

Charged Nariai black holes on the dark bubble

Vincent Van Hemelryck

Based on 2405.13679 with Ulf Danielsson



UPPSALA
UNIVERSITET

The dark bubble scenario

[Banerjee, Danielsson, Dibitetto, Giri, Schillo, 2018]

Alternative to string compactifications:
dS and accelerated expansion challenging

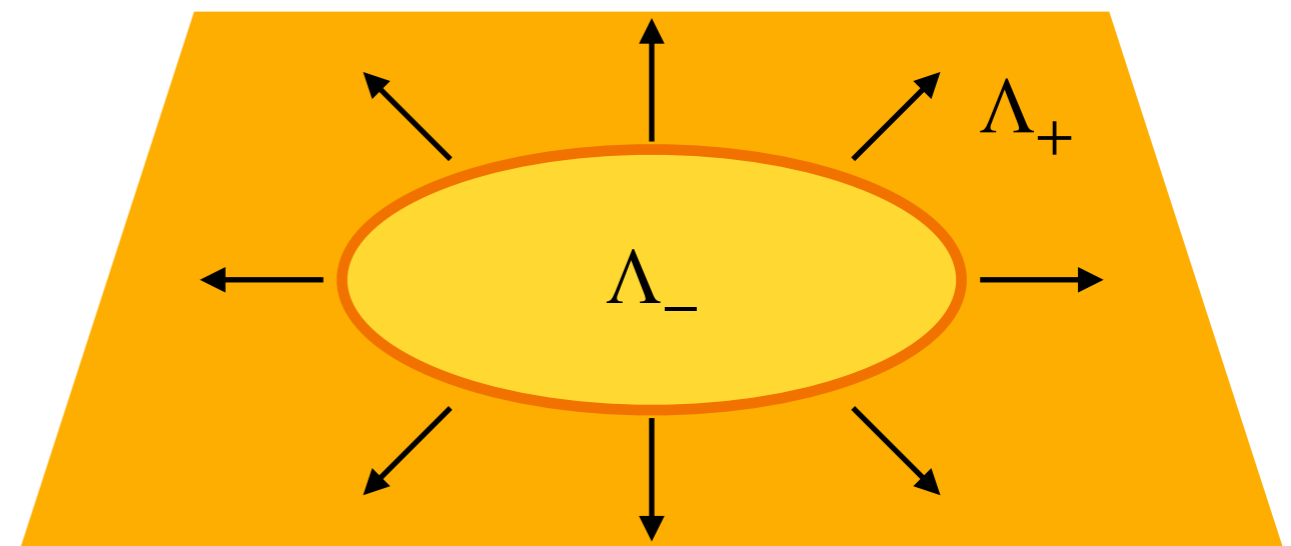
Braneworld model

Unstable AdS_5

Non-perturbative decay: brane nucleation

Codimension-one braneworld different from Randall-Sundrum

Accelerated FLRW cosmology on the bubble wall



$$\Lambda_- < \Lambda_+$$

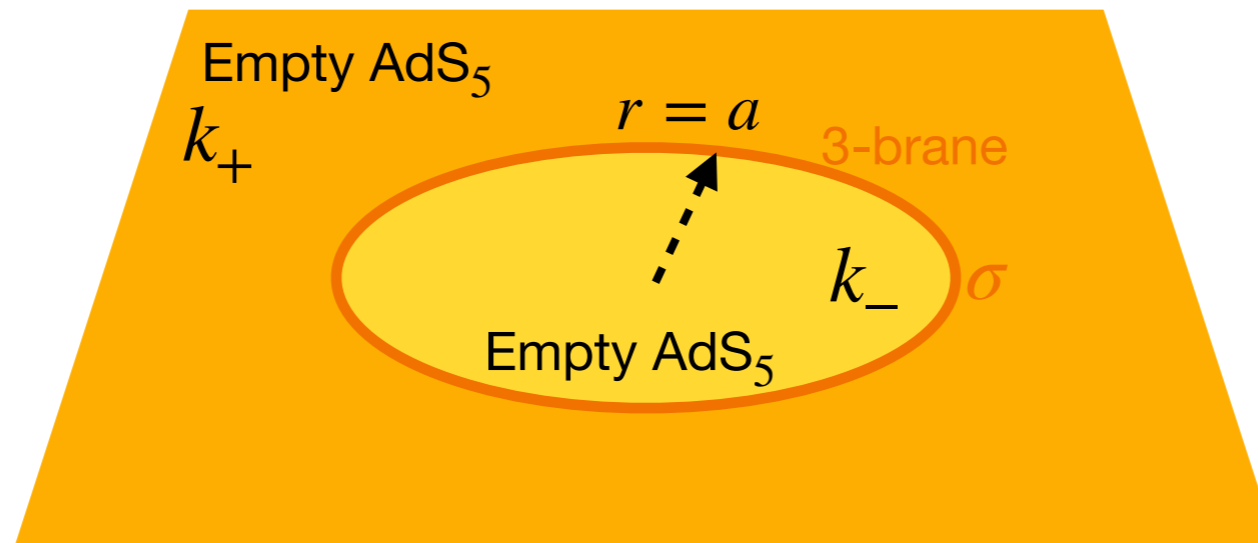
Dynamics of the bubble

[Banerjee, Danielsson, Dibitetto, Giri, Schillo, 2018]

Junction conditions

[Israel, 1966]

$$h_{mn}^- = h_{mn}^+$$



$$\Lambda_{\pm} = -4k_{\pm}^2$$

$$k_{\pm} = 1/L_{\text{AdS},\pm}$$

$$k_+ \lesssim k_-$$

$$k_{\pm}/M_{\text{Pl},5} \gg 1$$

$$\Delta K_{mn} - h_{mn} \Delta K = -M_{\text{Pl},5}^{-3} S_{mn}$$

In global coordinates, junction conditions = Friedmann equations

$$M_{\text{Pl},4}^2 = \frac{1}{2} \left(\frac{1}{k_+} - \frac{1}{k_-} \right) M_{\text{Pl},5}^3,$$

$$M_{\text{Pl},4}^2 \Lambda_4 = 3M_{\text{Pl},5}^3 (k_- - k_+) - \sigma$$

Gravity is stronger in lower dimensional theory \leftrightarrow compactifications

Small, positive induced c.c. when $\sigma \lesssim 3M_{\text{Pl},5}^3 (k_- - k_+)$

Some remarks

Physics studied on the brane includes

- (Massive) particles as hanging strings on the brane [Banerjee, Danielsson, Giri, 2020-21]
- Gravitational waves [Danielsson, Panizo, Tielemans, 2022]
- Electromagnetic waves [Basile, Danielsson, Giri, Panizo, 2023]
- ...

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What about black holes?

Black holes on the bubble?

Strings attached on brane bend the brane

→ manifested as particles backreacting with BH geometry

[Banerjee, Danielsson, Giri, 2020]

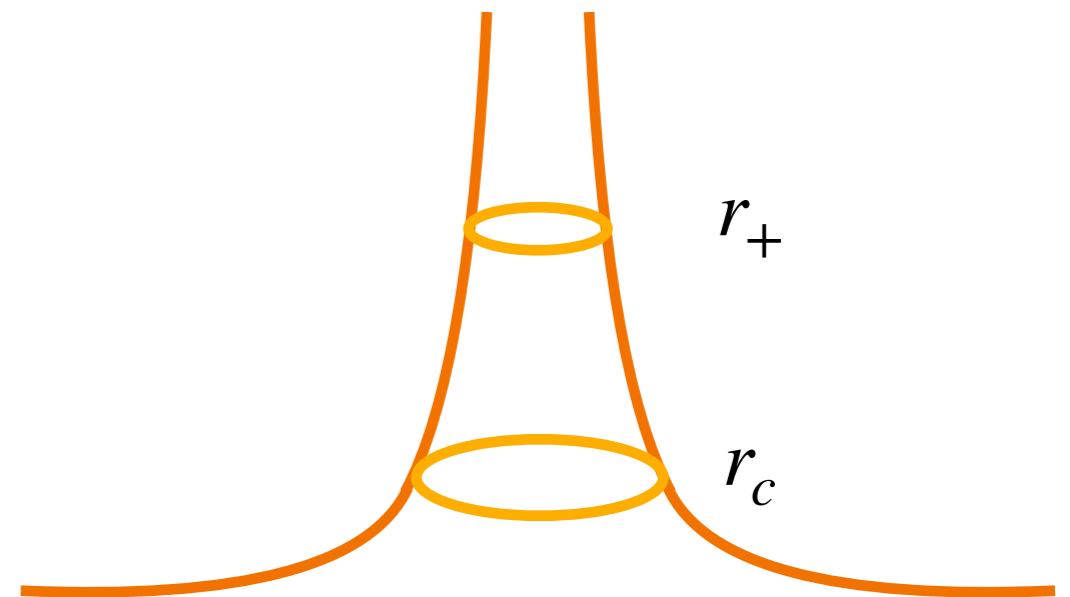
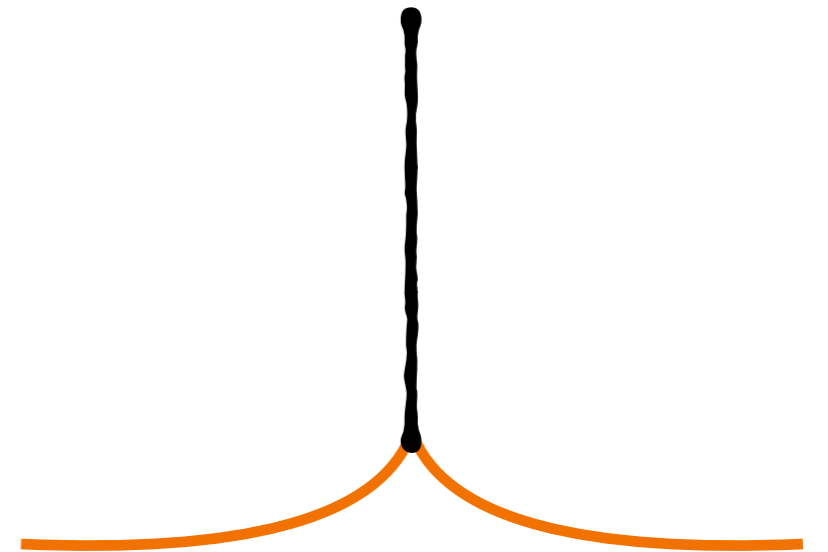
But these are no large black holes

What about large BHs?

→ more extreme bending, “thick strings”

“Take the textbook pictures of BHs seriously”

Goal: RN-dS as induced geometry



Black holes on the bubble?

[Danielsson, VVH, 2024]

Consider Nariai-limit

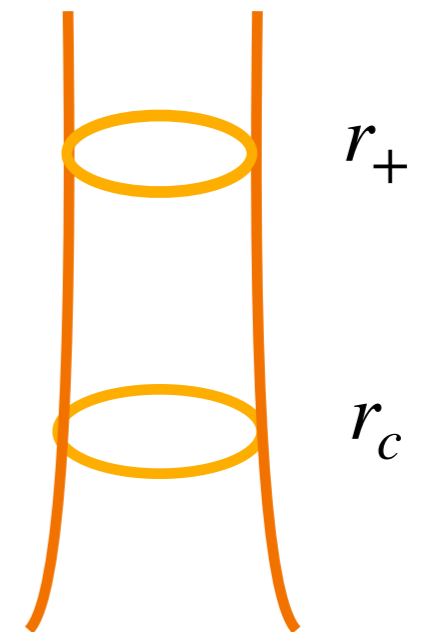
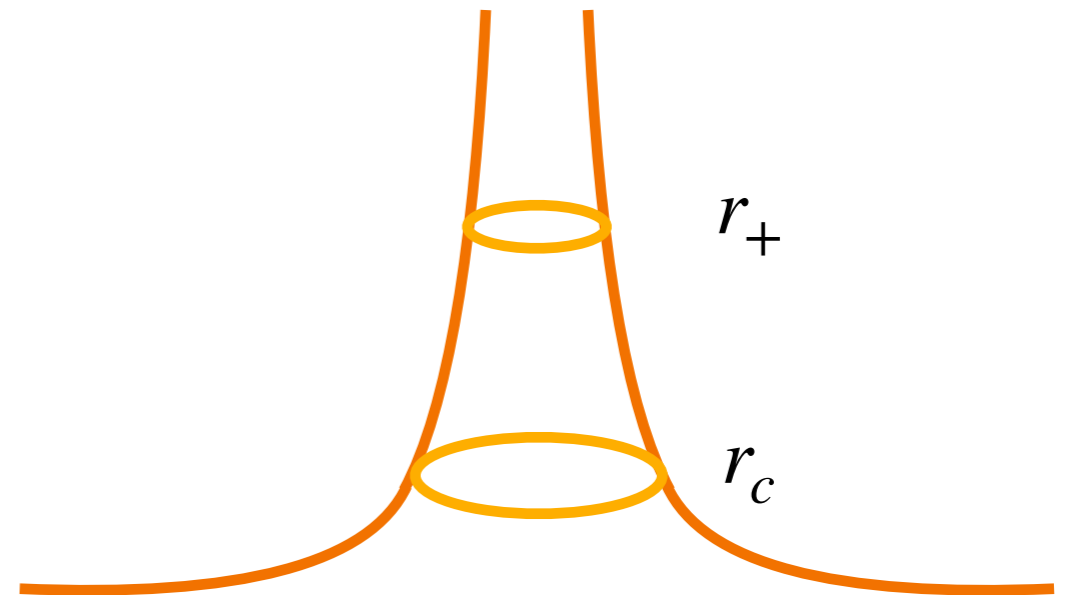
$$r_+ \rightarrow \leftarrow r_c$$

Two horizons become indistinguishable

Geometry becomes $dS_2 \times S^2$

In the brane embedding, local geometry becomes cylindrical

→ Cylindrical embedding necessary



Black holes on the bubble?

[Danielsson, VVH, 2024]

Consider Nariai-limit

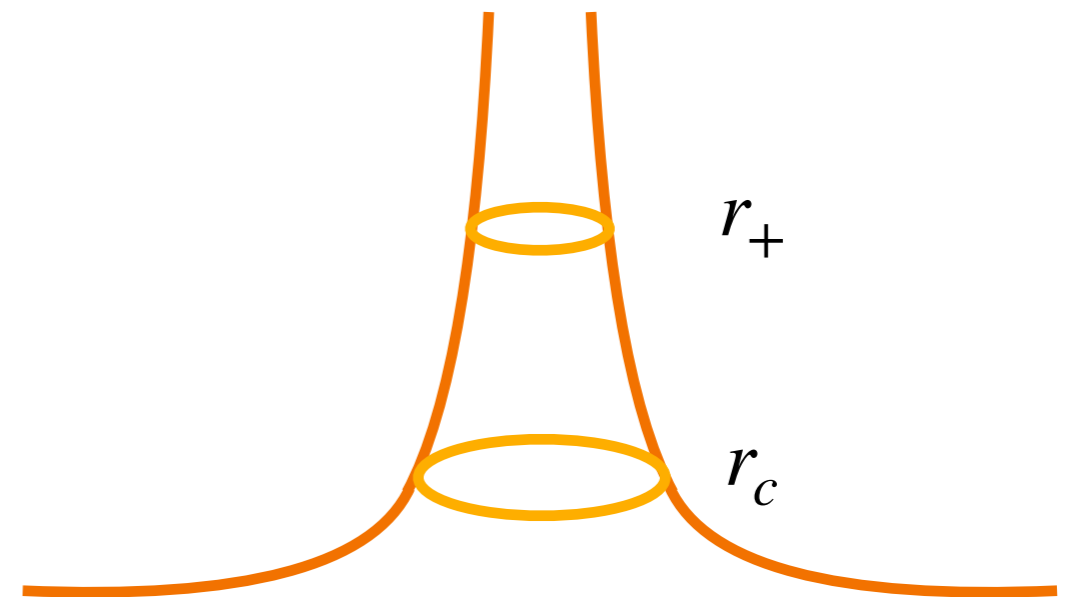
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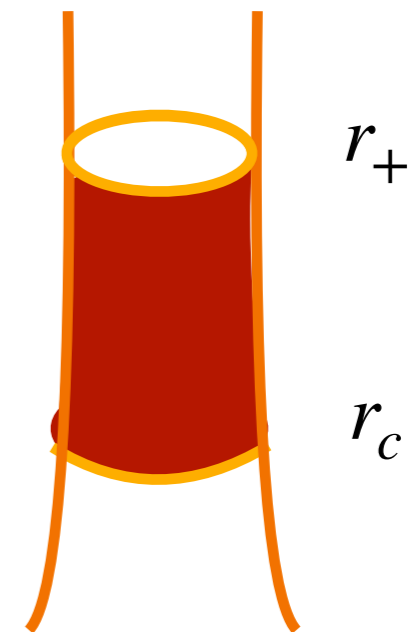
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Region of focus



Nariai geometry from black strings

[Danielsson, VVH, 2024]

5d background becomes a “magnetic” black string background in AdS_5

[Bernamonti, Caldarelli, Klemm, Olea, Sieg, 2007],
[Mann, Radu, Stelea, 2006],
...

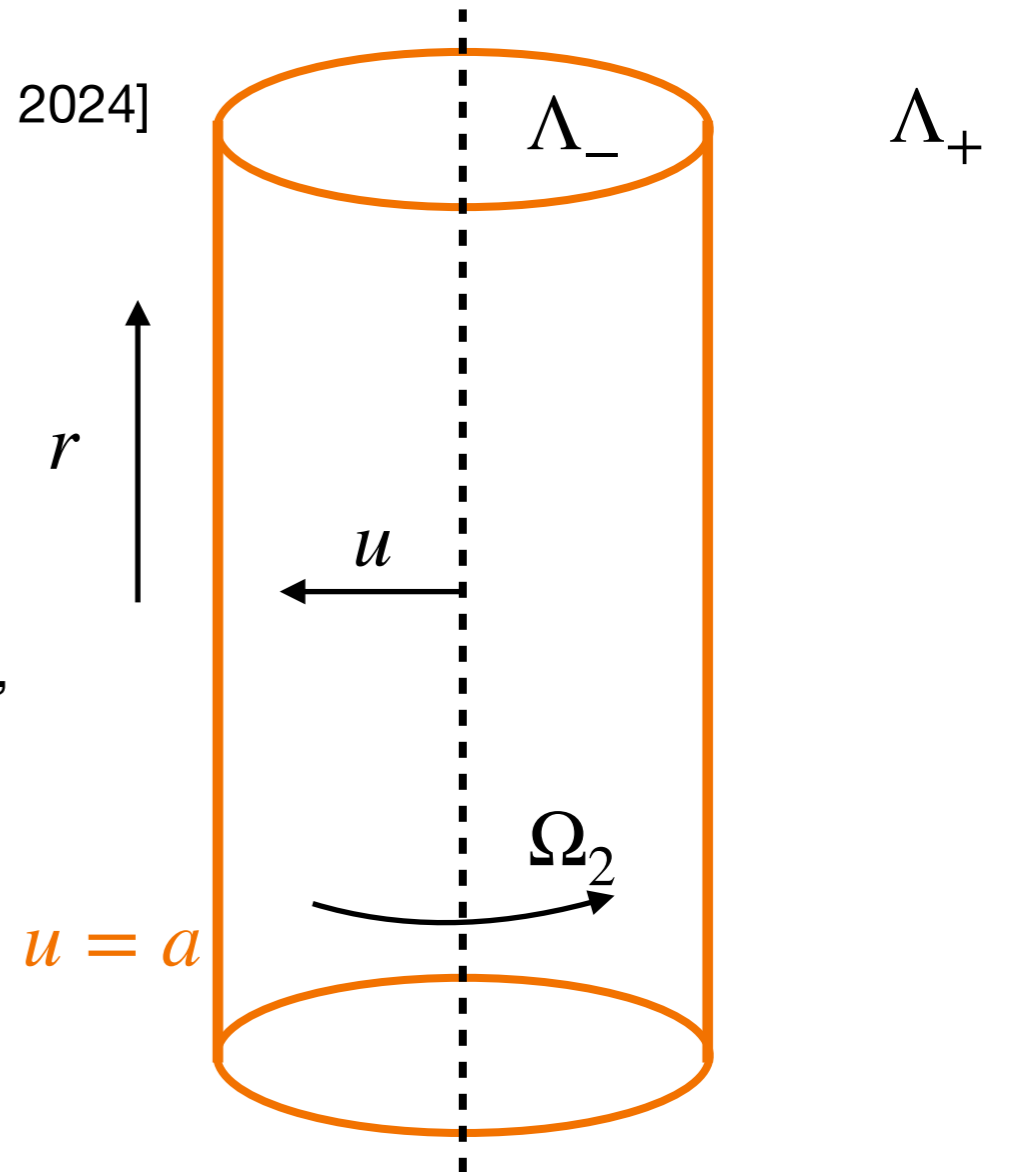
Glue such two backgrounds together

NSNS action

$$S_5 = \frac{1}{2\kappa_5^2} \int dx^5 \sqrt{-g_5} \left(R_5 + 12k^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{-(\phi-\phi_0)} |H_3|^2 \right)$$

$$ds_5^2 = f(u)(-dt^2 + dr^2) + u^2 d\Omega_2 + \frac{du^2}{w(u)}$$

$$\star_5 H_3 = q \text{vol}_{S^2}$$



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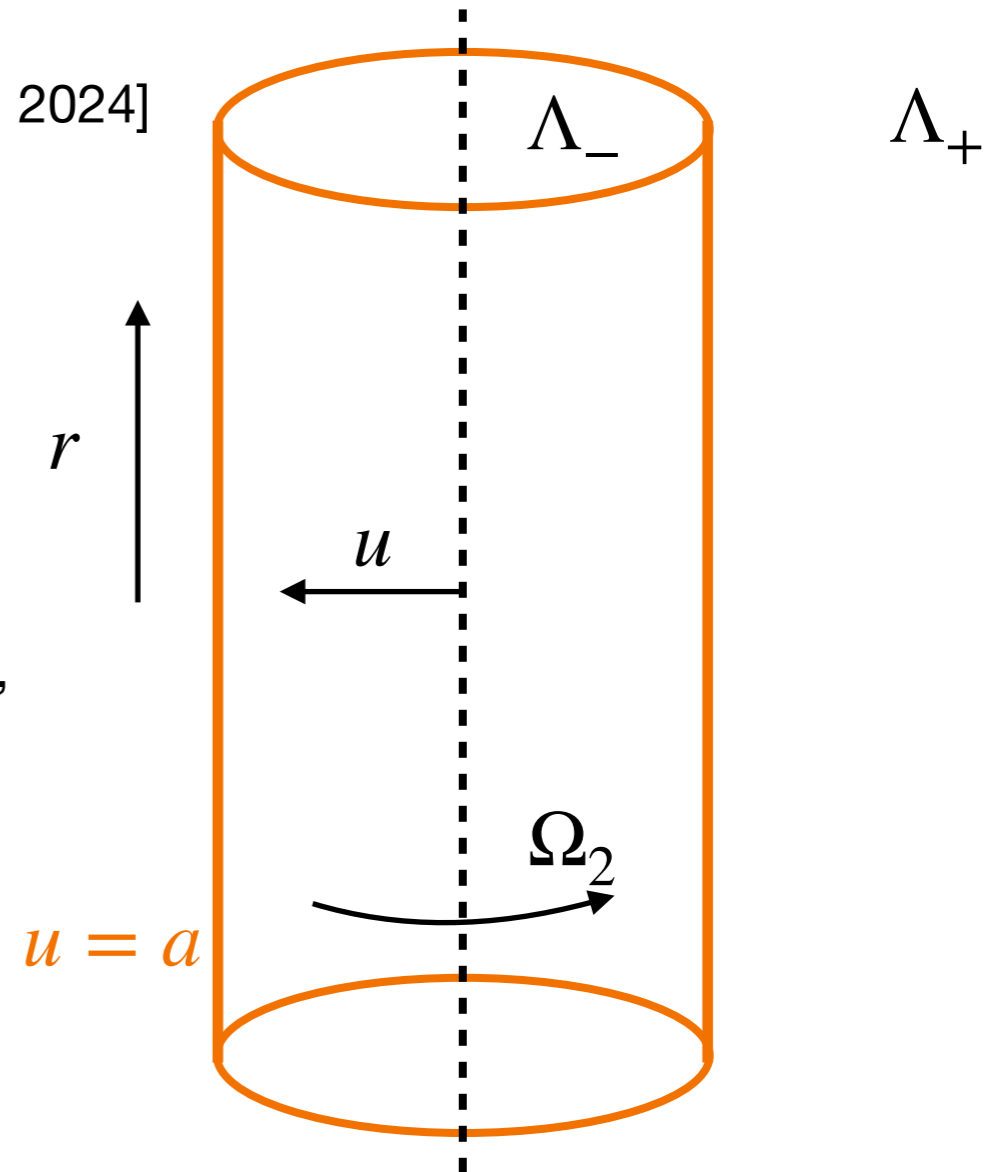
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$$ds_5^2 = f(u) \left(-(1 - k_2^2 r^2) dt^2 + \frac{dr^2}{1 - k_2^2 r^2} \right) + u^2 d\Omega_2 + \frac{du^2}{w(u)},$$

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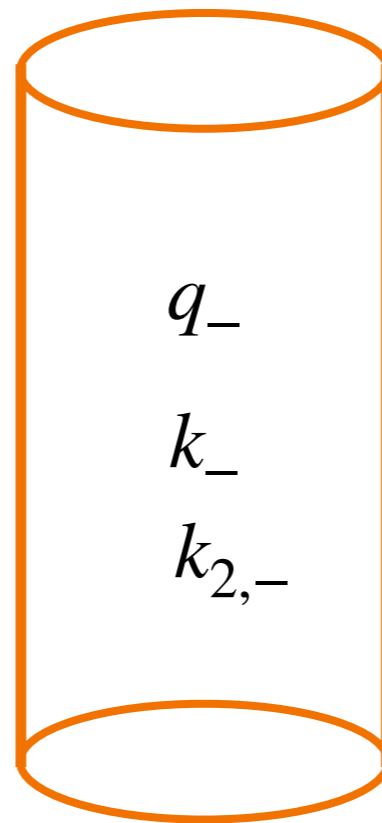


Glue two backgrounds together

DBI of D3-brane

$$S_{\text{brane}} = -\sigma \int dx^4 \sqrt{-\det(h_{mn} + \tau \mathcal{F}_{mn})} \text{ with } \tau \mathcal{F} = \tau F + B$$

$$d \star_5 H \propto \star_4 \mathcal{F} \wedge \delta(u - a) du$$



q_-

k_-

$k_{2,-}$

q_+

k_+

$k_{2,+}$

+ other parameters

Functions $f_{\pm}(u)$ and $w_{\pm}(u)$ not known analytically, double expansion required

in $\frac{k_{2\pm}^2}{k_{\pm}^2}$ and $\frac{1}{(k_{\pm}u)^2}$

Solving junction conditions

$$\begin{matrix} f_{\pm}(a) \\ w_{\pm}(a) \end{matrix}$$

$$(K_{mn}^+ - K_{mn}^-) - h_{mn}(K^+ - K^-) = -\kappa_5^2 S_{mn}$$

Einstein equations for Nariai
background!

$$\begin{aligned} -\frac{1}{a^2} &= -\frac{Q^2}{a^4} - \Lambda_4 \\ -k_2^2 &= +\frac{Q^2}{a^4} - \Lambda_4 \end{aligned}$$

Λ_4 and Q
functions of
 $\sigma, k_{\pm}, q_{\pm}, c_{r\pm}$

Solving junction conditions

$$f_{\pm}(a)$$

$$w_{\pm}(a)$$

$$(K_{mn}^+ - K_{mn}^-) - h_{mn}(K^+ - K^-) = -\kappa_5^2 S_{mn}$$

$$S = \int \sqrt{-g_4} \left(R_4 - 2\Lambda_4 - \frac{1}{2}|F|^2 \right)$$

$$ds_4^2 = -(1 - k_2^2 r^2) dt^2 + \frac{dr^2}{1 - k_2^2 r^2} + a^2 d\Omega_2$$

$$F = \frac{2Q}{a^2}$$

Einstein equations for Nariai background!

$$-\frac{1}{a^2} = -\frac{Q^2}{a^4} - \Lambda_4$$

$$-k_2^2 = +\frac{Q^2}{a^4} - \Lambda_4$$

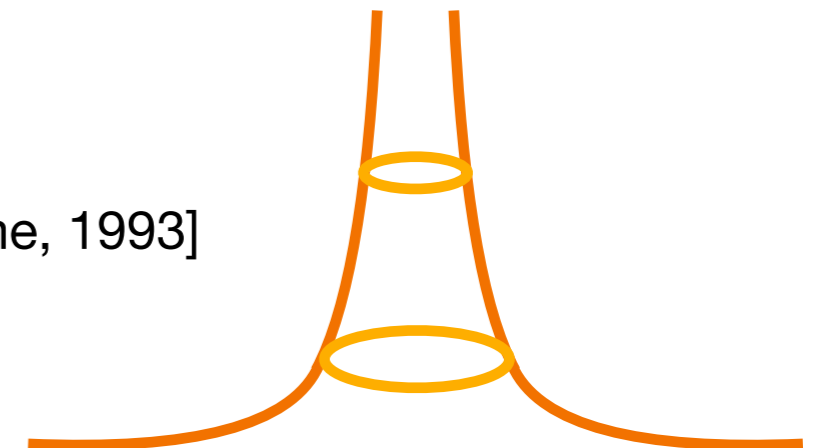
Λ_4 and Q
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Conclusions

- Charged Nariai geometry ($dS_2 \times S^2$) on the dark bubble
- Glueing modified magnetic black string solutions together
- Junction conditions \rightarrow 4d Nariai Einstein equations
- $\alpha_{\text{EM}} = \frac{3}{2} g_s$ from energy conservation as in [Danielsson, Panizo, 2023] from string embedding

Future directions:

- Instability analysis from 4d and 5d à la [Gregory, Laflamme, 1993]
- Going beyond Nariai limit



Thank you!

