Higgsing on $\mathrm{SU}(N)$ Symmetric Matter and its F-theory Realization

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Introduction

- Motivation: Higgsing $\mathrm{SU}(N)$ on 2-index symmetric matter yields a breaking $\mathrm{SU}(N) \to \mathrm{SO}(N)$; can we realize this in F-theory?
- Construct $\mathrm{SO}(3)$ model in 6D with $\mathbf{5}$ matter and no apparent Mordell–Weil torsional section
- Analyze via heterotic/F-theory duality and Sen limit
- Construct $\mathrm{U}(1)$ model with no Mordell–Weil generating section

Field theory: $\mathrm{SU}(3) \to \mathrm{SO}(3)$

Giving a VEV to the symmetric representation $\mathbf{6}$ of $\mathrm{SU}(3)$, the Higgsing is

$$\mathrm{SU}(3): \quad r \times \mathbf{6} + (g-r) \times \mathbf{8} + (18(1-g) + 6b_{\mathrm{SU}(3)} \cdot b_{\mathrm{SU}(3)} + r) \times \mathbf{3}$$
$$\downarrow$$
$$\mathrm{SO}(3): \quad (g-1) \times \mathbf{5} + (18 - 17g + 6b_{\mathrm{SU}(3)} \cdot b_{\mathrm{SU}(3)}) \times \mathbf{3}\,.$$

Here, $b_{\mathrm{SO}(3)} = 4b_{\mathrm{SU}(3)}$. The surviving generators are, e.g., $2\lambda_2, 2\lambda_5, 2\lambda_7$.

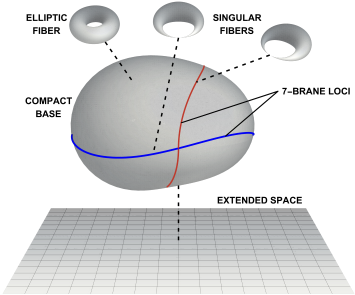
This should be distinguished from the $\mathfrak{su}(2)$ model that results from Higgsing via a fundamental VEV, which has spectrum

$$g \times \mathbf{3} + (16(1-g) + 6b_{\mathrm{SU}(3)} \cdot b_{\mathrm{SU}(3)}) \times \mathbf{2}$$

and $b_{\mathfrak{su}(2)} = b_{\mathrm{SU}(3)}$. The surviving generators are, e.g., $\lambda_1, \lambda_2, \lambda_3$.

F-theory overview

- Elliptically fibered Calabi–Yau n-fold Y:
  - Torus over each point in base B, $\pi\colon Y \to B$
  - Has a section, $\sigma\colon B \to Y$ s.t. $\pi \circ \sigma = \mathrm{Id}_B$
  - Complex structure τ encodes Type IIB axiodilaton



- Described by Weierstrass model: hypersurface

$$y^2 = x^3 + fxz^4 + gz^6$$

in $\mathbb{P}^{2,3,1}_{[x:y:z]}$ projective bundle, where f, g are sections of $-4K_B, -6K_B$

- Dictionary:
  - Codim.-one singularities (7-branes) $\longleftrightarrow$ nonabelian gauge algebras
  - Additional nontorsional rational sections $\longrightarrow$ $\mathfrak{u}(1)$ gauge algebras
  - Torsional rational sections $\longrightarrow$ global gauge group structure
  - Codim.-two singularities $\longrightarrow$ massless matter
    - Double point sing. of gauge divisor $\longrightarrow$ 2-index symmetric matter

Constructing the model

We realize $\mathrm{SO}(n)$ as an I_n singularity with unusual Tate monodromy, rather than $I^*_{\lceil \frac{n}{2} \rceil - 4}$.

We begin with an $\mathrm{SU}(3)_2$ model with symmetrics tuned on $\tilde{\sigma} = \sigma^2 - \frac{h\epsilon_1^2}{4}$ of class $2\tilde{S}$ over $B_2 = \mathbb{F}_m$ from [Anderson, Gray, Raghuram, Taylor '16], with spectrum

$$r \times \mathbf{6} + (m - 1 - r) \times \mathbf{8} + (6m + 36 + r) \times \mathbf{3}\,.$$

We then Higgs on a symmetric $\mathbf{6}$ by taking $\epsilon_1^2 \to h'$, yielding $\mathrm{SO}(3)$ tuned on $\tilde{\sigma} = \sigma^2 - \frac{hh'}{4}$ over $B_2 = \mathbb{F}_m$ with spectrum

$$(m - 2) \times \mathbf{5} + (7m + 35) \times \mathbf{3}\,.$$

F-theory SO(3) model

$$\tilde{\sigma} = \sigma^2 - \frac{hh'}{4}$$

$$f = -\frac{1}{48}\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big)^2 - \frac{1}{6}\big(9h'\lambda\nu + h\beta\phi_2 + (18\beta\lambda + 2\nu\phi_2)\sigma\big)\tilde{\sigma}$$
$$+ (f_4 + f_5\sigma)\tilde{\sigma}^2 + f_6\tilde{\sigma}^3$$

$$g = \frac{1}{864}\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big)^3$$
$$+ \frac{1}{72}\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big)\big(9h'\lambda\nu + h\beta\phi_2 + (18\beta\lambda + 2\nu\phi_2)\sigma\big)\tilde{\sigma}$$
$$\frac{1}{36}\big(81h'\lambda^2 + h\phi_2^2 + 36\lambda\phi_2\sigma - 3(f_4 + f_5\sigma)(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma)\big)\tilde{\sigma}^2$$
$$+ g_6\tilde{\sigma}^3$$

Interesting observation: this model seems to have no torsional section to explain its global structure $\mathrm{SO}(3) = \mathrm{SU}(2)/\mathbb{Z}_2$.

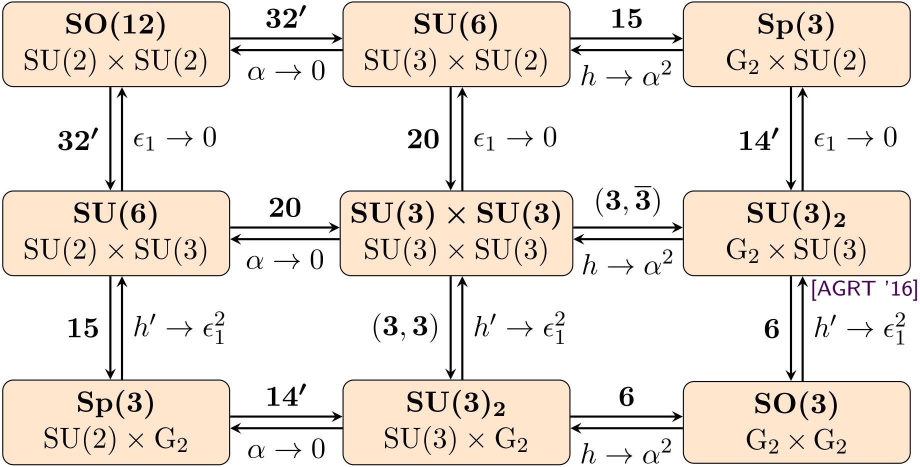
Heterotic/F-theory duality [Friedman, Morgan, Witten '97]

Heterotic on $X_n \longleftrightarrow$ F-theory on Y_{n+1}:

$$\begin{array}{ccc} X_2 = K3 & & Y_3 \xrightarrow{\ \mathbb{E}\ } B_2 = \mathbb{F}_m \\ \downarrow \mathbb{E} & & {\scriptstyle K3} \searrow \quad \downarrow \mathbb{P}^1 \\ B_1 = \mathbb{P}^1 & & B_1 = \mathbb{P}^1 \end{array}$$

- Break primordial $\mathrm{E}_8 \times \mathrm{E}_8$ by turning on gauge backgrounds over X_n in $H_a \times H_b \leq \mathrm{E}_8 \times \mathrm{E}_8$, preserving gauge group $G_a \times G_b$, where G_i is the commutant of H_i in E_8
- Mathematically, this is described by a complex vector bundle $\mathcal{V}_a \oplus \mathcal{V}_b$ over X_n with structure group $H_a \times H_b$
- Stable degeneration limit $\rightarrow$ Match degrees of freedom between F-theory and heterotic: Weierstrass parameters are Casimir invariants of gauge bundle structure group

SO(3) model web



$$(\lambda Z^3 + \nu XZ + \alpha Y)(\phi_2 Z^3 + \beta XZ + \epsilon_1 Y)$$

Sen limit

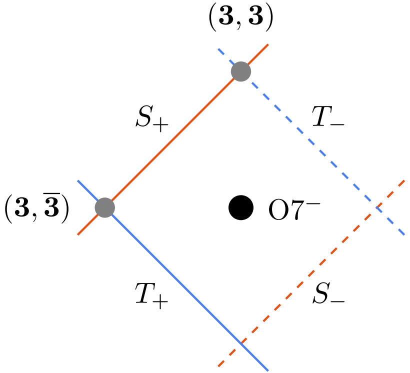
- Take limit in complex structure moduli space so that axiodilaton becomes constant almost everywhere
- Type IIB orientifold: double cover of F-theory base B_n branched over orientifold locus
- Discriminant locus of $\mathrm{SU}(3) \times \mathrm{SU}(3)$ model splits into a D7 stack and its orientifold image for each $\mathrm{SU}(3)$ factor, with each pair supporting a $\mathrm{U}(3)$, as well as an O7^{-}

IIB orientifold

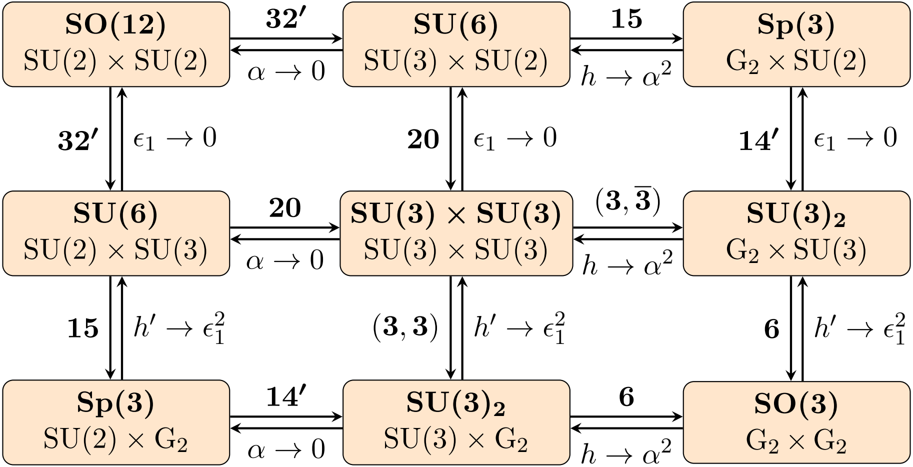
$$O7^-: \quad \alpha^2\beta^2 + 4\beta\nu\sigma + \nu^2\epsilon_1^2 = 0\,,$$

$$S_\pm = \{\sigma - \frac{1}{2}\alpha\epsilon_1 = \xi \mp (\alpha\beta + \nu\epsilon_1) = 0\}\,,$$

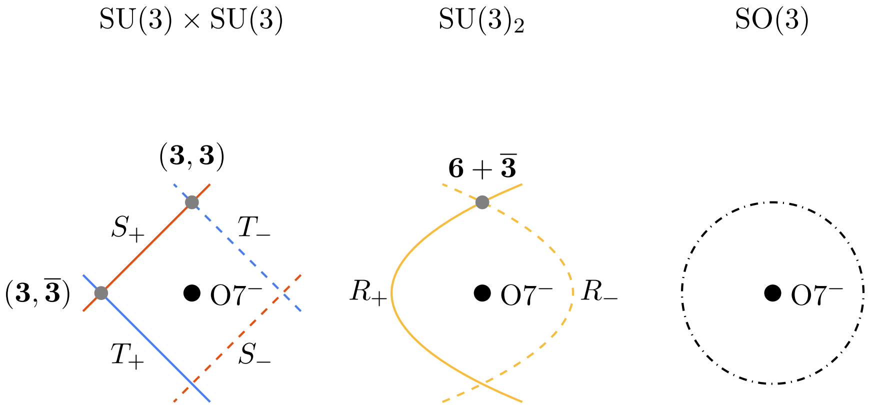
$$T_\pm = \{\sigma + \frac{1}{2}\alpha\epsilon_1 = \xi \pm (\alpha\beta - \nu\epsilon_1) = 0\}$$



SO(3) model web

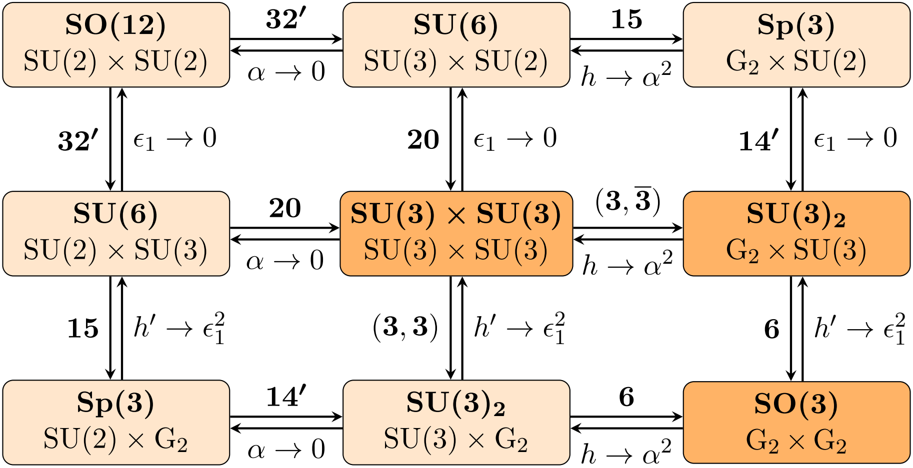


IIB orientifold

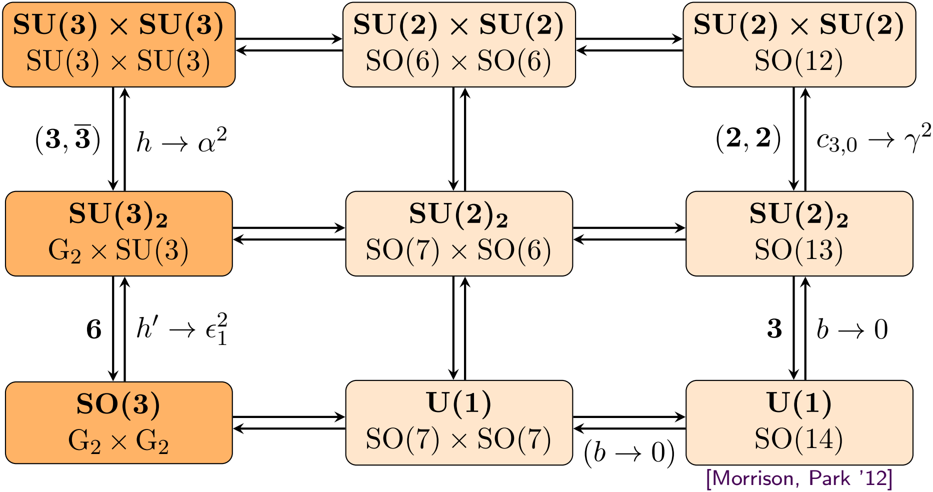


Here we can see the origin of the unusual Tate monodromy within gauge divisor of nonzero genus.

SO(3) model web

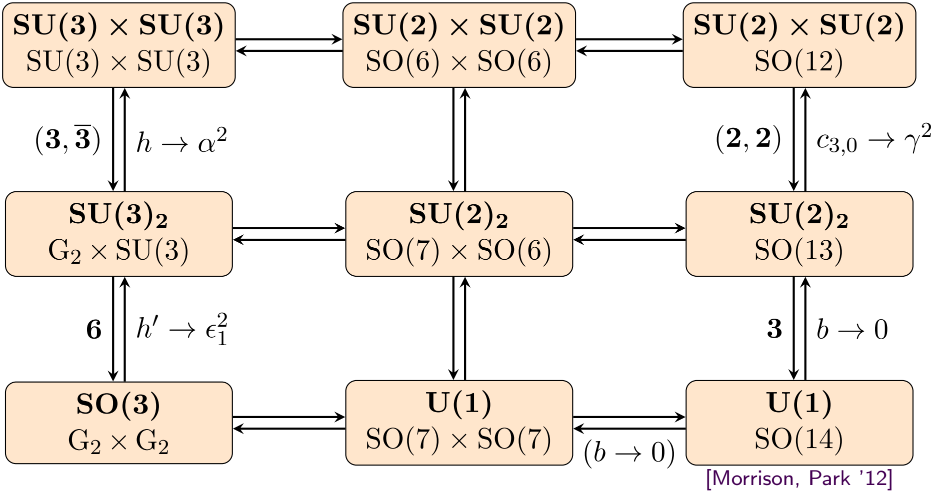


$\mathrm{U}(1)$ model web



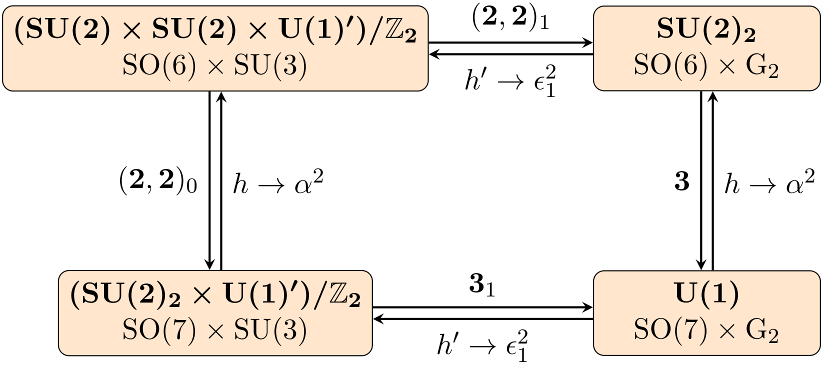
$b \longleftrightarrow$ Pfaffian of $\mathrm{SO}(14)$

$\mathrm{U}(1)$ model web



$b \longleftrightarrow$ Pfaffian of $\mathrm{SO}(14)$

$\mathrm{U}(1)$ model web



Locally supported $\mathrm{U}(1)$ with no Mordell–Weil generating section!

Conclusions

- First F-theory realization of Higgsing on symmetric matter (in the absence of $O7^+$), and of a model with 4-index symmetric matter
- Example of nontrivial nonabelian gauge group quotient structure with no associated torsional section
- Example of locally supported $\mathrm{U}(1)$ with no associated generating section
- Justified by heterotic and type IIB analyses
- Future directions:
  - Carry out full analysis in dual M-theory
  - Realize other $\mathrm{SU}(N) \to \mathrm{SO}(N)$
  - Realize analogues for exceptional groups

Thank you!

Field theory: $\mathrm{SU}(N) \to \mathrm{SO}(N)$

Consider a $\mathrm{SU}(N)$ 6D supergravity theory with gauge anomaly coefficient $b_{\mathrm{SU}(N)}$ and gravitational anomaly coefficient a. One anomaly free spectrum for such a model is

$$r \times \mathbf{Sym} + (b_{\mathrm{SU}(N)} \cdot b_{\mathrm{SU}(N)} + 2(1-g) + r) \times \mathbf{Anti} + (g-r) \times \mathbf{Adj} \\ + (16(1-g) + (8-N) b_{\mathrm{SU}(N)} \cdot b_{\mathrm{SU}(N)}) \times \mathbf{Fund}\,,$$

where $g = 1 + \frac{b_{\mathrm{SU}(N)} \cdot (b_{\mathrm{SU}(N)} + a)}{2}$ and $0 \leq r \leq g$ is the number of symmetrics.

This model can be Higgsed on the $\mathbf{Sym}$ representation, breaking $\mathrm{SU}(N) \to \mathrm{SO}(N)$, so long as $r \geq 1$, implying that $g \geq 1$ and $N \leq 8$.

Field theory: $\mathrm{SU}(N) \to \mathrm{SO}(N)$

The branching rule is

$$\begin{aligned}
\mathbf{Fund} &\to \boldsymbol{N}\,, \\
\mathbf{Anti} &\to \frac{\boldsymbol{N(N-1)}}{\mathbf{2}}\,, \\
\mathbf{Sym} &\to \mathbf{1} + \frac{\boldsymbol{(N-1)(N+2)}}{\mathbf{2}}\,, \\
\mathbf{Adj} &\to \frac{\boldsymbol{N(N-1)}}{\mathbf{2}} + \frac{\boldsymbol{(N-1)(N+2)}}{\mathbf{2}}\,.
\end{aligned}$$

In particular, consider $\mathrm{SU}(3) \to \mathrm{SO}(3)$, with branching rule

$$\begin{aligned}
\mathbf{3} &\to \mathbf{3}\,, \\
\mathbf{6} &\to \mathbf{1} + \mathbf{5}\,, \\
\mathbf{8} &\to \mathbf{3} + \mathbf{5}\,.
\end{aligned}$$

Heterotic/F-theory duality [Friedman, Morgan, Witten]

Heterotic on $X_n \longleftrightarrow$ F-theory on Y_{n+1}:

$$\begin{array}{ccc} X_n \\ \Big\downarrow{\scriptstyle \mathbb{E}} \\ B_{n-1} \end{array} \qquad\qquad \begin{array}{ccc} Y_{n+1} & \xrightarrow{\ \mathbb{E}\ } & B_n \\ & {\scriptstyle K3}\searrow & \Big\downarrow{\scriptstyle \mathbb{P}^1} \\ & & B_{n-1} \end{array}$$

Stable degeneration limit: weakly coupled limit of both theories, realized geometrically as

$$Y_{n+1} \to Y_{n+1}^{(1)} \cup_{X_n} Y_{n+1}^{(2)}\,,$$

where $Y^{(i)}$ are non-CY dP_9-fibered $(n+1)$-folds glued together along X_n.

Match degrees of freedom between heterotic and F-theory geometry: spectral (cameral) cover construction. Under the Fourier–Mukai transform, the heterotic gauge vector bundle data is equivalent to a pair (S, L_S), where S is a divisor in X_n and L_S is a rank-1 sheaf over S.

Heterotic gauge bundles

We are compactifying $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic on $K3$:

- To break the primordial $\mathrm{E}_8 \times \mathrm{E}_8$, we turn on gauge backgrounds over X_n in a subgroup $H_a \times H_b$ of $\mathrm{E}_8 \times \mathrm{E}_8$, preserving gauge group $G_a \times G_b$, where G_i is the commutant of H_i in E_8
- Mathematically, this is described by a complex vector bundle $\mathcal{V}_a \oplus \mathcal{V}_b$ over X_n with structure group $H_a \times H_b$
- This gauge configuration must have instanton number 24,

$$c_2(\mathcal{V}_a) + c_2(\mathcal{V}_b) = c_2(K3) = 24$$

- Matter is understood by breaking the E_8 adjoint into representations of $G_i \times H_i$
- For F-theory on $\mathbb{F}_m$:
  - $\mathcal{V}_a$ has $12 + m$ instantons and $\mathcal{V}_b$ has $12 - m$
  - G_a is supported on $\tilde{S}$ and G_b is supported on S

Heterotic dual

We can now understand the Higgsing chain in the heterotic dual: begin with $G_a = \mathrm{SU}(3) \times \mathrm{SU}(3)$, $H_a = \mathrm{SU}(3) \times \mathrm{SU}(3)$ ($\tilde{\sigma} = \sigma^2 - \frac{\alpha^2 \epsilon_1^2}{4}$), then Higgs on $(\mathbf{3}, \overline{\mathbf{3}})$ to get $G_a = \mathrm{SU}(3)_2$, $H_a = \mathrm{G}_2 \times \mathrm{SU}(3)$ ($\tilde{\sigma} = \sigma^2 - \frac{h\epsilon_1^2}{4}$).

The commutant of $\mathrm{SO}(3)$ in E_8 is $G_2 \times G_2$, so we see that we must enhance both factors of H_a to G_2. The spectral cover for $\mathrm{SU}(3) \times \mathrm{SU}(3)$ is

$$(\lambda Z^3 + \nu XZ + \alpha Y)(\phi_2 Z^3 + \beta XZ + \epsilon_1 Y)\,.$$

We know that the deformation $\alpha^2 \to h$ enhances the first factor, so $\epsilon_1^2 \to h'$ must enhance the other factor ($\tilde{\sigma} = \sigma^2 - \frac{hh'}{4}$).

Branching rule of $\mathrm{E}_8 \to \mathrm{SO}(3) \times \mathrm{G}_2 \times \mathrm{G}_2$:

$$\begin{aligned}\mathbf{248} \to (\mathbf{3},\mathbf{1},\mathbf{1}) + (\mathbf{5},\mathbf{1},\mathbf{7}) + (\mathbf{5},\mathbf{7},\mathbf{1}) + (\mathbf{3},\mathbf{7},\mathbf{7})\\ + (\mathbf{1},\mathbf{14},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{14})\end{aligned}$$

Sen limit

Take limit in complex structure moduli space so that axiodilaton becomes constant almost everywhere:

$$\lambda \to a\lambda\,, \quad \phi_2 \to a\phi_2\,, \quad f_4 \to af_4\,,$$
$$f_5 \to af_5\,, \quad f_6 \to a^2 f_6\,, \quad g_6 \to a^2 g_6\,.$$

In $\mathrm{SU}(3) \times \mathrm{SU}(3)$ model, this takes

$$\Delta \to a^2 \left(\sigma - \frac{1}{2}\alpha\epsilon_1\right)^3 \left(\sigma + \frac{1}{2}\alpha\epsilon_1\right)^3 (\alpha^2\beta^2 + 4\beta\nu\sigma + \nu^2\epsilon_1^2)\Delta' + \mathcal{O}(a^3)\,.$$

We see that there are stacks of D7s on $\{\sigma \pm \frac{1}{2}\alpha\epsilon_1 = 0\}$ and an $\mathrm{O7}^-$ at $\{\alpha^2\beta^2 + 4\beta\nu\sigma + \nu^2\epsilon_1^2 = 0\}$.

Type IIB orientifold: double cover of F-theory base B_n branched over orientifold locus, described as

$$\alpha^2\beta^2 + 4\beta\nu\sigma + \nu^2\epsilon_1^2 = \xi^2\,,$$

where $\xi \to -\xi$ is the orientifold involution.

IIB orientifold

Using

$$\alpha^2\beta^2 + 4\beta\nu\sigma + \nu^2\epsilon_1^2 = \xi^2\,,$$

we find

$$4\beta\nu\left(\sigma \pm \frac{1}{2}\alpha\epsilon_1\right) = (\xi - (\alpha\beta \mp \nu\epsilon_1))(\xi + (\alpha\beta \mp \nu\epsilon_1))\,.$$

So $\{\sigma - \frac{1}{2}\alpha\epsilon_1 = 0\}$ splits into a locus and its orientifold image,

$$S_\pm = \{\sigma - \frac{1}{2}\alpha\epsilon_1 = \xi \mp (\alpha\beta + \nu\epsilon_1) = 0\}\,,$$

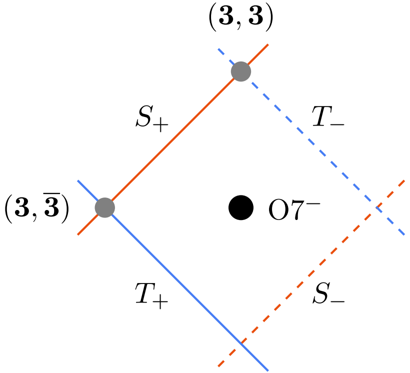
and $\{\sigma + \frac{1}{2}\alpha\epsilon_1 = 0\}$ splits into

$$T_\pm = \{\sigma + \frac{1}{2}\alpha\epsilon_1 = \xi \pm (\alpha\beta - \nu\epsilon_1) = 0\}\,.$$

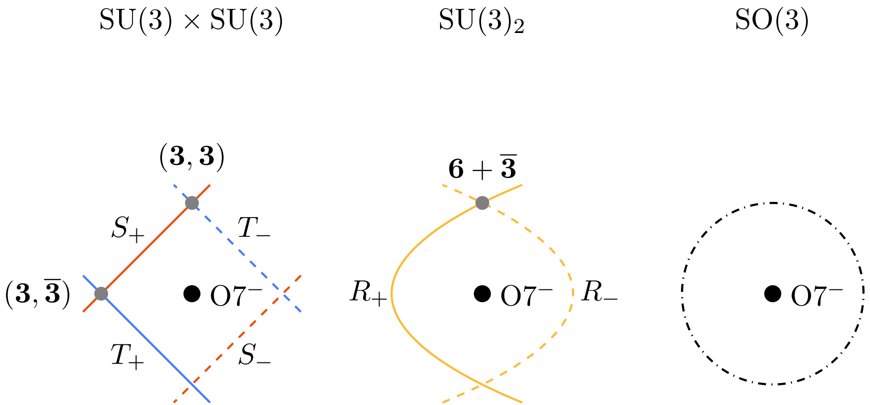
Each pair supports a $\mathrm{U}(3)$ factor.

IIB orientifold

$$O7^-: \quad \alpha^2\beta^2 + 4\beta\nu\sigma + \nu^2\epsilon_1^2 = 0\,,$$
$$S_\pm = \{\sigma - \frac{1}{2}\alpha\epsilon_1 = \xi \mp (\alpha\beta + \nu\epsilon_1) = 0\}\,,$$
$$T_\pm = \{\sigma + \frac{1}{2}\alpha\epsilon_1 = \xi \pm (\alpha\beta - \nu\epsilon_1) = 0\}$$



IIB orientifold



F-theory $\mathrm{SO}(3)$ model

$$\tilde{\sigma} = \sigma^2 - \frac{hh'}{4}$$

$$f = -\frac{1}{48}\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big)^2 - \frac{1}{6}\big(9h'\lambda\nu + h\beta\phi_2 + (18\beta\lambda + 2\nu\phi_2)\sigma\big)\tilde{\sigma}$$
$$+ (f_4 + f_5\sigma)\tilde{\sigma}^2 + f_6\tilde{\sigma}^3$$

$$g = \frac{1}{864}\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big)^3$$
$$+ \frac{1}{72}\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big)\big(9h'\lambda\nu + h\beta\phi_2 + (18\beta\lambda + 2\nu\phi_2)\sigma\big)\tilde{\sigma}$$
$$\frac{1}{36}\big(81h'\lambda^2 + h\phi_2^2 + 36\lambda\phi_2\sigma - 3(f_4 + f_5\sigma)(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma)\big)\tilde{\sigma}^2$$
$$+ g_6\tilde{\sigma}^3$$

F-theory local $\mathrm{U}(1)$ model

$$f = -\frac{1}{48}\Phi^2 + F_1\Sigma + (f_4 + f_5\sigma + f_6\Sigma)\Sigma^2 \qquad \Sigma = \sigma^2 - \frac{hh'}{4}$$

$$g = \frac{1}{864}\Phi^3 - \frac{1}{12}\Phi F_1\Sigma + G_2\Sigma^2 + g_6\Sigma^3$$

$$\Phi = h\beta^2 + h'\nu^2 + 4\beta\nu\sigma + 12h'\delta\lambda + \frac{8}{3}\sigma\delta\phi_2$$

$$\begin{aligned} F_1 = &-\frac{3}{2}h'\lambda\nu - \frac{1}{6}h\beta\phi_2 - 3\beta^2\delta\lambda + \frac{1}{3}\beta\nu\delta\phi_2 + \frac{1}{9}\delta^2\phi_2^2 + \frac{1}{2}(f_4 + f_5\sigma + f_6\Sigma)\delta\nu h' \\ &- g_6\delta^2 h' - \frac{1}{12}f_6\delta^2 h'\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big) - 3\beta\lambda\sigma - \frac{1}{3}\nu\phi_2\sigma \\ &+ (f_4 + f_5\sigma + f_6\Sigma)\beta\delta\sigma \end{aligned}$$

$$\begin{aligned} G_2 = &\frac{9}{4}h'\lambda^2 + \frac{1}{36}h\phi_2^2 + \phi_2\lambda\beta\delta - \frac{1}{9}\phi_2^2\delta\nu + \phi_2\lambda\sigma \\ &- \frac{1}{12}(f_4 + f_5\sigma + f_6\Sigma)\left(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma - 6h'\delta\lambda - \frac{4}{3}\delta\phi_2\sigma - 4\delta^2\beta\phi_2\right) \\ &+ \frac{1}{4}(f_4 + f_5\sigma + f_6\Sigma)^2\delta^2 h' + \frac{1}{12}f_6\Sigma\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big) \\ &+ \left(g_6 + \frac{1}{12}f_6\big(h\beta^2 + h'\nu^2 + 4\beta\nu\sigma\big)\right)\delta\big(\beta^2\delta + h'\nu + 2\beta\sigma\big) \end{aligned}$$