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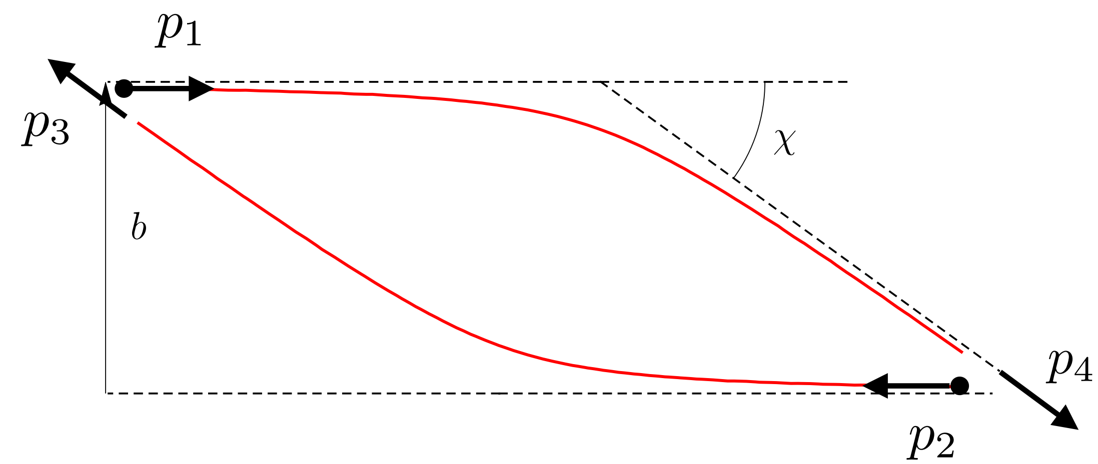
Gravitational Bremsstrahlung from reverse unitarity

Julio Parra-Martinez

w/ Ruf, Zeng [2005.04236] & w/ Herrmann, Ruf, Zeng [201X.XXXXX]

@ EFT methods from Bound States to Binary Systems 2020, U. Padova

Multiple approaches



- From scattering amplitudes to observables for the 2-body problem

Scattering amplitude

$\mathcal{M}(q,p)$

Eikonal Resumation

$\delta(b,p)$

EFT matching

$V(r,p)$

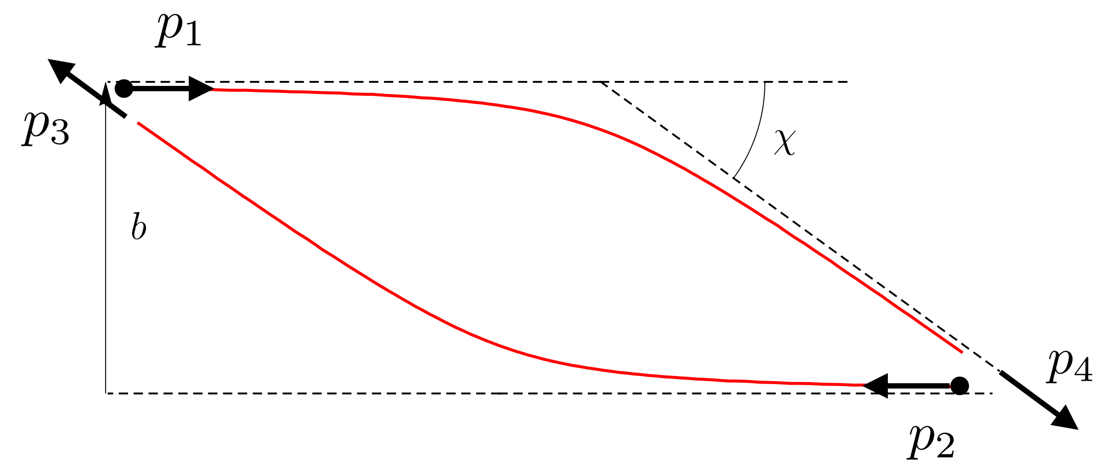
“in-in” (KMO’C)

see other talks today!

Gauge Invariant Observables

χ $\quad \Delta\Phi \quad E^{\rm rad} \quad J^{\rm rad}$

Multiple approaches



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my talk

Not the scenic route!

see other talks today!

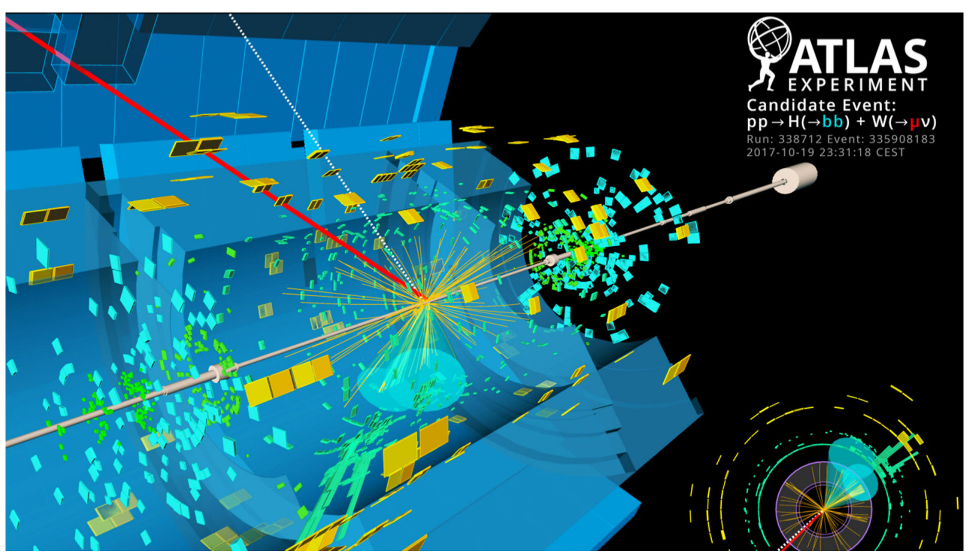
Gauge Invariant Observables

χ $\Delta\Phi$ E^{rad} J^{rad}

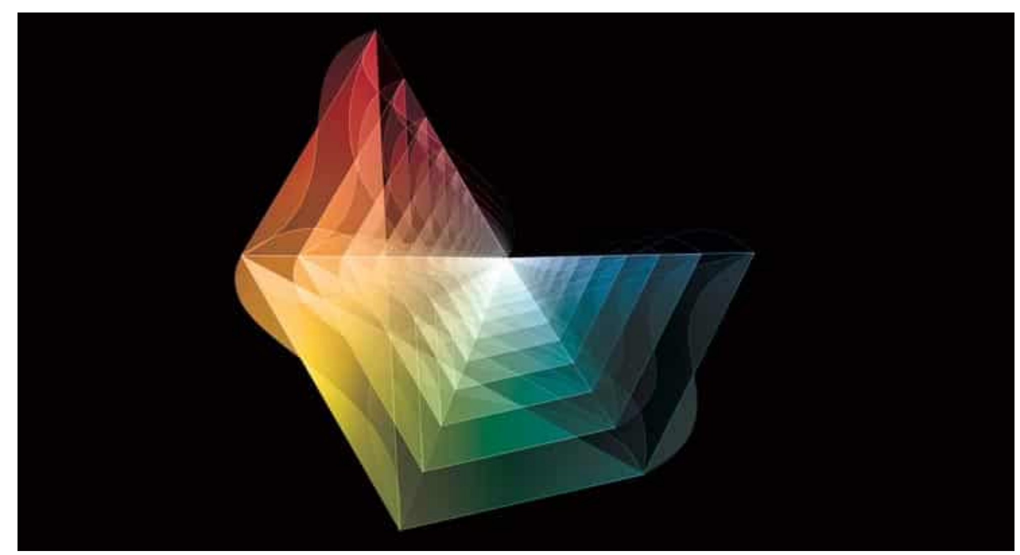
Lessons from amplitudes/EFT

- All orders results are not impossible $$\frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1 n \rangle \langle n1 \rangle}$$
- Logs/divergences can be predicted to all orders, resum!
- Toy models are useful:

To go here!



QCD



$\mathcal{N} = 4$ SYM

Start here!

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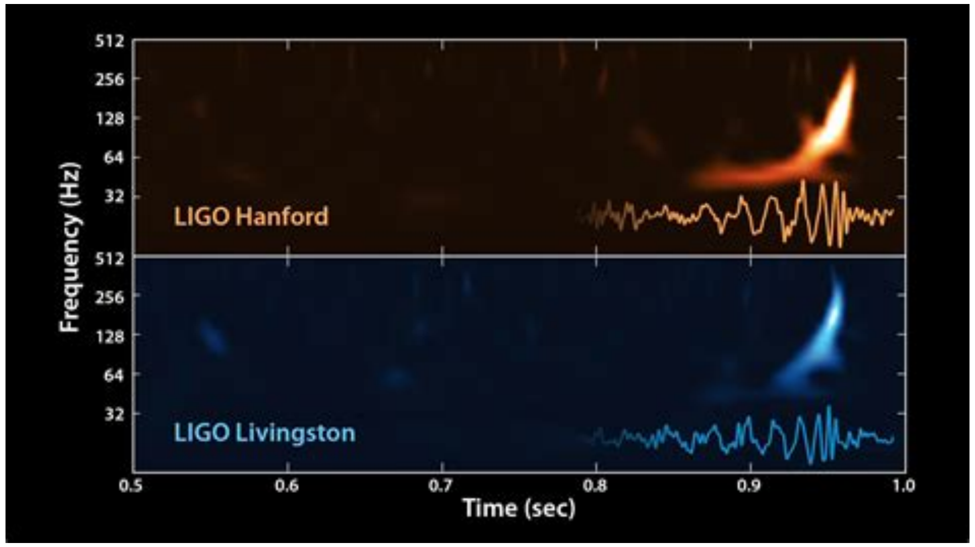
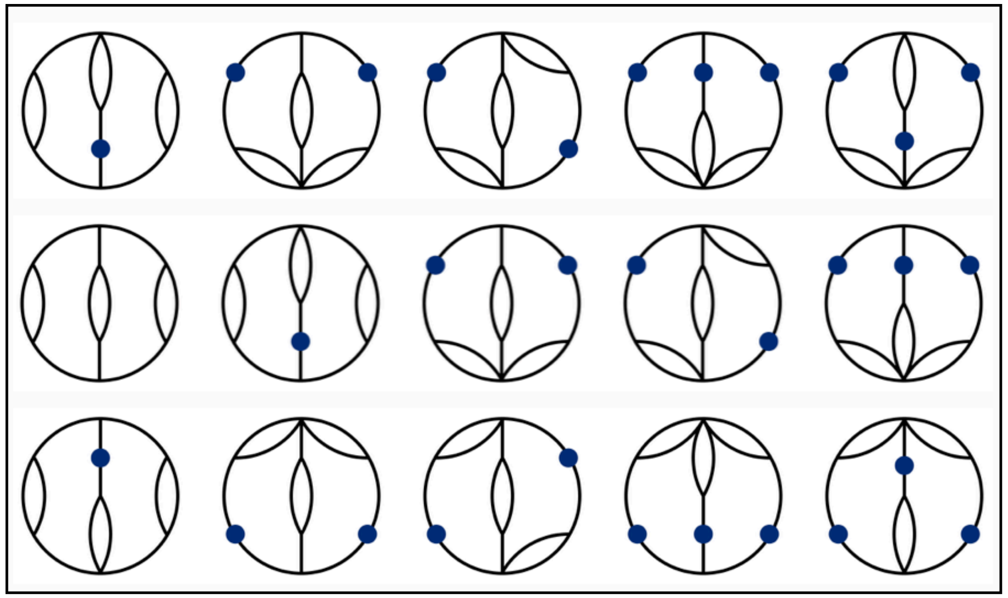




Einstein Gravity

Start here!

$\mathcal{N}=8$ SUGRA

A useful toy model

- $\mathcal{N} = 8$ graviton multiplet (massless) [Andrianopoli, D'Auria, Ferrara, Fre, Trigiante]

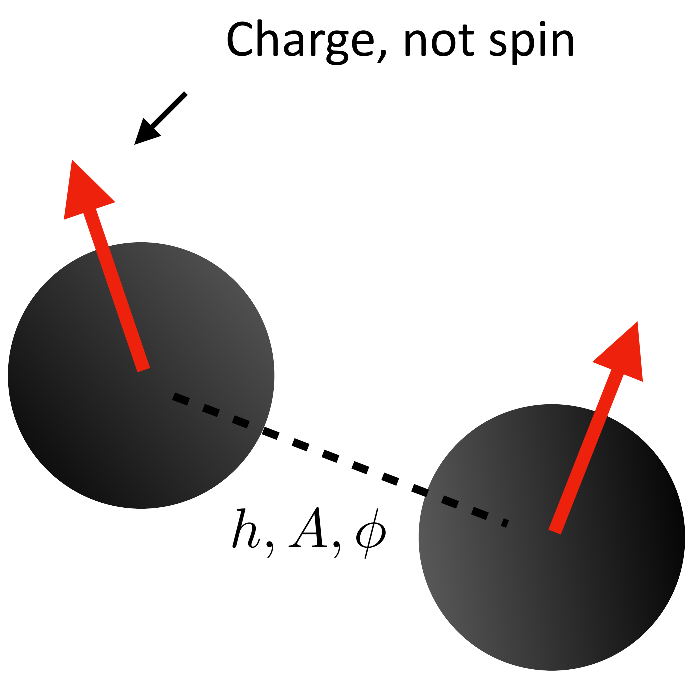
$$h_{\mu\nu}\,, \quad A^{\mu}_{IJ}\,, \quad \phi_{IJKL} \; + \text{ fermions}$$

- Extremal BH ~ half-BPS multiplet (focus on scalar component)

Central charges C_{IJ} [Caron-Huot, Zahraee]

$$C_1 = m_1 \begin{pmatrix} 0 & 1_{4\times 4} \\ -1_{4\times 4} & 0 \end{pmatrix}, \qquad C_2 = m_2 \begin{pmatrix} 0 & \Phi \\ -\Phi & 0 \end{pmatrix}$$

$$\Phi = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}, e^{i\phi_4}) \qquad \sum_i \phi_i = 0$$



- Force between BH depends on alignment of charges

- Two-body problem solved for aligned charges to $\mathcal{O}(v^2)$ all orders in G!

[Ferrell, Eardley; Gibbons, Ruback; Camps, Manton, Hadar]

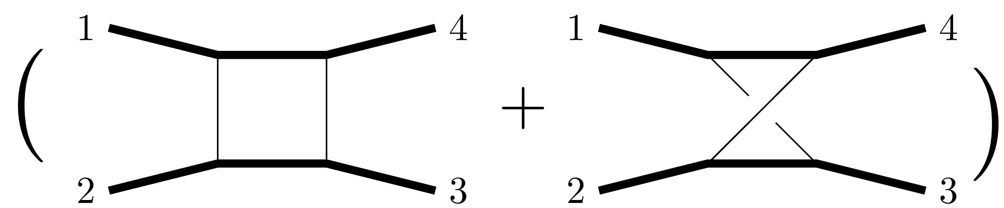
Simplicity in $\mathcal{N} = 8$ supergravity

$$\sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$t = -q^2$$

- One-loop integrand [Brink, Green, Schwarz; Caron-Huot, Zahraee]

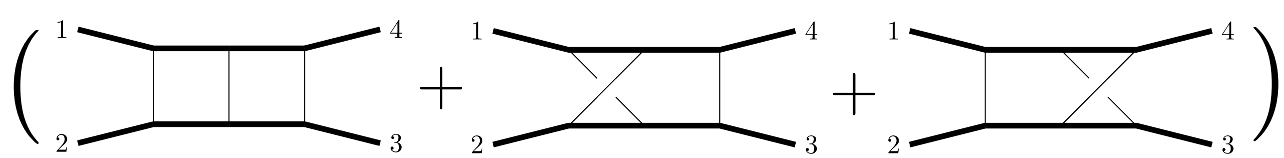
$$\mathcal{M}_4^{(1)} = -i(8\pi G)^2 16 m_1^4 m_2^4 (\sigma - \cos\phi)^4$$



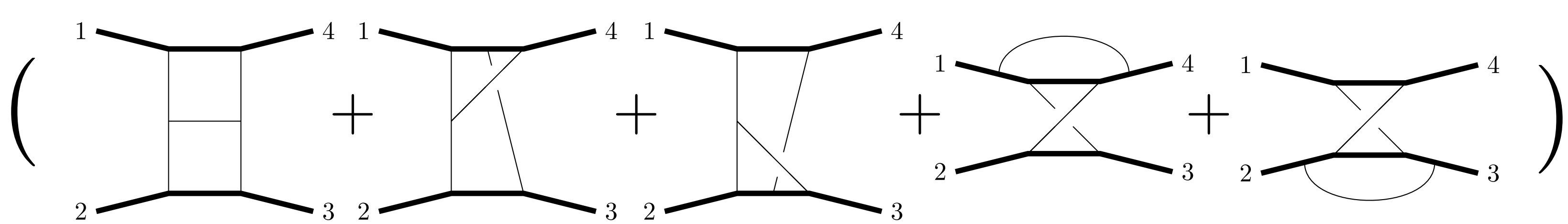
- Two-loop integrand [Bern, Dixon, Perelstein, Rozowski; JPM, Ruf, Zeng]

$$\mathcal{M}_4^{(2)} = -(8\pi G)^3 16 m_1^4 m_2^4 (\sigma - \cos\phi)^4$$

$$\Big[4 m_1^2 m_2^2 (\sigma - \cos\phi)^2$$



$$+(q^2)^2$$



Only scalar integrals

$$+ (2 \leftrightarrow 3) \Big]$$

- Loop integrand known up to five loops

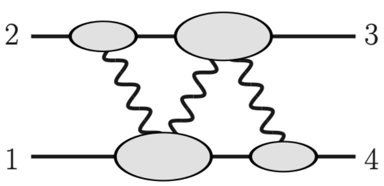
[Bern, Brink, Carrasco, Chen, Dixon, Edison, Green, Johansson, JPM, Kosower, Perelstein, Roiban, Rozowski, Schwarz, Zeng]

(Un)surprisingly simple!

State of matters

- In general, calculations can be separated in two parts:

  - Integrands - solved problem for near future (unitarity methods, double copy, simplified Feynman rules,...)



See Emil's, Zvi's talks

  - Integrals - real bottleneck (not a surprise for QCD friends)

- Today's talk, single scale integrals to all orders for observables that :

  1. are integrated over orbits and celestial sphere
  2. are independent of the phase of GW

Gravitons only inside cuts

e.g. scattering angle/impulse, energy loss

"inclusive enough"

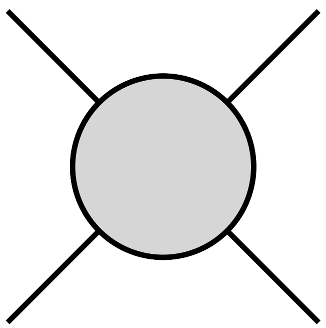
KMO’C (in-in) approach

[Kosower, Maybee, O’Connell]

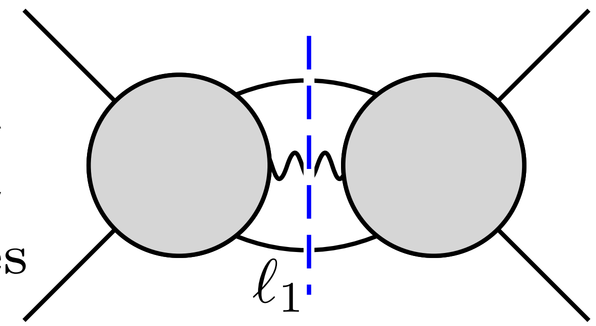
- Gauge invariant observables directly from amplitudes + unitarity cuts

Example 1: Impulse $$\Delta p_1^\mu = \int d^D q\, \delta(2u_1 \cdot q)\delta(2u_2 \cdot q)\, e^{ib\cdot q}\, \mathcal{I}^\mu$$

$$\mathcal{I}^\mu = q^\mu$$



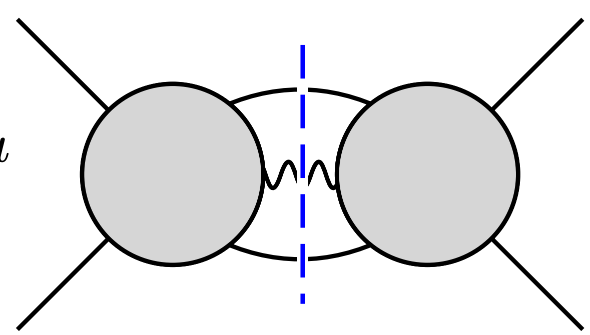
$$+ \quad \ell_1^\mu \sum_{\text{states}}$$



See David’s talk!

Example 2: Radiated momentum $$R^\mu = \int d^D q\, \delta(2u_1 \cdot q)\delta(2u_2 \cdot q)\, e^{ib\cdot q}\, \mathcal{R}^\mu$$

$$R^\mu = \sum_{\text{states}} k^\mu$$



$$\sim \int k^\mu |h_{\alpha\beta}|^2$$

Kernels are “inclusive enough”

See also [Maybee, O’Connell, Vines]

Integrals

Method of Regions

- Classical limit = Large angular momentum $\frac{m_i^2}{-q^2} \sim \frac{s}{-q^2} \sim J^2 \gg 1$
- Method of regions [Beneke, Smirnov] $|\boldsymbol{v}| = q^0/|\boldsymbol{q}|$

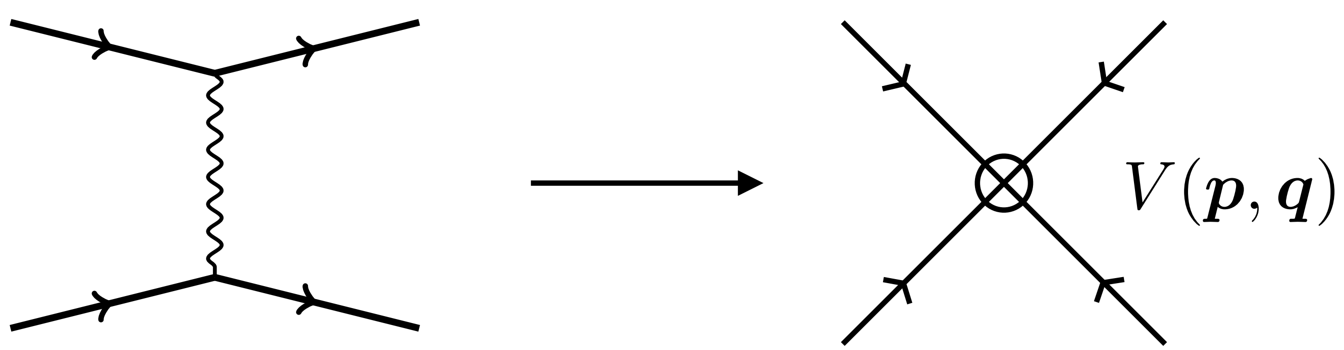
hard: $(\omega, \boldsymbol{\ell}) \sim (m, m)$

soft: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}|, |\boldsymbol{q}|) \sim J^{-1}(m|\boldsymbol{v}|, m|\boldsymbol{v}|)$

potential: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}||\boldsymbol{v}|, |\boldsymbol{q}|) \sim J^{-1}(m|\boldsymbol{v}|^2, m|\boldsymbol{v}|)$

radiation: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}||\boldsymbol{v}|, |\boldsymbol{q}||\boldsymbol{v}|) \sim J^{-1}(m|\boldsymbol{v}|^2, m|\boldsymbol{v}|^2)$

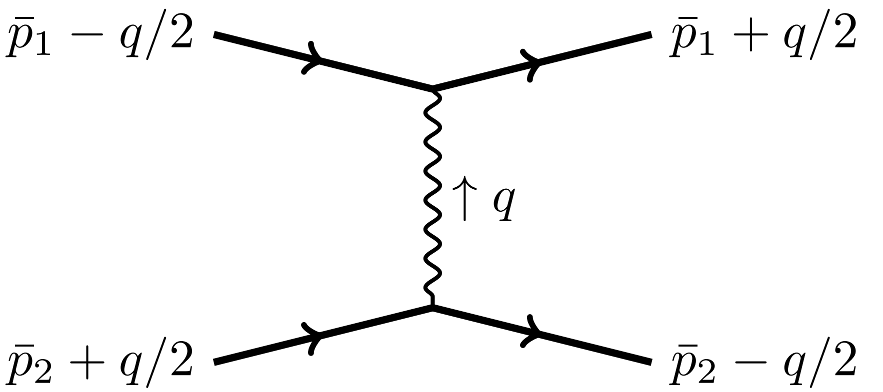
- Potential gravitons off-shell, mediate instantaneous interactions



Conservative dynamics

See Zvi's Talk!

Integration by parts (IBP)



- Special variables [Sudakov] $\bar{p}_i \cdot q = 0$

- Soft integrals (HQET-like)

$$\ell^2 \to \ell^2$$

$$(\ell + p_i)^2 - m_i^2 \to 2\ell \cdot u_i \,, \qquad u_i = \bar{p}_i / \bar{m}_i$$

- Homogeneous mass and q dependence trivialized - manifest power counting

- Result can be written in terms of a basis of *master integrals* via IBP

$$\int d^D \ell \frac{\partial}{\partial \ell^\mu} \frac{v^\mu}{\ell^2 (\ell - q)^2 \cdots} = 0$$

- Automated IBP tools exist (FIRE, Reduze, …) and can be applied to this problem.

Velocity differential equations

- Single variable! canonical form [Henn]

$$I(q,\bar{p}_i,\bar{m}_i) = (-q^2)^a\, I(y) \qquad\qquad y = \tfrac{\bar{p}_1\cdot\bar{p}_2}{\bar{m}_1\bar{m}_2} = \sigma + \mathcal{O}(q^2)$$

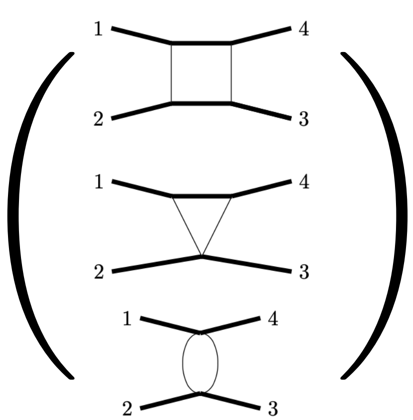
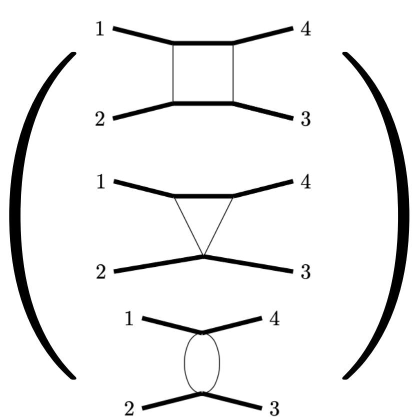
$$\mathrm{d}\vec{I}(y) = \epsilon \textstyle\sum_i A_i \mathrm{dlog}\,\alpha_i(y)\,\vec{I}(y)$$

- Symbol alphabet: $\{x, x+1, x-1\} \to$ new functions e.g. $\mathrm{Li}_2(1-x^2)$ ⟵ Relevant at $\mathcal{O}(G^4)$?

$$y = \tfrac{1+x^2}{2x} \qquad\qquad \log x \sim \mathrm{arcsinh}\sqrt{\tfrac{\sigma-1}{2}}$$

More generally, Harmonic polylogs (HPL) [Remiddi, Vermaseren] or worse!

Example:

d  $= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \epsilon\, \mathrm{d}\log x$ 

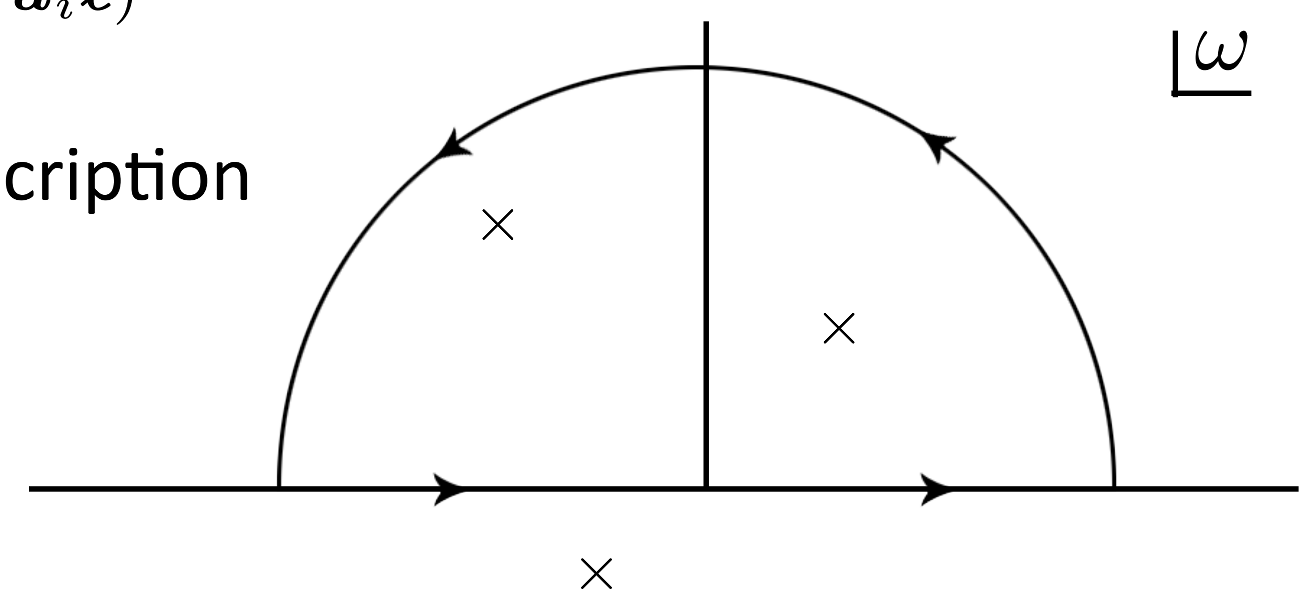
Static boundary conditions

- Radiation and potential regions split in near-static limit $v \ll 1$
- Only need to calculate appropriate boundary conditions (PN data), diff. eq. resum the velocity series.
- Potential region integral   with cut matter propagators

Graviton: $\frac{1}{\ell^2} = \frac{1}{\omega^2 - \boldsymbol{\ell}^2} = -\frac{1}{\boldsymbol{\ell}^2} - \frac{\omega^2}{(\boldsymbol{\ell}^2)^2} - \frac{\omega^4}{(\boldsymbol{\ell}^2)^3} + \cdots$

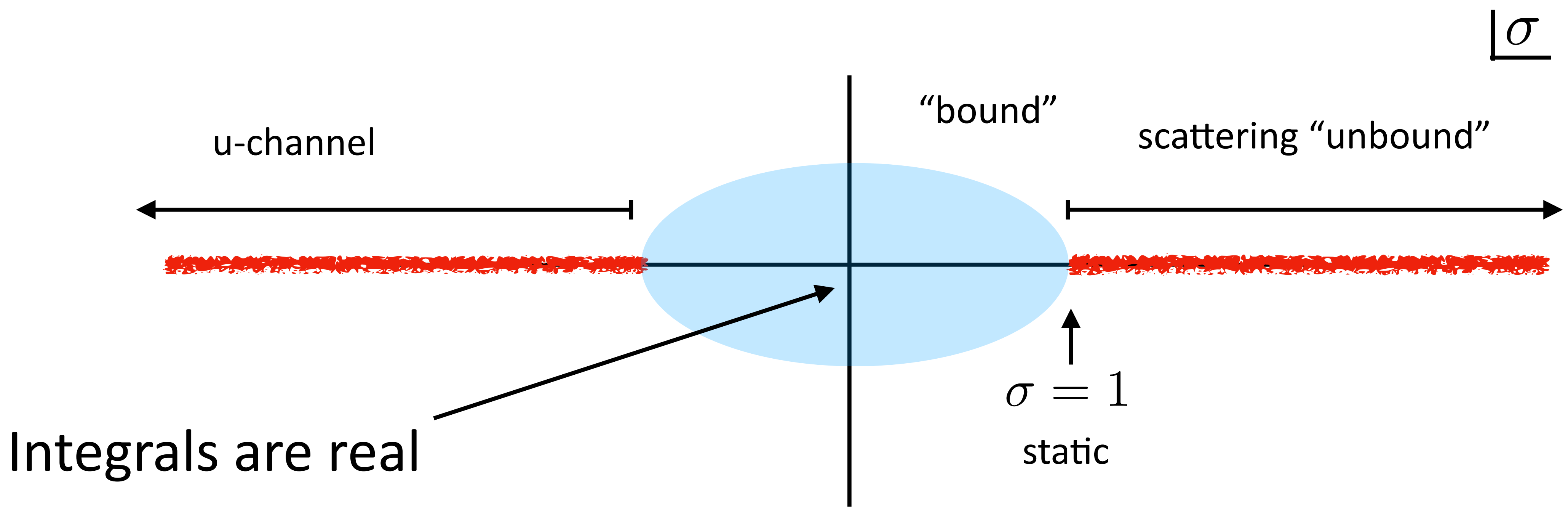
Matter: $\frac{1}{2u_i \cdot \ell} = \frac{1}{2(u_i^0\,\omega - \boldsymbol{u}_i \boldsymbol{\ell})}$

- Singular, evaluated by residue prescription
  (Similar to NRQCD/NRGR)

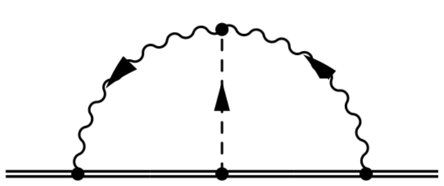


Full soft integrals

- Advantage over potential region: analyticity in velocity
- Advantage over massless integrals: Euclidean region!



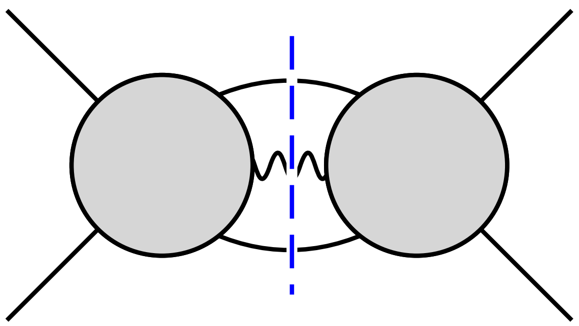
- Only a few boundary conditions are independent.
- No divergence from splitting potential + radiation (tails)



[image credit Porto, et al.]

Reverse unitarity

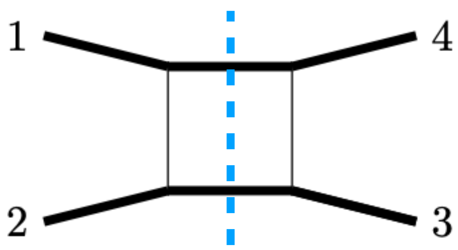
[Anastasiou, Melnikov]



- Phase space integrals - no problem!
- On-shell conditions are just like propagators

$$2\pi i\,\delta(2u_1\cdot\ell_1) = \frac{1}{2u_1\cdot\ell_1 - i\epsilon} - \frac{1}{2u_1\cdot\ell_1 + i\epsilon}$$

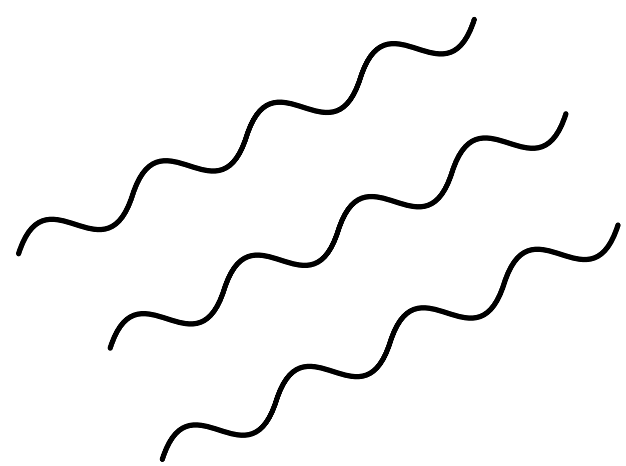
- Satisfy same type of differential equations as uncut integrals!

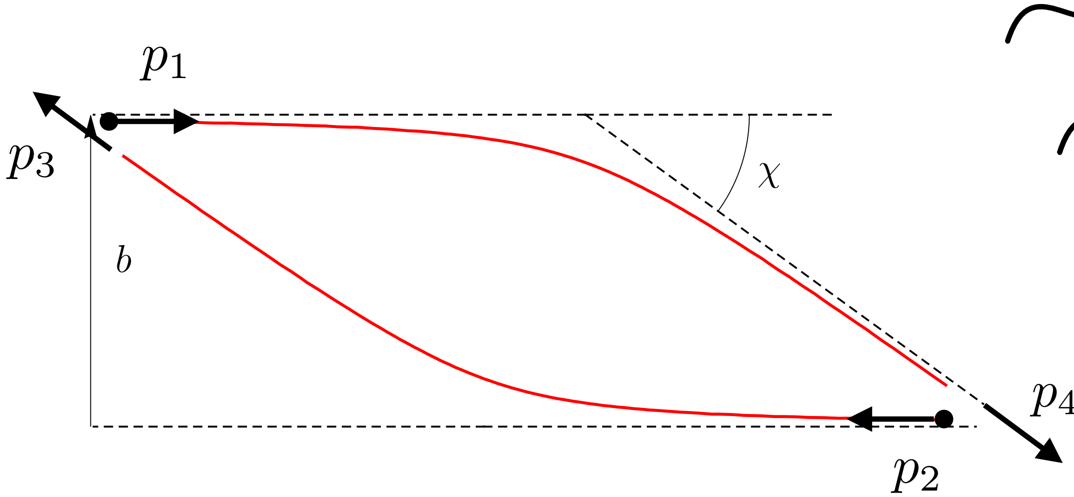
Simple example: $\frac{d}{d\sigma}$  $= 0$

Result equals static limit

- Importantly, we sum over full intermediate phase space!

Application:
Gravitational Bremsstrahlung





Conservative result

[Herrmann, JPM, Ruf, Zeng]

- KMO'C formulas can be used evaluating integrals in potential region. Only two-particle cuts necessary

$$\Delta p_1^\mu|_{\rm cons} = -\frac{16 G^3 m_1^2 m_2^2 \sigma^4}{\sigma^2 - 1} \frac{b^\mu}{b^4} \left(4\,\mathrm{arcsinh} \sqrt{\frac{\sigma - 1}{2}} + \frac{\sigma^2 s}{m_1 m_2 (\sigma^2 - 1)^{3/2}} \right)$$

High-energy log

see Heissenberg, Damour talks!

- Scattering angle: $\sin\frac{\chi}{2} = \frac{\sqrt{-\Delta p_1^2}}{2 p_{\rm cm}}$ matches result from eikonal/EFT

[JPM, Ruf, Zeng]

- Purely transverse due to "no-triangle" property at one loop

[Caron-Huot, Zahraee]

Two-loop radiative impulse

[Herrmann, JPM, Ruf, Zeng]

- Evaluating integrals in full soft region

Conservative

$$\Delta p_1^\mu = -\frac{16G^3 m_1^2 m_2^2 \sigma^4}{\sigma^2-1}\frac{b^\mu}{b^4}\left(-\frac{2\sigma^2}{\sigma^2-1}+\left(4-\frac{4\sigma\left(\sigma^2-2\right)}{\left(\sigma^2-1\right)^{3/2}}\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}+\frac{\sigma^2 s}{m_1 m_2\left(\sigma^2-1\right)^{3/2}}\right)$$
$$-\frac{4\pi G^3 m_1^2 m_2^2 \sigma^4}{(\sigma^2-1)^{3/2}}\frac{\sigma u_2^\mu - u_1^\mu}{|b|^3}\left(-\frac{2\sigma^2}{\sigma^2-1}-\left(4-\frac{4\sigma\left(\sigma^2-2\right)}{\left(\sigma^2-1\right)^{3/2}}\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}-4\log\left(\frac{1}{2}(1+\sigma-\sqrt{\sigma^2-1})\right)\right)$$

New function!

- Angle with radiation matches eikonal [Di Vecchia, Heissenberg, Russo, Veneziano]
- Contains more information!

Energy loss

- Radiated momentum $$R^\mu = -\Delta p_1 - \Delta p_2$$

- Energy loss in rest frame of one of the particles $$E^{\rm rad} = u_1 \cdot R = R^0$$

$$E^{\rm rad} = \frac{G^3 m_1^2 m_2^2}{|b|^3} \frac{4\pi\sigma^4}{(\sigma^2-1)^{1/2}} \left(-\frac{2\sigma^2}{\sigma^2-1} - \left(4 - \frac{4\sigma\left(\sigma^2-2\right)}{(\sigma^2-1)^{3/2}}\right) \operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}} - 4\log\left(\frac{1}{2}(1+\sigma-\sqrt{\sigma^2-1})\right)\right)$$

- Has expected mass dependence [Kovacs, Thorne; Bini, Damour, Geralico]

- High-energy limit as expected! $$E^{\rm rad} \sim 8\pi(2\log 2 - 1)\frac{G^3 m_1^2 m_2^2}{|b|^3}\sigma^3$$

$$\Delta E = (20.0 \pm 0.3)[(m_{\rm A} m_{\rm B})^2/b^3]\gamma^3 \ .$$

[Kovacs, Thorne]

Modulo coefficient…

Results in pure gravity underway…

stay tuned!

Conclusions

Comments about KMO'C approach

- Real time dynamics - Schwinger-Keldish

KMO'C!

  - Option 1: time ordered propagators/Feynman rules, unitarity cuts

$$\langle \text{in}|S^\dagger \mathcal{O} S|\text{in}\rangle = \int_{(\phi_1=\phi_2)_{t=+\infty}} D\phi_1 D\phi_2 \, \mathcal{O} \, e^{iS[\phi_1]-iS[\phi_2]}$$

  - Option 2: Keldish basis, causal (advanced, retarded) propagators, manifest classical limit

$$\phi_+ = \phi_1 + \phi_2 \sim \mathcal{O}(\hbar^0) \qquad \phi_- = \phi_1 - \phi_2 \sim \mathcal{O}(\hbar^1)$$

$$\int D\phi_- e^{i\phi_- \frac{\delta S[\phi_+]}{\delta \phi_+}} = \delta\left(\frac{\delta S[\phi_+]}{\delta \phi_+}\right)$$

c.f. Damour's talk

- Different from other approaches which calculate time-symmetric quantities. Here Feynman vs causal propagators just a choice of basis.

Conclusions

- KMO'C formalism captures real time dynamics in a way amenable to traditional methods for covariant integration
- Adapted modern techniques (IBP, canonical diff. eq.) for calculation of integrals relevant for classical gravity (or Heavy Particle EFTs). Single scale to all orders!
- In our example, different velocity regions captured in boundary conditions.
- Thanks to reverse unitarity: "inclusive enough observables" require no more complicated methods than virtual amplitude/scattering angle. Also in full soft region hereditary effects (tails) should not be an issue.
- Technology/automation ripe to go to higher orders!

Questions

- Can we adapt classical methods to use same integration techniques? (Yes! see Porto's talk)
- Can we have have our cake and eat it?
  - EFT: non-covariant integrals - integrand level subtractions
  - Eikonal, KMO'C: covariant integrals - no integrand level subtractions
- How much does the four-point amplitude know? Bounds states, decay rates, analytic continuation?
- Does EFT philosophy ("one scale at a time") + manifest power counting always imply one integration variable at a time?

Thank you!