High-Energy Fixed-Angle Meson Scattering and Holographic QCD

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Motivations

- String theory traces back to 1968, when Veneziano formulated the cross-symmetric, **Regge-behaved** amplitude for low-lying mesons:[1]

$$\mathcal{A}(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \tag{1}$$

- Later that year, experiments at SLAC observed **partonic behaviour** in inelastic electron-proton scattering in the high-energy fixed-angle regime
- Soon after the SLAC experiments, constituent counting rules provided a matching prediction [Brodsky, Farrar '73, Matveev, Muradian, Tavkhelidze '73, ...]
- In the high-energy fixed-angle regime:

Veneziano (String Theory)

$$\mathcal{A}(s,t) \sim f(\theta_s)^{-\alpha(s)}$$

Constituent Counting Rules (QCD)[2]

$$\mathcal{A}(s,t) \sim g(\theta_s) \times s^{2-\frac{1}{2}n}$$

[1] Veneziano explicitly constructed amplitudes for $\pi\pi \to \pi\omega$ and $\pi\eta \to \pi\rho$, and briefly looked at $\pi\pi \to \pi\pi$; Lovelace and Shapiro generalised his findings to the latter process

[2] Assuming helicity-conserving scattering processes

Synopsis

- We study high-energy fixed-angle scattering of mesons using a bottom-up holographic QCD model via the Polchinski–Strassler (PS) proposal for string amplitudes in AdS space with a hard IR cut-off [Polchinski, Strassler '02]

$$\textbf{PS Proposal}: \mathcal{A}(s,t,u) \sim \int d\text{vol} \times \mathcal{A}_{\text{string}}\left(\tilde{s},\tilde{t},\tilde{u}\right) \times \prod_{i=1}^{n} \psi_i \qquad (2)$$

- In our approach, 2-to-n pion amplitudes **agree** with constituent counting rules, while amplitudes with ρ-mesons are **zero** at *leading order* in s[3]
- We provide predictions for all isospin channels of $\pi\pi \to \pi\pi$ scattering
- For $\pi^+\pi^- \to \pi^+\pi^-$ scattering, we compare our predictions to exp. data
- Recent results on this subject can also be found in [Bianchi et al. '22][4]

[3] Herein, the leading order in s will always be dictated by the constituent counting rules
[4] The PS proposal was originally formulated for glueballs in [Polchinski, Strassler '02], while [Bianchi et al. '22] also used it to study meson-meson scattering

Background Material: A Holographic QCD Model

- Based on holography, we argue that the following set-up can partially capture the meson sector of a QCD-like theory at the **IR scale** given that $N_c \gg N_f$:[5]

$$S \sim \int d^{d+1}x \sqrt{|g|}\, \mathrm{Tr}\left[g^{MN} g^{PQ} F_{MP} F_{NQ}\right], \tag{3}$$

$$ds^2 = \frac{R_{d+1}^2}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2\right), \quad 0 \leq z \leq z_0, \tag{4}$$

$$F_{MN} = F_{MN}^a t^a = \partial_M A_N^a t^a - \partial_N A_M^a t^a + g f^{abc} A_M^b A_N^c t^a \tag{5}$$

- To recover d-dimensional physics from eq. (3), expand A_μ and A_z in terms of complete sets $\{\psi_n\}_{n\geq 1}$ and $\{\phi_n\}_{n\geq 0}$:

$$A_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu)\psi_n(z), \tag{6}$$

$$A_z(x^\mu, z) = \varphi^{(0)}(x^\mu)\phi_0(z) + \sum_{n=1}^{\infty} \varphi^{(n)}(x^\mu)\phi_n(z) \tag{7}$$

[5] This model is similar to [Son, Stephanov '04, Hirn, Sanz '05]

Background Material: Polchinski–Strassler Proposal

- For a gauge/string dual pair, the **PS proposal** relates a field theory amplitude in the high-energy fixed-angle regime to a string amplitude:[6]

$$\mathcal{A}(s,t,u) \sim \int d\text{vol} \times \mathcal{A}_{\text{string}}\left(\tilde{s},\tilde{t},\tilde{u}\right) \times \prod_{i=1}^{n} \psi_i \tag{8}$$

- Here, ψ_i are wavefunctions of the scattered states and $\tilde{s},\tilde{t},\tilde{u}$ are curved space Mandelstam variables, e.g., for 2-to-2 scattering with momenta $p^{(1)} \sim p^{(4)}$:

$$\tilde{s} = -g^{MN}\left(p_M^{(1)} + p_M^{(2)}\right)\left(p_N^{(1)} + p_N^{(2)}\right) \stackrel{!}{=} -g^{\mu\nu}\left(p_\mu^{(1)} + p_\mu^{(2)}\right)\left(p_\nu^{(1)} + p_\nu^{(2)}\right) \tag{9}$$

- Essentially, it is the **re-scaling of the string amplitude**[7] that accounts for recovering power-law behaviour in the high-energy fixed-angle regime

[6] As noted in [Polchinski, Strassler '02], while the LHS of eq. (8) describes high-energy fixed-angle field theory scattering, the RHS includes contributions from all stringy modes

[7] In other words, replacing $\eta^{\mu\nu}$ with $g^{\mu\nu}$ within each dot product in the amplitude

Results: Recovering Constituent Counting Rules (1/2)

- Take the string amplitude to be a **4-photon amplitude of superstrings**:

$$\mathcal{A}_{\text{string}} \propto K\left(k^{(1)\sim(4)}, \zeta^{(1)\sim(4)}\right) \times \frac{\Gamma(-\alpha' s)\,\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} \tag{10}$$

- In the high-energy fixed-angle regime:

$$\frac{\Gamma(-\alpha' s)\,\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} \sim s^{q_n} e^{-f(\theta)s} \tag{11}$$

- The factor K in the n-photon generalisation must scale as:

$$K\left(k^{(1)\sim(n)}, \zeta^{(1)\sim(n)}\right) \propto T^{\mu_1,\ldots\mu_n,\nu_1,\ldots,\nu_n}\, k^{(1)}_{\mu_1}\ldots k^{(n)}_{\mu_n}\, \zeta^{(1)}_{\nu_1}\ldots\zeta^{(n)}_{\nu_n} \tag{12}$$

- Taking $\eta^{MN} \to g^{MN}$, each term in K gets a factor of z^{2n} from contractions
- Each term in K also receives a power of $s^{\frac{n}{2}}$ from the n momenta
- We also get a factor of $s^{\frac{n_v}{2}}$ from vector meson polarizations[8]

[8] Axial vector mesons receive the same factor; higher spin states, get factors of $s^{\frac{\text{spin}}{2}}$

Results: Recovering Constituent Counting Rules (2/2)

- The AdS_{d+1} metric gives an explicit factor of $\sqrt{-g} = z^{-d-1}$
- The product of wavefunctions with $\psi_\pi \sim z^{d-3}$ and $\psi_\rho \sim z^{d-2}$ reads:

$$\prod_{i=1}^{n} \psi_i \sim z^{(d-3)n+n_v} \tag{13}$$

- Collecting everything together and applying the PS proposal:

$$\mathcal{A} \sim \int_0^{z_0} dz \sqrt{-g} \times \mathcal{A}_{\text{string}} \left(\tilde{s}, \tilde{t}, \tilde{u}\right) \times \prod_{i=1}^{n} \psi_i \tag{14}$$

$$\sim \int_0^{z_0} \left[dz z^{-d-1}\right] \left[(z^2 s)^{q_n} e^{-f(\theta_i) z^2 s}\right] \left[z^{2n} s^{\frac{1}{2}(n+n_v)}\right] \left[z^{(d-3)n+n_v}\right] \tag{15}$$

$$\sim s^{\frac{d}{2} - \frac{d-2}{2} n} \times \int_0^{\sqrt{\alpha' s}} d\tilde{z} \, F(\theta_i; \tilde{z}) \tag{16}$$

Results: A Novel Prediction for Vector Mesons

- The previous counting argument **does not generalise** to ρ-mesons!
- Due to a **gauge symmetry** in our approach, amplitudes with ρ-mesons become zero at leading order in s dictated by the constituent counting rules
- The disagreement manifests itself explicitly in the factor K:

$$K^{\text{naïve}}[\pi\pi \to \rho\rho] \sim K^{\text{naïve}}[\pi\rho \to \pi\rho] \sim s^3 \text{ and } K^{\text{naïve}}[\rho\rho \to \rho\rho] \sim s^4, \quad (17)$$

$$K^{\text{actual}}[\pi\pi \to \rho\rho] \sim K^{\text{actual}}[\pi\rho \to \pi\rho] \sim s^2 \text{ and } K^{\text{actual}}[\rho\rho \to \rho\rho] \sim s^3 \quad (18)$$

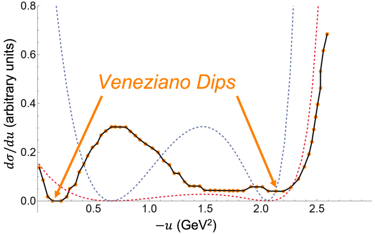
- Hence, our approach offers a **testable prediction** for future experiments:[9]

$$\frac{\mathcal{A}(\pi\pi \to \pi\pi)}{\mathcal{A}(\pi\pi \to \rho\rho)} \sim \frac{\mathcal{A}(\pi\pi \to \pi\pi)}{\mathcal{A}(\pi\rho \to \pi\rho)} \sim \frac{\mathcal{A}(\pi\pi \to \pi\pi)}{\mathcal{A}(\rho\rho \to \rho\rho)} \sim \mathcal{O}\left(s^{-1}\right) \quad (19)$$

[9] However, it may be more accurate to identify this as a prediction for a strongly coupled CFT

Results: Pion-Pion Scattering Phenomenology

- We compared our model with exp. data for $\pi^+\pi^- \to \pi^+\pi^-$ [Grayer et al. '74]
- Data available is at relatively low energies, going up to only $\sqrt{s} = 1.79$ GeV, while our predictions are expected to be valid when $\alpha' s \gg 1$[10]
- Also, our results are (roughly) a leading-order N_c result in the 't Hooft limit



The black line plots the exp. data and the dashed lines are the theoretical fit both for $\sqrt{s} = 1.79$ GeV; the two theoretical fits differ by the choice of a proportionality constant (the red dashed line is fitted to the lower end of the domain, while the blue dashed line fit covers the entire domain)

[10] The effective Regge slope α' is set to its phenomenological value of roughly 0.9 GeV^{-2}

Comments, Conclusions & Future Directions

- **Comments**: I did not describe how to construct pion-pion invariant scattering amplitudes, regularize them, or explicitly motivate the PS proposal
- **Conclusions**: we found a testable prediction, but our holographic model is expected to be dual to a strongly coupled CFT in the UV limit, while QCD has a weak and slowly varying coupling constant in this regime
- **Future Directions**: a top-down construction that recovers the PS proposal, cross-over to Regge regime, proton-proton scattering, and much more!

Thank You for Listening