Perturbations in Topological Star geometries

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Outline

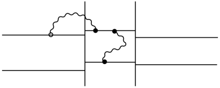
- Topic and physical motivation
  - Perturbations in Black Holes (BHs) and Exotic Compact Objects (ECOs) geometries
  - Focus on Topological star geometries
- Focus on Quasi normal modes (QNMs)
  - WKB approximation
  - Leaver method
  - Direct integration
  - Seiberg-Witten (SW) quantization
- These techniques have been used in order to study:
  - Topological stars (TS) scalar, vectorial and tensorial QNMs spectra
  - TS scalar charged QNMs and superradiant amplification (Not discussed here: see G. Sudano poster)

Most of the talk is based on our recent papers: 2305.15105, 2402.06621

and some work in progress with I. Bena, F. Morales, A. Ruiperez

Special thank also to: A. Cipriani, F. Fucito, A. Grillo, G. Sudano

qSW curves for BHS and fuzzballs perturbation theory



- Main result of $SU(2)$ $\mathcal{N}=2$ SYM: the dynamic is described by an ellyptic curve
$$qy^2 P_L(x) + yP_0(x) + P_R(x) = 0$$
$$P_{L,R} = (x - m_{1,3})(x - m_{2,4}) \quad , \quad P_0 = (x - a_1)(x - a_2)$$
- Branes configuration for $N = 2$ $SU(2)$ SYM (**Witten_1997**)
- The SW curve embedded in the $\Omega-$background $x \to \hat{x} = \hbar y \frac{d}{dy}$, $y \to \hat{y}$.
- Integrability condition of the quantum curve
$$P_0(a) = \cfrac{qM(a+\hbar)}{P_0(a+\hbar) - \cfrac{qM(a+2\hbar)}{P_0(a+2\hbar) - \dots}} + \cfrac{qM(a)}{P_0(a-\hbar) - \cfrac{qM(a-\hbar)}{P_0(a-2\hbar) - \dots}} \quad , \quad M(a) = P_L\left(a - \frac{\hbar}{2}\right) P_R\left(a - \frac{\hbar}{2}\right)$$
- from which you can perturbatively compute in q
$$a(u,q) = \sqrt{u} + \frac{q}{4\sqrt{u}}\left(\frac{4m_1 m_2 m_3}{4u - \hbar^2} + m_1 + m_2 + m_3 - \hbar\right) + \mathcal{O}(q^2)$$
$$u(a,q) = a^2 + q\left(\frac{1}{2}(\hbar - m_1 - m_2 - m_3) - \frac{2m_1 m_2 m_3}{4a^2 - \hbar^2}\right) + \mathcal{O}(q^2)$$
- The prepotential can be computed inverting quantum Matone relation $u = -q\partial_q \mathcal{F}$
- The other inequivalent cycle
$$a_D = -\frac{1}{2\pi i}\frac{\partial \mathcal{F}}{\partial a}$$
- Connection formulae (**Bonelli:2022ten**) in terms of these gauge quantities. After the imposition of b.c.s
$$a_D = n \in \mathbb{Z}$$

Topological star scalar perturbations

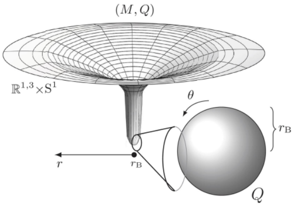
- Solution of Einstein-Maxwell theory in 5 dimensions symmetric under double Wick rotation (**Bah:2020pdz**)

$$t \to it, \quad y \to iy, \quad r_b \leftrightarrow r_s$$

$$ds^2 = -f_s dt^2 + \frac{dr^2}{f_s f_b} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + f_b dy^2$$

$$F^{(e)} = \frac{Q}{r^2} dr \wedge dt \wedge dy, \quad F^{(m)} = P \sin\theta\, d\theta \wedge d\phi$$

$$P^2 + Q^2 = \frac{3 r_s r_b}{2\kappa_5^2}, \quad f_{s,b} = 1 - \frac{r_{s,b}}{r}$$



$$(r-r_s)(r-r_b)R''(r) + (2r - r_s - r_b)R'(r) + \left(\frac{\omega^2 r^3}{r-r_s} - \frac{p^2 r^3}{r-r_b} - \ell(\ell+1)\right) R(r) = 0$$

- $r_s < r_b < 2r_s$: smooth and horizonless, and NO Gregory-Laflamme instability
- QNMs are resonant frequencies that dominate the GW ring-down signal in binary mergers: exponential decay (stability) or growth in time (instability):

$$\omega = \omega_R + i\omega_I \quad \to \quad \Phi \sim e^{-i\omega t} = e^{\omega_I t} e^{-i\omega_R t} \quad , \quad \omega_I \sim -1/\tau$$

- They could discriminate BHs from ECOs.

$$\psi''(r) + Q(r,\omega)\psi(r) = 0$$

- Different b.c.s (assuming $\Re(\omega) > 0$): $\psi_\infty(r) \underset{r \to \infty}{\sim} e^{i\omega r} r^{\lambda_\infty}$
  - BH: $\psi_0(r) \underset{r \to r_H}{\sim} e^{-i\omega_H r}$, *Smooth* : $\psi_0(r) \underset{r \to r_{cap}}{\sim} (r - r_{cap})^\lambda$ with $\lambda \in \mathbb{R}$

Topological star scalar perturbations

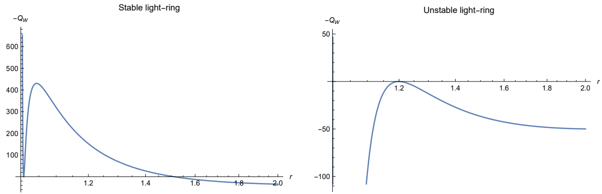


Figure: $\ell = 10$, $r_b = 1$, $r_s = 0.8$, $P_y = 0.25$. Stable (unstable) light-ring is at $\omega = 4.76906$ and $x = 0.0390789$ ($\omega = 5.0556$ and $x = 0.995483$).

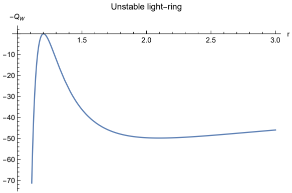


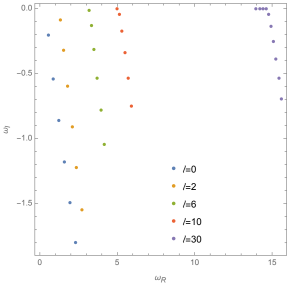
Figure: $\ell = 10$, $r_b = 1$, $r_s = 0.8$, $P_y = 0$. Unstable light-ring is at $\omega = 5.04324$ and $r = 1.20201$.

TS scalar QNMs spectrum

- In WKB viewpoint there are two families of QNMs: TS scalar perturbations are stable.

ℓ	Seiberg-Witten	Direct integration
0	0.24 − 0.058i	0.24 − 0.067i
2	1.23 − 0.089i	1.35 − 0.086i
6	3.15 − 0.10i	3.32 − 0.13i
10	5.11 − 0.037i	5.14 − 0.043i
30	14.76 − 0.048i	14.77 − 0.041i

Table: Prompt ring-down modes $r_b = 1$, $r_s = 0.8$, $P_y = 0.25$, $n = 0$



Leaver	Direct Integration
$13.9318975 - 8.2824 \times 10^{-14}$i	$13.9318968 - 2.3112 \times 10^{-14}$i
$14.1874938 - 1.2330 \times 10^{-9}$i	$14.1874940 - 1.2329 \times 10^{-9}$i
$14.420296 - 3.6398 \times 10^{-6}$i	$14.4202964 - 3.6399 \times 10^{-6}$i
14.6178752 − 0.001824i	14.6178768 − 0.001824i
14.7663184 − 0.041397i	14.7663207 − 0.041399i
14.9183936 − 0.136179i	14.9181194 − 0.135541i

Table: Metastable modes $r_b = 1$, $r_s = 0.8$, $P_y = 0.25$ $\ell = 30$ various n

TS metric and gauge field perturbations

- Linear perturbations of the metric and the gauge field (work in progress Bena, D.R., Morales, Ruiperez)
- Problem in the perturbations around magnetically charged backgrounds: decoupling between even and odd perturbations does not occur
- Here the problem is circumvented by dualizing the vector field into $C_{\mu\nu}$

$$R_{mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{1}{4}\left(F_{\mu\alpha\beta}F_{\nu}{}^{\alpha\beta}-\frac{1}{6}g_{\mu\nu}F_{\alpha\beta\gamma}F^{\alpha\beta\gamma}\right), \quad \nabla_{\mu}F^{\mu\nu\rho}=0, \quad F_{\mu\nu\rho}=3\partial_{[\mu}C_{\nu\rho]}$$

- Study linear perturbations around solutions of the Einstein equations with metric and 2-form given by

$$ds^2=-f_s dt^2+\frac{dr^2}{f_s f_b}+r^2(d\theta^2+\sin^2\theta d\phi^2)+f_b dy^2 \quad , \quad C=\frac{\sqrt{3r_s r_b}}{r}dt\wedge dy$$

- Perturbations of the background metric and the 2-form $g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}$, $C_{\mu\nu}=\bar{C}_{\mu\nu}+c_{\mu\nu}$
- Odd perturbations

$$Q_{\pm}=\frac{r^3}{(r-r_s)^2(r-r_b)}\left[\omega^2-\frac{\ell(\ell+1)}{r^2}+\frac{(2r_b+3r_s)(1\pm\gamma)+2r_s(\ell^2+\ell+1)}{2r^3}\right.$$
$$\left.-\frac{r_s\left[(2r_b+3r_s)(1\pm\gamma)+8r_b+\frac{3}{2}r_s\right]}{2r^4}+\frac{15r_b r_s^2}{4r^5}\right] \quad , \quad \gamma=\sqrt{1-\frac{12(\ell+2)(\ell-1)r_b r_s}{(2r_b+3r_s)^2}}$$

- Spherically symmetric ($\ell=0$) even perturbations

$$\frac{d}{dr}\Big[A(r)k'(r)\Big]-B(r)k(r)=0, \quad A(r)=\frac{(r-r_b)(r-r_s)}{W(r)^2}, \quad W(r)=p^2(4r-3r_s)-\omega^2(4r-3r_b)$$

$$B(r)=\frac{r^3\mathcal{W}(r)\left(p^2(r-r_s)-\omega^2(r-r_b)\right)+2(r-r_b)(r-r_s)\left(p^2(2r_b-r_s)-\omega^2(2r_s-r_b)\right)}{\mathcal{W}(r)^3(r-r_b)(r-r_s)}$$

TS gravitational induced odd perturbations

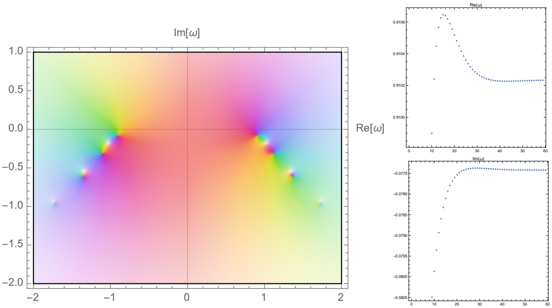


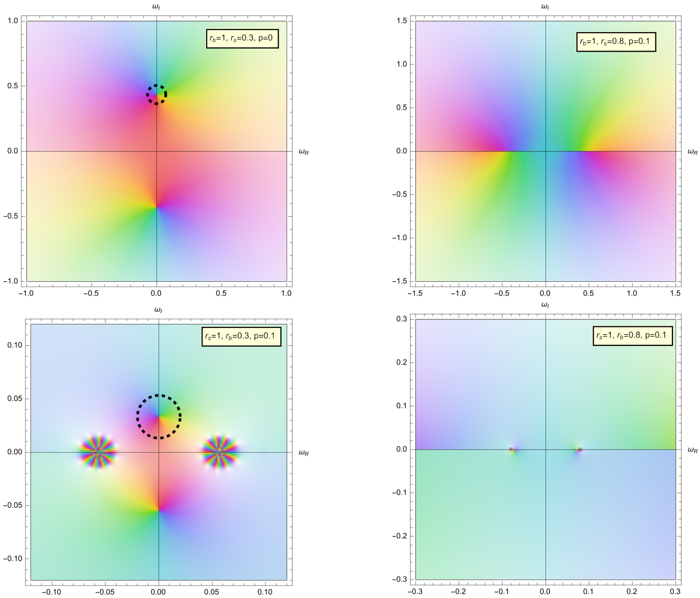
Figure: L) R_+ QNM's for top star with $r_b = 1$, $r_s = 0.8$, $\ell = 2$. R) Convergence of the continuous fraction as a function of the truncation order.

ℓ	Leaver($n = 0$)	Dir. Int.
2	$0.9102 - 0.0774i$	$0.9102 - 0.0774i$
3	$1.3939 - 0.0477i$	$1.3939 - 0.0477i$
4	$1.8618 - 0.0286i$	$1.8618 - 0.0286i$
5	$2.3241 - 0.0164i$	$2.3241 - 0.0164i$
8	$3.6952 - 0.0019i$	$3.6952 - 0.0019i$
10	$4.6003 - 0.0003i$	$4.6003 - 0.0003i$

n	Leaver ($\ell = 2$)
0	$0.9102 - 0.0774i$
1	$1.1201 - 0.3126i$
2	$1.3775 - 0.6018i$
3	$1.6711 - 0.9105i$
4	$1.9868 - 1.2224i$
5	$2.3415 - 1.5428i$

Table: ψ_+ QNM's for Top Stars with $r_b = 1$, $r_s = 0.8$.

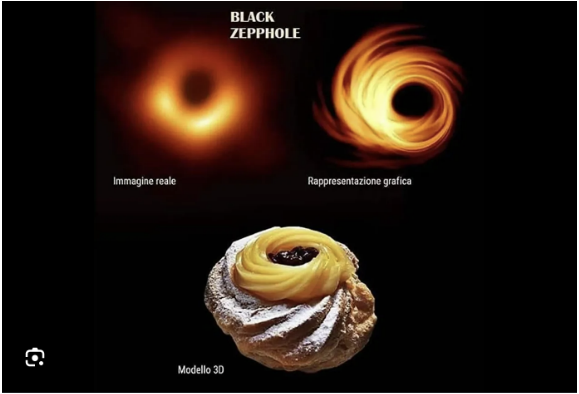
Gregory-Laflamme instabilities



Outlook

- Numerical simulation of the merging of two TSs
  - Study of the gravitational wave profile
- Hamiltonian neural networks
  - geodesic motion of non separable geometries
  - localization of the light rings
  - QNMs spectrum analysis
  - Our ambition is studying the non separable 3-charge circular fuzzball exploiting our results on (101)-superstratum (**Bena:2015bea**)

Thank you!



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ZEPPHOLE
Immagine reale
Rappresentazione grafica
Modello 3D