CORRELATORS AND OPE COEFFICIENTS IN ARGYRES-DOUGLAS THEORIES

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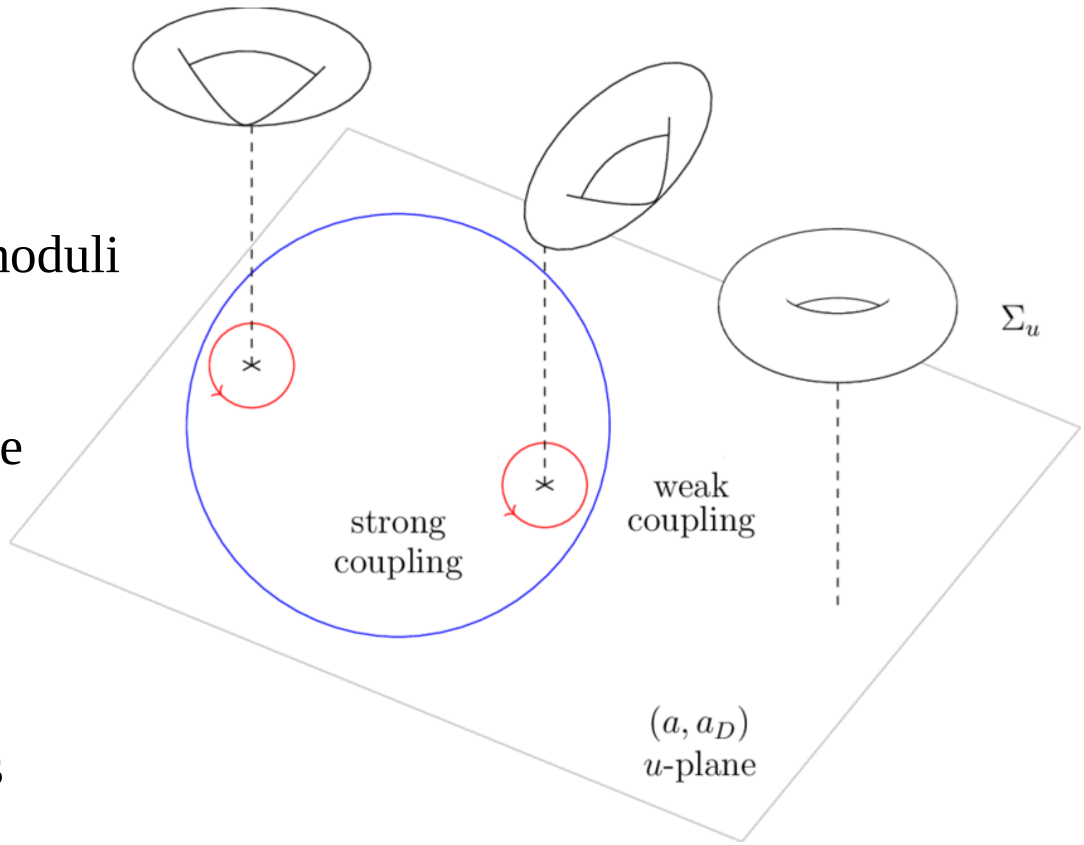
ARGYRES-DOUGLAS THEORIES

- These are 4-dimensional $\mathcal{N} = 2$ superconformal field theories and
  - without a Lagrangian description;
  - isolated;
  - strongly coupled;
- We focus on the Coulomb Branch (SSB of $U(1)_R$) of moduli space
- It is parametrized by the VEVs of CB operators (that are scalar chiral superconformal primaries)
- The study is devoted to rank-1 theories, meaning
  - the CB has complex dimension 1 (u is the coordinate)
  - the SW curve associated to each point of CB is a torus



ARGYRES-DOUGLAS THEORIES

- Argyres-Douglas (AD) theories are very special points on the CB, because:

  - from a geometrical side, the SW curve associated to them has both 1-cycles simultaneously shrinking

  - from a physical side, these points describe theories with mutually non-local degrees of freedom that are simultaneously massless

- This makes a local Lagrangian that could describe their interactions not possible

- At points where mutually non-local objects become simultaneously massless the theory is interacting and conformal

- AD theories are in particular superconformal and, since they are interacting and isolated, they are intrinsically strongly coupled

MOTIVATION AND EXTREMAL CORRELATORS

- We want to compute observable quantities, in particular OPE coefficients between CB operators

- It is a challenge: the **ideal goal** is finding an explicit expression for these quantities in terms of geometric objects (maybe not possible); at the moment we settle for **improving** the results I am going to show

- We indicate the CB operators as $\mathcal{O}_i$ ($i \in \mathbb{N}_0$ related to the R-charge)

- The OPE coefficients we are interested in are determined from the 2-points extremal correlators

$$G_{ij}(x) = \langle \mathcal{O}_i(x)\, \bar{\mathcal{O}}_j(0) \rangle$$

(notice that from the selection rule coming from the conservation of $U(1)_R$ part of R-symmetry at the superconformal point, the two-point functions involving only chiral primaries are trivial)

COMPUTATION WITH LOCALIZATION ON THE 4-SPHERE

- This technique furnishes a formula for the 2-points extremal correlator on the 4-sphere of radius R, $G_{ij}(2\pi R)$, for any rank

- It turns out that if $i \neq j$, then $G_{ij} = 0$, while for $i = j = n \geq 1$ there is the following expression

$$G_{nn}^{\mathrm{Loc}}(2\pi R) = \frac{\det\limits_{0\leq k,l\leq n} C_{kl}}{\det\limits_{0\leq k,l\leq n-1} C_{kl}}$$

[A.Grassi, Z.Komargodski, L.Tizzano, 'Extremal correlators and random matrix theory', JHEP 04 (2021) 214, [1908.10306]]

- The matrix C (two-point matrix model integral) is a $(n+1) \times (n+1)$ whose elements are

$$C_{kl} = \frac{\int_{\mathbb{R}} da\, O_k(a)\, \bar{O}_l(a)\, |Z_{\mathbb{R}^4}(a,R)|^2}{\int_{\mathbb{R}} da\, |Z_{\mathbb{R}^4}(a,R)|^2}$$

[A.Bissi, F.Fucito, A.Manenti, J.F.Morales, R.Savelli, 'OPE coefficients in Argyres-Douglas theories', JHEP 06 (2022) 085, [2112.11899]]

where
- a is related to u as $u \propto a^d$, where d is the conformal dimension of the CB operator
- O_k is the 1-point function on $\mathbb{R}^4$ deformed in a particular way dictated by the localization itself
- $Z_{\mathbb{R}^4}$ is the partition function on this space. We write it as $Z_{\mathbb{R}^4}(a,R) = e^{R^2\,\mathcal{F}(a,R)}$

COMPUTATION WITH LOCALIZATION ON THE 4-SPHERE

- At this point the OPE coefficient can be computed in the following way

$$\lambda_{ij,i+j} = \sqrt{\frac{G^{\text{Loc}}_{i+j,i+j}}{G^{\text{Loc}}_{ii} G^{\text{Loc}}_{jj}}}$$

- So, from this procedure, it is clear that everything consists in computing the matrix C_{kl},

- Following the passages in a particular 'approximation' that we are about to discuss, we get

$$C_{kl} = \frac{1}{(\alpha\, R)^{d(k+l)}} \frac{\Gamma\left(\frac{d}{2}(k+l) + \frac{3}{2}d - 1\right)}{\Gamma\left(\frac{3}{2}d - 1\right)} \qquad (1)$$

[A.Bissi, F.Fucito, A.Manenti, J.F.Morales, R.Savelli, 'OPE coefficients in Argyres-Douglas theories', JHEP 06 (2022) 085, [2112.11899]]

where α is a constant that depends from the theory, but it is not important in the determination of the OPE coefficients

LARGE RADIUS EXPANSION

- The prepotential can be written using the **large radius expansion**, according to which the radius of the 4-sphere is taken very ‘large’ (approaching the flat space)

$$\mathcal{F}(a,R) = \sum_{g=0}^{\infty} \mathcal{F}_g(a) R^{-2g} = \sum_{g=0}^{\infty} f_g \, a^{2-2g} \, R^{-2g}$$

[A.Bissi, F.Fucito, A.Manenti, J.F.Morales, R.Savelli, ‘OPE coefficients in Argyres-Douglas theories’, JHEP 06 (2022) 085, [2112.11899]]

- The result (1) is obtained including only $\mathcal{F}_0$ and $\mathcal{F}_1$
- The fact is that this expansion is only **formal**, due to the conformal nature of our original theory
- From a mathematical point of view, it means that the series is not perturbative, but asymptotic
- In principle, it is not true that $\mathcal{F}_{g\geq 2}$ terms are less important than $\mathcal{F}_0$ and $\mathcal{F}_1$
- The same argument is valid also for 1-point functions O_k, whose higher-order corrections are not known

EXAMPLES AND APPLICATIONS

- Three examples of rank-1 AD theories: $\mathcal{H}_0, \mathcal{H}_1, \mathcal{H}_2$ with $d = \frac{6}{5}, \frac{4}{3}, \frac{3}{2}$ respectively
- They are particular points of the moduli space of $\mathcal{N} = 2 \;\; SU(2)$ SQCD with $N_f = 1{,}2{,}3$ respectively
- At this point we can use **localization** formula (1) and all the other formulae in order to get the OPE coefficients. The first ones are reported in the table
- Another technique that can be used for this study is the **conformal bootstrap**
- This last one furnishes the window within which the OPE coefficients have to fall in
- Except for the smallest coefficient in $\mathcal{H}_0$, results obtained with the first method are inside the window

OPE COEFFICIENT	METHOD	$\mathcal{H}_0\left(d = \frac{6}{5}\right)$	$\mathcal{H}_1\left(d = \frac{4}{3}\right)$	$\mathcal{H}_2\left(d = \frac{3}{2}\right)$
λ^2_{112}	Loc. Conf. Boost.	2,098 2,142 ÷ 2,167	2,241 2,215 ÷ 2,359	2,421 2,298 ÷ 2,698
λ^2_{123}	Loc. Conf. Boost	3,300 3,192 ÷ 3,637	3,674 3,217 ÷ 4,445	4,175

[A.Bissi, F.Fucito, A.Manenti, J.F.Morales, R.Savelli, 'OPE coefficients in Argyres-Douglas theories', JHEP 06 (2022) 085, [2112.11899]]

LARGE R-CHARGE LIMIT

- We study G_{nn}^{Loc} with only $\mathcal{F}_0$ and $\mathcal{F}_1$ in the large R-charge limit (that is large n)

- The reasons to do it are

  1) the large radius expansion of above becomes a real perturbative expansion: from the saddle point method applied to the integral for C_{kl}, it can be seen that the largest part of the contribution derives from $a \gg \frac{1}{R}$

  2) we can compare the results of this limit with those obtained using the EFT dictionary

- This last strategy gives a formula for the extremal correlator that is perturbatively exact in n^{-1}

$$G_{nn}^{\text{EFT}} \simeq e^{n\,A}\, B\, \Gamma\left(dn + \frac{3}{2}d - \frac{1}{2}\right)$$

where A and B are theory-dependent constants that cannot be captured by the EFT technique

UNIVERSAL QUANTITIES

- In order to get rid of these constants, we have focused on the following universal quantities

$$G_{nn}^{U,\mathrm{Loc}} = \frac{G_{n+1,n+1}^{\mathrm{Loc}} G_{n-1,n-1}^{\mathrm{Loc}}}{\left(G_{nn}^{\mathrm{Loc}}\right)^2} \qquad G_{nn}^{U,\mathrm{EFT}} = \frac{G_{n+1,n+1}^{\mathrm{EFT}} G_{n-1,n-1}^{\mathrm{EFT}}}{\left(G_{nn}^{\mathrm{EFT}}\right)^2}$$

- Nowadays it is not possible to get an analytical expression of the correlator G_{nn}^{Loc} for AD theories: the integrals that come out using the Andréief identity for the determinant cannot be solved exactly

- Only a numerical study is reachable (another reason to eliminate the constants in our study)

- We expected that the difference between the two methods for the perturbative expansion of the universal quantities would start from n^{-3} term

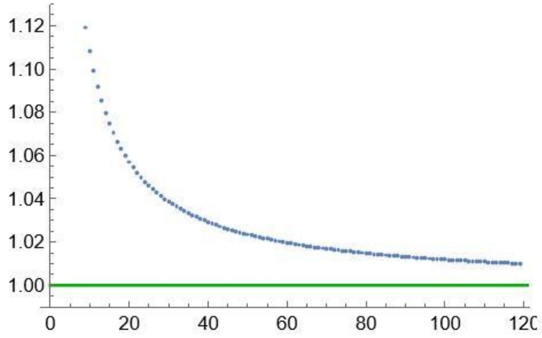
$$G_{nn}^{U,\mathrm{Loc}} = 1 + \frac{\alpha}{n} + \frac{\beta}{n^2} + \frac{\gamma}{n^3} + \mathcal{O}(n^{-4}) \qquad G_{nn}^{U,\mathrm{EFT}} = 1 + \frac{\alpha}{n} + \frac{\beta}{n^2} + \frac{\gamma_1}{n^3} + \mathcal{O}(n^{-4})$$

$$\alpha = d \qquad \beta = \frac{2 - 3d + d^2}{2} \qquad \gamma_1 = \frac{(d-1)^2\,(11 - 14d + 2d^2)}{12\,d}$$

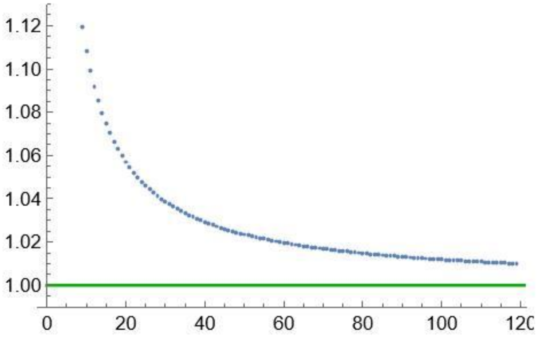
NUMERICAL STUDY FOR $\mathcal{H}_0$

[AC, R.Savelli, In preparation]

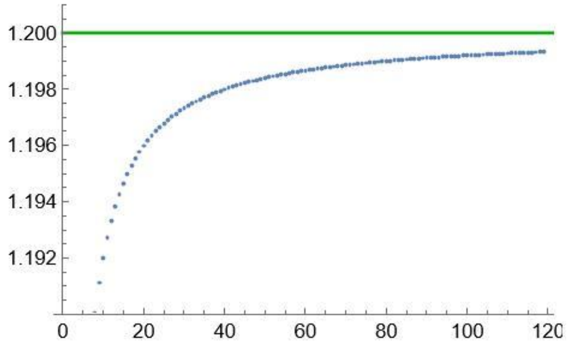
$G_{nn}^{U,\mathrm{Loc}}$



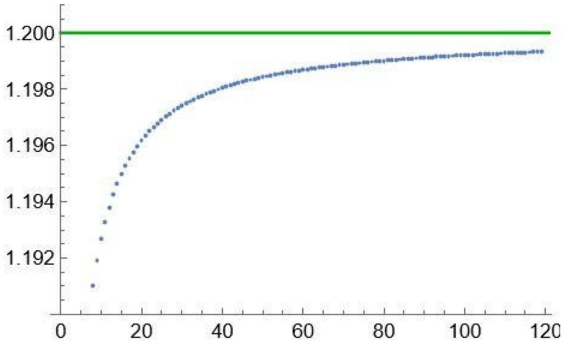
$G_{nn}^{U,\mathrm{EFT}}$



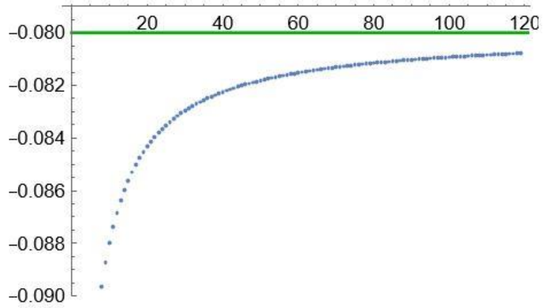
$\left(G_{nn}^{U,\mathrm{Loc}} - 1\right) \cdot n$



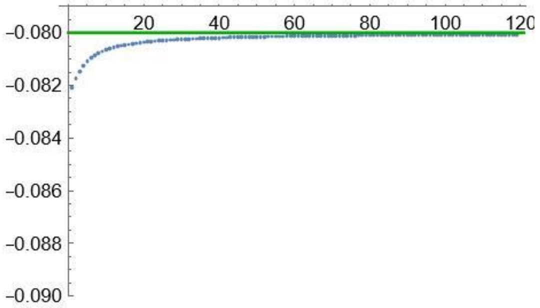
$\left(G_{nn}^{U,\mathrm{EFT}} - 1\right) \cdot n$



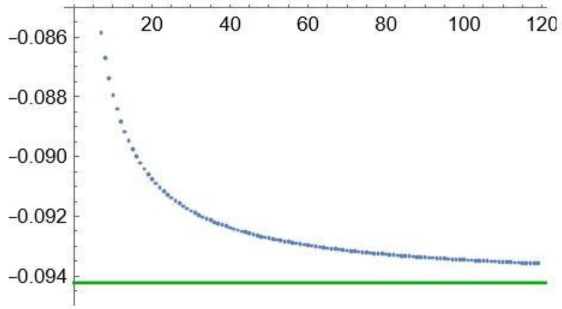
$$\left(G_{nn}^{U,\mathrm{Loc}} - 1 - \frac{6}{5\,n}\right) \cdot n^2$$



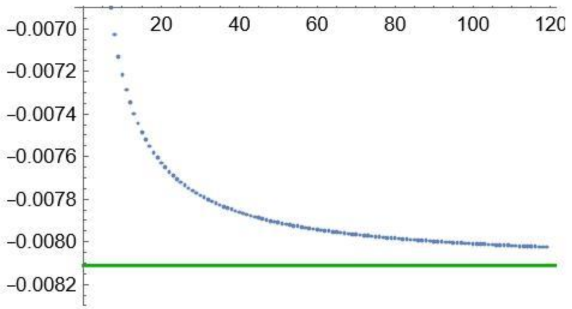
$$\left(G_{nn}^{U,\mathrm{EFT}} - 1 - \frac{6}{5\,n}\right) \cdot n^2$$



$$\left(G_{nn}^{U,\mathrm{Loc}} - 1 - \frac{6}{5\,n} + \frac{2}{25\,n^2}\right) n^3$$



$$\left(G_{nn}^{U,\mathrm{EFT}} - 1 - \frac{6}{5\,n} + \frac{2}{25\,n^2}\right) n^3$$



NUMERICAL RESULTS AND COMPARISON

- We managed to determine the coefficient of the n^{-3} term for $\mathcal{H}_0$ and $\mathcal{H}_1$

$$G_{nn}^{U,\mathrm{Loc}}(\mathcal{H}_0) = 1 + \frac{6}{5\,n} - \frac{2}{25\,n^2} - \frac{106}{1125\,n^3} + \mathcal{O}(n^{-4})$$

$$G_{nn}^{U,\mathrm{EFT}}(\mathcal{H}_0) = 1 + \frac{6}{5\,n} - \frac{2}{25\,n^2} - \frac{73}{9000\,n^3} + \mathcal{O}(n^{-4})$$

$$G_{nn}^{U,\mathrm{Loc}}(\mathcal{H}_1) = 1 + \frac{4}{3\,n} - \frac{1}{9\,n^2} - \frac{7}{324\,n^3} + \mathcal{O}(n^{-4})$$

$$G_{nn}^{U,\mathrm{EFT}}(\mathcal{H}_1) = 1 + \frac{4}{3\,n} - \frac{1}{9\,n^2} - \frac{37}{1296\,n^3} + \mathcal{O}(n^{-4})$$

- This behaviour is in agreement with *[A.Grassi, Z.Komargodski, L.Tizzano, 'Extremal correlators and random matrix theory', JHEP 04 (2021) 214, [1908.10306]]*, where the results are explicitly shown for SQCD with $N_f = 4 \ (d = 2)$ and reported for $\ln(G_{nn})$

PROPOSAL FOR IMPROVEMENT OF THE RESULTS

- In order to fix this mismatch, the **first step** we can do is including in the computation from localization also all the other terms in the prepotential

- Ansatz for the partition function that interpolates between the behaviour for large a (known) and small a (new contribution)

- The ansatz cannot change the coefficients of n^{-1} and n^{-2} in the universal quantity

- The first idea that has come in our mind is (setting $R = 1$)

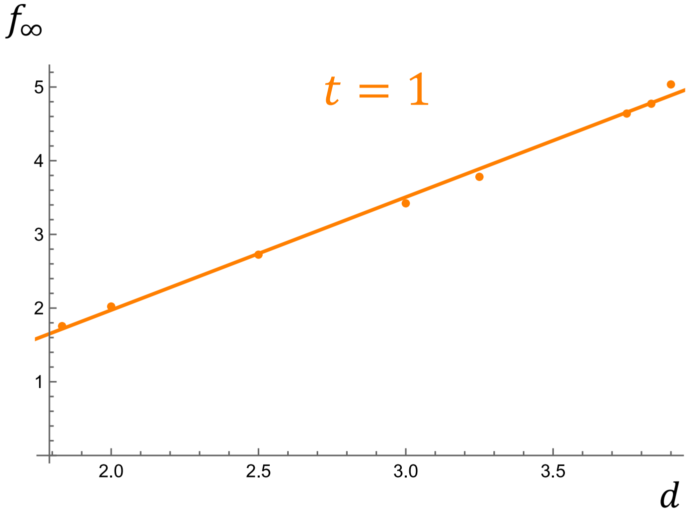
$$Z_{\mathbb{R}^4} = e^{\mathcal{F}_0}\, e^{\mathcal{F}_1}\ a^{-d\, f_\infty} \left(t + a^d\right)^{f_\infty}$$

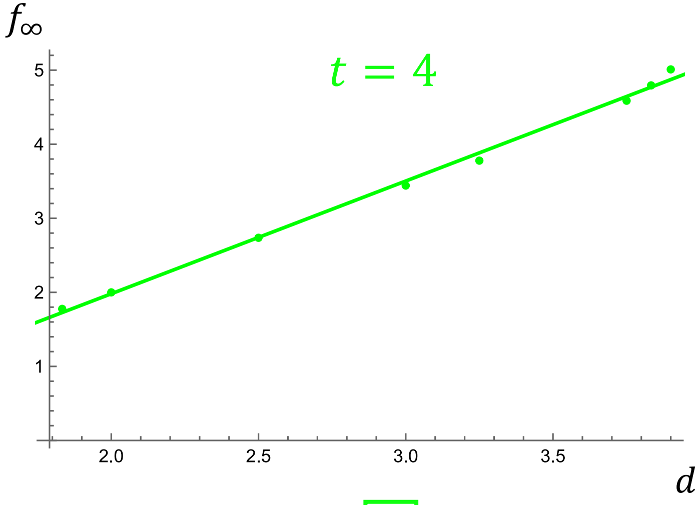
with $t > 0$.

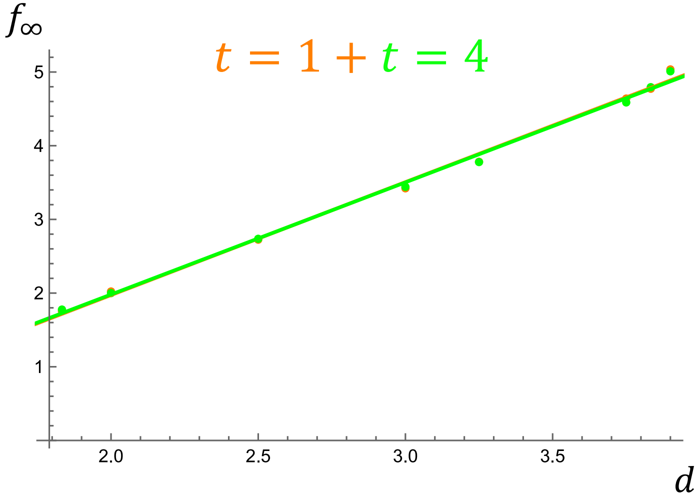
- At this point we chose the value of f_∞ for different $d \in [\frac{11}{6}, 4)$ (AD theories are below this interval) in such a way that the new coefficient of n^{-3} term is equal to the one from the EFT formula

NUMERICAL RESULTS FOR THE NEW ANSATZ

[AC, R.Savelli, In preparation]







$$f_\infty = m_1\, d + q_1$$
$$m_1 \simeq -1.09 \quad q_1 \simeq 1.53$$
$$\chi_1^2 = 0.0076$$

$$f_\infty = m_4\, d + q_4$$
$$m_4 \simeq -1.06 \quad q_4 \simeq 1.52$$
$$\chi_4^2 = 0.0072$$

$$f_\infty(\mathcal{H}_0) \simeq 0.75$$
$$f_\infty(\mathcal{H}_1) \simeq 0.96$$
$$f_\infty(\mathcal{H}_2) \simeq 1.21$$

CONCLUSIONS AND GOALS

- Up to now insertions have been treated classically

- Hence the **second step** we can do is considering SQCD with $N_f = 4$ and adding the contribution of instantons to the partition function, giving rise to a more complicated dependence from τ than the one just studied

- If the EFT formula

$$G_{nn}^{\mathrm{EFT}} \simeq e^{nA(\tau,\bar{\tau})}\, B(\tau,\bar{\tau})\, \Gamma\left(dn + \frac{3}{2}d - \frac{1}{2}\right)$$

is correct, then the appearance of instantons should not change the terms with integer powers in the perturbative expansion of the universal quantities

- We are trying to verify this for the theory just mentioned, where everything can be treated analytically

THANK YOU SO MUCH FOR YOUR ATTENTION!

LARGE RADIUS EXPANSION IN LARGE R-CHARGE LIMIT

- Large R-charge limit consists in taking n very large (where we remember that $n+1$ is the size of the matrix C_{kl})
- Assuming that there exists a saddle point for large a (hence we neglect $\mathcal{F}_{g\geq 2}$) and using the expressions of the ingredients entering in C_{nn} $(k=l=n)$, we get, from the numerator (setting $R=1$)

$$C_{nn} \simeq \int_{\mathbb{R}} da \;\, a^{2dn} \, e^{-2\pi \;\, \mathrm{Im}(\tau) \;\, a^2} a^{\beta} = \int_{\mathbb{R}} da \;\, e^{(2dn+\beta)\ln(a)-2\pi \;\, \mathrm{Im}(\tau) \;\, a^2}$$

where β is a real number

- By applying the saddle point method we gain

$$\frac{2dn+\beta}{a} - 4\pi \, \mathrm{Im}(\tau) \, a = 0 \Rightarrow a = \sqrt{\frac{2dn+\beta}{4\pi \, \mathrm{Im}(\tau)}} \simeq \sqrt{n}$$

finding a consistency with the initial assumption. Hence in the large R-charge limit $\mathcal{F}_{g\geq 2}$ are subleading w.r.t. $\mathcal{F}_0$ and $\mathcal{F}_1$

APPLICATION OF ANDREIEF IDENTITY

- Andrèief identity states that, given two sets of n functions $\{f_k(y); g_k(y)\}_{k=0}^{n-1}$ and a measure $d\mu(y)$, then

$$\det_{ab} \int d\mu(y)\, f_a(y) g_b(y) = \frac{1}{n!} \int \prod_{i=0}^{n-1} d\mu(y_i) \;\det_{ab}\big(f_a(y_b)\big)\; \det_{cd}\big(g_c(y_d)\big) \qquad (\#)$$

that is the identity relates a determinant of integrals to a multivariate integral over determinants

- In our case, we have to compute (modulo some constants that do not care in the comparison with the EFT formula)

$$\det_{kl} \int_{\mathbb{R}} da\, \left(a^d\right)^{k+l} a^{3(d-1)}\, e^{-a^2}$$

- Hence, by comparing with (#), we identify $d\mu(y) \leftrightarrow da\, e^{-a^2}\, a^{3(d-1)}$, $f_k(y) \leftrightarrow a^{dk}$, $g_l(y) \leftrightarrow a^{dl}$ (and, roughly, we replace every a with y_i) and hence we get, from the identity of the Vandermonde determinant

$$\det_{kp}\left(f_k(y_p)\right) = \det_{kp}\left(\left(y_p^d\right)^k\right) = \prod_{j<k}\left(y_j^d - y_k^d\right) \qquad \det_{ls}\left(g_l(y_s)\right) = \det_{ls}\left(\left(y_s^d\right)^l\right) = \prod_{j<k}\left(y_j^d - y_k^d\right)$$

APPLICATION OF ANDREIEF IDENTITY

- So our determinant becomes

$$\det_{kl}\int_{\mathbb{R}} da\,\left(a^d\right)^{k+l} a^{3(d-1)}\,e^{-a^2} = \frac{1}{n!}\int_{\mathbb{R}^n}\prod_{j=0}^{n-1} dy_i\;e^{-y_j^2}\,y_j^{3(d-1)}\prod_{j<k}\left(y_j^d - y_k^d\right)^2$$

- Applying the following change of variables (I will be sloppy on the interval of integration, which should be $\mathbb{R}_+^n$), $x_i = y_i^d$, then we get

$$\det_{kl}\int_{\mathbb{R}} da\,\left(a^d\right)^{k+l} a^{3(d-1)}\,e^{-a^2} = \frac{1}{n!}\int_{\mathbb{R}_+^n}\prod_{j=0}^{n-1} dx_i\;e^{-x_j^{\frac{2}{d}}}\,x_j^{2-\frac{2}{d}}\prod_{j<k}\left(x_j - x_k\right)^2$$

- If $d = 2$ these integrals can be solved in an analytical way, finding the known result for SQCD with $N_f = 4$ of *[A.Grassi, Z.Komargodski, L.Tizzano, 'Extremal correlators and random matrix theory', JHEP 04 (2021) 214, [1908.10306]]*; for generic d nowadays we cannot solve these integrals analytically