Low Energy Quantum General Relativity

1) Effective field theory – UV vs IR
  - non-analytic, non-local
  - what are quantum predictions?

2) Connections of EFT to unitarity
  - on-shell and non-local relations
  - dispersion relations, etc

See also
Emil's talk

3) Unitarity with unstable particles (beyond EFT)
  - who gets counted in the unitarity sum?
  - unstable ghosts in gravity
  - non-local effective Lagrangian description

With
Gabriel
Menezes

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

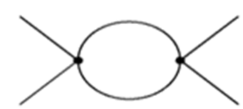
John Donoghue
EFT Methods
Padova, Oct. 2020

Why does Quantum General Relativity work?

1) The uncertainty principle

a) Unknown effects from high energy look local at low energy
- includes all UV divergences
- go into local terms in an effective Lagrangian

b) Known effects from low energy involve long distance propagation
- non-local in position space
- non-analytic in momentum space



2) **The energy expansion**
- expansion in powers of the energy/derivatives

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

Nature of the energy expansion:

Power counting theorem ($\Lambda = 0$) for quantum effects:
- one loop – extra $E^2 \sim \partial^2$
- two loop - $E^4 \sim \partial^4$

Vary from process to process

Amplitudes

$$Amp_i \sim Amp_i^{(0)} \left[1 + a_i G m \sqrt{-q^2} \; + \; b_i G q^2 \ln \frac{-q^2}{\mu^2} \; + \; c_i(\mu) G q^2 + ... \right]$$

Local terms
One loop divergences

Action

$$S = \int d^4x \sqrt{-g} \; \left[-\frac{2}{\kappa^2} R \; + \; c_1(\mu) R^2 \; + \; c_2(\mu) R_{\alpha\beta} R^{\alpha\beta} + ... \right]$$

Procedures:

1) General local Lagrangian, ordered by energy expansion

$$S = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R \;+\; c_1(\mu) R^2 \;+\; c_2(\mu) R_{\alpha\beta} R^{\alpha\beta} + \ldots \right]$$

2) Apply quantum field theory – perturbation theory
   Feynman-DeWitt
3) Renormalize Lagrangian

$$\Delta \mathcal{L}_0^{(1)} = \frac{1}{8\pi^2} \frac{1}{\epsilon} \left\{ \frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} \right\}$$

'tHooft
Veltman

4) Make quantum predictions
   - these come from low energy propagation

"Low energy theorems"
- independent of UV completion
- depend only on IR structure

What are the quantum predictions?

Not the divergences

- they come from the Planck scale
- unreliable part of theory

Not the parameters

- local terms in L
- we would have to measure them

Low energy propagation

- not the same as terms in the Lagrangian
- most always non-analytic dependence in momentum space
- can't be Taylor expanded – can't be part of a local Lagrangian
- long distance in coordinate space

$$\boxed{Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}}$$

Corrections to Newtonian Potential

JFD 1994
JFD, Holstein, Bjerrum-Bohr 2002
Khriplovich and Kirilin
Other references later

Here discuss scattering potential of two heavy masses – S matrix element.

$$\begin{aligned}\langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle\end{aligned}$$

Potential found using from

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura

What to expect:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

Classical expansion parameter

Quantum expansion parameter

Short range

Relation to momentum space:

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$

Classical

quantum

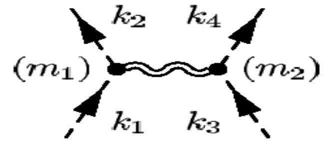
short range

Non-analytic

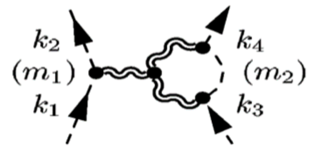
analytic

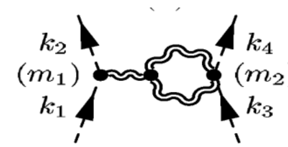
The calculation:

Lowest order:

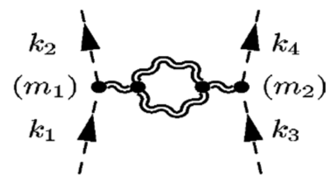


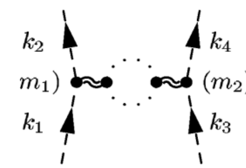
Vertex corrections:



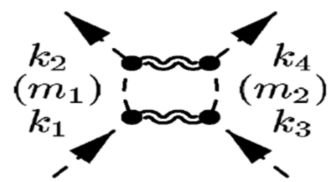


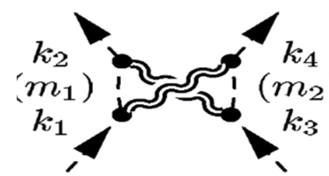
Vacuum polarization:
(Duff 1974)



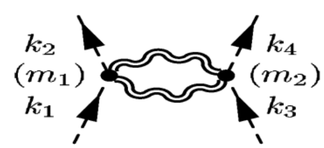


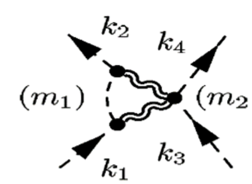
Box and crossed box





Others:





Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2 \right)$$

$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2$$

Sum diagrams:

$$\boxed{V(r) = -\frac{Gm_1 m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]}$$

Classical

Quantum correction

Comments

1) Both classical and quantum emerge from a one loop calculation
   - classical first done by Gupta and Radford, Iwasaki
   - **classical physics from quantum loops**

1) Unmeasurably small quantum correction:
   - best perturbation theory

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Radikowski, Duff, JFD; Muzinich and Vokos; Hamber and Liu; Akhundov, Bellucci, and Sheikh ; Khriplovich and Kirilin)
   -other potentials or mistakes

Dispersion relations

Emil will treat
modern techniques

Feinberg-Sucher(1960s)

Directly from on-shell intermediate states
- subtractions give analytic contributions

Bjerrum-Bohr
JFD
Vanhove

$$V(s,q^2) = -\frac{1}{\pi}\int_0^\infty dt \frac{1}{t-q^2}\rho(s,t) + \text{R.H.cut}\,.$$

$$\rho(s,t) = -\frac{1}{\pi}\int \frac{d\Omega_\ell}{4\pi}\, M^{\text{tree}}_{\mu\nu,\rho\sigma}(p_1,p_2,-\ell_2,\ell_1)\mathcal{S}^{\mu\nu,\alpha\beta}\mathcal{S}^{\rho\sigma,\gamma\delta}(M^{\text{tree}}_{\alpha\beta,\gamma\delta}(p_4,p_3,\ell_2,-\ell_1))^*$$

One finds:

$$\rho(s,t) = a_1(s)\frac{1}{\sqrt{t}} + a_2(s) + \ldots.$$

$$V(s,q^2) = \frac{1}{\pi}[a_1(s)\frac{\pi}{\sqrt{-q^2}} + a_2(s)\ln(-q^2) + \ldots.]$$

Reproduces previous results

1) Helicity basis – double copy – no ghosts
2) Covariant basis – also needs cuts from FDFP ghosts

One loop universality/soft theorem

Low
Gell-Mann
Goldberger
Weinberg
Gross
Jackiw

The result for potential is universal – spin independent

- explained by on-shell calculation

Tree level soft theorems

- Compton amplitudes and gravitational Compton amplitudes are universal at leading order
- Conservation of charge/energy and ang. mom.

One loop soft theorem

- E&M and gravitational potentials
- formed by square of Compton amplitudes
- quantum term down from classical by $\sqrt{-q^2}$
- first found in **direct calculations (!)** by Holstein and Ross

Quantum corrections to Reissner-Nordstrom and Kerr-Newman metrics

JFD
Holstein
Garbrecht
Konstantin

Metric around charged bodies,
without (RN) or with (KN) angular momentum

Quantum Electrodynamics calculation

-gravity is classical here

-but uses EFT logic

Metric determined by energy momentum tensor:

harmonic gauge

$$h_{\mu\nu}(x) \ = \ -16\pi G \int d^3y D(x-y)(T_{\mu\nu}(y) - \frac{1}{2}\eta_{\mu\nu}T(y))$$

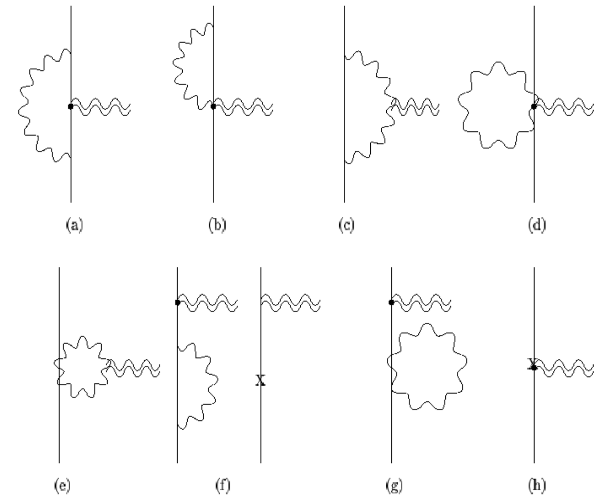
Logic:

looking for non-analytic terms again:

-long range propagation of photons

$$\begin{aligned} \text{metric} \ &\sim \ Gm \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{1}{\vec{q}^2} \left[1 - b\alpha \frac{\vec{q}^2}{m^2}\sqrt{\frac{m^2}{\vec{q}^2}} - \frac{\vec{q}^2}{m^2}\log(\vec{q}^2) - c\alpha \frac{\vec{q}^2}{m^2} + \ldots \right] \\ &\sim \ Gm \left[\frac{1}{r} + \frac{a\alpha}{mr^2} + \frac{b\alpha\hbar}{m^2r^3} + \frac{c\alpha}{m^2}\delta^3(x) + \ldots \right] \qquad (7) \end{aligned}$$

Calculation:



Boson:

$$< p_2|T_{\mu\nu}(x)|p_1 >= \frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{4E_2E_1}}\left[2P_\mu P_\nu F_1(q^2) + (q_\mu q_\nu - g_{\mu\nu}q^2)F_2(q^2)\right]$$

Fermion:

$$\begin{aligned}
< p_2|T_{\mu\nu}|p_1 > &= \bar{u}(p_2)\Big[F_1(q^2)P_\mu P_\nu \frac{1}{m} \\
&- F_2(q^2)(\frac{i}{4m}\sigma_{\mu\lambda}q^\lambda P_\nu + \frac{i}{4m}\sigma_{\nu\lambda}q^\lambda P_\mu) \\
&+ F_3(q^2)(q_\mu q_\nu - g_{\mu\nu}q^2)\frac{1}{m}\Big]u(p_1)
\end{aligned}$$

$$\begin{aligned}
F_1(q^2) &= 1 + \frac{\alpha}{4\pi}\frac{q^2}{m^2}\left(-\frac{8}{3} + \frac{3}{4}\frac{m\pi^2}{\sqrt{-q^2}} + 2\log\frac{-q^2}{m^2} - \frac{4}{3}\log\frac{\lambda}{m}\right) + \ldots \\
F_2(q^2) &= -\frac{1}{2} + \frac{\alpha}{4\pi}\left(-\Omega - \frac{26}{9} + \frac{m\pi^2}{2\sqrt{-q^2}} + \frac{4}{3}\log\frac{-q^2}{m^2}\right) + \ldots
\end{aligned}$$

where

$$\Omega = \frac{2}{\epsilon} - \gamma - \log\frac{m^2}{4\pi\mu^2}$$

Results:

-reproduce classical terms (harmonic gauge)

- quantum terms **universal**

$$\begin{aligned}
g_{00} &= 1 - \frac{2Gm}{r} + \frac{G\alpha}{r^2} - \frac{8G\alpha\hbar}{3\pi m r^3} + \ldots \\
g_{0i} &= (\frac{2G}{r^3} - \frac{G\alpha}{mr^4} + \frac{2G\alpha\hbar}{\pi m^2 r^5})(\vec{S}\times\vec{r})_i \\
g_{ij} &= -\delta_{ij} - \delta_{ij}\frac{2Gm}{r} + G\alpha\frac{r_i r_j}{r^2} + \frac{4G\alpha\hbar}{3\pi m r^3}\left(\frac{r_i r_j}{r^2} - \delta_{ij}\right) + \ldots
\end{aligned}$$

Physical intepretation:

- classical terms are just the classical field around charged particle

$$\begin{aligned} T_{00}^{EM}(\vec{r}) &= \frac{1}{2}E^2 = \frac{e^2}{32\pi^2 r^4} \\ T_{0i}^{EM}(\vec{r}) &= 0 \\ T_{ij}^{EM}(\vec{r}) &= -E_i E_j + \frac{1}{2}\delta_{ij}E^2 = -\frac{e^2}{16\pi^2 r^4}\left(\frac{r_i r_j}{r^2} - \frac{1}{2}\delta_{ij}\right) \end{aligned}$$

-reproduced in the loops expansion

- quantum terms are fluctuations in the electromagnetic fields

Beyond scattering amplitudes

Gravity much more than scattering
- but QFT techniques less developed

Non-local effective actions:
- most work done by Barvinsky, Vilkovisky and collab.
- covariant
- expansion in curvature

Others:
Starobinsky
Hartle, Hu

Note: This is a different expansion from EFT derivative expansion

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$ EFT

$$S_{curv} \sim \int d^4x \sqrt{-g} \;\; ... + c(\mu) R^2 + d R \log(\Box/\mu^2) R + R^2 \frac{1}{\Box} R + ... + R^{n+1} \frac{1}{\Box^n} R + ...$$ BV

Basic message:

We are used to the local derivative/energy expansion in GR

$$S_{grav} = \int d^4x \sqrt{-g}\left\{\Lambda + \frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \ldots\right\}$$

and we know that quantum corrections generate R^2 terms

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\}$$

but quantum content of GR is a non-local action:

$$\begin{aligned} S_{tot} = \int d^4x\, \sqrt{g}\, \frac{2}{\kappa^2}R & \\ & + \left[\bar{\alpha}R\log\left(\nabla^2/\Lambda_1^2\right)R + \bar{\beta}C_{\mu\nu\alpha\beta}\log(\nabla^2/\Lambda_2^2)C^{\mu\nu\alpha\beta}\right. \\ & \left. + \bar{\gamma}\left(R_{\mu\nu\alpha\beta}\log\left(\nabla^2\right)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log\left(\nabla^2\right)R^{\mu\nu} + R\log\left(\nabla^2\right)R\right)\right] + ... \end{aligned}$$

Renormalize R^2 parameters and generate non-local terms:

Barvinsky, Vilkovisky, Avrimidi

Perturbative running is contained in the R^2 terms

$$S_4 = \int d^4x \sqrt{g} \left[c_1(\mu) R^2 + c_2(\mu) R_{\mu\nu} R^{\mu\nu}\right]$$
$$+ \left[\bar{\alpha} R \log\left(\nabla^2/\mu^2\right) R + \bar{\beta} C_{\mu\nu\alpha\beta} \log(\nabla^2/\mu^2) C^{\mu\nu\alpha\beta}\right.$$
$$\left. + \bar{\gamma}\left(R_{\mu\nu\alpha\beta} \log\left(\nabla^2\right) R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} \log\left(\nabla^2\right) R^{\mu\nu} + R \log\left(\nabla^2\right) R\right)\right] \quad + \mathcal{O}(R^3)$$

Again running can all be packaged in non-local terms:

$$S_{tot} = \int d^4x \sqrt{g} \frac{2}{\kappa^2} R$$
$$+ \left[\bar{\alpha} R \log\left(\nabla^2/\Lambda_1^2\right) R + \bar{\beta} C_{\mu\nu\alpha\beta} \log(\nabla^2/\Lambda_2^2) C^{\mu\nu\alpha\beta}\right.$$
$$\left. + \bar{\gamma}\left(R_{\mu\nu\alpha\beta} \log\left(\nabla^2\right) R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} \log\left(\nabla^2\right) R^{\mu\nu} + R \log\left(\nabla^2\right) R\right)\right] + \ldots.$$

	α	β	γ	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\gamma}$
Scalar	$5(6\xi-1)^2$	-2	2	$5(6\xi-1)^2$	3	-1
Fermion	-5	8	7	0	18	-11
Vector	-50	176	-26	0	36	-62
Graviton	430	-1444	424	90	126	298

Coefficients of different fields. All numbers should be divided by $11520\pi^2$

Caveats: Limitations to the EFT

Limits in the UV are well known

Technical (?) limits in the IR

- gravity effects build up
- quantum calculations are perturbative
- don't have techniques to sum these yet
- also horizons etc

Can amplitudes program help?

- amazing success at classical corrections
- quantum effects often come along with classical
- quantum effects from classical loops?

Modern view of low energy quantum gravity:

We have a quantum theory of General Relativity

It has the form of an effective field theory

We can make predictions at low energy

The effective theory points to the need of a UV completion

We will need to find a more complete theory eventually
- but for now this EFT is part of our core theory

To be continued in Emil's talk…..

Unitarity of unstable particles:

$$\langle f|T|i\rangle - \langle f|T^\dagger|i\rangle = i\sum_j \langle f|T^\dagger|j\rangle\langle j|T|i\rangle$$

Who counts in unitarity relation?

- Veltman 1963
- only stable particles count
- they form asymptotic Hilbert space
- **do not** make any cuts on unstable resonances

UNITARITY AND CAUSALITY IN A RENORMALIZABLE FIELD THEORY WITH UNSTABLE PARTICLES

M. VELTMAN *)

This looks funny from free-field quantization

- interaction removes states from the Hilbert space

Also, we know some states are almost stable

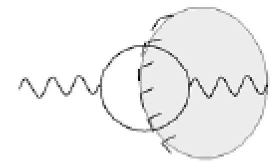
- can treat them as essentially stable
- Narrow Width Approximation (NWA)

Nevertheless, Veltman is correct

Cutkosky cutting rules

Obtain discontinuity by replacing propagator with:

$$\frac{i}{q^2 - m^2 + i\epsilon} \to 2\pi\delta(q^2 - m^2)\theta(q_0)$$



Also on far side of cut, use:

$$\frac{i}{q^2 - m^2 + i\epsilon} \to \frac{-i}{q^2 - m^2 - i\epsilon}$$

- some QFT texts statements are incomplete

Example – self energy

$$\mathrm{Disc}_2\ \Sigma(q) = \frac{\kappa^2 q^4 (N+1)}{2} \int \frac{d^4k}{(2\pi)^4}\ 2\pi\delta(k^2)\theta(k_0)\ 2\pi\delta((q-k)^2)\theta((q-k)_0)$$

Also can repackage this inserting:

$$\int \frac{d^4p}{(2\pi)^4}\,(2\pi)^4\delta^4(q-k-p) \;=\; 1$$

To have the equivalent expression:

$$\mathrm{Disc}_2\;\Sigma(q) = \frac{(N+1)}{2}\int \frac{d^4k}{(2\pi)^4}\,\frac{d^4p}{(2\pi)^4}\,(2\pi)^4\delta^4(q-k-p)|\mathcal{M}_2|^2 2\pi\delta(k^2)\theta(k_0)\;2\pi\delta((q-k)^2)\theta((q-k)_0)$$

Doing the integrals now yields

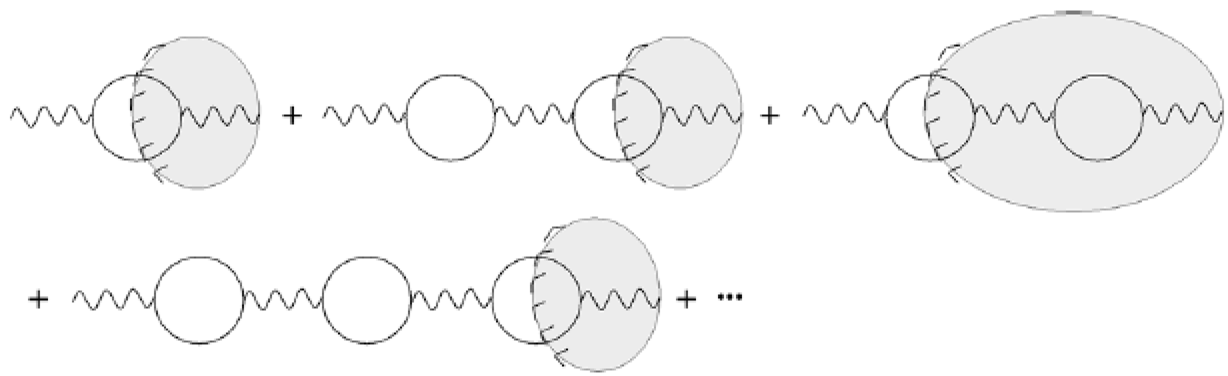
$$\mathrm{Disc}_2\;\Sigma(q) = \frac{(N+1)}{2}\int \frac{d^3k}{(2\pi)^3 2\omega_k}\,\frac{d^3p}{(2\pi)^3 2\omega_p}\,(2\pi)^4\delta^4(q-k-p)|\mathcal{M}_2|^2\quad .$$

Or

$$\mathrm{Disc}_2\;\Sigma(q) = 2q\Gamma_2(q)$$

The discontinuity is equivalent to the decay width at q^2

Cuts in a resonance propagator:

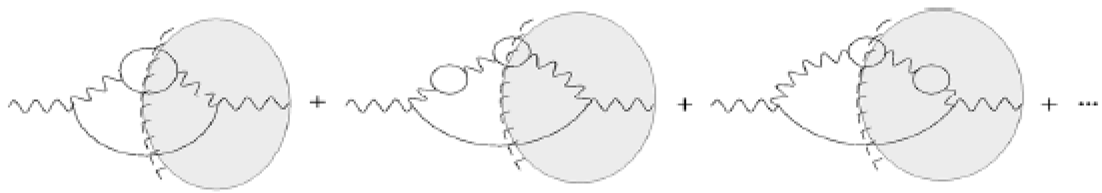


Bubble sum on each side of propagator:
- will c.c. propagators on the far side

$$\mathrm{Disc}\ D(q) = D(q)\ 2q\Gamma_2(q)\ D^*(q) = -2\ \mathrm{Im}[D(q)]$$

This is true no matter if normal resonance or Merlin modes

Loops within loops = resonance + stable cut



$$\mathrm{Disc}_3\ \Sigma(q) \ = \ \kappa^2 q^4 \int \frac{d^4k_1}{(2\pi)^4}\,\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(q-k_1)^2 - \frac{\kappa^2(q-k_1)^4}{2\xi^2}}\,\frac{(N+1)\kappa^2(q-k_1)^2}{2}$$
$$\times \quad 2\pi\delta(k_1^2)\theta(k_{10})\ 2\pi\delta(k_2^2)\theta(k_{20})\ 2\pi\delta((q-k_1-k_2)^2)\theta((q-k_1-k_2)_0)\frac{1}{(q-k_1)^2 - \frac{\kappa^2(q-k_1)^4}{2\xi^2}}$$

Identify matrix element

$$\mathcal{M}_3 = \kappa q^2 \kappa (q-k_1)^2 D(q-k_1)$$

and play similar games, to get expected unitarity relation

$$\mathrm{Disc}_3\ \Sigma(q) = 2q\Gamma_3(q)$$

Again result is independent of type of resonance

Bottom line:

- Discontinuities come from cuts on stable particles
- Resonances do not go onshell
- Do not make separate cuts on resonances

Narrow width approximation

Discontinuity in propagator was due to on-shell states only

$$\text{Disc } D(q) = D(q)\ 2q\Gamma(q)\ D^*(q) = \frac{2q\Gamma(q)}{(q^2 - m_r^2)^2 + (m_r\Gamma(q))^2}$$

But when Γ is small, this is highly peaked on the resonance,
Use:

$$\delta(x) = \lim_{\epsilon\to 0} \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2}$$

Limits to usual cutting rule:

$$\lim_{\Gamma\to 0} \text{Disc } D(q) = 2\pi\delta(q^2 - m_r^2)$$

In “three particle cut” , this is equiv. decay to resonance plus stable

Again, this result is independent of normal or Merlin resonance

Relevance for quantum gravity

- Beyond the EFT limit

- Consider the effective Lagrangian in basis:

$$S_{\rm quad} = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} R + \frac{1}{6 f_0^2} R^2 - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right]$$

- The graviton propagator gets modified by q^4 terms

- Spin zero portion *heads to* either a tachyon or a normal resonance
  - depends on sign of f_0^2
- Spin two portion *heads to* either a tachyon or an **unstable ghost**
  - depends on sign of ξ^2

- Perhaps new DOF for quantum gravity before hitting either of those
- **But can we survive with an unstable ghost? Unitarity?**

Unstable ghost pole in propagator (including self-energy)

$$iD(q) = \frac{i}{q^2 + i\epsilon - \frac{q^4}{M^2} + \Sigma(q)} \quad .$$

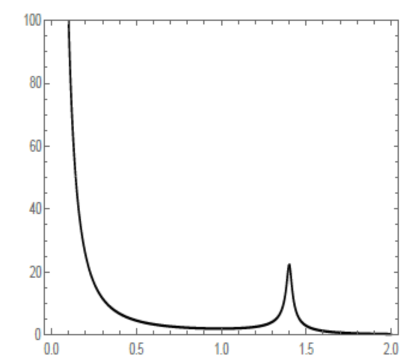
Above some threshold

$$\text{Im}\ \Sigma(q) = \gamma(q) \qquad\qquad \gamma(q) \geq 0$$

The high mass pole carries two minus sign differences:

$$\begin{aligned} iD_F(q) &= \frac{i}{q^2 - \frac{q^4}{M^2} + i\gamma(q)} \\ &= \frac{i}{\frac{q^2}{M^2}[M^2 - q^2 + i\gamma(q)(M^2/q^2)]} \\ &\sim \frac{-i}{q^2 - M^2 - i\gamma_M} \quad . \end{aligned}$$

Propagator contains two poles
- massless stable particle
- massive resonance ("ghost-like")



In both resonance and unstable ghost cases, **same** imaginary part:

$$iD(q) \sim \frac{Zi}{q^2 - m^2 + iZ\gamma}$$

$$\mathrm{Im}[D(q)] \sim \frac{-\gamma}{(q^2 - m^2)^2 + \gamma^2}$$

This is **time-reversed** version of a resonance propagator

- time reversal is anti-unitary
- causality violation on timescales of order γ^{-1}

Time reversed path integral: e^{-iS} instead of e^{iS}

Consider generating functions:

$$
\begin{aligned}
Z_\pm[J] &= \int [d\phi] e^{\pm i S(\phi, J)} \\
&= \int [d\phi] e^{\pm i \int d^4x [\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + J\phi]} \quad .
\end{aligned}
$$

Need to make this better defined – add

$$
e^{-\epsilon \int d^4x \phi^2 / 2}
$$

Solved by completing the square:

$$
Z_\pm[J] = Z[0] \exp\left\{ -\frac{1}{2} \int d^4x d^4y J(x)\ iD_{\pm F}(x-y) J(y) \right\}
$$

Yield propagator with specific analyticity structure

$$
iD_{\pm F}(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \frac{\pm i}{q^2 - m^2 \pm i\epsilon}
$$

Merlin modes:

-Merlin (the wizard in the tales of King Arthur) ages backwards

"Now ordinary people are born forwards in Time, if you understand what I mean, and nearly everything in the world goes forward too. (...) But I unfortunately was born at the wrong end of time, and I have to live backwards from in front, while surrounded by a lot of people living forwards from behind."

T. H. White *Once and Future King*

Note, there is a key distinction with usual nomenclature "ghosts"

- ghost is anything with a minus sign in the numerator
- these Merlin modes refer to crucial sign $-i\gamma$ in denominator in addition

Heuristic proof of unitarity with unstable ghosts

Unitarity works with stable particle as external states

Cuts through stable particle loops same for normal and Merlin resonances

Both normal and Merlin resonances described by same propagator
- all-in-one propagator

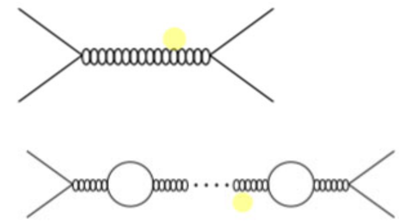
Veltman proved normal resonances satisfy unitarity to all orders

The Merlins will then also satisfy unitarity

The examples discussed above work also for Merlin resonances

Unitarity in the spin two channel

Do these features cause trouble in scattering?
- consider scattering in spin 2 channel



First consider single scalar at low energy:

$$i\mathcal{M} = \left(\frac{1}{2}V_{\mu\nu}(q)\right)\left[iD^{\mu\nu\alpha\beta}(q^2)\right]\left(\frac{1}{2}V_{\alpha\beta}(-q)\right)$$

$$\mathcal{M} = 16\pi\sum_{J=0}^{\infty}(2J+1)T_J(s)P_J(\cos\theta)$$

Results in

$$T_2(s) = -\frac{N_{\rm eff}s}{640\pi}\,\bar{D}(s).$$

$N_{\rm eff} = 1/6$ for a single scalar field

$$\bar{D}^{-1}(s) \;=\; \frac{1}{\tilde{\kappa}^2}\left\{1 - \frac{\tilde{\kappa}^2 s}{2\xi^2(\mu)} - \frac{\tilde{\kappa}^2 s N_{\rm eff}}{640\pi^2}\ln\left(\frac{s}{\mu^2}\right) + \frac{i\tilde{\kappa}^2 s N_{\rm eff}}{640\pi}\right\}$$

Satisfies elastic unitarity:

$$\mathrm{Im}T_2 = |T_2|^2.$$

This implies the structure

$$T_2(s) = \frac{A(s)}{f(s) - iA(s)} = \frac{A(s)[f(s) + iA(s)]}{f^2(s) + A^2(s)}$$

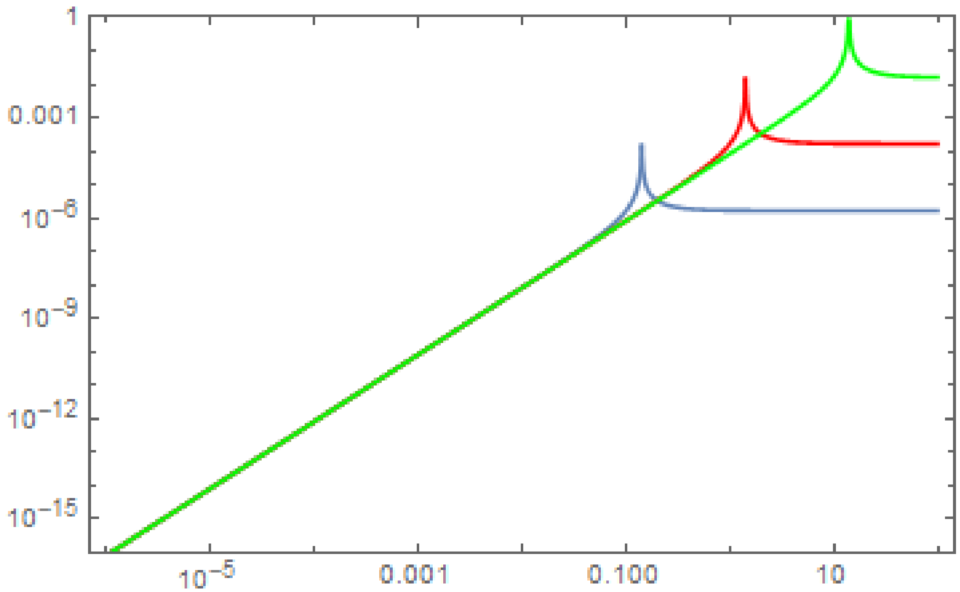
for any real *f(s)*

Signs and magnitudes work out for $A(s) = -\frac{N_{\mathrm{eff}}s}{640\pi}.$

Multi-particle problem:

- just diagonalize the J=2 channel
- same result but with general N

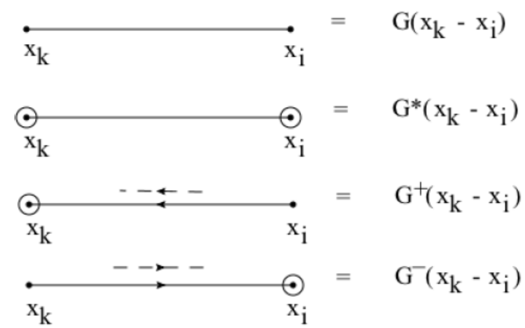
Scattering amplitude at weak coupling:



$\xi^2 = 0.1, 1, 10$

Formal proof of unitarity with unstable ghosts

With G. Menezes arXiv:1908.02416 in RD



Follows Veltman:
- circling rules
- largest time equation
- turns into derivation of cutting rules

Only difference is energy flow

$$
\begin{aligned}
-iG^*(x-x') &= \Theta(x_0 - x_0')G^-(x-x') + \Theta(-x_0 + x_0')G^+(x-x') \\
-i\widetilde{G}^*(x-x') &= \Theta(x_0 - x_0')\widetilde{G}^-(x-x') + \Theta(-x_0 + x_0')\widetilde{G}^+(x-x') \\
-i\widetilde{G}^*_{\rm GH}(x-x') &= \Theta(x_0 - x_0')\widetilde{G}^+_{\rm GH}(x-x') + \Theta(-x_0 + x_0')\widetilde{G}^-_{\rm GH}(x-x')
\end{aligned}
$$

Important point - all steps in Minkoswki space
- no analytic continuation employed

Non-local Effective Lagrangians for unstable particles

Beenakker, Berends, Chapovsky 2000

Build non-local effective actions

$$\mathcal{L}_{\rm NL} = \phi^*(x)\Sigma(x-y)U(x,y)\phi(y)$$

with $$U(x,y) = \exp\left[-ie\int_x^y d\omega^\mu A_\mu(\omega)\right]$$

For unitarity:

- local vertices are highly constrained
- non-local vertices can reflect resonance properties

G. Menezes is working on unitarity program for unstable particles

- both normal and Merlin particles together
- succeeding so far in NWA

Potential to open up new considerations of unitary theories

Gravity fits well with our other interactions in Core Theory

$$Z = \int_{\Lambda} [d\phi] \exp i \int d^4x \sqrt{-g} \Big[-\frac{1}{4} F^2 + \bar{\psi} i \not{D} \psi + \partial_\mu \phi^\dagger \partial^\mu \phi - \Gamma \bar{\psi} \psi \phi - V(\phi) - \Lambda - \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \Big]$$

GR is also a QFT - classic form of an EFT
We know how to handle quantum gravity in ordinary situations
Plenty more to understand!