Machine-Learning Yukawa couplings from String Theory

Learning Standard Model couplings on Calabi-Yau manifolds.

Kit Fraser-Taliente

UNIVERSITY OF OXFORD

based on work with Andrei Constantin, Andre Lukas, Thomas Harvey, and Burt Ovrut (2402.01615), (240X.XXXXX).

KFT is supported by a Gould-Watson scholarship

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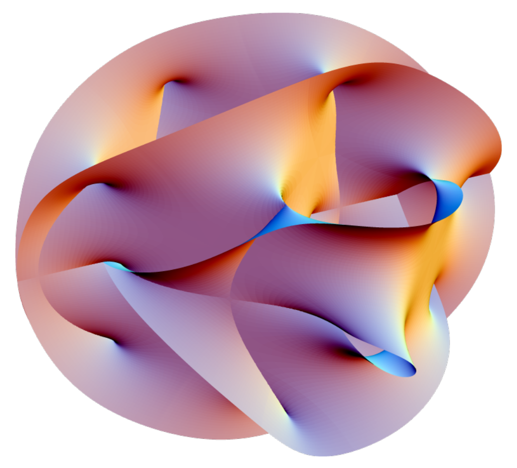
(Horava-Witten)

vector bundle V over

smooth CY3 X

(CY3 X S1/Z2)



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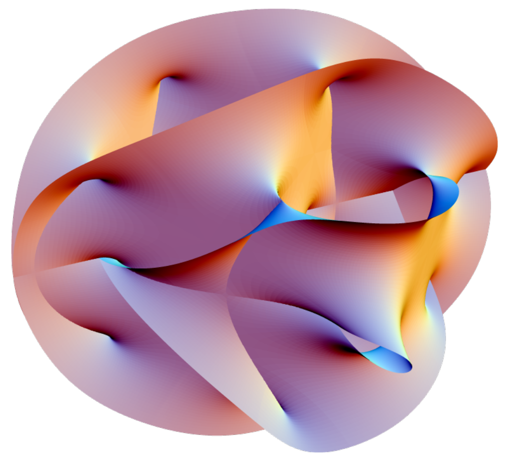

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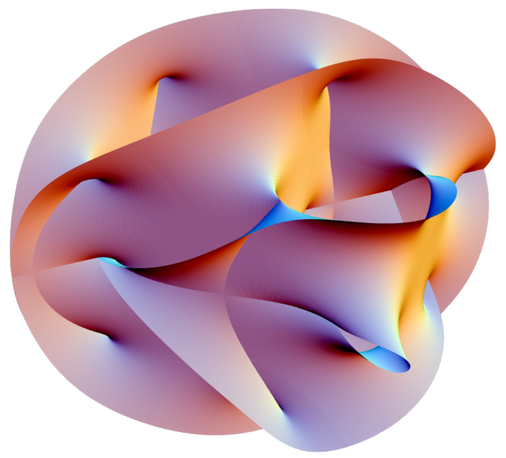

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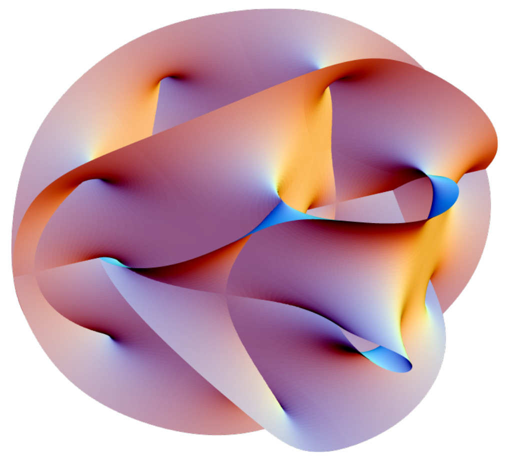

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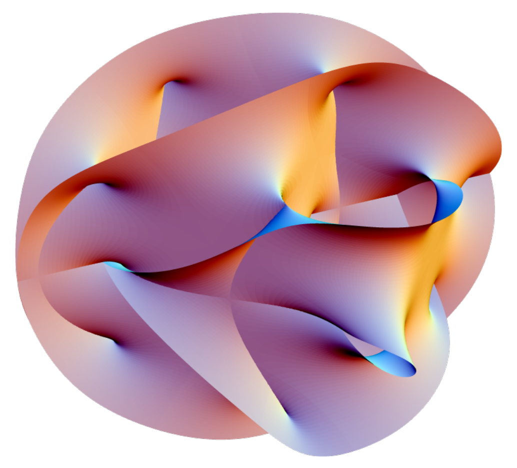

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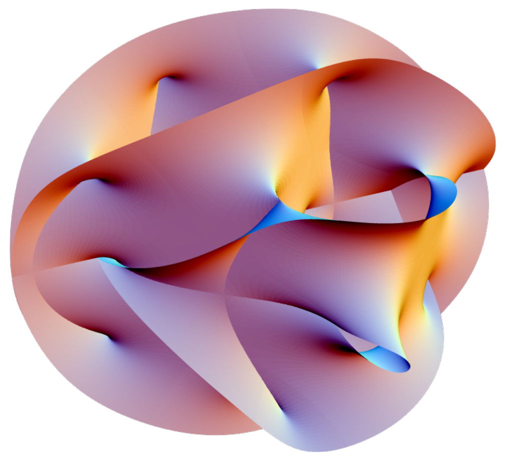

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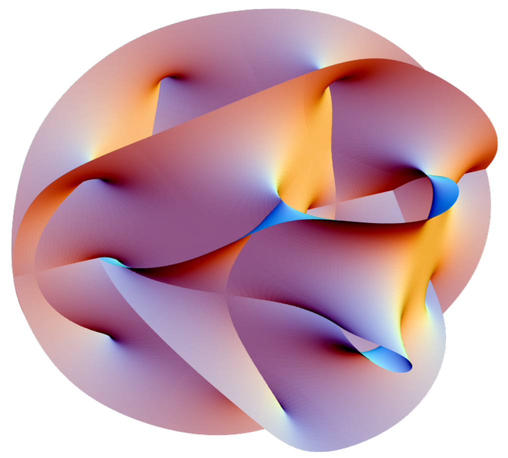

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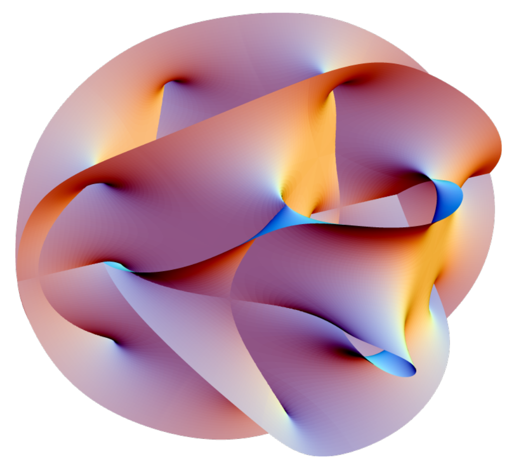

4D $N = 1$ chiral gauge theory

Caveat. Proof of concept.

heterotic string compactifications to $4D$ $N = 1$

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$$\mathscr{L} = -\, K_{I\bar{J}} \partial_\mu c^I \partial^\mu \bar{c}^{\bar{J}} - i K_{I\bar{J}} \bar{\chi}^{\bar{J}} \bar{\sigma}^\mu \partial_\mu \chi^I + e^{K_{mod}/2} (\lambda_{IJK} c^I \chi^J \chi^K + c\,.\,c.) + \ldots$$

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metric g

bundle metric $H \sim A^\mu$

harmonic forms ν_I

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N=1 heterotic compactifications

10D gauge theory

$N = 1$ SUGRA coupled to $N = 1$

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$(X, V, \tilde{V},$ 5-branes$, W)$

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SUSY variation $\stackrel{!}{=} 0$

$\Big\downarrow$ constrains

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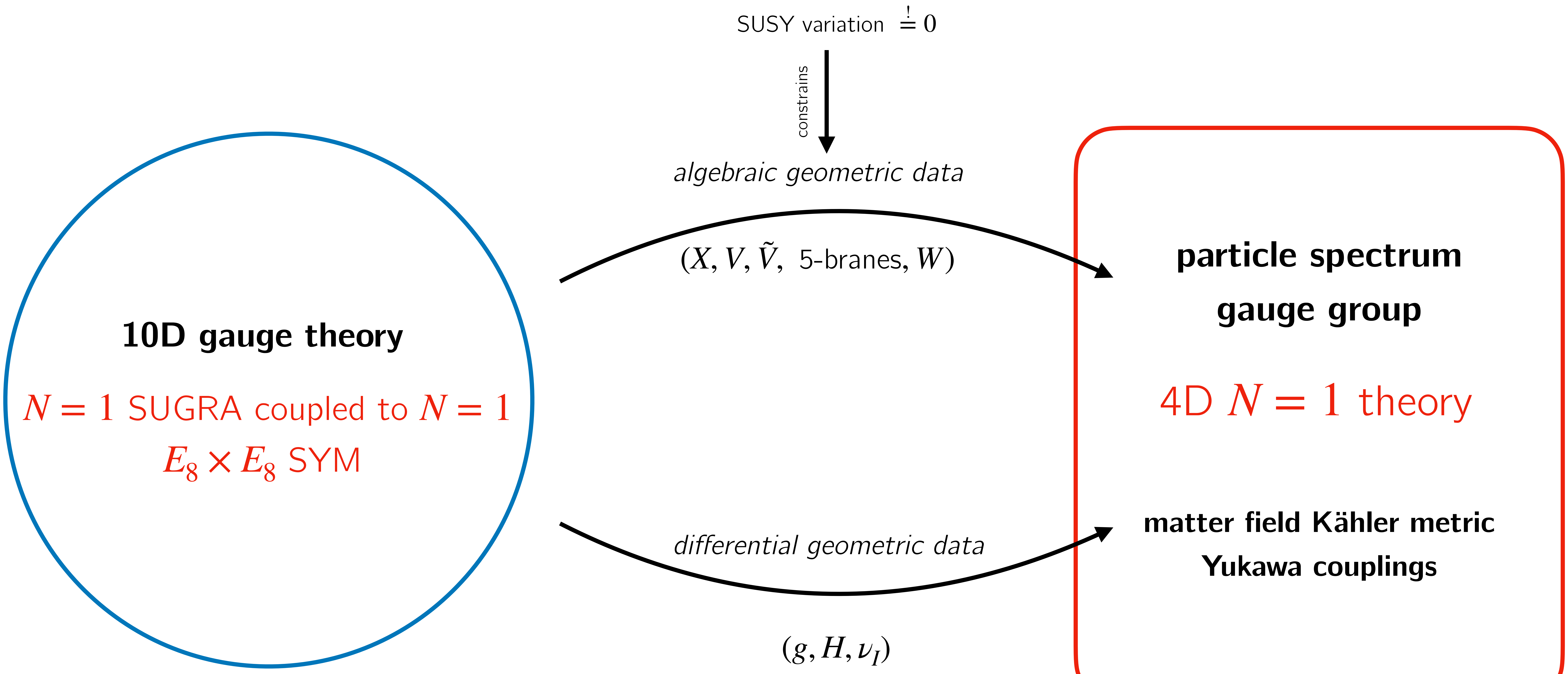

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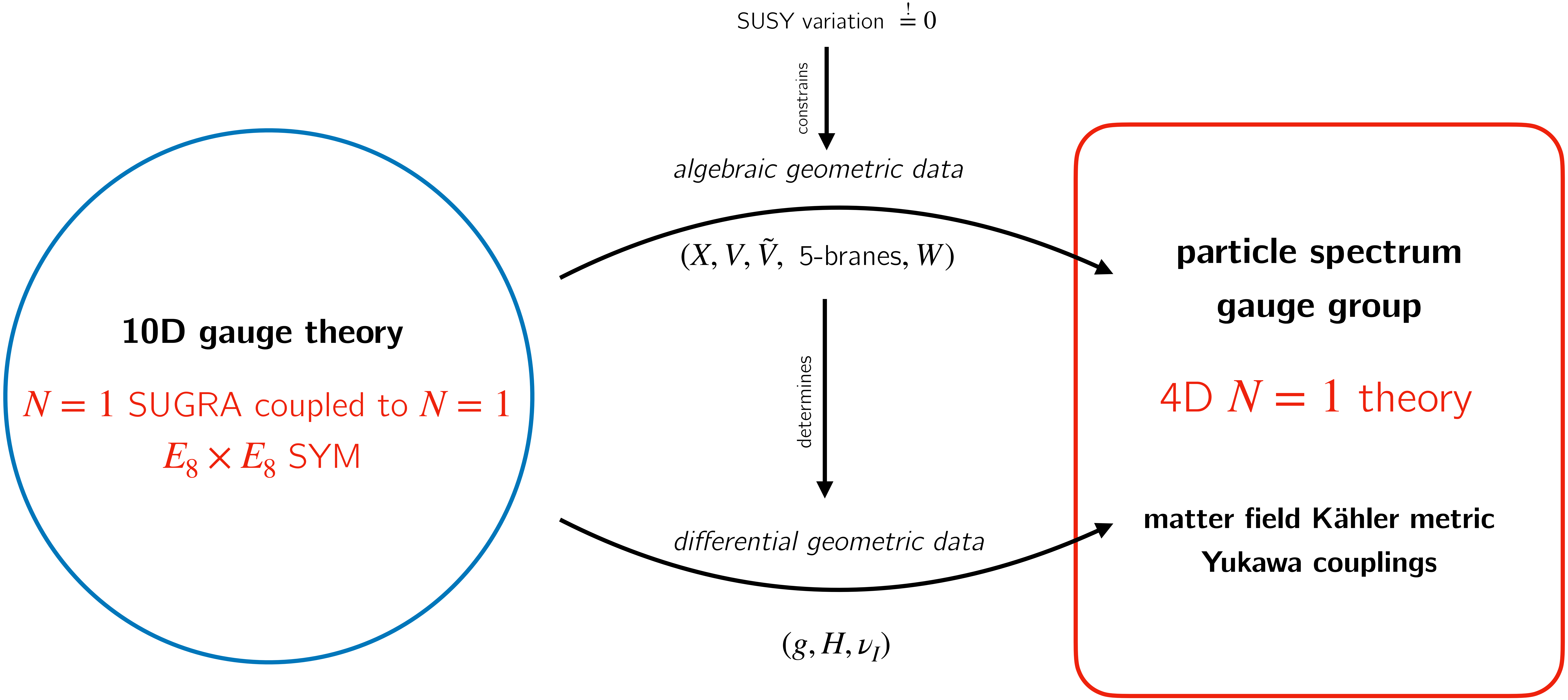

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2 and 3: just (bundle)-Poisson equations!

Solving PDEs with neural networks

general method: approximate $f \in C(X)$ with $\theta \in \mathbb{R}^N$, **NN architecture** specifies the map $\mathbb{R}^N \to C(X)$

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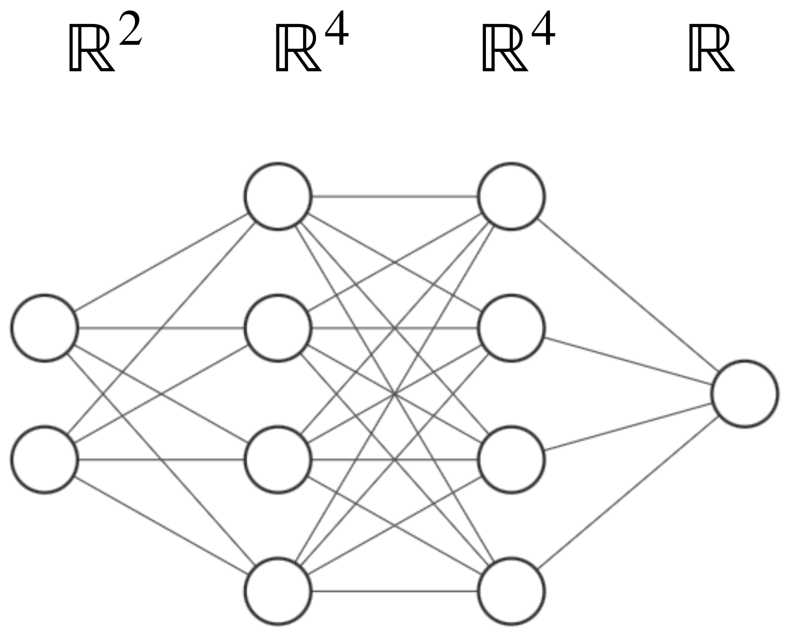

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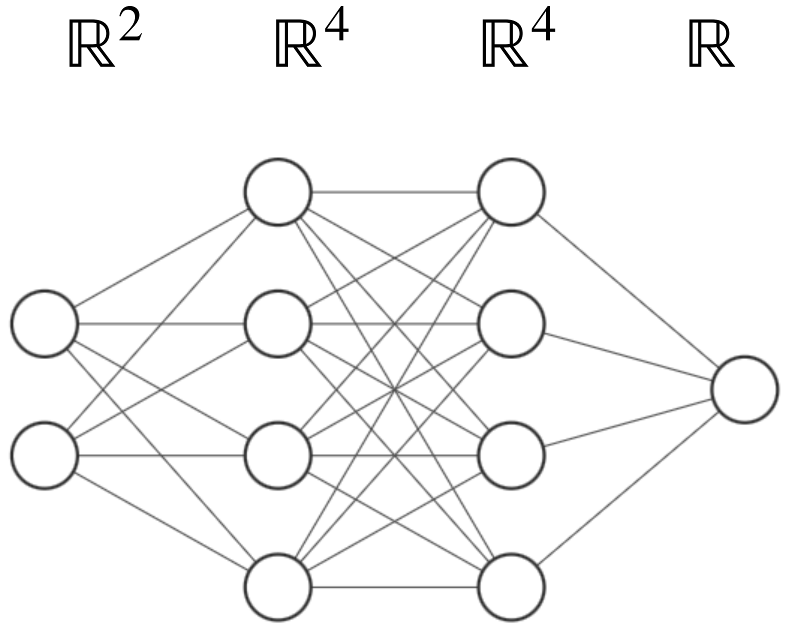

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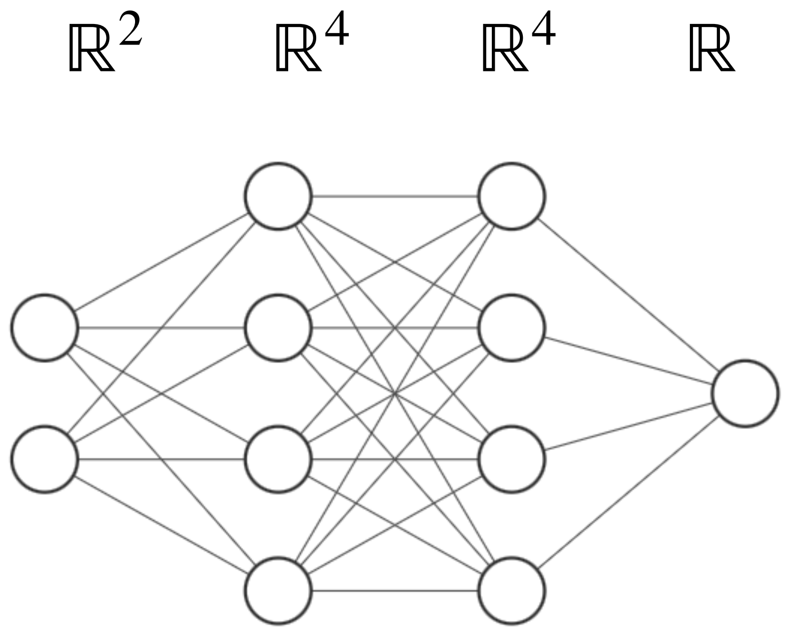

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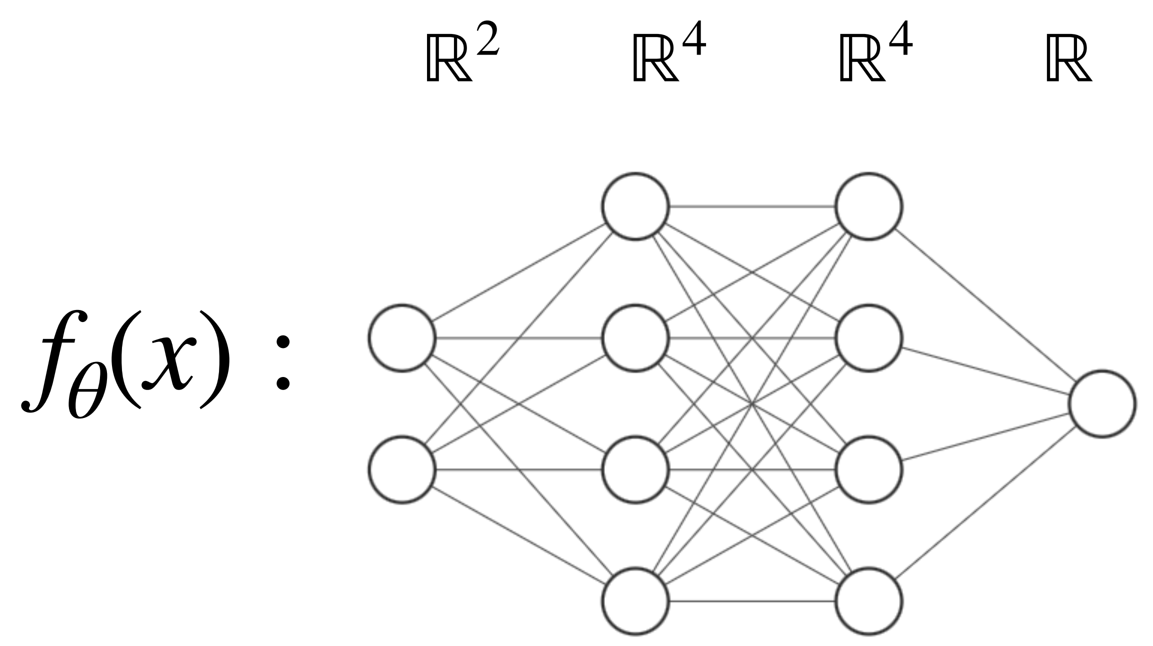

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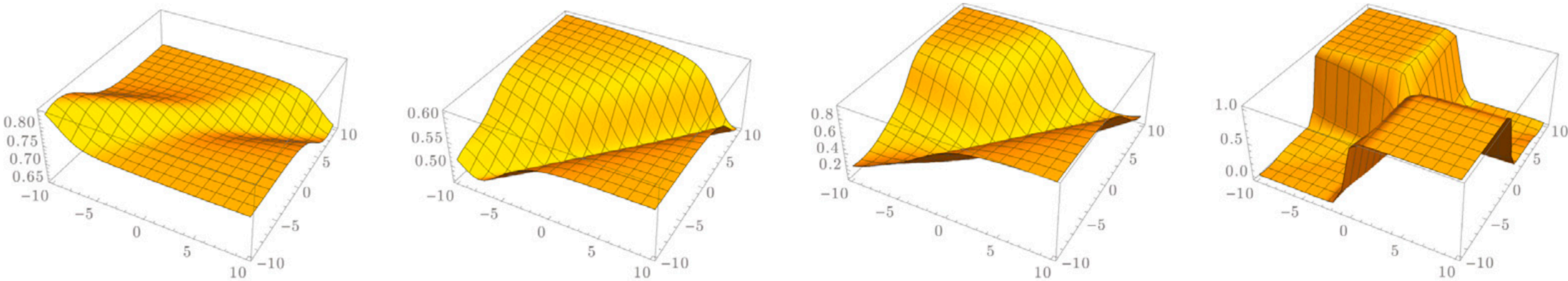
NN output after 0, 100, 200, 1000 training steps



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‘Loss functionals’ (to minimise)

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(real) correction to metric: $\phi_\theta : \mathbb{R}^m \to \mathbb{R}$

$$g_{CY,a\bar{b}} = g_{FS,a\bar{b}} + \partial_a \partial_{\bar{b}} \phi, \qquad \mathscr{L}_{\mathrm{MA}} \sim \left| 1 - \frac{1}{\kappa} \frac{\det g}{\Omega \wedge \bar{\Omega}} \right|_p$$

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$H^E = e^\beta H^E_{FS}$, solve $\Delta\beta = \rho_\beta$, $\qquad \mathscr{L}_{\text{HYM}} \sim \left| \Delta\beta - \rho_\beta \right|_p$

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($\bar{\partial}$-exact, complex) correction to one-form: σ bundle-valued

$$\hat{\sigma}_\theta : \mathbb{R}^m \to \mathbb{C}, \ \nu = \nu_{ref} + \bar{\partial}_{L_i} \sigma_\theta, \quad \mathscr{L}_{\text{one-form}} \sim \left| \Delta_{L_i} \sigma - \rho_\sigma \right|_p$$

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$$H^E = e^\beta H^E_{FS}, \text{ solve } \Delta\beta = \rho_\beta, \quad \mathscr{L}_{\text{HYM}} \sim \left| \Delta\beta - \rho_\beta \right|_p$$

($\bar{\partial}$-exact, complex) correction to one-form: σ bundle-valued

$$\hat{\sigma}_\theta : \mathbb{R}^m \to \mathbb{C}, \; \nu = \nu_{ref} + \bar{\partial}_{L_i} \sigma_\theta, \quad \mathscr{L}_{\text{one-form}} \sim \left| \Delta_{L_i} \sigma - \rho_\sigma \right|_p$$

design networks ϕ, β, σ that are ***by construction* good functions/sections** on your manifold

‘Loss functionals’ (to minimise)

(real) correction to metric: $\phi_\theta : \mathbb{R}^m \to \mathbb{R}$

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Good measure of success?

$$M_{MA}(\phi) = \int_X \left| 1 - \frac{\det g}{\Omega \wedge \Omega} \right|$$

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$$M_L(\beta \text{ or } \sigma) = \frac{\int_X \left| \Delta\beta - \rho_\beta \right|}{\int_X \left| \rho_\beta \right|}$$

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String model

String model

X : (smooth quotient of) **"tetraquadric"**, hypersurface in $\mathbb{P}_1 \times \mathbb{P}_1 \times \mathbb{P}_1 \times \mathbb{P}_1$

1-parameter family of polynomials: ψ

$$p = \sum_{\text{even}} x_\alpha^2 y_\beta^2 u_\gamma^2 v_\delta^2 + \psi_0 \sum_{\text{odd}} x_\alpha^2 y_\beta^2 u_\gamma^2 v_\delta^2$$
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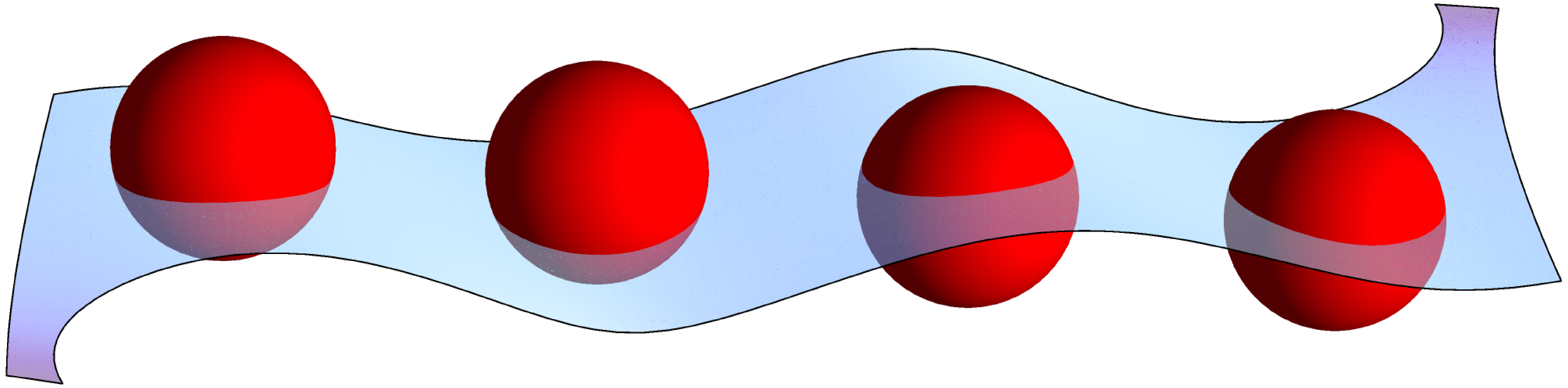

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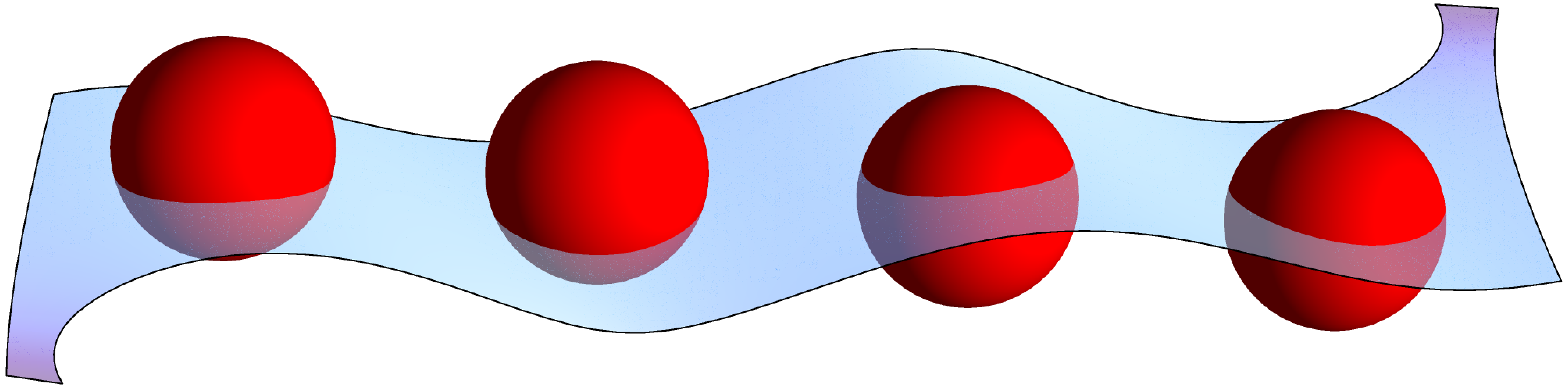

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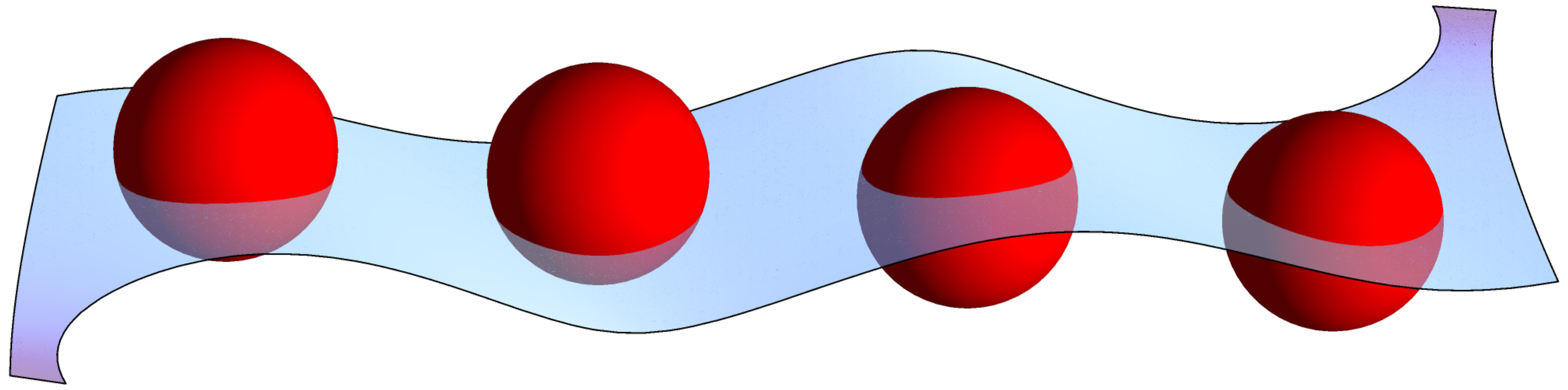

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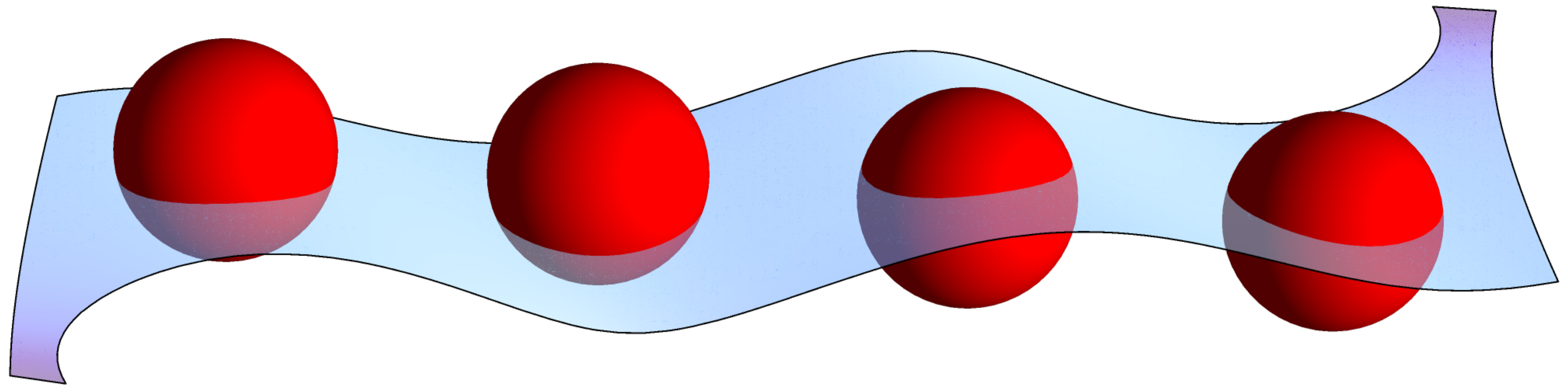

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perturbative operators at dimension-four $\supset$ up-type Yukawa couplings

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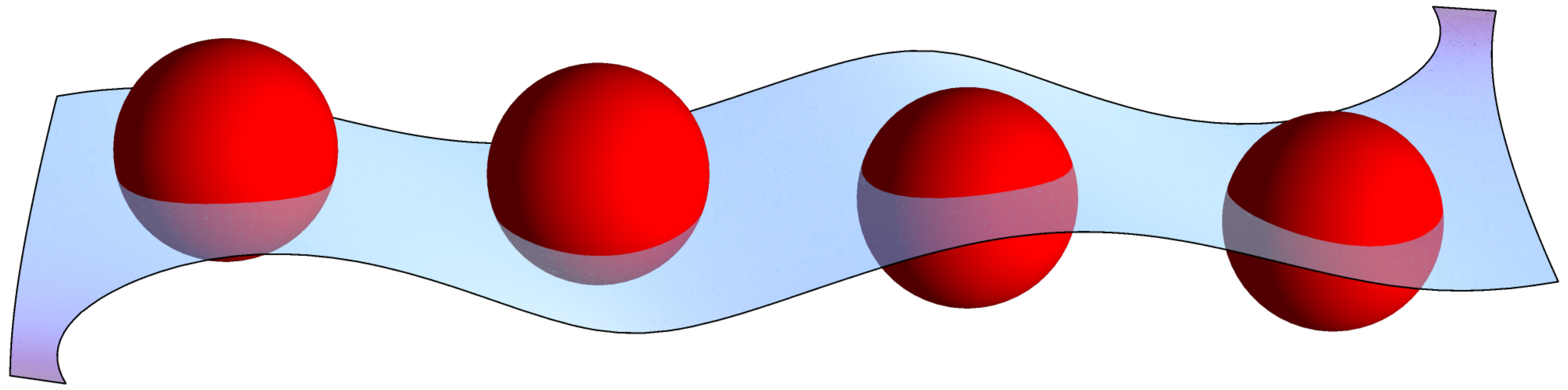

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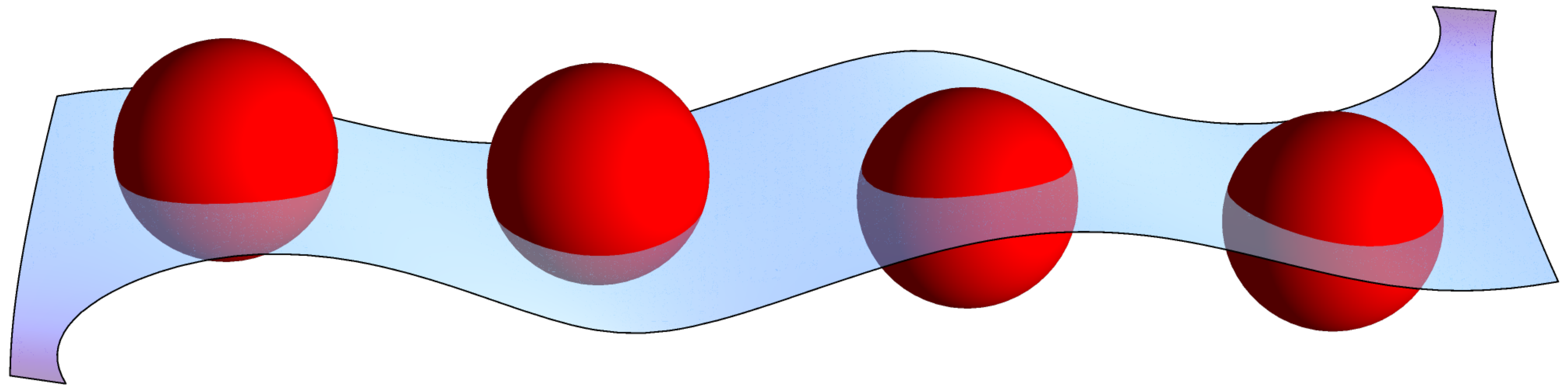

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- 11 feedforward neural networks,
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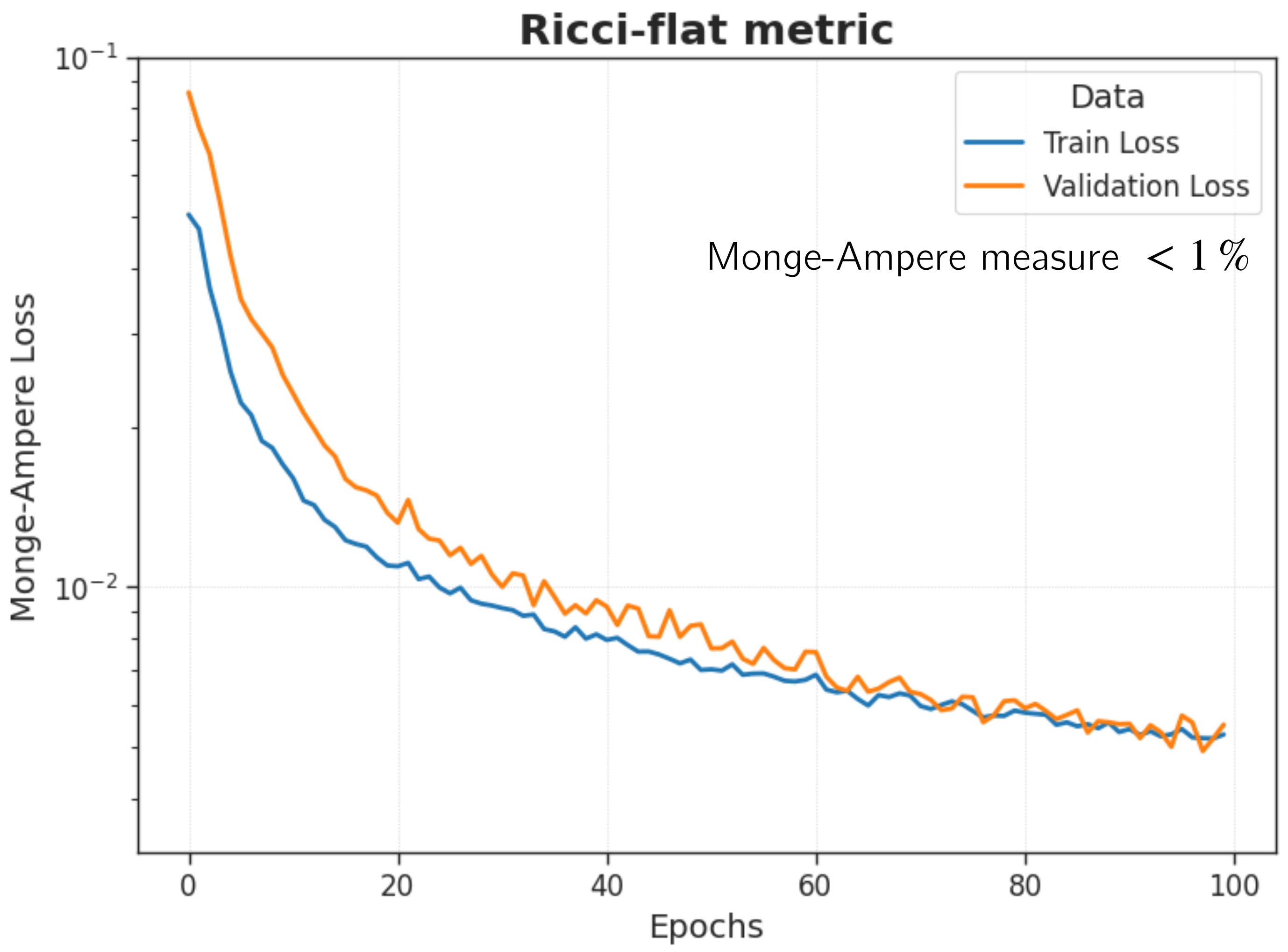

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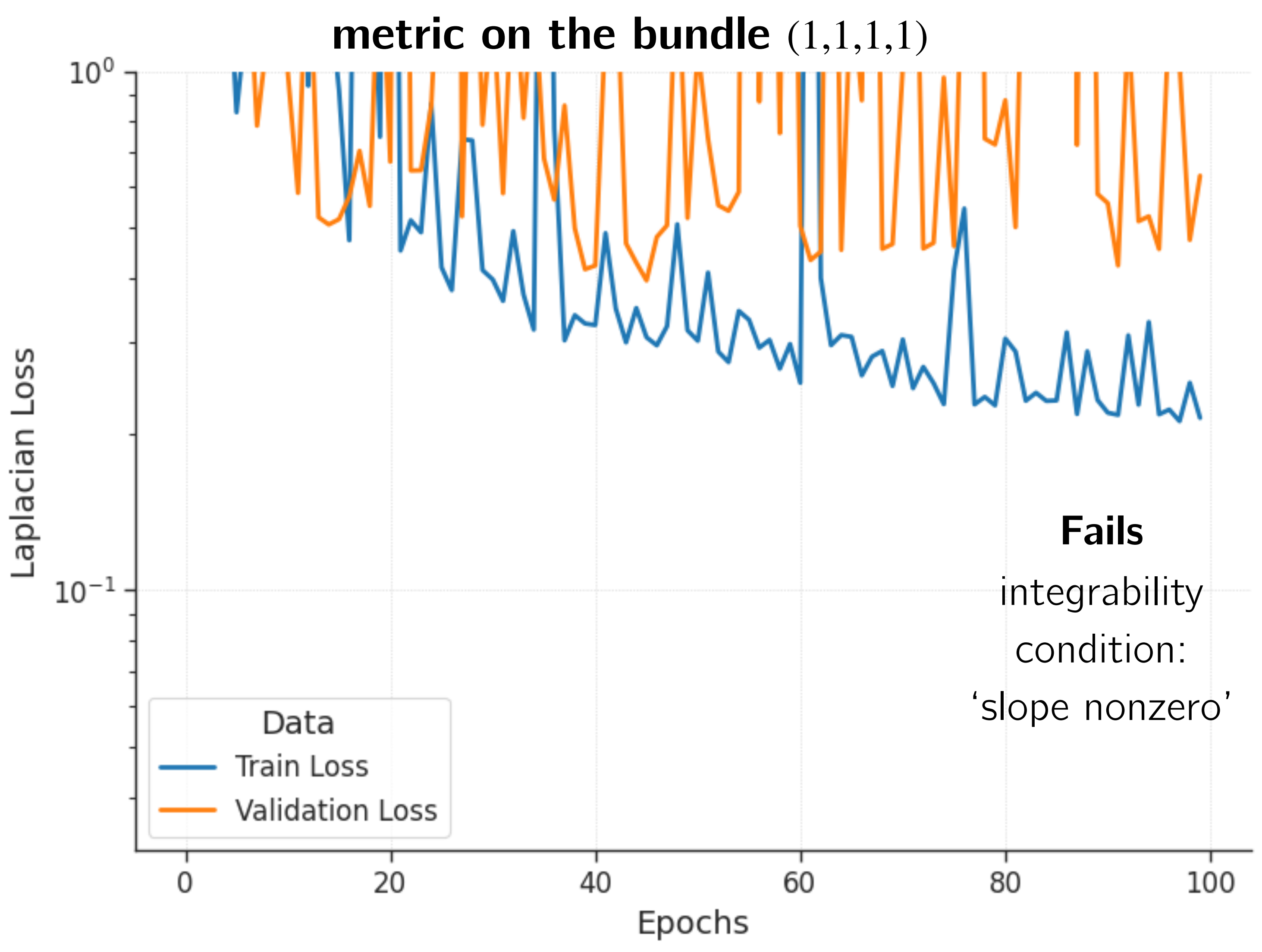

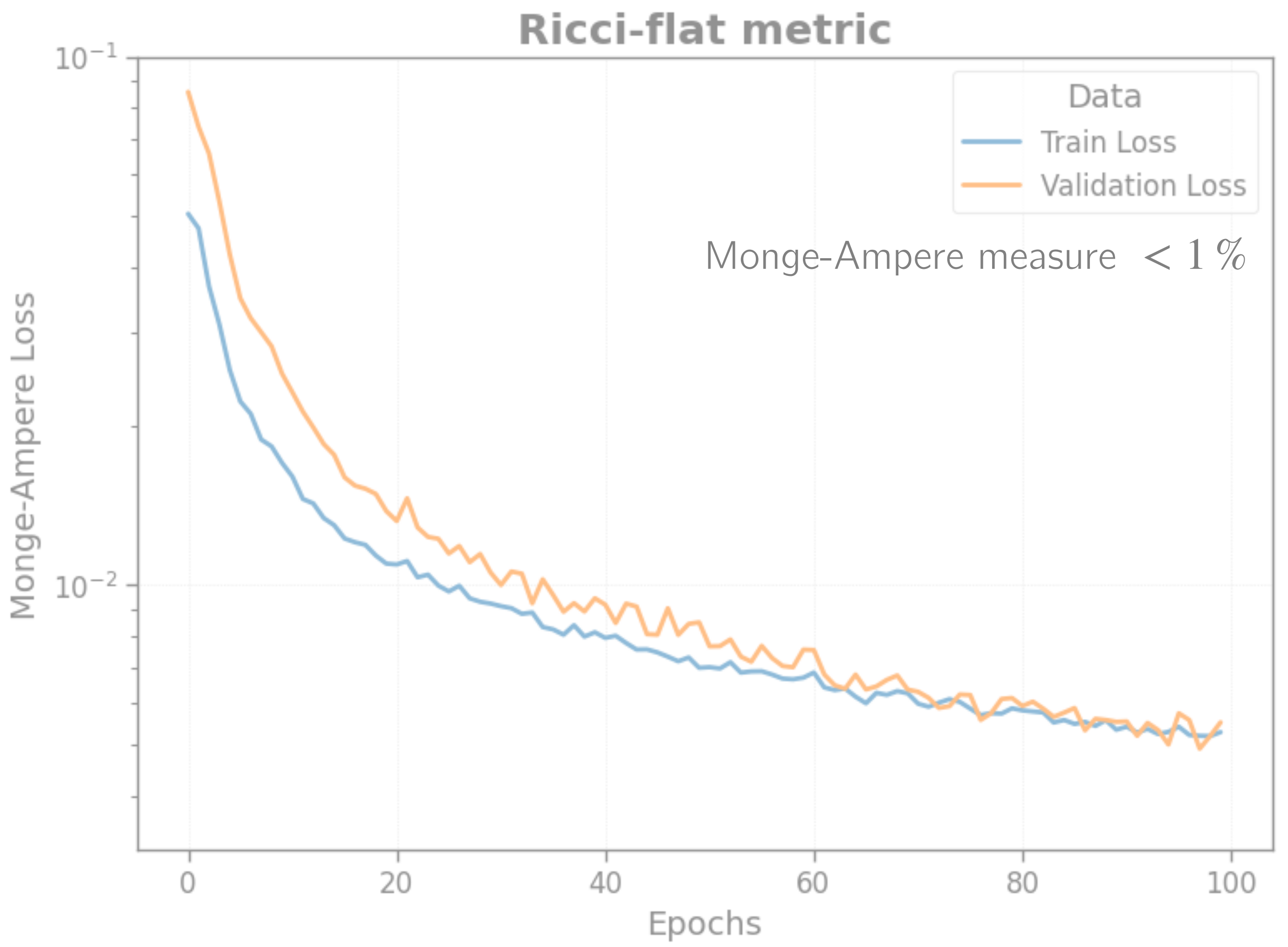

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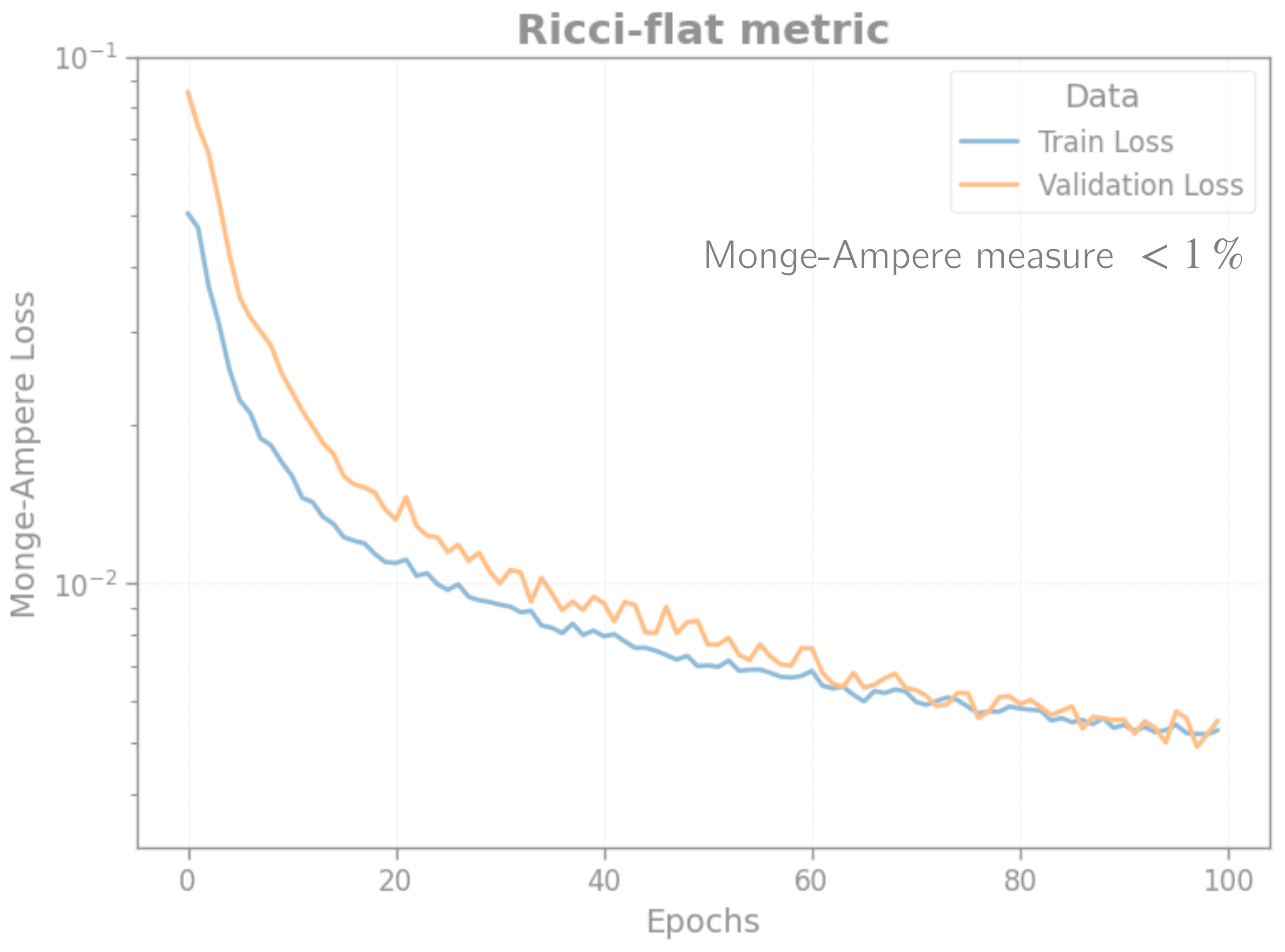

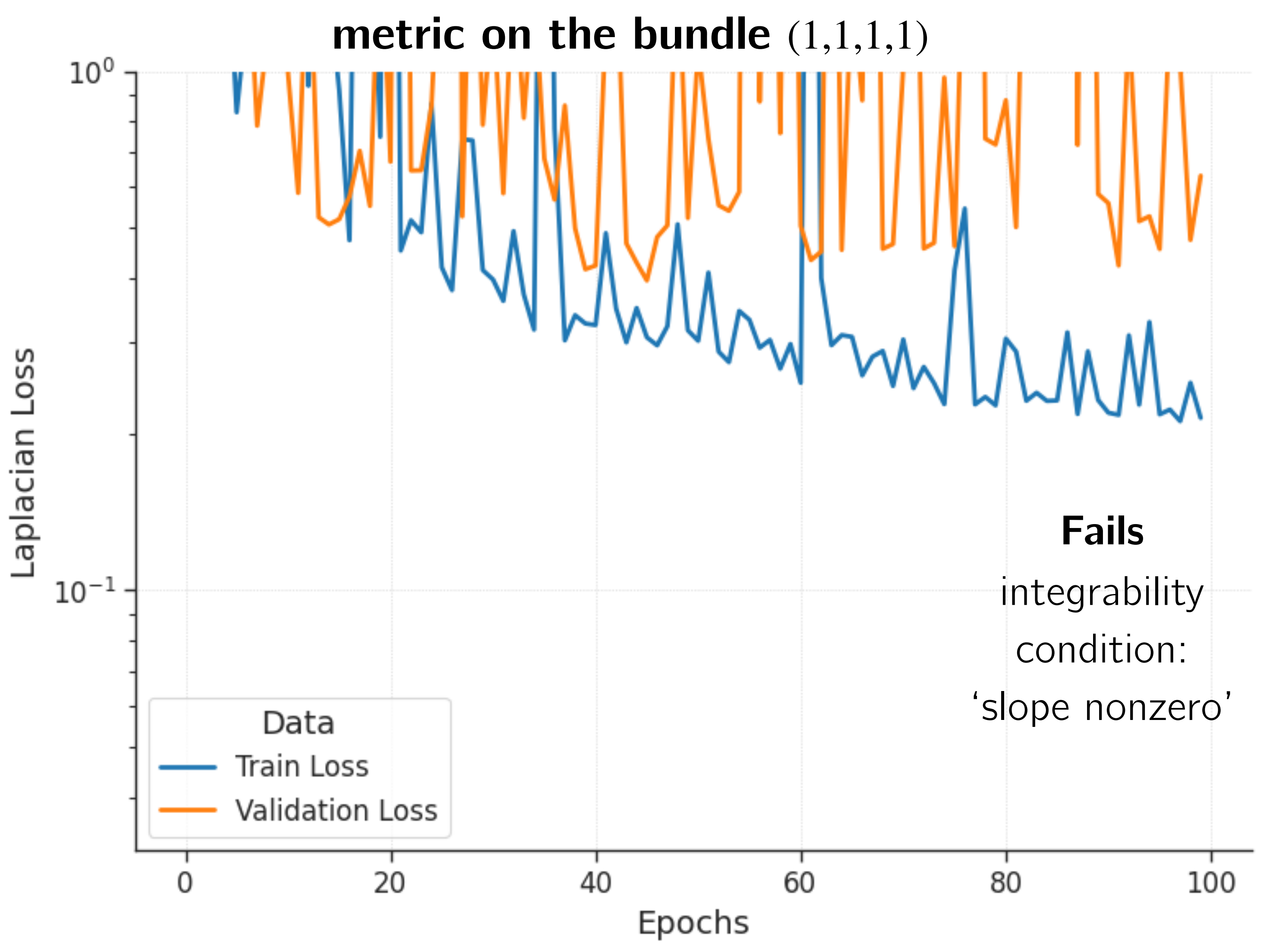

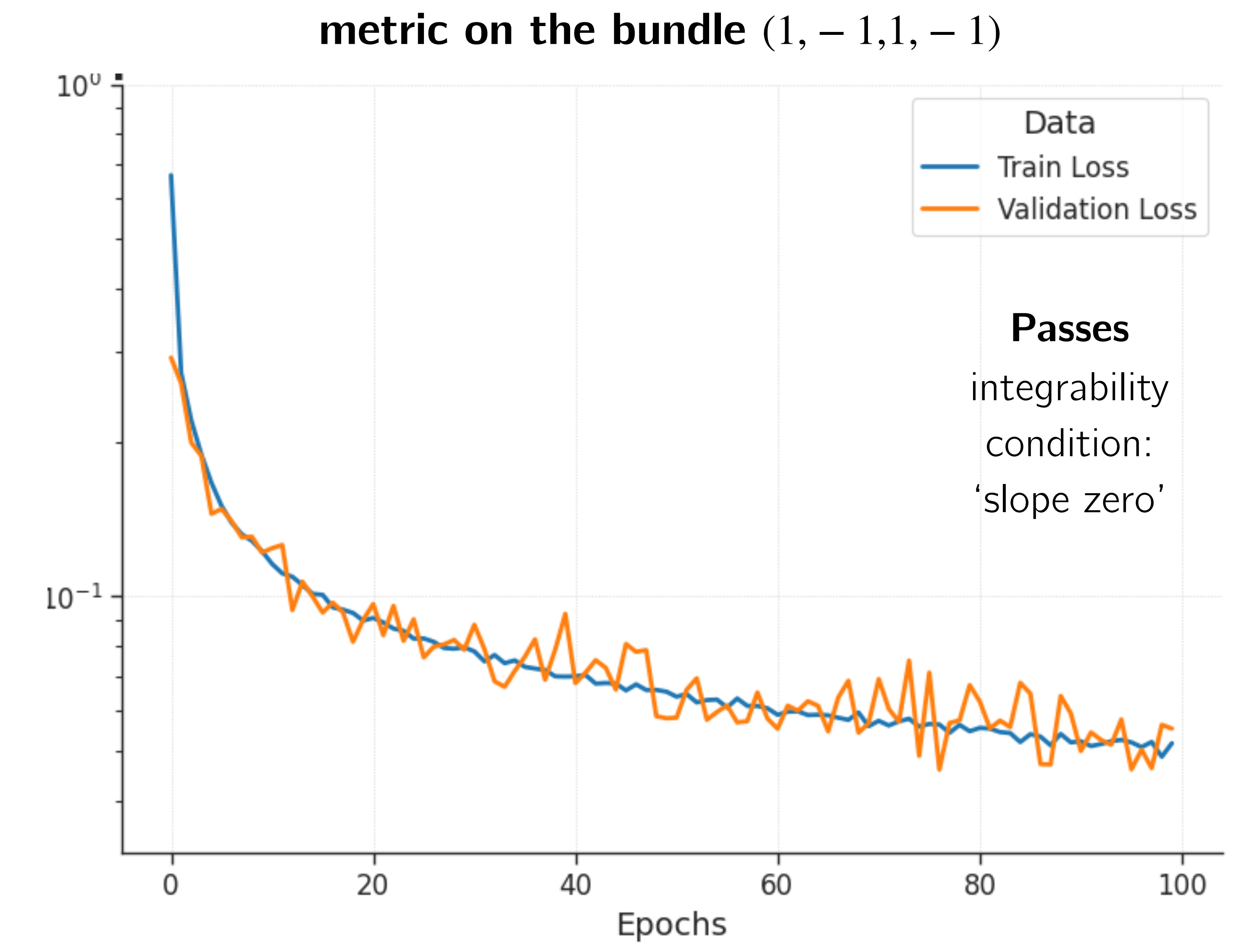

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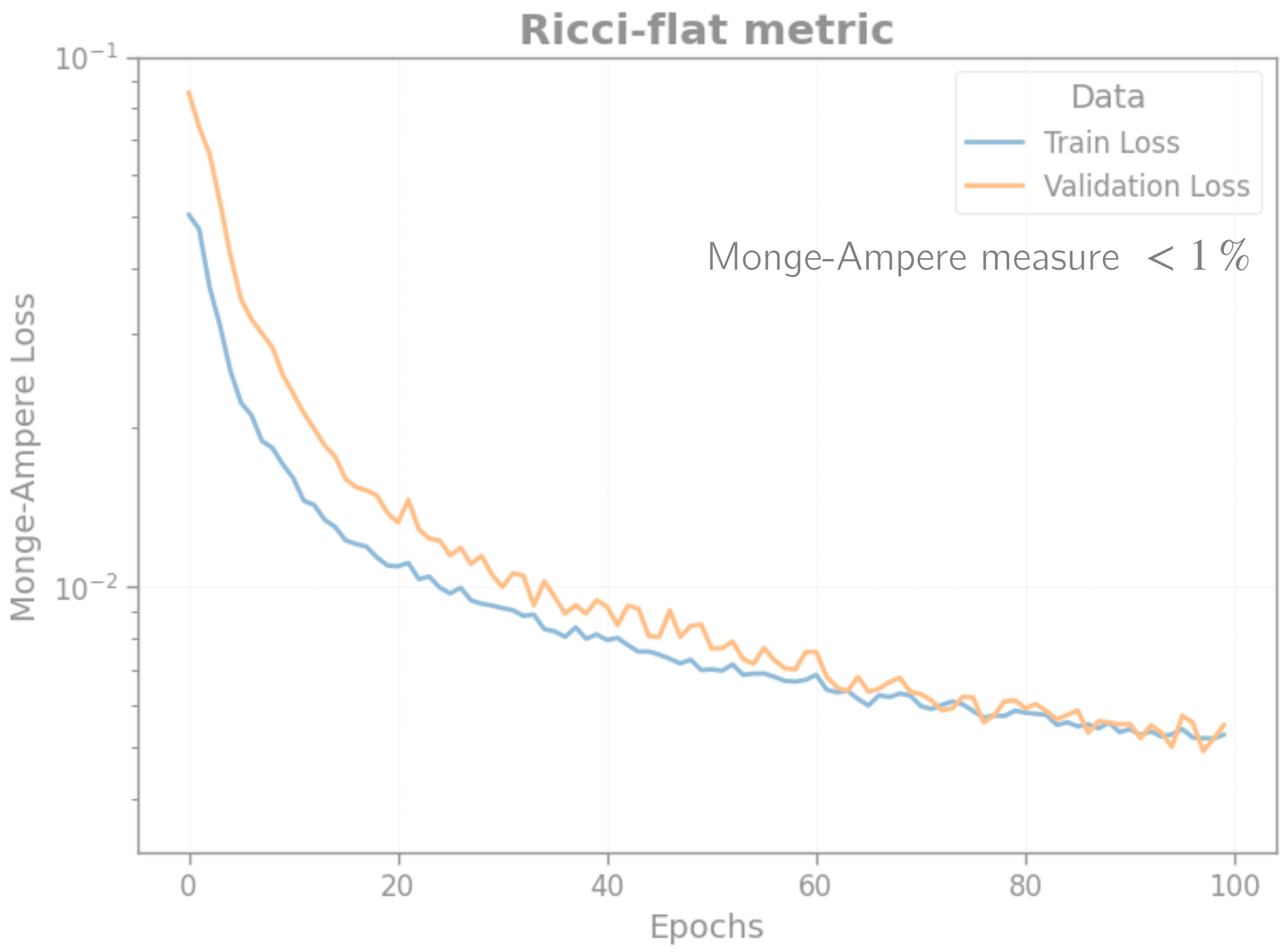
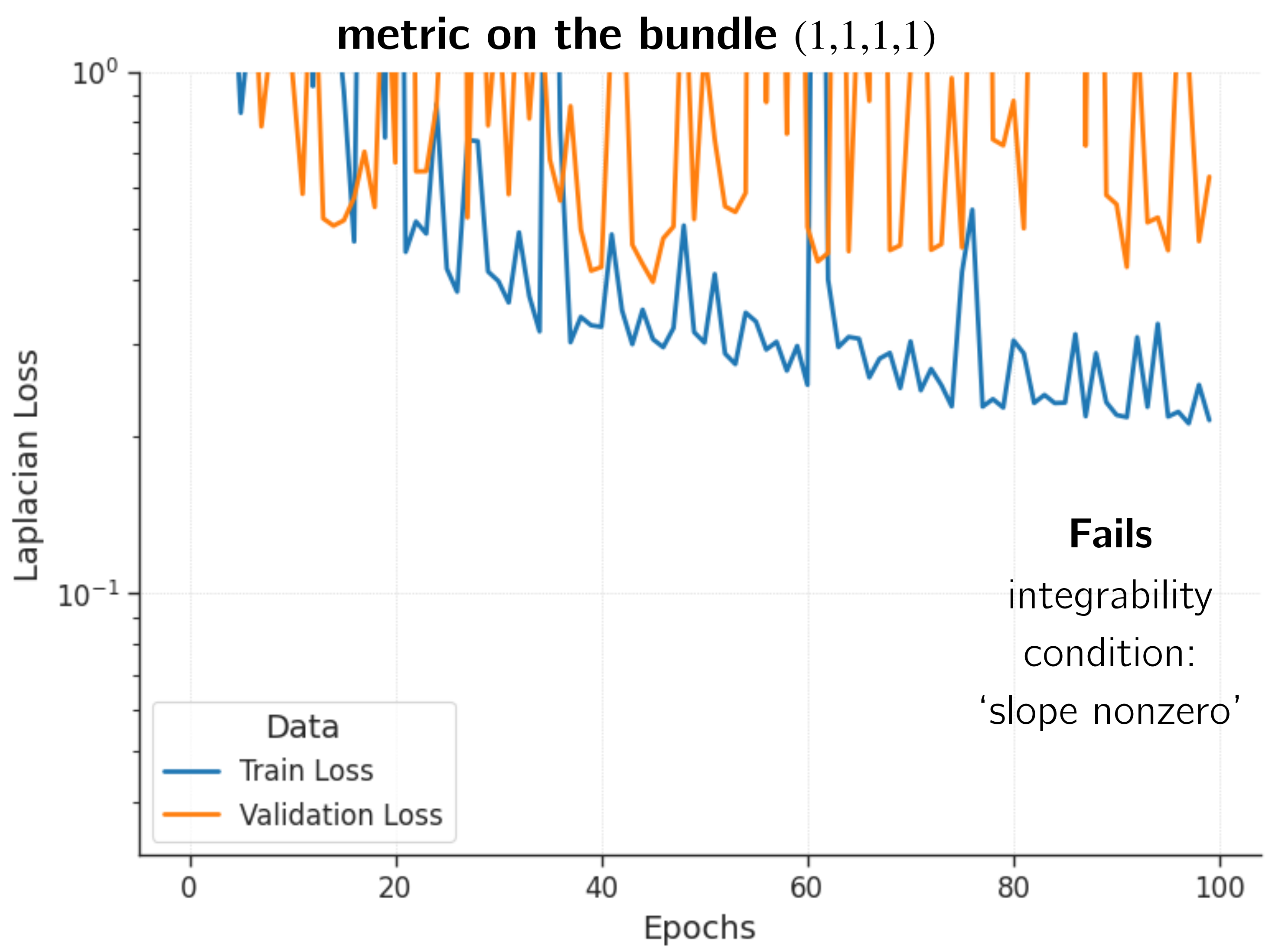
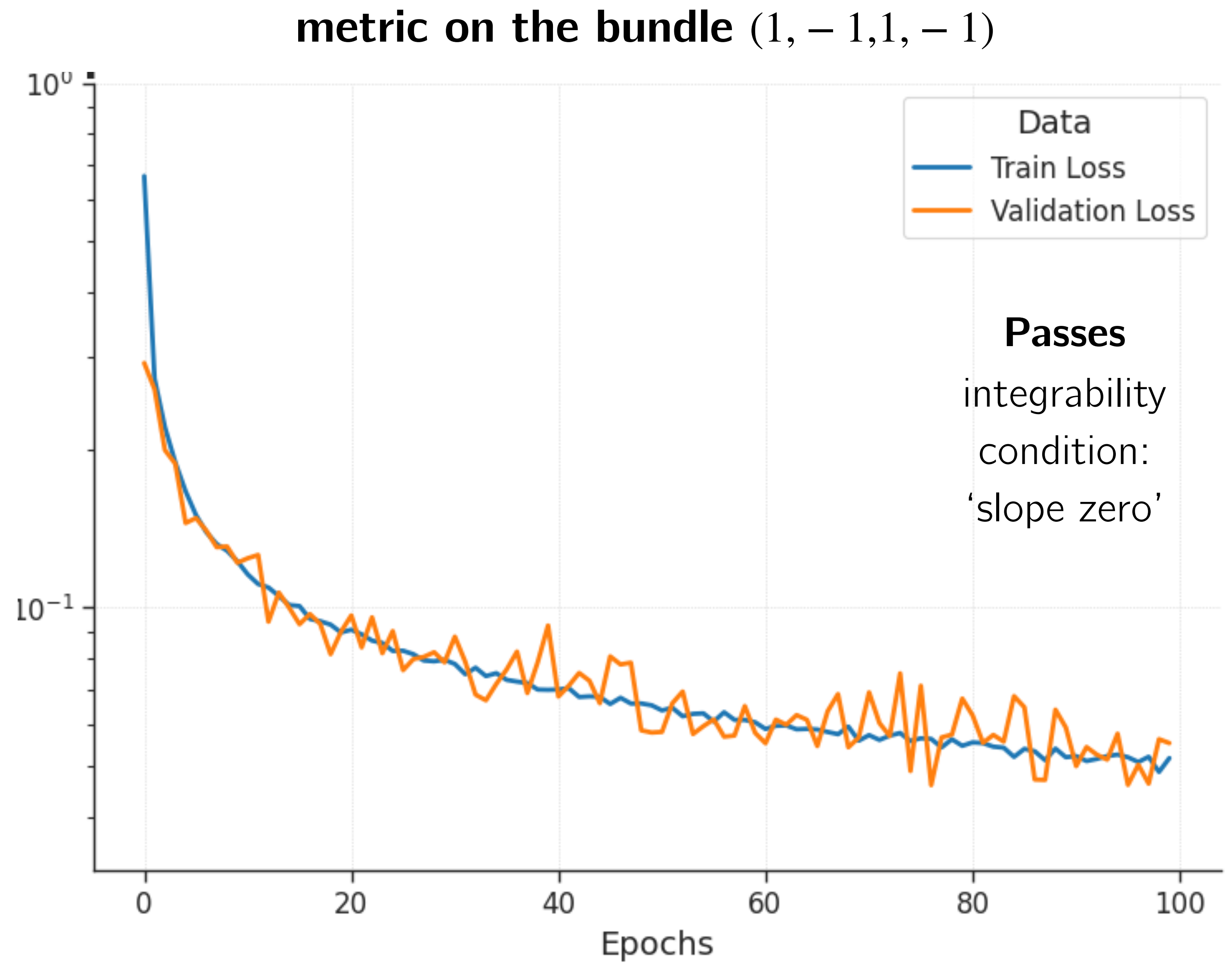


Laplacian measure for each bundle metric

	(0,2,-2,0)	(1,1,0,-2)	(-1,-3,2,2)
$\mathcal{M}_{\text{Laplacian}}(\beta)$	5%	4%	3%





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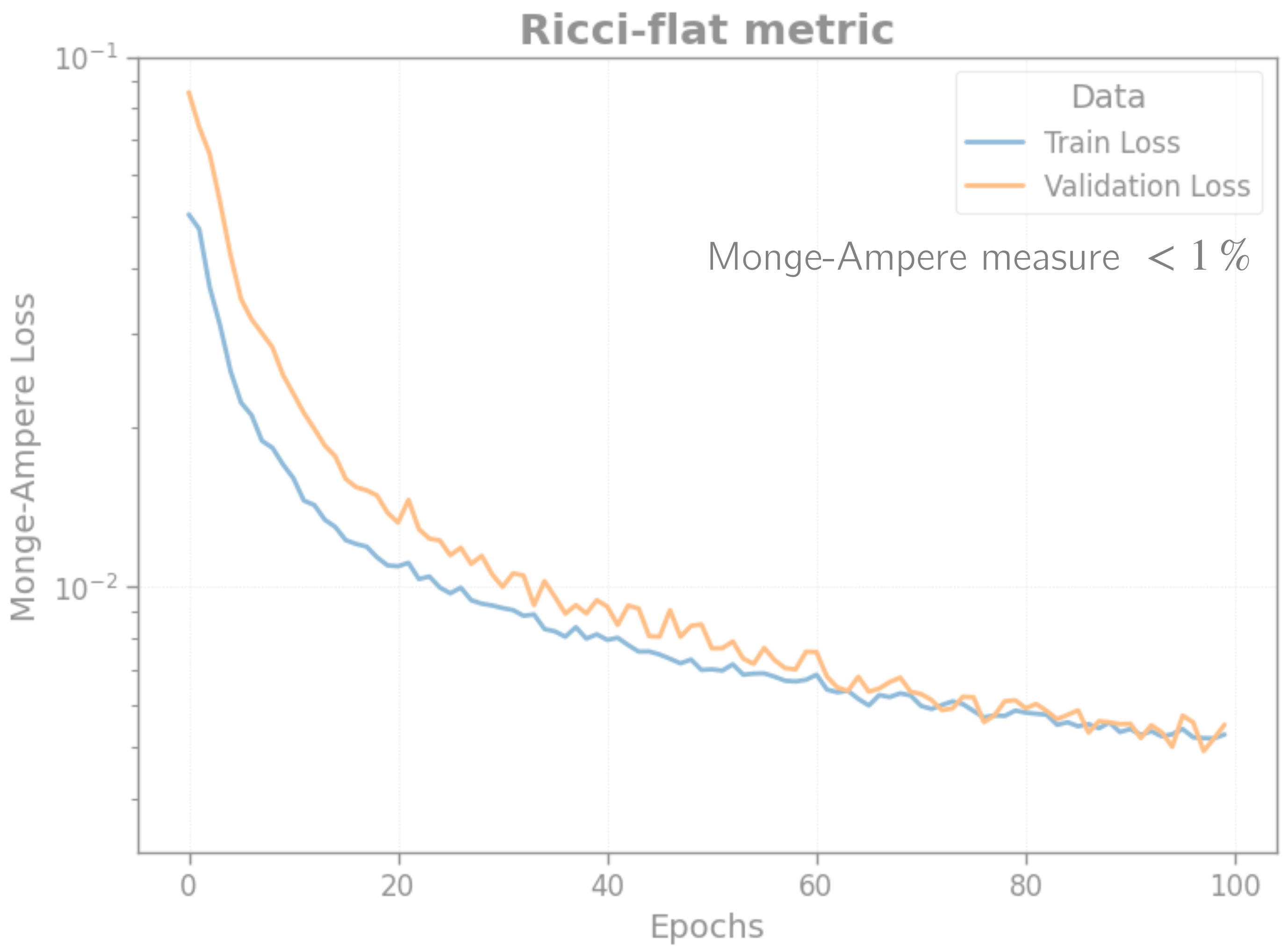

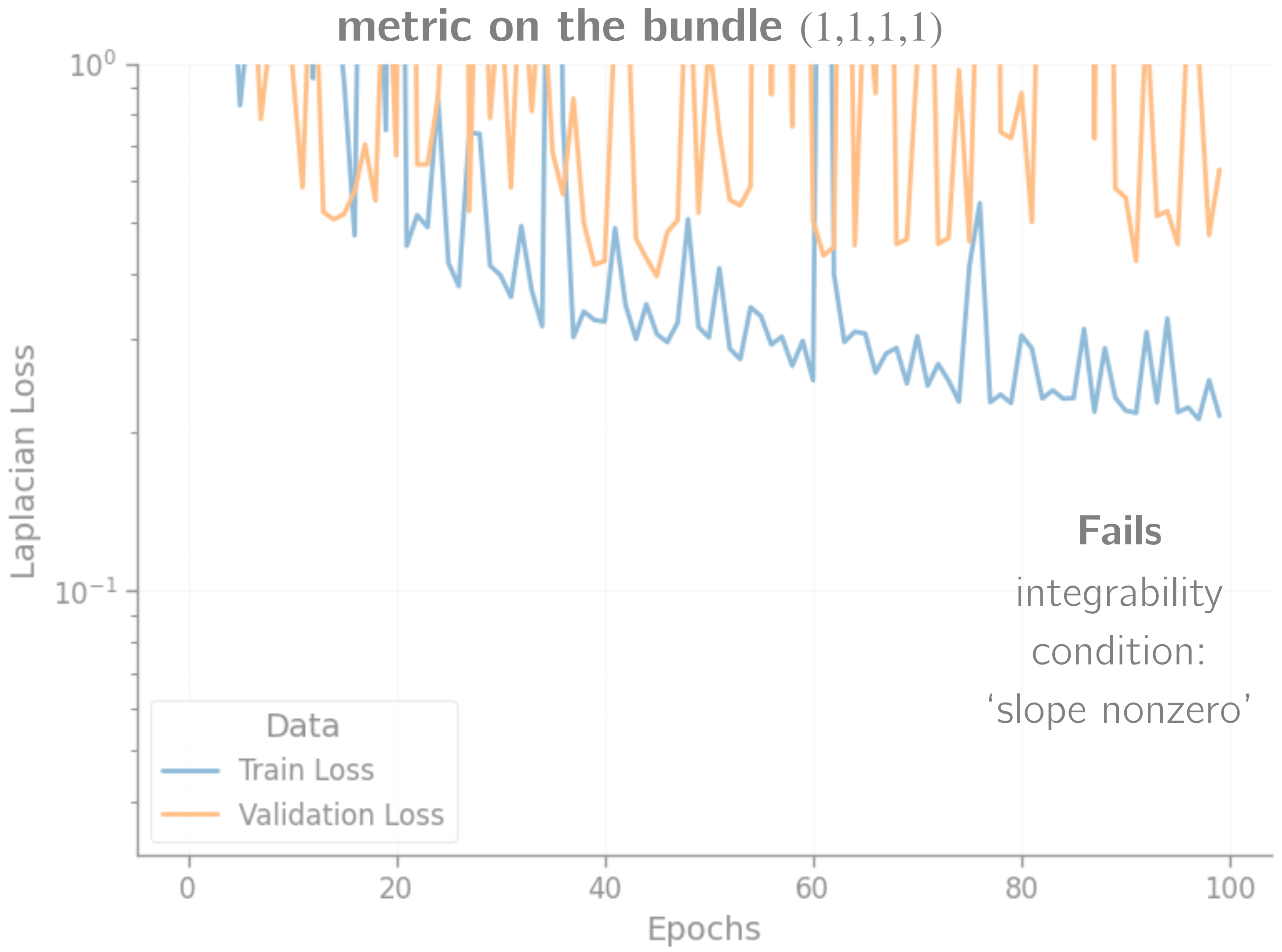

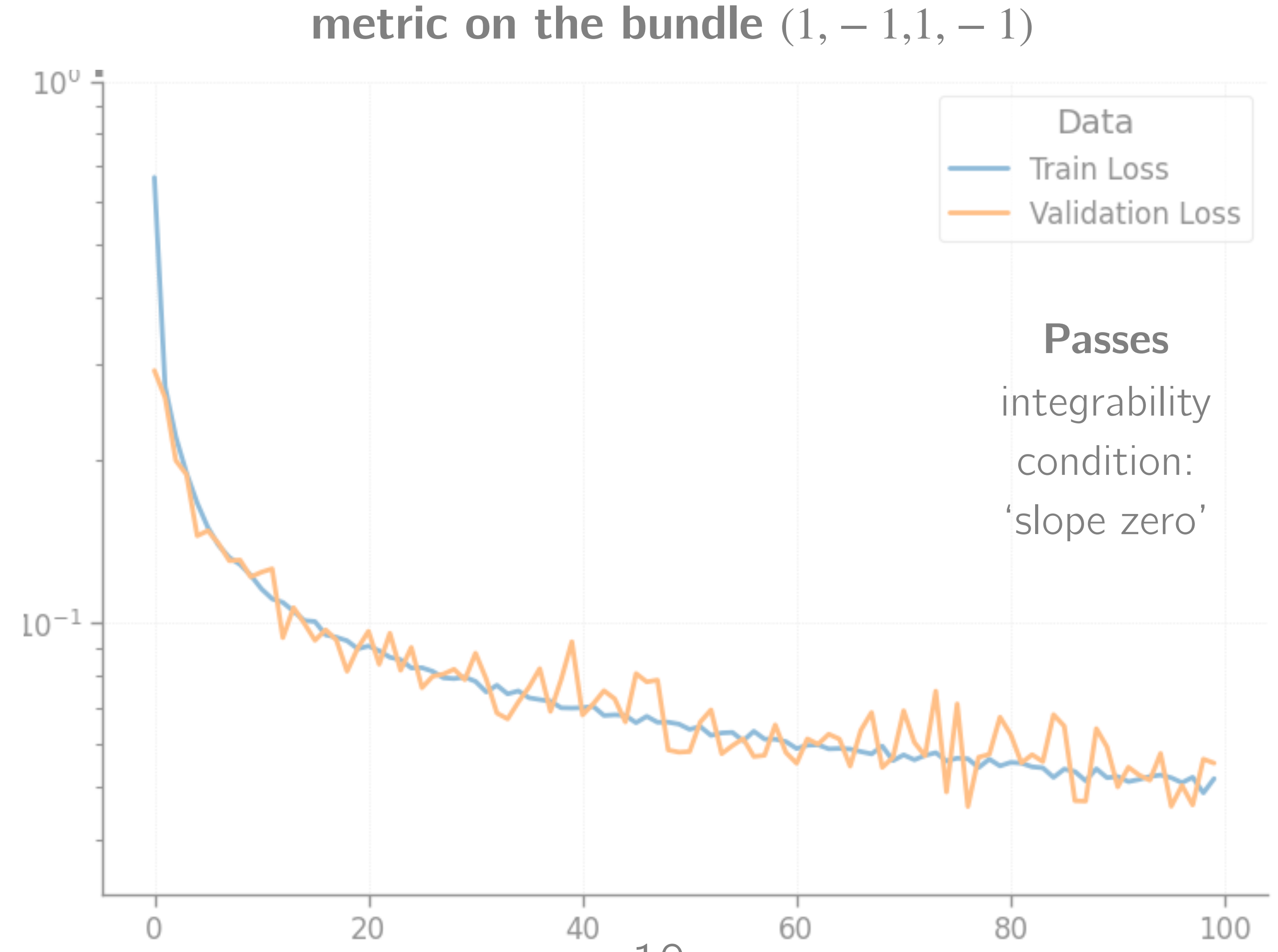

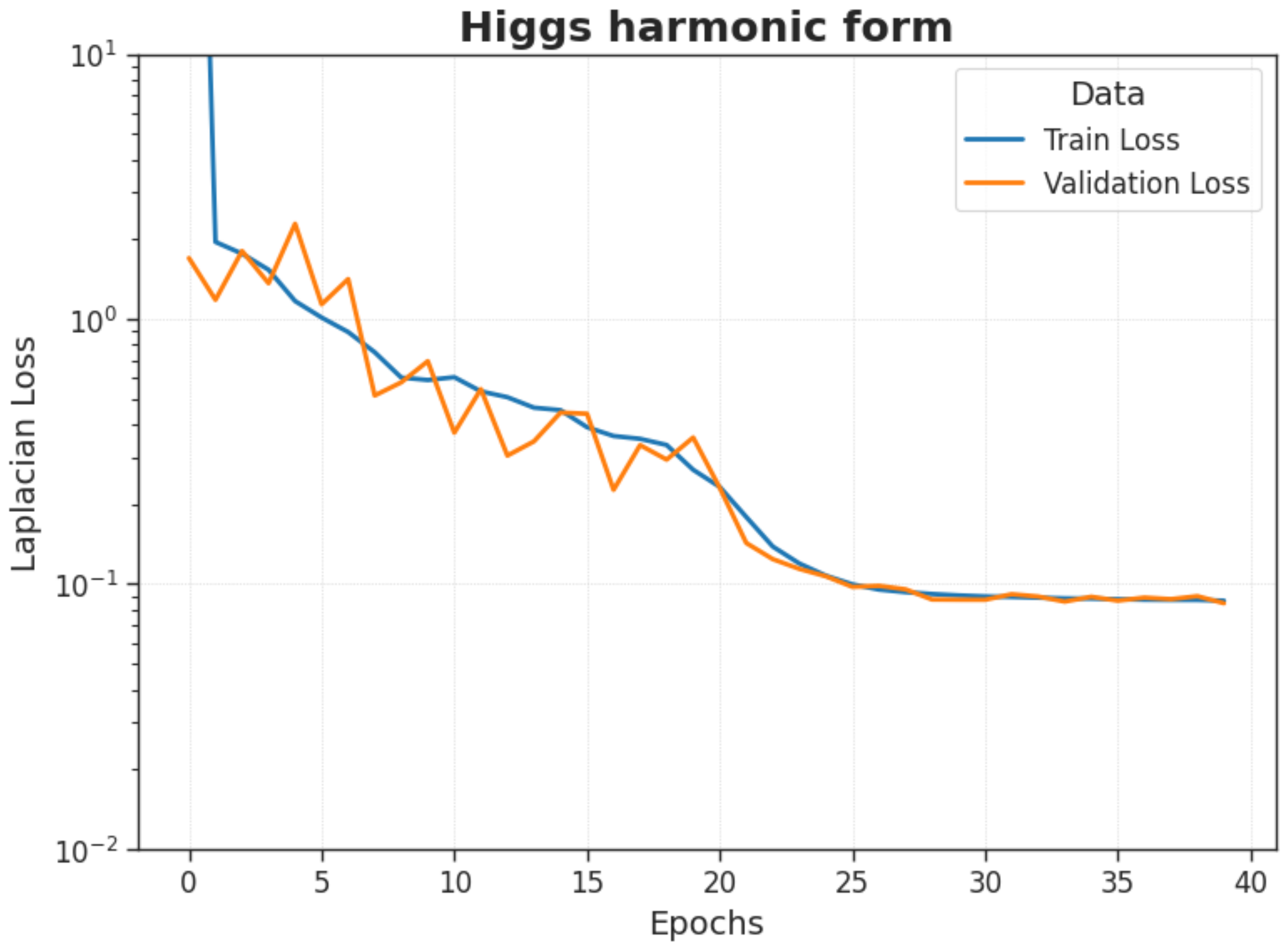

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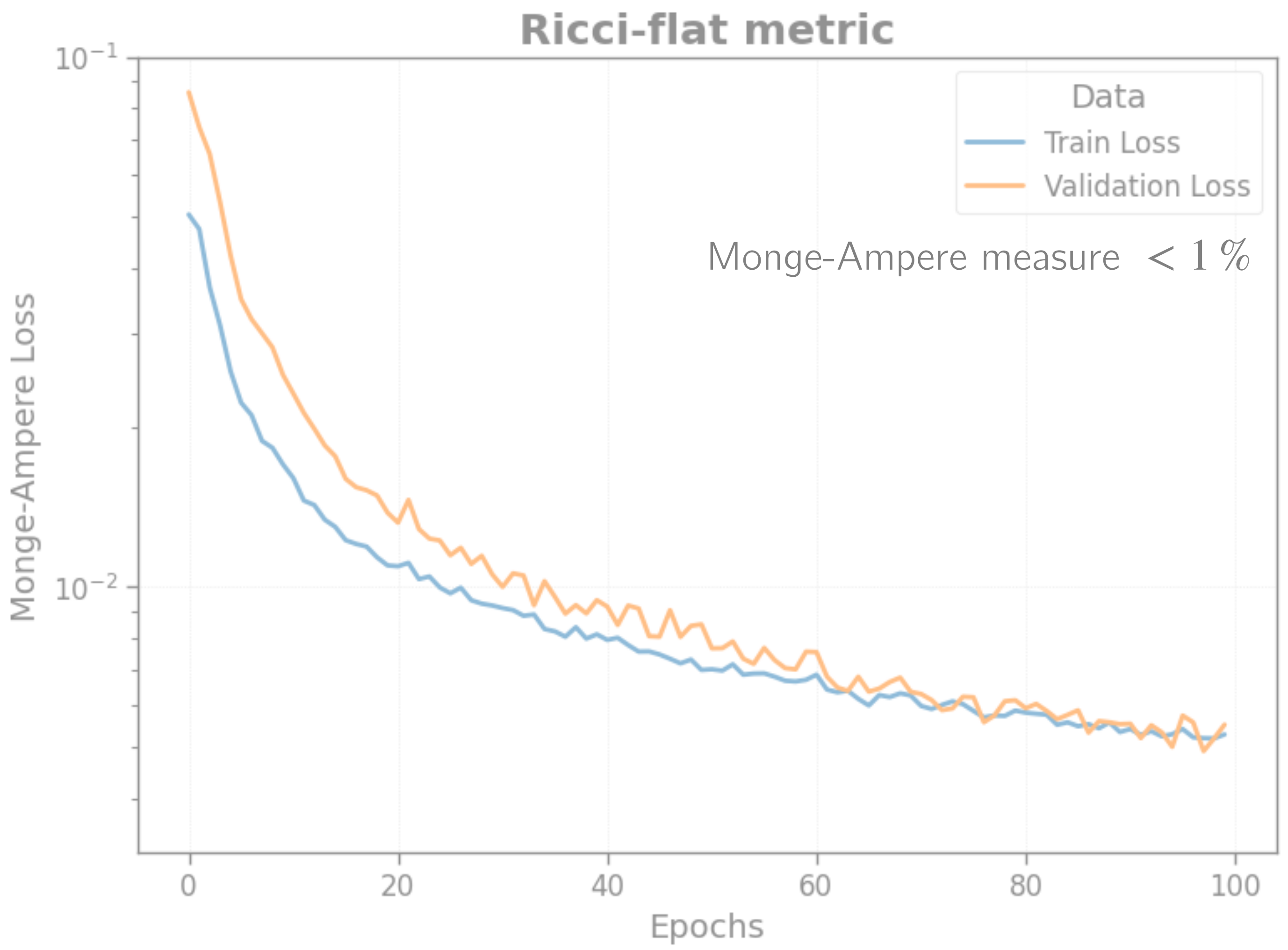

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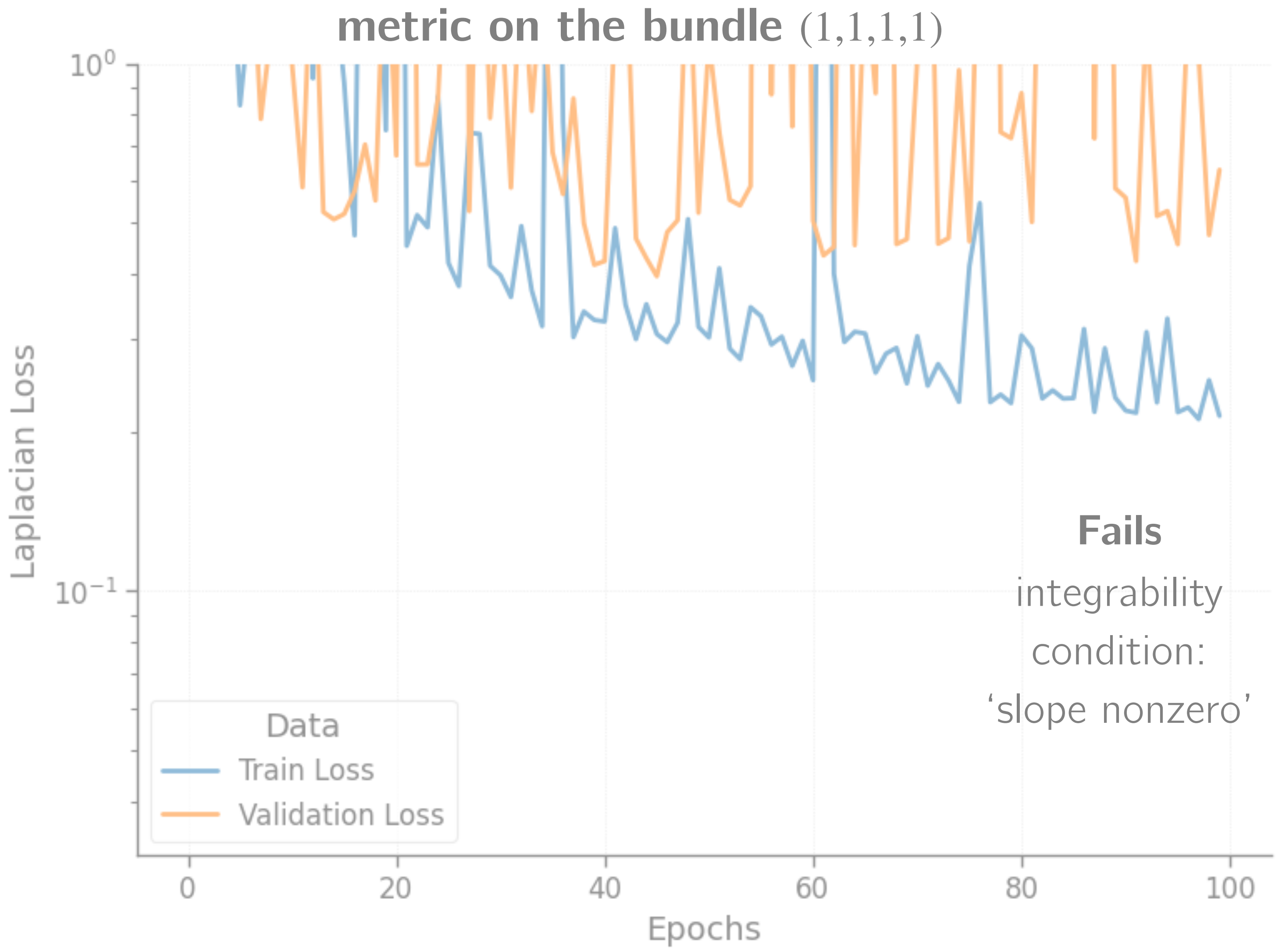

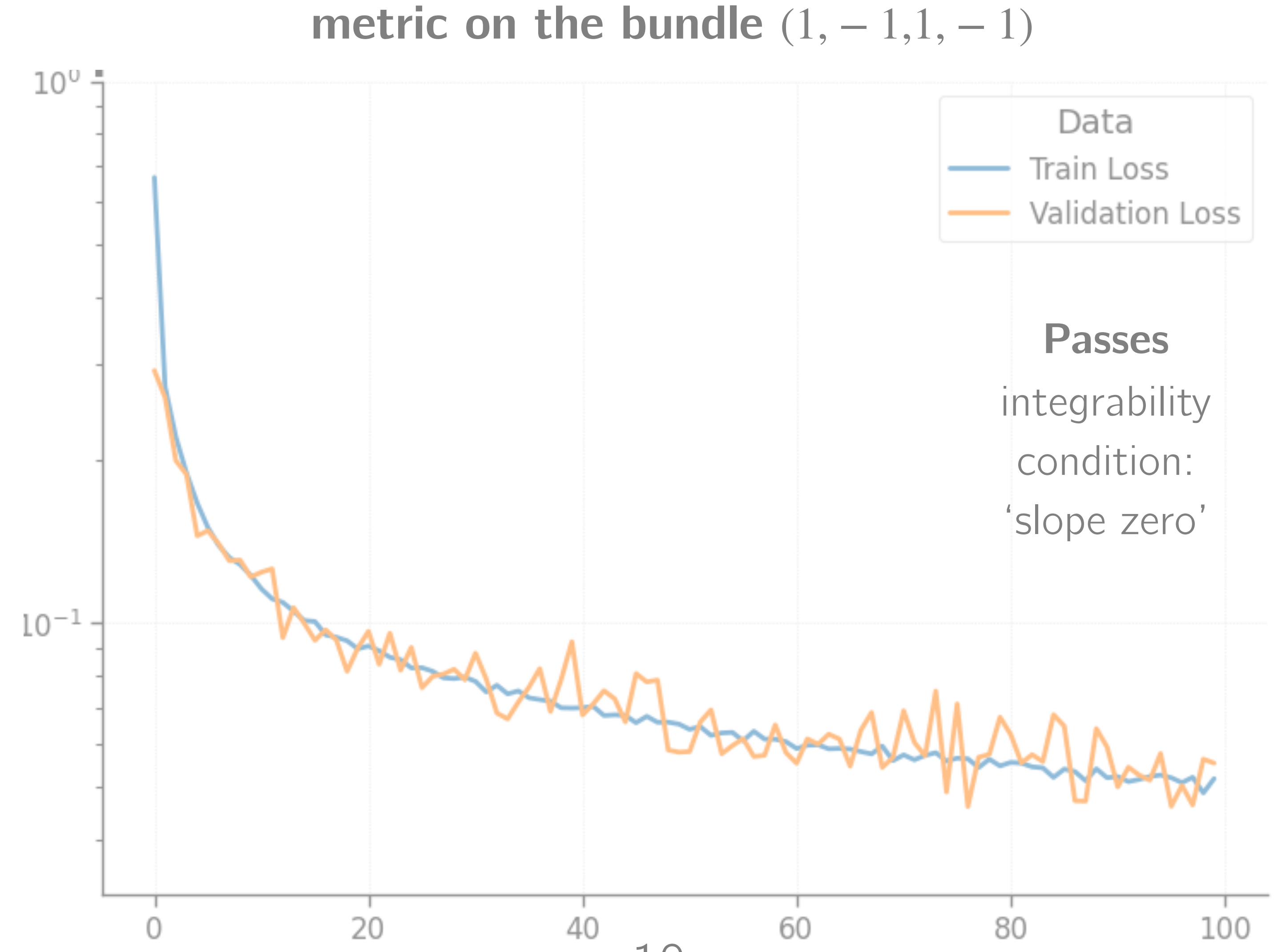

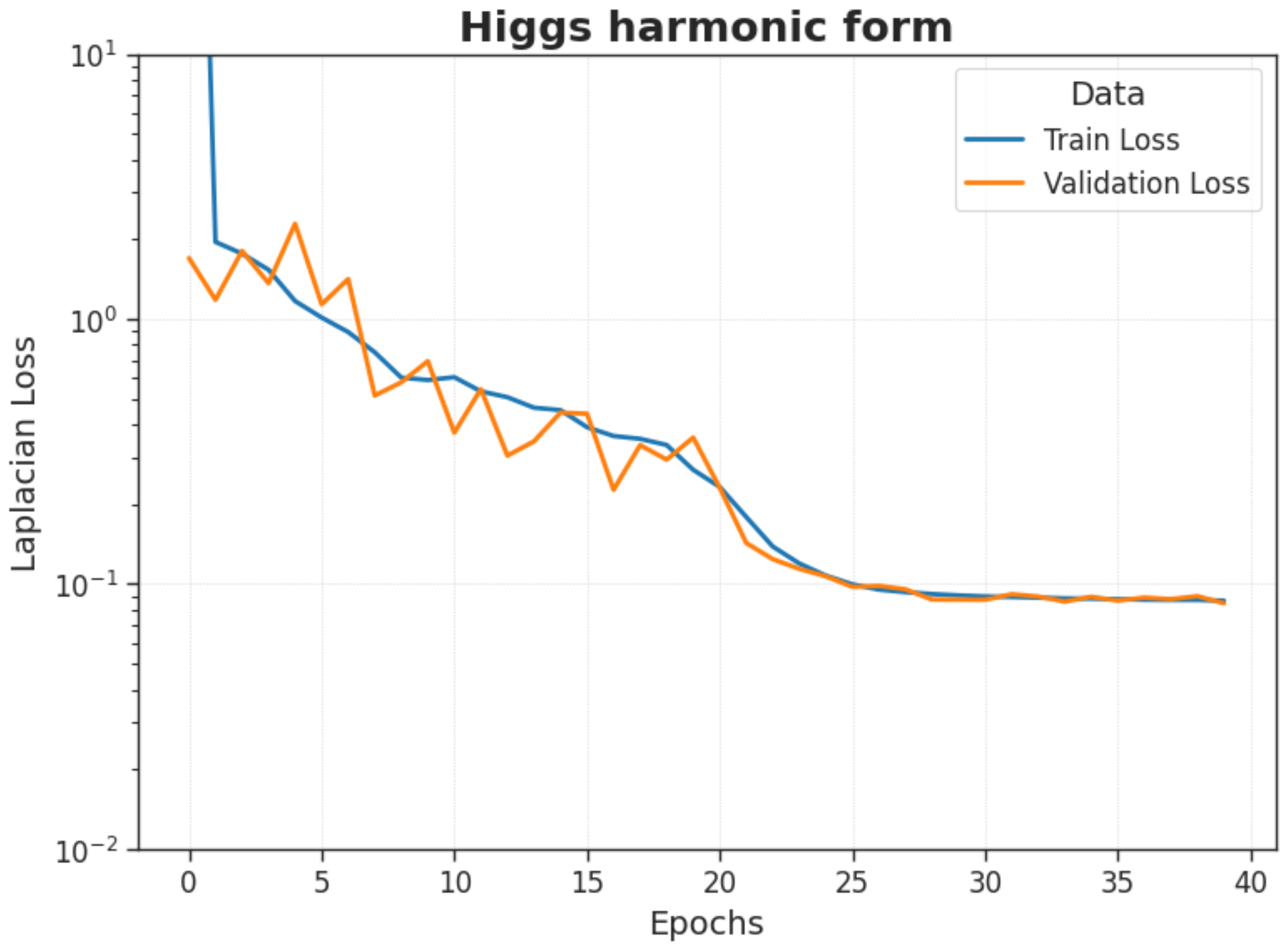

Laplacian measure for each 1-form

	H	Q3	U3	Q1	Q2	Q3	Q4
$M_{\text{Laplacian}}(\beta)$	3%	4%	5%	7%	8%	6%	7%

Masses as a function of moduli

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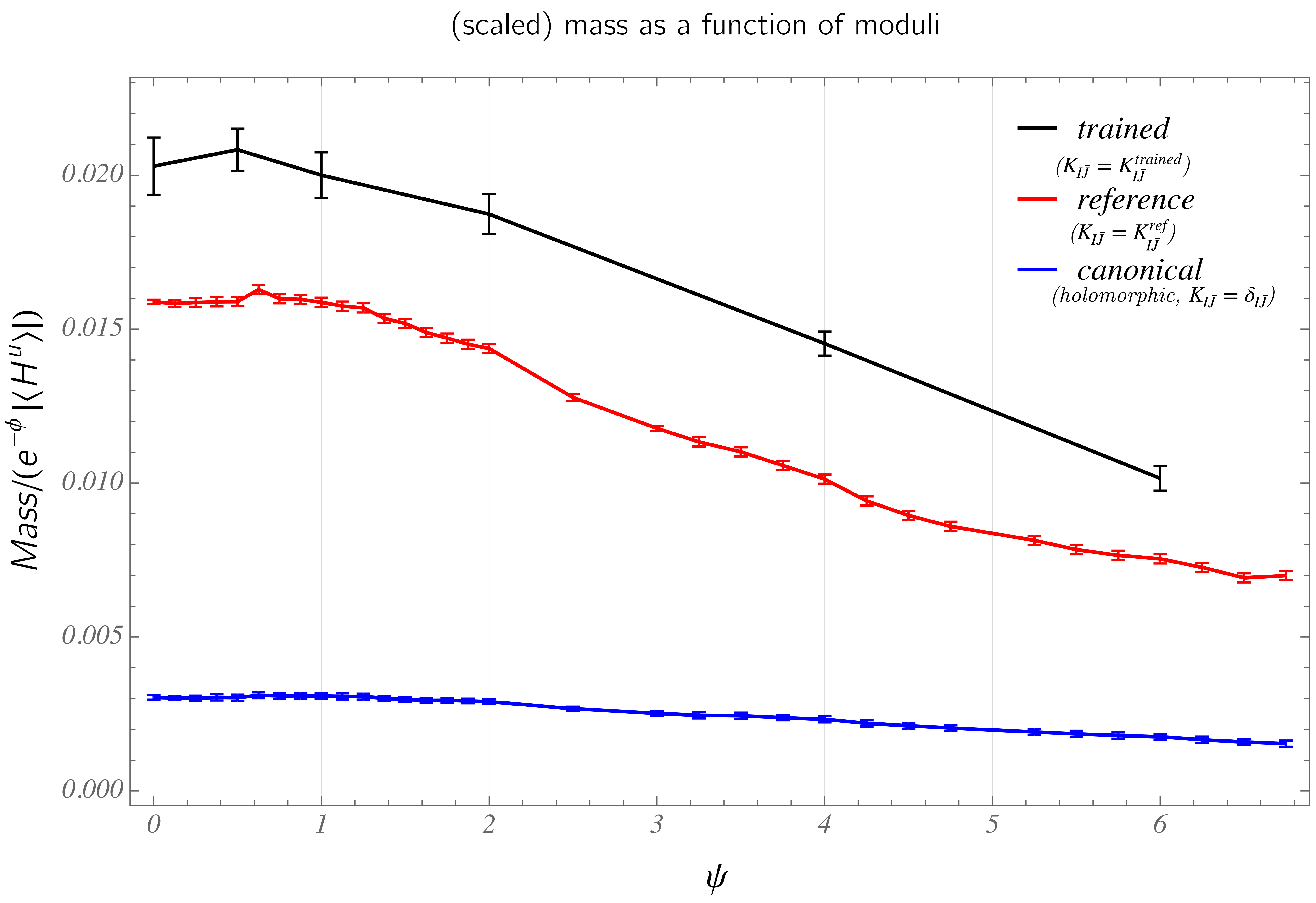

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Two nonzero masses **equal** up to numerical error

- *general polynomial* ***breaks*** *mass degeneracy*
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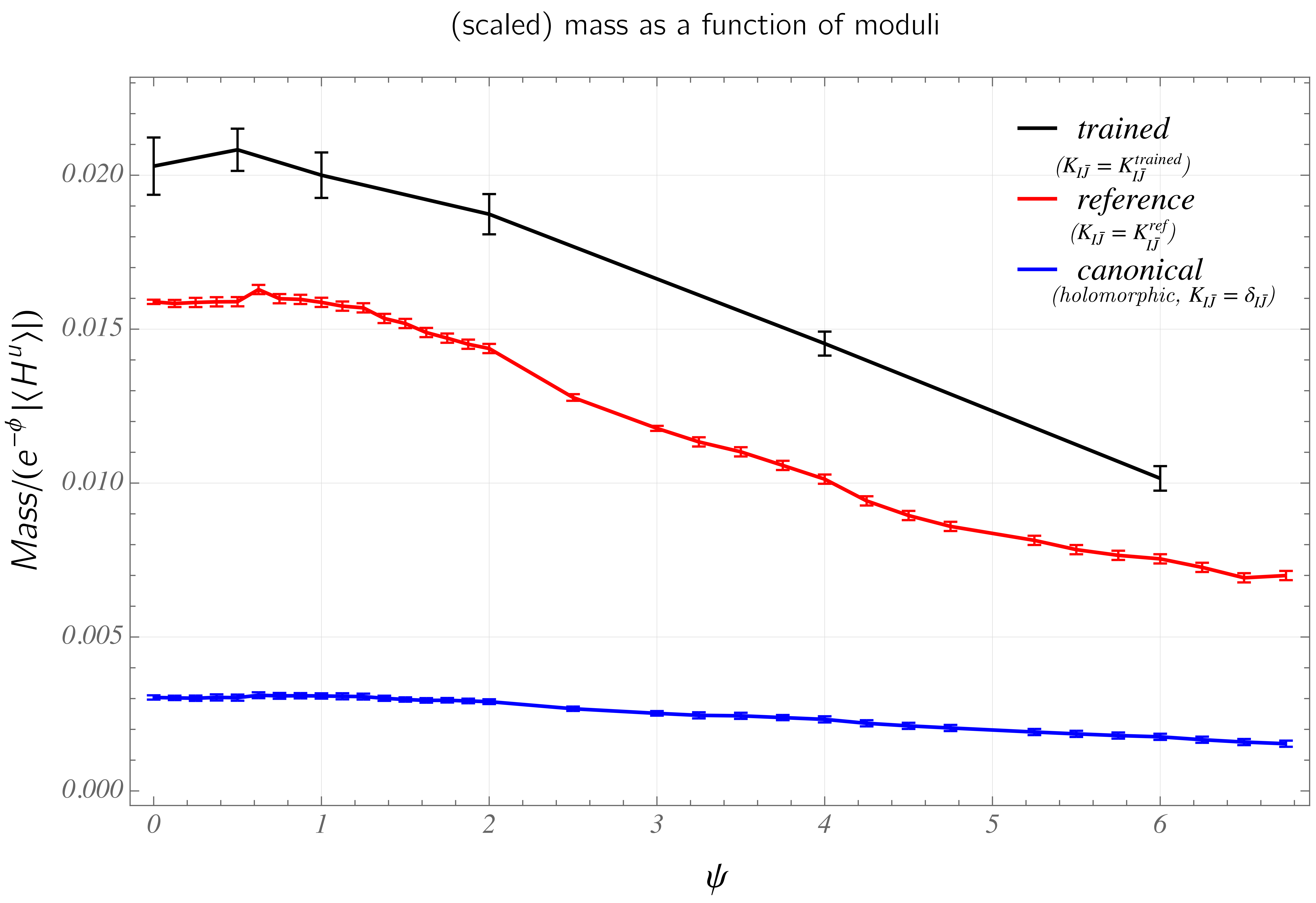

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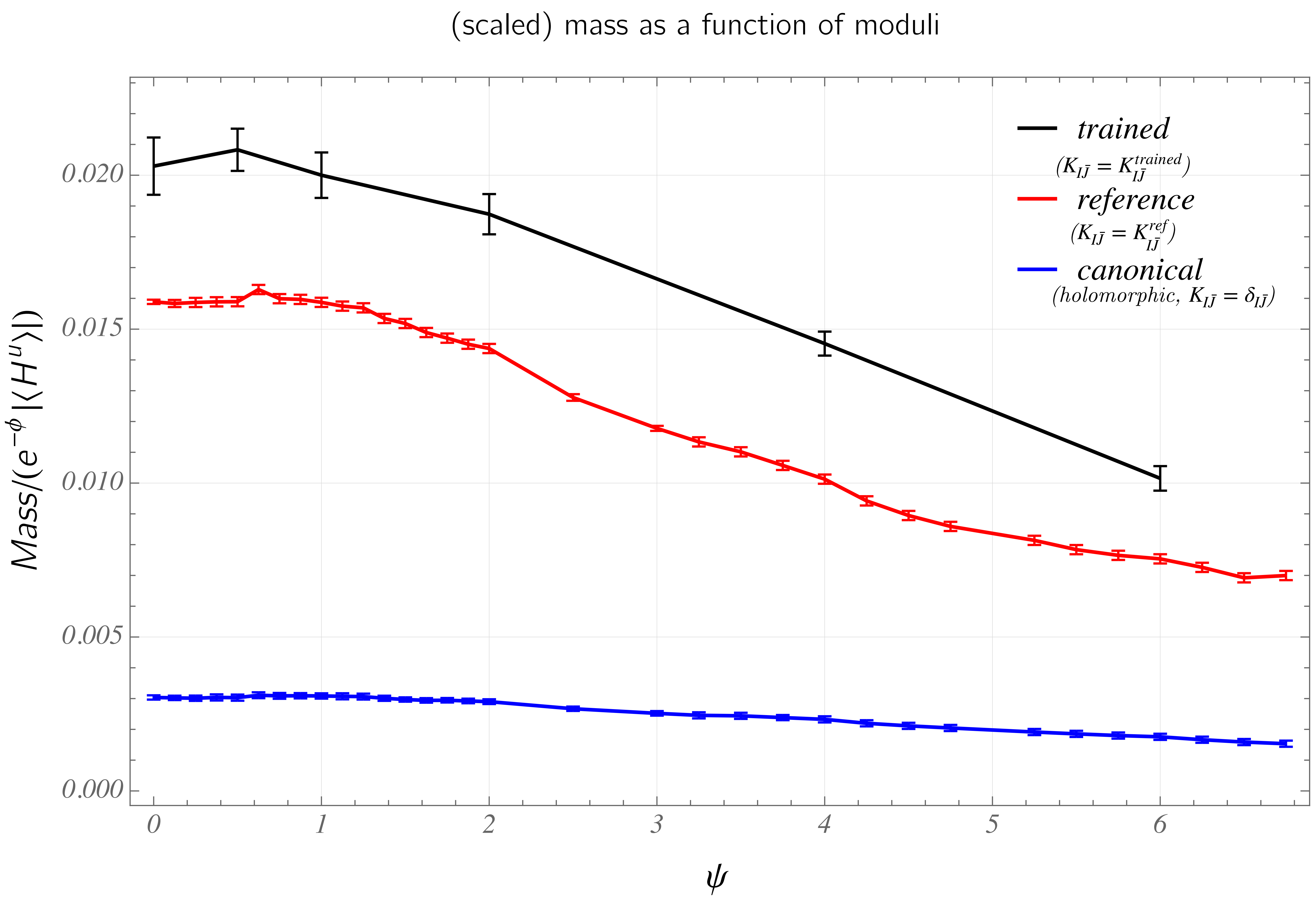
Tree-level physical masses for given moduli



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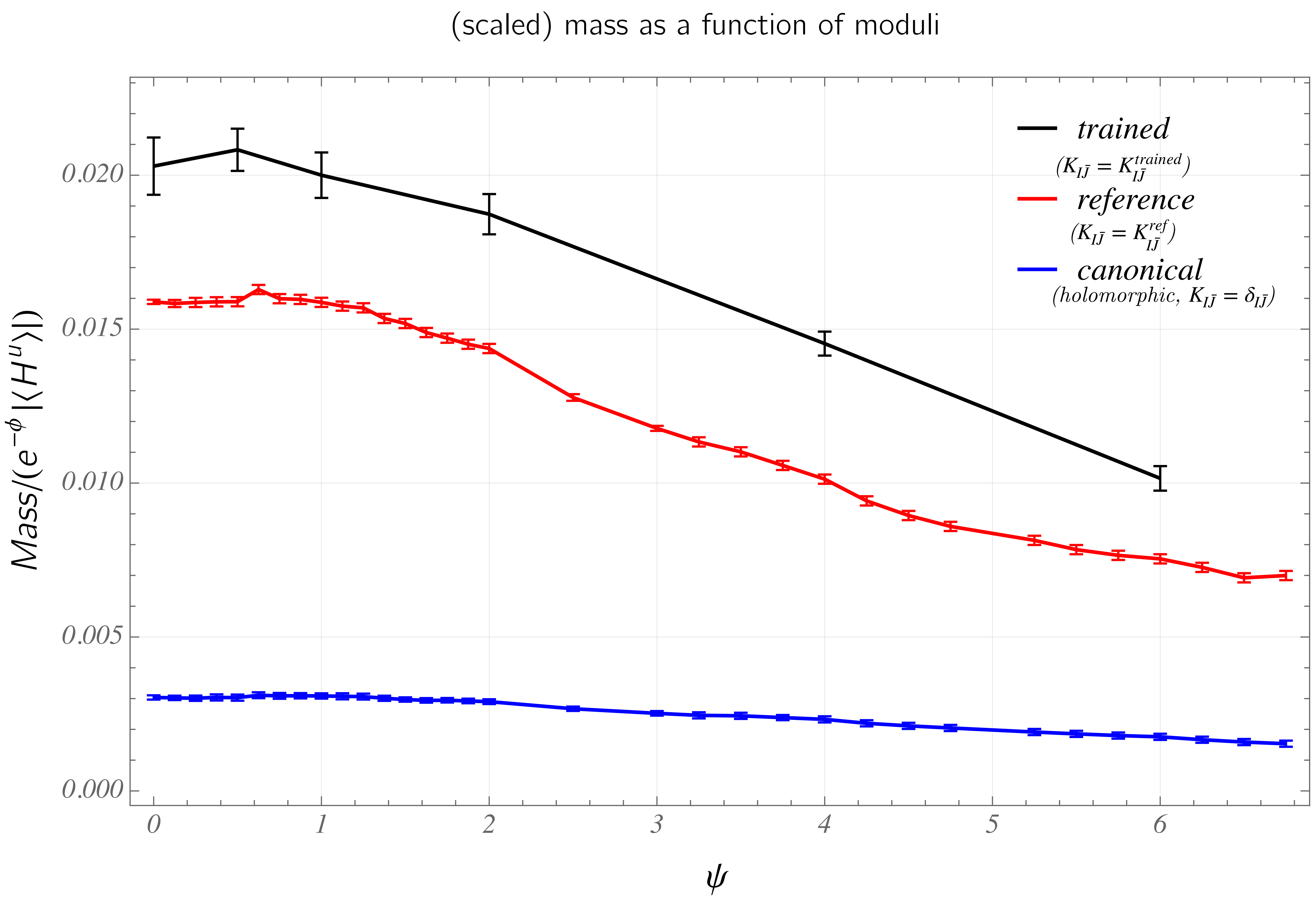
Tree-level physical masses for given moduli

Reference calculation $(\phi = \beta = \sigma = 0)$ *in red*

- *relatively good approximation* $\sim 25\,\%$
- ***only*** ~ 1 *min for 100,000 pts*
- ***closer** than the 'canonical' **unnormalised** holomorphic Yukawa couplings in blue*
- *enables exploration of moduli space*



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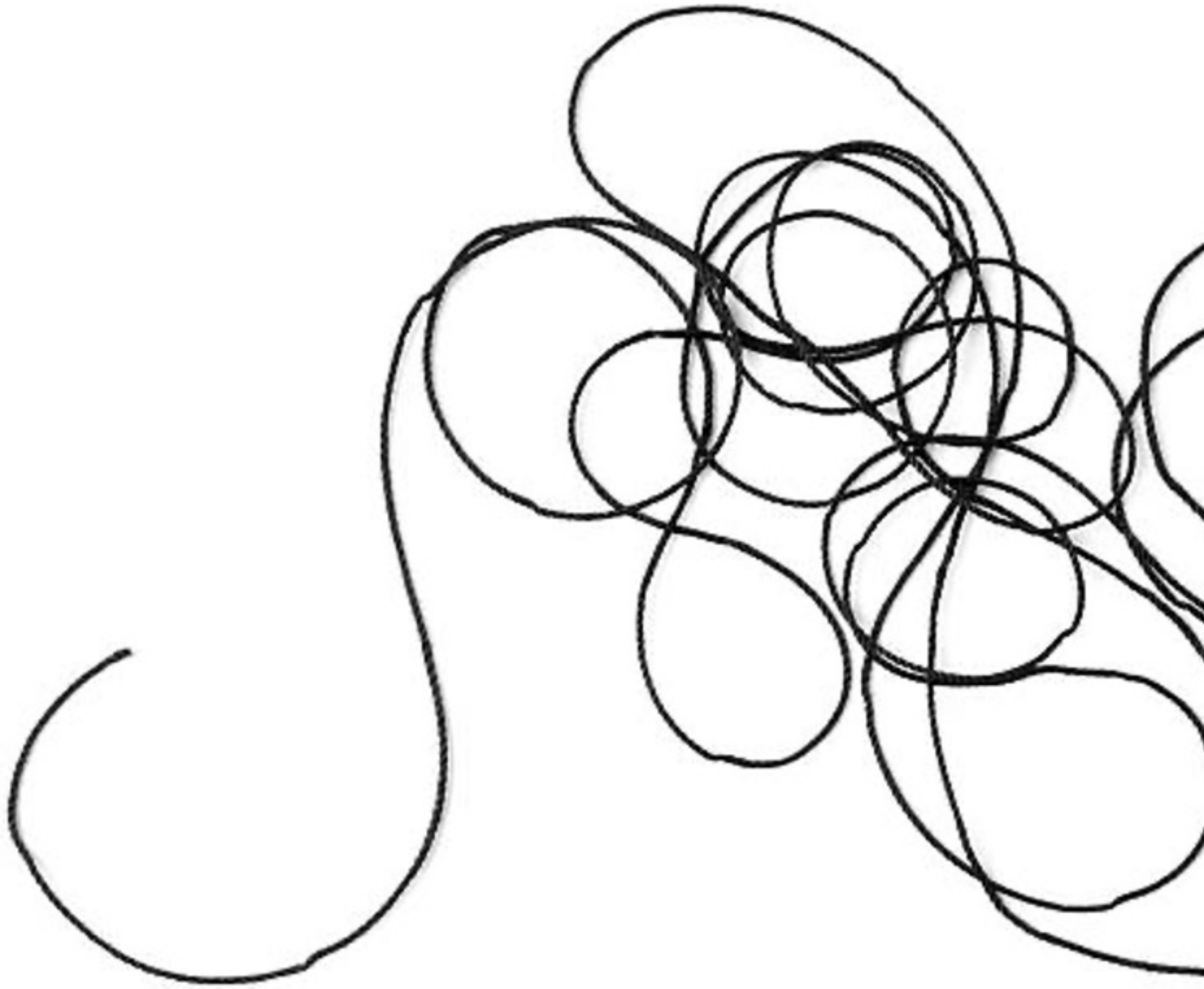

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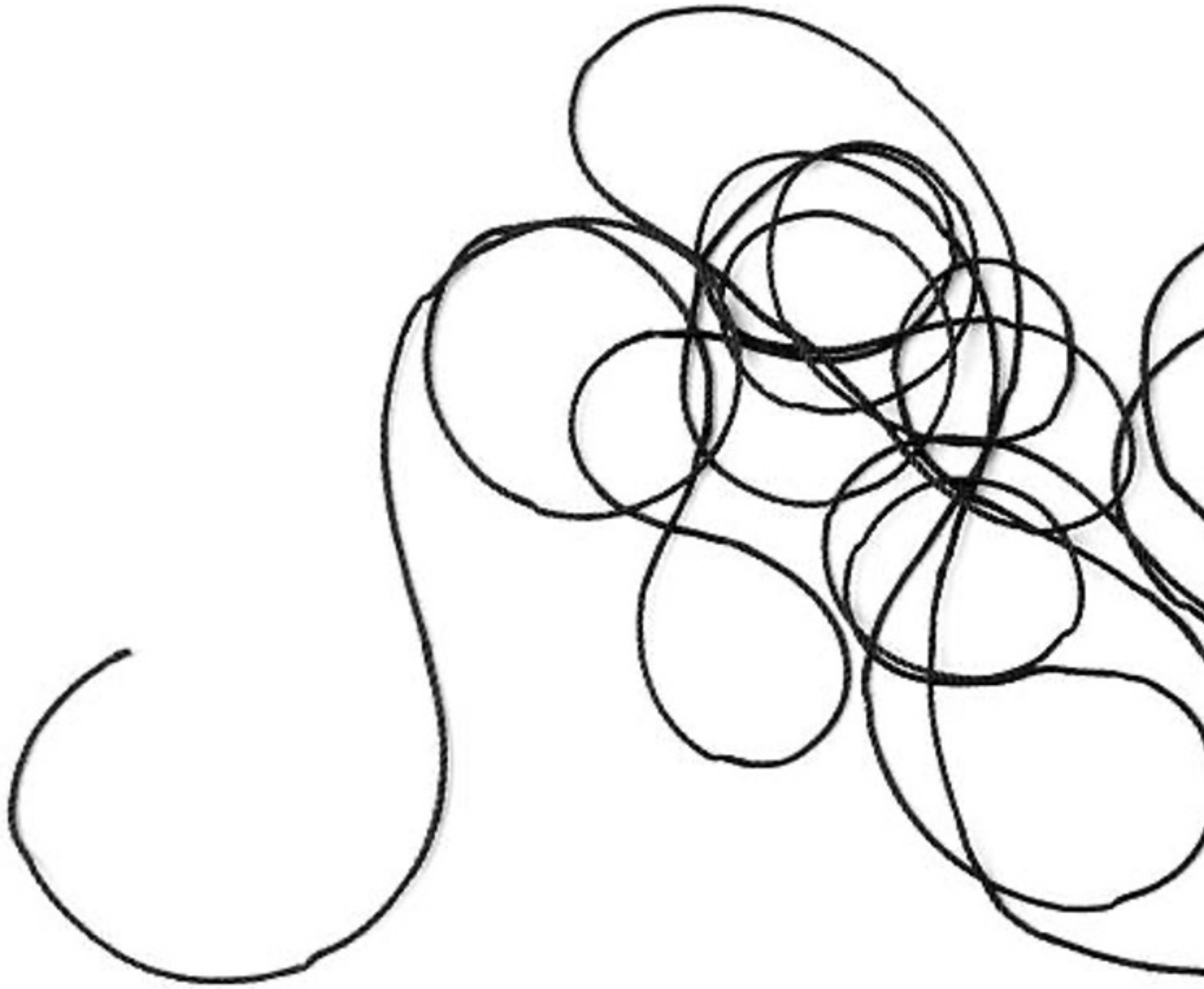

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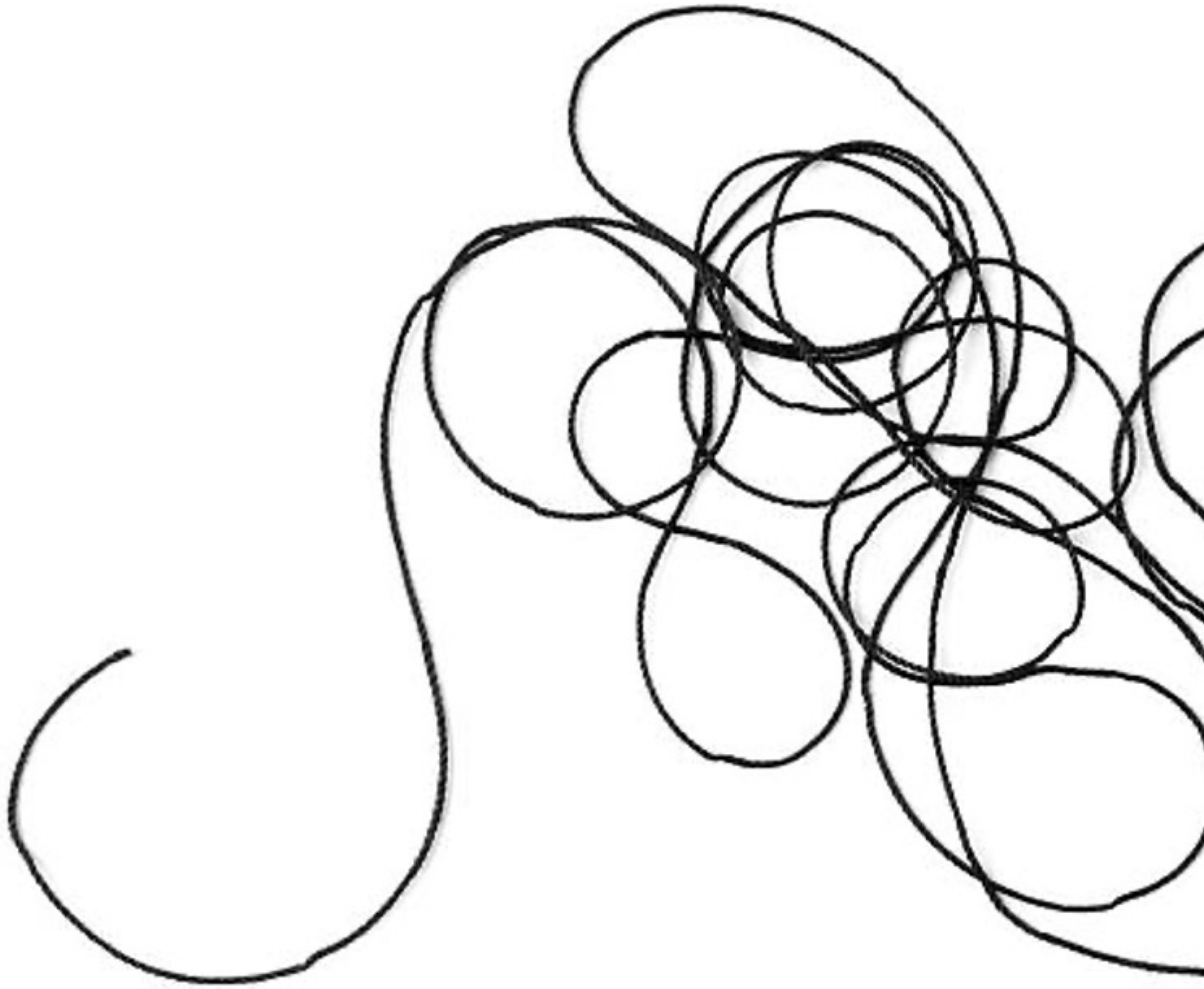
- *Able to compute CY masses to within* $< 5\,\%$ *(improved since publication: ultimately expect* $< 2\,\%$ *with unchanged training time)*
- systematic error analysis/ablation study **forthcoming**
- **easily generalised** to run on other manifolds/models.



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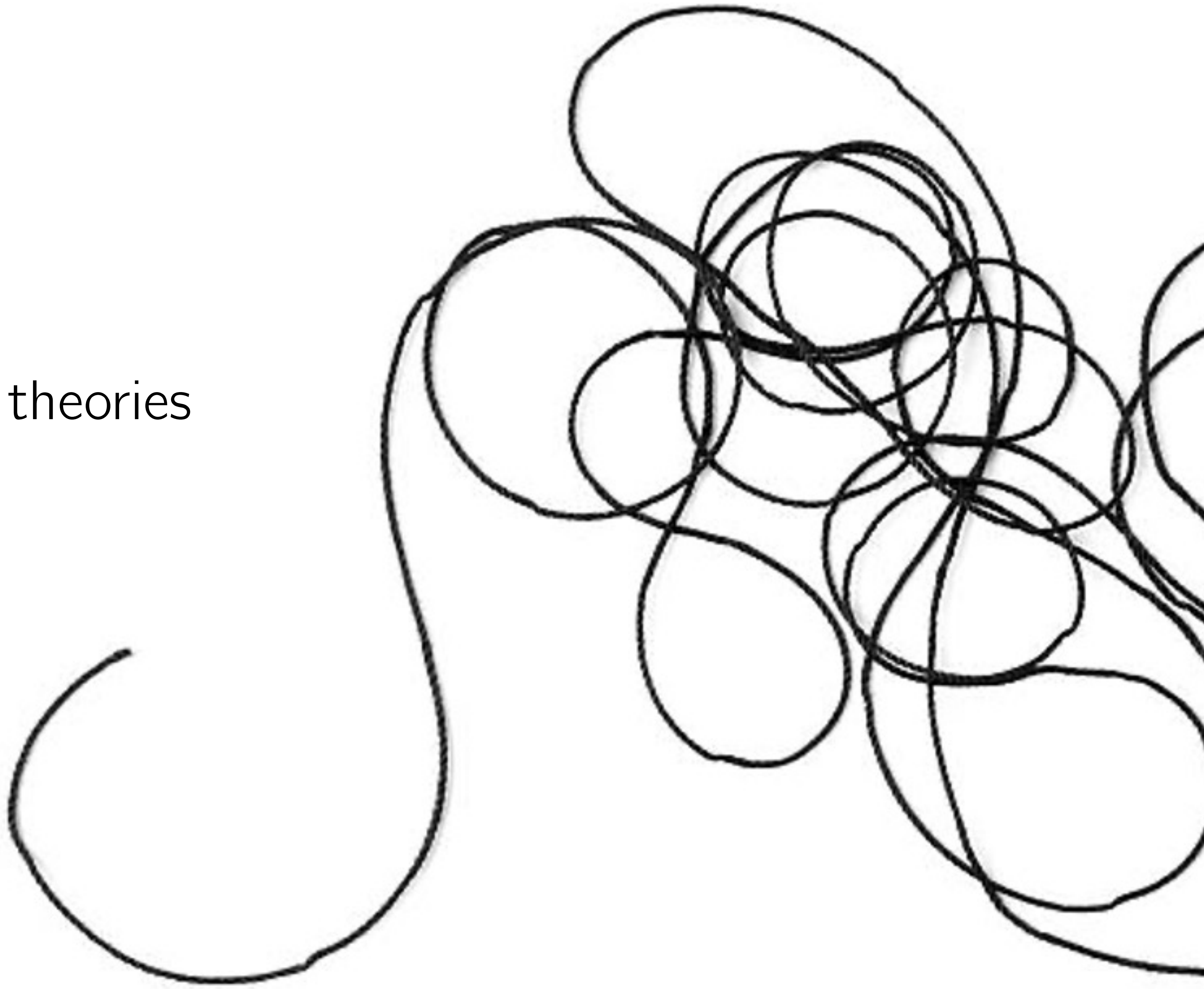

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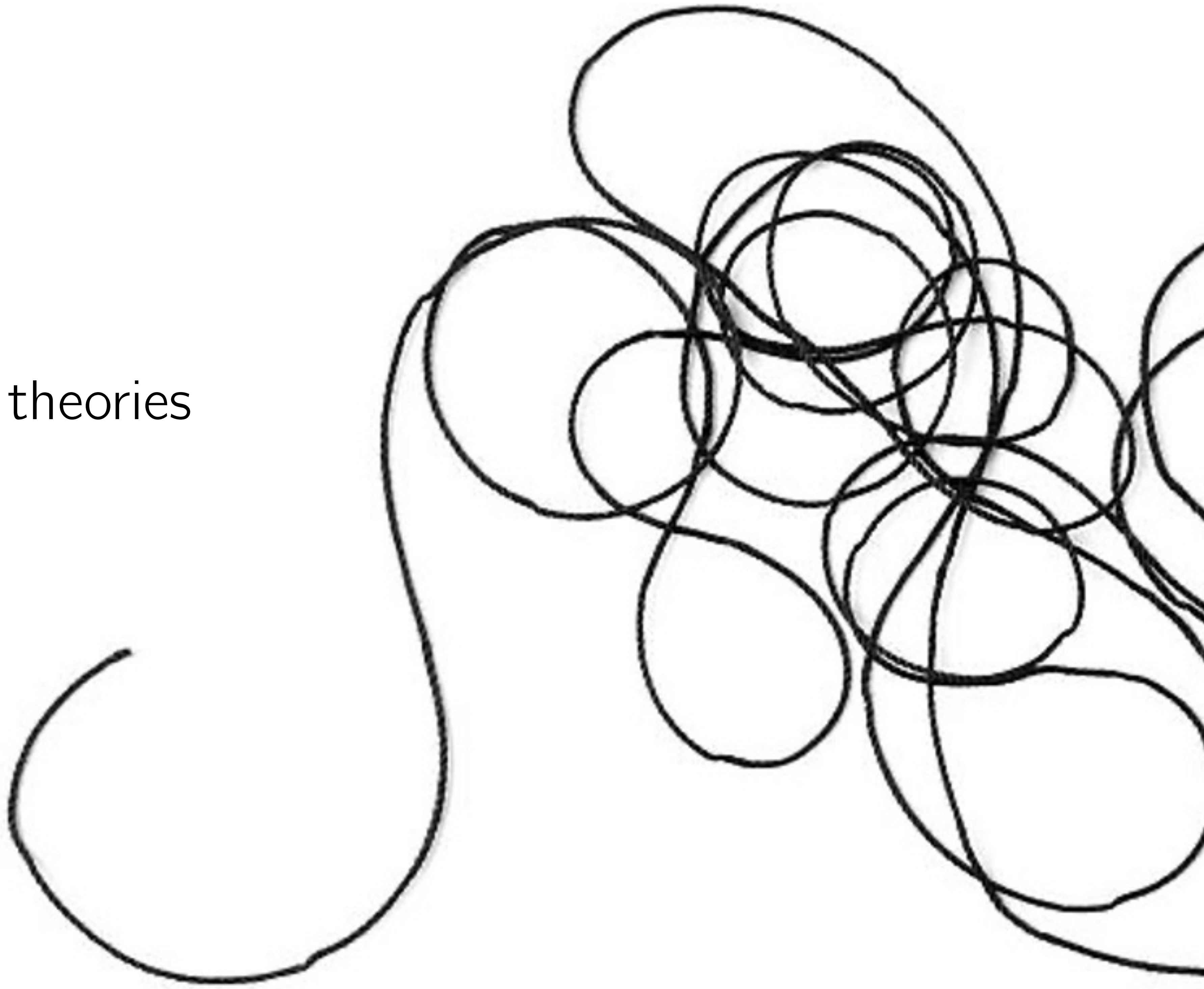

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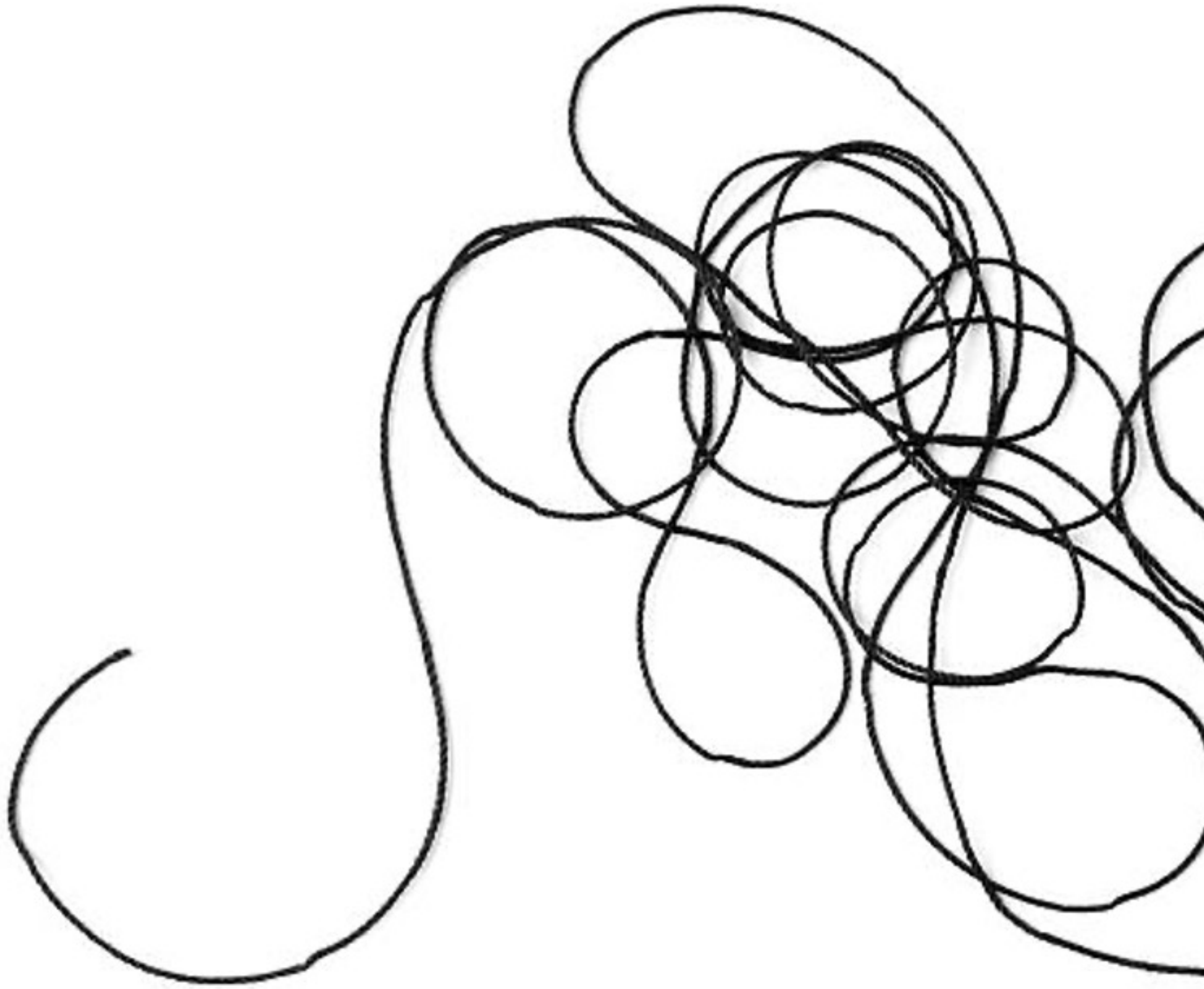
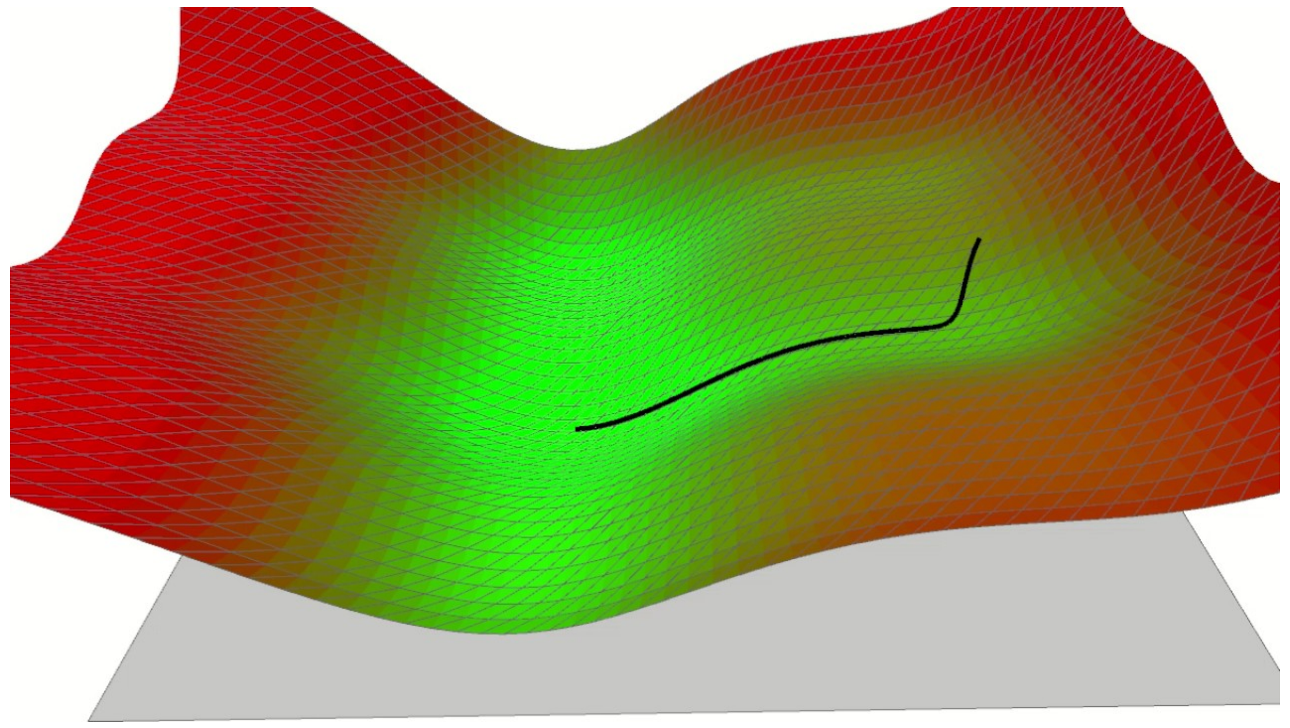


breaking the degeneracy!



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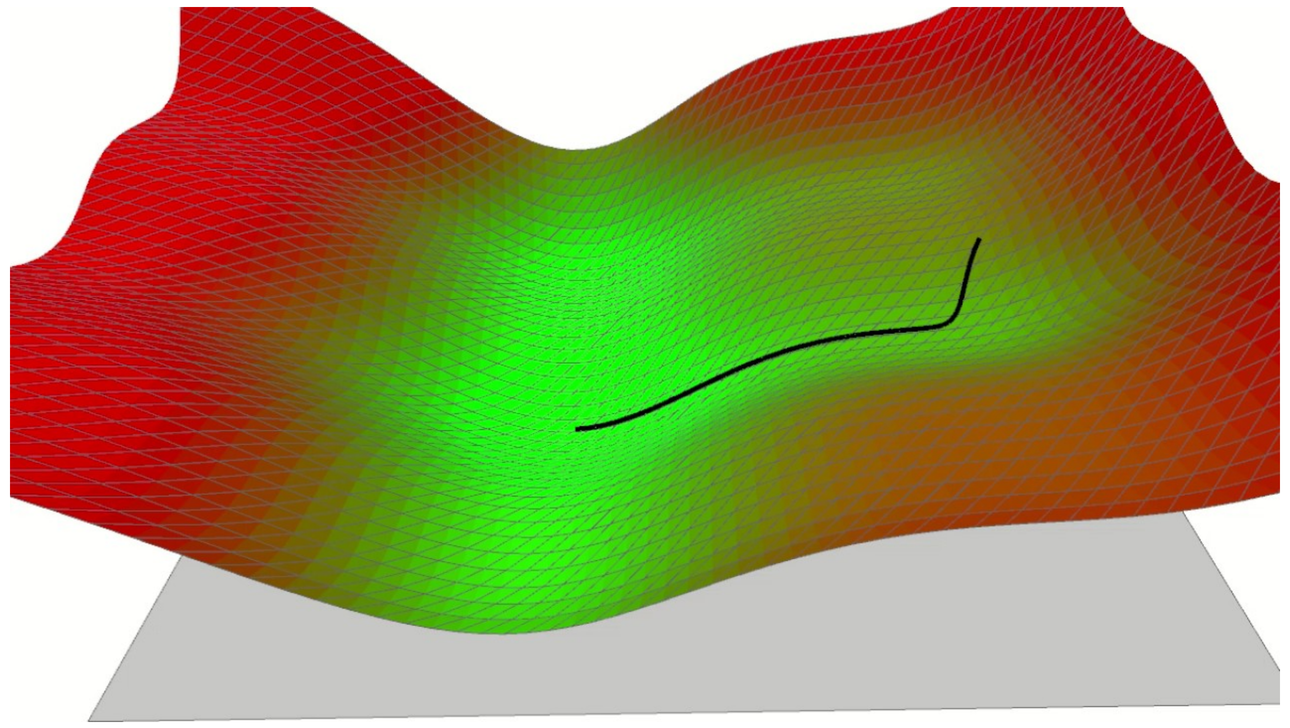

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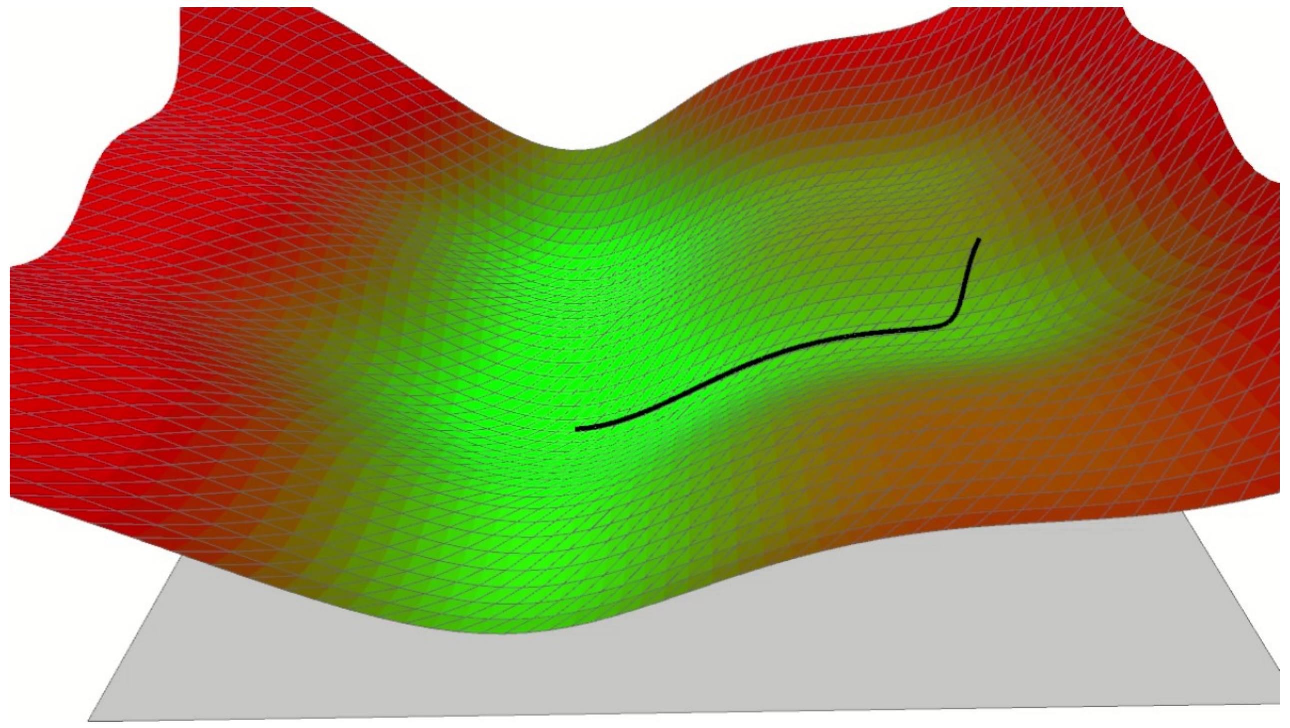

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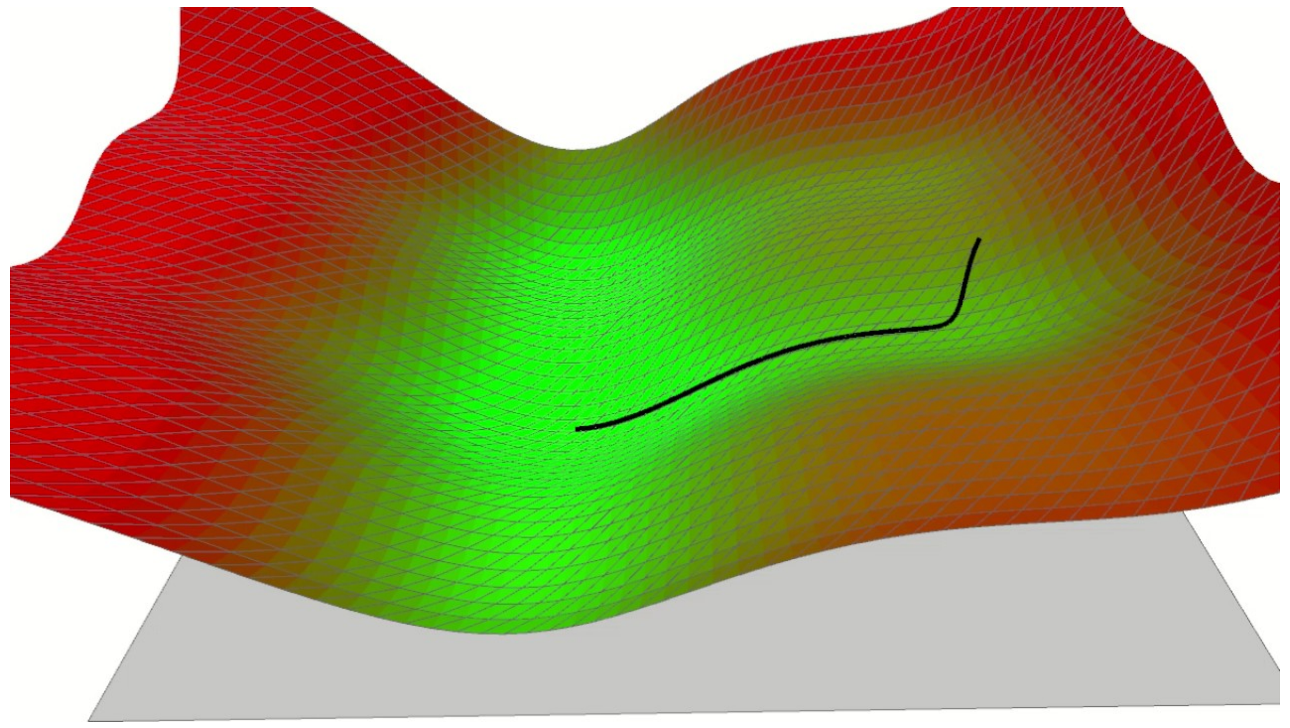

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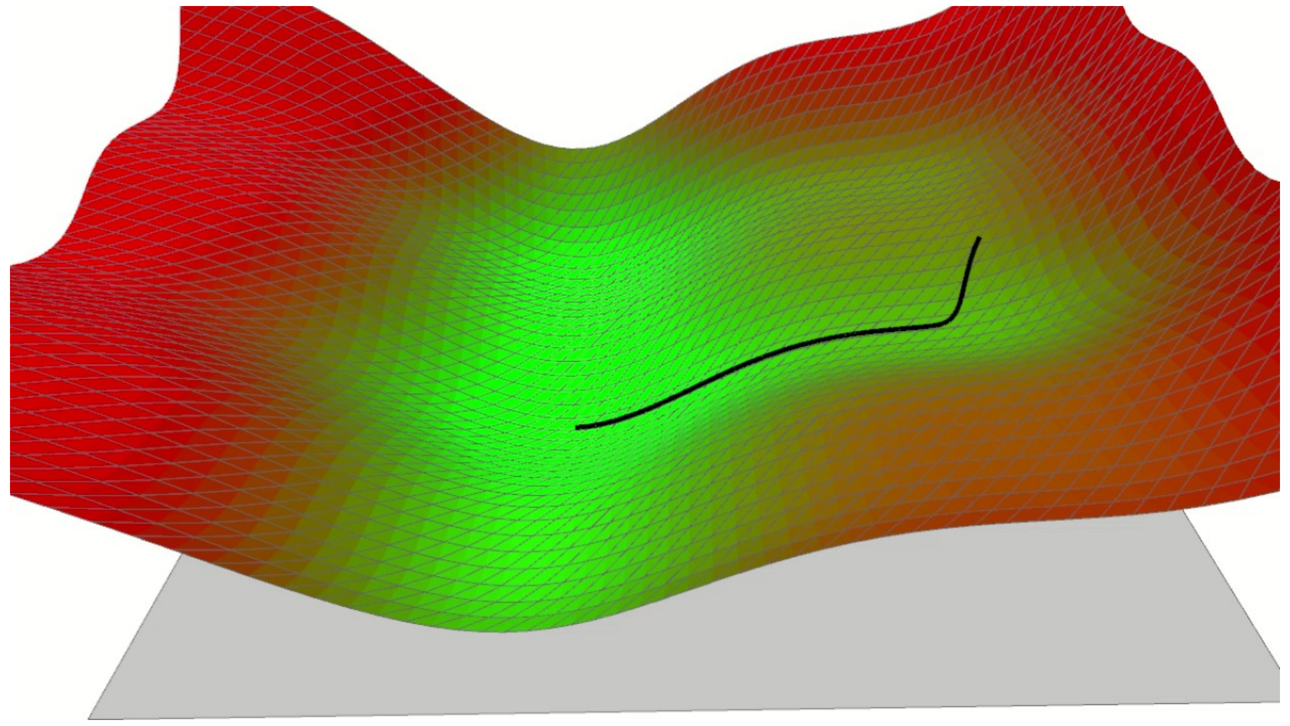

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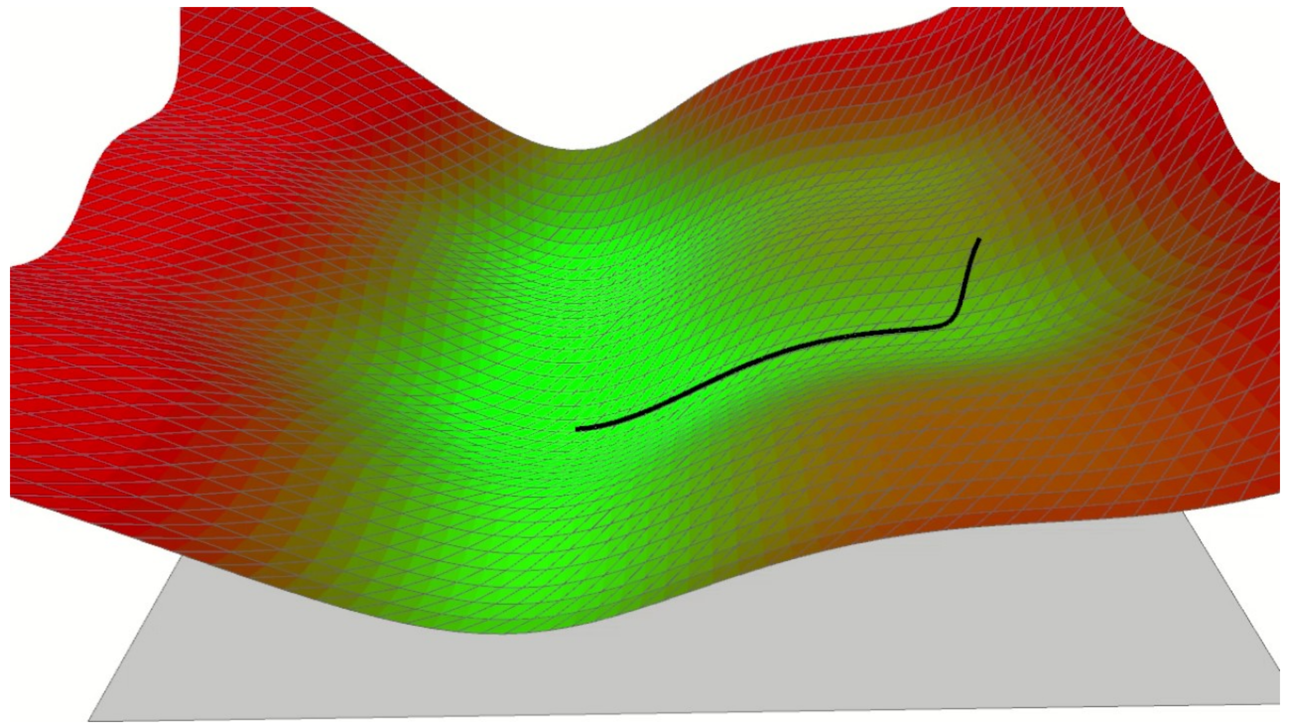

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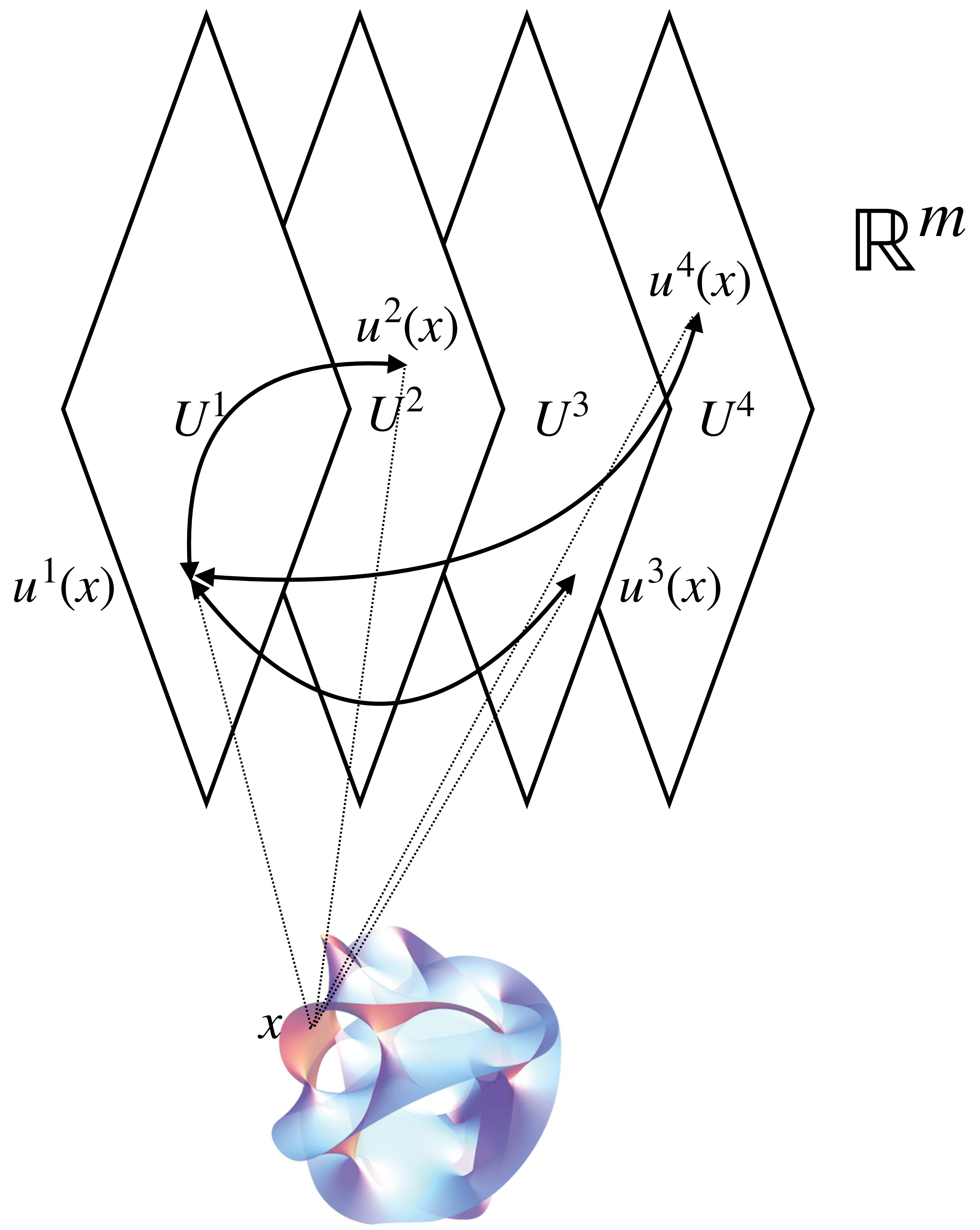

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$n-$manifold: defined over patches. embed patches in $\mathbb{R}^{m>n}$

consistently solve PDE over all patches

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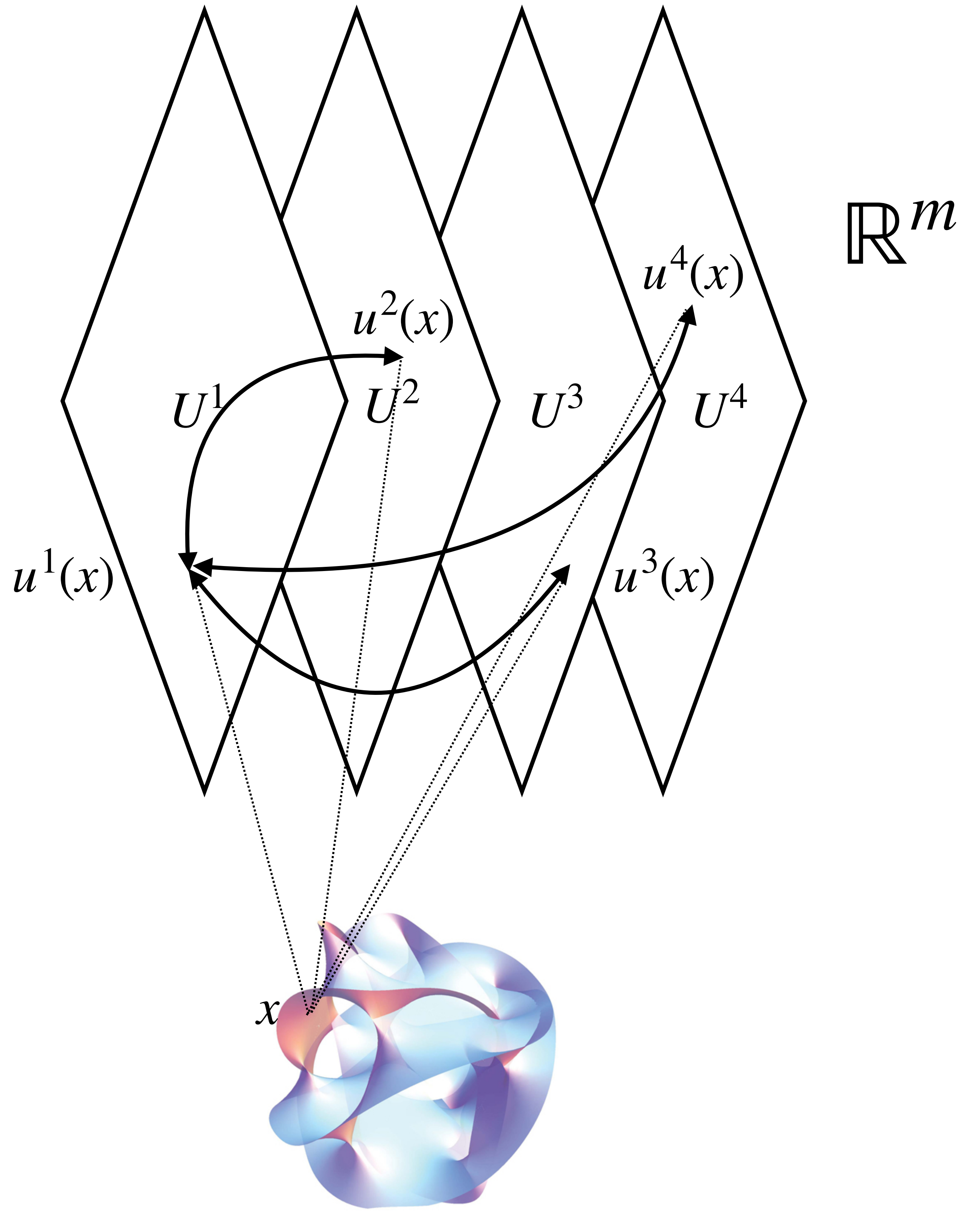
$\tilde{\phi}_\theta(u^s(x)) = \tilde{\phi}_\theta(u^t(x)) \;\; \forall$ patches $s, t \implies \tilde{\phi}_\theta$ is a **good function**



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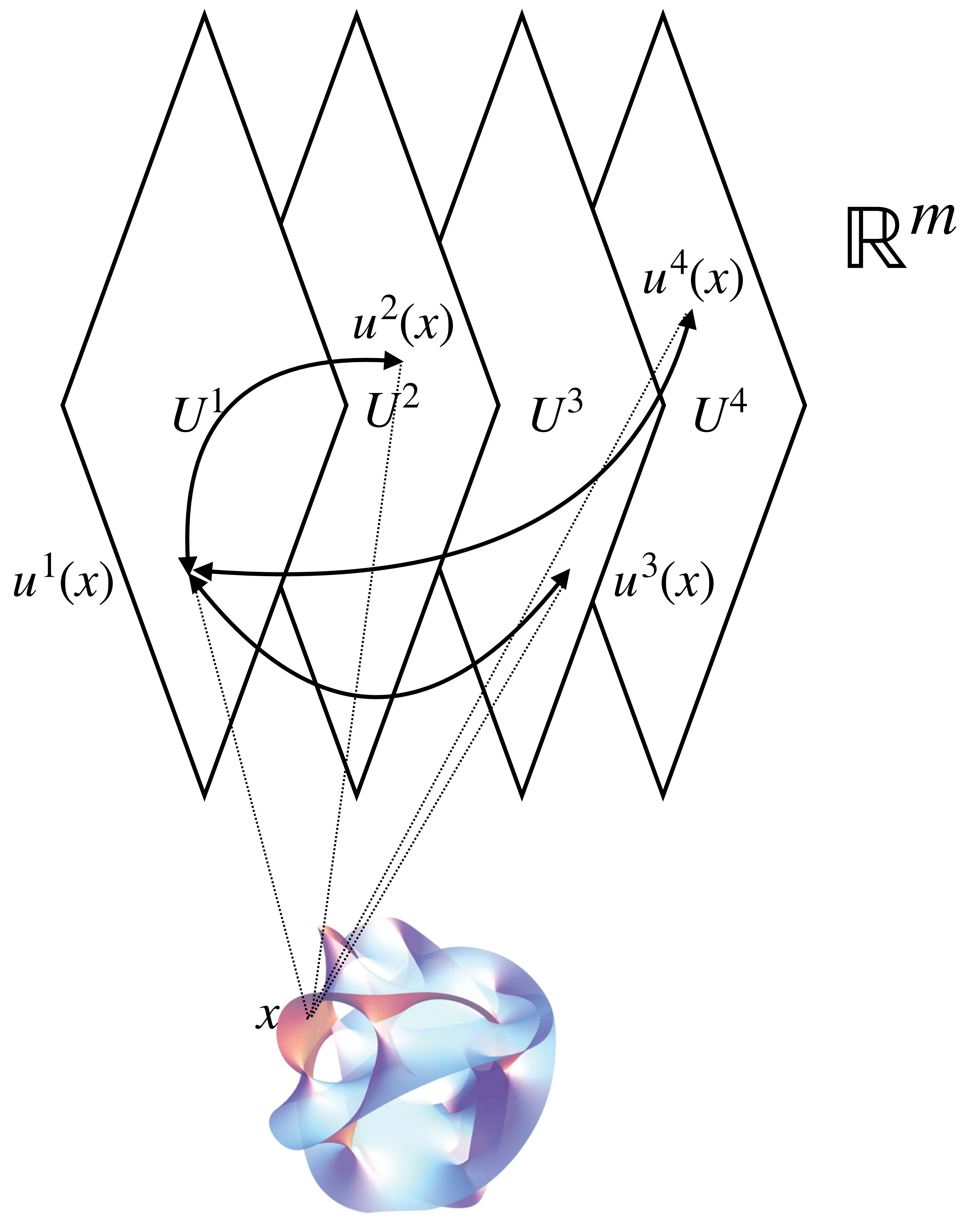

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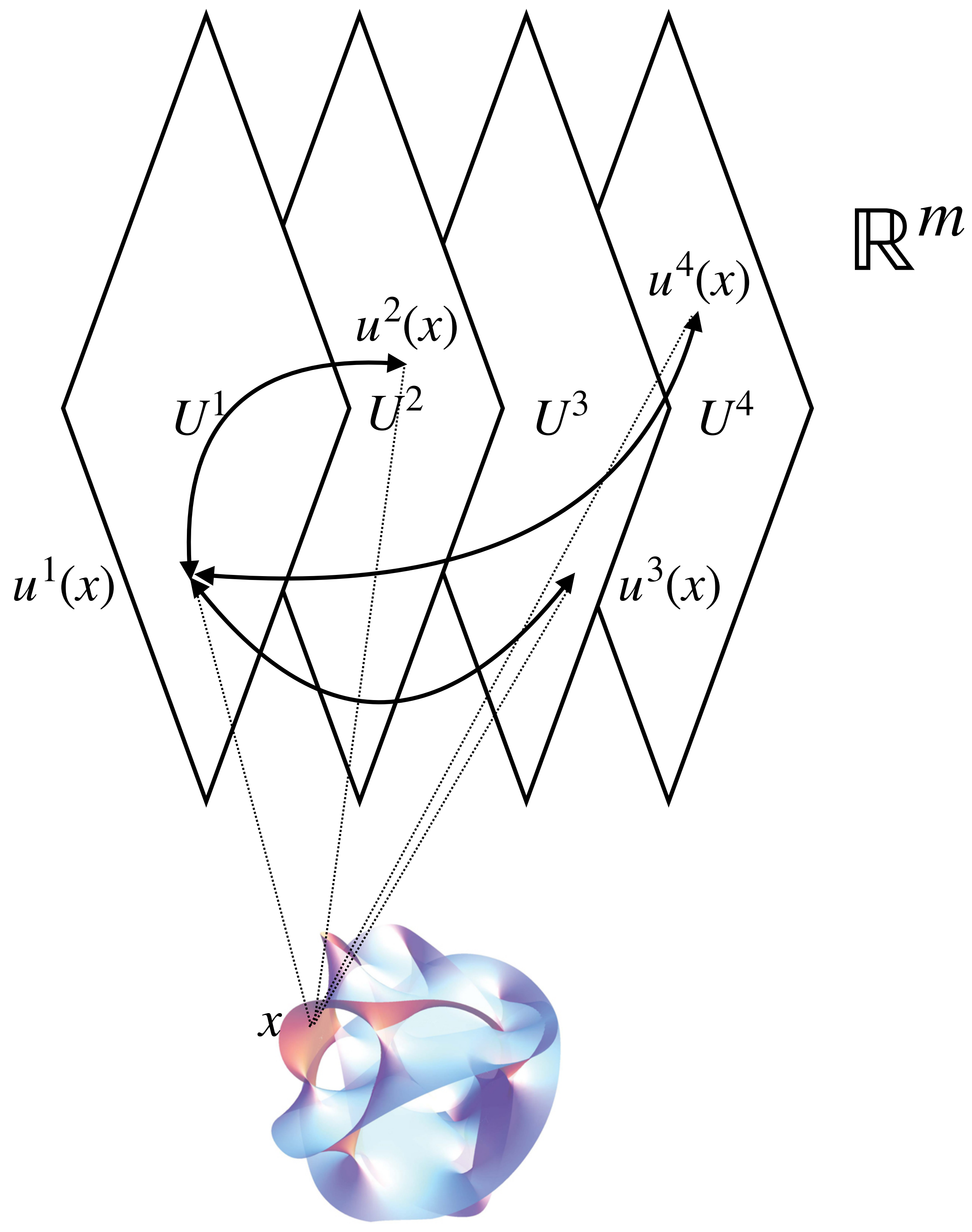

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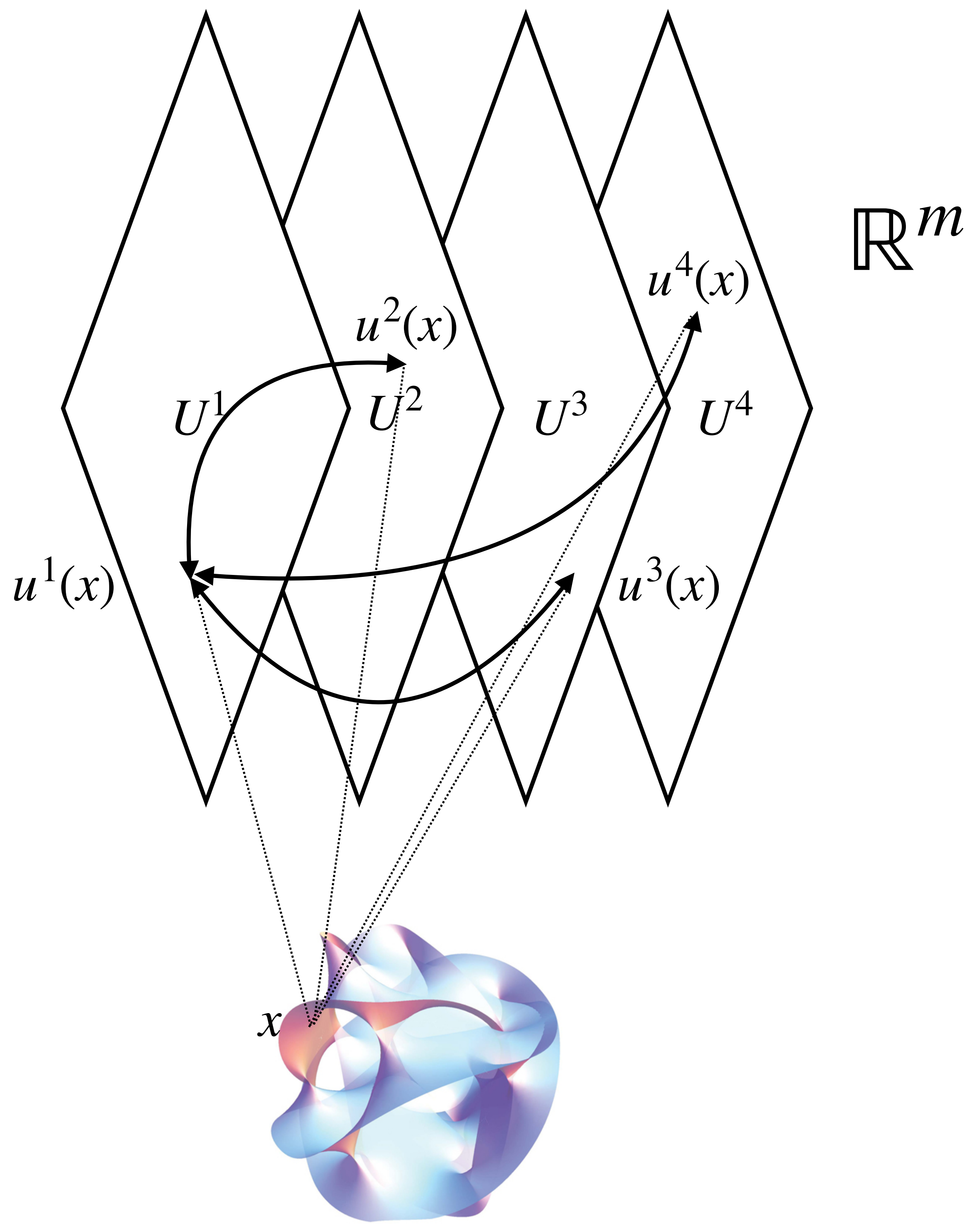

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