# STRING THEORY
# IN THE FIRST HALF OF THE UNIVERSE

## PART II

*See Part I,*
*by Fien Apers*

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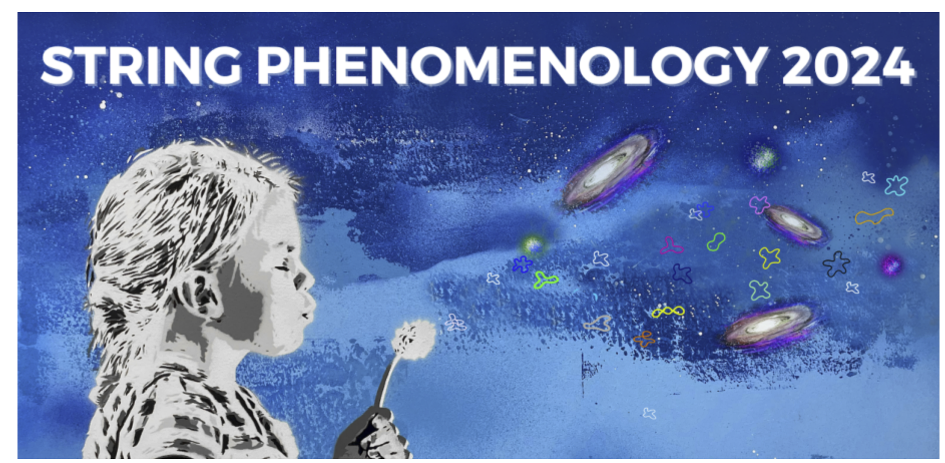
Utrecht University




*Based on:* *2401.04064 [Apers,Conlon,Copeland,Mosny,FR]*
*24xx.xxxxx [Ghoshal,FR,Villa]*

# THE FIRST HALF OF THE UNIVERSE (ON A LOG SCALE)

*String Theory motivates exotic cosmologies in the Early Universe*

*Unconstrained by **current** observations*

*Huge opportunity for ST !*

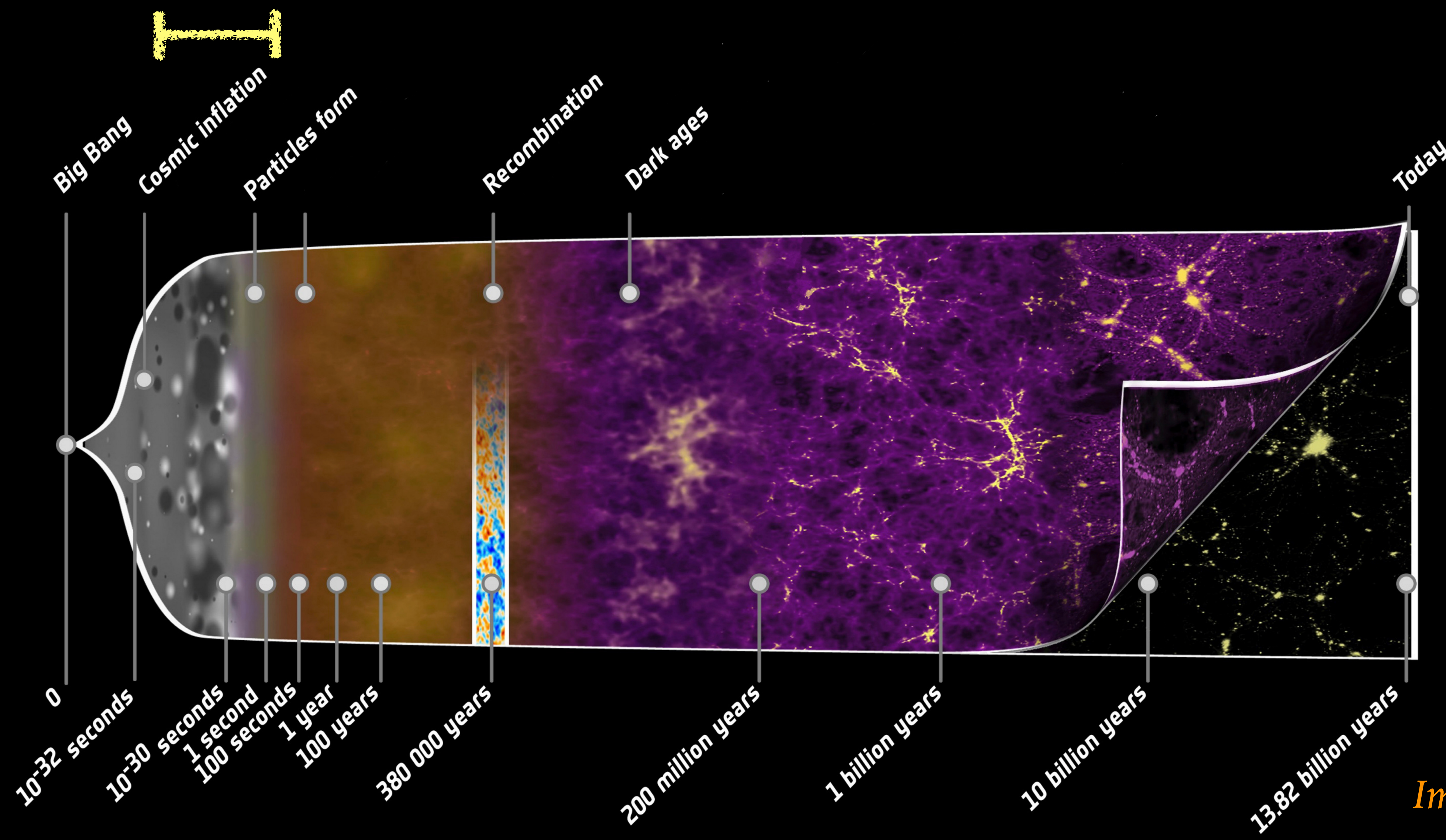



*Image from ESA*

*This talk:* *consequences for perturbations & cosmic strings/GWs*

# A NON-STANDARD PICTURE

*Example of <u>full cosmological history</u> compatible with ST*

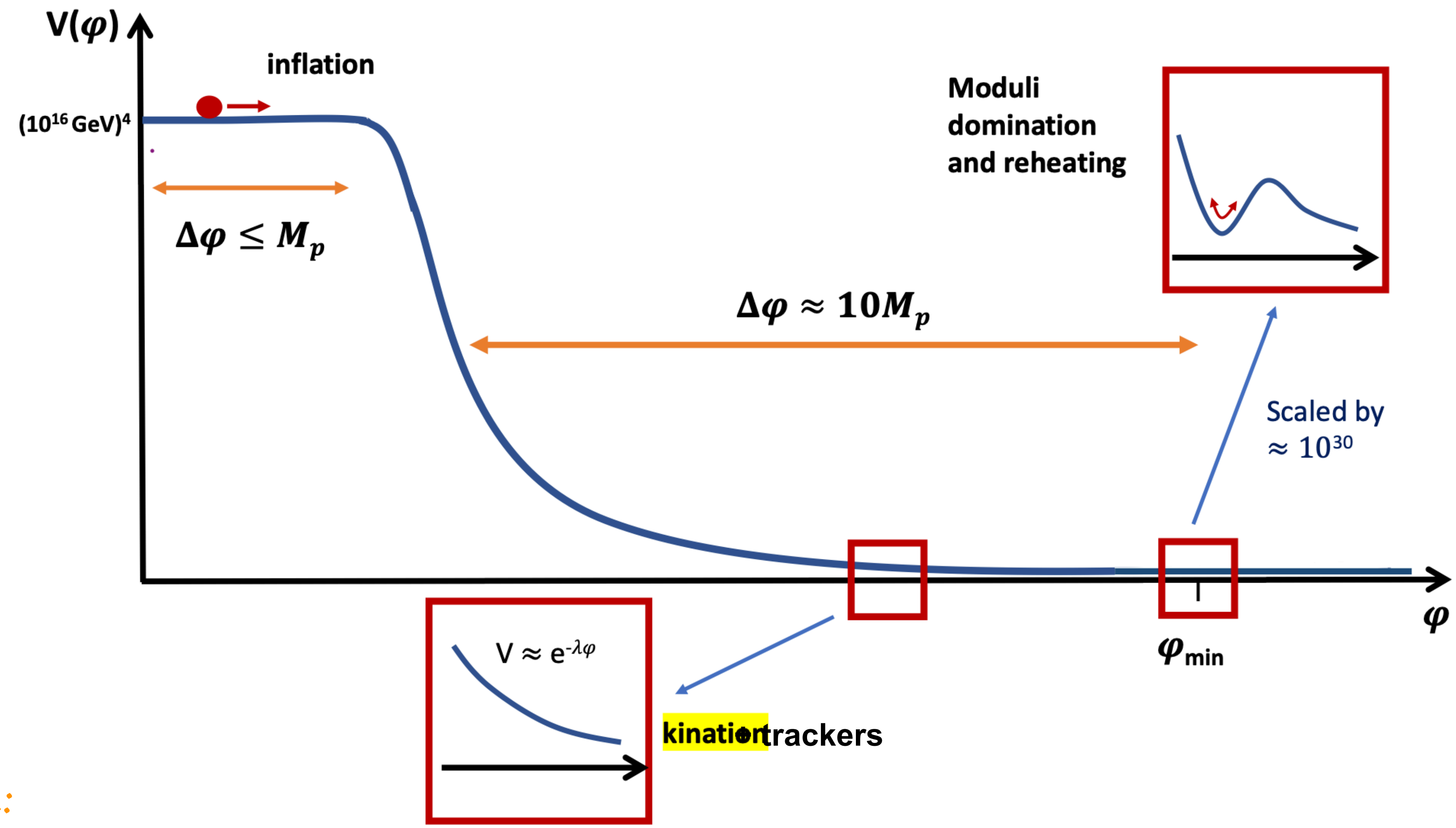



*Figure from:*

*[Cicoli,Conlon,Maharana,Quevedo,Parameswaran,Zavala '23] [Apers,Conlon,Copeland,Mosny,FR '24]*

*Candidate/example: <u>volume</u> modulus in LVS*

*(In general $\neq$ inflaton!)* *E.g. waterfall field* *[Burgess,Quevedo '22]*

# COSMOLOGICAL PERTURBATIONS 1

*Most general metric and matter pert. + gauge invariance*

*Newtonian gauge*:* *"Gravitational potential"*

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i x^j\right]$$

$$T^0_{\ 0} \equiv -(\bar\rho + \delta\rho) \quad T^0_{\ i} \equiv (\bar\rho + \bar P)\partial_i v \quad T^i_{\ j} \equiv (\bar P + \delta P)\delta^i_{\ j}$$

*Gauge invariant variables*

$$\Phi$$

$$\Delta = \frac{\delta\rho}{\bar\rho} + v\frac{\bar\rho'}{\bar\rho}$$

*Figure taken from [Baumann '22]*

**Table 6.1** Summary of the evolution of cosmological perturbations.

| | | radiation era | matter era |
|---|---|---|---|
| $\Phi$ | $k < \mathcal{H}$ | const | const |
| | $k > \mathcal{H}$ | $a^{-2}\cos(k\eta/\sqrt{3})$ | const |
| $\Delta_r$ | $k < \mathcal{H}$ | $a^2$ | $a$ |
| | $k > \mathcal{H}$ | $\cos(k\eta/\sqrt{3})$ | $\cos(k\eta/\sqrt{3}) + \text{const}$ |
| $\Delta_m$ | $k < \mathcal{H}$ | $a^2$ | $a$ |
| | $k > \mathcal{H}$ | $\ln a$ | $a$ |

# COSMOLOGICAL PERTURBATIONS 2

*Solve perturbed Einstein Eqs*

$$\nabla^2\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right) = \frac{a^2}{2M_P^2}\left(\delta\rho_k + \delta\rho_p + \delta\rho_f\right)$$

$$\Phi'' + 3\mathcal{H}\Phi' + \left(2\mathcal{H} + \mathcal{H}^2\right)\Phi = \frac{a^2}{2M_P^2}\left(\delta\rho_k - \delta\rho_p + (\gamma - 1)\delta\rho_f\right)$$

*+ scalar field EOMs*

$$\delta\phi_k'' + 2\mathcal{H}\delta\phi_k' + k^2\delta\phi_k + \lambda^2 a^2 \frac{V(\phi)}{M_P^2}\delta\phi_k = 4\phi'\Phi_k' + 2\lambda a^2 \frac{V(\phi)}{M_P}\Phi_k$$

*Sub-horizon* ← *Analytical solutions* → *Super-horizon*

$k\eta \gg 1$ | $k\eta \ll 1$

# COSMOLOGICAL PERTURBATIONS - RESULTS

*Kination:*

*Very fast!*

$$\Phi(\eta) = \frac{C_1}{k\eta} J_1(k\eta) + \frac{C_2}{k\eta} Y_1(k\eta)$$

$$\Delta(\eta) = -\frac{2iH_{\text{inf}}}{M_P\sqrt{\varepsilon_V k^3}} \frac{k\eta J_1(k\eta)}{J_0(k\eta_0)}$$

$\to$ $a(\eta)\sin(k\eta), \quad k\eta \gg 1$

$\to$ $a(\eta)^4, \quad k\eta \ll 1$

*Radiation Tracker:* $k\eta \ll 1$

*Very similar to radiation domination, O(1) diff*

$$\Phi(\eta) \simeq \frac{C_3 \sin\left(\frac{k\eta}{\sqrt{3}}\right) + C_4 \sin(k\eta + C_5)}{a(\eta)^2} \qquad \Delta \simeq C_3 \sin\left(\frac{k\eta}{\sqrt{3}}\right) + C_4 \sin(k\eta + C_5)$$

*Matter Tracker:* $k\eta \ll 1$

*Unlike matter domination, depends on* $\lambda$

$$\Phi \simeq C_6 a(\eta)^{\frac{1}{4}\left(\frac{\sqrt{25\lambda^2-72}}{\lambda}-5\right)} \sim a(\eta)^{-0.14} \qquad \Delta \simeq C_7 a(\eta)^{\frac{1}{4}\left(\frac{\sqrt{25\lambda^2-72}}{\lambda}-1\right)} \sim a(\eta)^{0.86}$$

*LVS* *LVS*

+ *various analytical results for transient epochs*

# TIME-DEPENDENT COUPLINGS

*Hallmark of String Theory: couplings depend on moduli*

$$\Phi_i \to \Phi_i(t)$$

*Time dependent couplings*

*Example:*

$$\frac{2\pi}{g^2_{\rm YM}} = \int_{\Sigma_i} e^{-\phi}\sqrt{g}$$

*Cycle volumes can vary with time*

*Relevant for close-to-critical couplings? Higgs mass, quartic…*

*THE coupling in String Theory*

$$m_s \sim \frac{M_P}{\sqrt{\mathcal{V}(t)}}$$

*time dependent !*

*See talk by Joe Conlon*

# COSMIC SUPERSTRINGS

*Cosmic **F-strings*** [Witten '85][Polchinski '88][Copeland,Myers,Polchinski '05]

*Conventional wisdom:*

*cosmic superstrings excluded by* $G\mu = \left(\frac{m_s}{M_P}\right)^2 < 10^{-7}$ *(CMB)*

*OK if* $\mathcal{V} \gtrsim 10^5 - 10^6$ ***after** CMB* ✓

*Can be tested with **Gravitational Waves*** [Ghoshal,FR,Villa'24]

***Distinctive new pheno** from time dependent tension* $\mu(t)$

*Also relevant for QFT strings (ex. axion strings) ?*

*Growing loops* | *Scaling regime?* | *High freq. GWs*

[Conlon,Copeland,Hardy,Sanchez Gonzalez'24] [Ghoshal,FR,Villa'24]

# GRAVITATIONAL WAVES FROM COSMIC SUPERSTRINGS

*Dynamical attractor:* *scaling regime*

$\mu(t)$ *"quasi" scaling*

*Long strings* $\rho_{\rm strings} = \dfrac{\mu}{L^2}$ $L = \xi t$

*Loops decay to GWs* $\dfrac{{\rm d}E}{{\rm d}t} = \Gamma G \mu^2$

*energy in GWs*

*redshift*

$$\Omega_{\rm GW}(f) \equiv \frac{f}{\rho_c}\left|\frac{d\rho_{\rm GW}\,(f,t_0)}{df}\right| = \sum_k \frac{f}{\rho_c}\int_{t_F}^{t_0} d\tilde{t}\,\frac{dE_{\rm GW}}{d\tilde{t}}\,\frac{dn(\tilde{f},\tilde{t})}{d\tilde{f}}\left(\frac{a(\tilde{t})}{a\,(t_0)}\right)^3$$

*oscillator modes*

*loop production*

*i) Information on cosmic history*

*E.g. volume kination* $\Omega_{GW} h^2 \sim f^5$

*ii) GW emission extended in time*

*signal extends to* ***low frequencies****

# OUTLOOK & FUTURE DIRECTIONS

*Exotic epochs from ST: vast and unexplored territory*

*Kination*

*Trackers*

*Dynamical system approach*

*GWs*

*Anomalous perturbation growth*

*time dependent couplings*

*Radiation seed / self perturbations*

*DM, baryogenesis*

*Primordial Black Holes*

*Axion dynamics*

# THANK YOU FOR YOUR ATTENTION!

# WHAT ABOUT THE DISTANCE CONJECTURE?

*Distance conjecture:* $\Delta\phi \gg 1$ ⟷ *towers of light states* ↓ *Invalidate EFT*

*[Ooguri, Vafa '06]*
*[Ooguri,Palti,Shiu,Vafa '19]*

## *Kinematics*

*KK modes become light, but so do* $m_\phi, \Lambda_\phi$ ✔

## *Dynamics*

*KK modes above Hubble* $m_{KK}(t) \gg H(t)$ ✔

*Cutoff is adiabatic* $\left|\frac{\mathrm{d}\Lambda(t)_{KK}}{\mathrm{d}t}\right| \ll \Lambda(t)^2_{KK}$ ✔

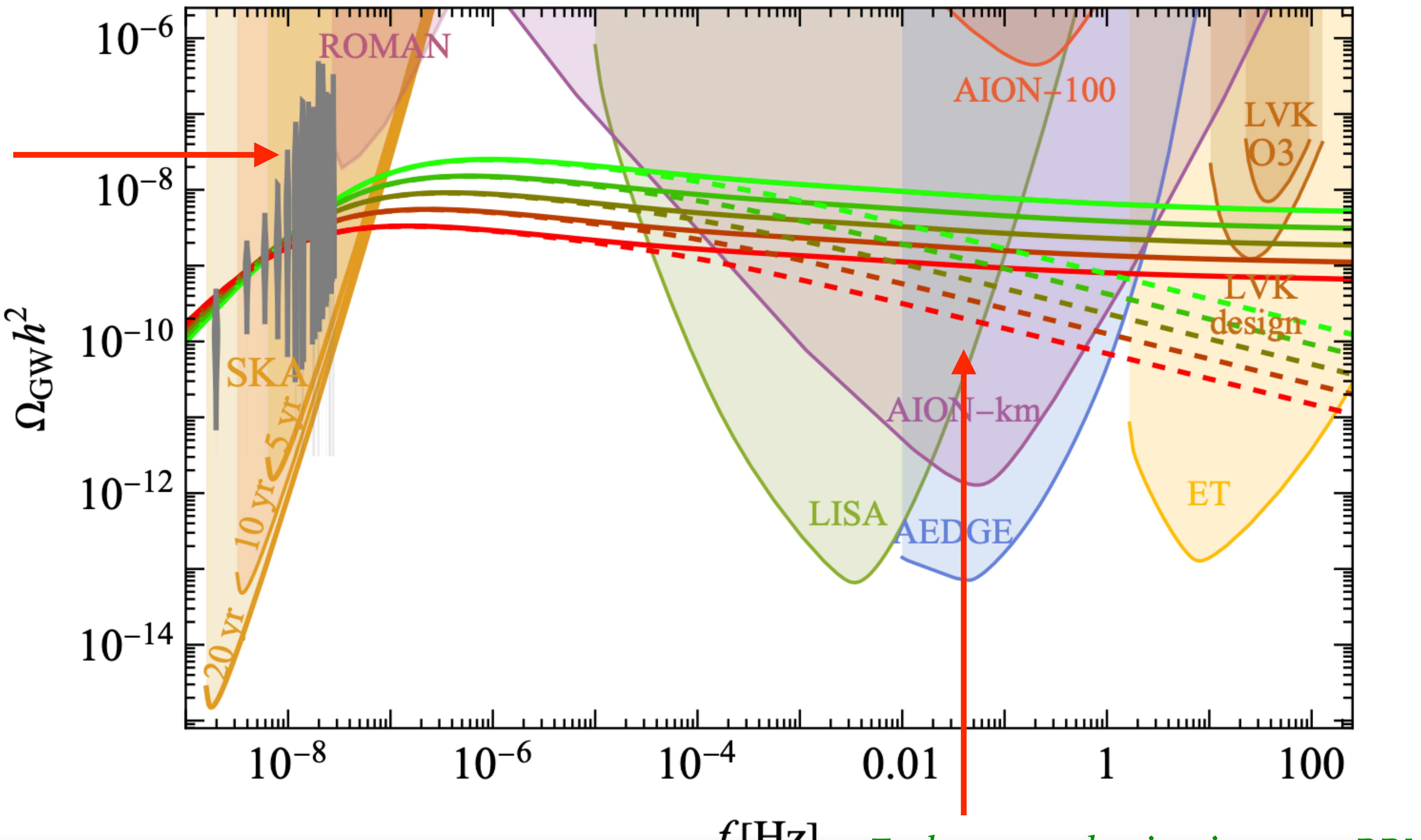
# GW FROM COSMIC STRINGS

[Ellis, Lewicki,Lin,Vaskonen '23]

*Different fits to Nanograv for cosmic superstrings*

$G\mu \sim 10^{-11} - 10^{-12}$

$P \sim 10^{-1} - 10^{-3}$




*Early matter domination up to BBN*

*For LVS*

$$t_{\rm reh} \sim \frac{1}{\Gamma_\Phi} = \left(\frac{1}{4\pi}\frac{m_\Phi^3}{M_P^2}\right)^{-1}$$

*Predicts deviation!*

# THE BACKGROUND COSMOLOGY

| **Epoch** | **a(t)** | $\eta$ | **Range of** $\eta$ | $\mathcal{H}=\frac{a'(\eta)}{a(\eta)}$ | **PE:KE:Rad** |
|---|---|---|---|---|---|
| Inflation | $e^{H_{inf}t}$ | $\sim -e^{-Ht}$ | $-\infty < \eta \lesssim 0 \sim \eta_0$ | $H_{inf}$ | $\frac{1}{2}$:$\frac{1}{2}$:$\epsilon$ (at end) |
| Kination | $t^{1/3}$ | $\eta \sim t^{2/3}$ | $\eta_0 \lesssim \eta \lesssim \frac{\eta_0}{\epsilon}$ | $\frac{1}{2\eta}$ | $\epsilon^{3/2}$ : $\frac{1}{2}$ : $\frac{1}{2}$ (at end) |
| Radiation domination: PE $\leq$ KE | $t^{1/2}$ | $\eta \propto t^{1/2}$ | $\frac{\eta_0}{\epsilon} \lesssim \eta \lesssim \frac{\eta_0}{\epsilon^{5/4}}$ | $\frac{1}{\eta}$ | $\epsilon^{1/2}$:$\epsilon^{1/2}$:1 (at end) |
| Radiation domination: PE $\geq$ KE | $t^{1/2}$ | $\eta \propto t^{1/2}$ | $\frac{\eta_0}{\epsilon^{5/4}} \lesssim \eta \lesssim \frac{\eta_0}{\epsilon^{11/8}}$ | $\frac{1}{\eta}$ | $\frac{1}{2}$:$\epsilon^{3/4}$:$\frac{1}{2}$ (at end) |
| Tracker | $t^{1/2}$ | $\eta \propto t^{1/2}$ | $\frac{\eta_0}{\epsilon^{11/8}} \lesssim \eta \lesssim m_\Phi^{-1/2}$ | $\frac{1}{\eta}$ | $\frac{3(2-\gamma)\gamma}{2\lambda^2}$:$\frac{3\gamma^2}{2\lambda^2}$: $1-\frac{3\gamma}{\lambda^2}$ |
| Matter domination | $t^{2/3}$ | $\eta \propto t^{1/3}$ | $m_\Phi^{-1/2} \lesssim \eta \lesssim \Gamma_\Phi^{-1/2}$ | $\frac{2}{\eta}$ | NA |
| Reheating to Standard Model | $t^{1/2}$ | $\eta \propto t^{1/2}$ | $\eta \gtrsim \Gamma_\Phi^{-1/2}$ | $\frac{1}{\eta}$ | 0:0:1 (at end) |

[Apers,Conlon,Copeland,Mosny,FR '24]

*Analytical treatment of transition epochs*

*Eg: kination to radiation domination:*

$$a(\eta) = a_0\sqrt{\frac{\eta}{\eta_0} + \frac{\varepsilon}{4}\left(\frac{\eta}{\eta_0}\right)^2} \qquad \mathcal{H}(\eta) = \frac{1}{2\eta}\frac{1+\frac{\varepsilon\eta}{2\eta_0}}{1+\frac{\varepsilon\eta}{4\eta_0}}$$

*Useful to analyse e.g. perturbations*

# EQUATIONS OF MOTION

$$\begin{cases} \ddot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + (d-1)H\dot{\phi}^i + \partial^i V = 0 \\ \frac{(d-1)(d-2)}{2}H^2 = \frac{1}{2}G_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V\left(\phi^i\right) + \rho_\gamma \\ \dot{\rho}_\gamma + 3\gamma H\rho_\gamma = 0 \end{cases}$$

*Generic fluid: matter, radiation…*

*Asymptotics of moduli space (large Vol, CS moduli…)*

$$G_{ij,I} = C_I \frac{\delta_{ij}}{\left(s^I\right)^2} \quad V\left(s^I, a^I\right) = V_0 \prod_{I=1}^{N} \frac{1}{\left(s^I\right)^{\lambda_I}}$$

*Simpler expressions, analytic treatment*

# DYNAMICAL SYSTEM REFORMULATION

$$x_i = \sqrt{\frac{C_i}{(d-1)(d-2)}}\frac{\dot{s}_i}{H s_i} \qquad y_i = \sqrt{\frac{C_i}{(d-1)(d-2)}}\frac{\dot{a}_i}{H s_i} \qquad w = \frac{1}{H}\sqrt{\frac{2\rho_\gamma}{(d-1)(d-2)}}$$

*saxions* *axions* *fluid*

$$\begin{cases} \dfrac{dx_i}{dM} = -\sqrt{\dfrac{(d-1)(d-2)}{C_i}}\left[{y_i}^2 - \dfrac{\lambda_i}{2}\left(1 - \displaystyle\sum_{j=1}^{N}\left({x_j}^2 + {y_j}^2\right) - w^2\right)\right] \\ \qquad -x_i(d-1)\left(1 - \displaystyle\sum_{j=1}^{N}\left({x_j}^2 + {y_j}^2\right) - \dfrac{\gamma}{2}w^2\right) \\ \dfrac{dy_i}{dM} = \sqrt{\dfrac{(d-1)(d-2)}{C_i}}x_i y_i - (d-1)\left(1 - \displaystyle\sum_{j=1}^{N}\left({x_j}^2 + {y_j}^2\right) - \dfrac{\gamma}{2}w^2\right)y_i \\ \dfrac{dw}{dM} = (d-1)w\left[\displaystyle\sum_{j=1}^{N}\left({x_j}^2 + {y_j}^2\right) + \dfrac{\gamma}{2}\left(w^2 - 1\right)\right] \end{cases}$$

*See talk by Flavio Tonioni*

*[Copeland,Liddle,Wands '98], [Collinucci,Nielsen, Van Riet '04]*
*[(Brinkmann),Cicoli,Dibitetto,Pedro'20 x2, 22],[Shiu,Tonioni,Tran '23] x 3, [FR '23]…*