On the landscape of 6d supergravities

Gregory J. Loges

String Pheno 2024

[2311.00868], [2402.04371], [2404.08845] w/ Yuta Hamada

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KEK

The plan

- 6d, $\mathcal{N} = (1,0)$ supergravity
- Enumerating anomaly-free models
- An overview of the landscape
- Consistent models for $T \leq 1$ and $T \gg 9$
- Conclusions

6d, $\mathcal{N} = (1, 0)$ supergravity

gravity : (g, Ψ, B^+)

vector : (A, λ)

tensor : (B^-, χ, ϕ)

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Pick $(T, G, \mathcal{H}_{\text{ch}})$:

- # tensors $T \geq 0$
- Gauge group $G = \prod_{i=1}^{k} G_i$
- Charged hypermultiplets $\mathcal{H}_{\text{ch}} = \bigoplus_{R \neq 1} n_R \times (R_1, \ldots, R_k)$

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We will take G non-abelian with any number of simple factors and allow for any hypermultiplet representations

Consistency conditions

- Local anomalies:
  $I_8 = \frac{1}{2}\Omega_{\alpha\beta} X_4^\alpha \wedge X_4^\beta , \quad X_4 = b_0 \operatorname{tr} \mathcal{R}^2 + \sum_i b_i \operatorname{tr} F_i^2$

$$H_{\mathrm{ch}} - V + 29T \leq 273$$

$$b_0 \cdot b_0 = 9 - T\,, \qquad b_i \cdot b_i = \frac{1}{3}\Big(\sum\nolimits_R n_R^i C_R - C_{\mathrm{Adj}}\Big)\,, \ldots$$

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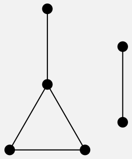
- "Positivity": $j \in \mathrm{SO}(1,T)/\,\mathrm{SO}(T)$ with $j \cdot b_i > 0$ so that all gauge kinetic terms have the correct sign

Strategy for enumerating models

[Hamada, GL – 23/24] , building upon [Avramis, Kehagias – 05] [Kumar, Park, Taylor – 10] [Becker et al. – 23] . . .



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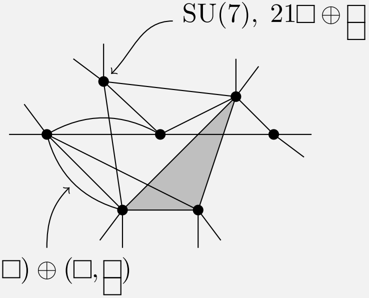

$$\mathcal{M} = \bigoplus_n \mathcal{M}_n^{\text{irr}}$$

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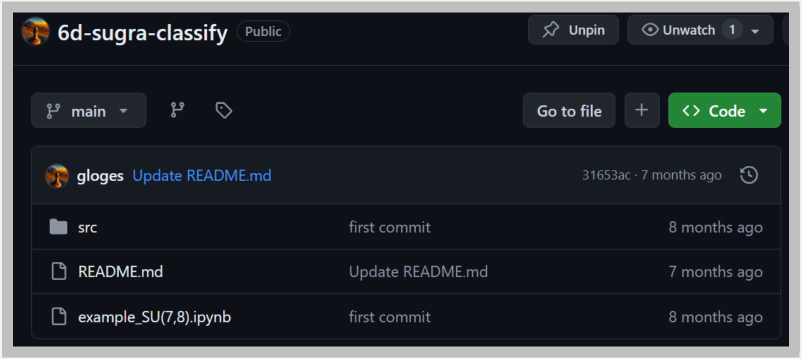
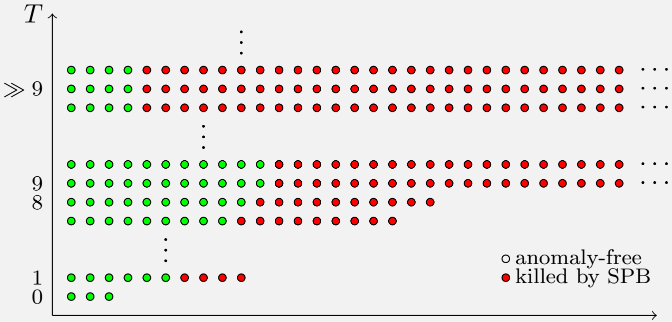


An overview of the landscape



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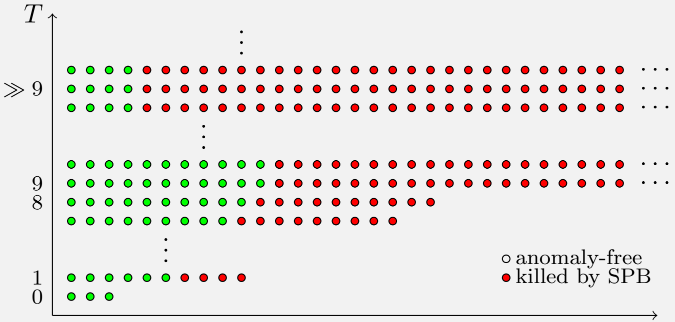

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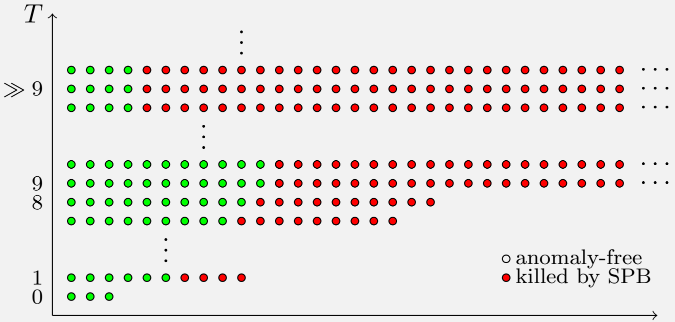

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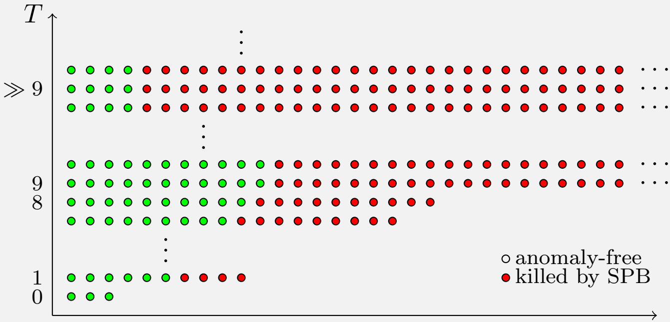

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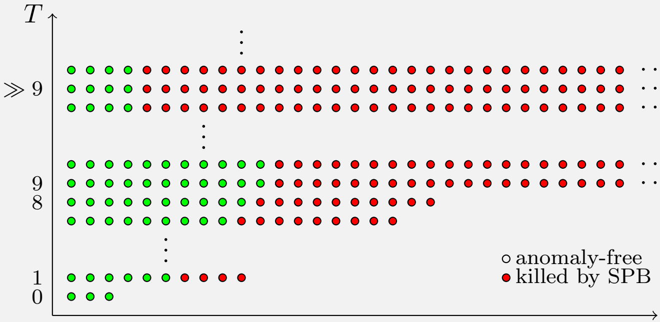

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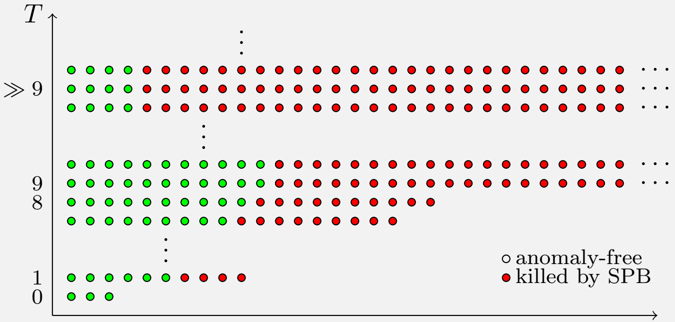

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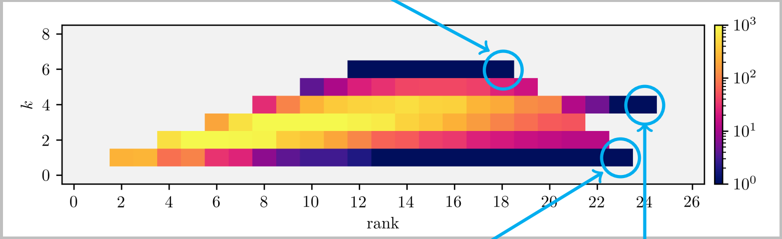
  (combining [Kim, Shiu, Vafa – 19] [Tarazi, Vafa – 21] [Hamada, GL – 23])
- Some consistent infinite families exist with T unbounded [GL – 24]

$T = 0$

Complete list of 19,847 anomaly-free models [Hamada, GL – 24]
(with no U(1), SU(2), SU(3), Sp(2) gauge factors)

$$\mathrm{SU}(4)^6 \quad + \quad (\underline{\mathbf{4}}, \underline{\mathbf{4}}, \underline{\mathbf{1}}^4) \oplus 3(\underline{\mathbf{6}}, \underline{\mathbf{1}}^5) \oplus \cdots$$



$$\mathrm{SU}(24) \quad + \quad 20 \times \underline{\mathbf{1}} \oplus 3 \times \underline{\mathbf{256}}$$

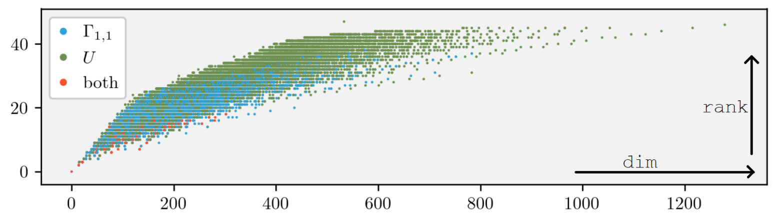
$$\mathrm{SU}(7)^4 \quad + \quad (\underline{\mathbf{7}}, \underline{\mathbf{7}}, \underline{\mathbf{1}}, \underline{\mathbf{1}}) \oplus (\underline{\mathbf{21}}, \underline{\mathbf{1}}^3) \oplus \cdots$$

Exactly 20 models have (3+)-charged hypers, e.g.

$$\mathrm{SU}(4) \times \mathrm{Sp}(4)^2 \quad + \quad (\underline{\mathbf{4}}, \underline{\mathbf{8}}, \underline{\mathbf{8}}) \oplus (\underline{\mathbf{20'}}, \underline{\mathbf{1}}, \underline{\mathbf{1}}) \oplus (\underline{\mathbf{1}}, \underline{\mathbf{42}}, \underline{\mathbf{1}}) \oplus (\underline{\mathbf{1}}, \underline{\mathbf{1}}, \underline{\mathbf{42}})$$

$T = 1$

Complete list of 608,355 anomaly-free models (with no U(1), SU(2), SU(3), Sp(2) gauge factors) [Hamada, GL – 24]



Swampland bounds

- BPS string probes & anomaly inflow: $Q \in \Gamma_{1,T}$, $k_i = Q \cdot b_i \geq 0$

$$\sum_i \frac{k_i \, \dim G_i}{k_i + h^\vee_{G_i}} \leq c_L \quad (= 3Q^2 + 9Q \cdot b_0 + 2)$$

[Kim, Shiu, Vafa – 19]

- Two swampland bounds proposed in [Tarazi, Vafa – 21] limit the allowed hypermultiplet representations:

$$\sum_{m=1}^{r} \lambda_m \leq k_i \,, \qquad \Delta_R = \frac{C_2(R)}{2(k_i + h_i^\vee)} \leq 1$$

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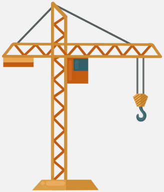
We find that $\approx$50% of anomaly-free models are consistent for $T = 1$
(Work in progress: for those which remain, how many/which can be matched to known string constructions?)

$T \gg 9$

- $\big[H_{\rm ch} - V + 28\,{\rm rk}(b_i \cdot b_j)\big](\mathcal{M}^{\rm irr}) > 0$ with only 8 exceptions $\implies$ any infinite family with $T \gg 9$ must be of a form like

$$G = G'(k) \times E_8^k\,, \qquad \mathcal{H}_{\rm ch} = (\mathcal{H}'_{\rm ch}(k), \underline{\mathbf{1}}^k)$$



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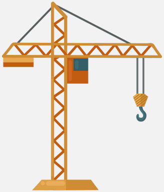

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- With a scaling argument,

$$Q \sim k^\alpha\,, \qquad b'_i \cdot b'_j \sim k^\beta\,, \qquad b_i^{E_8} \cdot b_j^{E_8} \sim k^0\,, \qquad \dots$$

  and using existence of j with $j \cdot j = 1$ and $j \cdot b_i > 0$, can show that a consistent infinite family…
  - with $n_+(b_i \cdot b_j) = 0$ must have $b_0 \in \text{span}(b_i)$ (fixes T)
  - with $n_+(b_i \cdot b_j) = 1$ must have $T - 9 + (b_0 \cdot b_i)(b \cdot b)^{-1}_{ij}(b_0 \cdot b_j) \geq 0$ bounded $\implies$ T must grow with k in a prescribed way

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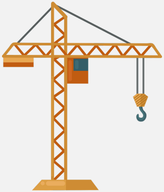

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- Idea: some interplay between the highly constrained form of the surviving families and potential global Dai-Freed anomalies

(e.g. see [Basile, Leone – 23])

Summary

- 6d SUGRA + local anomalies, unimodularity, positivity
- Graph algorithms + iterative construction of models
- Complete lists of anomaly-free models for $T \leq 1$ (modulo low-rank groups)
- Swampland bounds: *all* anomaly-free models are consistent for $T = 0$ and *half* are consistent for $T = 1$
- For $T \gg 9$ the swampland bounds grow stronger and stronger.
  The surviving infinite families are highly constrained.
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Proof of finiteness for all T may be within reach – stay tuned!

Thanks!

$$G = G_2 \times E_8^{4m}$$
$$\mathcal{H} = (140m + 440)(\underline{\mathbf{1}}; \underline{\mathbf{1}}^{4m}) \oplus (72m - 26)(\underline{\mathbf{7}}; \underline{\mathbf{1}}^{4m})$$
$$T = 12m + 1$$

$$\begin{aligned}
b_0 &= (3; 1, 1^{12m}) \\
b^{G_2} &= (3m + 2; -3m + 4, 1^{12m}) \\
b^{E_8}_{4r+1} &= (-1; 1, 0^{12r}, -1, \quad 1, \quad 1, \quad 1, 2, 2, 0, 0, 0, 0, 0, 0, 0^{12(m-r-1)}) \\
b^{E_8}_{4r+2} &= (-1; 1, 0^{12r}, \quad 1, -1, \quad 1, \quad 1, 0, 0, 2, 2, 0, 0, 0, 0, 0^{12(m-r-1)}) \\
b^{E_8}_{4r+3} &= (-1; 1, 0^{12r}, \quad 1, \quad 1, -1, \quad 1, 0, 0, 0, 0, 2, 2, 0, 0, 0^{12(m-r-1)}) \\
b^{E_8}_{4r+4} &= (-1; 1, 0^{12r}, \quad 1, \quad 1, \quad 1, -1, 0, 0, 0, 0, 0, 0, 2, 2, 0^{12(m-r-1)})
\end{aligned}$$