Uncovering the Non-supersymmetric Heterotic String Landscape

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String Phenomenology (25-06-2024)

Centro Culturale Altinate - San Gaetano - Padova



Based on:

- [2307.13745] [BF, M. Graña, H. Parra de Freitas, S. Sethi]
- Upcoming work: [BF, H. Parra de Freitas]

Some things being worked on:

- [BF, M. Graña, H. Parra de Freitas]
- [BF, I. Ruiz, I. Valenzuela]
- [S. Baines, V. Collazuol, BF, M. Graña, D. Waldram]



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Underlying **structure** for large number of space-time dimensions and supercharges.

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Most top-down evidence has been obtained from the supersymmetric landscape:

Could these nice structures and properties be just a **SUSY lamppost effect**?

Let's **break** SUSY and find out!

It is possible to **break** SUSY at the **string scale**

Recent resurgence in interest:

[Abel, Acharya, Aldazabal, Angelantonj, Basile, Baykara, Condeescu, Cribiori, Debray, Delgado, Diaz Avalos, Dudas, Faraggi, Florakis, Font, BF, Graña, Itoyama, Kaidi, Koga, Lanza, Leone, Matyas, Montero, Nakajima, Narain, Parameswaran, Parra de Freitas, Percival, Raucci, Robbins, Sagnotti, Sethi, Tarazi, Tonioni, Vafa, Wrase, etc.]

Most of the structure of SUSY landscape can be generalized under certain SUSY breakings! More compactifications, richer spectra

Possible ISSUES?

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 Many extrema of this potential found for non-SUSY heterotics but yet no minima Why is this? Try to identify general patterns!

Heterotic theories in 10D:

[Dixon, Harvey; Alvarez-Gaume, Ginsparg, Moore, Vafa; Kawai, Lewellen, Tye; Seiberg, Witten '86]

Theory	Gauge symmetry	Tachyons
HE	$E_8 \times E_8 \ltimes \mathbb{Z}_2$	0
HO	$rac{Spin(32)}{\mathbb{Z}_2}$	0
$O(16) \times O(16)$	$\frac{Spin(16)^2}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0
E_8	E_8	1
U(16)	$\frac{SU(15) \times U(1)}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	2
$(E_7 \times SU(2))^2$	$\frac{(E_7 \times SU(2))^2}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	4
$O(24) \times O(8)$	$\frac{Spin(24) \times Spin(8)}{\mathbb{Z}_2}$	8
$E_8 \times O(16)$	$E_8 \times Spin(16)$	16
O(32)	Spin(32)	32

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<u>Massless</u> **bosons**: G, B, ϕ plus gauge fields in the adjoint of the corresponding gauge group

<u>Massless fermions</u>: (e.g. O(16) x O(16)) Half of them transform in the spinor rep. of only one SO(16), or the other. and the rest transform in the bi-fundamental rep. of SO(16) x SO(16)

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Non-SUSY theories include a quantum potential:

$$\int d^{10}x \sqrt{-g} \Lambda \qquad \Lambda_{1-\text{loop}} = \int \frac{d^2\tau}{\tau_2^2} Z(\tau)$$

[Dixon, Harvey; Alvarez-Gaume, Ginsparg, Moore, Vafa; Kawai, Lewellen, Tye; Seiberg, Witten '86]

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 $\Lambda > 0$ for known non-tachyonic rigid theories [Baykara, Tarazi, Vafa '24]

 $\Lambda \rightarrow -\infty$ when **tachyons**

O(16) x O(16) on S¹



O(16) x O(16) on S¹



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States separated in **sectors**:

- <u>Untwisted</u>: Γ_v (gauge **bosons**, appear at boundaries) Γ_s (fermions)
- <u>Twisted</u>: Γ_c (fermions) Γ_0 (scalars or tachyons)

Gauge bosons $M_{\text{gauge boson}}^2 \ge 0$







Enhancement to non-abelian symmetry.

 $M_{\text{gauge boson}}^2 \ge 0$



0.0

0.0

0.5

1.0

 $A = (a, 0^7, a, 0^7)$

1.5

2.0

Enhancement to **non-abelian** symmetry. Classification from embeddings in Γ_{charge} Full rank: [BF, Graña, Parra de Freitas, Sethi '23] Rank reduced by 8: [BF, Parra de Freitas WIP]

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Enhancement to **non-abelian** symmetry at Classification from embeddings in Γ_{charge} Full rank: [BF, Graña, Parra de Freitas, Sethi '23] Rank reduced by 8: [BF, Parra de Freitas WIP] Part of an **infinite tower** of states (distance conjecture [Ooguri, Vafa, '07]) Related swampland conjectures in non-SUSY setup? [BF, Ruiz, Valenzuela WIP]

Fermions: very similar* to gauge bosons

*they do not necessarily become massless at fixed points of T-duality

What about the tachyons?

Tachyons

$M_{\rm scalar}^2$? Can be negative at some regions

 $M_{\text{scalar}}^2 = M_{\text{min}}^2 < 0$ could happen in the bulk or in the boundary of parameter space

 M^2 $M^2 \propto p_R^2 - 1$ R



Tachyons if $p_R^2 < 1$ massless scalars if $p_R^2 = 1$



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Part of an **infinite tower** of tachyons becoming **extremal**.

They signal a tachyonic higher dim. theory!

What happens to the quantum potential after compactifying?

Is it still positive? Is there some minimum?

One-loop potential

One-loop potential

$$\Lambda_{1-\text{loop}}(R, A) = \int \frac{d^2\tau}{\tau_2^2} Z(\tau, R, A)$$
Quantum potential now depends on compact geometry (R,A) $Z \sim \sum q^{p_L^2(R,A)} \bar{q}^{p_R^2(R,A)}$
One-loop potential

Quantum potential now depends on compact **geometry (R,A)** $Z \sim \sum_{k=1}^{N} q^{p_{L}^{2}(R,A)} \bar{q}^{p_{R}^{2}(R,A)}$



 $\Lambda_{1-\text{loop}}(\mathbf{R},\mathbf{A}) = \int \frac{d^2\tau}{\tau_2^2} \mathbf{Z}(\tau,\mathbf{R},\mathbf{A})$

Bosons, fermions and scalars give finite contributions, tachyons make it diverge.

 $A = (a, 0^7, a, 0^7)$



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No, it can be **negative** close to SUSY restoration decomp limits! ($\Lambda \rightarrow 0$)

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It is **extremized** at **maximal enhancement** points [Ginsparg, Vafa '87]

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- All extrema found to be **unstable** [BF, Graña, Parra de Freitas, Sethi '23] Satisfy **dS conjecture** $\frac{\min(\nabla_i \nabla_j V)}{V} \leq -O(1)$

[Andriot '18] [Ooguri, Palti, Shiu, Vafa '18]

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Dec. limit to **D < 9**: more extrema... still **not minima**? [BF, Graña, Parra de Freitas WIP] **SO(32) SUSY** string

8 (all rank 16 theories in 10D)

Affine subdiagrams of EDD:









[Baines, Collazuol, BF, Graña, Waldram]

Veronica Collazuol's talk on Thursday

Rank reduced non-SUSY theories

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Compactifying on torus gives more and more **parameters**

Compactifying on orbifolds gives **less parameters** (compared to full rank)

e.g. CHL string [Chaudhuri, Hockney, Lykken '95]

 \rightarrow more likely for stable points to exist!

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Systematic SUSY vacua construction carried out long ago [de Boer et al '01]

New non-SUSY theory with reduced rank found [Nakajima '23]

(without generic tachyons)

Can the SUSY structures and properties be generalized to non-SUSY string compactifications?

Systematic approach for non-SUSY theories

> 9D $O(16) \times O(16)$ non-SUSY string

10D $E_8 \times E_8$ SUSY string

SS reduction:

 $(-1)^F$ holonomy along some cycle (breaks SUSY) [Scherk, Schwarz '79]



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Rank reduction orbifold:

Holonomy (e.g. \mathbb{Z}_2) along some cycle, acting on internal CFT (e.g. $E_8 \times E_8 \rightarrow E_8$) (reduces rank of gauge symmetries, reduces number of parameters)

[Chaudhuri, Polchinski '95]



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We combine the two: SUSY **breaking** and rank **reduction**

Many possibilities already in 9D, even more for lower dimensions!

Non-SUSY theories where the rank is reduced by 8:

- E_8 string (D ≤ 10) \rightarrow Generic tachyons
- B_{IIb} string (D \leq 9) \rightarrow Some tachyon-free regions, some tachyonic regions
- B_{IIa} string (D \leq 9) \rightarrow Generic tachyons
- B_I string (D ≤ 8) \rightarrow Some tachyon-free regions, some tachyonic regions

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Parameter space: $\frac{O(8+d,d)}{O(\Gamma_{(8+d,d)}) \times O(8+d) \times O(d)}$, with $\Gamma_{(8+d,d)}$ depending on the theory

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For **9D** we can get all maximal **enhancement** points and their symmetries constructing their **Extended Dynkin Diagrams**:



For $D \leq 8$ it is done by finding embeddings in the charge lattice and respecting the conditions for the charges imposed by the **holonomies**.

Is the matter content similar to the O(16)xO(16) case?

(Fermions transforming in the spinor or bi-fundamental representations of the gauge group).

Matter content

E₈ theory on a circle (9D)



Gauge bosons with maximal enhancement $E_{8-n} \times Sp(n+1)$, Fermions in the **adjoint** of E_{8-n} Fermions in the **antisymmetric traceless** rep. of Sp(n+1)

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BIIb theory (9D)

Gauge bosons with maximal enhancement $E_{8-n} \times SO(2n+2)$, Fermions in the **adjoint** of E_{8-n} Fermions in the **symmetric traceless** rep. of SO(2n+2)For *n* odd, there are spinors in some **(bi)fundamental** rep.

B_{IIb} string in 9D:



7 maximal enhancements

3 points at infinity. Are the two points with \hat{E}_8 equivalent?

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7 maximal enhancements

10D decomp.:

- O(16)xO(16) string
- **3 points at infinity.** Are the two points with \hat{E}_8 equivalent? \rightarrow No! $E_8 \times E_8$ string
 - E₈ string

B_{IIb}: One-loop potential

Only 2 tachyon-free maximal enhancements:

• SO(16) x SU(2) with $\Lambda \sim 312$ (massless scalars \rightarrow knife-edge)

• **SO(18)** with $\Lambda \sim 308$ (no massless scalars)

Same order of magnitude as in 9D O(16)xO(16) string.

No minima, but could be used to **construct stable vacua**.

Many more possibilities when compactifying to lower dimensions.

By analyzing infinite distance limits, we discover connections between new and old theories:



- T-dual descriptions of same theory.
- Interpolating models connecting higher-dimensional theories

Conclusions and Outlook

- Novel corners in the non-SUSY landscape can be reached using known and simple techniques
- Subset of theories with rank reduced by **8**: interesting features:
 - → symplectic groups at 10D, fermions transforming in sym and anti-sym traceless rep.
- → interpolating models, **new** constructions
- → freezing relations between full rank SUSY and some of the reduced rank non-SUSY theories [WIP]
- One-loop **potential** behaves similar to the full-rank cases: could be **stabilized**
- Structure of **SUSY** landscape → **non-SUSY** counterparts

Grazie mille!

Gauge symmetry	A	N_v	N_s	N_c	N_0	$N_{ m t}$	Λ
$SU(5) \times SO(10)$	$\left(0^3, \frac{1}{5}^4, \frac{4}{5}\right)$	76	78	0	0	30	-∞
$SO(8) \times SO(10)$	$\left(0^4, \frac{1}{4}^3, \frac{3}{4}\right)$	80	80	80	0	16	-∞
$SU(2) \times SU(3) \times SO(12)$	$\left(0^2, \frac{1}{6}^5, \frac{5}{6}\right)$	84	88	64	0	52	$-\infty$
$SU(2) \times SO(16)$	$\left(\frac{1}{8}^{7}, -\frac{7}{8}\right)$	130	138	128	128	0	312
SO(18)	$\left(\frac{1}{6}^7, \frac{5}{6}\right)$	160	170	0	0	0	308
$E_6 \times SU(4)$	$\left(0^{5}, \frac{1}{3}^{2}, \frac{2}{3}\right)$	100	98	0	0	8	$-\infty$
$E_7 \times SU(2)^2$	$\left(0^6, \frac{1}{2}^2\right)$	146	142	112	224	4	$-\infty$
$E_8 \times SO(2)$	(0^8)	256	250	0	0	2	$-\infty$
$R = \sqrt{1 - A^2}$, Λ in units of $(4\pi^2 \alpha')^{-\frac{9}{2}}$							

$\boldsymbol{0}(16)$ string



 $O(16) \times O(16)$ on \mathbb{Z}_2 -orbifold \equiv HE on asym. orbifold $\equiv E_8$ on asym. orbifold

 $\begin{array}{l} \underline{\mathbf{CHL string}} \text{ can be obtained as the} \\ \overline{\mathbf{HE compactified on } S^1 \text{ with the } \mathbb{Z}_2} \\ \text{holonomy:} & \begin{cases} R: & E_8 \times E'_8 \to E'_8 \times E_8 \\ T: & x^9 \to x^9 + \pi R \end{cases} \\ \left|\varphi\right\rangle_{\text{untwisted}} = \frac{|w, n, \pi, \pi'\rangle + (-1)^n |w, n, \pi', \pi\rangle}{\sqrt{2}} \\ \left|\varphi\right\rangle_{\text{twisted}} = \left|w + \frac{1}{2}, n, \frac{\pi}{2}, \frac{\pi}{2}\right\rangle \end{cases} \\ \begin{array}{l} \underline{\mathbf{SIIb} \ string} \text{ can be obtained as the } O(16) \times O(16) \\ \text{compactified on } S^1 \text{ with the } \mathbb{Z}_2 \text{ holonomy:} \\ R: & SO(16) \times SO(16)' \to SO(16)' \times SO(16) \\ T: & x^9 \to x^9 + \pi R \end{cases} \\ \left|\varphi\right\rangle_{\text{untwisted}} = \frac{|w, n, \pi, \pi'\rangle + (-1)^n |w, n, \pi', \pi\rangle}{\sqrt{2}} \text{ with } + (-) \text{ for gauge bosons (spinors).} \\ \left|\varphi\right\rangle_{\text{twisted}} = \left|w + \frac{1}{2}, n, \frac{\pi}{2}, \frac{\pi}{2}\right\rangle \end{array}$









Infinite distance limits

8 points at infinity (decompactification limits to all the rank 16 10D strings)


WL	V	S	С	0
(0^{16})	$[A_1 + 2D_8; \mathbb{Z}_2]$	$(1, 128, 1) \\ (1, 1, 128)$	(1, 16, 16)	none
$\left(\frac{1}{2}^2, 0^{14}\right)$	$[A_1 + A_2 + D_6 + D_8; \mathbb{Z}_2]$	$(2, 1, 32, 1) \\(1, 1, 1, 128)$	$(1,\!1,\!12,\!16)$	none
$\left(\frac{1}{2}^3, 0^{13}\right)$	$A_4 + D_5 + D_8$	(1,1,128)	(1,10,16)	none
$\left(\frac{1}{2}^3, 0^5, \frac{1}{2}^3, 0^5\right)$	$[A_7 + 2D_5 ; \mathbb{Z}_4]$	none	$(1, 10, 10) \ (70, 1, 1)$	none
$(1,0^{15})$	$D_8 + D_9$	(1,128)	(16, 18)	$(128,1) \times 2$
$\left(\frac{1}{2}^4, 0^{12}\right)$	$[D_4 + D_5 + D_8; \mathbb{Z}_2]$	$(1, 1, 128) \\ (8, 10, 1)$	(8,1,16)	$(8, 16, 1) \times 2$
$\left(\frac{1}{2}^2, 0^6, \frac{1}{2}^2, 0^6\right)$	$[2A_1 + A_3 + 2D_6; \mathbb{Z}_2^2]$	$(2,1,1,32,1) \\ (1,2,1,1,32)$	$(1,1,1,12,12) \ (2,2,6,1,1)$	$ \begin{array}{ } (2,1,1,32,1)\times 2 \\ (1,2,1,1,32)\times 2 \end{array} $
$\left(\frac{1}{2}^{5}, 0^{3}, \frac{1}{4}^{7}, -\frac{1}{4}\right)$	$[A_{11} + E_6; \mathbb{Z}_3]$	none	none	$(143,1) \times 2$ $(1,78) \times 2$

Maximal enhancements

Group		Wilson line	Λ	$\lambda(H_{\Lambda}) imes R^2$
$\left[Spin(16)^2\right]/\mathbb{Z}_2 \times SU(2)$	1	0^{16}	431.354	$-306^{16}, 831$
$\left[Spin(16) \times Spin(12) \times SU(2)\right] / \mathbb{Z}_2 \times SU(3)$	$\frac{3}{4}$	$0^{14}, \frac{1}{2}^2$	383.516	$-307^{15}, 544^2$
$Spin(16) \times Spin(10) \times SU(5)$	$\frac{5}{8}$	$0^{13}, \frac{1}{2}^3$	359.196	$-569^5, -256^8, 355^4$
$\left[Spin(10)^2 \times SU(8)\right] / \mathbb{Z}_4$	$\frac{1}{4}$	$0^4, \frac{1}{2}^4, \frac{1}{4}^8$	303.778	-195^{17}
$Spin(18) \times Spin(16)$	$\frac{1}{2}$	$0^{15}, 1$	305.013	$-1283^8, 588^9$
$\left[Spin(16) \times Spin(10) \times Spin(8)\right] / \mathbb{Z}_{2}$	$\frac{1}{2}$	$0^{12}, \frac{1}{2}^4$	305.013	$-1283^4, -347^8, 588^5$
$\left[Spin(12)^2 \times SU(4) \times SU(2)^2\right] / \mathbb{Z}_2^2$	$\frac{1}{2}$	$0^6, \frac{1}{2}^2, 0^6, \frac{1}{2}^2$	305.013	$-1283^2, -347^{12}, 588^3$
$\left[E_6 \times SU(12)\right] / \mathbb{Z}_3$	$\frac{1}{8}$	$0^3, \frac{1}{2}^5, -\frac{1}{4}, \frac{1}{4}^7$	180.426	-72^{17}