Higher-curvature Gravities, EFTs and the Swampland

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Mainly based on the following works:
1) JHEP 11 (2019) 062, w/ P. Bueno, P. Cano & J. Moreno
2) JHEP 10 (2020) 125, w/ P. Cano
3) JHEP 07 (2022) 010, w/ P. Cano, A. Rivadulla & X. Zhang

Introduction

String effective actions include **higher-curvature** terms:

$$I_{\rm ST} = I_{\rm sugra} + \sum_{k=0}^{\infty} \sum_{n=1}^{\infty} g_s^{2k} \left(\alpha'\right)^n I_{n,k} \, .$$

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1. **Heterotic String Theory.** Truncating all Yang-Mills fields [Gross, Sloan '87; Bergshoeff, de Roo '89]:

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2. **IIB String Theory**. On $\mathcal{A}_5 \times S^5$ ansatz [Gubser, Klebanov, Tseytlin '98]:

$$I_{\mathrm{IIB}_{\mathcal{A}_5\times \mathrm{S}^5}}[g_{\mathrm{A}_5}] = \frac{1}{16\pi G}\int \mathrm{d}^5x \sqrt{|g_{\mathrm{A}_5}|}\left[R + \frac{12}{\ell^2} + \frac{\zeta(3)}{8}\alpha'^3 W^4\right] + \ldots\,,$$

up to subleading terms in α'. W^4 is certain combination of quartic invariants.

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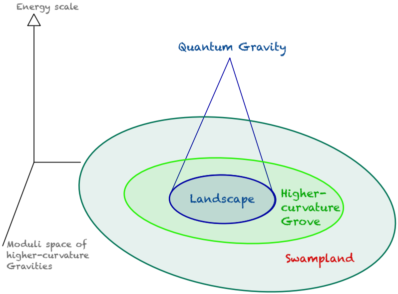

Within the terrible confines of the Swampland, a magical and mysterious Grove towards the Landscape there is.

Generalized Quasitopological Gravities

We focus on a **special class of theories** with intriguing properties on **static** and **spherically symmetric** ansätze:

$$\mathrm{d}s^2_{N,f} = -N(r)^2 f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-2}\,.$$

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Definition (Generalized Quasitopological Gravities)

Let $\mathcal{L}(g^{ab}, R_{abcd})$ be theory of pure gravity. It is a **Generalized Quasitopological Gravity** *(GQG)* [*e.g.* Oliva, Ray '10; Myers, Robinson '10; Bueno, Cano '16; Hennigar, Kubizňák, Mann '17; Bueno, Cano '17] *iff reduced Lagrangian $L_f = r^{D-2}\mathcal{L}|_{\mathrm{d}s^2_{1,f}}$:*

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GR is trivially a **GQG**. **Cubic** example in $D = 4$ [Bueno, Cano '16]:

$$\mathcal{P} = 12 R_a{}^c{}_b{}^d R_c{}^e{}_d{}^f R_e{}^a{}_f{}^b + R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab} - 12 R_{abcd} R^{ac} R^{bd} + 8 R_a^b R_b^c R_c^a\,.$$

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Any purely gravitational higher-curvature **theory** can be **mapped** via perturbative **field redefinitions** to a **GQG** [Bueno, Cano, Moreno, ÁM '19].

Type-IIB action at $\mathcal{O}(\alpha'^3)$ as a GQG

$$I_{\mathrm{IIB}_{\mathcal{A}_5\times \mathrm{S}^5}}[g_{\mathrm{A}_5}] = \frac{1}{16\pi G}\int \mathrm{d}^5x\sqrt{|g_{\mathrm{A}_5}|}\left[R+\frac{12}{\ell^2}+\frac{\zeta(3)}{8}\alpha'^3 W^4\right]+\dots\,,$$

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We perform the metric redefinition:

$$g_{ab}\to g_{ab}-\frac{\zeta(3)\alpha'^3}{8}\left(\hat{C}_{ab}-\frac{1}{3}\hat{C}g_{ab}\right)\,,$$

for a specific cubic-curvature symmetric tensor $\hat{C}_{ab}$ and $\hat{C}=\hat{C}^a_a$.

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for a specific cubic-curvature symmetric tensor $\hat{C}_{ab}$ and $\hat{C} = \hat{C}^a_a$. The **new theory** is a **GQG** with solution:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + \frac{r^2}{\ell^2}\mathrm{d}\vec{x}_3^2\,,$$

$$f(r) = \frac{r^2}{\ell^2}\left[1 - \frac{\omega^4}{r^4} + \gamma\left(\frac{25\omega^{12}}{2r^{12}} - \frac{15\omega^{16}}{2r^{16}}\right)\right] + \mathcal{O}(\gamma^2)\,,$$

where $\gamma = \frac{\zeta(3)\alpha'^3}{8\ell^6}$ and ω an integration constant related to the energy.

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The **thermodynamic** properties of the black hole are:

$$T(\omega) = \frac{\omega}{\pi\ell^2}\left(1 - \frac{15}{4}\gamma\right) + \mathcal{O}(\gamma^2)\,,$$
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Higher-curvature gravities belonging to the **swampland** can be extremely useful for **Quantum Gravity**!

Weak Gravity Conjecture for higher-curvature gravities

On the other hand, it is possible to use the **swampland program** to **constrain** the **couplings** of **higher-order terms** in effective actions **[*e.g.* Bellazzini, Lewandowski, Serra '19; Charles '19; Aalsma, Shiu '22]**.

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Example in $D = 4$ at **quadratic derivative order**:

$$\mathcal{L} = \alpha R F^2 + \beta R_{ab} F^{ac} F^{b}{}_{c} - (10\alpha + 2\beta) R_{abcd} F^{ab} F^{cd}\,.$$

Extremal Black Holes, EQGs and WGC

It is possible to analize which EQGs are consistent with **Weak Gravity Conjecture** [Arkani-Hamed, Motl, Nicolis, Vafa '06]: $P/M|_{\text{ext}}$ must not increase as mass increases[1].

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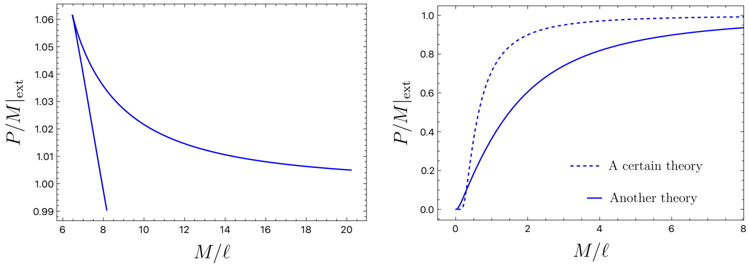

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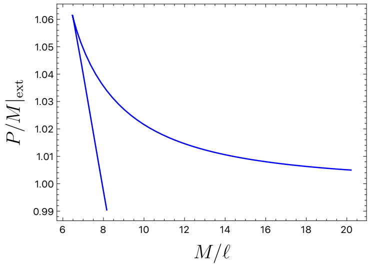

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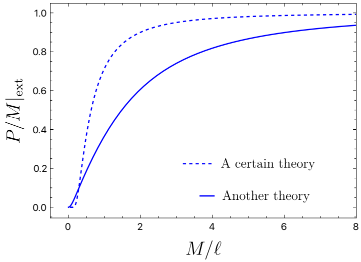

(b) WGC ✗

For EQGs consistent with WGC, **extremal BH** solutions do not exist below a **minimal mass**.

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Holographic EQGs and WGC

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$$\frac{\partial}{\partial n} S_n \leq 0\,, \quad \frac{\partial}{\partial n}\left(\frac{n-1}{n} S_n\right) \geq 0$$
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Grazie mille!