Recent Advances in pre Big Bang String Cosmology:
Bouncing solutions in non-perturbative String Theory

Speaker: **Eliseo Pavone**

Co-authors: **P. Conzinu, G. Fanizza, M. Gasperini, L. Tedesco, G. Veneziano**

String Phenomenology Padua 24-28/06/2024

Cosmological Challenges and String Theory

- Consistent theory of Quantum Gravity
- Avoid the Classical Big Bang Singularity
- Go beyond EFT approach to Inflation
- No need for ad-hoc fields to drive Inflation

What does String Theory suggest?

- The existence of a low energy massless multiplet $\{\phi, g_{\mu\nu}, B_{\mu\nu}\}$
- Pre Big-Bang scenario instead of Slow-Roll (Bouncing Cosmology)
- Higher order curvature corrections (α' expansion)
- Higher order string coupling expansion $g_s^2 = exp(\phi)$ (Genus Expansion)
- Additional simmetry in cosmological space-time with d abelian isometries, O(d,d) invariance in the field space (Continuous generalisation of T-duality)

Main Topics

- General criteria to have bouncing solutions using Hohm Zwiebach (HZ) action (all order α' action) starting from perturbative vacuum of String Theory

Main Topics

- General criteria to have bouncing solutions using Hohm Zwiebach (HZ) action (all order α' action) starting from perturbative vacuum of String Theory

- Dilaton stabilisation, FLRW and de-Sitter attractor from non-perturbative dilaton potential

Main Topics

- General criteria to have bouncing solutions using Hohm Zwiebach (HZ) action (all order α' action) starting from perturbative vacuum of String Theory

- Dilaton stabilisation, FLRW and de-Sitter attractor from non-perturbative dilaton potential

- Isotropisation mechanism via α' corrections and Dilaton potential

All order α' O(d,d) invariant HZ String Action in a anisotropic cosmology (Bianchi-I)

$$S = \int L dt = -\frac{1}{2}\int dt N e^{-\bar{\phi}} \left(N^{-2}\dot{\bar{\phi}}^2 + F(N^{-1}\dot{\beta}_i) + 2(\alpha')^{(d-1)/2} V(\phi) \right)$$

All order α′ O(d,d) invariant HZ String Action in a anisotropic cosmology (Bianchi-I)

$$S = \int L dt = -\frac{1}{2}\int dt N e^{-\bar{\phi}} \left(N^{-2}\dot{\bar{\phi}}^2 + F(N^{-1}\dot{\beta}_i) + 2(\alpha')^{(d-1)/2} V(\phi) \right)$$

$$ds^2 = N^2(t)dt^2 - \sum_i \delta_{ij} e^{2\beta_i} dx^i dx^j$$

$$F = -N^{-2}\sum_i \dot{\beta}_i^2 + \mathcal{O}(\alpha') = -\sum_i H_i^2 + \mathcal{O}(\alpha')$$ Even function of the Hubble functions

$$\bar{\phi} = \phi - \sum_i \beta_i$$ Shifted Dilaton

All order α' O(d,d) invariant HZ String Action in a anisotropic cosmology (Bianchi-I)

$$S = \int L dt = -\frac{1}{2}\int dt N e^{-\bar{\phi}} \left(N^{-2}\dot{\bar{\phi}}^2 + F(N^{-1}\dot{\beta}_i) + 2(\alpha')^{(d-1)/2} V(\phi) \right)$$

$$ds^2 = N^2(t)dt^2 - \sum_i \delta_{ij} e^{2\beta_i} dx^i dx^j$$

$$F = -N^{-2}\sum_i \dot{\beta}_i^2 + \mathcal{O}(\alpha') = -\sum_i H_i^2 + \mathcal{O}(\alpha')$$ Even function of the Hubble functions

$$\bar{\phi} = \phi - \sum_i \beta_i$$ Shifted Dilaton

Low Energy isotropic solution (Asymptotic Past Triviality)

$$\phi \sim -\left(1+\sqrt{d}\right)\ln(-t)$$

$$H = \dot{\beta} = \frac{1}{\sqrt{d}(-t)}$$

All order α′ O(d,d) invariant HZ String Action in a anisotropic cosmology (Bianchi-I)

$$S = \int L dt = -\frac{1}{2}\int dt N e^{-\bar{\phi}} \left(N^{-2}\dot{\bar{\phi}}^2 + F(N^{-1}\dot{\beta}_i) + 2(\alpha')^{(d-1)/2} V(\phi) \right)$$

$$ds^2 = N^2(t)dt^2 - \sum_i \delta_{ij} e^{2\beta_i} dx^i dx^j$$

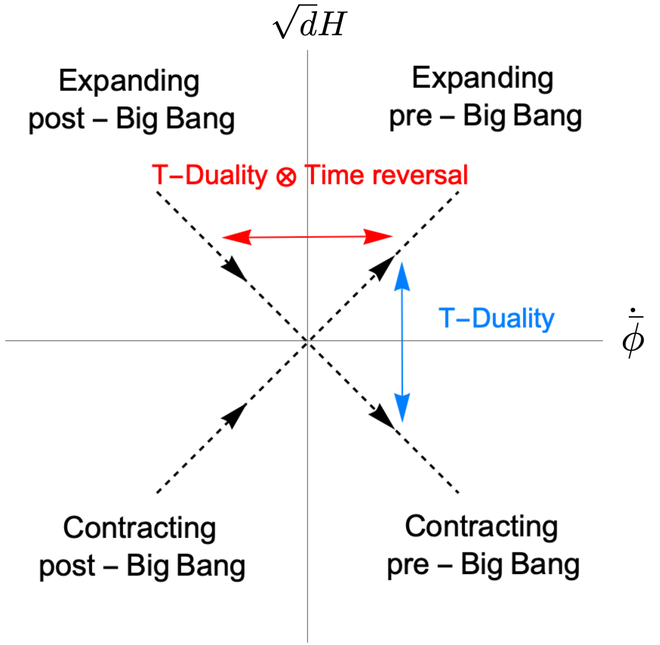
$$F = -N^{-2}\sum_i \dot{\beta}_i^2 + \mathcal{O}(\alpha') = -\sum_i H_i^2 + \mathcal{O}(\alpha')$$ Even function of the Hubble functions

$$\bar{\phi} = \phi - \sum_i \beta_i$$ Shifted Dilaton

Low Energy isotropic solution (Asymptotic Past Triviality)

$$\phi \sim -\left(1+\sqrt{d}\right)\ln(-t)$$

$$H = \dot{\beta} = \frac{1}{\sqrt{d}(-t)}$$



The Routhian formalism and the all order α' Hamiltonian.

Legendre transform only a subset of the variables of the Lagrangian β_i

$$\mathcal{R}(N,\bar{\phi},\pi_i) = \sum_i \pi_i \dot{\beta}_i - L = N e^{\bar{\phi}} \left[\frac{1}{2} N^{-2} \dot{\bar{\phi}}^2 + h(z_i) + V(\bar{\phi} + \sum_i \beta_i) \right]$$

$$h(z_i) \equiv \frac{1}{2} \left(F - \sum_i \dot{\beta}_i \frac{\partial F}{\partial H_i} \right) = \frac{1}{2} \sum z_i^2 + \mathcal{O}(\alpha') + \dots \qquad \frac{\partial h}{\partial z_i} = H_i$$

The Routhian formalism and the all order α' Hamiltonian.

Legendre transform only a subset of the variables of the Lagrangian β_i

$$\mathcal{R}(N,\bar\phi,\pi_i) = \sum_i \pi_i\dot\beta_i - L = Ne^{\bar\phi}\left[\frac{1}{2}N^{-2}\dot{\bar\phi}^2 + h(z_i) + V(\bar\phi + \sum_i \beta_i)\right]$$

$$h(z_i) \equiv \frac{1}{2}\left(F - \sum_i \dot\beta_i \frac{\partial F}{\partial H_i}\right) = \frac{1}{2}\sum z_i^2 + \mathcal{O}(\alpha') + \ldots \qquad \frac{\partial h}{\partial z_i} = H_i$$

EoM

$$\dot{\bar\phi}^2 = 2h(z_i) + 2V, \qquad \dot z_i = z_i\dot{\bar\phi} - \frac{\partial V}{\partial \phi}, \qquad \ddot{\bar\phi} = \sum_i z_i \frac{\partial h}{\partial z_i} + \frac{\partial V}{\partial \phi}$$

The Routhian formalism and the all order α' Hamiltonian.

Legendre transform only a subset of the variables of the Lagrangian β_i

$$\mathcal{R}(N,\bar{\phi},\pi_i) = \sum_i \pi_i \dot{\beta}_i - L = N e^{\bar{\phi}} \left[\frac{1}{2} N^{-2} \dot{\bar{\phi}}^2 + h(z_i) + V(\bar{\phi} + \sum_i \beta_i) \right]$$

$$h(z_i) \equiv \frac{1}{2} \left(F - \sum_i \dot{\beta}_i \frac{\partial F}{\partial H_i} \right) = \frac{1}{2} \sum z_i^2 + \mathcal{O}(\alpha') + \ldots \qquad \frac{\partial h}{\partial z_i} = H_i$$

EoM

$$\dot{\bar{\phi}}^2 = 2h(z_i) + 2V, \qquad \dot{z}_i = z_i \dot{\bar{\phi}} - \frac{\partial V}{\partial \phi}, \qquad \ddot{\bar{\phi}} = \sum_i z_i \frac{\partial h}{\partial z_i} + \frac{\partial V}{\partial \phi}$$

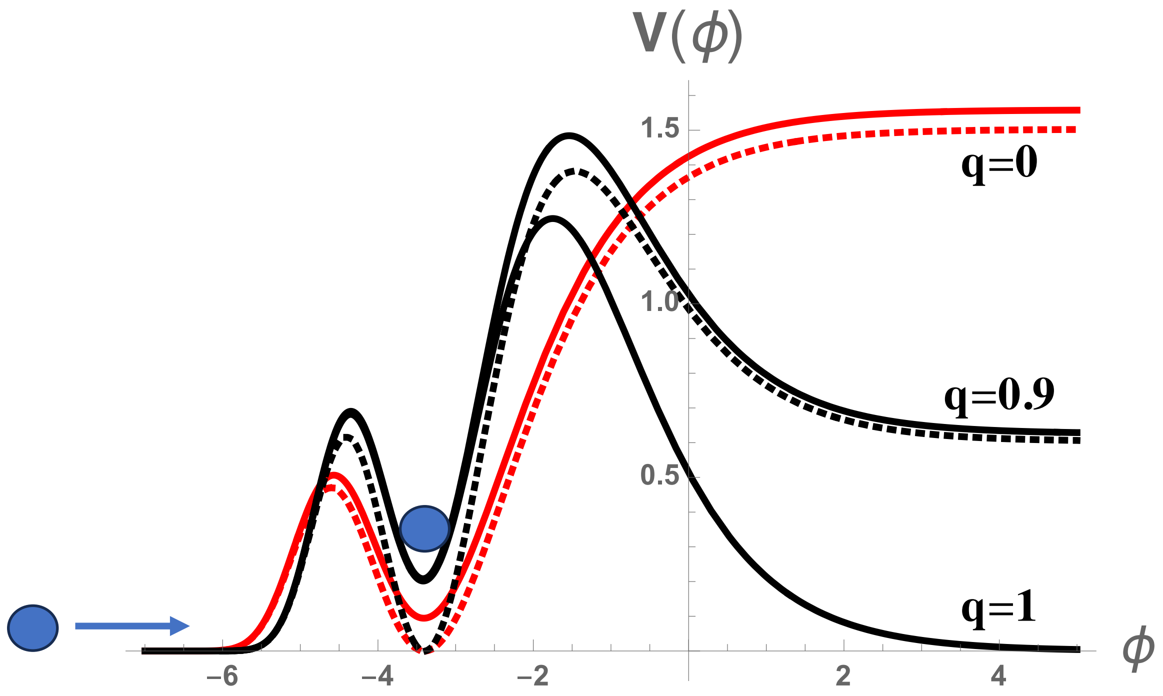
Advantages:

- Specify only two functions to have a non-perturbative description of the system.
- Simple 'hamiltonians' capture all α' corrections and generate bouncing solutions if they have another zero except the trivial one. In general they come from non-holomorphic F. In the isotropic case we used:

$$h(z) = \frac{d}{2}(z^2 - \alpha' \frac{z^4}{2})$$

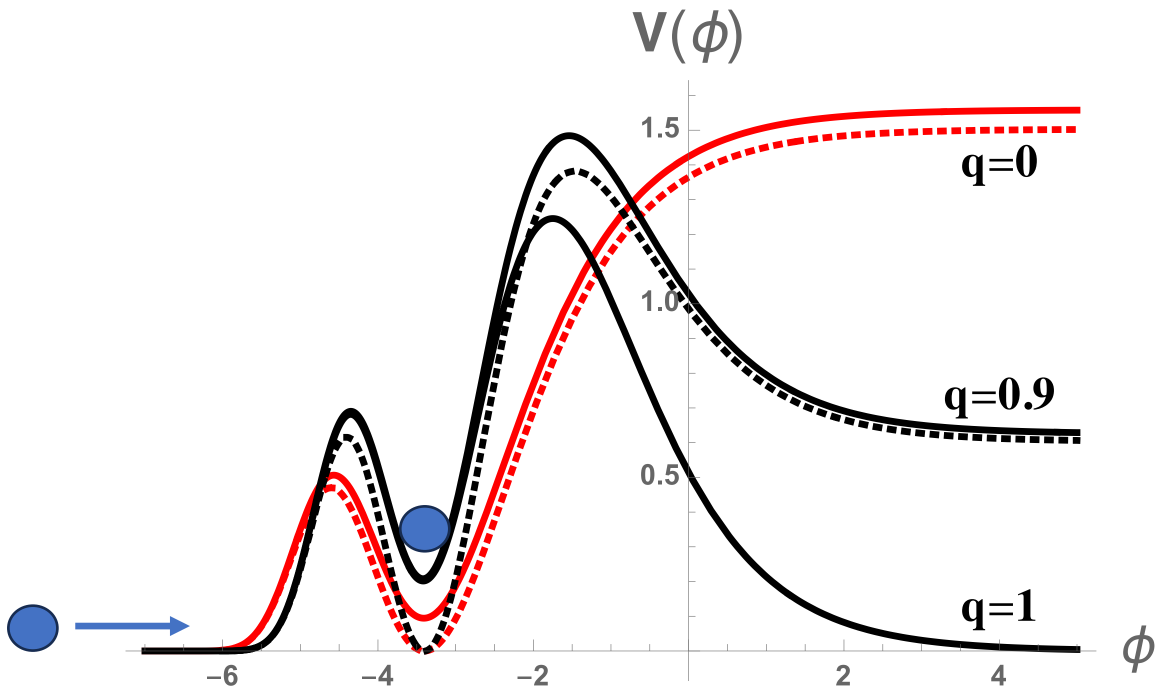
The phenomenological non-perturbative dilaton potential

Dilaton Potential with Istantonic behaviour not captured by string coupling expansion (Non-perturbative Potential)



The phenomenological non-perturbative dilaton potential

Dilaton Potential with Istantonic behaviour not captured by string coupling expansion (Non-perturbative Potential)

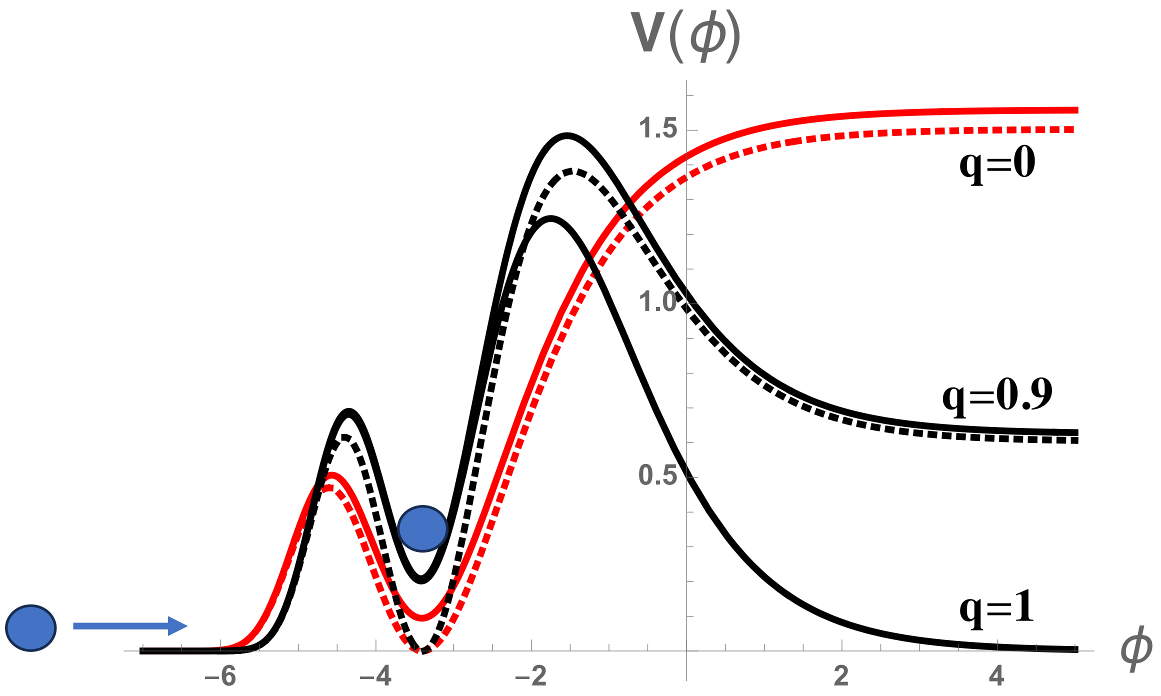


The phenomenological non-perturbative dilaton potential

Dilaton Potential with Istantonic behaviour not captured by string coupling expansion (Non-perturbative Potential)

$$V(\phi) = A\, e^{-B(\phi)/\beta} \left[\left(c^2 - B(\phi)\right)^2 + \delta B(\phi) \right] \left[1 - q\, B^{-1}(\phi)\right]$$

$$B(\phi) = \frac{1 + \alpha\, g_s^2}{\alpha\, g_s^2} = \frac{1 + \alpha\, e^{\phi}}{\alpha\, e^{\phi}}$$

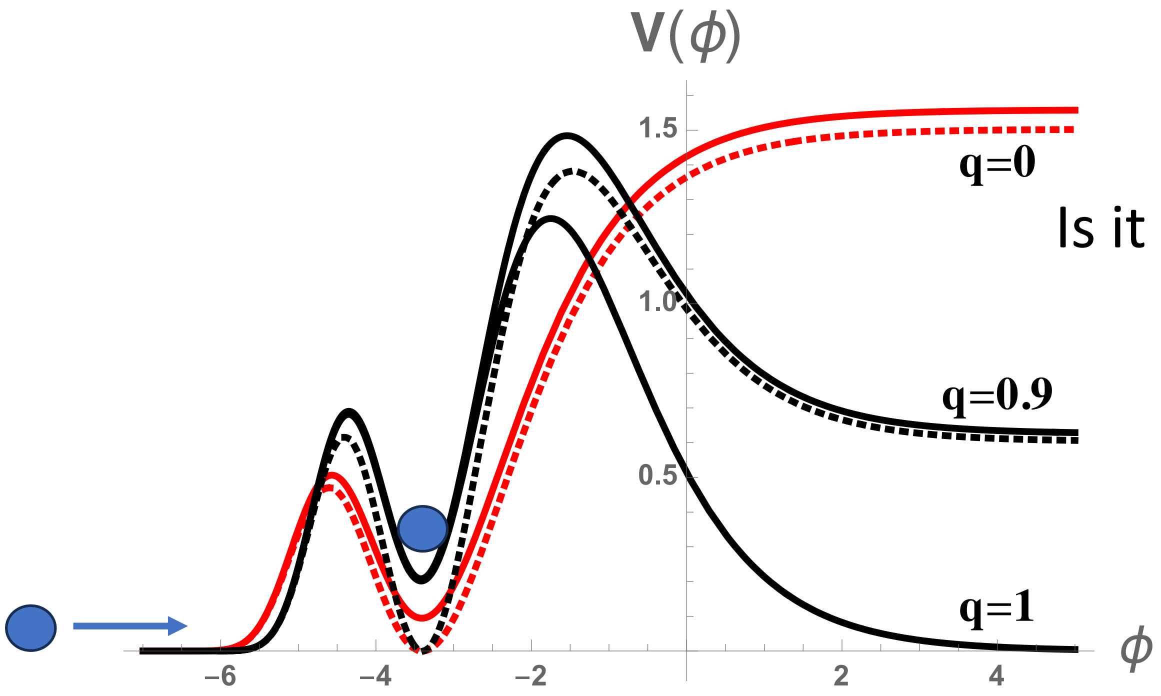


The phenomenological non-perturbative dilaton potential

Dilaton Potential with Istantonic behaviour not captured by string coupling expansion (Non-perturbative Potential)

$$V(\phi) = A\, e^{-B(\phi)/\beta} \left[\left(c^2 - B(\phi)\right)^2 + \delta B(\phi) \right] \left[1 - q\, B^{-1}(\phi) \right]$$

$$B(\phi) = \frac{1 + \alpha\, g_s^2}{\alpha\, g_s^2} = \frac{1 + \alpha\, e^{\phi}}{\alpha\, e^{\phi}}$$



Is it possible to reach a final state with a frozen dilaton?

- Stabilisation of the dilaton required to have Einstenian final cosmology
- What is the final geometry?
- Is the final configuration an attractor?

Dilaton Stabilisation and FLRW attractor

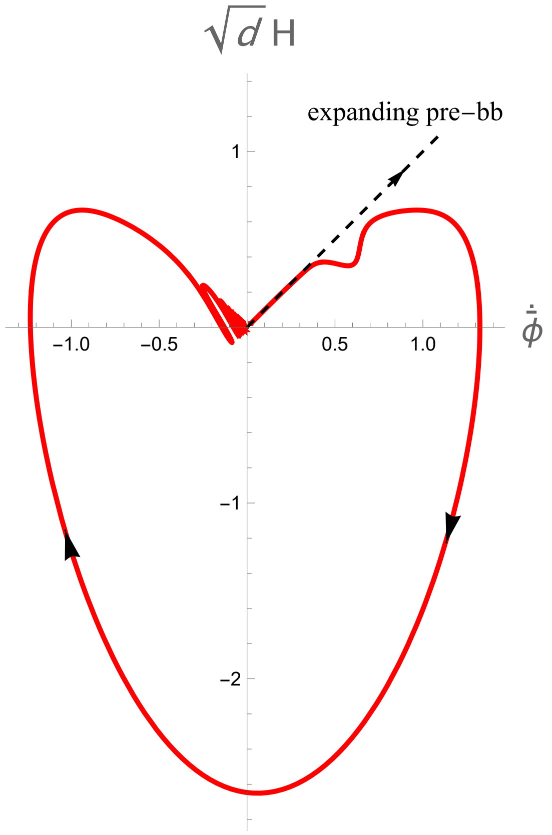
Asymptotic Matter FLRW geometry $\delta=0$

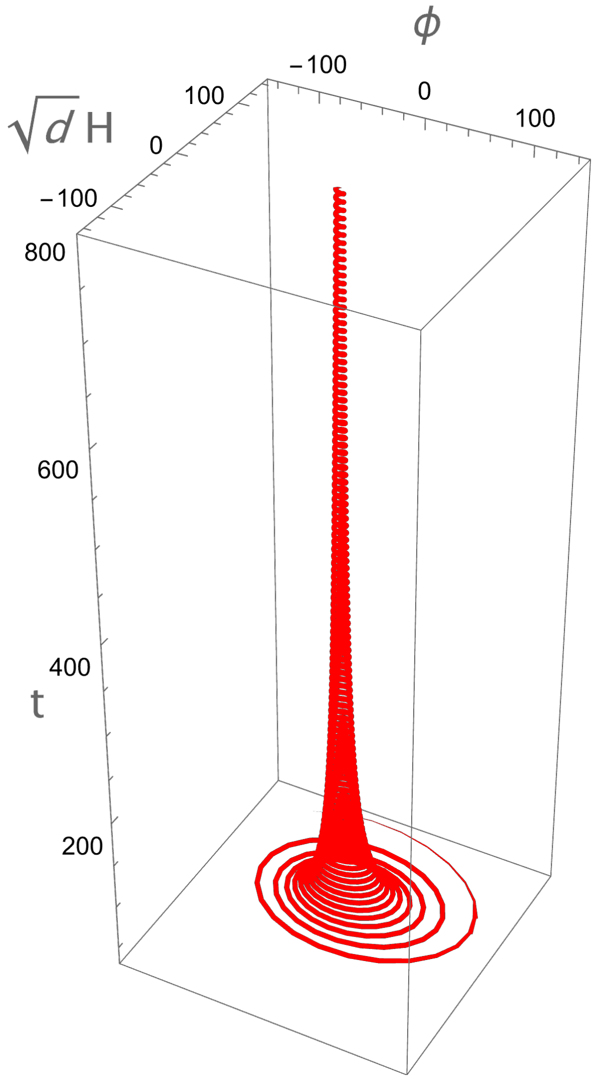
$$V_0 = 0$$

Dilaton Stabilisation and FLRW attractor

Asymptotic Matter FLRW geometry δ=0

$$V_0 = 0$$





Dilaton Stabilisation and FLRW attractor

Asymptotic Matter FLRW geometry δ=0

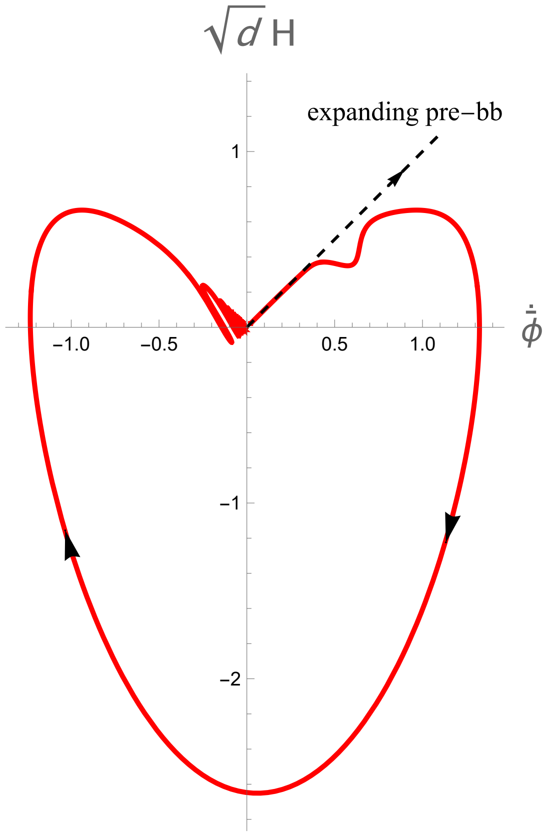
$$V_0 = 0$$

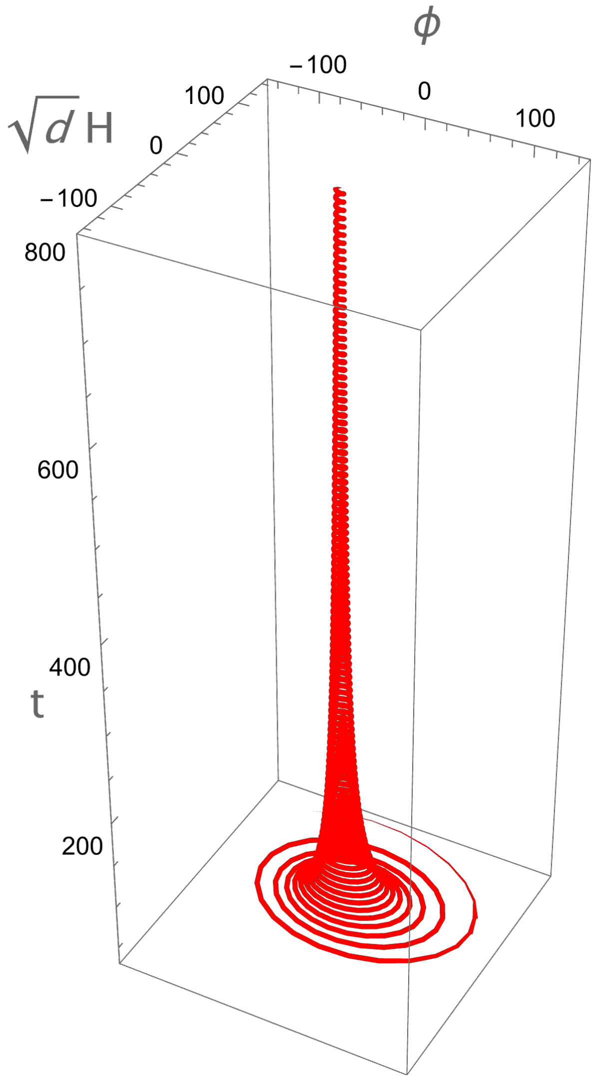
$$\phi(t) = \quad \phi_0 + \frac{2(d-1)}{t\omega\sqrt{d}} \sin(\omega t + \theta)$$

$$H(t) = \quad \frac{2}{td}\left[1 + \sqrt{d}\cos(\omega t + \theta)\right]$$

$$\omega = m\sqrt{d-1}$$

$$m^2 = V''(\phi_m)$$





Dilaton Stabilisation and de-Sitter attractor

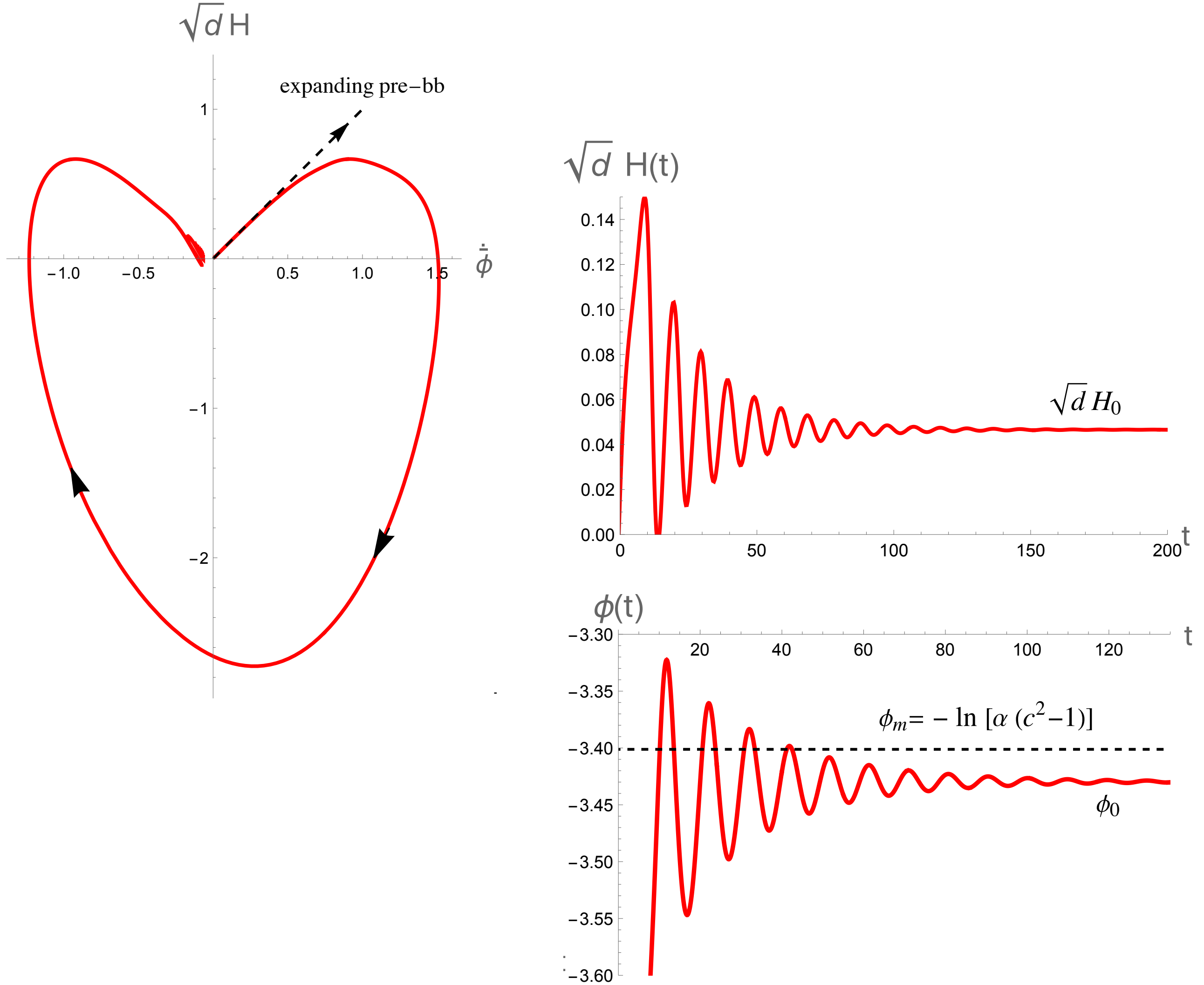
Asymptotic de-Sitter geometry $\delta \neq 0$

$$V_0 = V(\phi_m) > 0$$

Dilaton Stabilisation and de-Sitter attractor

Asymptotic de-Sitter geometry δ ≠ 0

$$V_0 = V(\phi_m) > 0$$



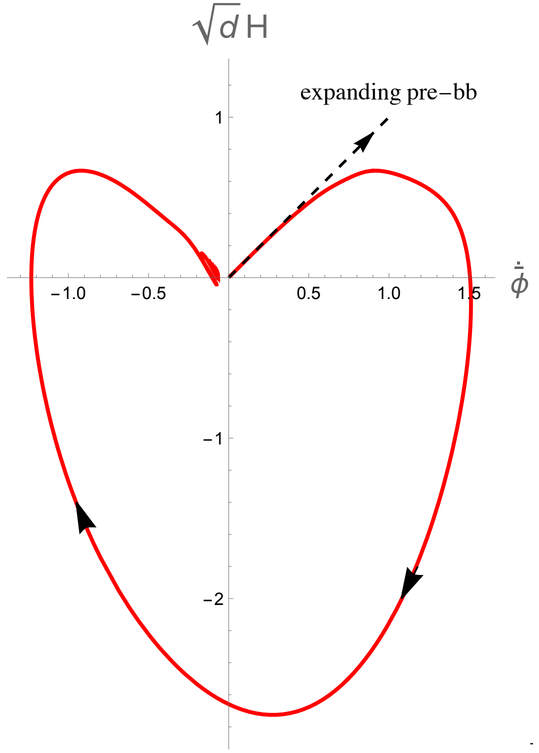
Dilaton Stabilisation and de-Sitter attractor

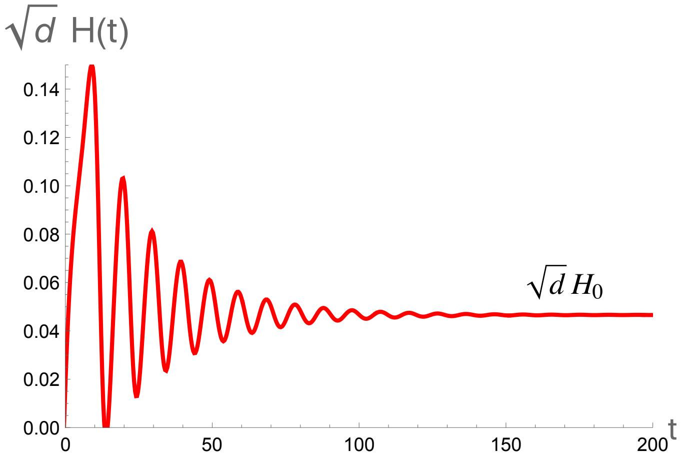
Asymptotic de-Sitter geometry $\delta \neq 0$

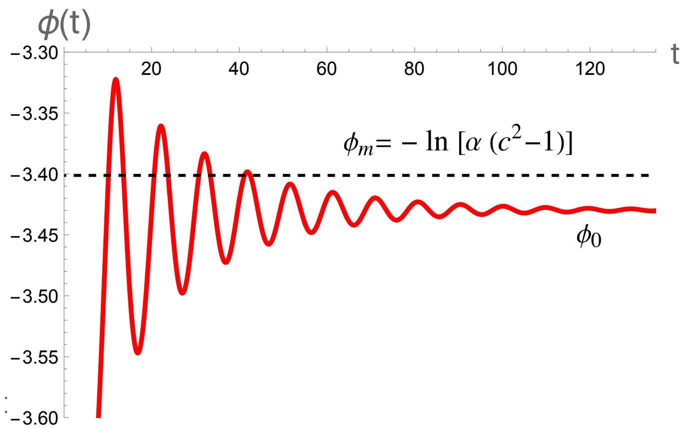
$$V_0 = V(\phi_m) > 0$$

$$\phi_0 \simeq \phi_m - \frac{2V_0}{m^2(d-1)}$$

$$H_0 \simeq \left[\frac{2V_0}{d(d-1)}\right]^{1/2}$$







Dilaton Stabilisation and de-Sitter attractor

Asymptotic de-Sitter geometry δ ≠ 0

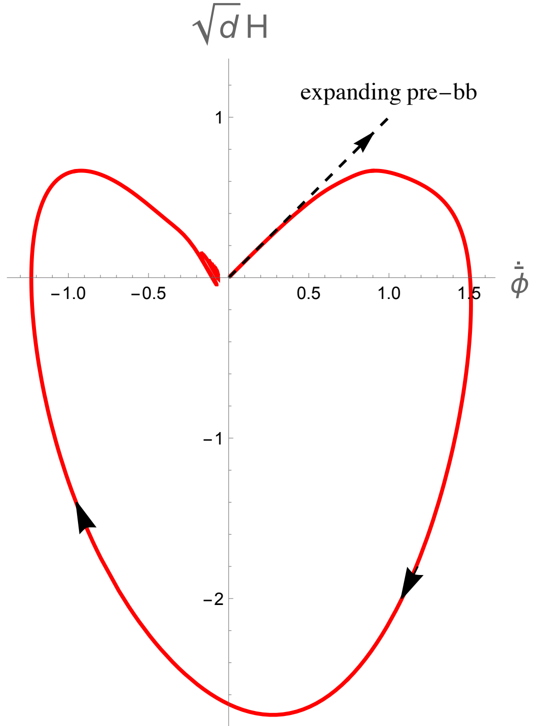
$$V_0 = V(\phi_m) > 0$$

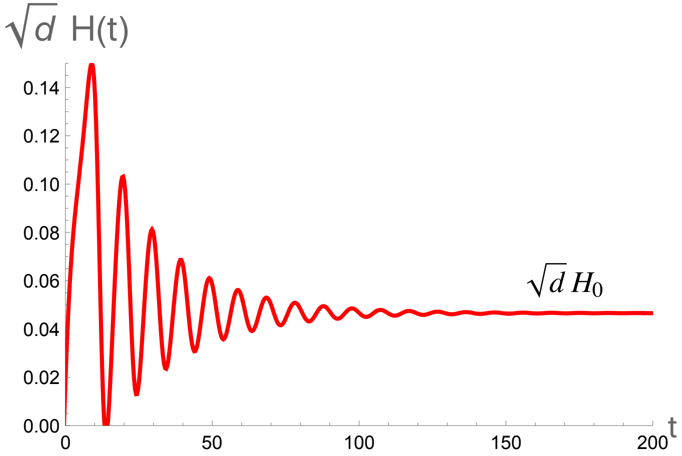
$$\phi_0 \simeq \phi_m - \frac{2V_0}{m^2(d-1)}$$

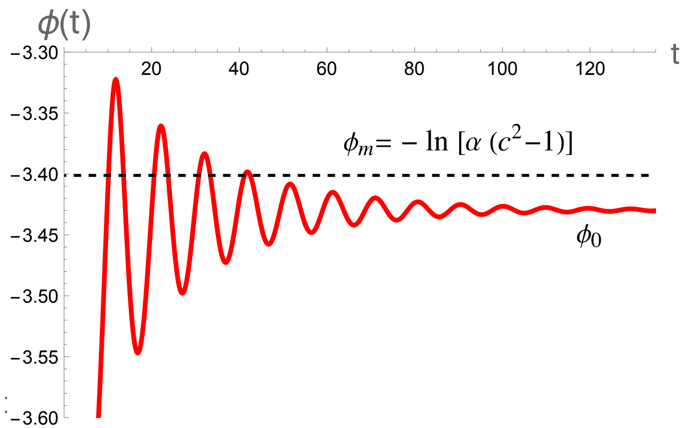
$$H_0 \simeq \left[\frac{2V_0}{d(d-1)}\right]^{1/2}$$

$$\left[\frac{\partial}{\partial\phi}\left(V e^{\frac{2\phi}{d-1}}\right)\right]_{\phi=\phi_0} = 0,$$

E-frame dilaton potential







Minimum of the dilaton potential in the E-frame (The dilaton is a minimally coupled scalar in the E-frame)

Dilaton Stabilisation and de-Sitter attractor

Asymptotic de-Sitter geometry δ ≠ 0

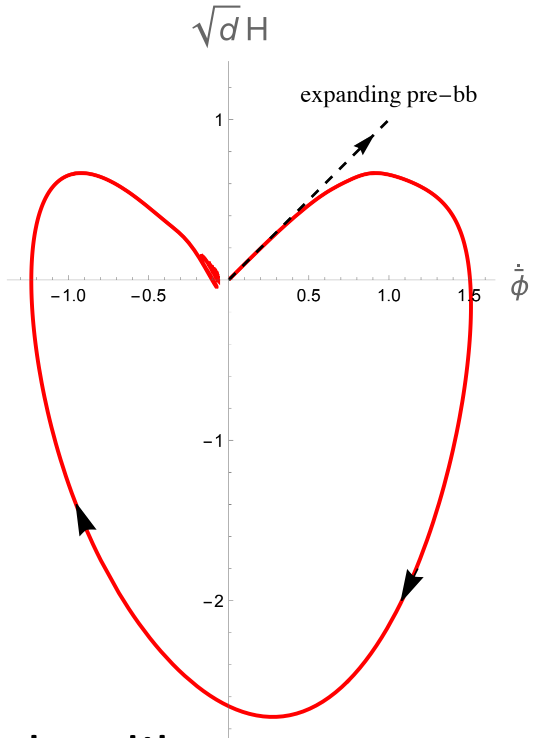
$$V_0 = V(\phi_m) > 0$$

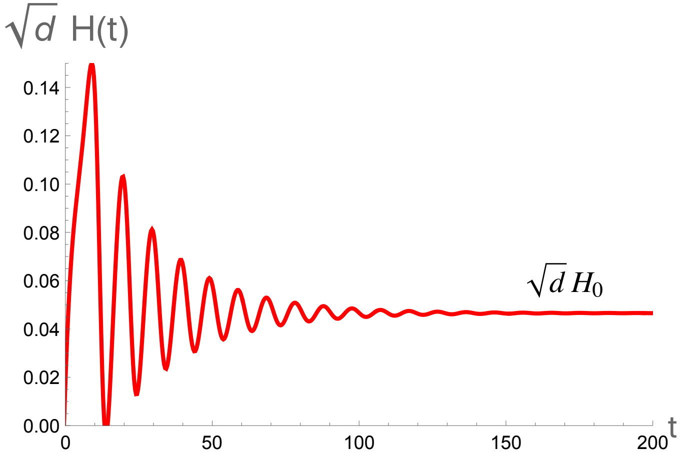
$$\phi_0 \simeq \phi_m - \frac{2V_0}{m^2(d-1)}$$

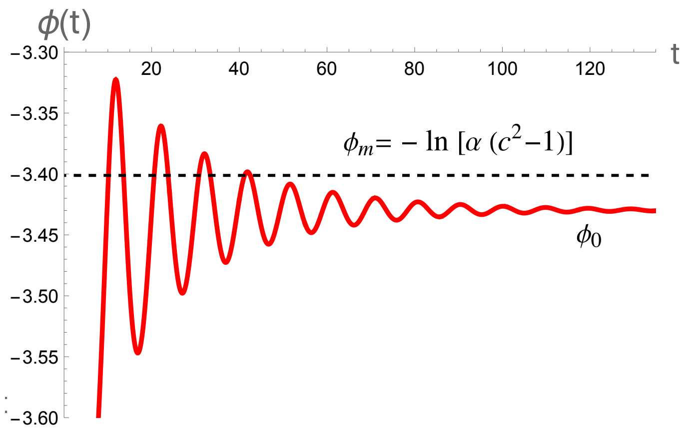
$$H_0 \simeq \left[\frac{2V_0}{d(d-1)}\right]^{1/2}$$

$$\left[\frac{\partial}{\partial\phi}\left(V e^{\frac{2\phi}{d-1}}\right)\right]_{\phi=\phi_0} = 0$$

E-frame dilaton potential







Minimum of the dilaton potential in the E-frame (The dilaton is a minimally coupled scalar in the E-frame)

Is this solution a stable attractor?

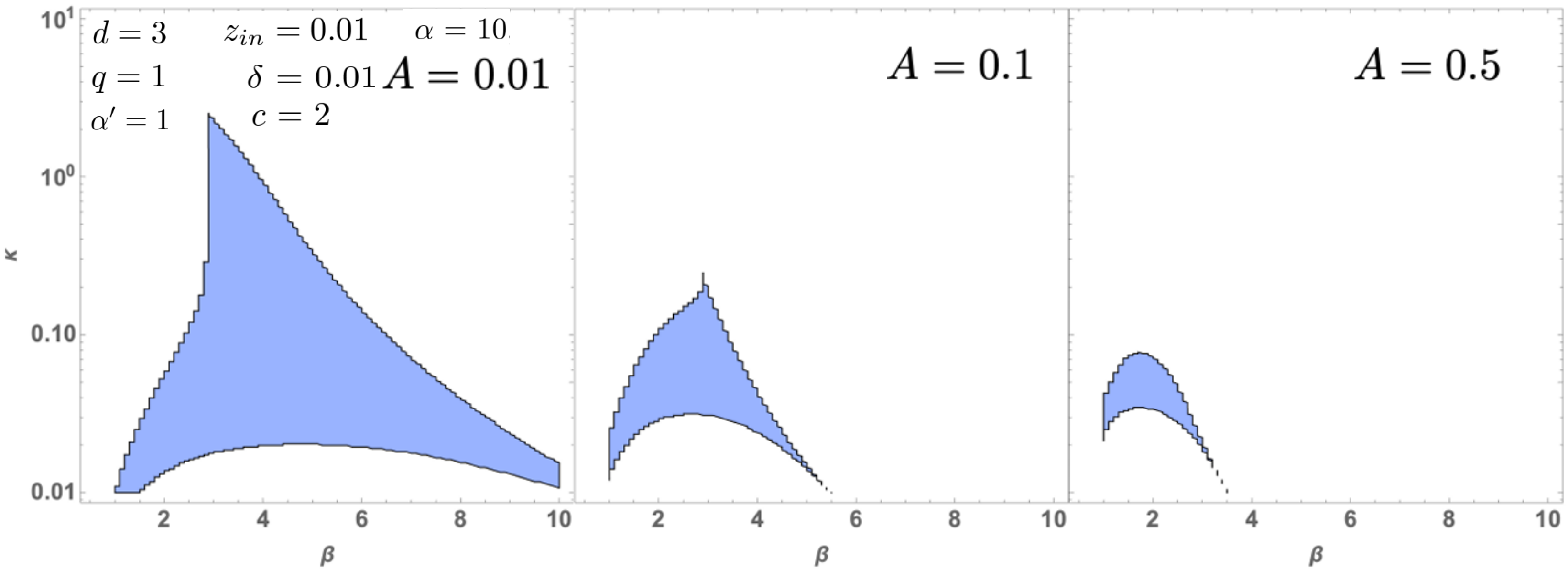
Numerical analysis of the attraction basin: Isotropic case

β controls the height of the first peak.

- Conserved quantity from the EoM used to fix the initial condition of the dilaton.

$$e^{-\overline{\phi}} z = \kappa^{-1}$$

Variation of the height of the first peak (β) and the amplitude A



Numerical analysis of the attraction basin: Isotropic case

β controls the height of the first peak.

- Conserved quantity from the EoM used to fix the initial condition of the dilaton.

$$e^{-\overline{\phi}}z = \kappa^{-1}$$

q controls the asymptotic behaviour of the dilaton potential.

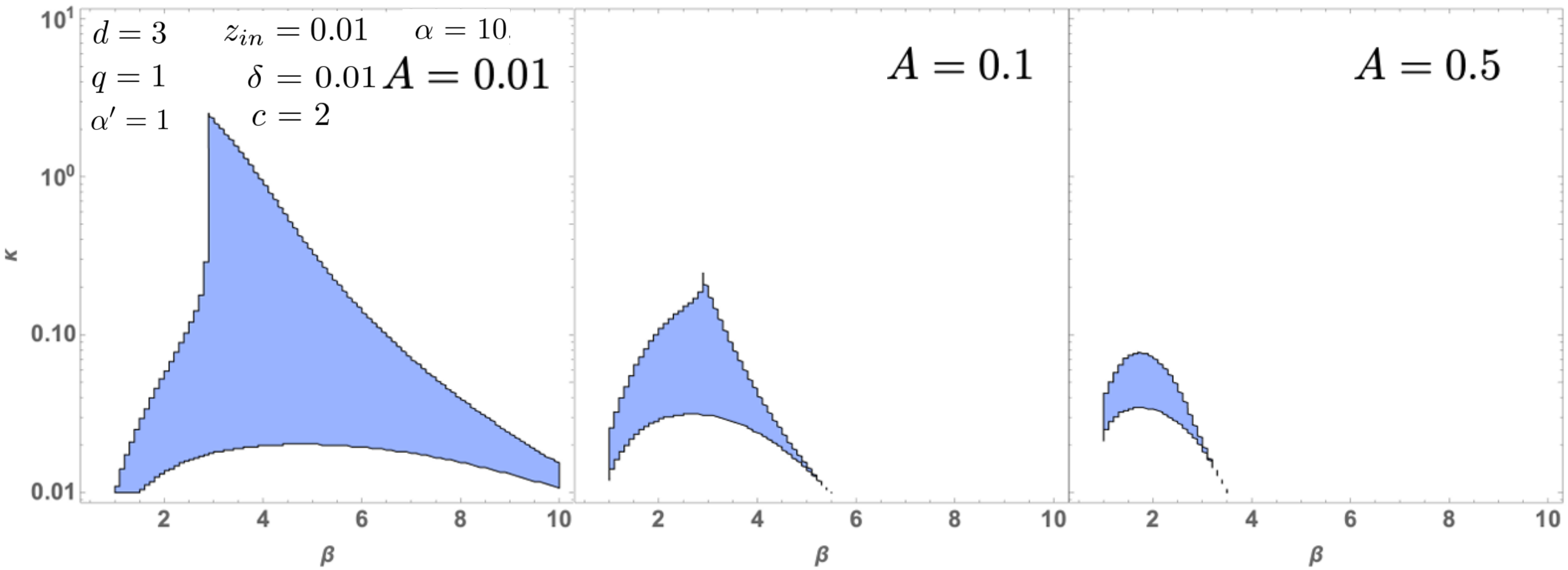
- **q**=1: Asymptotically vanishing potential
- $0 \leq$ **q** < 1: Asymptotically constant potential

(blue): de Sitter, stabilised dilaton

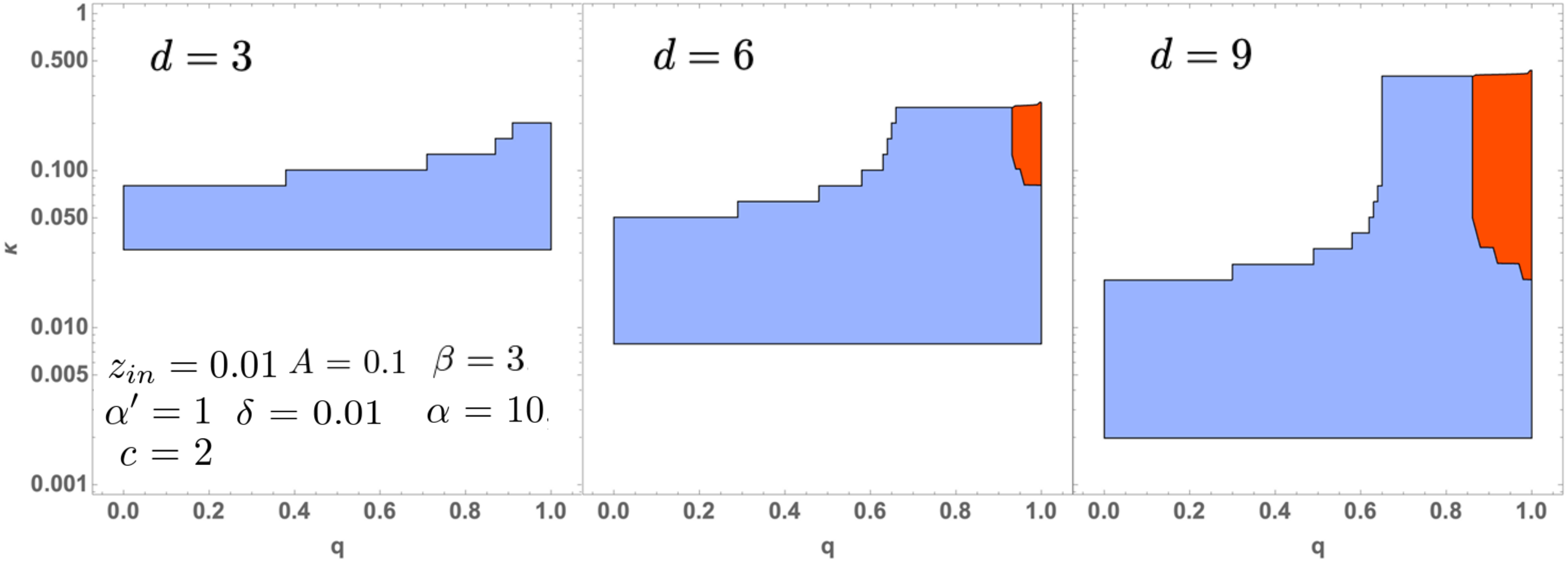
(orange): Growing dilaton, Runaway

(white): Time-reversal post Bounce, decreasing dilaton

Variation of the height of the first peak (β) and the amplitude A



Variation of the asymptotic behaviour (q) and the number of spatial dimensions d



Anisotropic case: Isotropisation mechanism

Whenever there is a late-time attractor with constant ϕ and z_i the attractor must be isotropic, i.e $z_i = z = z_0$, and consequently $H_i = H = H_0$.

Anisotropic case: Isotropisation mechanism

Whenever there is a late-time attractor with constant ϕ and z_i the attractor must be isotropic, i.e $z_i = z = z_0$, and consequently $H_i = H = H_0$.

From the 2° EoM

$$\left(\sum_j H_j\right) z_i + \frac{\partial V}{\partial \phi} = 0$$

Anisotropic case: Isotropisation mechanism

Whenever there is a late-time attractor with constant ϕ and z_i the attractor must be isotropic, i.e $z_i = z = z_0$, and consequently $H_i = H = H_0$.

From the 2° EoM $$\left(\sum_j H_j\right) z_i + \frac{\partial V}{\partial \phi} = 0$$ Independent from i ! Consistent with the other two EoM

Anisotropic case: Isotropisation mechanism

Whenever there is a late-time attractor with constant ϕ and z_i the attractor must be isotropic, i.e $z_i = z = z_0$, and consequently $H_i = H = H_0$.

From the 2° EoM $$\left(\sum_j H_j\right) z_i + \frac{\partial V}{\partial \phi} = 0$$ Independent from i ! Consistent with the other two EoM

Case study: Bianchi I geometry with two isotropic *d* and *n* dimensional subspaces

$$h(z_i) = \frac{1}{2}\sum_i z_i^2 - \frac{\alpha'}{4}\sum_i z_i^4$$

Anisotropic ''Hamiltonian'' consistent at the first order in α′ with the perturbative result obtained for F in heterotic string theory

Anisotropic case: Isotropisation mechanism

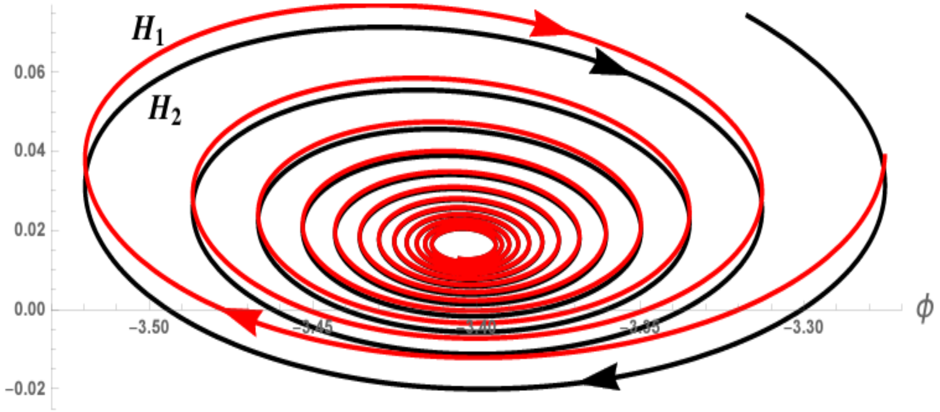
Whenever there is a late-time attractor with constant ϕ and z_i the attractor must be isotropic, i.e $z_i = z = z_0$, and consequently $H_i = H = H_0$.

From the 2° EoM $$\left(\sum_j H_j\right) z_i + \frac{\partial V}{\partial \phi} = 0$$ Independent from i ! Consistent with the other two EoM

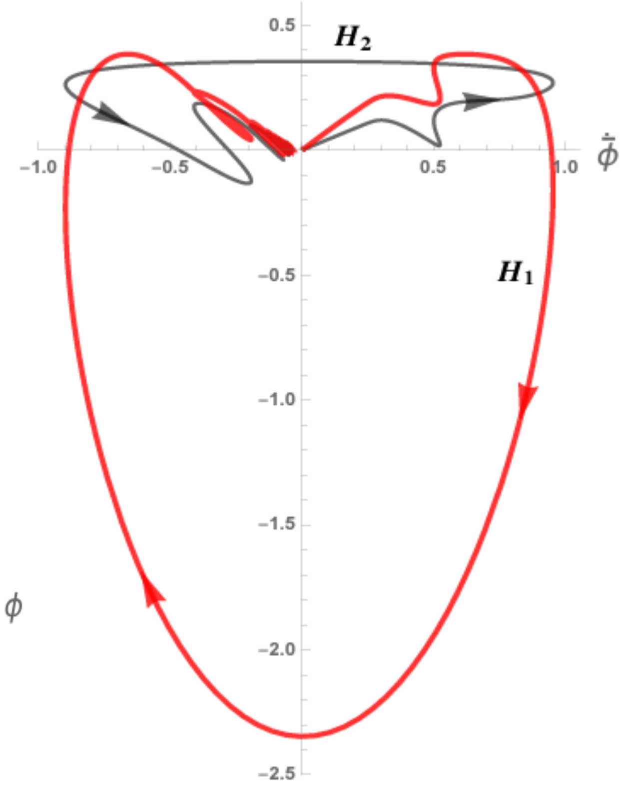
Case study: Bianchi I geometry with two isotropic *d* and *n* dimensional subspaces

$$h(z_i) = \frac{1}{2}\sum_i z_i^2 - \frac{\alpha'}{4}\sum_i z_i^4$$

Anisotropic ''Hamiltonian'' consistent at the first order in α′ with the perturbative result obtained for F in heterotic string theory



$d = 1 \quad n = 2$



Anisotropic case: numerical analysis of the attraction basin

- Conserved quantity

$$\kappa = e^{\phi}\left[\frac{d\,H_1 + n\,H_2}{\sqrt{d+n}}\right]^{-(1+d\,\gamma_1+n\,\gamma_2)}$$

Anisotropic case: numerical analysis of the attraction basin

- Constant quantity

$$\kappa = e^{\phi}\left[\frac{d\,H_1 + n\,H_2}{\sqrt{d+n}}\right]^{-(1+d\,\gamma_1+n\,\gamma_2)}$$

Past-asymptotic low curvature solutions.

- Scale factors:

$$a_i \sim (-t)^{-\gamma_i}$$

- Hubble functions:

$$H_i = \frac{\gamma_i}{t}$$

- Kasner condition

$$\sum \gamma_i^2 = 1$$

Anisotropic case: numerical analysis of the attraction basin

- Constant quantity

$$\kappa = e^{\phi}\left[\frac{d\,H_1 + n\,H_2}{\sqrt{d+n}}\right]^{-(1+d\,\gamma_1+n\,\gamma_2)}$$

ϵ : Anisotropy parameter

Past-asymptotic low curvature solutions.

- Scale factors:

$$a_i \sim (-t)^{-\gamma_i}$$

- Hubble functions:

$$H_i = \frac{\gamma_i}{t}$$

- Kasner condition

$$\sum \gamma_i^2 = 1$$

$$\gamma_1 = \sqrt{(1-n\,\epsilon)/(d+n)}$$

$$\gamma_2 = \sqrt{(1+d\,\epsilon)/(d+n)}$$

Anisotropic case: numerical analysis of the attraction basin

- Constant quantity

$$\kappa = e^{\phi}\left[\frac{d\,H_1 + n\,H_2}{\sqrt{d+n}}\right]^{-(1+d\,\gamma_1+n\,\gamma_2)}$$

ϵ : Anisotropy parameter

Past-asymptotic low curvature solutions.

- Scale factors:

$$a_i \sim (-t)^{-\gamma_i}$$

- Hubble functions:

$$H_i = \frac{\gamma_i}{t}$$

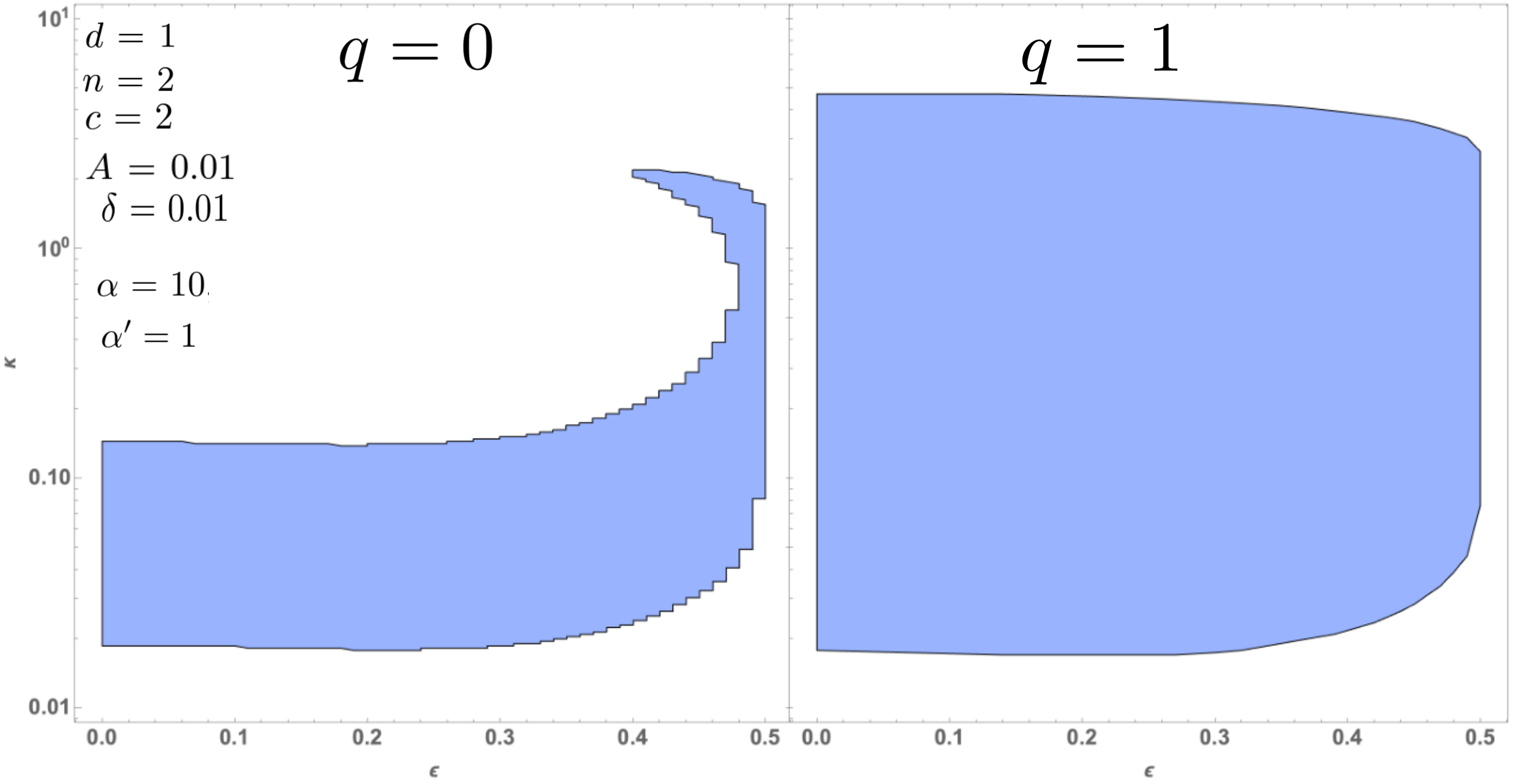
- Kasner condition

$$\sum \gamma_i^2 = 1$$

$$\gamma_1 = \sqrt{(1-n\,\epsilon)/(d+n)}$$

$$\gamma_2 = \sqrt{(1+d\,\epsilon)/(d+n)}$$

de Sitter attraction basin in the $\{\epsilon, \kappa\}$ space



Take-home messages

- Regular bounce if the reduced hamiltonian has another non trivial zero
- Non-perturbative potential to stabilize the dilaton
- FLRW or de-Sitter are attractors
- Isotropisation of the final geometry

Thanks for your attention!

E-mail: eliseo.pavone@ba.infn.it

This presentation was based on:

P. Conzinu, G.Fanizza, M.Gasperini, E. Pavone, L. Tedesco, G. Veneziano, *From the string vacuum to FLRW or de Sitter via α' corrections* ,JCAP **12**, 019 (2023).