Calabi-Yau threefold flops as quiver varieties from monopole deformations

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SISSA

Warm-up: The Conifold

The Conifold can be defined as:

$$x^2 + y^2 = z^2 - w^2, \quad (x, y, z, w) \in \mathbb{C}^4$$

The equation can be interpreted as a fibration of a local K3 ($A_1 : x^2 + y^2 = z^2$) over $\mathbb{C}$.

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In Type IIA String Theory:

Complex deformations of the ADE singularity are encoded in the background profile of $\Phi \in Adj(g)$, $g \in \{A, D, E\}$.

The Conifold can be obtained by turning on: $\Phi(w) = \begin{pmatrix} w & 0 \\ 0 & -w \end{pmatrix} \in A_1$,

Which gives rise to: $x^2 + y^2 = \det(z1 - \Phi(w)) = z^2 - w^2$

Non-toric flop geometries

In general:

To a given $\Phi(w)$ corresponds a non trivial K3 fibration along with its resolution pattern [Collinucci, De Marco, Sangiovanni, Valandro]:

$$\Phi \neq const: \qquad X_2^g \times \mathbb{C}_{\{w\}} \xrightarrow[\Phi(w)]{} \begin{array}{ccc} \mathrm{X}_2^{g,def} & \longrightarrow & \mathrm{X}_3^g(\Phi) \\ & & \downarrow \\ & & \mathbb{C}_{\{w\}} \end{array}$$

We exploit this construction to study:

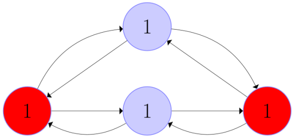
Simple threefold flops $\leftrightarrow$ Non-toric generalizations of the Conifold

Non-toric flop geometries: Examples

$A_3 \times \mathbb{C}_w \quad \Rightarrow \quad$ Reid Pagoda

$$x^2 + y^2 = z^4, \quad \forall\, w \quad \Rightarrow \quad x^2 + y^2 = \det(z1 - \Phi) = z^4 - w^2 \quad (x, y, z, w) \subset \mathbb{C}^4$$

$$\Phi(w) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ w & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -w & 0 \end{pmatrix} \in A_3 \quad \longrightarrow$$



Our aim: Recover the threefold geometry from the moduli space of a $D2$-brane probe.

Quiver Varieties from Monopole Deformations

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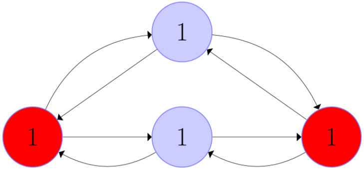

$W_{N=4}$

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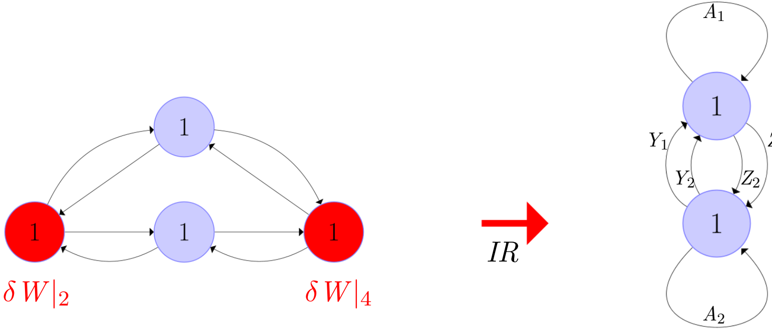


$$W_{N=4} + \delta W(W_2^{\pm}, W_4^{\pm}, \phi_2, \phi_4)$$

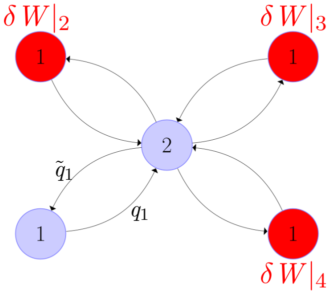
$$W_{eff} = (A_1 - A_2)(Y_1 Z_1 + Y_2 Z_2 + 2A_1 A_2)$$

Monopole deformations have been studied [Giacomelli, Collinucci, Valandro, Savelli; Benini, Benvenuti, Pasquetti]

Quiver Varieties from Monopole Deformations

$D_4 \times \mathbb{C}_w \quad \Rightarrow \quad$ Brown-Wemyss Threefold

$x^2 + y^2 z = z^3, \quad \forall\, w \quad \Rightarrow \quad x^2 + y^2 z = (z - w)(z w^2 + (z - w)^2) \quad (x, y, z, w) \subset \mathbb{C}^4$



$$W = W_{N=4} + \delta W(W_2^{\pm}, W_3^{\pm}, W_4^{\pm}, \phi_2, \phi_3, \phi_4)$$

Integrating out massive degrees of freedom...

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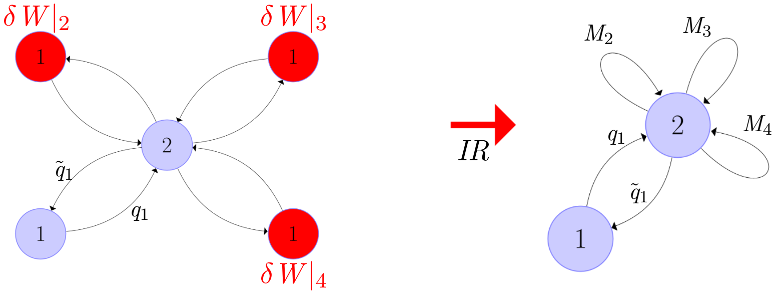

$$W = W_{N=4} + \delta W(W_2^{\pm}, W_3^{\pm}, W_4^{\pm}, \phi_2, \phi_3, \phi_4) \qquad W_{\mathit{eff}} = F(M_2, M_3, M_4, q_1, \tilde{q}_1)$$

Integrating out massive degrees of freedom...

Claim: Given the EFT on a trivially fibered ADE surface X_2^g, the theory on $X_3^g(\Phi)$ is obtained by adding a $N = 2$ preserving deformation:

$$W = W_{N=4} + \mathrm{Tr}(\Phi(\{\phi_i\})\mu), \quad \mu \rightarrow \text{g-moment map}, \quad \phi_i \rightarrow \text{CB chirals}$$

$$W \quad \underset{IR}{\longrightarrow} \quad W_{\mathit{eff}} \quad \Rightarrow \quad \mathrm{X}_3^g(\Phi)\text{'s defining equation in } \mathbb{C}^4$$

Analogous results have been obtained through different approaches [Witten, Klebanov; Cachazo, Vafa, Katz; Gubser, Nekrasov, Shatashvili]

Conclusions and Outlook

- We provide a simple recipe to extract the $N = 2$ superpotential of a large class of non toric CY threefolds.

- We propose a physical explanation for a non-commutative geometry algorithm that derives the quiver and the relations of the threefold from the non-affine [Cachazo, Katz, Vafa] and the affine [Karmazin] Dynkin diagram of the starting ADE algebra.

- We aim at applying our technique to simple flops of any length.

Thanks!