Chiral anomalies and their implications in 10d non-supersymmetric $\mathfrak{SP}(16)$ gauge theories



Work in progress with Ling Lin

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String Phenomenology 2024 Padova, June 25







[Basile, Debray, Delgado, and Montero '23]





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There are 3 non-supersymmetric 10d models without tachyons: Heterotic, Sagnotti [Alvarez-Gaume, Ginsparg, Moore, Vafa '86/ Dixon, Harvey '86] [Sagnotti '95/ Sagnotti '97]





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Sugimoto model The (other) orientifold of Type IIB

- Anomaly cancellation: O9 + 16D9 \implies Brane supersymmetry breaking
- Open string sector: $\mathfrak{sp}(16)$ gauge algebra.
- Chiral massless spectrum: Gravitino, Dilatino and 495 (anti-symmetric).
- Two dual RR gauge fields B_2 and $B_6 \implies$ charged D1 and D5- branes.

[Antoniadis, Dudas, Sagnotti '99/ Angelantonj '00/ Aldazabal, Uranga '99/ Angelantonj, Antoniadis, D'Appollonio, Dudas, A. Sagnotti '00]

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SUSY theories are (perturbative) anomaly free

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Form of the chiral spectrum

Global structure of the gauge group





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- **Bottom up**: constrain 10d $\mathfrak{sp}(16)$ gravitational QFTs by means of anomaly cancellation



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Quantified via the Anomaly Polynomial: $I_{d+2} = X_{d+2} + X_2 \wedge X_d + X_4 \wedge X_{d-2} + \dots$

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Anomaly inflow: the coupling term of a defect

contributes to its anomaly and must be canceled by dynamical d.o.f. on world-volume.

'' p+1

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[Callan, Harvey '85/ Blum, Harvey '94/ Freed, Harvey, Minasian, Moore '98/ Lawrie, Schäfer-Nameki, Weigand '17/ Kim, Shiu, Vafa '19/ Angelantonj, Bonnefoy, Condeescu, Dudas '20/ Martucci, Risso, Weigand '23/ Hamada, Vafa '21/ Bedroya, Hamada, Montero, Vafa '21/ Hamada, Loges '23 & '24/ Loges '24/...]















D5-brane



 $I_8 = X_8 + X_4 \wedge Y_4 \xrightarrow{\text{descent}} -\delta S_{D5}.$





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Bottom-up perspective

Sugimoto model: a priori $Sp(16)/\mathbb{Z}_2$ is allowed by 10d spectrum, only the brane fundamental d.o.f. forces the gauge group to be Sp(16).

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• $Sp(16)/\mathbb{Z}_2$ has no 1-form symmetry anomaly.

• The chiral spectrum must be \mathbb{Z}_2 invariant

$$\implies I_{12} = n_1 I_{121} + n_g I_{12g} + n_{\Box} I_{12}$$

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[Apruzzi, Dierigl, Lin '20/ Cvetič, Dierigl, Lin, Zhang '20]







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Green-Schwarz: $X_{12} = 0 \implies n_{\Box} = 0 \text{ and } n_1 = 495n_g - 496n_{\Box}$

Constraints for $\mathfrak{sp}(16)$ **theories**



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$$\tilde{X}_4 = \frac{1}{2} [trR^2 - \beta trF^2] \quad \textbf{Imposing } I_{12} = \tilde{X}_4 \land \tilde{X}_8 \implies n_g = n_{\Box} \text{ and } \beta = 1.$$

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Anomaly inflow: The 1-brane forces $n_{\rm f}$

$$P_{2g} + n_{\Box} I_{12\Box} + n_{\Box} I_{12\Box}.$$

$$P_{\Box} = 0 \quad \text{and} \quad n_1 = 495n_a - 49$$

A For the structure of $\mathfrak{sp}(16)$ this is the only possible factorization $\implies \exists B_2$ and charged 1 and 5 branes

$$\square^W \neq 0$$
 and the 5-brane requires $n_{\square} = 1$

 $I_{12} = n_1 I_{121} + n_q I_{12}$



(2) Sp(16) is THE group of this class of theories!

$$_{2g} + n_{I_{12}} + n_{I_{12}} + n_{I_{12}}$$

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Results

- (1) The only possible (minimal) 10d realization of $\mathfrak{sp}(16)$ is the Sugimoto model!



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Anomaly cancellation in space-time



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 \times Obstruction to anomaly inflow onto 1-brane





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Outlook

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Thank you!

