)^2\left(1 - \frac{12}{7}\frac{(v\cdot q)^2}{q^2}\right) + \frac{1}{7}m^4\left(1 - 3\frac{(v\cdot q)^2}{q^2} + 6\frac{(v\cdot q)^4}{q^4}\right)\right].$$

Goldberger/Rothstein 20

How does this scale relative to canonical EFT corrections to (elastic) Newtonian scattering?

Donoghue 94





$$\frac{d\sigma_N}{dq^2} = \frac{4\pi r_s^2}{q^4}\frac{\left[(v\cdot p)^2 - \frac{1}{2}m^2\right]^2}{(v\cdot p)^2 - m^2}\quad (1 + O(q^2/M_{pl}^2))$$

To compare need to integrate inelastic result

$$\frac{d\sigma_H}{d\sigma_N} \sim \frac{q^2}{m_{Pl}^2},$$

EFFECTS OF HAWKING RADIATION SAME ORDER AS LO CORRECTIONS TO NEWTON.