Minimalism in $\mathcal{N} = 2$ moduli spaces from orbifold constructions

– with *least* moduli and *zero* quantum corrections

Guoen Nian

Based on arXiv:2407.xxxxx

with G. Gkountoumis[1], C. Hull[2], S. Vandoren[1]

[1] *Institute for Theoretical Physics, Utrecht University*

[2] *The Blackett Laboratory, Imperial College London*

Utrecht University

Outline

1. Compactification
2. A minimal moduli space
3. No perturbative correction – triality
4. No non-pertubative correction
5. Conclusion and outlook

Start with a type IIB freely acting asymmetric orbifold compactification

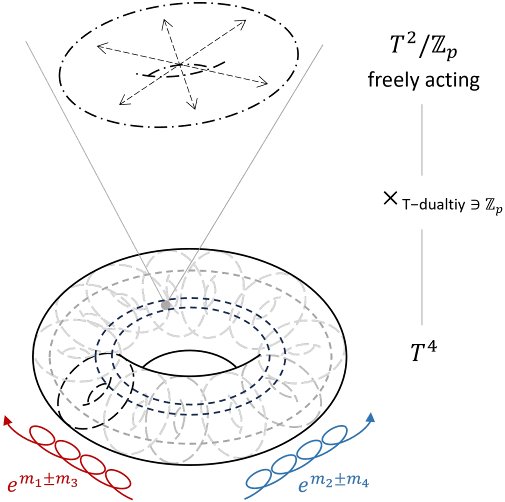
$$\mathbb{R}^{1,3} \times \left(T^4 \times T^2\right)/\mathbb{Z}_p\,.$$

Here $\mathbb{Z}_p$ acts as twists on T^4 and a shift on T^2:

$$\begin{aligned} W_L^1 &\to e^{i(m_1+m_3)}\, W_L^1\,, \\ W_L^2 &\to e^{i(m_1-m_3)}\, W_L^2\,, \\ W_R^1 &\to e^{i(m_2+m_4)}\, W_R^1\,, \\ W_R^2 &\to e^{i(m_2-m_4)}\, W_R^2\,; \\ Z_1 &\to Z_1 + 2\pi\mathcal{R}_5/p\,, \end{aligned}$$

with $p(m_{1,2} \pm m_{3,4}) = 2\pi n$, $n \in \mathbb{Z}$.

[Gkountoumis, Hull, Stemerdink, Vandoren, '23]



Compactification

1. As a T-fold compactification, the low-energy SUGRA theory is at the minimum of a Scherk–Schwarz potential. [Dabholkar, Hull, '03]

2. Each of the four mass parameters $m_{1,2,3,4} \neq 0 \pmod{2\pi}$
   $\Rightarrow$ 2 massive gravitini
   $\Rightarrow$ 8 supercharges spontaneously broken.
   We focus on $\mathcal{N} = 2$ by taking $m_4 = 0, m_{1,2,3} \neq 0$.

Compactification

1. As a T-fold compactification, the low-energy SUGRA theory is at the minimum of a Scherk–Schwarz potential. [Dabholkar, Hull, '03]

2. Each of the four mass parameters $m_{1,2,3,4} \neq 0 \pmod{2\pi}$
   $\Rightarrow$ 2 massive gravitini
   $\Rightarrow$ 8 supercharges spontaneously broken.
   We focus on $\mathcal{N} = 2$ by taking $m_4 = 0, m_{1,2,3} \neq 0$.

3. By adjusting the mass parameters, some untwisted moduli can be projected out
   $\Rightarrow$ moduli space with a small dimension.

4. The moduli space can be **classically exact** according to duality symmetries and other properties of the theory.

A minimal moduli space

- No Ramond–Ramond field ($\Rightarrow$ no D-brane):

$$m_i \pm m_j \neq 0 \pmod{2\pi}\ , \quad \forall\, i,j = 1,2,3,4.$$

- No hypermultiplet: $m_1 \pm m_2 \pm m_3 \neq 0 \pmod{2\pi}$.

[Baykara, Hamada, Tarazi, Vafa, '23; Gkountoumis, Hull, Vandoren, '24]

Masses satisfying above conditions can be chosen as

$$m_1 = \frac{2\pi}{3}\ , \quad m_2 = \frac{\pi}{2}\ , \quad m_3 = \frac{\pi}{3}\ , \quad (m_4 = 0)\ .$$

This corresponds to a $\mathbb{Z}_{12}$-orbifold with D_4 Narain lattice on T^4. Its classical moduli space is

$$\mathcal{M} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,2)}{\mathrm{SO}(2)\times\mathrm{SO}(2)} \simeq \left(\frac{\mathrm{SL}(2)}{\mathrm{U}(1)}\right)_S \times \left(\frac{\mathrm{SL}(2)}{\mathrm{U}(1)}\right)_T \times \left(\frac{\mathrm{SL}(2)}{\mathrm{U}(1)}\right)_U .$$

A minimal moduli space

The moduli are

$$S = a + ie^{-2\phi_4}\,, \quad T = \frac{1}{\alpha'}\left(b + i\sqrt{\det g}\right)\,, \quad U = \frac{g_{12}}{g_{11}} + i\frac{\sqrt{\det g}}{g_{11}}\,.$$

This is indeed an STU-model with prepotential $F = i(X^0)^2 STU$ classically. With most general quantum corrections, the prepotential is

$$F = \left(X^0\right)^2 \left[iSTU + f^{(1)}(T, U) + f^{(\mathrm{np})}(e^{-2\pi i S, T, U}, T, U)\right]\,.$$

A minimal moduli space

The moduli are

$$S = a + ie^{-2\phi_4}\,, \quad T = \frac{1}{\alpha'}\left(b + i\sqrt{\det g}\right)\,, \quad U = \frac{g_{12}}{g_{11}} + i\frac{\sqrt{\det g}}{g_{11}}\,.$$

This is indeed an STU-model with prepotential $F = i(X^0)^2 STU$ classically.
With most general quantum corrections, the prepotential is

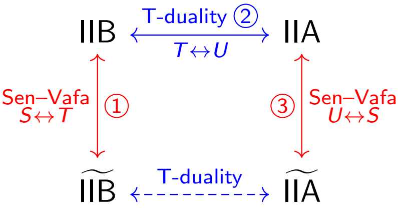
$$F = \left(X^0\right)^2 \left[iSTU + f^{(1)}(T,U) + f^{(\mathrm{np})}(e^{-2\pi i S,T,U}, T, U)\right]\,.$$

Axion shift symmetries of T, U restrict the quantum corrections to be at most power 3 [Cecotti, Ferrara, Girardello, '88]:

$$f^{(1)} = ip_3(T,U) + \lambda\,.$$

The prefactors of $f^{(\mathrm{np})}$ are the same.

No perturbative correction – triality



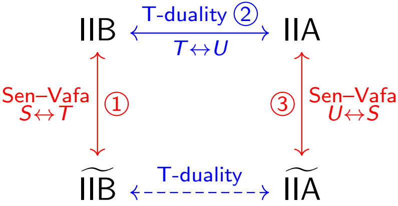
① There is a Sen–Vafa dual $\widetilde{\text{IIB}}$ theory with the same orbifold but

$$\widetilde{m}_1 = 0, \quad \widetilde{m}_2 = \frac{\pi}{2}, \quad \widetilde{m}_3 = \frac{\pi}{3}, \quad \widetilde{m}_4 = \frac{2\pi}{3} .$$

The spectra including the moduli are the same up to $S \leftrightarrow T$, i.e.

$$\widetilde{S} = T, \quad \widetilde{T} = S .$$

No perturbative correction – triality



① There is a Sen–Vafa dual $\widetilde{\text{IIB}}$ theory with the same orbifold but

$$\widetilde{m}_1 = 0, \quad \widetilde{m}_2 = \frac{\pi}{2}, \quad \widetilde{m}_3 = \frac{\pi}{3}, \quad \widetilde{m}_4 = \frac{2\pi}{3}.$$

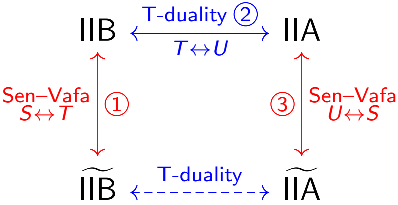
The spectra including the moduli are the same up to $S \leftrightarrow T$, i.e.

$$\widetilde{S} = T, \quad \widetilde{T} = S.$$

The prepotential of the effective theory of $\widetilde{\text{IIB}}$ is

$$\begin{aligned}\widetilde{F} &= (X^0)^2 \left[i\widetilde{S}\widetilde{T}U + \widetilde{f}^{(1)}(\widetilde{T}, U) + \widetilde{f}^{(\text{np})}(e^{-2\pi i \widetilde{S}, \widetilde{T}, U}, \widetilde{T}, U) \right] \\ &= (X^0)^2 \left[iSTU + \widetilde{f}^{(1)}(S, U) + \widetilde{f}^{(\text{np})}(e^{-2\pi i S, T, U}, S, U) \right] \stackrel{!}{=} F,\end{aligned}$$

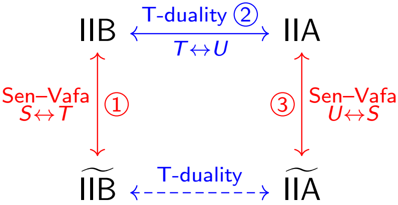
which requires $\widetilde{f}^{(1)}(S, U) = f^{(1)}(T, U)$.
Hence the 1-loop correction $f^{(1)}$ **has no T-dependence**.



② The T-duality along the shift-circle of T^2 exchanges T and U and gives the corresponding type IIA theory. Denote its prepotential to be

$$F' = (X^0)^2 \left[iSTU' + f'^{(1)}(T', U') + f'^{(\text{np})}(e^{-2\pi i X}, T', U') \right] .$$

No perturbative correction – triality



② The T-duality along the shift-circle of T^2 exchanges T and U and gives the corresponding type IIA theory. Denote its prepotential to be

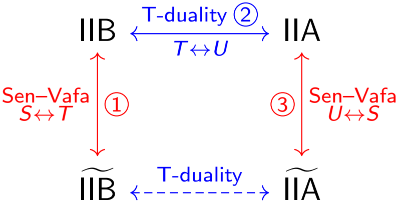
$$F' = (X^0)^2 \left[iSTU' + f'^{(1)}(T', U') + f'^{(\text{np})}(e^{-2\pi i X}, T', U') \right] .$$

Same as the indication of the IIB Sen–Vafa dual pair, ③ shows F' being T'-independent. Hence,

$$F' \stackrel{!}{=} F \quad \Rightarrow \quad f'^{(1)}(U') = f'^{(1)}(T) \stackrel{!}{=} f^{(1)}(U)$$

$\Rightarrow$ The 1-loop correction $f^{(1)}$ **has no U-dependence**. $f^{(1)} = \lambda$.

No perturbative correction – triality



② The T-duality along the shift-circle of T^2 exchanges T and U and gives the corresponding type IIA theory. Denote its prepotential to be

$$F' = \left(X^0\right)^2 \left[iSTU' + f'^{(1)}(T', U') + f'^{(\mathrm{np})}(e^{-2\pi i X}, T', U') \right] .$$

Same as the indication of the IIB Sen–Vafa dual pair, ③ shows F' being T'-independent. Hence,

$$F' \stackrel{!}{=} F \quad \Rightarrow \quad f'^{(1)}(U') = f'^{(1)}(T) \stackrel{!}{=} f^{(1)}(U)$$

$\Rightarrow$ The 1-loop correction $f^{(1)}$ **has no U-dependence**. $f^{(1)} = \lambda$.
The constant λ is proportional to the Euler characteristic χ, [Pope, Sohnius, Stelle, '87; Candelas, De La Ossa, Green, Parkes, '91] which is zero for our freely-acting toroidal orbifold. Such that

$$f^{(1)} = 0 .$$

No non-pertubative correction

- As mentioned earlier, we turned off all Ramond–Ramond field hence there is no D-instanton.
- The worldsheet instantons and NS-instantons exist but cannot correct the moduli space. Note that the gravitini are massive rather than projected out, which means the supersymmetry is **spontaneously broken** from 32 to 8 supercharges.
  Branes must break at least 16 to be BPS, giving instanton contribution to the effective action
  $$S_{\text{inst}} \sim \int \chi^{16}\mathcal{R}^{(16)}\,,$$
  with χ fermionic zero-modes. The derivative is too high to correct the moduli space metric. [Becker, Becker, Strominger,'95; Lawrence, Nekrasov, '97]

Conclusion and outlook

- We construct an $\mathcal{N} = 2$, $D = 4$ theory with only three moduli from freely-acting asymmetric orbifold compactification, with exact worldsheet CFT and low-energy Scherk–Schwarz reduction description.
- We further argue that the theory is classically exact.
  The 1-loop correction is eliminated by the triality of STU moduli and the compact space topology;
  the instanton correction is eliminated due to high supersymmetry of the action.
- Relaxing the no-hypermultiplet condition we can get larger exact moduli space;
  Models with more vector multiplets can be constructed, e.g.:
  $m_1 = \frac{2\pi}{3}$, $m_2 = \frac{\pi}{2}$, $m_3 = \frac{2\pi}{3}$ gives 5 moduli S, T, U, V, W.
  Utilizing the T-duality between V and W we can argue the similar exactness of moduli space.

Thank you!