Efficiency in F-Theory: `FTheoryTools`

Martin Bies

RPTU Kaiserslautern-Landau

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Based on work with A. Turner, M. E. Mikelsons, and the `OSCAR` team.

Introduction ([Weigand 18'] and references therein)

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Discrete gauge factors	Weil–Châtelet group
Chiral matter	G_4-flux
Vector-like matter	Deligne cohomology, root bundles, refined bases [Li, Taylor '23]

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- This complexity obstructs progress in F-theory:
  - Imposes large computational overhead for analyzing models.
  - Makes it harder for newcomers to enter the field.

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- Yet more details:

  M. Bies, and A. Turner, *F-Theory Applications*, chapter in book "The Computer Algebra System OSCAR: Algorithms and Examples", Sept. 2024, ISSN 1431-1550.

FTheoryTools – New Features Since Summer 2023

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    Klevers, Pena, Oehlmann, Piragua, Reuter '14,
    16 hypersurface and 16 **Weierstrass models** over arbitrary base. Gauge groups:
    $SU(3)^3/\mathbb{Z}_3$, $(SU(2))^4/\mathbb{Z}_2 \times U(1)$, $\dots$, $U(1) \times \mathbb{Z}_2$, $\mathbb{Z}_3$.

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  - "A Quadrillion Standard Models from F-theory"
    Cvetič, Halverson, Lin, Liu, Tian '19.
    702 families of hypersurface models with $SU(3) \times SU(2) \times U(1)/\mathbb{Z}_6$
    Each family encoded in triangulations of certain 3d reflexive polytopes [Kreuzer Skarke '98].
    Includes root bundle data. ([M.B. Cvetič, Donagi, Ong '23], [M.B. '23], [M.B, Cvetič, Donagi, Ong '22], [M.B., Cvetič, Liu '21], [M.B., Cvetič, Donagi, Liu, Ong '21])

Code demonstration is in order!

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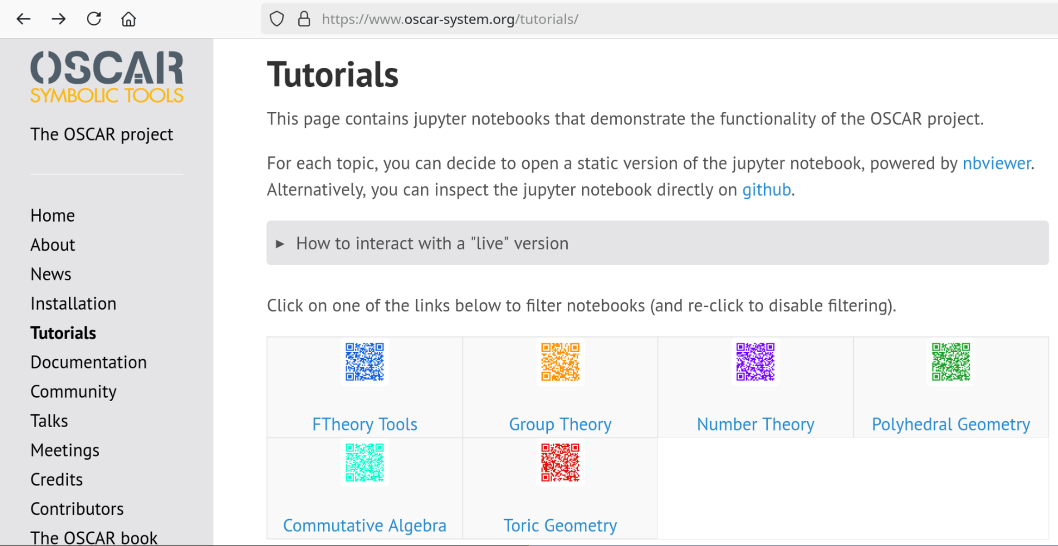

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FTheoryToolsInOSCAR.ipynb

Code — Notebook — Julia 1.9.2

FTheoryTools

- Authors: Martin Bies, Mikelis Emils Mikelsons, Andrew P. Turner
- Version: OSCAR version 1.1.0
- Last modified: June 21st, 2024

This tutorial provides a brief introduction to the *FTheoryTools* component of *OSCAR*.

The aim is to describe some basic constructions. In order to facilitate this introduction, we are very selective in what we present. For a full description of the available functionality, see https://docs.oscar-system.org/stable/Experimental/FTheoryTools/introduction/. It is worth mentioning that *FTheoryTools* is still (as of June 21st, 2024) in its experimental stage in *OSCAR*. Consequently, major changes may happen in the future.

For this tutorial, no prior knowledge of *OSCAR* is necessary.

Content

[1]:
```
using Oscar
```

```
  ___   ____   ____    _    ____
 / _ \ / ___| / ___|  / \  |  _ \   |  Combining ANTIC, GAP, Polymake, Singular
| | | |\___ \| |     / _ \ | |_) |  |  Type "?Oscar" for more information
| |_| | ___) | |___ / ___ \|  _ <   |  Manual: https://docs.oscar-system.org
 \___/ |____/ \____/_/   \_\_| \_\  |  1.1.0 #release-1.1 d0b75de 2024-06-21
```

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FTheoryToolsInOSCAR.ipyn ● +

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1 An SU(5) Tate model from scratch

1.A Construction

We begin by showing how an F-theory model can be created within the F-theory tools. This is showcase this by looking at an $SU(5)$ Tate model over an arbitrary base. To construct this model, we begin with a $\mathbb{P}^{2,3,1}$ fibration over a complex n-dimensional base manifold B, such that the homogeneous coordinates $[x : y : z]$ transform as

$$x \in H^0(B, \bar{K}_B^{\otimes 2})\,, \quad y \in H^0(B, \bar{K}_B^{\otimes 3})\,, \quad z \in H^0(B, \mathcal{O}_B)\,,$$

with K_B the canonical bundle of the base. A Tate model is realized as a hypersurface in this $(n+2)$-dimensional ambient space given by the locus

$$y^2 + a_1 xyz + a_3 yz^3 = x^3 + a_2 x^2 z^2 + a_4 x z^4 + a_6 z^6\,,$$

with $a_i \in H^0(B, \bar{K}_B^{\otimes i})$. These data define an elliptic fibration X over the base B.

For the SU(5) Tate model, we tune the parameters a_i of the model so they become proportional to specific powers of w, with $w = 0$ a divisor in the base B:

$$a_1 = w^0 a_{1,0}\,, \quad a_2 = w^1 a_{2,1}\,, \quad a_3 = w^2 a_{3,2}\,, \quad a_4 = w^3 a_{4,3}\,, \quad a_6 \equiv 0\,.$$

Certainly, we could fix a base. In this case we talk about one explicit geometry/F-theory model. Typically, people in the literature like to study families of models in that the base is left mostly unconstrained. Even so, one would at the very least fix the dimension of the base. We shall follow this example and fix $\dim(B) = 3$.

With `FTheoryTools`, we can create this model with the following steps. First, we create a ring (which you may read as an auxiliary base coordinate ring) whose indeterminates are the parameters needed to define the Tate model in question. Here these parameters are a_{10}, a_{21}, a_{32}, a_{43}, a_{65} and w. So we create the following ring:

[2]:
```
auxiliary_base_ring, (a10, a21, a32, a43, a65, w) = QQ["a10", "a21", "a32", "a43", "a65", "w"]
```

[2]:
```
(Multivariate polynomial ring in 6 variables over QQ, QQMPolyRingElem[a10, a21, a32, a43, a65, w])
```

Note that this command not only defines the polynomial ring, but also defines symbols a10, a21, \dots, w which refer to the indeterminates of this polynomial ring. This will be important momentarily.

As mentioned above, the model parameters a_{ij} transform as sections of the line bundle $\overline{K}_B^{\otimes i} \otimes \mathcal{O}_B(\{w=0\})^{\otimes(-j)}$. Certainly, we must inform `FTheoryTools` about this transformation behavior. To this en, we record the powers i and j in the following matrix:

[3]:
```
auxiliary_base_grading = [1 2 3 4 6 0; 0 -1 -2 -3 -5 1]
```

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Thank you for your attention!