Bounds and Dualities of Type-II Little String Theories

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Based on:

2405.03877	with	P. Oehlmann and F. Rühle
(& 2312.13347	with	C. Lawrie)

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Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Motivation

What are the gravity-decoupling limits of 6d $\mathcal{N} = (1, 0)$ theories?

If interacting theory, two UV possibilities:

Superconformal Field Theories (SCFTs)	Little String Theories (LSTs)
Have no scale	Have a scale M_{LST}
Local theories	Non-local theories
All curves are contractible	Have a non-contractible curve

Motivation

TODAY: focus on LSTs

What are the generic properties of LSTs?

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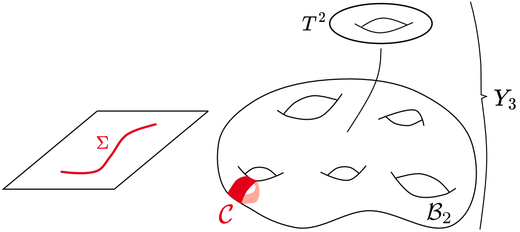
Local theories	Non-local theories
	Have a non-contractible curve

Geometric Engineering

F-theory on elliptically-fibered CY $Y_3 \to B_2$

$\mathbb{R}^6 \times Y_3$

Little string from
D3-branes



- LST: F-theory on Y_3

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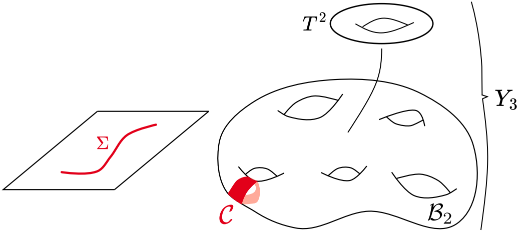

- LST: F-theory on Y_3

  1) Y_3 is elliptically fibered

  2) $\exists!$ 0-curve $\mathcal{C}_{\mathsf{LST}} \cdot \mathcal{C}_{\mathsf{LST}} = 0$

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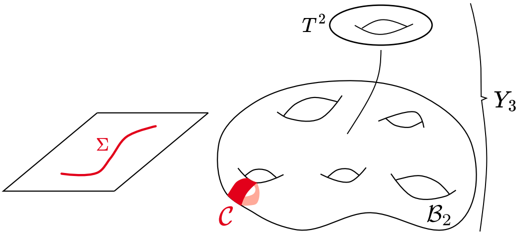

- LST: F-theory on Y_3

Only two types of bases:

[Bhardwaj, Del Zotto, Heckman, Morrison, Rudelius, Vafa '19]

[Ogiso| Kollar| Wilson '93,'94,'12]

Heterotic : $B_2 \sim \mathbb{P}^1 \times \mathbb{C}$ Type II : $B_2 \sim (T^2 \times \mathbb{C})/\Lambda$

T-duality

Due to their stringy nature: LSTs have T-duality

$$\mathcal{K}_1 \overset{\text{T-dual}}{\longleftrightarrow} \mathcal{K}_2 \qquad \Leftrightarrow \qquad \mathcal{K}_1\big|_{S^1} = \mathcal{T}_{5d} = \mathcal{K}_2\big|_{S^1}$$

Important to study LSTs in general

So far mostly with Heterotic LSTs

[Del Zotto, Ohmori, '20]

[Del Zotto, Liu, Oehlmann, '20]^2

[Bhardwaj, '22]

[Mansi, Lawrie, '23]

[Mansi, Sperling, '23]

[Del Zotto, Fazzi, Giri, '23]

[Ahmed, Oehlmann, Rühle '24]

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Proposed T-duality invariants:

$$\text{dim}(\text{CB}_{5d}) = \#(\text{vector}_{6d}) + \#(\text{tensor}_{6d})$$

Flavour rank: $\text{rk}(\mathfrak{f})$

5d HFS: $D_{6d}^{(1)} \times D_{6d}^{(2)} \rightsquigarrow D_{5d}^{(1)}$

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2-group const $\left(\text{Poincaré} \times SU(2)_R \times \mathcal{F}\right)^{(0)} \times U(1)_{\text{LST}}^{(1)}$

$$B_2^{\text{LST}} \longrightarrow B_2^{\text{LST}} + d\Lambda_1 + \kappa_P\, d\omega + \kappa_R\, dA_R + \kappa_F\, dA_F$$

Not all independent, and can help in the hunt for LSTs!

Bounding the Flavour Rank

[FB, Oehlmann, Rühle ’24]

Consider LST with flavour $\mathfrak{f} = \oplus_A \mathfrak{f}^A$

Worldsheet analysis of BPS string associated with 0-curve:

[Kim, Shiu, Vafa ’19]

$$\text{rk}\,\mathfrak{f} \quad \leq \quad 9\,\kappa_P + 2$$

$\Rightarrow$ Flavour bounded by 2-group constants

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Het : $\kappa_P = 2$ rk $\mathfrak{f} \leq 20$ Type II : $\kappa_P = 0$ rk $\mathfrak{f} \leq 2$

Hunting for Type-II LSTs

We know:

Base given by Kodaira singularity $\mathfrak{g}_B$

$$\mathcal{C}_I \cdot \mathcal{C}_J = -A(\widehat{\mathfrak{g}}_B)_{IJ}$$

Fibres very constrained: rk $\mathfrak{f} \leq 2$

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Type-II LSTs labelled by pair of algebras $\mathcal{K}(\mathfrak{g}_F, \mathfrak{g}_B)$:

1) Base Kodaira singularity: $\mathfrak{g}_B$

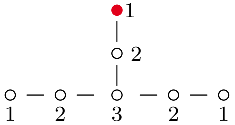
2) Fibre Kodaira singularity: $\mathfrak{g}_F$

3) Read off geometry

[FB, Oehlmann, Rühle '24]

Hunting for Type-II LSTs

Example: $\mathcal{K}(III\,,\mathfrak{e}_6)$



$$\begin{array}{ccccc} & & 2 & & \\ & & 2 & & \\ 2 & 2 & 2 & 2 & 2 \end{array}$$

Hunting for Type-II LSTs

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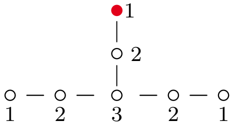

$\overset{\mathfrak{su}_2}{2}$

2

$2\ \ 2\ \ 2\ \ 2\ \ 2$

Affine node: ord$(f, g, \Delta) = (\ \ 1,\ \ 2,\ \ 3)$

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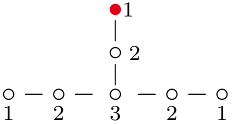

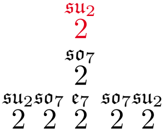

Affine node: ord$(f, g, \Delta) = (\ 1,\ 2,\ 3)$

i-th node: ord$(f, g, \Delta) = (\ell_i 1, \ell_i 2, \ell_i 3)$

ℓ_i = Kac label

Hunting for Type-II LSTs

Example: $\mathcal{K}(III\,,\mathfrak{e}_6)$

$$
\begin{array}{ccccc}
 & & \overset{\mathfrak{su}_2}{2} & & \\
 & & \overset{\mathfrak{so}_7}{2} & & \\
\overset{\mathfrak{su}_2}{2} & \overset{\mathfrak{so}_7}{2} & \overset{\mathfrak{e}_7}{2} & \overset{\mathfrak{so}_7}{2} & \overset{\mathfrak{su}_2}{2}
\end{array}
\quad \longrightarrow \quad
\begin{array}{ccccccccc}
 & & & & \overset{\mathfrak{su}_2}{2} & & & & \\
 & & & & \overset{\mathfrak{so}_7}{3} & & & & \\
 & & & & \overset{\mathfrak{su}_2}{2} & & & & \\
 & & & & 1 & & & & \\
\overset{\mathfrak{su}_2}{2} & \overset{\mathfrak{so}_7}{3} & \overset{\mathfrak{su}_2}{2} & 1 & \overset{\mathfrak{e}_7}{8} & 1 & \overset{\mathfrak{su}_2}{2} & \overset{\mathfrak{so}_7}{3} & \overset{\mathfrak{su}_2}{2}
\end{array}
$$

Paper:
Explicit construction via double elliptic fibration in Tate form.

T-dual Pairs

$$\mathcal{K}(\mathfrak{g}_F\,,\mathfrak{g}_B) \quad \overset{\text{T-dual}}{\longleftrightarrow} \quad \mathcal{K}(\mathfrak{g}_B\,,\mathfrak{g}_F)$$

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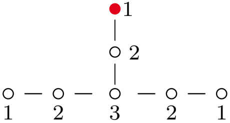
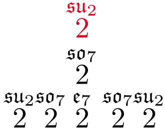
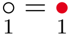




$\mathcal{K}(\mathfrak{e}_6\,, III)$:



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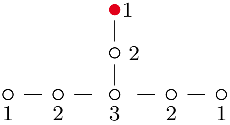

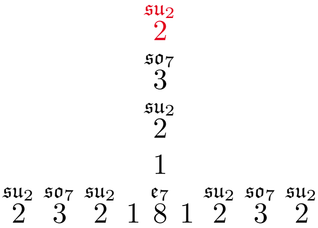

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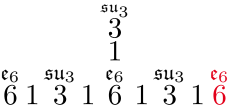

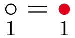

$\kappa_P = 0$ $\quad \kappa_R = 133$ $\quad$ dim(CB) $= 34$

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Higher-form symmetries

$$D^{(1)} = Z(\mathfrak{g}_F), \qquad D^{(2)} = Z(\mathfrak{g}_B)$$

$$D^{(1)}_{6d} \times D^{(2)}_{6d} \rightsquigarrow D^{(1)}_{5d}$$

$\mathfrak{g}_F, \mathfrak{g}_B$ not II, III, IV:

$$\dim(\mathrm{CB}_{5d}) = h^\vee_{\mathfrak{g}_F} h^\vee_{\mathfrak{g}_B} - 1 \qquad \kappa_R = \Gamma_{\mathfrak{g}_F} \Gamma_{\mathfrak{g}_B}$$

$$\Gamma_{\mathfrak{g}} = h^\vee(h^\vee - 2)/(\mathrm{rk}\mathfrak{g} - 1)$$

Conclusions

LSTs are constrained by T-duality invariants

$$\text{rk}\,\mathfrak{f} \leq 9\,\kappa_P + 2$$

Constructed all T-dual pairs for Type-II LSTs

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Interesting group-theoretical patterns

$\longrightarrow$ Some quantities do not depend on details of quiver.

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Thank you for your attention