

On a new democratic, $SL(2,R)$ -invariant formulation of type IIB

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String Phenomenology 2024 – Padova - 27/06/2024

Based on arXiv:2401.00549



"la Caixa" Foundation



Setup:

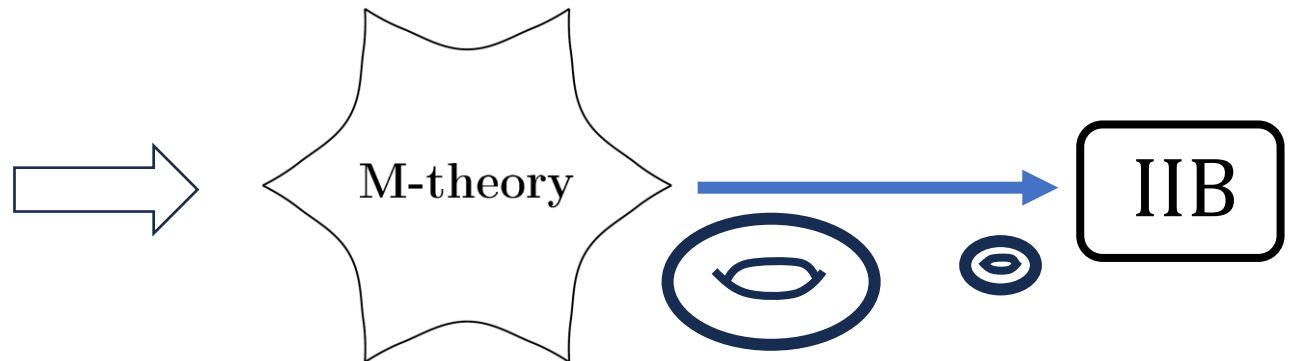
Effective Bosonic sector of
type IIB

$$\Rightarrow \left\{ g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(0)}, C_{\mu\nu}^{(2)}, C_{\mu\nu\rho\sigma}^{(4)} \right\}$$

Democratic formulation

$$\Rightarrow \left\{ C_{\mu\dots\sigma}^{(6)}, C_{\mu\dots\sigma}^{(8)} \right\}$$

$SL(2, R)$ Duality



In a Nutshell

We propose a new **democratic** formulation of **type IIB** (bosonic) effective action with explicit **$SL(2, R)$** duality

[M.Z., T. Ortin, C. Gomez-Fayren, G. Giorgi, J. J. Fernandez-Melgarejo 24]

Motivations:

D7 branes couplings, fluxes compactifications, it's funny, ...

Why is it not trivial?

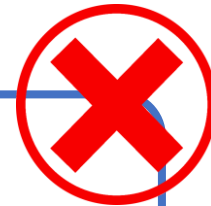
Introduction of magnetic
duals of RR fields

$$G^{(2n+1)} = (-1)^n \star G^{(9-2n)}$$

[E. Bergshoeff, R. Kallosh, T. Ortin, D.
Roest, A. Van Proyen 01]



$SL(2, R)$ Duality



Democratic formulation



How do fields transform?

$$\left\{ g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(0)}, C_{\mu\nu}^{(2)}, C_{\mu\nu\rho\sigma}^{(4)} \right\} \longrightarrow \left\{ g_{E\mu\nu}, \tau, \mathcal{B}_i, D \right\}$$

$$D = C^{(4)} - \frac{1}{2} B \wedge C^{(2)}$$

$$\tau = C^{(0)} + ie^{-\phi}$$

$$\mathcal{B}_i = \begin{pmatrix} C^{(2)} \\ B \end{pmatrix}$$

$$\begin{array}{lcl} g_{E\mu\nu}, D & \longrightarrow & \mathbf{1} \\ \tau & \longrightarrow & \frac{a\tau + b}{c\tau + d} \\ \mathcal{B}_i & \longrightarrow & \mathbf{2} \end{array}$$

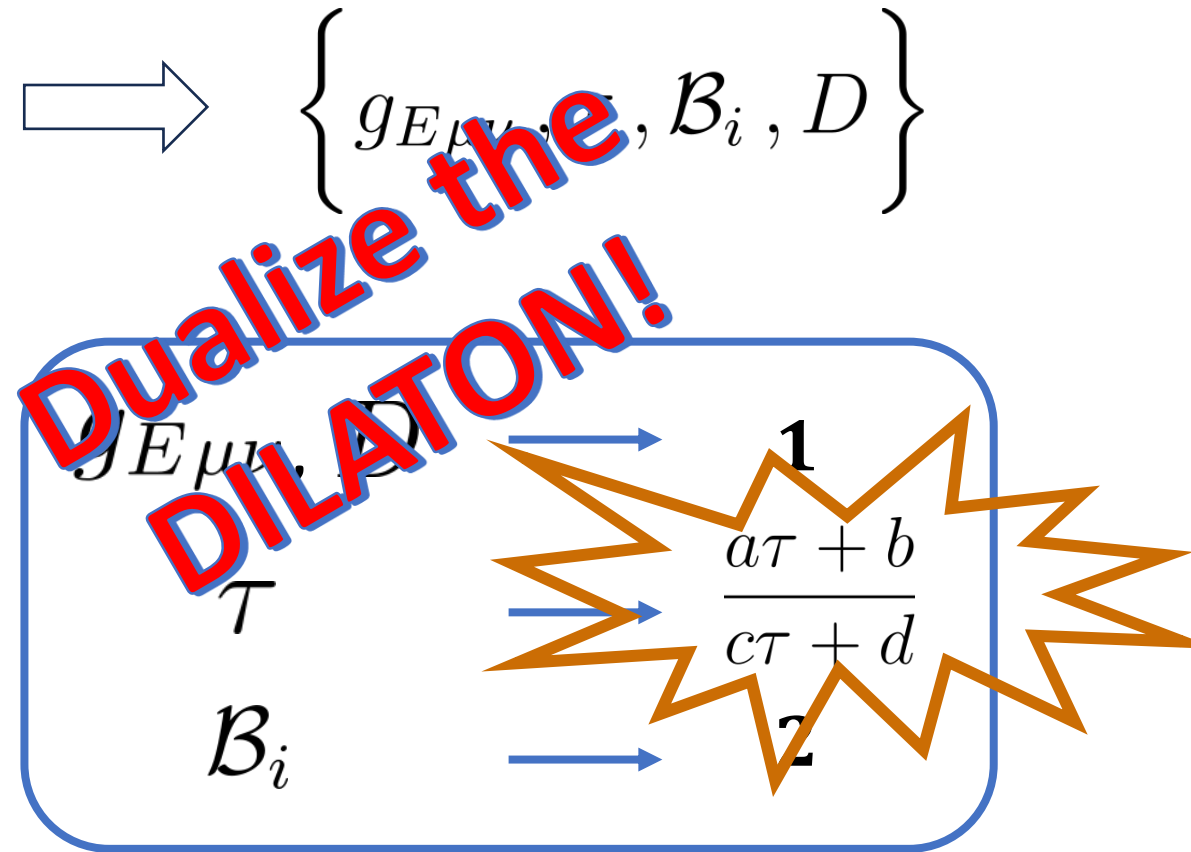
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$$\mathcal{B}_i = \begin{pmatrix} C^{(2)} \\ B \end{pmatrix}$$



Scalar Fields dualization in a toy model

Action depends **only** on the **field strength**



Poincare' duality

$$L = \frac{1}{2} d\phi \wedge \star d\phi \longrightarrow d \star d\phi = 0 \longrightarrow G = \star d\phi$$

Scalar fields **couplings**

$$L = \frac{1}{2} g_{xy}(\phi) d\phi^x \wedge \star d\phi^y \longrightarrow D \star d\phi^x = 0 \longrightarrow ?$$

Idea 1

$$g_{xy}(\phi) \text{ Isometries} \longrightarrow d \star K_A = 0 \longrightarrow G_A = \star K_A$$
$$K_A = K_A^x \frac{\partial}{\partial \phi^x}$$

We can define the **duals** via the scalar metric **isometries**!

Application

$$S[e^a, \phi^x] = \int (-1)^{d-1} \star (e^a \wedge e^b) \wedge R_{ab} + \frac{1}{2} g_{xy}(\phi) d\phi^x \wedge \star d\phi^y$$

In the case of scalar manifolds with the structure of **symmetric coset spaces** G/H

$$S_{\text{Dem}}[e^a, \phi^x, C_A] = \int \left\{ (-1)^{d-1} \star (e^a \wedge e^b) \wedge R_{ab} + \frac{1}{4} g_{xy} d\phi^x \wedge d\phi^y \right. \\ \left. + \frac{(-1)^d}{4} \mathfrak{M}^{AB} G_A \wedge \star G_B - \frac{(-1)^d}{2} g^{AB} G_A \wedge \hat{k}_B \right\},$$

Application

$$S[e^a, \phi^x] = \int (-1)^{d-1} \star (e^a \wedge e^b) \wedge R_{ab} + \frac{1}{2} g_{xy}(\phi) d\phi^x \wedge \star d\phi^y$$

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$$\mathfrak{M}^{AB} = g^{AC} g^{BD} k_C^x k_D^y g_{xy}$$

NEW

Democratic Type IIB fields

$$G/H = \text{SU}(1, 1)/\text{U}(1)$$

2 scalars: $\phi, C^{(0)}$

3 isometries: $C_A^{(8)} \longrightarrow$

Triplet transforming
in the adjoint of
 $\text{SL}(2, \mathbb{R})$

$$\left\{ g_{E\mu\nu}, \tau, \mathcal{B}_i, D, \mathcal{B}^i, C_A^{(8)} \right\}$$

Democratic Type IIB action

$$\begin{aligned}
 S_{\text{Dem}}[e^a, \tau, \mathcal{B}_i, D, \mathcal{B}^i, C_A] = & \int \left\{ - \star (e^a \wedge e^b) \wedge R_{ab} + \frac{1}{4} g_{xy} d\phi^x \wedge \star d\phi^y + \frac{1}{4} \mathcal{M}^{ij} \mathcal{H}_i \wedge \star \mathcal{H}_j \right. \\
 & + \frac{1}{4} \mathcal{F} \wedge \star \mathcal{F} + \frac{1}{4} \mathcal{M}_{ij} \mathcal{H}^i \wedge \star \mathcal{H}^j + \frac{1}{4} \mathfrak{M}^{AB} G_A \wedge \star G_B \\
 & \left. - \frac{1}{2} g^{AB} G_A \wedge \star \hat{k}_B + \frac{1}{4} \varepsilon^{ij} \mathcal{D} \wedge \mathcal{H}_i \wedge \mathcal{H}_j \right\} . \quad G_A = \star \hat{k}_A
 \end{aligned}$$

$$\mathcal{H}_i = \mathcal{M}_{ij} \star \mathcal{H}^j$$

$$\begin{aligned}
 G_A = & dC_A + \frac{1}{2} (T_A)^i{}_k \mathcal{B}_i \wedge \mathcal{H}^k - \frac{1}{2} (T_A)^k{}_i \mathcal{B}^i \wedge \mathcal{H}_k \\
 & - \frac{1}{24} (T_A)^i{}_k \varepsilon^{kj} \varepsilon^{mn} \mathcal{B}_i \wedge \mathcal{B}_j \wedge \mathcal{B}_m \wedge \mathcal{H}_n
 \end{aligned}$$

$$\mathcal{H}^i = d\mathcal{B}^i - \varepsilon^{ij} \mathcal{B}_j \wedge (d\mathcal{D} - \frac{1}{6} \varepsilon^{kl} \mathcal{B}_k \wedge \mathcal{H}_l)$$

$$(\mathcal{M}_{ij}) \equiv \frac{1}{\Im \tau} \begin{pmatrix} |\tau|^2 & \Re \tau \\ \Re \tau & 1 \end{pmatrix}$$

Idea 2

$$L = \frac{1}{2} g_{xy}(\phi) d\phi^x \wedge \star d\phi^y + \dots$$

$$K_A = K_A^x \frac{\partial}{\partial \phi^x}$$



$$d \star K_A + \dots = 0$$

If the isometries are part of the a duality group we can use **Noether-Gaillard-Zumino** theorem

$$dC_A = \star K_A + \dots$$