Physically normalised Yukawa Couplings

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Background

arXiv 2401.15078 hep-th (v2 eta $\sim$ 1 week)

- Heterotic compactification on Calabi–Yau 3-fold (X, g, V).
- Want $(\mathbf{27})^3$ couplings describing EFT interactions.
- Physical field normalisation determined by geometry of X, V.
- Consider $V = T_X$, two different numerical approaches:
  - Exact numerics via special geometry / Kodaira–Spencer map.
  - Approximate harmonic forms $\longrightarrow$ general holomorphic bundle V?
- Excellent agreement even close to singularities in complex structure moduli space.

Couplings

- $\mathcal{N} = 1$ theory descending from heterotic compactification $V \to X$,

$$S_{\text{eff}} \supset \int d^4x \left(\int d^4\theta \, K(\Phi, \overline{\Phi}) + \int d^2\theta \, W(\Phi) \right) .$$

- Kinetic terms in Kähler potential determine field normalisation,

$$K(\Phi, \overline{\Phi}) \supset G_{a\overline{b}} \Phi^a \overline{\Phi}^b , \quad G_{a\overline{b}} \sim (a, b) = \int_X a \wedge \bar{\star}_V b \; ,$$
$$a, b \in \mathcal{H}^1(X, V) \;\; \text{harmonic} \; .$$

- Couplings originate from superpotential $W(\Phi) \supset \tilde{\kappa}_{abc} \Phi^a \Phi^b \Phi^c$.

Couplings

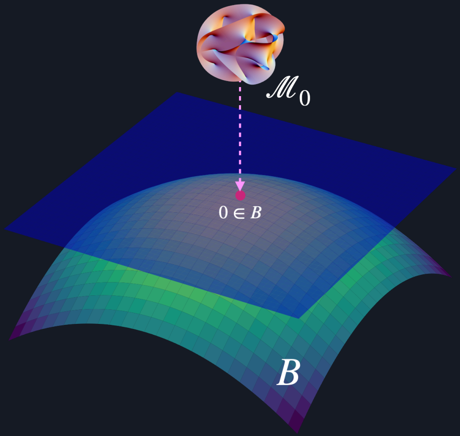
- Identify matter fields with harmonic $(0,1)$ bundle-valued forms $a \in H^1(X, V_r)$.
- Holomorphic couplings quasi-topological:

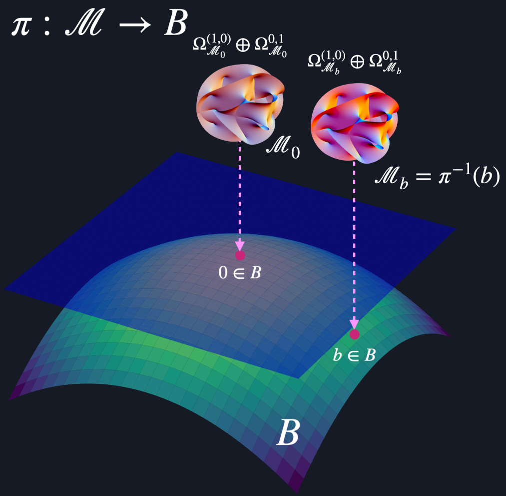
$$\tilde{\kappa}(a,b,c) = \int_X \Omega \wedge \Omega\left(\{a,b,c\}\right) \ .$$

- Rotate to eigenbasis of $G_{a\bar{b}}$, find **physically normalised couplings** κ_{abc} describing low-energy field interactions, masses etc.
- Standard embedding, eigenbasis $\{a_k\}_{k=1}^{h^{(2,1)}}$,

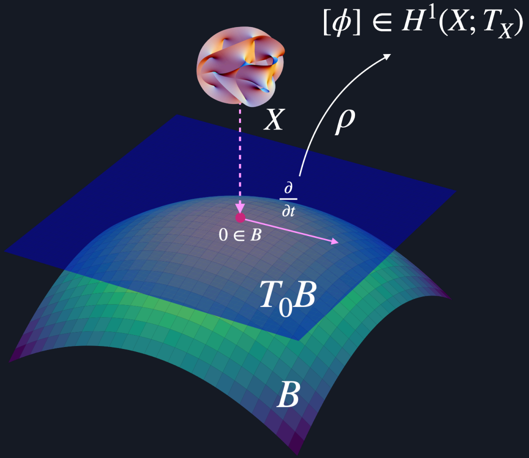
$$\kappa_{abc} = \frac{\int_X \Omega \wedge \Omega_{\mu\nu\rho}\left(a^\mu \wedge b^\nu \wedge c^\rho\right)}{\sqrt{\lambda_a \lambda_b \lambda_c} \int_X \Omega \wedge \overline{\Omega}} \ .$$

$$\pi : \mathscr{M} \to B$$




$\pi : \mathscr{M} \to B$
$\Omega^{(1,0)}_{\mathscr{M}_0} \oplus \Omega^{0,1}_{\mathscr{M}_0}$
$\Omega^{(1,0)}_{\mathscr{M}_b} \oplus \Omega^{0,1}_{\mathscr{M}_b}$
$\mathscr{M}_0$
$\mathscr{M}_b = \pi^{-1}(b)$
$0 \in B$
$b \in B$
B

$$\pi : \mathscr{M} \to B$$



CY moduli space

- Deformations of T_X integrate to c.s. deformations for CYs, $(H^1(X; T_X) \simeq H^{2,1}(X))$.
- c.s. moduli space Weil–Petersson metric is the natural pairing,

$$\langle a, b\rangle_{\mathrm{WP}} \triangleq (a, b) = \int_X a \wedge \overline{\star_g} b, \quad R_{i\bar{j}}(g) = 0, \quad a, b \in \mathcal{H}^1(X; T_X)\ .$$

- Special geometry: elide computation of g.

CY moduli space

Special geometry: elide computation of g. The following are equivalent:

- With harmonic forms, ☹

$$\langle a, b\rangle_{\mathrm{WP}} \propto -\int_{X_t} \Omega(\mathcal{H}\rho(a)) \wedge \overline{\Omega(\mathcal{H}\rho(b))}$$

- Cup products on $H^{p,q}$ only, ☺ [Keller, Lukic, (2012)] - hypersurfaces.

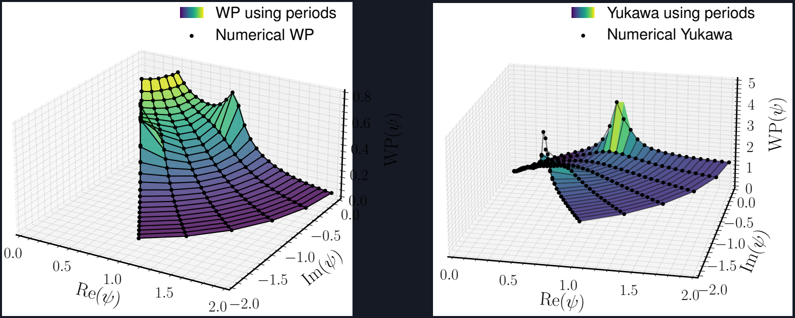
$$\langle a, b\rangle_{\mathrm{WP}} \propto \frac{\left(\left.\frac{\mathrm{d}\Omega_t}{\mathrm{d}t^a}\right|_{t=0}, \left.\frac{\mathrm{d}\Omega_t}{\mathrm{d}t^b}\right|_{t=0} \right)}{(\Omega, \Omega)} + \frac{\left| \left(\Omega, \left.\frac{\mathrm{d}\Omega_t}{\mathrm{d}t}\right|_{t=0} \right) \right|^2}{(\Omega, \Omega)^2} .$$

Example: Mirror of $\mathbb{P}^5[3,3]$

- Calabi–Yau X in deformation space $\mathbb{P}^5[3,3]$ ($h^{2,1} = 73$, $h^{1,1} = 1$):

$$x_0^3 + x_1^3 + x_2^3 - 3\psi\, x_3 x_4 x_5 = 0\,, \qquad x_3^3 + x_4^3 + x_5^3 - 3\psi\, x_0 x_1 x_2 = 0\,.$$

- Mirror $\tilde{X}$ is a blowup of finite quotient of same zero locus.



(a) Weil–Petersson metric

(b) Normalized Yukawa coupling

Example: Tian-Yau quotient

- Complete-intersection CY $X \hookrightarrow \mathbb{P}^3 \times \mathbb{P}^3$ given by zero locus, take $\mathbb{Z}_3$ quotient.

$$\frac{1}{3}\sum_{a=0}^{3} x_a^3 = \frac{1}{3}\sum_{a=0}^{3} y_a^3 = \sum_{a=0}^{3} x_a y_a = 0\,, \quad h^{2,1}(X/\mathbb{Z}_3) = 9. \qquad (1)$$

- Construct orthonormal basis of polynomial deformations,

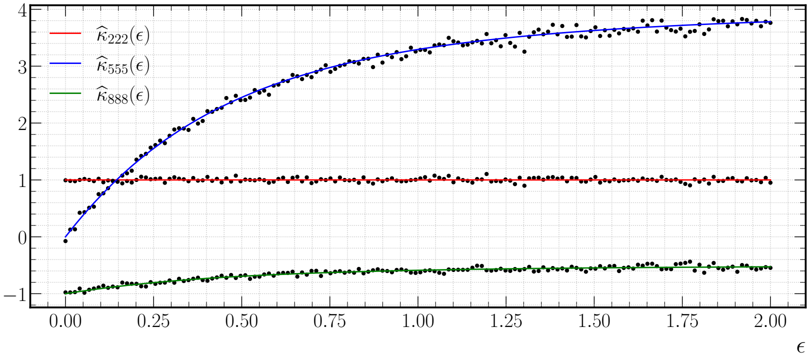
$$\{\lambda_i\} \leftrightarrow H^{(2,1)}(X/\mathbb{Z}_3)\ .$$

- Study moduli dependence along deformation

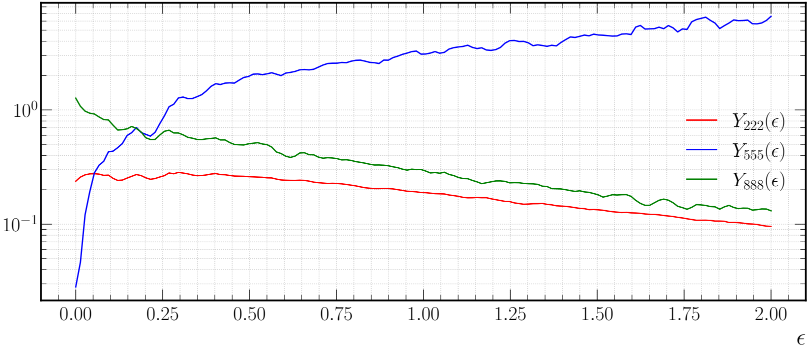
$$p_2 = x_0 y_0 + x_1 y_1 + (1+\epsilon)(x_2 y_2 + x_3 y_3) = 0\,, \epsilon \in \mathbb{R}\ .$$

- Compare quasi-topological couplings against existing literature [Candelas, Kalara, Mohapatra (87)].

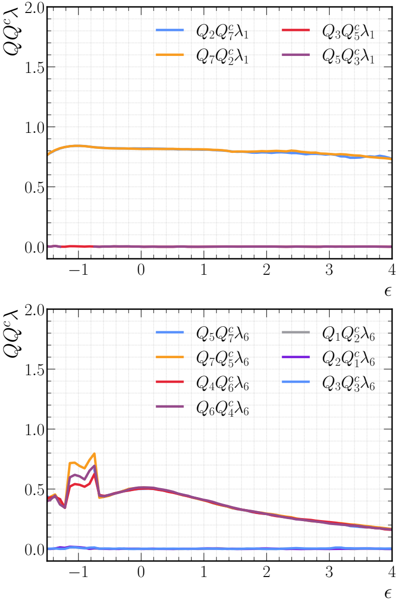
Topological cubic couplings



Normalised cubic couplings



More normalised couplings



Machine–learning on CYs: patterns

Want to: approximately solve PDEs governing tensor fields on CY $X \hookrightarrow \mathcal{A}$, respecting topology of X.

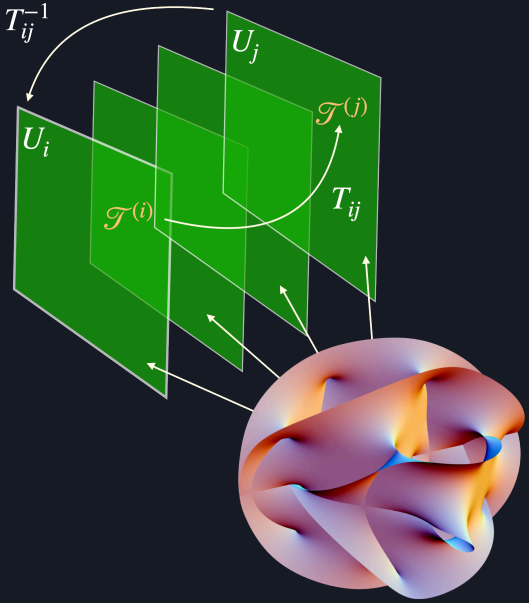
(a) Reduce problem to learning a global **function** $u(x) \in C^\infty(X)$ s.t.

$$\mathcal{F}\left(u(x), \partial u(x), \cdots, \partial^{(n)} u(x)\right) = 0 \text{ locally.}$$

(b) Parameterise u_θ, variational formulation:

$$\mathcal{F} = 0 \longleftrightarrow \min_{\theta \in \Theta} \mathscr{L}(\cdots; \theta) \ .$$

Is this generic?


T_{ij}^{-1}
U_j
$\mathscr{T}^{(j)}$
U_i
T_{ij}
$\mathscr{T}^{(i)}$

Machine–learning on CYs: antipatterns

- Multiple losses $\sum_i c_i \mathscr{L}_i(\cdots; \theta)$.
- Loss as a proxy for 'downstream' tasks.
- Hypotheses which are not global by construction.
- Hypotheses which are sensitive to choices of hyperparameters.

Is this generic?

Harmonic form learning

- Tensor field ansätze must be globally defined on $X \hookrightarrow \mathcal{A}$.
  - Local computations should glue b/w patches.
- Want harmonic bundle-valued forms $\eta \in \mathcal{H}^{(0,1)}_{\bar{\partial}}(X; V)$, $\Delta_{\bar{\partial}_V}\eta = 0$.
- e.g. $V = T_X$, pick complex moduli t, parameterise η as $\bar{\partial}_V$-exact correction from Kodaira-Spencer rep.:

$$\eta = \phi + \overline{\partial}_V \mathfrak{s} \in \mathcal{H}^{(0,1)}_{\overline{\partial}}(X; V), \quad [\phi] \in \mathrm{Im}(\rho_t), \mathfrak{s} \in \Gamma(T_X) \;.$$

Harmonic form learning

- Natural variational formulation:

$$\tilde{\eta}_\lambda = \phi + \bar{\partial}_V \mathfrak{s}(\cdot;\lambda), \quad \lambda = \operatorname*{argmin}_{\lambda' \in \Lambda} \Delta_g \tilde{\eta}(\cdot;\lambda').$$

- $\tilde{\eta} \in H^*(X; T_X)$ by construction, natural objective:

$$\mathscr{L}(\lambda) \triangleq \left(\bar{\partial}_V^\dagger \eta_\lambda, \bar{\partial}_V^\dagger \eta_\lambda\right) = \int_{X_t} \bar{\partial}_V^\dagger \eta_\lambda \wedge \overline{\star_V} \bar{\partial}_V^\dagger \eta_\lambda$$

- Reduces to finding globally defined section $\mathfrak{s}$ of $V = T_X$.

Globally defined ansätze

- Reduces to finding globally defined section $\mathfrak{s}$ of $V = T_X$.
  - **Find basis of sections for algebraic $\mathcal{O}(k)$-twisted forms.**
  - Parameterise coefficients with global parameterised function.
- Dualise, twist Euler sequence:

$$0 \to \Omega_{\mathbb{P}^n}(1) \to \bigoplus_{j=0}^{n} \mathcal{O} \to \mathcal{O}(1),$$

- Basis of $\Omega_{\mathbb{P}^n}(1)$ given by

$$\{\alpha^{ij} = Z^i dZ^j - Z^j dZ^i \mid i \neq j\}.$$

- Künneth-like formula on products $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_a}$

$$\Omega^{(k)}_{X \times Y}(m) \simeq \bigoplus_{i+j=k} \Omega^{(i)}_X(m) \otimes \Omega^{(j)}_Y(m) \ .$$

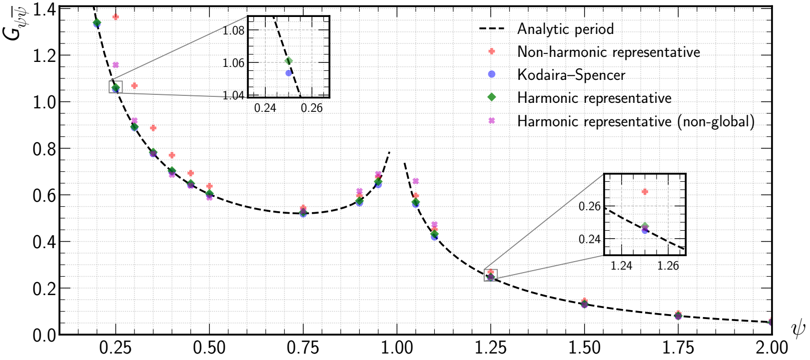
Globally defined ansätze

- Reduces to finding $h^{2,1}$ globally defined sections $\mathfrak{s}^{(h)}$ of $V = T_X$.
  - Find basis of sections for algebraic $\mathcal{O}(1)$-twisted forms.
  - **Parameterise coefficients with global parameterised function.**

$$\mathfrak{s}(Z) = \psi_{abcd}(Z)\frac{(\iota^*\overline{\alpha})^{\bar{a}\bar{b}}_{\bar{\nu}} Z^c Z^d}{\|Z\|^2} \cdot g^{\mu\bar{\nu}} \cdot \frac{\partial}{\partial z^\mu} \in \Gamma(T_X), \quad \psi \in C^\infty(X).$$

- Need a recipe for constructing reps $\in H^1(X;V)$ and sections $s \in \Gamma(V)$ for general V!

Example: Mirror of $\mathbb{P}^5[3,3]$



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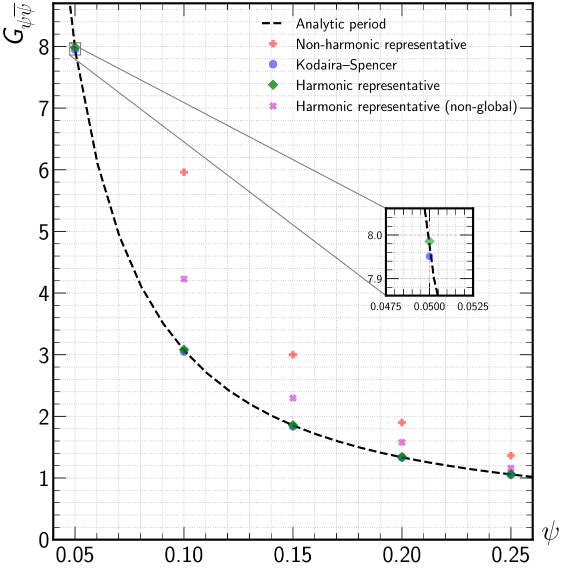

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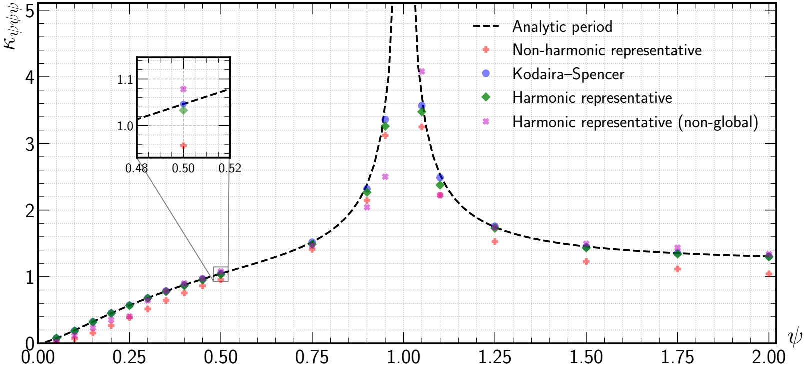

Conclusions/Outlook

`DONE`

- Numerical compactification data now obtainable for any toric variety under standard embedding.
- Demonstrated machine–learned harmonic forms fairly trustworthy.

`TODO`

- Generalise approximation of bundle-valued harmonic forms to holomorphic vector bundles $V \to X$.
- Release of efficient `Jax` package for:
  - Numerical differential geometry on Calabi–Yaus.
  - Coarse-grained map:

$$\{\text{Geometric compactification data}\} \longrightarrow \{\text{Phenomenological data}\}$$

Thanks/Credits

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C. Mishra, J. Tan.*

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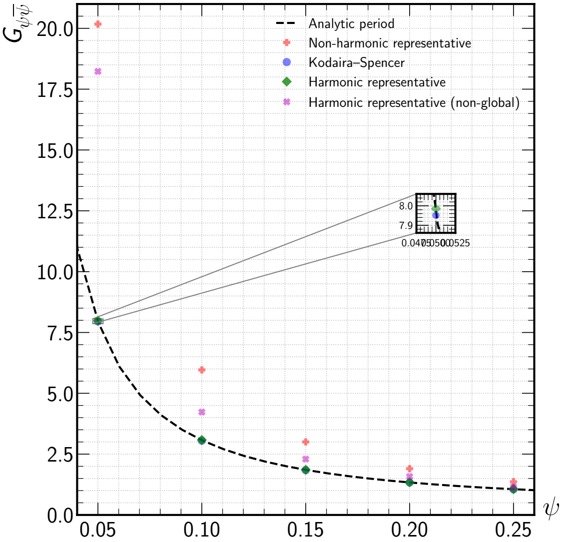
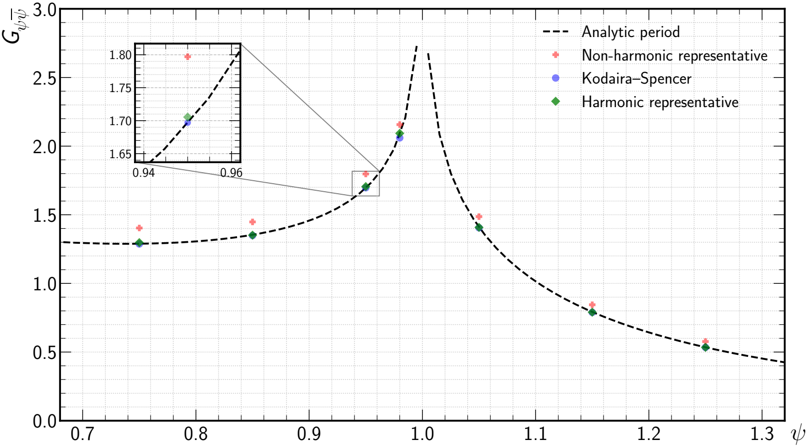
> I don't like it that they're not calculating anything. ... why are the masses of the various particles such as quarks what they are? All these numbers ... have no explanations in these string theories – absolutely none!
>
> — Richard Feynman

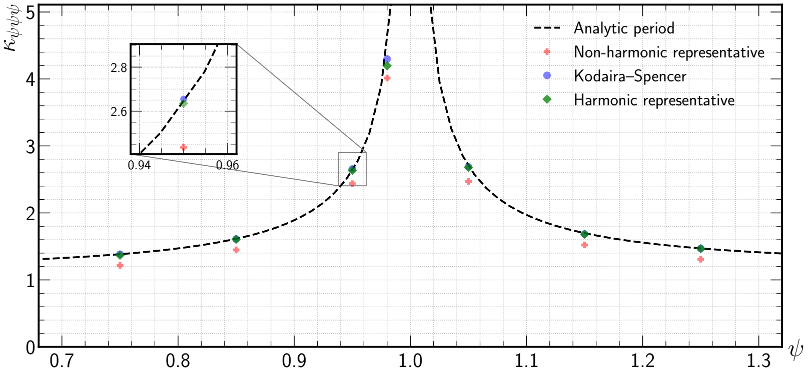
Example: Mirror of $\mathbb{P}^5[3,3]$



Example: Mirror of $\mathbb{P}^7[2,2,2,2]$



Example: Mirror of $\mathbb{P}^7[2,2,2,2]$



Example: Mirror of $\mathbb{P}^5[3,3]$